



Steps of Wonder

by Marsha Landau

Illustrated by S. D. Monahan

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“We’re the first ones back to the bus,” noted Tonnica.

“Now what?” asked Daniel.

“We wait,” said Caroline.

“This is boring,” said Tonnica.

Caroline laughed. “We just spent the afternoon at the Botanic Garden seeing AMAZING things, and now that we have to wait a few minutes you’re bored.”

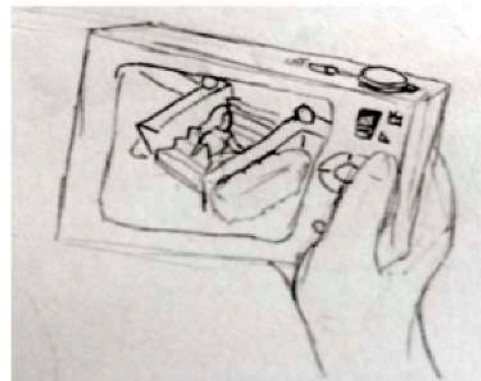
Daniel scrunched his face and, in his best imitation of their teacher Ms. Mathis, announced “Boys and girls! There’s no such thing as BORED in this class. It’s time to NOTICE and WONDER.”

“Wow! You sound just like her,” said Tonnica through her giggles.

“That’s not really such a bad idea,” said Caroline.

The group began looking around.

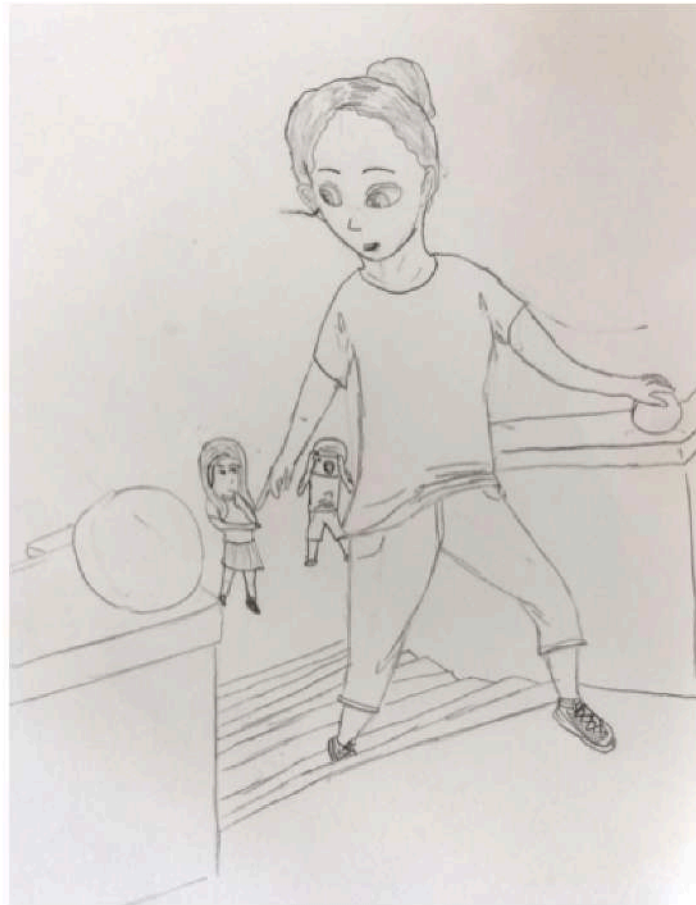
“Over there! Did you see that girl?”
asked Eric, still looking through his digital camera. “She just ran up those stairs, sometimes two at a time, sometimes only one. I think I shot a photo. Wait—I actually got a short video!”



“So?” said Tonnica.

“Good NOTICING,” said Caroline. “I WONDER how many different ways there are to go up the stairs if you can take them one step or two steps at a time.”

Tonnica went over and climbed the stairs one at a time, counting. “There are ten stairs,” she said, “and there’s the first way to climb them—all one-steps. Ten of them.”



Daniel dashed up the stairs two at a time. “All two-steps. Five of them. So there are two ways to climb the stairs. Big deal. That wasn’t much of a problem!”

“Wait a minute,” said Caroline, standing at the bottom of the stairs. “I can do a one-step OR a two-step—they don’t have to be all the same. How about this?” she asked, and alternated: 1, 2, 1, 2, 1, 2, 1.

“Oh, my gosh!” exclaimed Tonnica. “I can use three two-steps and four one-steps but in a different order—like this!” And she demonstrated a new pattern: 1, 1, 2, 2, 2, 1, 1.

“How about this—2, 2, 2, 1, 1, 1, 1?” said Eric.

“There are other ways to arrange the 1s and 2s,” said Caroline. “Lots of ways! I’m beginning to WONDER whether this problem is too big for us... and I just realized we can climb ten stairs by using just two 2s and six 1s—and there’s lots of ways to arrange those. This is getting hard!”

Ms. Mathis arrived just at that moment, along with the buzz of the rest of the fifth grade as they were herded toward the bus by the parent chaperones.

“Do you mean to say that something is *challenging*, Caroline?” asked Ms. Mathis as her students quieted to listen to their teacher. “Tell me more about it.”

“Our group found an interesting problem we have no idea how to solve!” explained Caroline.

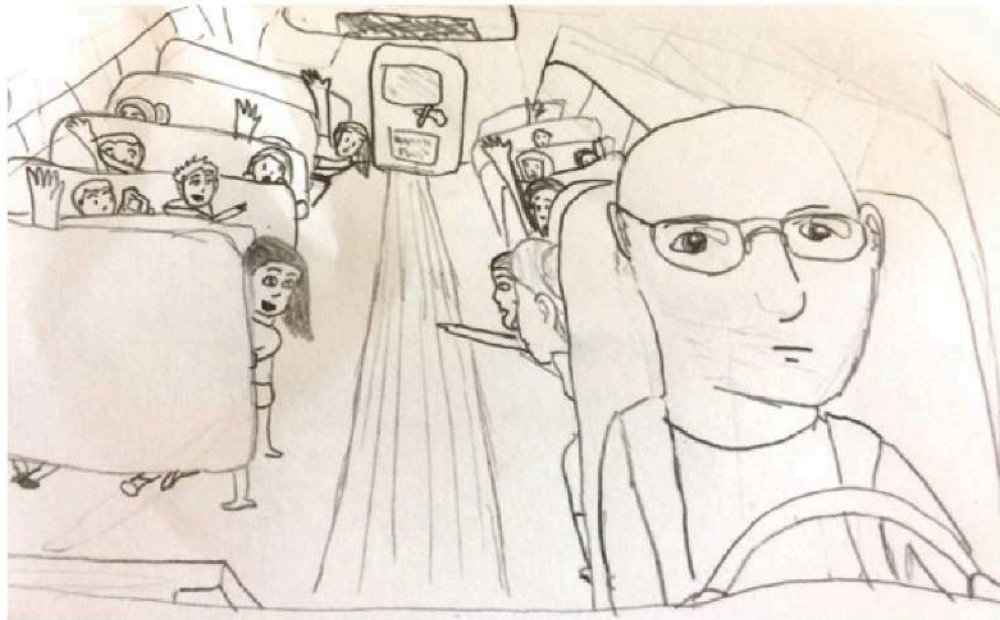
“In how many DIFFERENT ways can you climb ten stairs taking 1 or 2 steps at a time?”

“At first we thought there were two ways: all 1-steps or all 2-steps,” explained Tonnica, “but then we realized there could combinations of 1s and 2s, and different orders of 1s and 2s were different ways. “

Everyone began talking at once! Some students were holding up ten fingers, then folding down 1 or 2 fingers at a time to find more ways. But no one was keeping track to see whether each new way was different from the ones they had.

By the time everyone was seated aboard the bus to return to school, it was so noisy Ms. Mathis had to use her whistle to ask for silence. “Boys and girls,” she said, “I

need your attention. This is clearly an interesting problem. Let's brainstorm a bit on the bus and when we get back to our classroom we'll make a plan. Now who remembers our problem solving strategies? We're certainly going to need them to solve this problem."



Hands were waving, and before the bus pulled up to the door of the school they had made this list:

1. Guess, check, and revise.
2. Use a picture or diagram.
3. Act it out.
5. Make a systematic list.
6. Simplify the problem.
7. Look for a pattern.

Back in their classroom, Tonnica remembered the ways they had already found by ACTING IT OUT while they were waiting:

1, 1, 1, 1, 1, 1, 1, 1, 1, 1

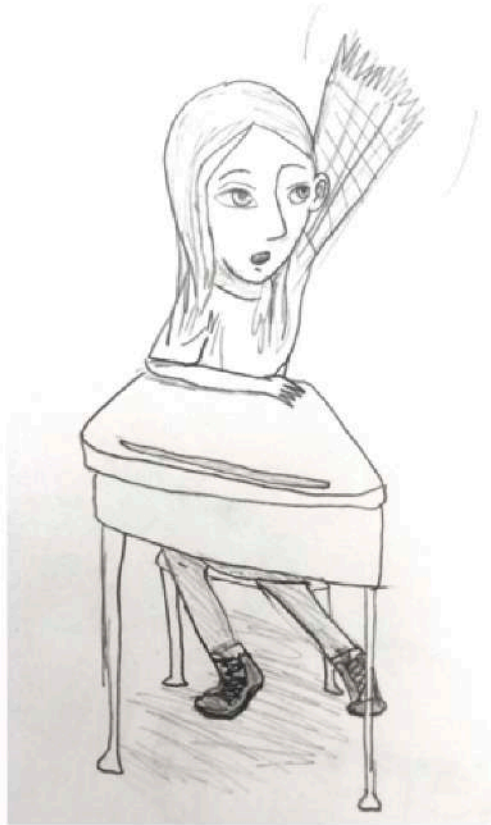
2, 2, 2, 2, 2

1, 2, 1, 2, 1, 2, 1 but the order of the 1s and 2s could be changed into many more ways

2, 2, 1, 1, 1, 1, 1, 1 but many more orders are possible

“Not a very systematic list,” observed Daniel, “but I have an idea about how to organize it. I’ll need help.” A group of students quickly volunteered.

“What about pictures?” asked Eric. “I have a video of the stairs at the Botanic Garden. We can grab a still and use that.”



Caroline’s hand was waving frantically. When Ms. Mathis nodded at her, Caroline exclaimed, “What if we ALL brought in pictures of stairs and used them to investigate the problem? That might help!”

“Yeah, right,” blurted Taylor, “help us have 23 problems instead of one!”

Everyone laughed and Caroline blushed and shrank into her seat.

“Taylor, that’s a warning for blurting and rudeness,” said Ms. Mathis. “And, frankly, I think Caroline’s idea is a good one. Instead of the math homework I assigned

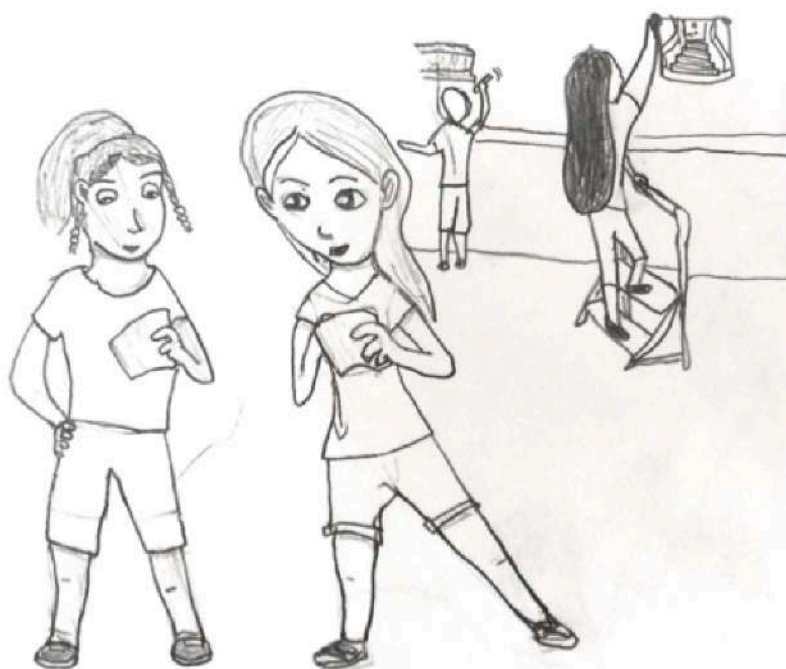
this morning, I'd like each of you to photograph a set of stairs in your neighborhood, print it, and bring it in tomorrow. We are going to investigate different ways to climb stairs going 1 step or 2 steps at a time. If your stairs have only a small number of steps, try to solve the problem for that number of stairs. Does everyone understand what to do? Can someone repeat the assignment? Nathan?"

"There are 3 steps to my front door," said Nathan. "So I should bring in a photo, and try to count all the different ways to go up the 3 stairs if I go 1 or 2 steps at a time... right?"

"There are NO steps in our house because my grandma is in a wheelchair," said Jordan. "Can I bring in a picture from a magazine instead?"

"Yes and yes," said Ms. Mathis as the bell rang to end the day. "See you tomorrow!"

The next day the children were excited to share their pictures and their numbers.



“Let’s line up in order,” suggested Caroline.

Taylor nodded in agreement, and waited to be called on to speak. “Maybe, if we make a table, there will be a pattern!”

“Excellent idea, Taylor. You may come to the whiteboard and start the table,” said Ms. Mathis.

<u>number of stairs</u>	<u>number of ways</u>
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

“I don’t have a picture, but I know that for one step there is only one way. I’ll write that in the table,” said Taylor.

Then the class began to collect their data.

“I’ve got two!” said Michelle. “And there are two ways: 1,1 and 2. Mine was easy!”

Next Nathan reported that with three steps, there were three ways: 1,1,1; 1,2; 2,1.

Taylor recorded the results as they were reported:

<u>number of stairs</u>	<u>number of ways</u>
1	1
2	2
3	3
4	
5	
6	
7	
8	
9	
10	

“I think I see a pattern already!” called out Michelle. “I predict that for four steps there will be four ways! Who has four?”

“I do,” said Tonnica. “But it’s not four ways—I’ve got 1,1,1,1; 2,1,1; 1,2,1; 1,1,2; 2,2—so FIVE ways. Sorry, Michelle.”

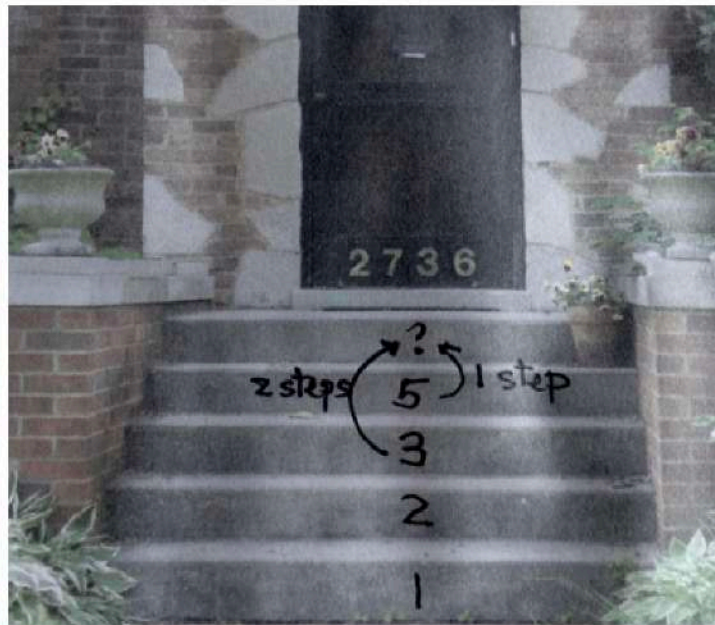
“I have five steps,” said Daniel. “I had to be very systematic about how I wrote the answers, see?”



First I thought about what numbers I could use to make 5, and then I thought about how to arrange them. I systematically moved the 2 around to get four arrangements of one 2 and three 1s, and then I moved the 1 around to get three arrangements of one 1 and two 2s.”

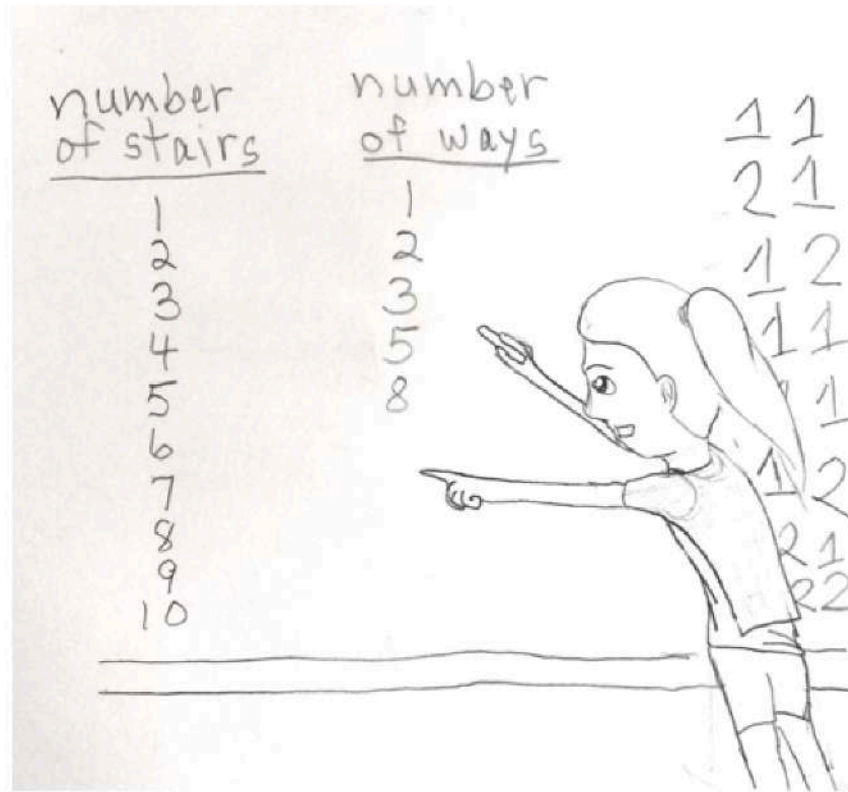
“Also,” Daniel continued, “I really got into this problem, so I figured out all the ways to do 1, 2, 3, and 4. Then I wrote the number of ways to get to each step right on my photo. See?”

Daniel got drowned out with all the excitement in the room. He still tried to show his picture and finish his thought, even though no one (except Ms. Mathis) was paying attention to him.



“I noticed that to get to the top step, you must be coming from the third step and taking a 2-step or from the fourth step and taking a 1-step. There were 3 ways to get to step 3 and 5 ways to get to step 4. And $3+5=8$.”

“Okay, 8,” said Taylor, as she wrote 8 next to the 5 in the table:



“Who has six?”

“I brought a picture of six,” answered Jordan, “but I’m not sure about the ways.

Here’s what I have:

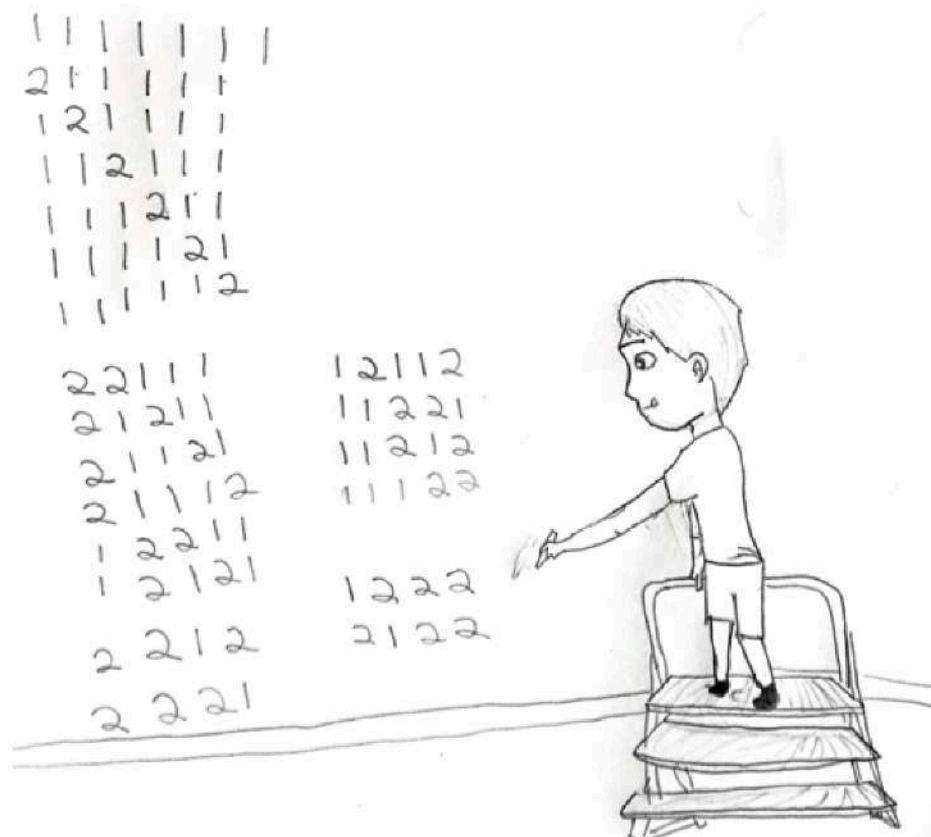
1 1 1 1 1 1	2 1 1 1 1	2 1 1 1 1
2 2 2	1 1 2 1 1	1 1 2 1 1
1 1 1 1 2	1 2 2 1	1 2 1 1 1
2 2 1 1	2 1 1 2	1 1 1 2 1

That’s twelve, but I’m not sure.”

“Wait! I did six, too,” said Caroline. “I can compare my list with Jordan’s.”

“Who did seven?” asked Ms. Mathis.

“I did,” said Eric. “And I’m sure that there are 21 ways:



The challenging part was using two 2s and three 1s.”

“We’ve got 13 ways for 6 steps,” Caroline and Jordan said in unison.

“I missed 2121,” said Jordan. “While we were comparing, I started thinking about the zero steps to get into my house. There IS a way to get in—shouldn’t we put zero in the table, with ONE way in the second column?”

“I like your thinking, Jordan,” said Ms. Mathis. “Let’s add that row at the top of our table.”

number of stairs	number of ways
1	1
2	2
3	3
4	5
5	8
6	13
7	21
8	
9	
10	

Other students showed their pictures for 8 and 9, and Eric had the picture of the 10 steps at the Botanic Garden, but no one had managed to count the ways.

“Good work, boys and girls. Counting for so many steps truly IS daunting! I think it’s time to study the table and look for a pattern,” said Ms. Mathis. “What do you notice? Work in pairs.”



The class got very quiet for a minute while everyone copied the table into their notebooks. Then the room began to hum. Everyone was startled when the bell rang.

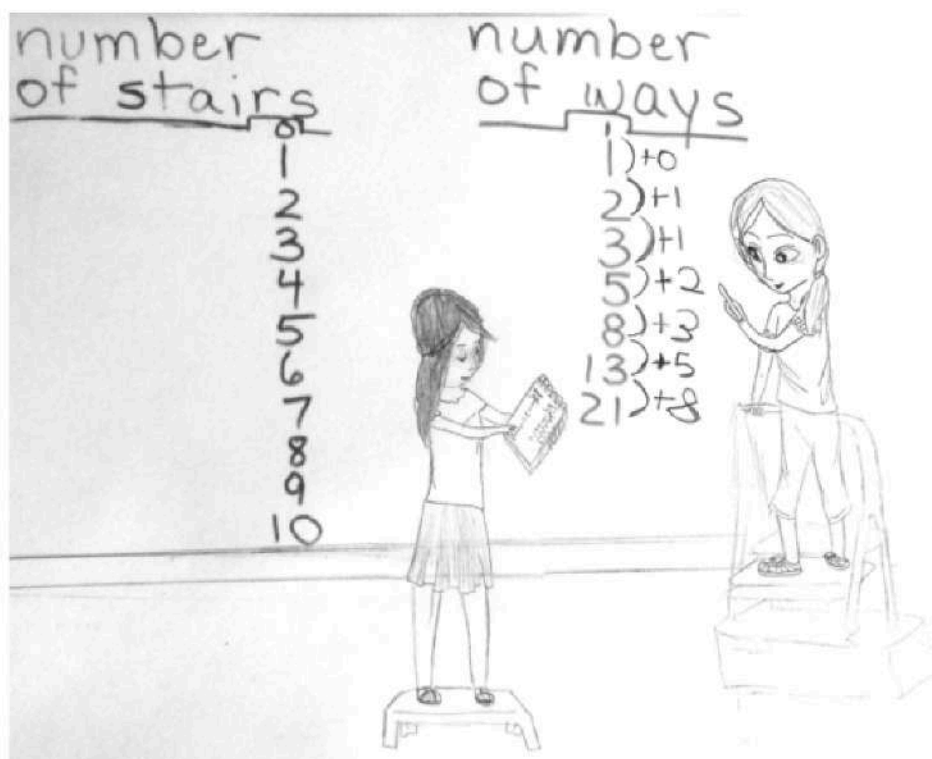
“We’ll pick this up tomorrow,” said Ms. Mathis. “See if you can bring in a conjecture about the data we’ve collected.”

The next day, Ms. Mathis asked for conjectures. Hands went up, but Taylor, not waiting to be called on, practically yelled, “I searched the internet for the pattern and...”

Ms. Mathis cut Taylor off immediately. “Stop! That’s your second warning for blurting this week—you are making it harder for your classmates to learn. And internet searches are no substitute for doing your own thinking. Now who wants to share?”

Caroline and Jordan began waving their hands. When Ms. Mathis called on them, they said “There IS a pattern!”

“We figured out how many ways were added each time there was one more step. Something amazing happened! Look!”

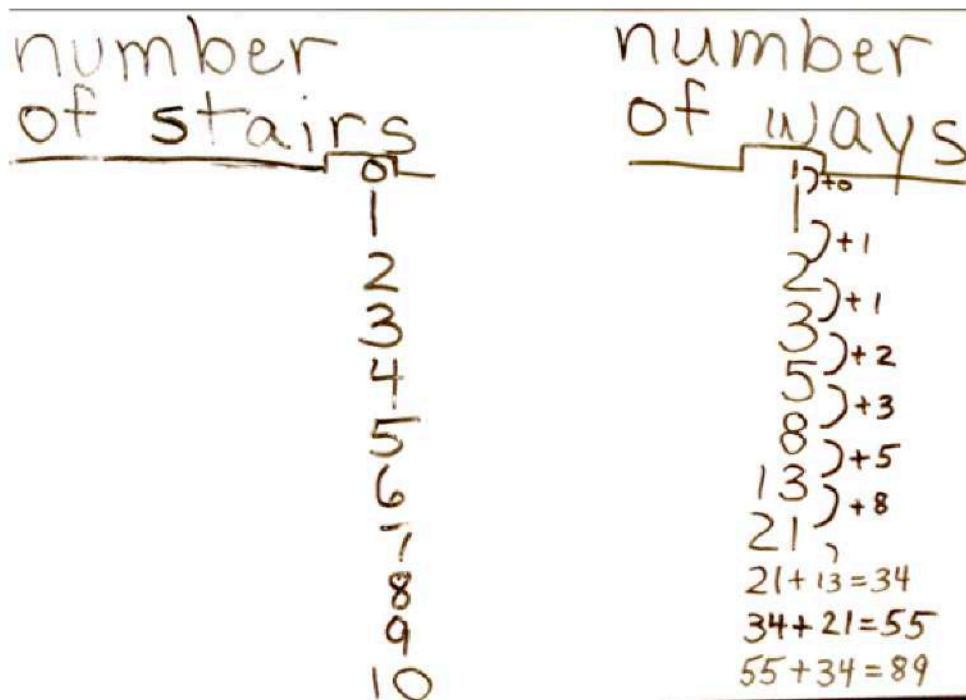


“What’s so amazing?” asked Nathan. “After the zero those make the same mystery pattern we already had in the second column!”

“Wow!” exclaimed Daniel. “Those ARE the same numbers we already had in the second column—so, if we keep on going... add 13... add 21...”

“That’s exactly what we did,” said Caroline.

“Can I come up and finish the table?” asked Jordan, already out of her seat.



“Hold on a sec,” said Daniel. “This fits with what I noticed yesterday—for each new step you add the ways you got to each of the last two steps because then you can just go up a 1-step or a 2-step. Cool!”

“Good work, class!” said Ms. Mathis. “This pattern, that builds on itself, where each number in the pattern is the sum of the two previous numbers, is called... Taylor?”

“The Fibonacci Sequence!” declared Taylor.



“More tomorrow,” said Ms. Mathis.

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Note: “Notice and Wonder” is a teaching approach used in all subject areas. For example,
see

<http://wonderteacher.com/learning-to-notice-visual-storytelling/>

<http://mathforum.org/pow/samples/MathForumNoticeWonderRecordSheet.pdf>

<http://corelaboratewa.org/what-do-you-notice-what-do-you-wonder/>

<https://amiealbrecht.com/2016/08/21/notice-and-wonder-sense-making/>

<https://buildingmathematicians.wordpress.com/tag/notice-and-wonder/>