A FRACTIONATED FAIRY TALE

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Once upon a time a young girl named Katharine lived in a snug cottage not far from the castle, where her mother stitched the royal quilts.

Katharine's mother brought home scraps of fabric for Katharine, who made beautiful miniature quilts. Quilt-making was easy and fun for Katharine but she longed for something more challenging to put her mind to. *If only I could get an education,* Katharine thought, *I'd be able to find my true calling.*



Every night, Katharine's mother reported on life at the castle. The favorite topic was the unseemly behavior of Prince Henry, just the same age as Katharine and the only child of the King and Queen. Katharine loved listening to stories about Henry. Most of the royal staff described Henry as "bossy" and "demanding" and "downright unpleasant."

Henry's parents were always too busy for him. They solved every problem by hiring a Royal Somebody-Or-Other to fix it. He had tutors for everything from art to zoology, even for activities like swimming and chess.

Katharine thought Henry was just plain lonely. She felt sorry for him.

One day, Katharine's mother told her that Prince Henry proclaimed he would eat only peanut butter and grape jelly sandwiches on perfectly square white bread and that each sandwich must be cut into two equal-sized pieces.

The Royal Sandwich Chef cut Prince Henry's sandwich the only ways he knew: into two long rectangles, or two right triangles.



When his parents took no notice of him, Prince Henry marched down to the Royal Kitchen and announced, "Every sandwich must be cut into two equal-sized pieces in a DIFFERENT way--or I will not eat."

The Royal Sandwich Chef told the King and Queen.

They went straight to Prince Henry's chamber.

"What is this nonsense about your sandwiches?" asked the King.

"The Royal Sandwich Chef knows only two ways to cut your sandwiches. If you won't eat them you will starve!" said the Queen, flitting around Henry like a mother hen rounding up a stray chick.



"Then I shall starve," said Henry. "My sandwich must look different every time, or I refuse to eat."

"Well," said the Queen, "we need a Royal Sandwich Designer. But where will we find one?"

"Let's hold a Royal Sandwich Designing Contest," suggested Henry.

"Good idea!" said the King.

"We must work out the rules," said the Queen.

So the three of them put their heads together.

Prince Henry's parents had not spent this much time with him in years. He bit his lip to keep from smiling.

The Royal Guards posted the rules all about the kingdom.

Royal Sandwich Designer Contest Tomorrow! - Contestants will form a single line in mont of the Castle at 9:00 A.M. - Contestants will in turn, show the Royal Mathematican their a their designs for cutting a square sandwich into two equal-sized parts. -One gold coin will be awarded for each NEW aesign. -Successful contestants may return to the line to submit additional new designs. The final contestant will become the new Royal Sandwich Designer.

When Katharine saw the contest rules, she leaped around the cottage and then bounded up to her loft to draw some sandwich designs before bedtime.



The next morning Katharine waited near the end of the line. She was the only child in the contest.

At exactly nine o'clock, the Royal Herald announced the start.

The first entry was a square cut into two equal-sized rectangles.

"Oh, dear! Oh, dear!" the Royal Mathematician muttered under his breath. "The Royal Chef knows this design already. But the rules do not exclude it. Oh, my! We must pay one gold coin."

The contestant received a coin and departed.

The Royal Mathematician turned the paper around for all to see. About half the contestants saw that it matched their only design, so they left.

The next contestant's design was two right triangles.

The Royal Mathematician grumbled again, then awarded a gold coin.

The remaining contestants groaned when they saw the triangles. All but three people stepped out of the line!

The first, a young man, handed a paper to the Royal Mathematician. He studied it, then shook his head sadly.

He showed the design to the crowd, saying, "These two pieces are NOT equal in size. No coin."



The next contestant was Katharine. She handed a design to the Royal Mathematician.

"Good job!" he said, as a grin spread across his face. "We finally have a NEW design-and a very INTERESTING one, I must say."

He showed Katharine's design:



Then the Royal Mathematician sent it to the Royal Chef so the prince could be served breakfast at once.

While the throng cheered, Katharine collected her gold coin and returned to the line.

Suddenly, trumpets blared. Prince Henry came outside to watch the contest. He was eating his newly cut sandwich.

The other contestant gave her drawing to the Royal Mathematician.

"Excellent! Two equal-sized trapezoids," he said.

He handed her a gold coin. The people applauded as he displayed the design.



A hush fell over the spectators as the woman walked away. Katharine was alone in line.

The Royal Mathematician said, "Young lady, with one more new design, you will become the Royal Sandwich Designer. But I hope you have MANY ways to cut sandwiches into halves, or our prince will starve."

Katharine showed another pattern to the Royal Mathematician.

He quickly gave her a gold coin, then held up the new design and declared, "Katharine is indeed our Royal Sandwich Designer!"



The crowd roared their approval.

"I have a lot more," she said, handing the Royal Mathematician a stack of papers. "I use my quilting squares and triangles to make my designs, and grid paper to record them. See?"

Then she took her quilting pieces from her bag and spread them out on the Royal Mathematician's table.



Henry ran over to have a look and started making his own designs while Katharine explained her methods.

At that moment the King and Queen came out of the castle.

They watched Katharine and Henry. They had never seen Henry look so happy.

The King said, "Katharine will be paid three gold coins a day as the Royal Sandwich Designer."

"We are so grateful to Katharine for saving our son's life," said the Queen. "She shall be an Honorary Princess. She is welcome to go wherever she wishes in the castle."

Everyone cheered.

"Katharine, do you have anything to say?" asked the King.

"Yes," said Katharine. "I want to thank my mother for making it possible for me to win this contest."

Katharine's mother nodded proudly from the front row.

"And I want to thank Prince Henry for coming up with this splendid sandwich-cutting problem," Katharine continued. "I have been looking for a challenging problem to put my mind to. Prince Henry is surely the most talented problem poser in the kingdom."

Prince Henry beamed.

"With your permission," she looked toward the King and Queen, "I would like to study at the castle with Prince Henry and the royal tutors. May I?"

"Pleeeeease say yes," begged Prince Henry.

"Of course!" said the King and Queen.

So Katharine visited the castle every day and studied with Henry. They became good friends.

Katharine and Henry planned sandwich parties for the village children. The King and Queen attended every one, without fail.

And they all lived happily ever after.



The End

Suggestions for Using the Story to Develop Mathematical Thinking

Introduce the accompanying red and yellow quilting pieces to the child.



To begin, arrange 8 red squares and 8 yellow squares in the frame to duplicate the rectangle pattern used by the Royal Chef. Have the child count the squares with you so that they agree that half a sandwich contains a total of 8 square units.

Next, have the child cover one of the small squares with two triangles to show that each triangle is one-half the size of a square unit.

Arrange a combination of squares and triangles to duplicate the triangular pattern used by the Royal Chef. Count the wholes and halves to check that each half of the sandwich contains a total of 8 square units.

To get a design for halves of the sandwich, Katharine made sure that she used a total of exactly 8 square units to make each piece. Remember, you can use a combination of squares and triangles.

Make several designs for halves of the sandwich using the squares and triangles, checking that each half of the sandwich contains a total of 8 square units.

Congruence

Note that the two pieces might be the same shape as well as the same size. Then the halves are **congruent.** Here are some additional designs with congruent parts:



The two halves do NOT have to be congruent. There are many ways to make non-congruent halves. Here are some examples:

