Version 01

Trigonometry I

Apprenticeship and Workplace Mathematics (Grade 10/Literacy Foundations Level 7)

OPEN SCHOOL BC

Course History

New, March 2012

Project Partners

This course was developed in partnership with the Distributed Learning Resources Branch of Alberta Education and the following organizations:

- Black Gold Regional Schools
- Calgary Board of Education
- Edmonton Public Schools
- Peace Wapiti School Division No. 76
- Pembina Hills Regional Division No. 7
- Rocky View School Division No. 41

Project Management: Jennifer Riddel

Content Revisions: Jennifer Riddel, Ester Moreno

Edit: Leanne Baugh, Monique Brewer

Math Edit:
Learning Centre of the Greater Victoria School District Continuing Education Program: Nigel Cocking, Keith Myles, Bill Scott
School District 47, Powell River: Tania Hobson
OSBC: Christina Teskey

Module Tests: Barb Lajeunesse, Michael Finnigan (SD 34)

Copyright: Ilona Ugro

Production Technicians: Sharon Barker, Dennis Evans, Brian Glover

Art Coordination: Christine Ramkeesoon

Media Coordination: Janet Bartz

Art: Cal Jones

Flash Programming: Sean Cunniam

Narration Recording: MOH Productions and Neil Osborne

Voice Talent: Felix LeBlanc, Kate Eldridge, Wendy Webb and MOH Productions

Advisors: JD Caudle (Yukon Territory), Randy Decker (SD 40), Bev Fairful (Yukon Territory), Sonya Fern (SD 62), Sandra Garfinkel (SD 39), Richard Giroday (SD 58), Sharon Hann (SD 39), Tim Huttemann (SD 20), Dan Laidlaw (SD 73), Heather Lessard (SD 53), Gloria Lowe (SD 6), Jan Malcolm (SD 36), Christina Teskey (OSBC), Jennifer Waughtal (SD 57), Ray Wong (SD 91)

Table of Contents

Viewing Your PDF Learning Package

This PDF Learning Package is designed to be viewed in Acrobat. If you are using the optional media resources, you should be able to link directly to the resource from the pdf viewed in Acrobat Reader. The links may not work as expected with other pdf viewers.

Download Adobe Acrobat Reader:
http://get.adobe.com/reader/

Section Organization

This section on Trigonometry is made up of several lessons.

Lessons

Lessons have a combination of reading and hands-on activities to give you a chance to process the material while being an active learner. Each lesson is made up of the following parts:

Essential Questions

The essential questions included here are based on the main concepts in each lesson. These help you focus on what you will learn in the lesson.

Focus

This is a brief introduction to the lesson.

Get Started

This is a quick refresher of the key information and skills you will need to be successful in the lesson.

Activities

Throughout the lesson you will see three types of activities:

- Try This activities are hands-on, exploratory activities.
- Self-Check activities provide practice with the skills and concepts recently taught.
- Mastering Concepts activities extend and apply the skills you learned in the lesson.

You will mark these activities using the solutions at the end of each section.

Explore

Here you will explore new concepts, make predictions, and discover patterns.

Bringing Ideas Together

This is the main teaching part of the lesson. Here, you will build on the ideas from the Get Started and the Explore. You will expand your knowledge and practice your new skills.

Lesson Summary

This is a brief summary of the lesson content as well as some instructions on what to do next.

At the end of each section you will find:

Solutions
This contains all of the solutions to the Activities.

Appendix
Here you will find the Data Pages along with other extra resources that you need to complete the section. You will be directed to these as needed.

Glossary
This is a list of key terms and their definitions.

Throughout the section, you will sefe the following features:

Icons
Throughout the section you will see a few icons used on the left-hand side of the page. These icons are used to signal a change in activity or to bring your attention to important instructions.

AWM online resource (optional)
This indicates a resource available on the internet. If you do not have access, you may skip these sections.

Solutions

Calculator

My Notes

The column on the outside edge of most pages is called "My Notes". You can use this space to:

- write questions about things you don't understand.
- note things that you want to look at again.
- draw pictures that help you understand the math.
- identify words that you don't understand.
- connect what you are learning to what you already know.
- make your own notes or comments.

Materials and Resources

There is no textbook required for this course.

You will be expected to have certain tools and materials at your disposal while working on the lessons. When you begin a lesson, have a look at the list of items you will need. You can find this list on the first page of the lesson, right under the lesson title.

In general, you should have the following things handy while you work on your lessons:

- a scientific calculator
- a ruler
- a geometry set
- Data Pages (found in the Appendix)

Trigonometry I

Photo by Roca © 2010

Hikers and skiers who are out on snowy slopes need to be careful to avoid areas that are high risk for avalanches. Many of these outdoor enthusiasts carry a device called a *clinometer*. A clinometer can be used to measure the angle of inclination of a slope. Hikers and skiers can measure the incline of the slopes around them and avoid risky areas.

Hikers and skiers aren't the only ones who use clinometers. Surveyors, geologists, sailors, city planners, and engineers use clinometers to measure distances indirectly.

How can measuring an angle of incline help one determine an unknown distance? Well, the angle measurements are used together with a mathematical concept called *trigonometry*. Trigonometry, as you will discover in this section, involves relationships arising from the sides and angles of right triangles.

In this section you will:

- apply similarity to right triangles.
- generalize patterns from similar right triangles.
- build and use a clinometer to measure distances indirectly.
- apply the trigonometric ratios tangent and sine to solve problems.

Lesson A
The Tangent Ratio

To complete this lesson, you will need:

- a piece of cardboard (8 ½ by 11")
- a drinking straw
- tape
- scissors
- a paper clip or other small object to use as a weight
- the clinometer template from the appendix
- protractor
- metric ruler
- a calculator

In this lesson, you will complete:

- 9 activities

Essential Questions

- What is the tangent ratio?
- How is the tangent ratio used to find unknown sides and angles in right triangles?

My Notes

Focus

Photo by Ewan Chesser © 2010 and photo by Joe Gough © 2010

Imagine building a tree house. Some measurements, such as the height of the tree, would be difficult to take—it's not easy (or safe) to climb a tree with a tape measure in your hand!

Luckily, we have trigonometry! In the construction industry, surveying allows people to use trigonometry to calculate unknown lengths so that direct measurements are not required.

In this lesson you'll create a device for measuring angles and you'll begin your study of trigonometry.

Get Started

My Notes

Photo by Pitroviz © 2010

In the following activity you will make a **clinometer**. A clinometer is a device that you put to your eye to see how high an object is off the ground. Clinometers are often used in forestry to measure tree heights or elevation changes.

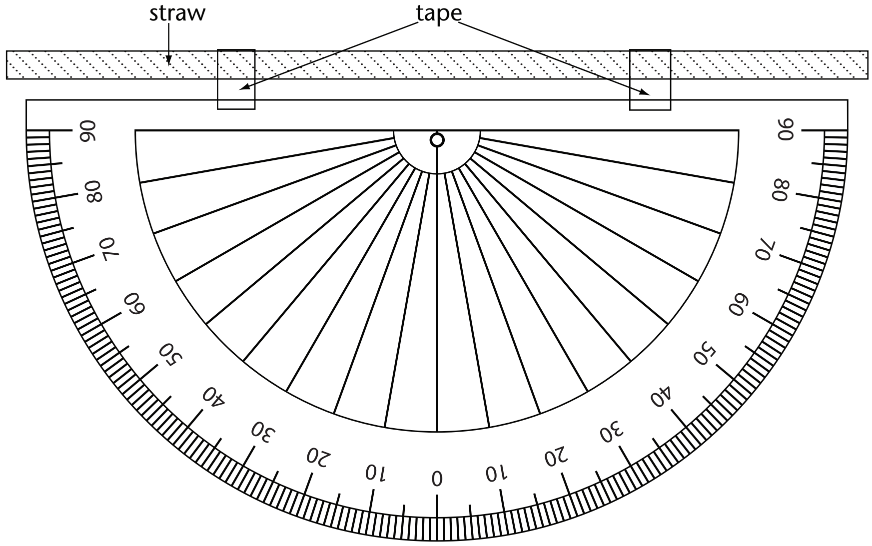
Activity 1
Try This

Step 1: Get the “Clinometer Template” from the appendix, and cut it out.

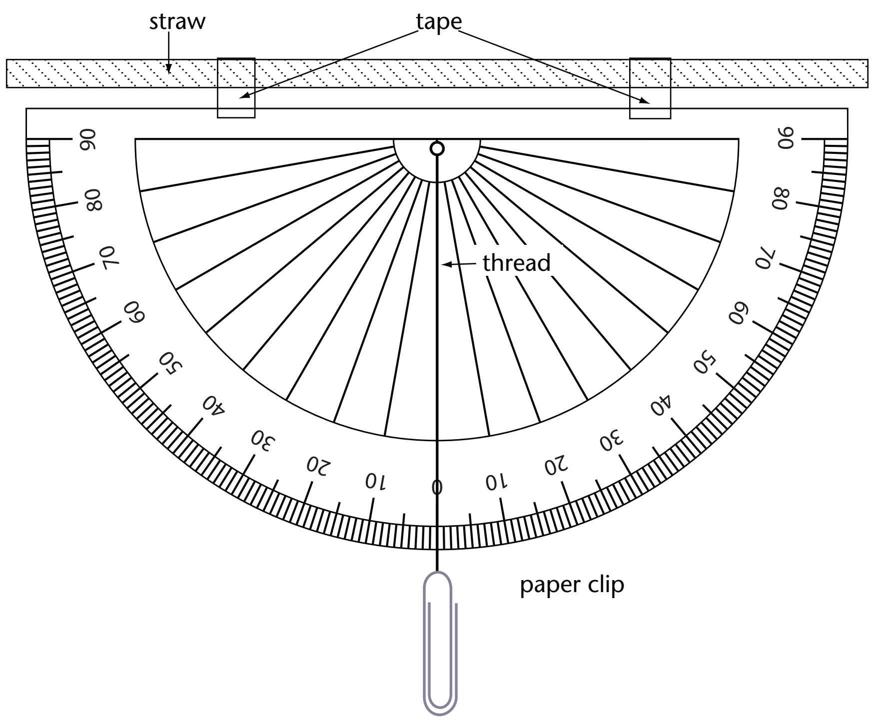
Step 2: Glue the template to a sheet of cardboard. Then, cut out the semicircular protractor.

My Notes

Step 3: Tape a drinking straw to the base of the protractor as shown.

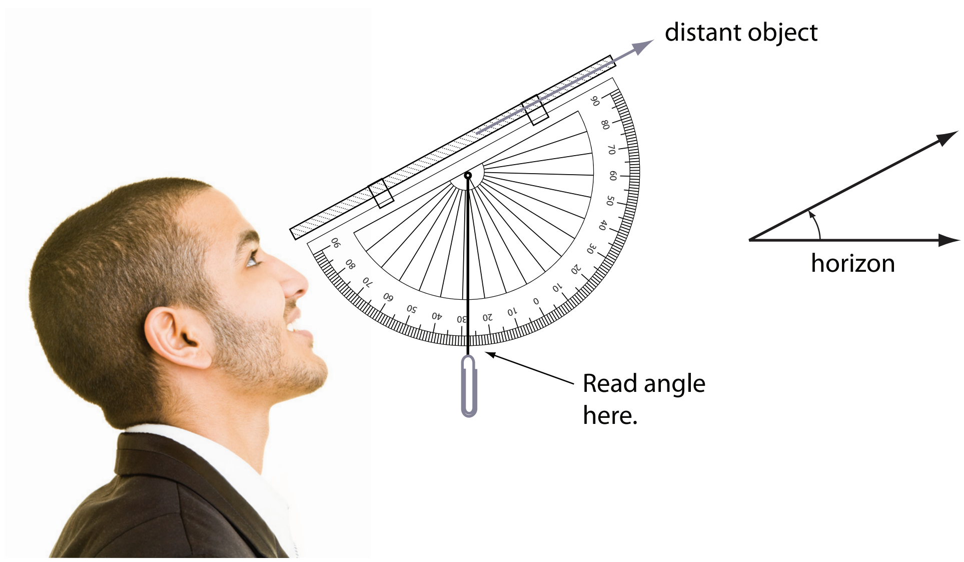


Step 4: Poke a small hole through the crosshairs at the centre of the protractor's base. Pass one end of your thread through the hole. Knot and tape that end of the thread so it will not slip back through. Tie a weight (such as a paper clip) to the free end of the thread.

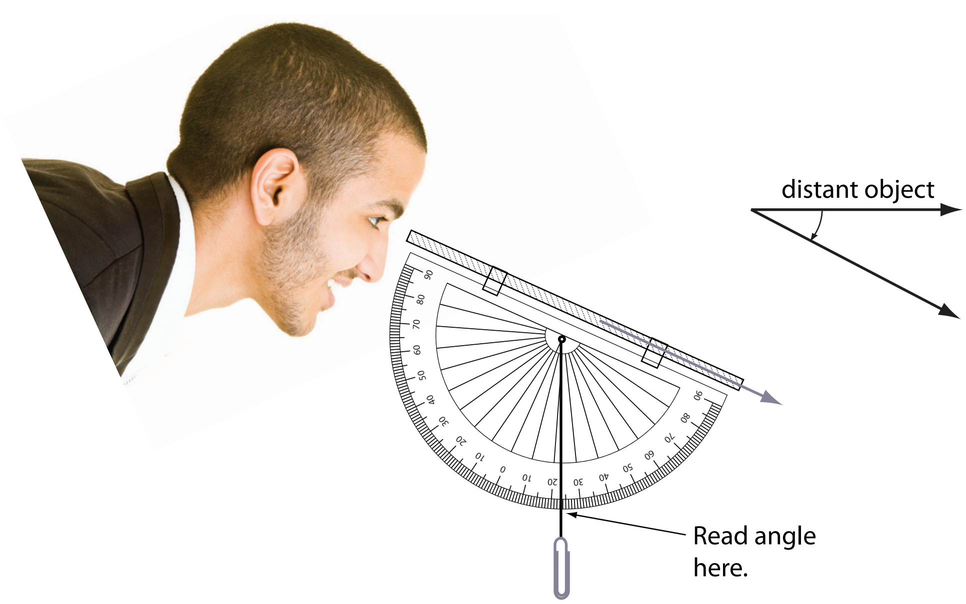


Step 5: Let the weight swing free. When the straw is horizontal, the thread should lie along the zero degree line as shown in the diagram.

When you look through the straw at an object higher than you are, your partner should be able to read the **angle of elevation**. When the weight hangs down and is motionless, the string will lie along a particular angle. This is the angle of elevation.



When you look through the straw at an object lower than you are, your partner should be able to read the angle of depression from the scale when the weight hangs down motionlessly.



My Notes

My Notes

Practise looking through the straw and measuring angles of elevation and depression.

Questions

1. What is the greatest angle of elevation you can measure?

2. What is the greatest angle of depression you can measure?

Keep your clinometer: you will need it to complete the next lesson.

Turn to the solutions at the end of the section and mark your work.

Ratio Skills

An important algebra skill you will use repeatedly throughout this section involves equal ratios. You will now review how to determine if ratios are equal.

Example 1

Are $\frac{10}{15}$ and $\frac{8}{12}$ equal ratios?

Solution

Method 1: Reduce each fraction (ratio) to its simplest terms.

$$\frac{10}{15} = \frac{10}{15} \div \frac{5}{5} = \frac{2}{3} \qquad \frac{8}{12} = \frac{8}{12} \div \frac{4}{4} = \frac{2}{3}$$

Since each ratio reduces to $\frac{2}{3}$, the ratios are equal.

My Notes

Method 2: Compare the cross products.

$$\frac{10}{15} \times \frac{8}{12}$$

10 × 12 = 120

15 × 8 = 120

Because the cross products are equal, the ratios are equal.

The Cross Product Principle

The previous example illustrates the following principle in algebra.

If $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$.

and

If $ad = bc$ then $\frac{a}{b} = \frac{c}{d}$.

This is the *cross product principle*.

Activity 2
Self-Check

In each of the following questions, use both methods from the example to compare each pair of ratios.

1. Are $\frac{7}{14}$ and $\frac{4}{8}$ equal ratios?

   Method 1 **Method 2**

My Notes

2. Are $\frac{3}{9}$ and $\frac{6}{15}$ equal ratios?

Method 1

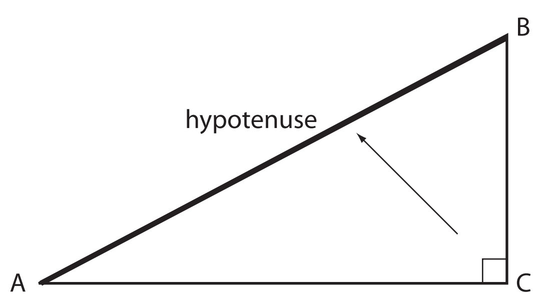
Method 2

Turn to the solutions at the end of the section and mark your work.

Explore

Right Triangle Side Names

The longest side of a right triangle is called the **hypotenuse**. It is the side that *does not* touch the right angle. It is the side that is opposite the largest angle in the triangle, the right angle.



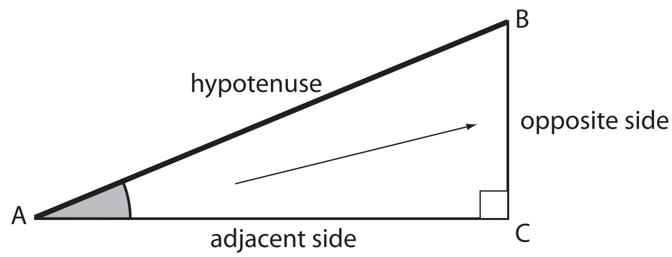
The two shorter sides are named depending on the acute angle you choose to use as your reference angle. In the diagram, the acute ∠A will be the reference angle.

My Notes

Before you move on in this section, you must be able to name the side **adjacent** and the side **opposite** to either one of the acute angles of a right triangle. The remaining side is called the **hypotenus**e of the right triangle. (The hypotenuse will be bolded black in many of the following examples.)

Look at the diagram of the right triangle ΔABC. For ∠A, the opposite side is side BC.

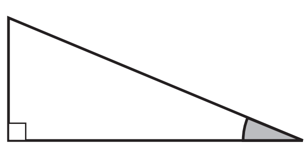
The side adjacent to ∠A is side AC.



Activity 3
Self-Check

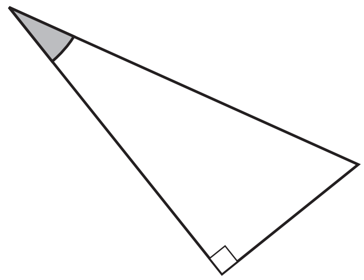
For each of the following right triangles an angle has been shaded. This is your angle of reference. Using this angle label the opposite and adjacent side.

1.

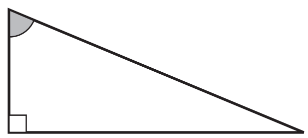


My Notes

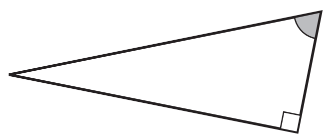
2.



3.



4.



Turn to the solutions at the end of the section and mark your work.

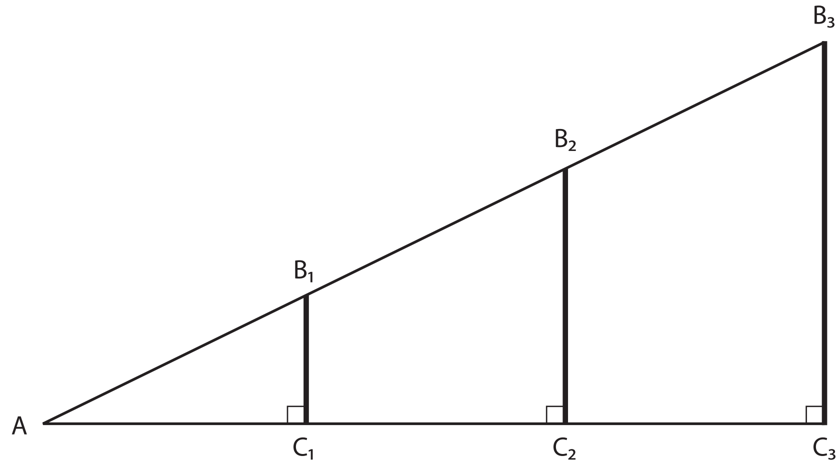
Activity 4
Try This

In this activity, you will explore the ratio of two sides within a right triangle. You will compare this ratio to ratios involving the same pair of sides from other right triangles. For this comparison you will use your skills with similar right triangles.

You will need your metric ruler and calculator.

Consider the right triangles in the following diagram.

My Notes



Step 1: On a full piece of paper, draw a similar diagram as the one above. Make an enlargement of it, so that the diagram takes up the entire page. Label the diagram using the same lettering.

Step 2: For right triangle ΔAB_1C_1, measure to the nearest millimetre and record the length of sides B_1C_1 and AC_1 in the table below.

For right triangle ΔAB_2C_2, measure to the nearest millimetre and record the length of sides B_2C_2 and AC_2 in the table below.

For right triangle ΔAB_3C_3, measure to the nearest millimetre and record the length of sides B_3C_3 and AC_3 in the table below.

Step 3: Complete the last column of the table using your calculator.

Right triangle	Side opposite $\angle A$ (nearest mm)	Side adjacent to $\angle A$ (nearest mm)	$\frac{\text{opposite side}}{\text{adjacent side}}$ (to 2 decimal places)
ΔAB_1C_1	$B_1C_1 =$	$AC_1 =$	$\frac{B_1C_1}{AC_1} =$
ΔAB_2C_2	$B_2C_2 =$	$AC_2 =$	$\frac{B_2C_2}{AC_2} =$
ΔAB_3C_3	$B_3C_3 =$	$AC_3 =$	$\frac{B_3C_3}{AC_3} =$

Questions

My Notes

1. What pattern do you see in the table?

____

____

____

2. Are these three triangles ΔAB_1C_1, ΔAB_2C_2, and ΔAB_3C_3 similar? Explain.

____

____

____

____

____

____

Turn to the solutions at the end of the section and mark your work.

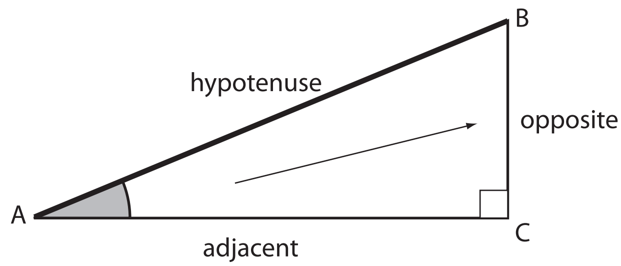
Bringing Ideas Together

In the Explore, you looked at right triangles and the ratio of the length of the opposite side to the length of the adjacent side ($\frac{\text{opposite side}}{\text{adjacent side}}$). This ratio is an example of a *trigonometric ratio*.

Trigonometry comes from two Greek words which mean *triangle measure*. Trigonometry is the branch of mathematics based originally on determining sides and angles of triangles, particularly right triangles.

My Notes

In Explore, when a particular acute angle in set of similar triangles was selected, the ratio $\frac{\text{opposite side}}{\text{adjacent side}}$ was the same value regardless of the size of the similar right triangles.



This ratio is called the **tangent ratio**.

The tangent ratio always depends on the acute angle selected. In this case, the acute angle that was selected was ∠A—not ∠B. The acute angle has to be identified in order to write a tangent ratio.

So, tangent $\angle A = \frac{\text{opposite side}}{\text{adjacent side}}$.

The angle is missing!

It is incorrect to write: tangent $= \frac{\text{opposite side}}{\text{adjacent side}}$

It is often written more simply as:

$$\tan A = \frac{\text{opp}}{\text{adj}} \text{ where } A = \text{measure of } \angle A$$

Example 2

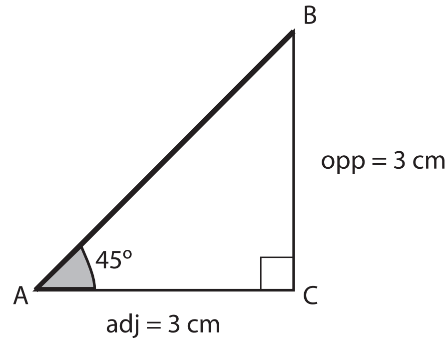
Draw a right triangle with an acute angle of 45° Determine tan 45°.

Solution

It does not matter how large you draw the triangle, as all right triangles with a 45° degree angle will be similar triangles.

Once you draw the triangle, shade in the angle of 45°. Then label the opposite side and the adjacent side and measure each.

My Notes



$$\text{tangent } \angle A = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\tan A = \frac{\text{opp}}{\text{adj}}$$

$$\tan 45° = \frac{3\text{ cm}}{3\text{ cm}}$$

$$\tan 45° = 1$$

A ratio of 1 means that, in a right triangle with a 45° angle, the opposite side and the adjacent side are always equal. So if the opposite side were 10 cm, the adjacent side would also be 10 cm long.

Example 3

To view another example, go and open *Tangent Solution* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/ma10_tangent.html*).

My Notes

Activity 5
Self-Check

Use the method of the previous two examples to complete the following table. Use 10 cm for the length of the adjacent side.

Complete the following table.

Angle	Adjacent Side	Opposite Side	Tangent of an Angle
10°	10 cm		tan 10° =
20°	10 cm		tan 20° =
30°	10 cm		tan 30° =
40°	10 cm		tan 40° =
45°	10 cm	10 cm	tan 45° = 1
50°	10 cm		tan 50° =

Turn to the solutions at the end of the section and mark your work.

Calculating the Tangent Ratio

Fortunately, it is possible to determine these ratios without actually having to draw a right triangle. You can also use your calculator.

My Notes

Example 4

Find each tangent ratio using your calculator.

1. tan 45°

2. tan 30°

Solution

There are several different ways angle may be expressed. Make sure your calculator is set to degree mode.

1. To find tan 45°, press this sequence of keys. If you do not obtain the answer shown, consult your calculator manual for help.

Tan	4	5	)	=

   In your calculator display window, you may see the solution as:

   ```
   tan(45)
                 1
   ```

   So, tan 45°= 1

2. To find tan 30°, press this sequence of keys. If you do not obtain the answer shown, consult your manual for help.

Tan	3	0	)	=

   In your calculator display window, you may see the solution as:

   ```
   tan(30)
        .5773502692
   ```


Activity 6
Self-Check

My Notes

Use your calculator to complete the following table.

Complete the following table. Round your answers to 4 decimal places.

Angle	Tangent Ratio
10°	
20°	
30°	0.5774
40°	
45°	
50°	
60°	
70°	
80°	

Turn to the solutions at the end of the section and mark your work.

My Notes

Solving Problems with the Tangent Ratio

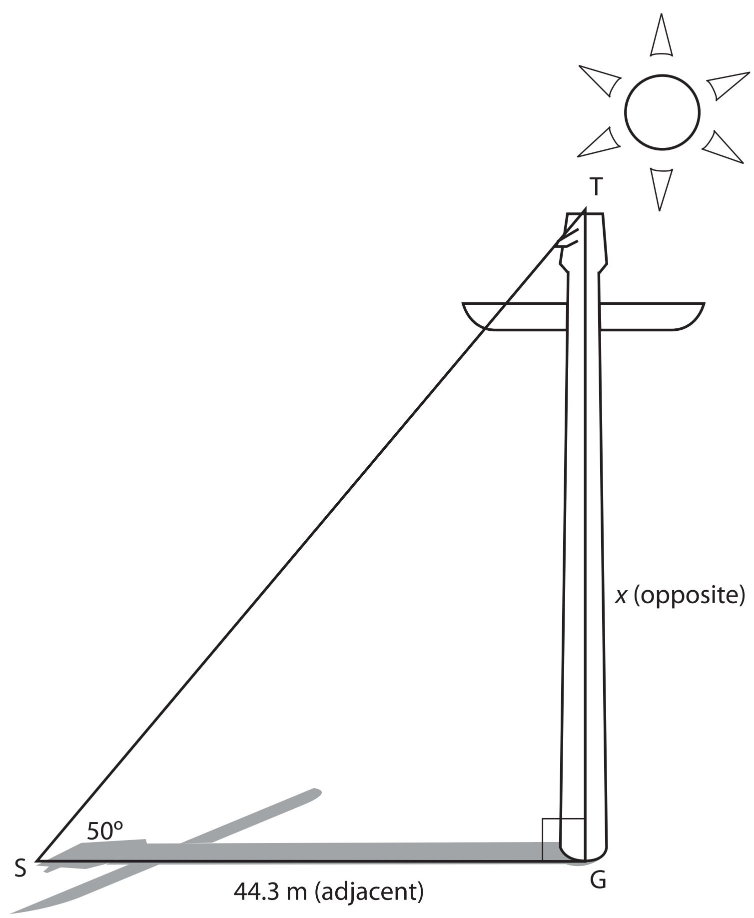
In the next example, you will use your calculator to solve a problem involving the tangent ratio.

Example 5

The Kwakwaka'wakw totem pole at Alert Bay, British Columbia, is one of the world's tallest totem poles. The pole casts a shadow along the ground of 44.33 m in length. The angle of elevation of the sun is 50°. Calculate the height of the pole to the nearest metre.

Solution

Draw a diagram. Start by drawing a right triangle with an angle of 50°. On the triangle draw in the totem pole, the sun and the shadow, where they would logically be. Then put in the length of 44.3 m and label the side you are trying to find with an "*x*."



Let x be the height of the totem pole.

My Notes

The totem pole is perpendicular to the ground at G.

The angle S at the tip of the shadow is 50°. This is the angle an observer would look through a clinometer to see the sun at point T above the pole.

$$\tan S = \frac{\text{opposite side}}{\text{adjacent side}}$$

Start by writing the tangent ratio equation.

$$\tan 50° = \frac{x}{44.3}$$

Fill in the measurements you know and your unknown.

$$(44.3)\tan 50° = \frac{x}{44.3}(44.3)$$

To isolate x, multiply both sides by 44.3.

$$(44.3)\tan 50° = \frac{x}{\cancel{44.3}}\cancel{(44.3)}$$

Cancel the 44.3 in the numerator with the 44.3 in the denominator.

$(44.3)\tan 50° = x$

To find x, use your calculator and do the following keystrokes:

4	4	.	3	×	Tan	5	0	)	=

Your calculator should display the value of x as: 52. 79468415 . . . (If you do not obtain the answer shown, consult your manual for help.)

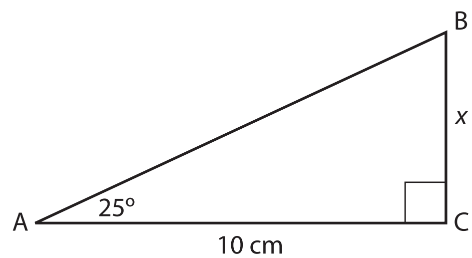
The totem pole is approximately 53 m high.

My Notes

Activity 7
Self-Check

Use the method outlined in Example 5 to solve this question.

Solve for x. Round to one decimal place.



Turn to the solutions at the end of the section and mark your work.

Finding Angles Using the Tangent Ratio

My Notes

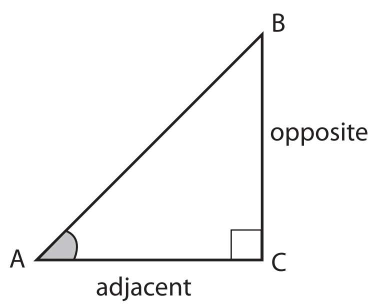
So far, you have learned to find the length of an unknown side in a right triangle, so long as you are given an acute angle and another length. But how can you find an unknown angle, using the tangent ratio?

We have learned that when the tangent of an angle equals 1, then the opposite and adjacent lengths are the same.

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = 1.$$

So the ratio of opposite and adjacent lengths could be: $\frac{8}{8}$ *or* $\frac{12}{12}$ *or* $\frac{6}{6}$.

But what is the size of angle A?



Draw a right triangle with equal adjacent and opposite sides. Measure the angle and you will see that $\angle A = 45°$.

But if tan A = 2, what would $\angle A$ be?

Example 6

Find $\angle A$, if tan A = 2.

Solution

Method 1. Before drawing a triangle, you need to understand what the given information tells you.

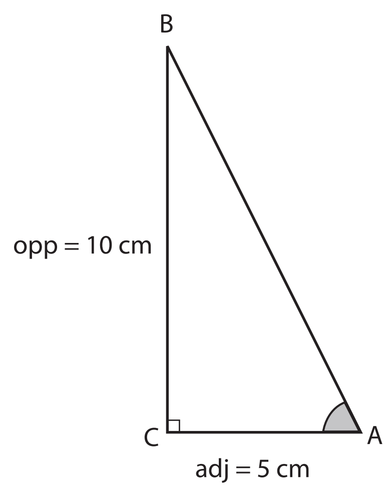
Since $\tan A = \frac{\text{opposite side}}{\text{adjacent side}}$ and tan A = 2 then $\frac{\text{opposite side}}{\text{adjacent side}} = 2$.

This means that the opposite length divided by the adjacent length is 2. There are lots of possibilities for these two lengths, so long as one divided by the other equals 2.

My Notes

Some options are $\frac{12}{6}$ *or* $\frac{8}{4}$ *or* $\frac{10}{5}$ since all these ratios divide to equal 2.

Draw a right triangle where the opposite side is always twice as long as the adjacent side. For example, if the adjacent side is 5 cm long, then the opposite side is 10 cm long.



Measure ∠A with your protractor. ∠A should measure approximately 63˚. That is, ∠A ≈ 63º.

This answer makes sense.

When tan A = 1, ∠A = 45°.

When tan A = 2, ∠A ≈ 63º.

The greater the tangent ratio, the greater the angle measure.

Method 2. Use your calculator.

To find an angle from its tangent ratio, use your calculator and do the following keystrokes:

[INV or 2nd] [Tan^{-1}] [2] [)] [=]

Your calculator should display the value of the angle as: 63.43494882 . . . (If you do not obtain the answer shown, consult your manual for help.)

My Notes

So, ∠A ≈ 63°.

Note: The symbol $\tan^{-1}$ means you are working backwards from the ratio to find the angle. It is read as, "inverse tangent."

You should write out your solution as follows:

$\tan A = 2$	Start by writing the equation.
$\tan^{-1}(\tan A) = \tan^{-1}(2)$	To solve for A, "inverse tan" both sides.
~~$\tan^{-1}(\tan$~~ $A) = \tan^{-1}(2)$	Inverse tangent and tangent cancel each other out.
$\angle A = \tan^{-1}(2)$	Key this in your calculator, to find the angle.
$\angle A \approx 63°$	

My Notes

Activity 8
Self-Check

Practice finding angles given the tangent ratio.

1. Find ∠A given tan A = 0.5.

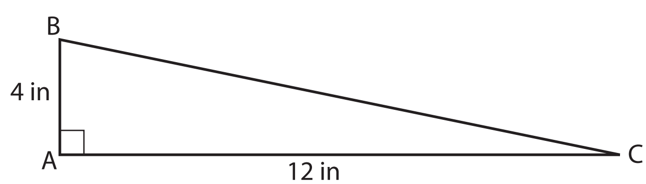
   Use both methods as shown in Example 6 to find the angle.

My Notes

2.  Use your calculator to find each angle from its tangent ratio. Round your answers to the nearest tenth.

tan A	∠A
0.3456	
4	
0.827	
2.5678	
$\frac{5}{6}$	

3. A garage roof rises 4 inches for a run of 12 inches. To the nearest degree, what is the angle of the roof?

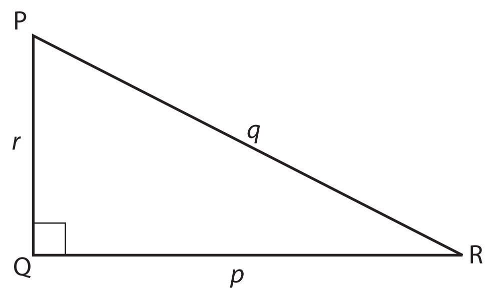


My Notes

Turn to the solutions at the end of the section and mark your work.

Activity 9
Mastering Concepts

Use the given triangle to answer the questions. Capital letters refer to the angles, and lower case letters refer to lengths.



1. For ΔPQR, write the ratios for tan P and tan R.

2. How are ∠P and ∠R related?

3. How are tan P and tan R related?

My Notes

Turn to the solutions at the end of the section and mark your work.

My Notes

Lesson Summary

Photo by photographer2222 © 2010

In the design of hockey sticks, the angle that a stick makes with the ice is a crucial consideration of the stick design. Therefore, trigonometric ratios, such as the tangent ratio, also play a key role in hockey stick design. The skills of determining angles and lengths using ratios are necessary in order to design an optimal stick.

In this lesson you explored the definition of the tangent as the ratio of the side opposite to the side adjacent for a given acute angle in a right triangle. You used this ratio to find missing sides and missing angles.

Lesson B
Using Tangents to Solve Problems

To complete this lesson, you will need:

- your clinometer from Lesson A
- a measuring tape
- a calculator

In this lesson, you will complete:

- 4 activities

Essential Questions

- How is the tangent ratio used to solve a variety of practical problems?

My Notes

Focus

Photo by gary718 © 2010

The Mayan Pyramid of Kulkulkan at Chichen Itza on the Yukatan Peninsula in Mexico is a UNESCO world heritage site. The pyramid was built over a thousand years ago, and its architecture is a mix of both art and mathematics. The temple faces the four directions representing the seasons. The 91 steps on each side and the platform at the top total 365—the number of days in the year.

The temple is famous for the shadows that slowly ascend and descend the staircases during the spring and fall equinoxes. These shadows represent mythical feathered snakes with their stone carved heads at the bottom of the stairs. And the shadows signal the time to plant and harvest the crops.

Did you know that the stairs on each face are inclined at 45° to the horizontal? During this lesson you will see how this bit of information and knowledge of the tangent ratio can help you find the height of this pyramid.

Get Started

My Notes

When you use the tangent ratio to solve problems, you will often be setting up equal ratios and solving for an unknown. In Lesson A, the unknown variable was in the numerator. However, sometimes the unknown variable will end up in the denominator.

In this activity you will review the algebra you will need to solve equations where the unknown or variable appears in the denominator of one of those ratios.

Example 1

Solve for x.

$$\frac{2}{5}=\frac{4}{x}$$

Solution

$$\frac{2}{5}=\frac{4}{x}$$

Cross multiply.

$$2x=(5)(4)$$
$$2x=20$$
$$\frac{2x}{2}=\frac{20}{2}$$
$$x=10$$

You can check your answer by substituting back into the original equation,

Left Side	Right Side
$\frac{2}{5}$	$\frac{4}{x}$ $\frac{4}{10}$ $\frac{4 \div 2}{10 \div 2}$ $\frac{2}{5}$
LS = RS	

My Notes

Activity 1
Self-Check

1. Solve for x.

$$\frac{5}{11} = \frac{3}{x}$$

2. Solve for x.

$$\frac{6}{15} = \frac{3}{2x}$$

Turn to the solutions at the end of the section and mark your work.

Explore

My Notes

In Lesson A you built a clinometer. A clinometer can be used to measure the angle of elevation or depression between you and a distant object. The angle can then be used to determine the height of the object. In the next activity, you will use your clinometer to determine the height of an object of your choice (tree, building, etc.).

Activity 2
Try This

You will need your clinometer, a tape measure, and your calculator. If possible, work with a partner.

Step 1: Go outside and choose an object that you would like to measure. (The object should be something that has a height that would be difficult to determine with a tape measure. Examples include a power pole, a tall tree, or a building.) The object should be on a flat, level stretch of ground, because you will need to take measurements with your tape measure along the ground.

Describe your object here: ______________________________

Step 2: From the base of the object, measure out a convenient distance. For a tree that might be 10 m tall, try not to be farther than 29 m from the object. Use your judgment.

Record your distance from the object here: __________________

My Notes

Step 3: At the distance you selected, with the help of your partner, use your clinometer to determine the angle of elevation to the top of the object. Measure the angle several times and average your results to ensure accuracy.

Angle of Elevation 1: _______

Angle of Elevation 2: _______ Average Angle of Elevation: _______

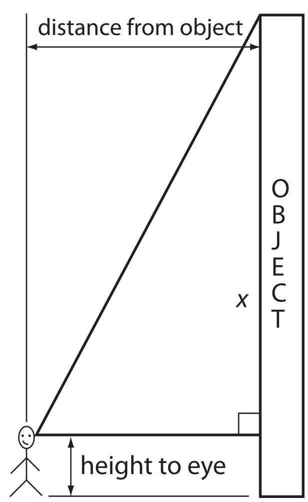
Angle of Elevation 3: _______

Step 4: Measure the height of your clinometer, by measuring from your feet to your eye.

Clinometer height: ____________________

Questions

1. Draw a diagram similar to the one below which describes your situation.



Draw

My Notes

2. Use the tangent ratio to calculate the height of the object. Show all steps.

3. Why did you need the height of your eye above the ground?

___

___

___

___

4. How accurate is your calculated answer? What could you have done to improve the accuracy?

___

___

___

___

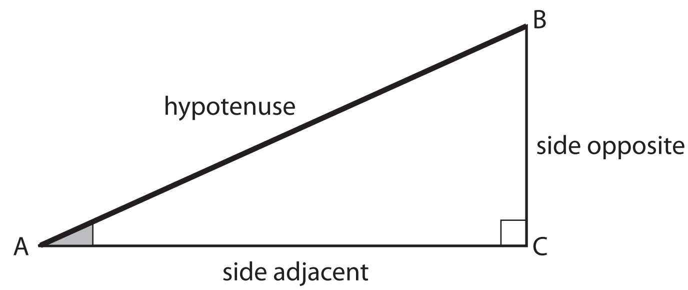
___

Turn to the solutions at the end of the section and mark your work.

My Notes

Bringing Ideas Together

In Explore, you used one measurement (the angle of elevation) to find another measurement (the height of the object). By doing this, you practised **indirect measurement**. You created a triangle similar to the one shown below and you used the tangent ratio to find the opposite side of the right triangle.



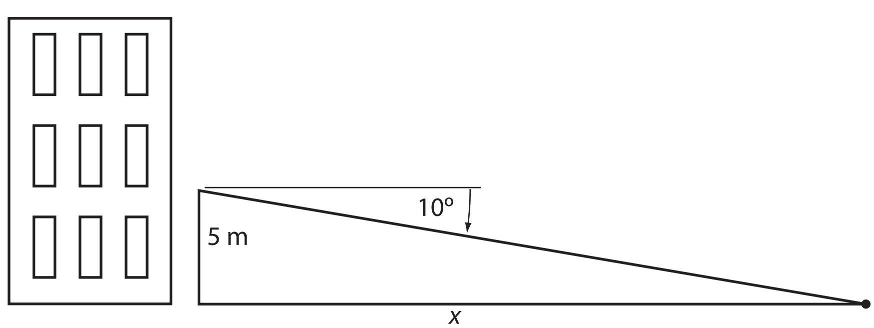
Now you'll learn how and when to use the tangent ratio to find the adjacent side of the reference angle. You'll also review some geometry skills and the skills you acquired in Lesson A.

Example 2

From a school window 5 m above the ground, a student measures the angle of depression to a fire hydrant located across the schoolyard. If the angle of depression is 10°, how far is the fire hydrant from the school? Round to the nearest metre.

Solution

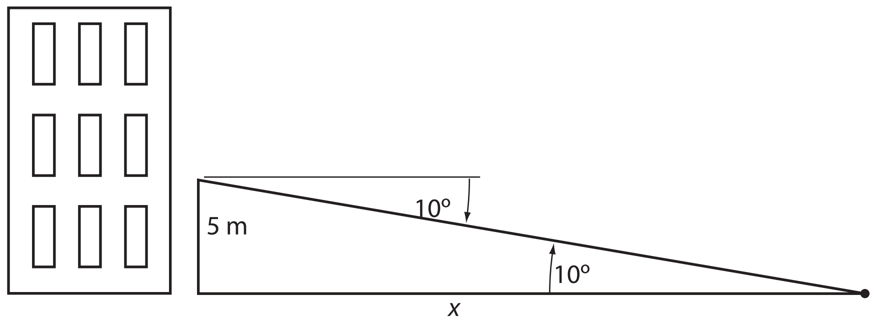
Draw a diagram.



My Notes

Method 1: In this method, "x" will be in the denominator.

The angle of depression is measured from a horizontal line. The ground is also assumed to be horizontal. This means both these lines are parallel. This means the angle of depression is equal to the angle of elevation from the hydrant.



In fact, the angle of elevation and the angle of depression are always equal because they are alternate interior angles.

Use the 10° angle as the reference angle. Notice, the 5 m side is the side opposite the reference angle.

$\tan 10° = \dfrac{\text{opposite side}}{\text{adjacent side}}$	Set up the formula.
$\tan 10° = \dfrac{5}{x}$	Plug in the given unknown and known length.
$x(\tan 10°) = 5$	Cross multiply.
$\dfrac{x(\tan 10°)}{\tan 10°} = \dfrac{5}{\tan 10°}$	Divide both sides by tan10°.
$\dfrac{x\cancel{(\tan 10°)}}{\cancel{\tan 10°}} = \dfrac{5}{\tan 10°}$	Cancel tan10° on both sides.
$x = \dfrac{5}{\tan 10°}$	

To find tan "x," press this sequence of keys. If you do not obtain the answer shown, consult your calculator manual or ask your teacher for help.

5	÷	Tan	1	0	)	=

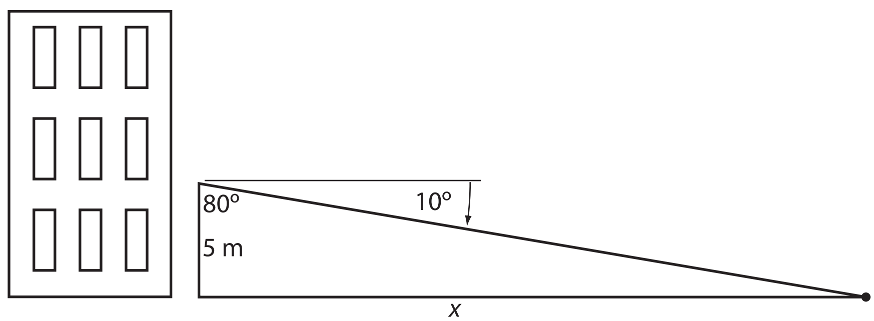
My Notes

Answer: $x = 28.3564091...$

The fire hydrant is approximately 28 m from the school as measured along the ground.

Method 2: In this method, "x" will be in the numerator.

You don't use the angle of 10°. It is the angle of depression, and in this case, is not within the right triangle. (The angle of depression is the angle an observer looks down from the horizontal to see an object below.) The angle adjacent to it in the right triangle is 80° since the two angles must add up to 90°. The two angles are *complementary*.



Use the 80° angle as the reference angle. Notice, now the 5-m side is the adjacent side.

$$\tan 80° = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\tan 80° = \frac{x}{5}$$

$$5(\tan 80°) = x$$

$$28.3564091... = x$$

The fire hydrant is approximately 28 m from the school as measured along the ground.

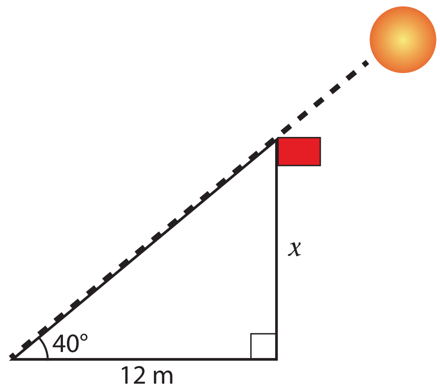
Example 3

A flagpole casts a shadow 12 m long when the angle of elevation of the sun is 40°. How tall is the flagpole? Round to the nearest tenth.

My Notes

Solution

Start by drawing a diagram. Note that the angle of elevation is drawn from the horizontal (where the shadow lies) up to where the sun would be. We know that this angle is 40º. We also know that the length of the shadow is 12 m. Since we are trying to find the height of the flagpole, we'll label that "*x*."



The unknown height, *x*, is opposite the 40º angle. The 12 m shadow length is adjacent to the 40º angle. Since we're dealing with opposite and adjacent sides, we'll use the tangent ration to solve for *x*.

$$\tan 40° = \frac{\text{opp}}{\text{adj}}$$

$$\tan 40° = \frac{x}{12}$$

$$(12)\tan 40° = \frac{x}{12}(12)$$

$$(12)\tan 40° = x$$

$$x = (12)\tan 40°$$

$$x = (12)(0.8390...)$$

Don't round yet! Leave the number in your calculator.

$$x = 10.0691...$$

$$x \approx 10.1 \text{ m}$$

The flagpole is approximately 10.1 m tall.

To see an animated version of this, go and look at *Flagpole Solution* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/ma10_flagpole.html*).

My Notes

Activity 3
Self-Check

Apply the tangent ratio to find unknown sides and angles. For each question, draw a diagram and show calculations.

1. A helicopter ambulance is dispatched from the airport to an accident scene 20 km north and 5 km east. The helicopter pilot needs to know what direction to head in to arrive at the scene most directly. How many degrees east of north must he fly?

2. An airplane flying at an altitude of 1000 m is headed towards the North Battleford airport. The angle of depression is 3°. How is the plane from the airport? (Give the answer to the nearest 100 m, measured along the ground.)

My Notes

My Notes

3. Maxine is building a set of stairs with a $7\frac{1}{2}$ inch riser and a 10 inch tread. To the nearest tenth of a degree, what is the staircase angle? Regulations specify that the angle should not be more than 42°. Is Maxine within specifications?

4. A ship's navigator sees a lighthouse on the top of a vertical cliff. The light on the top of the lighthouse is 50 m above the sea. The navigator measures the angle of elevation of the light to be 1.5°. If the navigator's charts say to stay at least 2 km from the foot of the cliff, is the ship safe?

My Notes

My Notes

5. A sign on a hill warns truckers to test their brakes because the slope is 10%. A slope of 10% means that the road rises 10 m for a horizontal distance of 100 m. To the nearest degree, what angle is the road surface inclined to the horizontal?

6. An airplane takes off from a runway and climbs at a 3° angle to the horizontal. If it maintains this rate of climb, what is the plane's altitude above the ground to the nearest 100 m when the aircraft is 30 km away, measured along the ground, from the point it took off from the runway?

My Notes

Turn to the solutions at the end of the section and mark your work.

My Notes

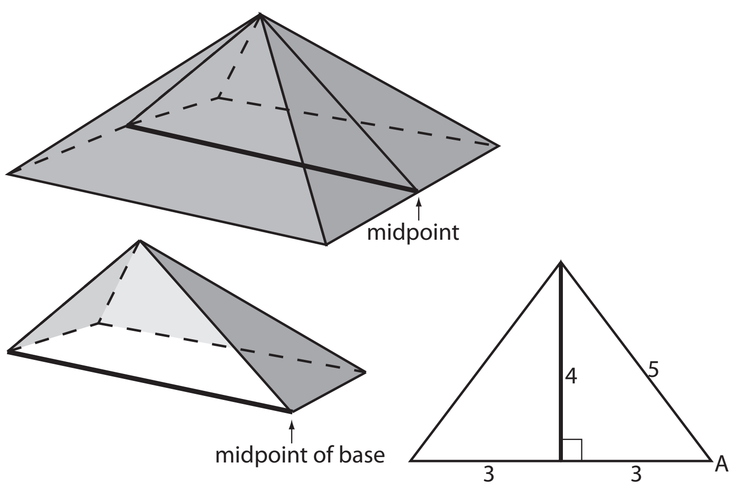
Activity 4
Mastering Concepts

Photo by ARCHITECTE® © 2010

The Pyramid of Khafre and the other pyramids at Giza, near Cairo, have stood for over 4500 years as monuments to the genius of the Ancient Egyptians. For centuries, these pyramids were the tallest human-made structures on the planet.

Did You Know?

If you were to slice the Khafre pyramid in half vertically from the apex at the top to the base, the triangular cross-section would be two, 3-4-5 right triangles.



My Notes

You will recall that the 3-4-5 right triangle was used by Egyptian surveyors. Do you remember the significance of these numbers? They make up a Phythagorean triple and indicate a right triangle.

Determine the angle, ∠A, to the nearest degree of the slant side of Khafre's Pyramid.

Turn to the solutions at the end of the section and mark your work.

My Notes

Lesson Summary

This photograph is of the head of the feathered serpent at the foot of the Temple of Kulkulkan, at the Mayan city of Chichen Itza, Mexico. This monument was designed so that the shadow cast by the stone ramp along the stairs becomes the body of the serpent moving up and down the stairs. This occurs during the spring and fall equinoxes.

Photo by David Davis © 2010

This shadow effect is a testament to the mathematical genius of the Mayan architects, stone masons, and artisans.

In this lesson you reviewed the definition of the tangent as the ratio of the opposite side to the adjacent side for a given acute angle in a right triangle. You used this ratio to find missing sides and missing angles in a variety of practical problems.

Lesson C
The Sine Ratio

To complete this lesson, you will need:

- your clinometer from Lesson A
- a tape measure
- a metric ruler
- a protractor
- a calculator

In this lesson, you will complete:

- 7 activities

Essential Questions

- What is the sine ratio?
- How is the sine ratio used to find unknown sides and angles in right triangles?

My Notes

Focus

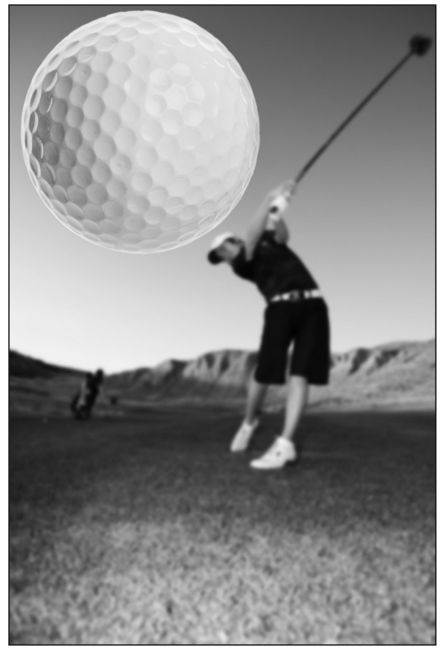


Photo by Stephen Mcsweeny © 2010

For most golfers, the game is a challenge. Not even the best golfers in the world always hit their shots on the fairway. Regardless of how often the player practices at the driving range, during the actual game the ball seems to have a mind of its own. Instead of going straight down the fairway, the ball will hook or slice wildly, travelling at a wicked angle to the desired path, and end up in the rough, bush, or nestled at the bottom of a sand trap.

Even a small angle off course can result in trouble. Suppose your average drive is 250 yards. If the ball ends up 5° off course, how many yards will it end up off the desired path? Surely it can't be too far! After all, 5° is a small angle, isn't it?

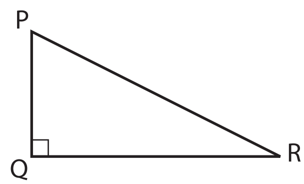
To answer this question, you will need to apply trigonometry and a new ratio called the sine function.

Get Started

My Notes

In this example you will review the names of the sides of a right triangle, relative to one of its acute angles. When an acute angle is specified in a right triangle, the acute angle is called the **reference angle**.

Example 1



1. Label the sides of ΔPQR using the letters *p*, *q*, and *r*.

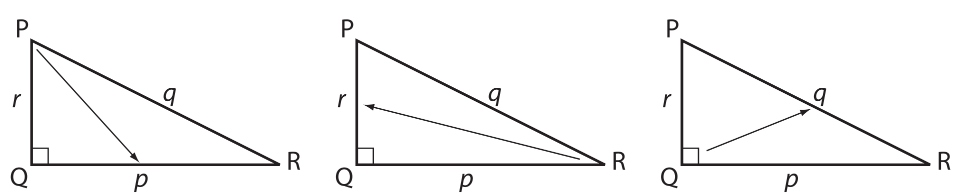
2. In relation to ∠R, which side is the side opposite? The side adjacent? The hypotenuse?

3. In relation to ∠P, which side is the side opposite? The side adjacent? The hypotenuse?

4. Regardless of which acute angle is the reference angle, which side always has the same name or label?

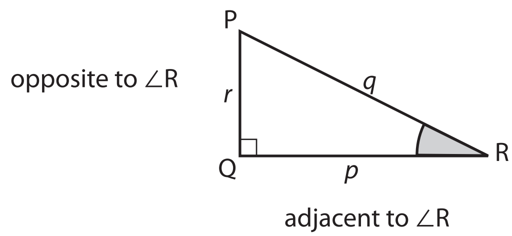
Solution

1. Side *p* is across from ∠P. Side *r* is across from ∠R. Side *q* is across from ∠Q.

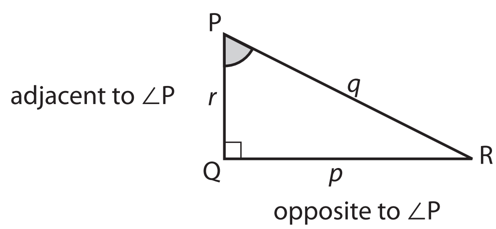


My Notes

2. In relation to ∠R, side r is the side opposite, side p is the side adjacent, and side q is the hypotenuse.



3. In relation to ∠P, side p is the side opposite, side r is the side adjacent, and side q is the hypotenuse.

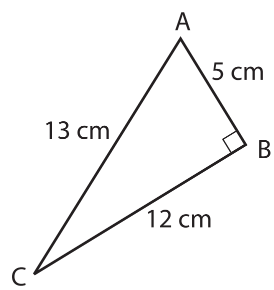


4. Side q, the longest side, is always the hypotenuse.

Activity 1
Self-Check

My Notes

Use the diagram below to answer the following questions.



1. What single letter could be used to name:

    a. side AC? ______________

    b. side BC? ______________

    c. side AB? ______________

2. What is the length of the side opposite ∠A? ________________

    The length of the side adjacent to ∠A is ____ .

3. What is the length of the side opposite ∠C? ________________

    The length of the side adjacent to ∠C is ____ .

4. What is the length of the hypotenuse? ________________

Turn to the solutions at the end of the section and mark your work.

My Notes

Explore

In the previous two lessons, you explored the tangent ratio:

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

You discovered that this ratio, is the same value for right triangles which are similar. If you know the angle, you can determine the ratio. And, if you know the ratio, you can determine the angle.

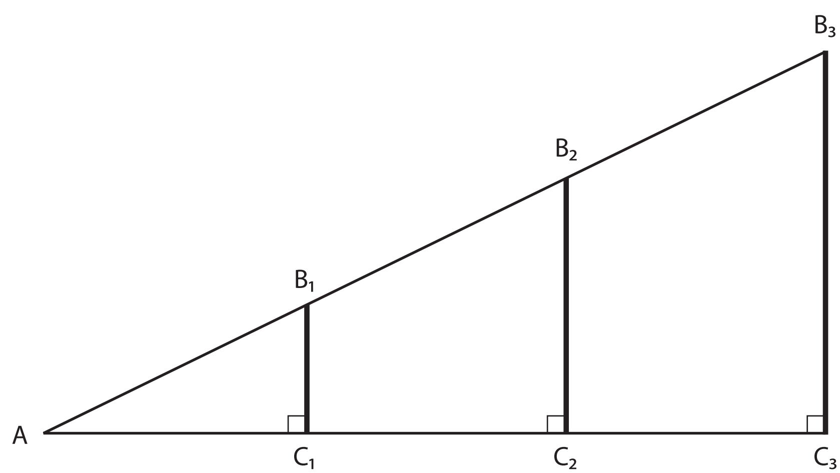
Are there other trigonometric ratios?

Activity 2
Try This

In this activity you will explore another trigonometric ratio involving a different pair of sides within a right triangle. You will compare this ratio with ratios involving the same pair of sides from other right triangles. For this comparison you will use your knowledge of similar right triangles.

You will need your metric ruler, protractor or square, and calculator.

Consider the right triangles in the following diagram.



My Notes

Step 1: On a full piece of paper, draw a diagram similar to the previous one. Enlarge the diagram so that it takes up the entire page. Label the diagram using the same lettering as shown.

Step 2: For right triangle AB_1C_1, measure to the nearest millimetre and record the length of sides B_1C_1 and AB_1 in the table below.

For right triangle AB_2C_2, measure to the nearest millimetre and record the length of sides B_2C_2 and AB_2 in the table below.

For right triangle AB_3C_3, measure to the nearest millimetre and record the length of sides B_3C_3 and AB_3 in the table below.

Step 3: Complete the last column of the table using your calculator.

Right triangle	Side opposite ∠A (nearest mm)	Hypotenuse (nearest mm)	$\frac{\text{opposite side}}{\text{hypotenuse}}$ (to 2 decimal places)
ΔAB_1C_1	$B_1C_1 =$	$AB_1 =$	$\frac{B_1C_1}{AB_1} =$
ΔAB_2C_2	$B_2C_2 =$	$AB_2 =$	$\frac{B_2C_2}{AB_2} =$
ΔAB_3C_3	$B_3C_3 =$	$AB_3 =$	$\frac{B_3C_3}{AB_3} =$

Questions

1. What pattern do you see in the table?

My Notes

2. Are these three triangles ΔAB_1C_1, ΔAB_2C_2, and ΔAB_3C_3 similar? Explain.

____

____

____

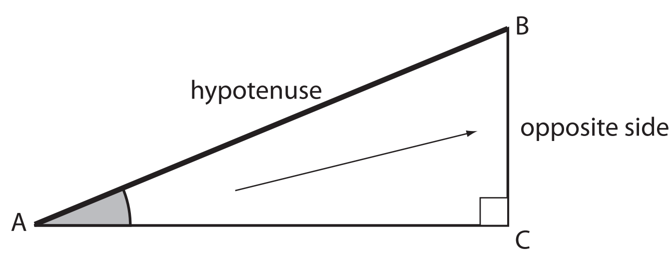
____

Turn to the solutions at the end of the section and mark your work.

Bringing Ideas Together

In the Explore, you examined the ratio in a right triangle of the side opposite to the hypotenuse of a given acute angle. This ratio is another example of a trigonometric ratio. You saw that for similar right triangles and a particular acute angle, the ratio $\frac{\text{opposite side}}{\text{hypotenuse}}$ was the same value regardless of the size of the right triangles.

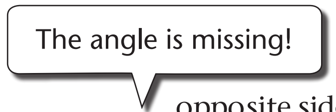
This ratio is called the **sine ratio**.



The sine ratio always depends on the acute angle selected. In this case, the acute angle that was selected was ∠A—not ∠B. The acute angle has to be identified in order to write a sine ratio.

So, sine $\angle A = \frac{\text{opposite side}}{\text{hypotenuse}}$

My Notes



It is incorrect to write: sine $= \dfrac{\text{opposite side}}{\text{hypotenuse}}$

It is often written more simply as:

$$\sin A = \frac{\text{opp}}{\text{hyp}} \text{ where } A = \text{measure of } \angle A$$

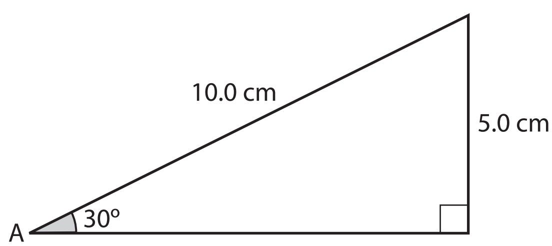
Example 2

Draw a right triangle with an acute angle of 30°. Determine sin 30° to two decimal places.

Solution

It doesn't matter how large you draw the triangle because all right triangles with a 30° degree angle will be similar triangles. However, a hypotenuse of 10 cm will make the calculation easier.

Once you draw the triangle, shade in the angle of 30°. Then label the opposite side and hypotenuse and measure each to the to the nearest millimetre.



$$\text{sine } \angle A = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 30° = \frac{5.0 \text{ cm}}{10.0 \text{ cm}}$$

$$\sin 30° = 0.50$$

My Notes

A ratio of 0.5 means that in a right triangle with a 30° angle, the opposite side will be one-half the length of the hypotenuse. For instance, if the opposite side were 30 inches, the hypotenuse would be 60 inches long, and $\frac{30 \text{ in}}{60 \text{ in}} = 0.50$.

Example 3

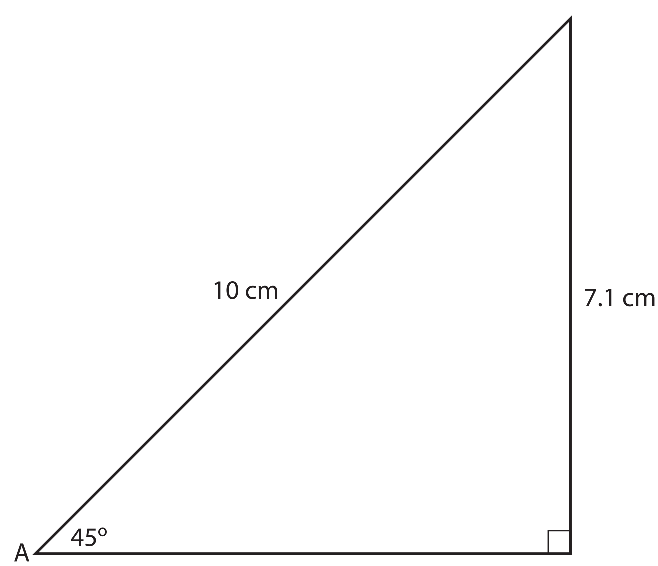
Measure the sides of a right triangle with an acute angle of 45°. Use these measurements to determine sin 45° to two decimal places.

Solution

Using your protractor, draw a right triangle with an acute angle of 45°. (The triangle can be big or small since, similar triangles will all have the same tangent ratio of 45°.) You may wish to draw a right triangle with a hypotenuse of 10 cm to simplify the calculation.

Shade the angle that is 45°. Label the opposite side and the hypotenuse and then measure these lengths, accurate to the nearest millimetre.

If you chose to draw the hypotenuse a length of 10 cm, your triangle would look like this:



My Notes

Put your measurements into the equation:

$$\text{sine } \angle A = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 45° = \frac{7.1 \text{ cm}}{10.0 \text{ cm}}$$

$$\sin 45° = 0.71$$

The ratio should come out close to the following value 0.707 106 781 186 55... to be accurate.

(Note: the answer is only approximate since measurements were only correct to the nearest millimetre.)

This ratio means the opposite side is shorter than the hypotenuse and is 0.71 times as long. So, if the hypotenuse were 20 cm long, the opposite side would be about 20 × 0.71 cm = 14.2 cm long.

My Notes

Activity 3
Self-Check

Use the method of the previous two examples to complete the following table. Use 10 cm for the hypotenuse.

Complete the following table.

Angle	Hypotenuse	Opposite Side	Sine of an Angle
10°	10 cm		sin 10° =
20°	10 cm		sin 20° =
30°	10 cm	5.0 cm	sin 30° = 0.50
40°	10 cm		sin 40° =
45°	10 cm		sin 45° = 0.71
50°	10 cm		sin 50° =
60°	10 cm		sin 60° =
70°	10 cm		sin 70° =
80°	10 cm		sin 80° =

Turn to the solutions at the end of the section and mark your work.

Calculating the Sine Ratio

Look at the table of sine ratios you prepared in Activity 3.

What did you notice about the value of the sine ratio?

____

What happens to the sine ratio as you move from an angle of 0° to 80°?

____

What you should have noticed is that the sine ratio is always between 0 and 1. And that as the angles increase from 0° to 80°, the sine ratio increases.

You should remember two benchmarks: sin 30° = 0.50 and sin 45° = 0.71. These benchmarks, or referents, will help you assess whether an answer is reasonable or not.

Example 4

Find each sine ratio correct to four decimal places using your calculator.

1. sin 45°

2. sin 30°

Solution

There are several different ways angle may be expressed. Make sure your calculator is set to degree mode.

1. To find sin 45°, press this sequence of keys. If you do not obtain the answer shown, consult your calculator manual or ask your teacher for help.

   [Sin] [4] [5] [)] [=]

My Notes

My Notes

In your calculator display window, you may see the solution as:

sin(45)
0.7071067812...

So, sin 45° ≈ 0.7071

2. To find sin 30°, press this sequence of keys.

sin	3	0	)	=

In your calculator display window, you may see the solution as:

sin(30)
0.5

So, sin 30° = 0.5

My Notes

Activity 4
Self-Check

Use your calculator to complete the following table. Round your answers to four decimal places.

Angle	Sine Ratio
10°	
20°	
30°	0.5000
40°	
45°	
50°	
60°	
70°	
80°	

Turn to the solutions at the end of the section and mark your work.

My Notes

Solving Problems Using the Sine Ratio

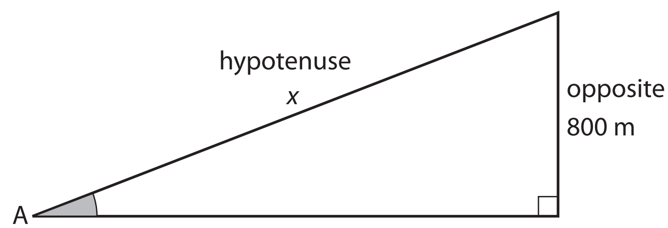
In the next example, you will apply your calculator skills to help you solve a problem involving the sine ratio.

Example 5

Suppose a ski run has a slope of 38°. After completing the run a skier has dropped in elevation by 800 m. How long was the ski run? Round your answer to the nearest 100 m.

Solution

Draw a diagram. Start by drawing a right triangle with an angle of 38°. On the triangle, label the side that would be the ski run length, "x", and label the elevation drop as 800 m.



Since the problem involves the opposite side and the hypotenuse, the sine ratio will be used to set up an equation.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 38° = \frac{800}{x}$$

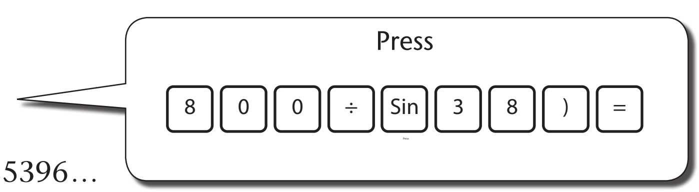
$$x \sin 38° = 800$$

$$\frac{x \sin 38°}{\sin 38°} = \frac{800}{\sin 38°}$$

$$x = \frac{800}{\sin 38°}$$

$$x = 1299.415396\ldots$$

$$x \approx 1300$$

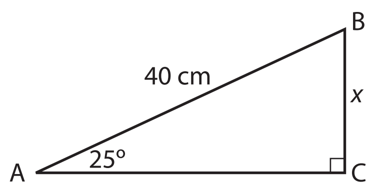


The length of the ski run would be approximately 1300 m in length or 1.3 km.

Activity 5
Self-Check

My Notes

Solve for *x*. Round to two decimal places.



Turn to the solutions at the end of the section and mark your work.

My Notes

Finding Angles Using the Sine Ratio

So far, you have learned to find the length of an unknown side in a right triangle, so long as you are given an acute angle and another length. But how can you find an unknown angle, using the sine ratio?

For instance, suppose you are told that the sine of a given acute angle in a right triangle is 0.6. What is the angle?

Example 6

If sin A = 0.6, determine ∠A to the nearest degree.

Solution

Method 1: Draw a triangle and measure the angle with a protractor.

First we need to figure out what to draw.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin A = 0.6$$

This information is given in the question.

Therefore:

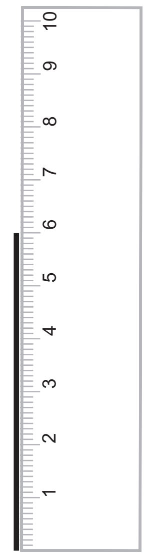
$$0.6 = \frac{\text{opp}}{\text{hyp}}$$

Now, if we let the hypotenuse be 10 cm long, how long would the opposite side be?

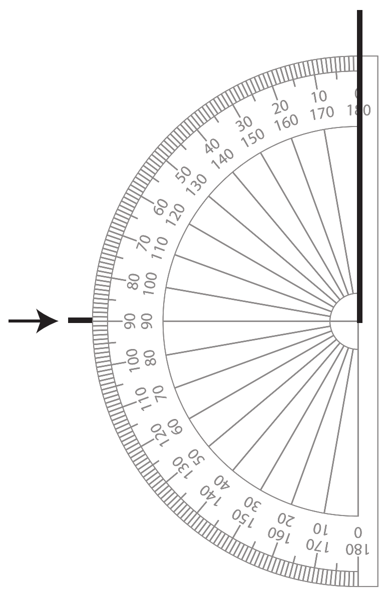
$$\begin{aligned} 0.6 &= \frac{\text{opp}}{\text{hyp}} \\ 0.6 &= \frac{\text{opp}}{10} \\ (10)0.6 &= \frac{\text{opp}}{10}(10) \\ 6 &= \text{opp} \end{aligned}$$

The opposite side is 6 cm long.

Now that we know the dimensions of the legs of the triangle, we can draw it. Start with the opposite side. Use a ruler to draw a 6 cm line.



Next, line up your protractor to draw a 90º angle at the end of the opposite side.

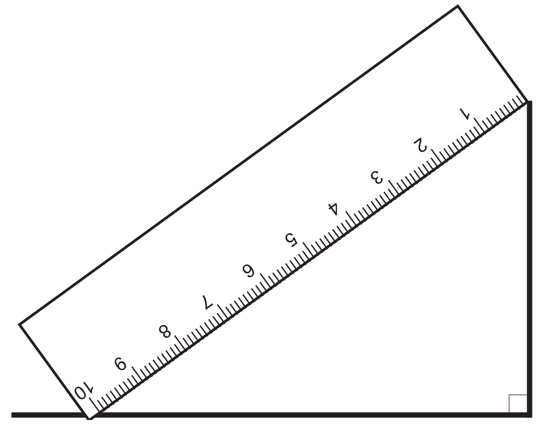


Now draw the adjacent side. We don't yet know its length, so make sure you draw a long enough line to meet with the hypotenuse.

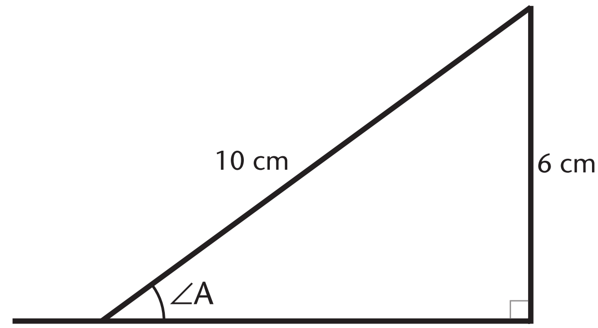
My Notes

My Notes

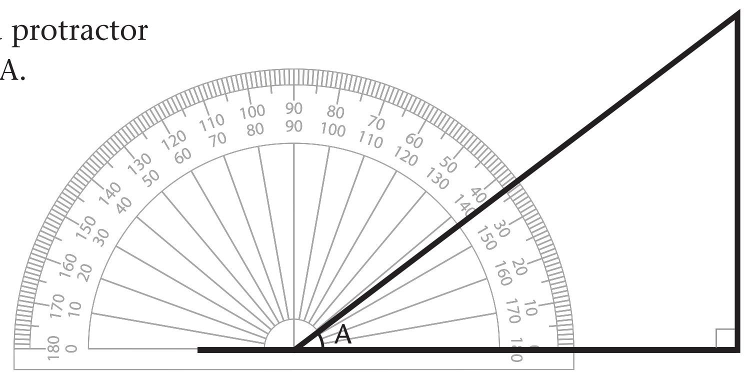
Using a ruler, draw the hypoteneuse: it is 10 cm long.



You now have a right triangle with ∠A.



You can use a protractor to measure ∠A.



∠A ≈ 37°

Now we'll try another method to solve the problem and to check the accuracy of our drawing.

Method 2: Using a calculator to determine the angle.

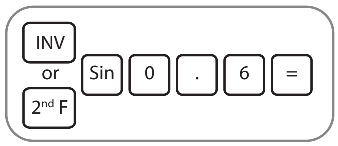
To find ∠A, we need to work backwards from the ratio to the angle.

My Notes

$$\sin A = 0.6$$
$$\sin^{-1}(\sin A) = \sin^{-1}(0.6)$$
$$\angle A = \sin^{-1}(0.6)$$

We isolate a by taking the inverse sin of both sides.

Now, we'll use a calculator to solve for $\angle A$. Make sure your calculator is in "degree" mode. The order you press the buttons will depend on the calculator you have. Try this out on your calculator and make sure you get the same answer. Check with your teacher or have a look at your instruction manual if you're not sure.



The answer is:

$\angle A = 36.869897...$
$\angle A \approx 37^\circ$

This is the same answer we got by solving with Method 1.

Activity 6
Self-Check

Practice finding angles given the sine ratio.

1. Find $\angle A$ given sin A = 0.8.

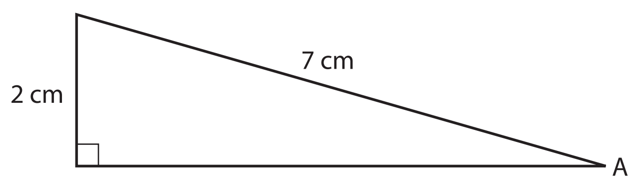
   Use both methods as shown in Example 6 to find the angle.

My Notes

2.  Use your calculator to find each angle from its sine ratio. Round your answers to the nearest tenth.

sin A	∠A
0.1257	
0.7826	
0.9000	
$\frac{2}{3}$	
$\frac{3}{4}$	

3. Find ∠A to the nearest tenth of a degree.

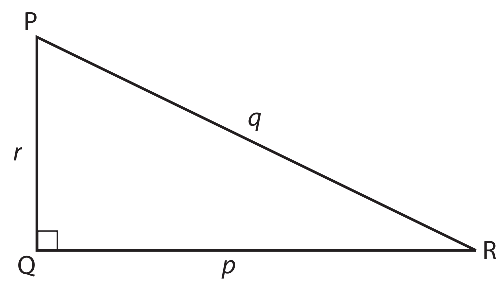


Turn to the solutions at the end of the section and mark your work.

My Notes

Activity 7
Mastering Concepts

Use the diagram below to answer the questions that follow.



1. For ΔPQR write the ratios for sin P and sin R.

My Notes

2. How are ∠P and ∠R related?

____

____

3. How are *p* and *r* related?

____

____

Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes

Photo by Maxim Petrichuk © 2010

In the introduction to this lesson, you were asked to think about the mathematics behind a drive in golf. The distance the drive ends up from its intended path is a function of the sine ratio.

Did you know that the height of the incline track designed for cycling also depends on the sine of the incline's angle? The angle is designed to maximize safety while allowing competitors to maintain speed.

Regardless of the sport you enjoy, trigonometry plays a major role in the design of the equipment, courts, and/or terrain.

In this lesson you explored the definition of the sine function as the ratio of the opposite side to the hypotenuse for a given acute angle in a right triangle. You used this ratio to find missing sides and missing angles.

Lesson D
Using Sines to Solve Problems

To complete this lesson, you will need:

- your clinometer from Lesson A
- a measuring tape
- a calculator

In this lesson, you will complete:

- 4 activities

Essential Questions

- How is the sine ratio used to solve a variety of practical problems?

My Notes

Focus

Photo by psamtik © 2010

Have you travelled over a bridge today? The engineers who design bridges must account for the forces acting on the bridge at different angles. People and cars moving across the bridge make up one such force. There is also the force of gravity and forces created by changing weather conditions. Engineers use trigonometry to make sure that the structure of the bridge will be able to withstand the forces that act upon it.

In this lesson, you'll use the sine ratio to solve a variety of problems.

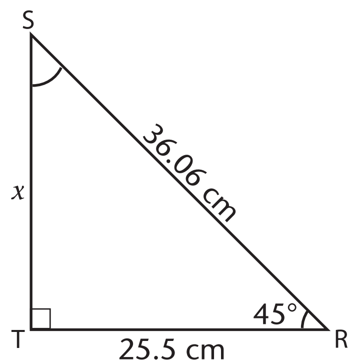
Get Started

In this activity you will review how the sine ratio is used to find either the opposite side or the hypotenuse, given the other side and an acute angle in a right triangle. Also, you will review how to find the acute angle if both the opposite side and hypotenuse of a right triangle are given.

Activity 1
Try This

My Notes

1. Find the missing sidelength by filling in the blanks in the given calculations.



$$\sin R = \frac{\square}{\text{hypotenuse}}$$

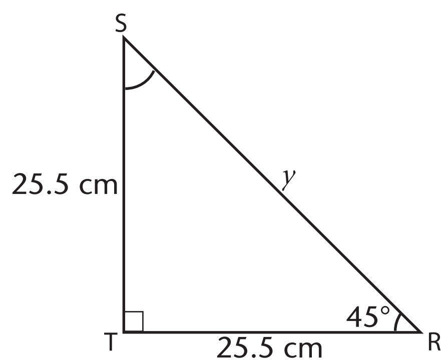
$$\sin 45° = \frac{x}{\square}$$

$$x = (\square)(\square)$$

$$x = 0.707(\square)$$

$$x = \square \text{ cm}$$

2. Find the missing sidelength by filling in the blanks in the given calculations.



$$\sin R = \frac{\text{opposite}}{\square}$$

$$\text{hypotenuse} = \frac{\square}{\square}$$

$$y = \frac{\square}{\square}$$

$$y = \frac{\square}{0.707}$$

$$y = \square \text{ cm}$$

3. Look at the steps for finding the two lengths in Questions 1 and 2. How are the calculations different?

____

____

____

____

____

____

My Notes

4. You may see the calculations for finding an angle are displayed as follows:

$$\sin R = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin R = \frac{12}{17.4}$$
$$\angle R = 43.6°$$

Throughout this section, we have used the inverse of sine to find angles. Add in the step that is missing that shows when to apply the inverse of sine to find the angle.

Turn to the solutions at the end of the section and mark your work.

Explore

My Notes

By now you should be fairly familiar with how your clinometer works. In the next activity you will determine the height of a hill in your neighbourhood. It could be along a straight road in the countryside or along a valley near where you live.

Activity 2
Try This

In this activity you will need your clinometer, a tape measure, and your calculator. Work with a partner, if possible.

Step 1: In your neighbourhood, select a hill along a straight stretch of road or overlooking a valley. You are going to determine the hill's height by using the sine ratio. The hill you select should be as uniform in slope as possible (not too bumpy).

Step 2: From the base of the hill, measure the hill's angle of elevation using your clinometer. You may wish to measure the angle several times and average your results to ensure accuracy.

Angle of Elevation 1: _______

Angle of Elevation 2: _______ Average Angle of Elevation: _______

Angle of Elevation 3: _______

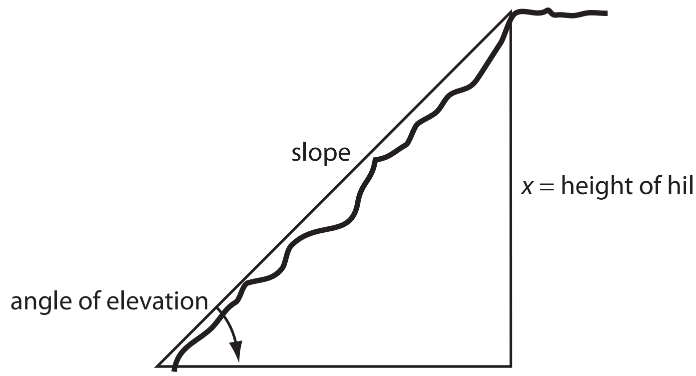
Step 3: You will also need to measure the length of the hill's slope. For a short slope you can use a measuring tape. For longer slopes you may have to pace off the distance, or if the hill is along a straight stretch of road, it may be possible to use the odometer of a car or truck. Then, complete the following questions.

Record the length of the hill's slope here: ________________
(include units)

My Notes

Questions

1. Using your measurements, draw and label a diagram which describes your situation similar to this one.



2. Use the sine ratio to calculate the height of the hill (x). Show all steps.

My Notes

3. How accurate is your calculated answer? What could you have done to improve the accuracy?

Turn to the solutions at the end of the section and mark your work.

My Notes

Bringing Ideas Together

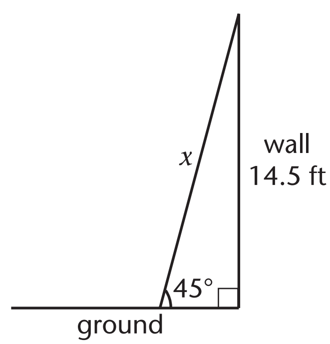
In Explore you worked with indirect measurement and the sine ratio. You used the sine ratio to find the height of the selected hill. You are now going to use the sine ratio to solve another type of word problem.

Example 1

A ladder leans against a vertical wall. The foot of the ladder is positioned for safety at an angle of 75° with the ground. The ladder reaches 14.5 feet up the wall. To the nearest foot, what is the length of the ladder?

Solution

Start by drawing a picture using the information given in the question.



Since we're dealing with the opposite side and the hypotenuse, we'll use the sine function.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 75^{\circ} = \frac{14.5}{x}$$

$$(x)\sin 75^{\circ} = \frac{14.5}{x}(x)$$

$$(x)\sin 75^{\circ} = 14.5$$

$$\frac{(x)\sin 75^{\circ}}{\sin 75^{\circ}} = \frac{14.5}{\sin 75^{\circ}}$$

$$x = \frac{14.5}{\sin 75^{\circ}}$$

$$x = 15.01150461\ldots$$

$$x \approx 15\text{ft}$$

To the nearest foot, the ladder is 15 ft long.

My Notes

Example 2

A mountain trail is used by cyclists to train. The trail drops 235 m from the trail head to the bottom over a trail distance of 1.1 km. To the nearest degree, what is the average slope of this trail?

Photo by BORTEL Pavel © 2010

Solution

Since this problem mentions two measurements with different units, you must decide which units you would prefer to work in.

The two measurements are: 1.1 km and 235 m.

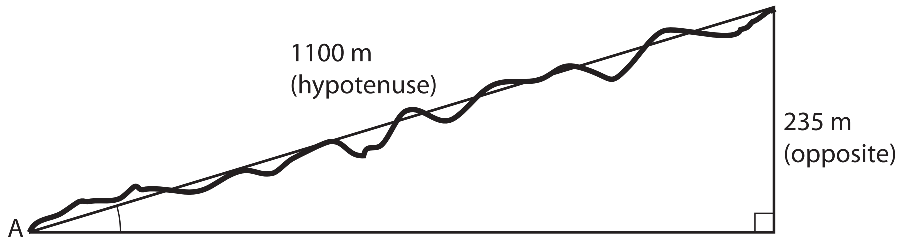
For this example, we will convert these so that they are both in metres.

$$1.1 \text{ km} = 1.1 \times 1000 \text{ metres} = 1100 \text{ metres}$$

If you wish to review metric length conversions, go and look at *SI Length Conversion* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/lengthconver/lengthConvert.htm*).

My Notes

Draw a diagram and label it with measurements that are in the same units.



Let ∠A be the required angle.

Substitute into the formula.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{235}{1100}$$

$$\angle A = \sin^{-1}\left(\frac{235}{1100}\right)$$

$$\angle A = 12.3355380509406\ldots$$

The average slope of the trail is approximately 12°.

Activity 3
Self-Check

My Notes

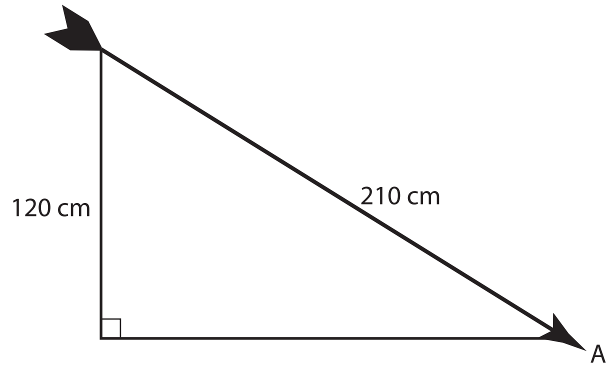
Draw a diagram and show calculations for each problem.

1. Kiteboarding is gaining popularity as an extreme sport. Kiteboarders wear a harness attached to the kite by a line that's often 30 m in length. It is possible for the kiteboarder to reach speeds of nearly 100 km/h. If the kite's angle of elevation in the photograph is 45°, and the line is 30 m long, how high above the water is the kite? Round to the nearest metre.

Photo by Dmitry Kosterev © 2010

My Notes

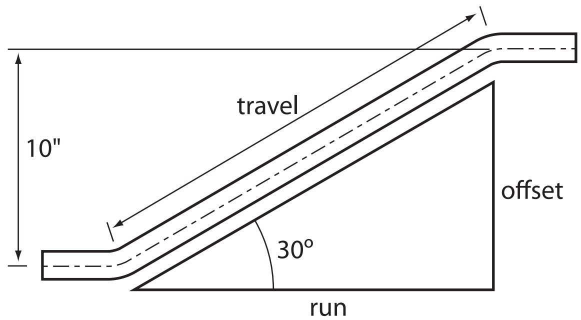
2. If a javelin thrower's toss is to count, the javelin must strike the ground with its point. The following diagram shows a women's javelin embedded point first in the ground with 210 cm of its length above ground.



The end of the javelin is 120 cm above the ground. To the nearest degree, what angle does the javelin make with the ground?

My Notes

3. Electricians and plumbers often must offset pipes to go through openings or to go above or below obstacles.



These tradespeople use the sine ratio to calculate the travel. What is the travel in this diagram, if the offset is 10 inches and the offset angle is 30°? Round to the nearest tenth of an inch.

My Notes

4. A guy wire will be used to support a vertical telephone pole. What must the length of the guy wire be, if one end is to be attached to the pole at a point 10 feet above the ground, and the other end, which is fastened to a stake, is to make a 45° angle with the ground? Express your answer to the nearest inch.

5. A ramp10 ft long is used to load items from the ground to a truck flatbed 3 feet above the ground. To the nearest degree, what angle does the ramp make with the ground?

My Notes

Turn to the solutions at the end of the section and mark your work.

My Notes

Activity 4
Mastering Concepts

Photo by Chas © 2010

From time to time, when grain storage is limited, farmers will store grain in conical piles in their fields. Depending on how long the grain remains in these piles, the grain may be left uncovered or will be covered with a tarp. Wheat will form a conical pile with an angle of 25° between the slant side and the horizontal (ground). This angle is called *the angle of repose*. The angle of repose varies from one kind of grain to another. Corn, for example, has a 22° angle of repose.

Suppose a conical pile of wheat is 3 m in height.

1. What is the length of the slant side of the conical pile? Round your answer to one decimal place.

My Notes

2. What is the radius of the pile? Round your answer to one decimal place.

3. What is the surface area of a tarp needed to cover the exposed wheat? Round your answer to the nearest square metre. Hints: Use SA = πrs (The cone-shaped tarp cone, geometrically speaking, has no bottom face.)

Turn to the solutions at the end of the section and mark your work.

My Notes

Lesson Summary

Photo by Triff © 2010

David Thompson (1770–1857) was a man who had many jobs associated with the fur trade—explorer, surveyor, and map maker. Thompson showed he was good at mathematics from an early age. He studied trigonometry, algebra, geography, and astronomy. At the age of 14, David Thompson joined the Hudson's Bay Company in Canada after leaving his home in England.

Over his lifetime, David Thompson mapped almost four million square kilometres of land that was later to become part of Canada and the United States. He accomplished this great feat by using trigonometry. He is considered one of the best surveyors who ever lived.

A surveyor is someone who measures topographical features of the Earth, determines land boundaries, and helps to create maps. Modern-day surveyors use Global Positioning System (GPS) technology to help them in their work. This technology is based on trigonometric principles.

In this lesson you reviewed the definition of the sine function as the ratio of the opposite side to the hypotenuse for a given acute angle in a right triangle. You used this ratio to find missing sides and missing angles in a variety of practical problems.

Trigonometry I —Appendix

TABLE OF CONVERSIONS

1 inch	≈	2.54 centimetres
1 foot	≈	30.5 centimetres
1 foot	≈	0.305 metres
1 foot	=	12 inches
1 yard	=	3 feet
1 yard	≈	0.915 metres
1 mile	=	1760 yards
1 mile	≈	1.6 kilometres
1 kilogram	≈	2.2 pounds
1 litre	≈	1.06 US quarts
1 litre	≈	0.26 US gallons
1 gallon	≈	4 quarts
1 British gallon	≈	$\frac{6}{5}$ US gallon

FORMULAE

Temperature

$$C = \frac{5}{9}(F - 32)$$

Trigonometry

(Put your calculator in Degree Mode)

- Right triangles

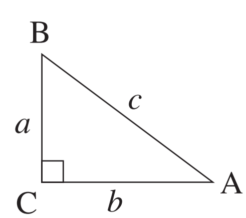
  Pythagorean Theorem

  $$a^2 + b^2 = c^2$$

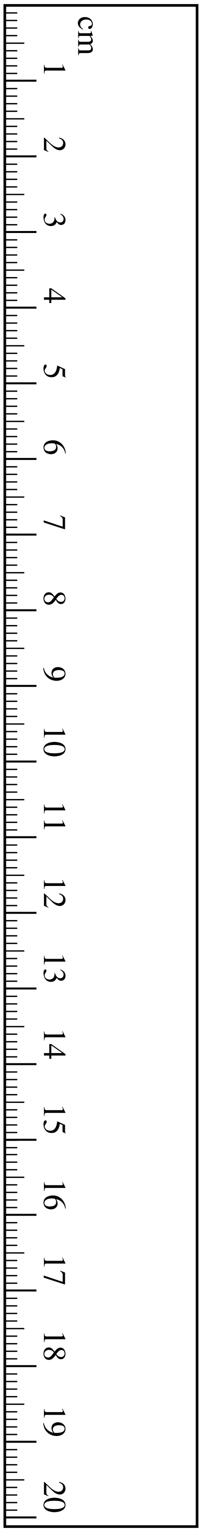
  $$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

  $$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

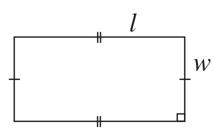
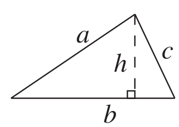
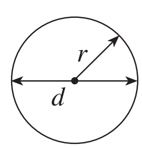
  $$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$



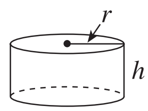
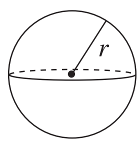
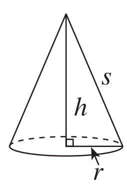
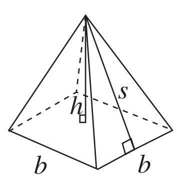
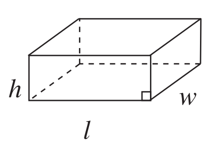
GEOMETRIC FORMULAE

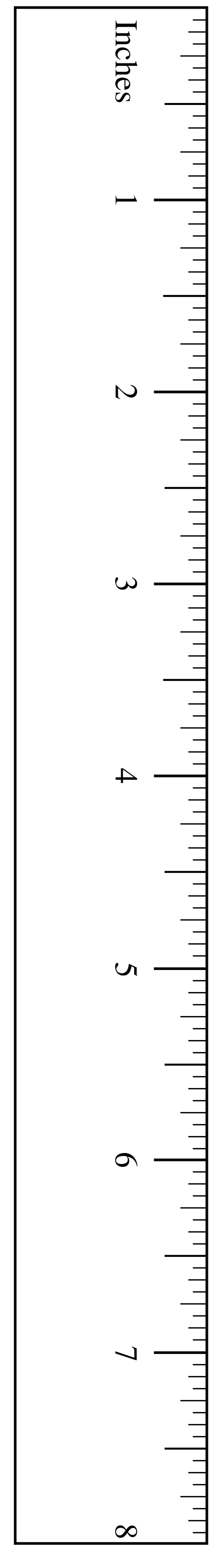


Key Legend	
l = length	P = perimeter
w = width	C = circumference
b = base	A = area
h = height	SA = surface area
s = slant height	V = volume
r = radius	
d = diameter	

Geometric Figure	Perimeter	Area
Rectangle 	$P = 2l + 2w$ **or** $P = 2(l + w)$	$A = lw$
Triangle 	$P = a + b + c$	$A = \frac{bh}{2}$
Circle 	$C = \pi d$ **or** $C = 2\pi r$	$A = \pi r^2$

Note: Use the value of **π** programmed in your calculator rather than the approximation of 3.14.

Geometric Figure	Surface Area
Cylinder 	$A_{top} = \pi r^2$ $A_{base} = \pi r^2$ $A_{side} = 2\pi rh$ $SA = 2\pi r^2 + 2\pi rh$
Sphere 	$SA = 4\pi r^2$ **or** $SA = \pi d^2$
Cone 	$A_{side} = \pi rs$ $A_{base} = \pi r^2$ $SA = \pi r^2 + \pi rs$
Square-Based Pyramid 	$A_{triangle} = \frac{1}{2}bs$ (for each triangle) $A_{base} = b^2$ $SA = 2bs + b^2$
Rectangular Prism 	$SA = wh + wh + lw + lw + lh + lh$ **or** $SA = 2(wh + lw + lh)$
General Right Prism	SA = the sum of the areas of all the faces
General Pyramid	SA = the sum of the areas of all the faces



Note: Use the value of **π** programmed in your calculator rather than the approximation of 3.14.

Canada Pension Plan Contributions | **Cotisations au Régime de pensions du Canada**

Weekly (52 pay periods a year) | Hebdomadaire (52 périodes de paie par année)

Pay Rémunération From - De	To - À	CPP RPC	Pay Rémunération From - De	To - À	CPP RPC	Pay Rémunération From - De	To - À	CPP RPC	Pay Rémunération From - De	To - À	CPP RPC
358.11	358.31	14.40	372.66	372.85	15.12	387.20	387.40	15.84	401.75	401.94	16.56
358.32	358.51	14.41	372.86	373.05	15.13	387.41	387.60	15.85	401.95	402.14	16.57
358.52	358.71	14.42	373.06	373.25	15.14	387.61	387.80	15.86	402.15	402.35	16.58
358.72	358.91	14.43	373.26	373.46	15.15	387.81	388.00	15.87	402.36	402.55	16.59
358.92	359.11	14.44	373.47	373.66	15.16	388.01	388.20	15.88	402.56	402.75	16.60
359.12	359.32	14.45	373.67	373.86	15.17	388.21	388.41	15.89	402.76	402.95	16.61
359.33	359.52	14.46	373.87	374.06	15.18	388.42	388.61	15.90	402.96	403.15	16.62
359.53	359.72	14.47	374.07	374.26	15.19	388.62	388.81	15.91	403.16	403.36	16.63
359.73	359.92	14.48	374.27	374.47	15.20	388.82	389.01	15.92	403.37	403.56	16.64
359.93	360.12	14.49	374.48	374.67	15.21	389.02	389.21	15.93	403.57	403.76	16.65
360.13	360.33	14.50	374.68	374.87	15.22	389.22	389.42	15.94	403.77	403.96	16.66
360.34	360.53	14.51	374.88	375.07	15.23	389.43	389.62	15.95	403.97	404.16	16.67
360.54	360.73	14.52	375.08	375.27	15.24	389.63	389.82	15.96	404.17	404.37	16.68
360.74	360.93	14.53	375.28	375.48	15.25	389.83	390.02	15.97	404.38	404.57	16.69
360.94	361.13	14.54	375.49	375.68	15.26	390.03	390.22	15.98	404.58	404.77	16.70
361.14	361.34	14.55	375.69	375.88	15.27	390.23	390.43	15.99	404.78	404.97	16.71
361.35	361.54	14.56	375.89	376.08	15.28	390.44	390.63	16.00	404.98	405.17	16.72
361.55	361.74	14.57	376.09	376.28	15.29	390.64	390.83	16.01	405.18	405.38	16.73
361.75	361.94	14.58	376.29	376.49	15.30	390.84	391.03	16.02	405.39	405.58	16.74
361.95	362.14	14.59	376.50	376.69	15.31	391.04	391.23	16.03	405.59	405.78	16.75
362.15	362.35	14.60	376.70	376.89	15.32	391.24	391.44	16.04	405.79	405.98	16.76
362.36	362.55	14.61	376.90	377.09	15.33	391.45	391.64	16.05	405.99	406.18	16.77
362.56	362.75	14.62	377.10	377.29	15.34	391.65	391.84	16.06	406.19	406.39	16.78
362.76	362.95	14.63	377.30	377.50	15.35	391.85	392.04	16.07	406.40	406.59	16.79
362.96	363.15	14.64	377.51	377.70	15.36	392.05	392.24	16.08	406.60	406.79	16.80
363.16	363.36	14.65	377.71	377.90	15.37	392.25	392.45	16.09	406.80	406.99	16.81
363.37	363.56	14.66	377.91	378.10	15.38	392.46	392.65	16.10	407.00	407.19	16.82
363.57	363.76	14.67	378.11	378.31	15.39	392.66	392.85	16.11	407.20	407.40	16.83
363.77	363.96	14.68	378.32	378.51	15.40	392.86	393.05	16.12	407.41	407.60	16.84
363.97	364.16	14.69	378.52	378.71	15.41	393.06	393.25	16.13	407.61	407.80	16.85
364.17	364.37	14.70	378.72	378.91	15.42	393.26	393.46	16.14	407.81	408.00	16.86
364.38	364.57	14.71	378.92	379.11	15.43	393.47	393.66	16.15	408.01	408.20	16.87
364.58	364.77	14.72	379.12	379.32	15.44	393.67	393.86	16.16	408.21	408.41	16.88
364.78	364.97	14.73	379.33	379.52	15.45	393.87	394.06	16.17	408.42	408.61	16.89
364.98	365.17	14.74	379.53	379.72	15.46	394.07	394.26	16.18	408.62	408.81	16.90
365.18	365.38	14.75	379.73	379.92	15.47	394.27	394.47	16.19	408.82	409.01	16.91
365.39	365.58	14.76	379.93	380.12	15.48	394.48	394.67	16.20	409.02	409.21	16.92
365.59	365.78	14.77	380.13	380.33	15.49	394.68	394.87	16.21	409.22	409.42	16.93
365.79	365.98	14.78	380.34	380.53	15.50	394.88	395.07	16.22	409.43	409.62	16.94
365.99	366.18	14.79	380.54	380.73	15.51	395.08	395.27	16.23	409.63	409.82	16.95
366.19	366.39	14.80	380.74	380.93	15.52	395.28	395.48	16.24	409.83	410.02	16.96
366.40	366.59	14.81	380.94	381.13	15.53	395.49	395.68	16.25	410.03	410.22	16.97
366.60	366.79	14.82	381.14	381.34	15.54	395.69	395.88	16.26	410.23	410.43	16.98
366.80	366.99	14.83	381.35	381.54	15.55	395.89	396.08	16.27	410.44	410.63	16.99
367.00	367.19	14.84	381.55	381.74	15.56	396.09	396.28	16.28	410.64	410.83	17.00
367.20	367.40	14.85	381.75	381.94	15.57	396.29	396.49	16.29	410.84	411.03	17.01
367.41	367.60	14.86	381.95	382.14	15.58	396.50	396.69	16.30	411.04	411.23	17.02
367.61	367.80	14.87	382.15	382.35	15.59	396.70	396.89	16.31	411.24	411.44	17.03
367.81	368.00	14.88	382.36	382.55	15.60	396.90	397.09	16.32	411.45	411.64	17.04
368.01	368.20	14.89	382.56	382.75	15.61	397.10	397.29	16.33	411.65	411.84	17.05
368.21	368.41	14.90	382.76	382.95	15.62	397.30	397.50	16.34	411.85	412.04	17.06
368.42	368.61	14.91	382.96	383.15	15.63	397.51	397.70	16.35	412.05	412.24	17.07
368.62	368.81	14.92	383.16	383.36	15.64	397.71	397.90	16.36	412.25	412.45	17.08
368.82	369.01	14.93	383.37	383.56	15.65	397.91	398.10	16.37	412.46	412.65	17.09
369.02	369.21	14.94	383.57	383.76	15.66	398.11	398.31	16.38	412.66	412.85	17.10
369.22	369.42	14.95	383.77	383.96	15.67	398.32	398.51	16.39	412.86	413.05	17.11
369.43	369.62	14.96	383.97	384.16	15.68	398.52	398.71	16.40	413.06	413.25	17.12
369.63	369.82	14.97	384.17	384.37	15.69	398.72	398.91	16.41	413.26	413.46	17.13
369.83	370.02	14.98	384.38	384.57	15.70	398.92	399.11	16.42	413.47	413.66	17.14
370.03	370.22	14.99	384.58	384.77	15.71	399.12	399.32	16.43	413.67	413.86	17.15
370.23	370.43	15.00	384.78	384.97	15.72	399.33	399.52	16.44	413.87	414.06	17.16
370.44	370.63	15.01	384.98	385.17	15.73	399.53	399.72	16.45	414.07	414.26	17.17
370.64	370.83	15.02	385.18	385.38	15.74	399.73	399.92	16.46	414.27	414.47	17.18
370.84	371.03	15.03	385.39	385.58	15.75	399.93	400.12	16.47	414.48	414.67	17.19
371.04	371.23	15.04	385.59	385.78	15.76	400.13	400.33	16.48	414.68	414.87	17.20
371.24	371.44	15.05	385.79	385.98	15.77	400.34	400.53	16.49	414.88	415.07	17.21
371.45	371.64	15.06	385.99	386.18	15.78	400.54	400.73	16.50	415.08	415.27	17.22
371.65	371.84	15.07	386.19	386.39	15.79	400.74	400.93	16.51	415.28	415.48	17.23
371.85	372.04	15.08	386.40	386.59	15.80	400.94	401.13	16.52	415.49	415.68	17.24
372.05	372.24	15.09	386.60	386.79	15.81	401.14	401.34	16.53	415.69	415.88	17.25
372.25	372.45	15.10	386.80	386.99	15.82	401.35	401.54	16.54	415.89	416.08	17.26
372.46	372.65	15.11	387.00	387.19	15.83	401.55	401.74	16.55	416.09	416.28	17.27

Employee's maximum CPP contribution for the year 2009 is $2,118.60 B-6 La cotisation maximale de l'employé au RPC pour l'année 2009 est de 2 118,60 $

Employment Insurance Premiums — Cotisations à l'assurance-emploi

Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE
333.24	333.81	5.77	374.86	375.43	6.49	416.48	417.05	7.21	458.10	458.67	7.93
333.82	334.39	5.78	375.44	376.01	6.50	417.06	417.63	7.22	458.68	459.24	7.94
334.40	334.97	5.79	376.02	376.58	6.51	417.64	418.20	7.23	459.25	459.82	7.95
334.98	335.54	5.80	376.59	377.16	6.52	418.21	418.78	7.24	459.83	460.40	7.96
335.55	336.12	5.81	377.17	377.74	6.53	418.79	419.36	7.25	460.41	460.98	7.97
336.13	336.70	5.82	377.75	378.32	6.54	419.37	419.94	7.26	460.99	461.56	7.98
336.71	337.28	5.83	378.33	378.90	6.55	419.95	420.52	7.27	461.57	462.13	7.99
337.29	337.86	5.84	378.91	379.47	6.56	420.53	421.09	7.28	462.14	462.71	8.00
337.87	338.43	5.85	379.48	380.05	6.57	421.10	421.67	7.29	462.72	463.29	8.01
338.44	339.01	5.86	380.06	380.63	6.58	421.68	422.25	7.30	463.30	463.87	8.02
339.02	339.59	5.87	380.64	381.21	6.59	422.26	422.83	7.31	463.88	464.45	8.03
339.60	340.17	5.88	381.22	381.79	6.60	422.84	423.41	7.32	464.46	465.02	8.04
340.18	340.75	5.89	381.80	382.36	6.61	423.42	423.98	7.33	465.03	465.60	8.05
340.76	341.32	5.90	382.37	382.94	6.62	423.99	424.56	7.34	465.61	466.18	8.06
341.33	341.90	5.91	382.95	383.52	6.63	424.57	425.14	7.35	466.19	466.76	8.07
341.91	342.48	5.92	383.53	384.10	6.64	425.15	425.72	7.36	466.77	467.34	8.08
342.49	343.06	5.93	384.11	384.68	6.65	425.73	426.30	7.37	467.35	467.91	8.09
343.07	343.64	5.94	384.69	385.26	6.66	426.31	426.87	7.38	467.92	468.49	8.10
343.65	344.21	5.95	385.27	385.83	6.67	426.88	427.45	7.39	468.50	469.07	8.11
344.22	344.79	5.96	385.84	386.41	6.68	427.46	428.03	7.40	469.08	469.65	8.12
344.80	345.37	5.97	386.42	386.99	6.69	428.04	428.61	7.41	469.66	470.23	8.13
345.38	345.95	5.98	387.00	387.57	6.70	428.62	429.19	7.42	470.24	470.80	8.14
345.96	346.53	5.99	387.58	388.15	6.71	429.20	429.76	7.43	470.81	471.38	8.15
346.54	347.10	6.00	388.16	388.72	6.72	429.77	430.34	7.44	471.39	471.96	8.16
347.11	347.68	6.01	388.73	389.30	6.73	430.35	430.92	7.45	471.97	472.54	8.17
347.69	348.26	6.02	389.31	389.88	6.74	430.93	431.50	7.46	472.55	473.12	8.18
348.27	348.84	6.03	389.89	390.46	6.75	431.51	432.08	7.47	473.13	473.69	8.19
348.85	349.42	6.04	390.47	391.04	6.76	432.09	432.65	7.48	473.70	474.27	8.20
349.43	349.99	6.05	391.05	391.61	6.77	432.66	433.23	7.49	474.28	474.85	8.21
350.00	350.57	6.06	391.62	392.19	6.78	433.24	433.81	7.50	474.86	475.43	8.22
350.58	351.15	6.07	392.20	392.77	6.79	433.82	434.39	7.51	475.44	476.01	8.23
351.16	351.73	6.08	392.78	393.35	6.80	434.40	434.97	7.52	476.02	476.58	8.24
351.74	352.31	6.09	393.36	393.93	6.81	434.98	435.54	7.53	476.59	477.16	8.25
352.32	352.89	6.10	393.94	394.50	6.82	435.55	436.12	7.54	477.17	477.74	8.26
352.90	353.46	6.11	394.51	395.08	6.83	436.13	436.70	7.55	477.75	478.32	8.27
353.47	354.04	6.12	395.09	395.66	6.84	436.71	437.28	7.56	478.33	478.90	8.28
354.05	354.62	6.13	395.67	396.24	6.85	437.29	437.86	7.57	478.91	479.47	8.29
354.63	355.20	6.14	396.25	396.82	6.86	437.87	438.43	7.58	479.48	480.05	8.30
355.21	355.78	6.15	396.83	397.39	6.87	438.44	439.01	7.59	480.06	480.63	8.31
355.79	356.35	6.16	397.40	397.97	6.88	439.02	439.59	7.60	480.64	481.21	8.32
356.36	356.93	6.17	397.98	398.55	6.89	439.60	440.17	7.61	481.22	481.79	8.33
356.94	357.51	6.18	398.56	399.13	6.90	440.18	440.75	7.62	481.80	482.36	8.34
357.52	358.09	6.19	399.14	399.71	6.91	440.76	441.32	7.63	482.37	482.94	8.35
358.10	358.67	6.20	399.72	400.28	6.92	441.33	441.90	7.64	482.95	483.52	8.36
358.68	359.24	6.21	400.29	400.86	6.93	441.91	442.48	7.65	483.53	484.10	8.37
359.25	359.82	6.22	400.87	401.44	6.94	442.49	443.06	7.66	484.11	484.68	8.38
359.83	360.40	6.23	401.45	402.02	6.95	443.07	443.64	7.67	484.69	485.26	8.39
360.41	360.98	6.24	402.03	402.60	6.96	443.65	444.21	7.68	485.27	485.83	8.40
360.99	361.56	6.25	402.61	403.17	6.97	444.22	444.79	7.69	485.84	486.41	8.41
361.57	362.13	6.26	403.18	403.75	6.98	444.80	445.37	7.70	486.42	486.99	8.42
362.14	362.71	6.27	403.76	404.33	6.99	445.38	445.95	7.71	487.00	487.57	8.43
362.72	363.29	6.28	404.34	404.91	7.00	445.96	446.53	7.72	487.58	488.15	8.44
363.30	363.87	6.29	404.92	405.49	7.01	446.54	447.10	7.73	488.16	488.72	8.45
363.88	364.45	6.30	405.50	406.06	7.02	447.11	447.68	7.74	488.73	489.30	8.46
364.46	365.02	6.31	406.07	406.64	7.03	447.69	448.26	7.75	489.31	489.88	8.47
365.03	365.60	6.32	406.65	407.22	7.04	448.27	448.84	7.76	489.89	490.46	8.48
365.61	366.18	6.33	407.23	407.80	7.05	448.85	449.42	7.77	490.47	491.04	8.49
366.19	366.76	6.34	407.81	408.38	7.06	449.43	449.99	7.78	491.05	491.61	8.50
366.77	367.34	6.35	408.39	408.95	7.07	450.00	450.57	7.79	491.62	492.19	8.51
367.35	367.91	6.36	408.96	409.53	7.08	450.58	451.15	7.80	492.20	492.77	8.52
367.92	368.49	6.37	409.54	410.11	7.09	451.16	451.73	7.81	492.78	493.35	8.53
368.50	369.07	6.38	410.12	410.69	7.10	451.74	452.31	7.82	493.36	493.93	8.54
369.08	369.65	6.39	410.70	411.27	7.11	452.32	452.89	7.83	493.94	494.50	8.55
369.66	370.23	6.40	411.28	411.84	7.12	452.90	453.46	7.84	494.51	495.08	8.56
370.24	370.80	6.41	411.85	412.42	7.13	453.47	454.04	7.85	495.09	495.66	8.57
370.81	371.38	6.42	412.43	413.00	7.14	454.05	454.62	7.86	495.67	496.24	8.58
371.39	371.96	6.43	413.01	413.58	7.15	454.63	455.20	7.87	496.25	496.82	8.59
371.97	372.54	6.44	413.59	414.16	7.16	455.21	455.78	7.88	496.83	497.39	8.60
372.55	373.12	6.45	414.17	414.73	7.17	455.79	456.35	7.89	497.40	497.97	8.61
373.13	373.69	6.46	414.74	415.31	7.18	456.36	456.93	7.90	497.98	498.55	8.62
373.70	374.27	6.47	415.32	415.89	7.19	456.94	457.51	7.91	498.56	499.13	8.63
374.28	374.85	6.48	415.90	416.47	7.20	457.52	458.09	7.92	499.14	499.71	8.64

Yearly maximum insurable earnings are $42,300
Yearly maximum employee premiums are $731.79
The premium rate for 2009 is 1.73 %

Le maximum annuel de la rémunération assurable est de 42 300 $
La cotisation maximale annuelle de l'employé est de 731,79 $
Le taux de cotisation pour 2009 est de 1,73 %

C-3

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the provincial table

Retenues d'impôt fédéral
En vigueur le 1er janvier 2009
Hebdomadaire (52 périodes de paie par année)
Cherchez aussi les retenues d'impôt dans la table provinciale

Pay Rémunération		Federal claim codes/Codes de demande fédéraux										
		0	1	2	3	4	5	6	7	8	9	10
From De	Less than Moins de	Deduct from each pay Retenez sur chaque paie										
335	339	44.65	15.55	12.70	7.00	1.30						
339	343	45.20	16.10	13.25	7.55	1.85						
343	347	45.80	16.65	13.80	8.10	2.45						
347	351	46.35	17.20	14.35	8.65	3.00						
351	355	46.90	17.75	14.90	9.25	3.55						
355	359	47.45	18.35	15.50	9.80	4.10						
359	363	48.00	18.90	16.05	10.35	4.65						
363	367	48.60	19.45	16.60	10.90	5.25						
367	371	49.15	20.00	17.15	11.45	5.80	.10					
371	375	49.70	20.55	17.70	12.05	6.35	.65					
375	379	50.25	21.15	18.30	12.60	6.90	1.20					
379	383	50.80	21.70	18.85	13.15	7.45	1.80					
383	387	51.40	22.25	19.40	13.70	8.00	2.35					
387	391	51.95	22.80	19.95	14.25	8.60	2.90					
391	395	52.50	23.35	20.50	14.85	9.15	3.45					
395	399	53.05	23.95	21.10	15.40	9.70	4.00					
399	403	53.60	24.50	21.65	15.95	10.25	4.60					
403	407	54.20	25.05	22.20	16.50	10.80	5.15					
407	411	54.75	25.60	22.75	17.05	11.40	5.70					
411	415	55.30	26.15	23.30	17.65	11.95	6.25	.55				
415	419	55.85	26.75	23.90	18.20	12.50	6.80	1.15				
419	423	56.40	27.30	24.45	18.75	13.05	7.40	1.70				
423	427	57.00	27.85	25.00	19.30	13.60	7.95	2.25				
427	431	57.55	28.40	25.55	19.85	14.20	8.50	2.80				
431	435	58.10	28.95	26.10	20.45	14.75	9.05	3.35				
435	439	58.65	29.50	26.70	21.00	15.30	9.60	3.95				
439	443	59.20	30.10	27.25	21.55	15.85	10.20	4.50				
443	447	59.80	30.65	27.80	22.10	16.40	10.75	5.05				
447	451	60.35	31.20	28.35	22.65	17.00	11.30	5.60				
451	455	60.90	31.75	28.90	23.25	17.55	11.85	6.15	.50			
455	459	61.45	32.30	29.50	23.80	18.10	12.40	6.75	1.05			
459	463	62.00	32.90	30.05	24.35	18.65	12.95	7.30	1.60			
463	467	62.60	33.45	30.60	24.90	19.20	13.55	7.85	2.15			
467	471	63.15	34.00	31.15	25.45	19.80	14.10	8.40	2.70			
471	475	63.70	34.55	31.70	26.05	20.35	14.65	8.95	3.30			
475	479	64.25	35.10	32.30	26.60	20.90	15.20	9.55	3.85			
479	483	64.80	35.70	32.85	27.15	21.45	15.75	10.10	4.40			
483	487	65.40	36.25	33.40	27.70	22.00	16.35	10.65	4.95			
487	491	65.95	36.80	33.95	28.25	22.60	16.90	11.20	5.50			
491	495	66.50	37.35	34.50	28.85	23.15	17.45	11.75	6.10	.40		
495	499	67.05	37.90	35.10	29.40	23.70	18.00	12.35	6.65	.95		
499	503	67.60	38.50	35.65	29.95	24.25	18.55	12.90	7.20	1.50		
503	507	68.20	39.05	36.20	30.50	24.80	19.15	13.45	7.75	2.05		
507	511	68.75	39.60	36.75	31.05	25.40	19.70	14.00	8.30	2.65		
511	515	69.30	40.15	37.30	31.65	25.95	20.25	14.55	8.90	3.20		
515	519	69.85	40.70	37.90	32.20	26.50	20.80	15.15	9.45	3.75		
519	523	70.40	41.30	38.45	32.75	27.05	21.35	15.70	10.00	4.30		
523	527	71.00	41.85	39.00	33.30	27.60	21.95	16.25	10.55	4.85		
527	531	71.55	42.40	39.55	33.85	28.20	22.50	16.80	11.10	5.45		
531	535	72.10	42.95	40.10	34.45	28.75	23.05	17.35	11.70	6.00	.30	
535	539	72.65	43.50	40.70	35.00	29.30	23.60	17.90	12.25	6.55	.85	
539	543	73.20	44.10	41.25	35.55	29.85	24.15	18.50	12.80	7.10	1.40	
543	547	73.80	44.65	41.80	36.10	30.40	24.75	19.05	13.35	7.65	2.00	
547	551	74.35	45.20	42.35	36.65	31.00	25.30	19.60	13.90	8.25	2.55	
551	555	74.90	45.75	42.90	37.25	31.55	25.85	20.15	14.50	8.80	3.10	

British Columbia provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the federal table

Retenues d'impôt provincial de la Colombie-Britannique
En vigueur le 1er janvier 2009
Hebdomadaire (52 périodes de paie par année)
Cherchez aussi les retenues d'impôt dans la table fédérale

Pay / Rémunération		Provincial claim codes/Codes de demande provinciaux										
		0	1	2	3	4	5	6	7	8	9	10
From / De	Less than / Moins de	Deduct from each pay / Retenez sur chaque paie										
	343	*	.00									
343	345	9.30	.20									
345	347	9.45	.35									
347	349	9.60	.50									
349	351	9.80	.65									
351	353	9.95	.80									
353	355	10.10	.95									
355	357	10.25	1.15	.10								
357	359	10.40	1.30	.25								
359	361	10.55	1.45	.40								
361	363	10.75	1.60	.60								
363	365	10.90	1.75	.75								
365	367	11.05	1.90	.90								
367	369	11.20	2.10	1.05								
369	371	11.35	2.25	1.20								
371	373	11.50	2.40	1.35								
373	375	11.70	2.55	1.55								
375	377	11.85	2.70	1.70								
377	379	12.00	2.90	1.85								
379	381	12.15	3.05	2.00								
381	383	12.30	3.20	2.15	.10							
383	385	12.45	3.35	2.30	.25							
385	387	12.65	3.50	2.50	.45							
387	389	12.80	3.65	2.65	.60							
389	391	12.95	3.85	2.80	.75							
391	393	13.10	4.00	2.95	.90							
393	395	13.25	4.15	3.10	1.05							
395	397	13.40	4.30	3.30	1.20							
397	399	13.60	4.45	3.45	1.40							
399	401	13.75	4.60	3.60	1.55							
401	403	13.90	4.80	3.75	1.70							
403	405	14.05	4.95	3.90	1.85							
405	407	14.20	5.10	4.05	2.00							
407	409	14.35	5.25	4.25	2.15	.10						
409	411	14.55	5.40	4.40	2.35	.30						
411	413	14.70	5.55	4.55	2.50	.45						
413	415	14.85	5.75	4.70	2.65	.60						
415	417	15.00	5.90	4.85	2.80	.75						
417	419	15.15	6.05	5.00	2.95	.90						
419	421	15.30	6.20	5.20	3.10	1.05						
421	423	15.50	6.35	5.35	3.30	1.25						
423	425	15.65	6.50	5.50	3.45	1.40						
425	427	15.80	6.70	5.65	3.60	1.55						
427	429	15.95	6.85	5.80	3.75	1.70						
429	431	16.10	7.00	5.95	3.90	1.85						
431	433	16.25	7.15	6.15	4.10	2.00						
433	435	16.45	7.30	6.30	4.25	2.20	.15					
435	437	16.60	7.45	6.45	4.40	2.35	.30					
437	439	16.75	7.65	6.60	4.55	2.50	.45					
439	441	16.90	7.80	6.75	4.70	2.65	.60					
441	443	17.05	7.95	6.90	4.85	2.80	.75					
443	445	17.20	8.10	7.10	5.05	2.95	.90					
445	447	17.40	8.25	7.25	5.20	3.15	1.10					
447	449	17.55	8.40	7.40	5.35	3.30	1.25					
449	451	17.70	8.60	7.55	5.50	3.45	1.40					

*You normally use claim code "0" only for non-resident employees. However, if you have non-resident employees who earn less than the minimum amount shown in the "Pay" column, you may not be able to use these tables. Instead, refer to the "Step-by-step calculation of tax deductions" in Section "A" of this publication.

*Le code de demande «0» est normalement utilisé seulement pour les non-résidents. Cependant, si la rémunération de votre employé non résidant est inférieure au montant minimum indiqué dans la colonne «Rémunération», vous ne pourrez peut-être pas utiliser ces tables. Reportez-vous alors au «Calcul des retenues d'impôt, étape par étape» dans la section «A» de cette publication.

Solutions

Lesson A: The Tangent Ratio

Lesson A: Activity 1: Try This

1. The greatest angle of elevation is 90°.

2. The greatest angle of depression is 90°.

Lesson A: Activity 2: Self-Check

1. **Method 1**: Reduce each fraction (ratio) to its simplest terms.

$$\frac{7}{14} = \frac{7}{14} \div \frac{7}{7} = \frac{1}{2} \qquad \frac{4}{8} = \frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$$

Since each ratio reduces to $\frac{1}{2}$, the ratios are equal.

Method 2: Compare the cross products (multiply the numerator of one fraction by the denominator in the other).

$$\frac{7}{14} = \frac{4}{8}$$

$7 \times 8 = 56$

$14 \times 4 = 56$

Because the cross products are equal, the ratios are equal.

2. **Method 1**: Reduce each fraction (ratio) to its simplest terms.

$$\frac{3}{9} = \frac{3}{9} \div \frac{3}{3} = \frac{1}{3} \qquad \frac{6}{15} = \frac{6}{15} \div \frac{3}{3} = \frac{2}{5}$$

When both ratios are reduced to their simplest terms, the result is two ratios that are clearly different. Since these resulting ratios are different, the original ratios are not equal to each other.

Method 2: Compare the cross products.

$$\frac{3}{9} = \frac{6}{15}$$

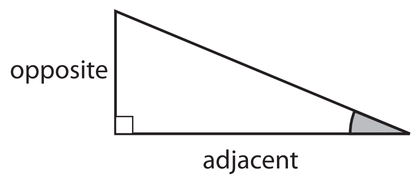
3 × 15 = 45

9 × 6 = 54

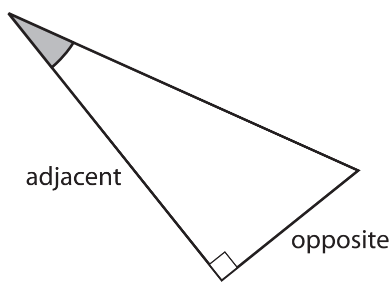
Because the cross products are not equal, the ratios are not equal.

Lesson A: Activity 3: Self-Check

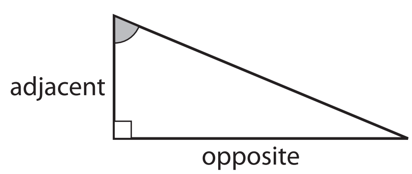
1.



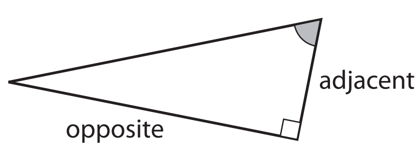
2.



3.



4.



Lesson A: Activity 4: Try This

Answers will vary based on the triangles in the diagram you drew. The sample answer below is based on the diagram in the activity.

Right triangle	Side opposite ∠A (nearest mm)	Side adjacent to ∠A (nearest mm)	$\frac{\text{opposite side}}{\text{adjacent side}}$ (to 2 decimal places)
ΔAB_1C_1	B_1C_1 = **17 mm**	AC_1 = **35 mm**	$\frac{B_1C_1}{AC_1} =$ **0.49**
ΔAB_2C_2	B_2C_2 = **34 mm**	AC_2 = **70 mm**	$\frac{B_2C_2}{AC_2} =$ **0.49**
ΔAB_3C_3	B_3C_3 = **51 mm**	AC_3 = **104 mm**	$\frac{B_3C_3}{AC_3} =$ **0.49**

1. Answers will vary. The ratios of the opposite side to the adjacent side is the same/or very close in all three triangles.

2. Yes.

   Answers will vary. Ratios of corresponding sides of similar triangles are equal.

   The right triangles are similar because they share a common acute angle.

Lesson A: Activity 5: Self-Check

Angle	Adjacent Side	Opposite Side	Tangent of an Angle
10°	10 cm	**1.8 cm**	tan 10° = $\frac{1.8}{10}$ = **0.18**
20°	10 cm	**3.6 cm**	tan 20° = $\frac{3.6}{10}$ = **0.36**
30°	10 cm	**5.8 cm**	tan 30° = $\frac{5.8}{10}$ = **0.58**
40°	10 cm	**8.4 cm**	tan 40° = $\frac{8.4}{10}$ = **0.84**
45°	10 cm	10 cm	tan 45° = 1
50°	10 cm	**11.9 cm**	**tan 50° = $\frac{11.9}{10}$ = 1.19**

Lesson A: Activity 6: Self-Check

Angle	Tangent Ratio
10°	**0.1763**
20°	**0.3640**
30°	0.5774
40°	**0.8391**
45°	**1**
50°	**1.1918**
60°	**1.7321**
70°	**2.7475**
80°	**5.6713**

Lesson A: Activity 7: Self-Check

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\tan 25° = \frac{x}{10}$$

$$(10)\tan 25° = \frac{x}{10}(10)$$

$$(10)\tan 25° = x$$

To find x, use your calculator and do the following keystrokes:

1 0 × Tan 2 5) =

Your calculator should display the value of x as: 4.6630765 . . . (If you do not obtain the answer shown, consult your manual for help.)

The value of x is approximately 4.7 cm.

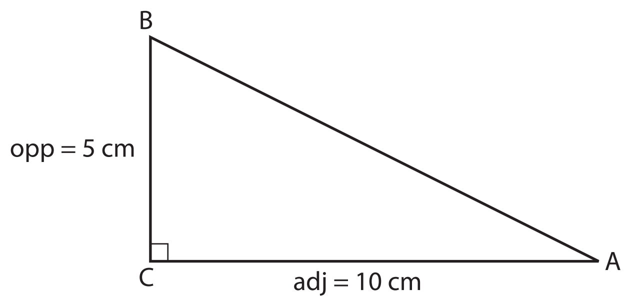
Lesson A: Activity 8: Self-Check

1. **Method 1**. Draw a right triangle, and measure the angle using a protractor.

   Since $\tan A = \dfrac{\text{opposite side}}{\text{adjacent side}}$ and tan A = 0.5, then, $\dfrac{\text{opposite side}}{\text{adjacent side}} = 0.5$.

   This means: Opposite side = 0.5 × Adjacent side.

   Draw any right triangle with the opposite side half as long as the adjacent side. For instance, if the adjacent side is 10 cm, then the opposite side is 5 cm. The size of the triangle will not matter, since all triangles for which tangent is 0.5, will be similar triangles.



   Measure ∠A with your protractor.
   ∠A ≈ 27º.
   This answer makes sense.
   When tan A = 1, ∠A = 45°.
   When tan A = ½ , ∠A ≈ 27º.
   The smaller the tangent ratio, the smaller the angle.

   Method 2. Use your calculator.

   To find an angle from its tangent, strike these keys.

INV or 2^{nd}	Tan^{-1}	0	.	5	)	=

   Your display should show 26.565051 18...
   So, ∠A ≈ 27º.
   You should write out your solution as follows:
   tan A = 2
   $\angle A = \tan^{-1}(2)\tan^{-1}(\tan A) = \tan^{-1}(0.5)$
   ~~$\tan^{-1}(\tan$~~ $A) = \tan^{-1}(0.5)$
   $\angle A = \tan^{-1}(0.5)$
   ∠A ≈ 27º

2.

tan A	∠A
0.3456	**19.1°**
4	**76.0°**
0.827	**39.6°**
2.5678	**68.7°**
$\frac{5}{6}$	**39.8°**

3. $\tan C = \frac{\text{opposite side}}{\text{adjacent side}}$

$\tan C = \frac{4}{12}$

$\angle C = \tan^{-1}\left(\frac{4}{12}\right)$

$\angle C = 18.43494882...$

The pitch of the garage roof is about 18 degrees.

Lesson A: Activity 9: Mastering Concepts

1. $\tan P = \frac{\text{opp}}{\text{adj}} = \frac{p}{r}$ (side *p* is opposite Angle P)

$\tan R = \frac{\text{opp}}{\text{adj}} = \frac{r}{p}$ (side *r* is opposite Angle R)

2. ∠P and ∠R are complementary angles. The sum of their measures is 90°.

3. tan P and tan R are reciprocals of each other. $\tan P = \frac{p}{r}$ and $\tan R = \frac{r}{p}$. (The ratios are flips of each other.)

Lesson B: Using Tangents to Solve Problems

Lesson B: Activity 1: Self-Check

1. $\frac{5}{11} = \frac{3}{x}$

$5x = (11)(3)$ — Cross multiply.

$5x = 33$

$\frac{5x}{5} = \frac{33}{5}$

$x = \frac{33}{5}$ or $6\frac{3}{5}$

2. $\frac{6}{15} = \frac{3}{2x}$

$(2x)(6) = (15)(3)$ — Cross multiply.

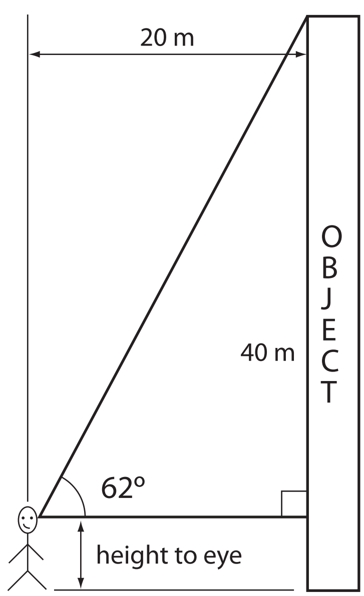
$12x = 45$

$\frac{12x}{12} = \frac{45}{12}$

$x = 3\frac{9}{12}$ or $3\frac{3}{4}$

Lesson B: Activity 2: Try This

1. A similar drawing with measurements included.



2. Answers will vary. A sample solution is provided below.

   If your angle of elevation was 62° and that your distance from the object is 20 m, your solution would be as follows:

$$\text{tangent A} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 62^\circ = \frac{x}{20}$$

The side opposite the 60° angle is the unknown height *x*.

$$\frac{x}{20} = \tan 62^\circ$$

$$x = 20(\tan 62^\circ)$$

$$x = 37.6145...$$

$$x \approx 38 \text{ m}$$

   This is the height of the object from the viewer's eye to the top of the object. The height of the viewer's eye would still need to added. Assume the height of the eye (height of the clinometer) is 2 m.

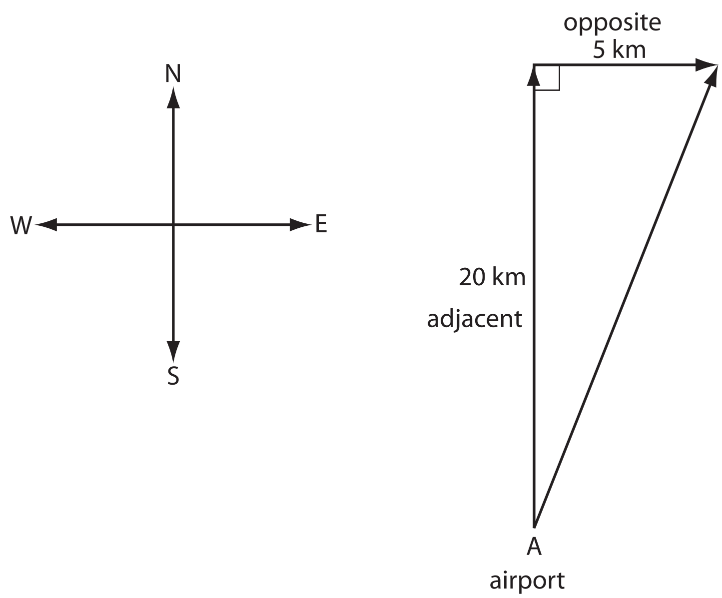
   The object is approximately 38 m + 2 m (height of clinometer) or 40 m high.

3. The height from the tangent calculation is from the viewer's eye to the top of the object. To find the height from the ground, the height from the ground to the viewer's eye would need to be added on.

4. Accuracy is affected by:
   - the accuracy of the clinometer reading
   - how level the ground is (Is the distance to the object measured by using the tape along a horizontal distance?)
   - rounding
   - the accuracy of the measurement of how high the clinometer is above the ground
   - the accuracy of the measured distance from your position to the object

Lesson B: Activity 3: Self-Check

1. Draw a diagram, and label the required angle with a letter. Label the sides using opposite and adjacent.



$\angle A$ is the required angle. Use $\angle A$ as the reference angle.

$$\tan \angle A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \angle A = \frac{5}{20}$$

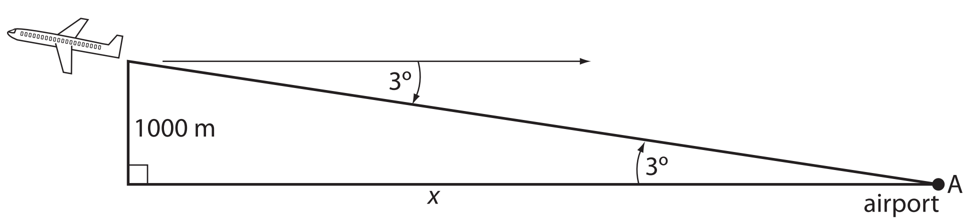
The side opposite $\angle A$ is 5 m.

$$\angle A = \tan^{-1}\left(\frac{5}{20}\right)$$

$$\angle A = 14.03624347\ldots$$

The helicopter pilot should head 14° east of north.

2. Draw a diagram.



Remember, the angle of elevation and the angle of depression are equal.

Let "x" be the distance from the airport at A.

Use the tangent ratio to set up an equation:

$$\tan \angle A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 3° = \frac{1000}{x}$$

The side opposite 3° is 1000 m.

$$x \tan 3° = 1000$$

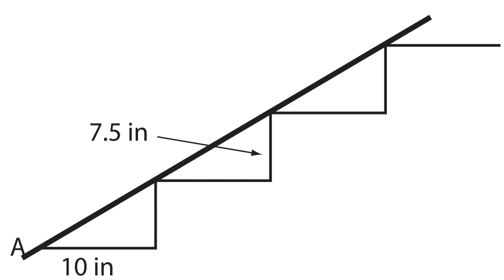
$$\frac{x \tan 3°}{\tan 3°} = \frac{1000}{\tan 3°}$$

$$x = \frac{1000}{\tan 3°}$$

$$x = 19\,081.13\,669\ldots$$

The plane is approximately 19 100 m from the airport.

3. Draw a diagram.



Let ∠A be the staircase angle.

Use the tangent ratio to set up an equation:

$$\tan \angle A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \angle A = \frac{7.5}{10}$$

The side opposite ∠A is 7.5 m.

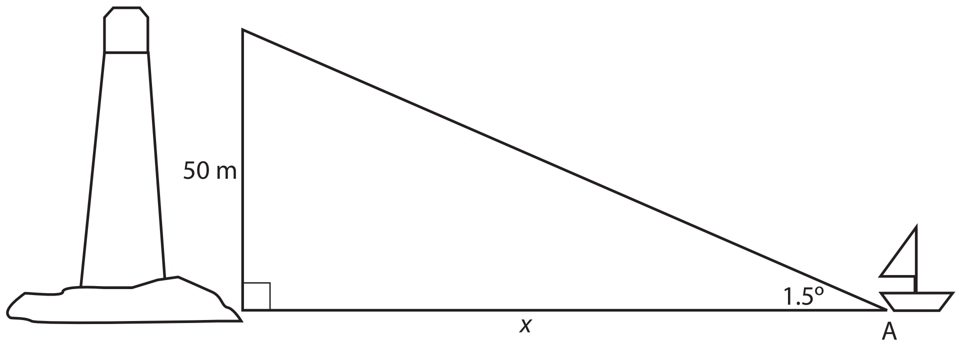
$$\angle A = \tan^{-1}\left(\frac{7.5}{10}\right)$$

$$\angle A = 36.86989765\ldots$$

The staircase angle is approximately 36.9° to the horizontal.

This is less than 42°, so Maxine is within the specifications.

4. Draw a diagram. Let x be the distance from ship to shore.



Use the tangent ratio to set up an equation:

$$\tan \angle A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 1.5° = \frac{50}{x}$$

The side opposite 5° is 50 m.

$$x \tan 1.5° = 50$$

$$\frac{x \tan 1.5°}{\tan 1.5°} = \frac{50}{\tan 1.5°}$$

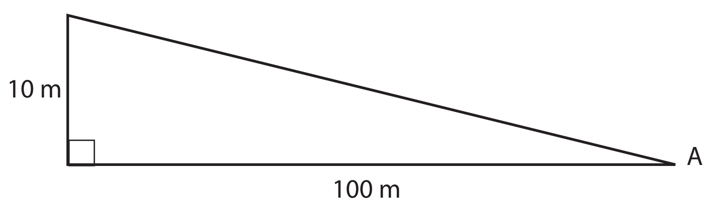
$$x = \frac{5}{\tan 1.5°}$$

$$x = 1909.422965\ldots$$

The measurement of 50 was in metres, so this answer is also in metres. The ship is approximately 1909 metres from the shore.

This means the ship is not safe.

5. Draw a diagram.



Use the tangent ratio to set up an equation:

$$\tan \angle A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \angle A = \frac{10}{100}$$

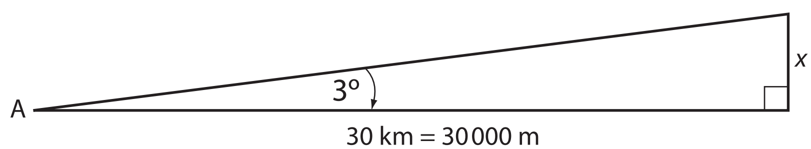
The side opposite ∠A is 10 m.

$$\angle A = \tan^{-1}\left(\frac{10}{100}\right)$$

$$\angle A = 5.710593137\ldots$$

The road is inclined at approximately 6° to the horizontal.

6. Draw a diagram. Let x be the height above the ground.



Use the tangent ratio to set up an equation:

$$\tan 3° = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 3° = \frac{x}{30\ 000}$$

The side opposite 3° is the unknown variable, "x."

$$30\ 000(\tan 3°) = x$$

$$1572.233378\ldots = x$$

The plane is approximately 1600 m above the ground.

Lesson B: Activity 4: Mastering Concepts

Use the tangent ratio to set up an equation:

$$\tan \angle A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \angle A = \frac{4}{3}$$

The side opposite ∠A is 4 m.

$$\angle A = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\angle A = 53.13010235\ldots$$

The slanted side of the Pyramid of Khafre is inclined at just over 53° to the horizontal.

Lesson C: The Sine Ratio

Lesson C: Activity 1: Self-Check

1. a. Side AC = *b*
   b. Side BC = *a*
   c. Side AB = *c*

2. The length of the side opposite ∠A is 12 cm. The length of the side adjacent to ∠A is 5 cm.

3. The length of the side opposite ∠C is 5 cm. The length of the side adjacent to ∠C is 12 cm.

4. 13 cm

Lesson C: Activity 2: Try This

Answers will vary based on the triangles in the diagram you drew. The sample answer below is based on the diagram in the activity.

Right triangle	Side opposite ∠A (nearest mm)	Hypotenuse (nearest mm)	$\frac{\text{opposite side}}{\text{hypotenuse}}$ (to 2 decimal places)
ΔAB_1C_1	B_1C_1 = **1.7 mm**	AB_1 = **3.9 mm**	$\frac{B_1C_1}{AB_1} = \mathbf{0.44}$
ΔAB_2C_2	B_2C_2 = **3.4 mm**	AB_2 = **7.7 mm**	$\frac{B_2C_2}{AB_2} = \mathbf{0.44}$
ΔAB_3C_3	B_3C_3 = **5.1 mm**	AB_3 = **11.5 mm**	$\frac{B_3C_3}{AB_3} = \mathbf{0.44}$

1. Answers will vary. The ratios of the opposite side to the hypotenuse is the same or very close in all three triangles.

2. Yes. Ratios of corresponding sides of similar triangles are equal. The right triangles are similar because they share a common acute angle.

Lesson C: Activity 3: Self-Check

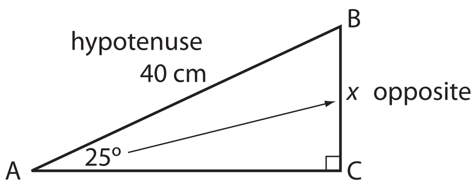
Angle	Hypotenuse	Opposite Side	Sine of an Angle
10°	10 cm	**1.7 cm**	sin 10° = $\frac{\mathbf{1.7}}{\mathbf{10}}$ = **0.17**
20°	10 cm	**3.4 cm**	sin 20° = $\frac{\mathbf{3.4}}{\mathbf{10}}$ = **0.34**
30°	10 cm	5.0 cm	sin 30° = 0.50
40°	10 cm	**6.4 cm**	sin 40° = $\frac{\mathbf{6.4}}{\mathbf{10}}$ = **0.64**
45°	10 cm	**7.1 cm**	sin 45° = 0.71
50°	10 cm	**7.7 cm**	sin 50° = $\frac{\mathbf{7.7}}{\mathbf{10}}$ = **0.77**
60°	10 cm	**8.7 cm**	sin 60° = $\frac{\mathbf{8.7}}{\mathbf{10}}$ = **0.87**
70°	10 cm	**9.4 cm**	sin 70° = $\frac{\mathbf{9.4}}{\mathbf{10}}$ = **0.94**
80°	10 cm	**9.8 cm**	sin 80° = $\frac{\mathbf{9.8}}{\mathbf{10}}$ = **0.98**

Lesson C: Activity 4: Self-Check

Angle	Sine Ratio
10°	**0.1736**
20°	**0.3420**
30°	0.5000
40°	**0.6428**
45°	**0.7071**
50°	**0.7660**
60°	**0.8660**
70°	**0.9397**
80°	**0.9848**

Lesson C: Activity 5: Self-Check

Label the side "x" as opposite and the length of 40 cm as hypotenuse.



Since the problem involved "opposite" and "hypotenuse" use the sine ratio to set up an equation.

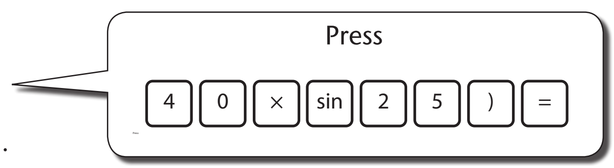
$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 25^\circ = \frac{x}{40}$$

$$\frac{x}{40} = \sin 25^\circ$$

$$(40)\frac{x}{40} = (40)\sin 25^\circ$$

$$x = (40)\sin 25^\circ$$



$$x = 16.90473047\ldots$$

$$x \approx 16.90$$

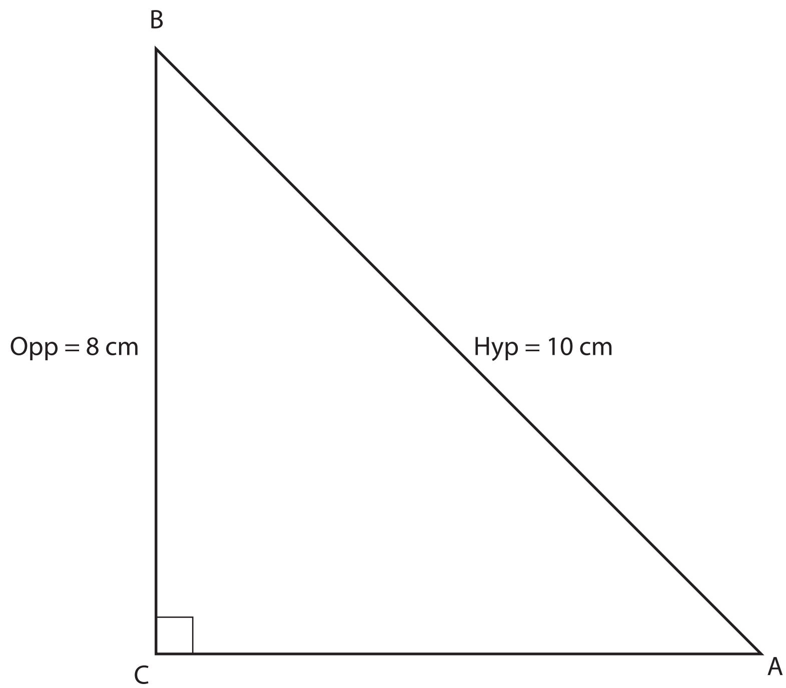
The value of x is approximately 16.90 cm.

Lesson C: Activity 6: Self-Check

1. **Method 1**. Draw a right triangle, and measure the angle using a protractor.

   Since sin A = 0.6 and $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$, we need to write 0.6 as a ratio in order to know how long to draw the two lengths in the right triangle.

   Since, 0.6 = 6 tenths = $\frac{6}{10}$. Then the opposite side can be 6 cm long and the hypotenuse should be 10 cm in length.



Measure ∠A with your protractor.

∠A ≈ 53º

Method 2. Use your calculator.

To find an angle from its sine, strike these keys.

INV or 2^{nd}	Sin^{-1}	0	.	8	)	=

Your display should show 53.13010235...

So, ∠A ≈ 53º.

You should write out your solution as follows.

sin A = 0.8

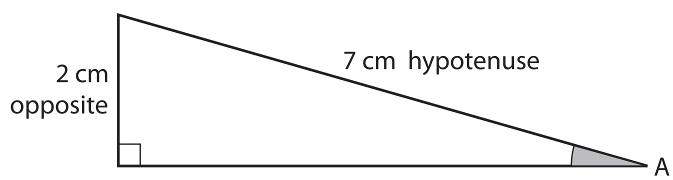
∠A = $\sin^{-1}(0.8)$

∠A ≈ 53º

2.

sin A	∠A
0.1257	**7.2°**
0.7826	**51.5°**
0.9000	**64.2°**
$\frac{2}{3}$	**41.8°**
$\frac{3}{4}$	**48.6°**

3. Shade in angle A, and name the sides with lengths.



Since the triangle involves "opposite" and "hypotenuse," use the sine ratio to set up an equation.

$$\sin\ A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\ A = \frac{2}{7}$$

$$\angle A = \sin^{-1}\left(\frac{2}{7}\right)$$

$$\angle A = 16.6015495990202\ldots$$

$$\angle A = 16.6°$$

Lesson C: Activity 7: Mastering Concepts

1. $\sin P = \frac{\text{opp}}{\text{hyp}} = \frac{p}{q}$ — side *p* is opposite Angle P.

   $\sin R = \frac{\text{opp}}{\text{hyp}} = \frac{r}{q}$ — side *r* is opposite Angle R.

2. ∠P and ∠R are complementary angles. The sum of their measures is 90°.

3. Sides *p* and *r* form the right angle in the right triangle.

Lesson D: Using Sines to Solve Problems

Lesson D: Activity 1: Try This

1.

$$\sin R = \frac{\boxed{\text{opposite}}}{\text{hypotenuse}}$$

$$\sin 45^\circ = \frac{x}{\boxed{36.06}}$$

$$x = \left(\boxed{\sin 45^\circ}\right)\left(\boxed{36.06}\right)$$

$$x = 0.707\boxed{36.06}$$

$$x = \boxed{25.5}\ \text{cm}$$

2.

$$\sin R = \frac{\text{opposite}}{\boxed{\text{hypotenuse}}}$$

$$\text{hypotenuse} = \frac{\boxed{\text{opposite}}}{\boxed{\text{sinR}}}$$

$$y = \frac{\boxed{25.5}}{\boxed{\sin 45^\circ}}$$

$$y = \frac{\boxed{25.5}}{0.707}$$

$$y = \boxed{36.06}\ \text{cm}$$

3. When finding the opposite side, the unknown is in the numerator and the calculations involve multiplication. When finding the hypotenuse, the unknown is in the denominator and the calculations involve division.

4. $\sin R = \frac{\text{opposite}}{\text{hypotenuse}}$

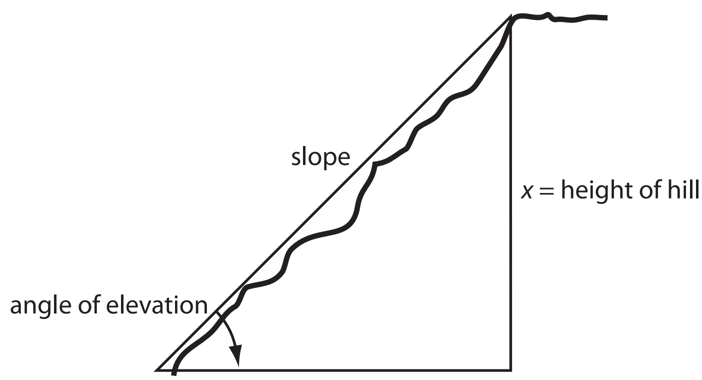
$$\sin R = \frac{12}{17.4}$$

$$\angle R = \sin^{-1}\left(\frac{12}{17.4}\right)$$

$$\angle R = 43.6^\circ$$

Lesson D: Activity 2: Try This

1. Your diagram should be similar to this one but should include all your measurements.



2. The solution will follow this format.

   Substitute your values into the formula below:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{x}{\text{slope}}$$

$$x = \text{slope}(\sin A)$$

3. It is unlikely that the answer is particularly accurate for three reasons:
   - difficulty using the clinometer and the questionable accuracy of the angle of elevation
   - difficulty measuring the length of the hill
   - the hill’s lack of uniform slope

Lesson D: Activity 3: Self-Check

1. Draw a diagram.

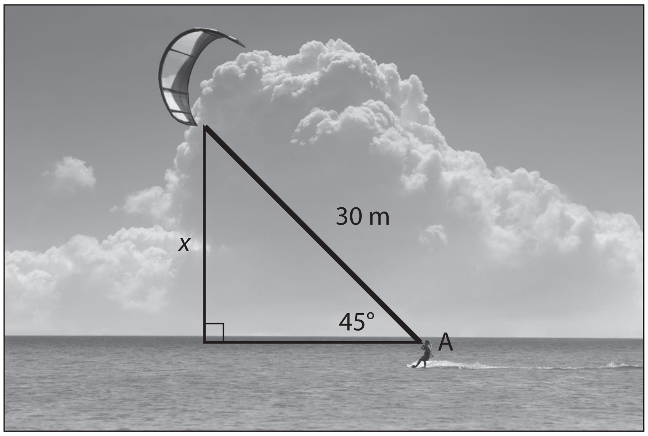


Photo by Dmitry Kosterev © 2010

Let "x" be the height the kite is above the water.

$$\sin 45° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 45° = \frac{x}{30}$$

$$30(\sin 45°) = x$$

$$21.2132\ldots = x$$

The kite is approximately 21 m above the water.

2. Plug the given values into the sine ratio. Let ∠A be the required angle.

$$\sin\ A = \frac{\text{opposite}}{\text{hypotenuse}}$$

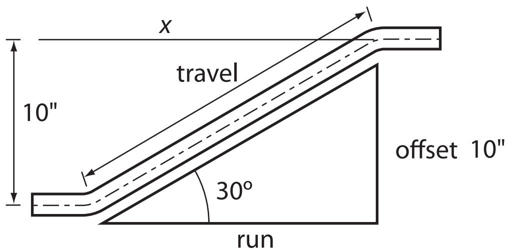
$$\sin\ A = \frac{120}{210}$$

$$\angle A = \sin^{-1}\left(\frac{120}{210}\right)$$

$$\angle A = 34.8499045790465\ldots$$

The javelin makes an angle of approximately 35° with the ground.

3. Let x be the travel. The offset is 10 inches.



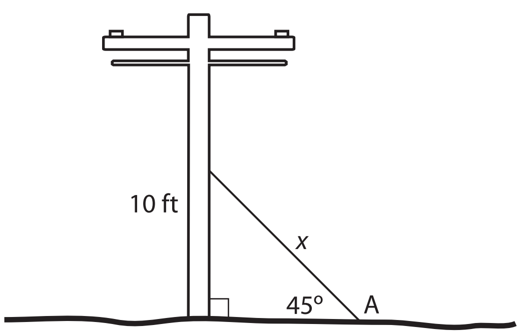
Use the sine ration to set up an equation.

$$\sin 30° = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 30° = \frac{10}{x}$$
$$x \sin 30° = 10$$
$$x = \frac{10}{\sin 30°}$$
$$x = 20$$

The travel will be 20 inches in length.

4. Draw a diagram.

Let x be the length of the guy wire.



Use the sine ratio to set up an equation.

$$\sin 45° = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 45° = \frac{10}{x}$$
$$x \sin 45° = 10$$
$$x = \frac{10}{\sin 45°}$$
$$x = 14.142135623731\ldots$$

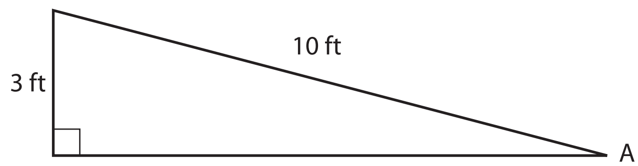
The guy wire is 14 feet and 0.1421356... of a foot long. Convert this fraction of a foot to inches to give an appropriate final answer.

Remember, 1 foot = 12 inches

So 0.1421356... × 12 = 1.705627485... inches, or approximately 2 inches.

The guy wire will be approximately 14 feet 2 inches in length.

5. Draw a diagram. Let ∠A be the required angle.



Use the sine ratio to set up an equation.

$$\sin\ A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\ A = \frac{3}{10}$$

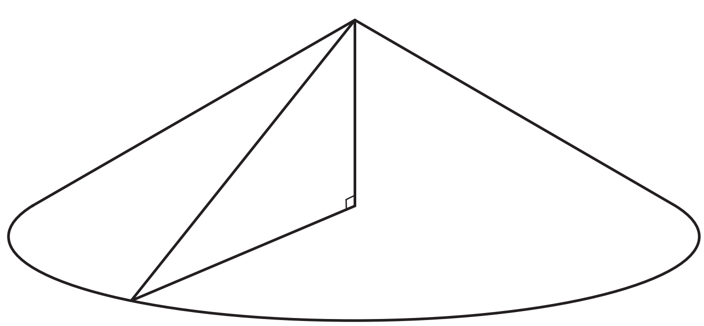
$$\angle A = \sin^{-1}\left(\frac{3}{10}\right)$$

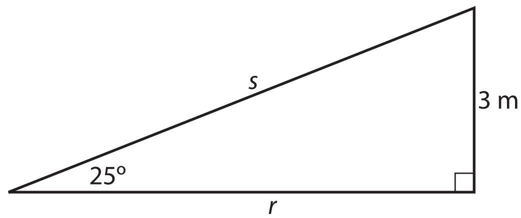
$$\angle A = 17.45760312\ldots$$

The ramp is inclined at approximately 17° to the horizontal.

Lesson D: Activity 4: Mastering Concepts

1. Draw a diagram.





Let *s* be the length of the slant side.

Use the sine ratio to set up an equation.

$$\sin 25° = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 25° = \frac{3}{s}$$
$$s \sin 25° = 3$$
$$x = \frac{3}{\sin 25°}$$
$$x = 7.0986047494575\ldots$$

The slant side is approximately 7.1 m in length.

2. Let *r* be the radius.

   Use the tangent ratio to set up an equation.

$$\tan 25° = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 25° = \frac{3}{r}$$
$$r \tan 25° = 3$$
$$r = \frac{3}{\tan 25°}$$
$$r = 6.43352076152868\ldots$$

   The radius of the pile is approximately 6.4 m.

3. SA = πrs
   SA = $\pi(6.4)(7.1)$
   SA = 142.75397017912...

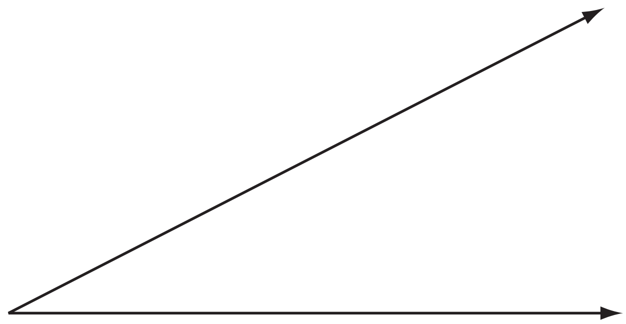
   The surface area of the tarp needed to cover the wheat is approximately 143 m^2.

Glossary

acute angle

an angle greater than 0° but less than 90°

For example, this is an acute angle.



adjacent angles

angles which share a common vertex and lie on opposite sides of a common arm

adjacent side

the side next to the reference angle in a right triangle. (The adjacent side cannot be the hypotenuse.)

alternate exterior angles

exterior angles lying on opposite sides of the transversal

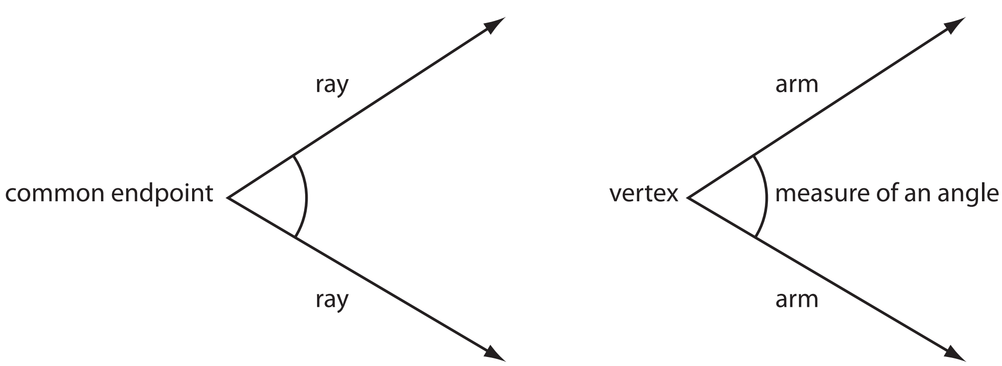
alternate interior angles

interior angles lying on opposite sides of the transversal

angle

a geometric shape formed by two rays with a common endpoint

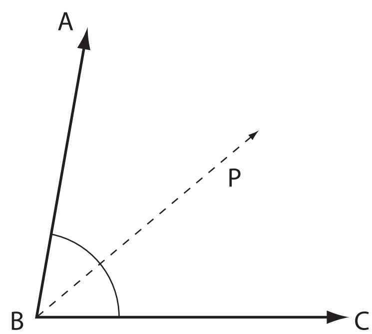
Each ray is called an *arm of the angle*. The common endpoint of the arms of the angle is the vertex of the angle.



angle of depression
an angle below the horizontal that an observer must look down to see an object that is below the observer

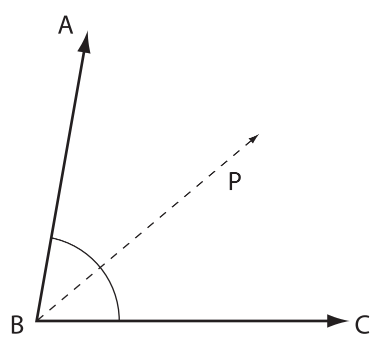
angle of elevation
the angle above the horizontal that an observer must look to see an object that is higher than the observer

bisect
divide into two congruent (equal in measure) halves



bisector
a line or ray which divides a geometric shape into congruent halves

Ray BP is a bisector of ∠ABC, since it bisects ∠ABC into two congruent halves.



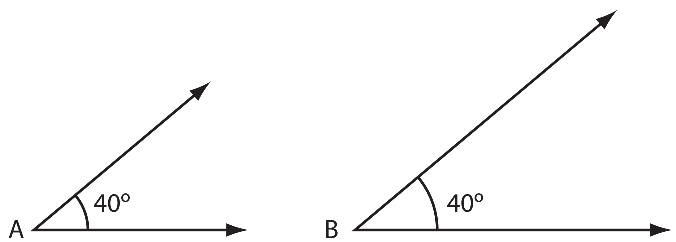
∠ABP ≅ ∠PBC

clinometer
a device for measuring angles to distant objects that are higher or lower than your position

complementary angles
two angles with measures that add up to 90°

One angle is called the *complement* to the other.

congruent angles
angles with the same measure



In the diagram ∠A = 40° and ∠B = 40°. So, ∠A and ∠B are congruent.

There is a special symbol for "is congruent to." The congruence symbol is ≅.

So, you can write ∠A ≅ ∠B.

corresponding angles
angles in the same relative positions when two lines are intersected by a transversal

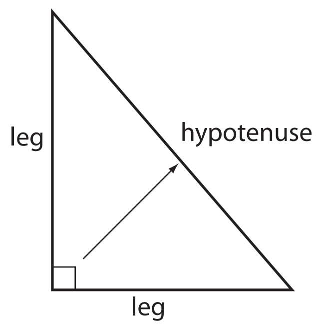
cosine ratio
the ratio of the length of the side adjacent to the reference angle, to the length of the hypotenuse of the right triangle

exterior angles
angles lying outside two lines cut by a transversal

full rotation
an angle having a measure of 360°

This is a full rotation angle.

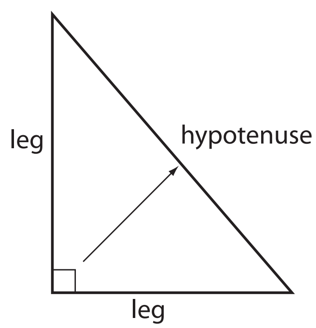
hypotenuse
in a right triangle, the side opposite the right angle; the longest side in a right triangle



indirect measurement
taking one measurement in order to calculate another measurement

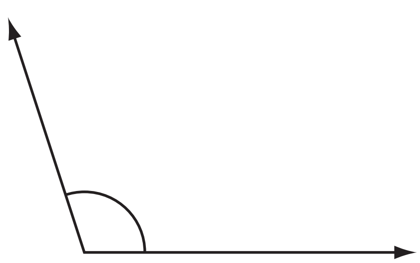
interior angles
angles lying between two lines cut by a transversal

leg
one of the two sides of a right triangle that forms the right angle



obtuse angle
an angle greater than 90° but less than 180°

For example, this is an obtuse angle.



opposite side
the side across from the reference angle in a right triangle

parallel
lines that are the same distance apart everywhere: they never meet

perpendicular
lines that meet at right angles

polygon
a many-sided figure

A triangle is a polygon with three sides, a quadrilateral is a polygon with four sides, and so on.

proportion
the statement showing two ratios are equal

Pythagorean Theorem
for any right triangle, the square of the hypotenuse is equal to the sum of the squares of the two legs

Pythagorean triple
three whole numbers, which represent the lengths of the sides of a right triangle

There are an infinite number of such triples.

reference angle
an acute angle that is specified (example, shaded) in a right triangle

referent
an object or part of the human body you can refer to when estimating length or distance

reflex angle
an angle having a measure greater than 180° but less than 360°

This is an example of a reflex angle.

regular polygon
a polygon with all its angles equal in measure and all its sides equal in measure

right angle
one quarter of a complete rotation. It is 90° in measure.

scale factor
the number by which the length and the width of a figure is multiplied to form a larger or smaller similar figure

similar figures
figures with the same shape but not necessarily the same size

A figure similar to another may be larger or smaller

sine ratio
the ratio of the length of the side opposite to the reference angle, over the hypotenuse of the right triangle

solve a right triangle
to find all the missing sides and angles in a right triangle

straight angle
one half a rotation; an angle 180°

This is a straight angle.

straightedge
a rigid strip of wood, metal, or plastic having a straightedge used for drawing lines

When a ruler is used without reference to its measuring scale, it is considered to be a straightedge.

supplementary angles
two angles, which add up to 180°

In a pair of supplementary angles, one angle is the supplement to the other.

symmetry
the property of being the same in size and shape on both sides of a central dividing line

tangent ratio
the ratio of the length of the side opposite to the selected acute angle, to the length of the side adjacent to the selected acute angle in a right triangle

transversal
a line that cuts across two or more lines

trigonometry
the branch of mathematics based originally on determining sides and angles of triangles, particularly right triangles

vertically opposite angles
angles lying across from each other at the point where two lines intersect

Vertically opposite angles are also referred to as *opposite angles*.

Clinometer Template