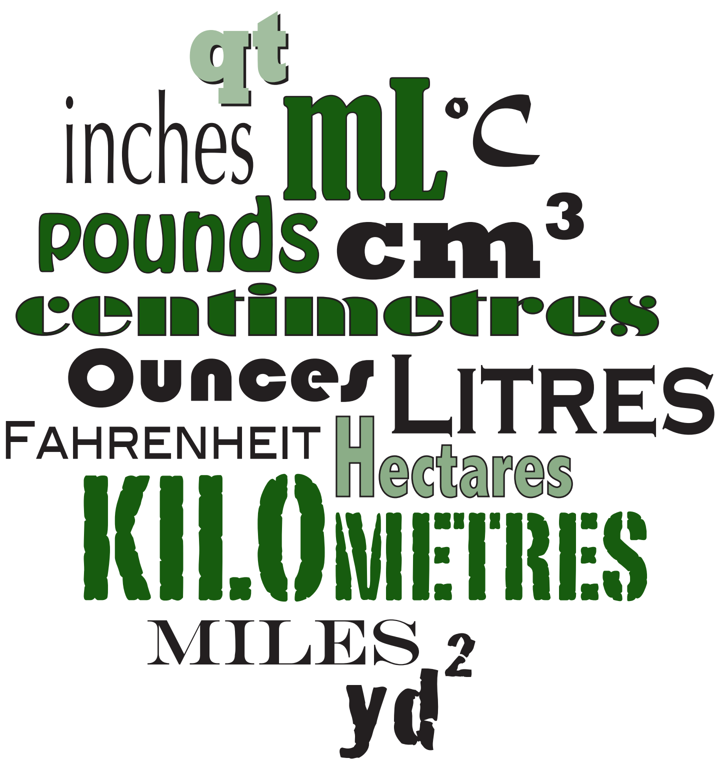
Version 01

Trigonometry II

Apprenticeship and Workplace Mathematics (Grade 10/Literacy Foundations Level 7)



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Course History

New, March 2012

Project Partners

This course was developed in partnership with the Distributed Learning Resources Branch of Alberta Education and the following organizations:

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- Calgary Board of Education
- Edmonton Public Schools
- Peace Wapiti School Division No. 76
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Viewing Your PDF Learning Package

This PDF Learning Package is designed to be viewed in Acrobat. If you are using the optional media resources, you should be able to link directly to the resource from the pdf viewed in Acrobat Reader. The links may not work as expected with other pdf viewers.

Download Adobe Acrobat Reader:
http://get.adobe.com/reader/

Section Organization

This section on Trigonometry is made up of several lessons.

Lessons

Lessons have a combination of reading and hands-on activities to give you a chance to process the material while being an active learner. Each lesson is made up of the following parts:

Essential Questions

The essential questions included here are based on the main concepts in each lesson. These help you focus on what you will learn in the lesson.

Focus

This is a brief introduction to the lesson.

Get Started

This is a quick refresher of the key information and skills you will need to be successful in the lesson.

Activities

Throughout the lesson you will see three types of activities:

- Try This activities are hands-on, exploratory activities.
- Self-Check activities provide practice with the skills and concepts recently taught.
- Mastering Concepts activities extend and apply the skills you learned in the lesson.

You will mark these activities using the solutions at the end of each section.

Explore

Here you will explore new concepts, make predictions, and discover patterns.

Bringing Ideas Together

This is the main teaching part of the lesson. Here, you will build on the ideas from the Get Started and the Explore. You will expand your knowledge and practice your new skills.

Lesson Summary

This is a brief summary of the lesson content as well as some instructions on what to do next.

At the end of each section you will find:

Solutions
This contains all of the solutions to the Activities.

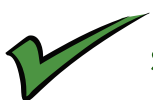
Appendix
Here you will find the Data Pages along with other extra resources that you need to complete the section. You will be directed to these as needed.

Glossary
This is a list of key terms and their definitions.

Throughout the section, you will see the following features:

Icons
Throughout the section you will see a few icons used on the left-hand side of the page. These icons are used to signal a change in activity or to bring your attention to important instructions.

AWM online resource (optional)
This indicates a resource available on the internet. If you do not have access, you may skip these sections.



Solutions

Calculator

My Notes

The column on the outside edge of most pages is called "My Notes". You can use this space to:

- write questions about things you don't understand.
- note things that you want to look at again.
- draw pictures that help you understand the math.
- identify words that you don't understand.
- connect what you are learning to what you already know.
- make your own notes or comments.

Materials and Resources

There is no textbook required for this course.

You will be expected to have certain tools and materials at your disposal while working on the lessons. When you begin a lesson, have a look at the list of items you will need. You can find this list on the first page of the lesson, right under the lesson title.

In general, you should have the following things handy while you work on your lessons:

- a scientific calculator
- a ruler
- a geometry set
- Data Pages (found in the Appendix)

Trigonometry II

Photo by Sergei Bachlakov © 2010

Imagine the excitement of carrying the Olympic torch! More than 12 000 torch-bearers from all parts of Canada carried the flame during the 106-day relay that started in Victoria, British Columbia. The torch route went through all of the territories and provinces of Canada, and ended in Vancouver, the host city of the 2010 Winter Olympic Games. Wayne Gretzky had the honour of lighting the outdoor cauldron—what a spectacular event!

Take a close look at that cauldron. Its design incorporates many of angles and triangles! The Winter Olympics required many new facilities, and these buildings and structures were designed by architects who applied mathematics. One of their mathematical tools is trigonometry.

In this section you will:

- apply similarity to right triangles.
- generalize patterns from similar right triangles.
- apply the trigonometric ratios tangent, sine, and cosine to solve problems.

Lesson A
The Cosine Ratio

To complete this lesson, you will need:

- a metric ruler
- a protractor
- a calculator

In this lesson, you will complete:

- 7 activities

Essential Questions

- What is the cosine ratio?
- How is the cosine ratio used to find unknown sides and angles in right triangles?

My Notes

Focus

Photo by Dainis Derics © 2010

The 2010 Olympic bobsleigh and skeleton track at Whistler, British Columbia is 1450 m in length with a vertical drop of 152 m and an average slope of 10.5°. In the women's skeleton event, the combined mass of the competitor and sled cannot exceed 92 kg. Part of this combined weight pushes the runners against the track and part of the weight pushes the sled down the slope. On average, what percentage of the mass pushes the sled down the track? At the end of this lesson you will have the trigonometric skills to answer this question.

Get Started

Do you remember what you learned in previous lessons about the relationship between the angles in a triangle?

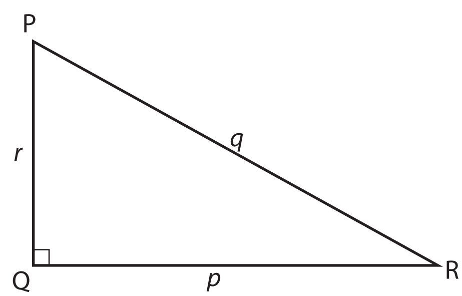
The measures of the angles in any triangle add up to 180º. In Activity 1 you will review the relationship between the two acute angles of a right triangle.

Activity 1
Try This

My Notes

You will need a protractor and a straightedge to complete this activity.

Step 1: Use your protractor and straightedge to draw a right triangle PQR of any size. An example is shown below.



Did You Know?

The letters that are used to name triangles are usually placed alphabetically. But the letters could be placed in any order. For example, the previous triangle could have been named any of the following: ΔQRP, ΔPRQ, or ΔRQP.

Step 2: Measure ∠P and ∠R to the nearest degree.

∠P = ________ ∠R = ________

Questions

1. Determine ∠P + ∠R.

My Notes

2. Will the sum of the two acute angles of a right triangle always be the same as the answer you obtained in Question 1? Explain.

3. What name is given to a pair of angles with this sum?

4. In right ΔABC, ∠A and ∠B are acute angles. If ∠B = 47°, what is the size of ∠A? Draw a diagram to support your answer.

5. What is the complement of a 60° angle?

My Notes

Turn to the solutions at the end of the section and mark your work.

Explore

In previous lessons, you explored the sine and tangent ratios. For a given acute angle in a right triangle, the tangent ratio was defined as

$\frac{\text{opposite}}{\text{adjacent}}$, and the sine ratio was defined as $\frac{\text{opposite}}{\text{hypotenuse}}$.

You discovered that for similar triangles, the values of the tangent ratios were the same regardless of how large or small the right triangles were. This was also true for the sine ratio. So, if you knew the angle, you could determine the ratio. And, if you knew the ratio, you could determine the angle.

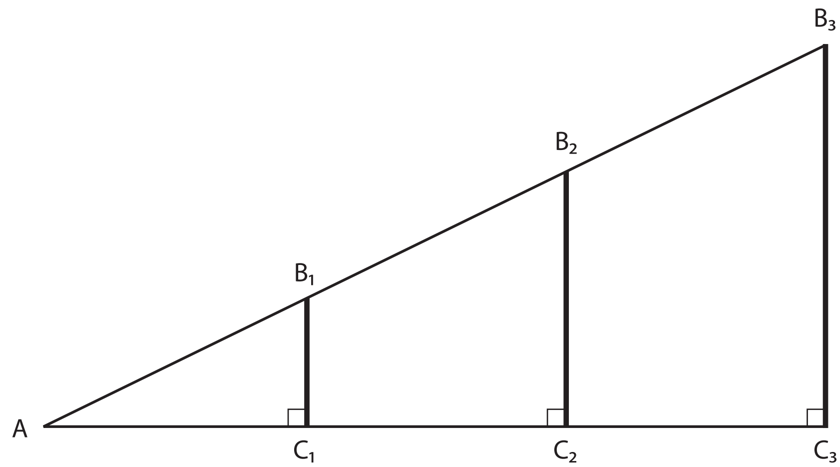
Activity 2
Try This

In this activity you will explore one other trigonometric ratio involving a different pair of sides within a right triangle. You will compare this ratio with ratios involving the same pair of sides from other right triangles. For this comparison you will use your knowledge of similar right triangles.

You will need your metric ruler, protractor, and calculator.

My Notes

Consider the right triangles in the following diagram.



Step 1: On a full piece of paper, draw a diagram similar to the one above. Make an enlargement of it, so that the diagram takes up the entire page. Label the diagram using the same lettering.

Step 2: For right triangle ΔAB_1C_1, measure to the nearest millimetre and record the length of sides AC_1 and AB_1 in the table below.

For right triangle ΔAB_2C_2, measure to the nearest millimetre and record the length of sides AC_2 and AB_2 in the table below.

For right triangle ΔAB_3C_3, measure to the nearest millimetre and record the length of sides AC_3 and AB_3 in the table below.

Step 3: Complete the last column of the table using your calculator.

Right triangle	Side adjacent $\angle A$ (nearest mm)	Hypotenuse (nearest mm)	$\frac{\text{adjacent side}}{\text{hypotenuse}}$ (to 2 decimal places)
ΔAB_1C_1	$AC_1 =$	$AB_1 =$	$\frac{AC_1}{AB_1} =$
ΔAB_2C_2	$AC_2 =$	$AB_2 =$	$\frac{AC_2}{AB_2} =$
ΔAB_3C_3	$AC_3 =$	$AB_3 =$	$\frac{AC_3}{AB_3} =$

My Notes

Questions

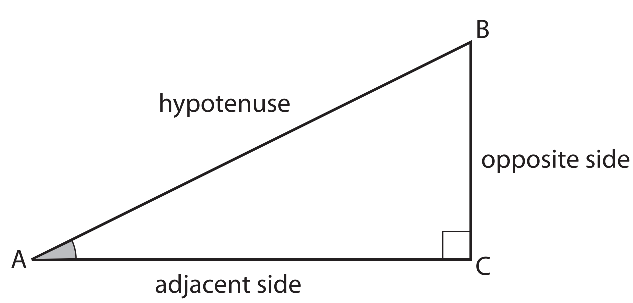
1. What pattern do you see in the table?

2. Are these three triangles, ΔAB_1C_1, ΔAB_2C_2, and ΔAB_3C_3, similar? Explain.

Turn to the solutions at the end of the section and mark your work.

Bringing Ideas Together

In Explore you examined the ratio in right triangles of the side adjacent to the hypotenuse. This ratio is another example of a trigonometric function. You saw that for similar right triangles and a particular acute angle, the ratio $\frac{\text{adjacent side}}{\text{hypotenuse}}$ was the same value regardless of the size of the right triangle.



This ratio is called the **cosine ratio**.

My Notes

The sine ratio always depends on the acute angle selected. In this case, the acute angle that was selected was ∠A—not ∠B. The acute angle has to be identified in order to write a cosine ratio.

So, cosine $\angle A = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

The angle is missing!

It is incorrect to write: Cosine $= \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

It is often written more simply as:

$\cos A = \dfrac{\text{adj}}{\text{hyp}}$ where A = measure of ∠A

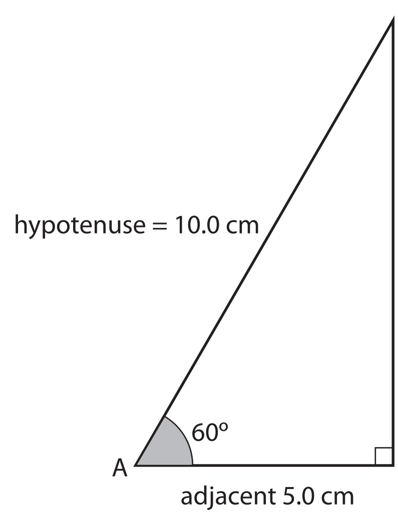
Example 1

Draw a right triangle with an acute angle of 60°. Determine cosine 60° to two decimal places.

Solution

It doesn't matter how large you draw the triangle since all right triangles with a 60° angle will be similar in shape. However, a hypotenuse of 10 cm will make the calculation easier.

Once you draw the triangle, shade in the angle of 60°. Then label the adjacent side and the hypotenuse and measure each to the to the nearest millimetre.



My Notes

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 60° = \frac{5 \text{ cm}}{10 \text{ cm}}$$

$$\cos 60° = 0.5$$

A ratio of 0.5 means that in a right triangle with a 60° angle, the adjacent side will be one-half the length of the hypotenuse. For instance, if the adjacent side were 30 inches, the hypotenuse would be 60 inches long, and $\frac{30 \text{ in}}{60 \text{ in}} = 0.50$.

Example 2

Measure the sides of a right triangle with an acute angle of 45°. Use these measurements to determine cos 45° to two decimal places.

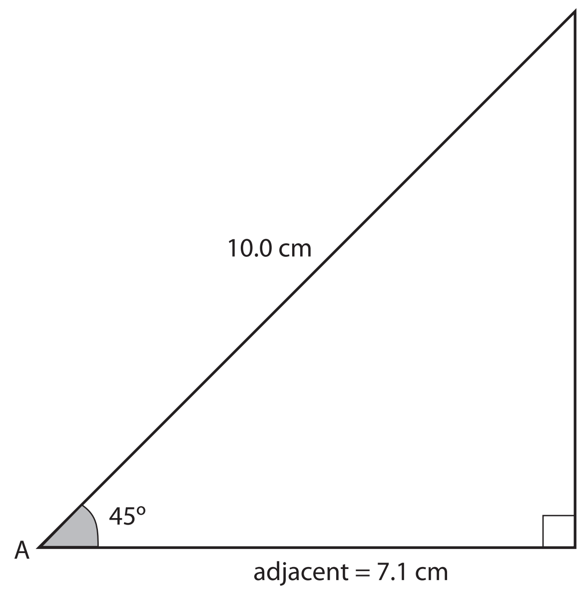
Solution

Using your protractor, draw a right triangle with an acute angle of 45°. (The triangle can be big or small since similar right triangles with a 45° angle will all have the same tangent ratio.) You may wish to draw a right triangle with a hypotenuse of 10 cm to simplify the calculation.

Shade the angle that is 45°. Label the side adjacent and the hypotenuse and then measure these lengths, accurate to the nearest millimetre.

My Notes

If you chose to draw the hypotenuse a length of 10 cm, your triangle would look like this:



$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 45° = \frac{7.1 \text{ cm}}{10.0 \text{ cm}}$$

$$\cos 45° = 0.71$$

The ratio should come out close to the following value 0.70710678118655 . . . to be accurate.

Note: This answer is only approximate since measurements were only correct to the nearest millimetre.

This ratio means the adjacent side is shorter than the hypotenuse and is 0.71 times as long. So, if the hypotenuse were 20 cm long, the adjacent side would be about 20 × 0.71 cm = 14.2 cm long.

Activity 3
Self-Check

My Notes

Use the method of the previous two examples to complete the following table. Use 10 cm for the hypotenuse.

Complete the following table.

Angle	Hypotenuse	Adjacent Side	Cosine of an Angle
10°			cos 10° =
20°			cos 20° =
30°			cos 30° =
40°			cos 40° =
45°		7.1 cm	cos 45° = 0.71
50°			cos 50° =
60°		5.0 cm	cos 60° = 0.50
70°			cos 70° =
80°			cos 80° =


Turn to the solutions at the end of the section and mark your work.

My Notes

Calculating the Cosine Ratio

Look at the table of cosine ratios you prepared in Activity 3. What did you notice about the value of the cosine ratio?

____

What happens to the cosine ratio as you move from 0° to 80°?

____

What you should have noticed is that because the hypotenuse is always longer than the side adjacent, the cosine ratios for angles between 0° and 90° will be between 0 and 1.

For angles close to 0° in size, the side adjacent is almost as long as the hypotenuse, so the cosine ratio is close to 1 in value. As the angle increases in size, the side adjacent decreases in length in relation to the hypotenuse, so the cosine ratio decreases in value. For angles close to 90°, the cosine ratio is almost 0.

Cosine and Sine Ratios are Related

You will now explore some other relationships that exist between the cosine and sine ratios.

Use your calculator to fill in the sine ratios in the table. Round your answers to 2 decimal places.

For example, to calculate sin 80º press the following keys on your calculator:

Sin	8	0	)	=

You should see the following display:

sin(80)

0.984807753

So sin 80 ≈ 0.98.

Angle	Cosine Ratio	Angle	Sine Ratio
10°	0.98	80°	
20°	0.94	70°	
30°	0.87	60°	
40°	0.77	50°	
50°	0.64	40°	
60°	0.50	30°	
70°	0.34	20°	
80°	0.17	10°	

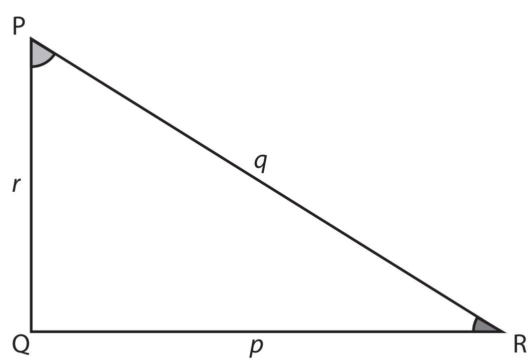
My Notes

What pattern can you see between the cosine and sine ratios?

____

Let's look at how the cosine ratio and the sine ratio are related.

Consider the following right triangle.



Notice that: cosine $\angle R = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{p}{q}$.

Similarly, sine $\angle P = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{p}{q}$.

Since both trigonometric ratios are equal to $\frac{p}{q}$, cos R = sin P.

My Notes

Since a general right-angled triangle was used, this relationship between the two acute angles is true for any right triangle. In fact, since ∠P and ∠R are complementary angles, the cosine of an angle is the sine of its complementary angle. In other words, the cosine of an angle and the sine of the angle's complement are equal.

> The **cosine** is just the **sine** of the complementary angle. That's where the name cosine comes from.

Example 3

Find the value of each of the following. Express each value so that it is correct to 4 decimal places. Use your calculator.

1. cos 45°
2. Use your result from (1), to find the sine ratio of the complement of 45°.
3. cos 60°
4. Use your result from (3), to find the sine ratio of the complement of 60°.

Solution

There are several different ways angle may be expressed. Make sure your calculator is set to degree mode.

1. To find cos 45°, press this sequence of keys. If you do not obtain the answer shown, consult your calculator manual or ask your teacher for help.

   [cos] [4] [5] [)] [=]

   In your calculator display window, you may see the solution as:

   cos(45)
   0.7071067812...

   So, cos 45° ≈ 0.7071

2. The complement of 45° is also 45° since 45°+ 45°= 90°.

   So, sin 45°= cos 45° ≈ 0.7071

3. To find cos 60°, press this sequence of keys. If you do not obtain the answer shown, consult your calculator manual or ask your teacher for help.

   | cos | 6 | 0 |) | = |

   You should see:

   cos(60)
   0.5

   So, cos 60° = 0.5

4. The complement of 60° is 30°, since 60° + 30° = 90°.

   Since cosine is just the sine of the complement angle, then sin 30° = cos 60° = 0.5000.

My Notes

My Notes

Activity 4
Self-Check

Use your calculator to complete the following table. Round your answers to 4 decimal places.

Angle	Cosine Ratio
10°	
20°	
30°	
40°	
45°	0.7071
50°	
60°	0.5000
70°	
80°	

Turn to the solutions at the end of the section and mark your work.

Solving Problems Using the Cosine Ratio

My Notes

In the next example you will apply your calculator skills to help you solve a problem involving the cosine ratio. There is not much difference between applying the cosine ratio and applying the sine ratio. All you have to remember is that for the cosine ratio, the adjacent side is used rather than the opposite side.

Example 4

Photo by EML © 2010

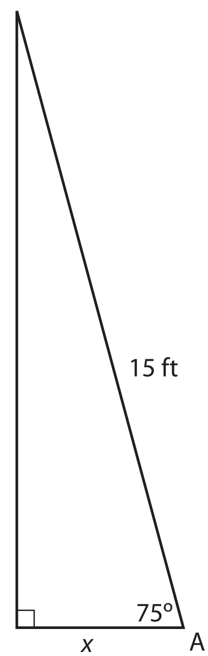
Angela is painting the exterior of her house. On one side, there is a 5-foot wide flower garden along the wall. Angela does not want to place the foot of the ladder on her flowers. However, for safety, the ladder must be positioned at a 75° angle with the ground. The ladder is 15 feet long. Can Angela avoid damaging her flowers if she places the ladder at the correct angle?

Solution

Draw a diagram. Start by drawing a right triangle with an angle of 75°. On the triangle, label the side that would be the ladder as 15 ft, and the unknown side "*x*."

Let *x* be the distance the foot of the ladder is away from the wall when the ladder leans at 75° to the ground.

My Notes



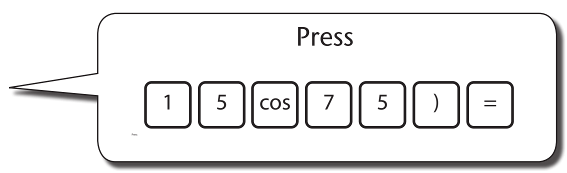
Since the problem involves the adjacent side and the hypotenuse, the cosine ratio will be used to set up an equation.

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 75^\circ = \frac{x}{15}$$

$$15(\cos 75^\circ) = x$$

$$3.882285677\ldots = x$$



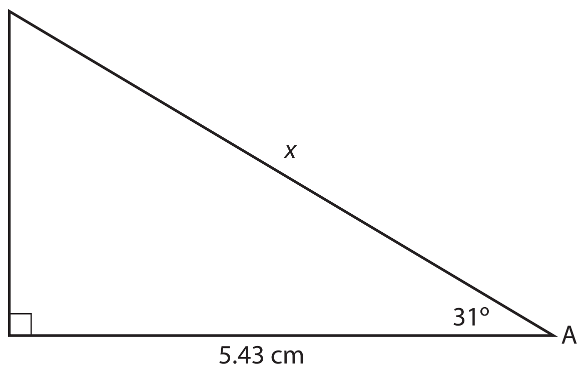
The foot of the ladder must be approximately 3.9 m from the wall. Angela will not be able to avoid her flowers. However, with a longer ladder she could avoid the flowers.

Activity 5
Self-Check

My Notes

Use the method outlined in Example 4 to solve this question.

Solve for *x*. Round to two decimal places.



Turn to the solutions at the end of the section and mark your work.

My Notes

Finding Angles Using the Cosine Ratio

You have just seen how an unknown side in a right triangle can be determined using the cosine ratio. You found the cosine ratio from the given measure of an angle. But can you go in the other direction? Can you find the measure of an angle from the cosine ratio?

For instance, suppose you were told that the cosine of an angle in a right triangle is 0.8. What is the angle?

Example 5

If cos A = 0.8, determine ∠A to the nearest degree.

Solution

Method 1: Draw a triangle and measure the angle with a protractor.

First we need to figure out what to draw.

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$
$$\cos A = 0.8$$

Therefore:

$$0.6 = \frac{\text{adj}}{\text{hyp}}$$

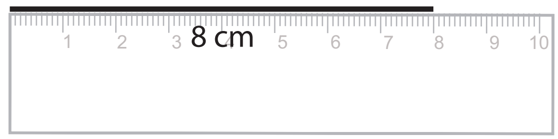
Now, if we let the hypotenuse be 10 cm long, how long would the adjacent side be?

$$\begin{aligned} 0.6 &= \frac{\text{adj}}{\text{hyp}} \\ 0.6 &= \frac{\text{adj}}{10} \\ (10)0.6 &= \frac{\text{adj}}{10}(10) \\ 6 &= \text{adj} \end{aligned}$$

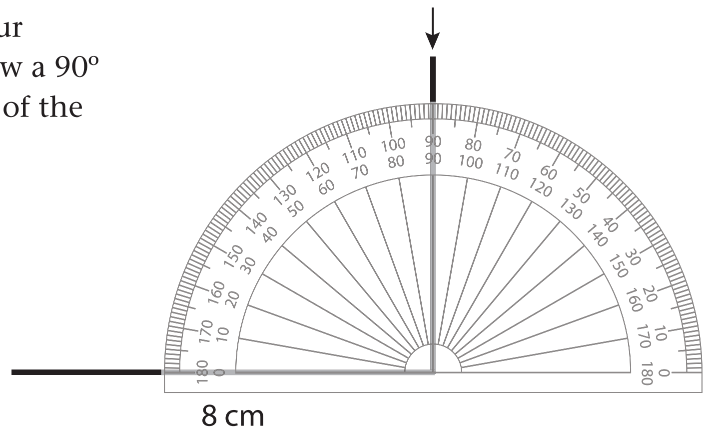
The adjacent side is 6 cm long.

My Notes

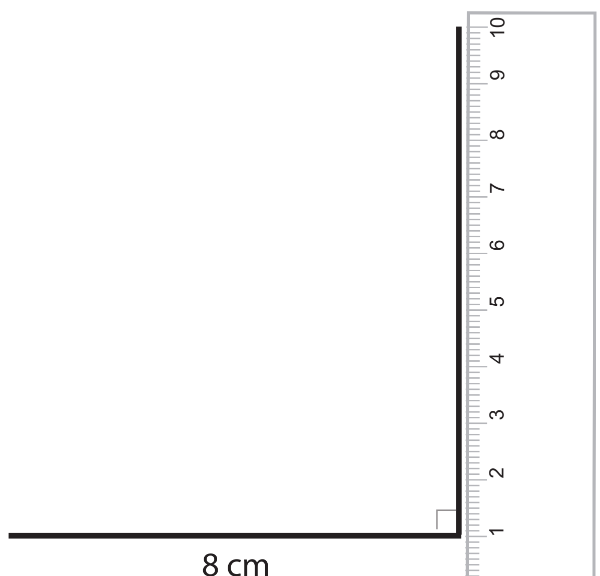
Now that we know the dimensions of two sides of the right triangle, we can draw it. Start with the adjacent side. Use a ruler to draw an 8 cm line.



Next, line up your protractor to draw a 90º angle at the end of the adjacent side.

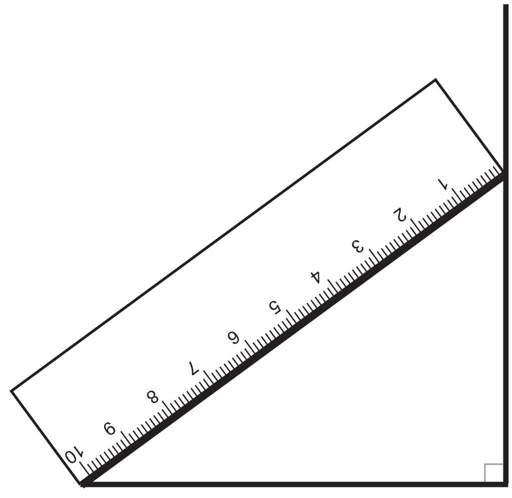


Now draw the opposite side. We don't yet know its length, so make sure you draw a long enough line to meet with the hypotenuse.

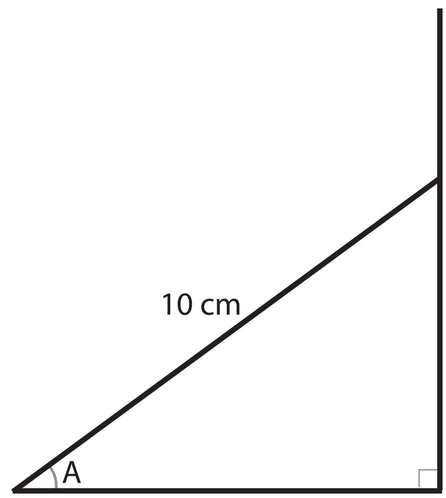


My Notes

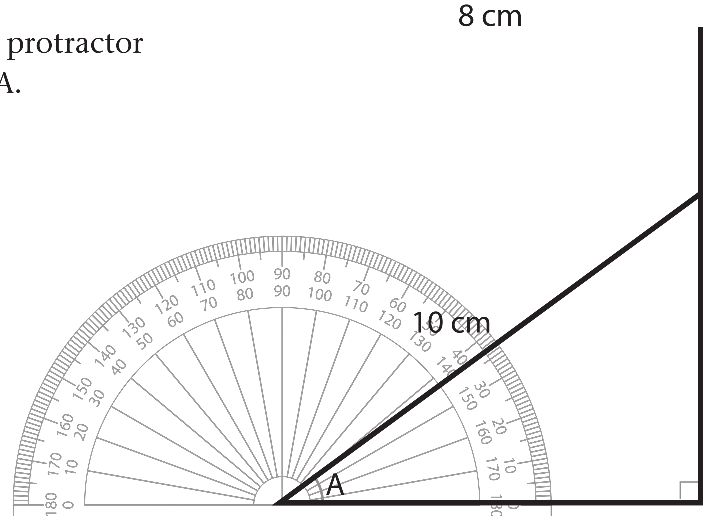
Using a ruler, draw the hypoteneuse: it is 10 cm long.



You now have a right triangle with ∠A.



You can use a protractor to measure ∠A.



My Notes

$\angle A \approx 37°$

Now we'll try another method to solve the problem and to check the accuracy of our drawing.

Method 2: Using a calculator to determine the angle.

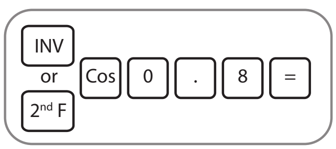
To find ∠A, we need to work backwards from the ratio to the angle.

$$\begin{aligned} \cos A &= 0.6 \\ \cos^{-1}(\cos A) &= \cos^{-1}(0.6) \\ \angle A &= \cos^{-1}(0.6) \end{aligned}$$

> We isolate a by taking the inverse sin of both sides.

Now, we'll use a calculator to solve for ∠A. Make sure your calculator is in "degree" mode. The order you press the buttons will depend on the calculator you have. Try this out on your calculator and make sure you get the same answer.

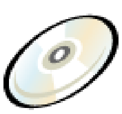
Check with your teacher or have a look at your instruction manual if you're not sure.



The answer is:

$$\begin{aligned} \angle A &= 36.869897\ldots \\ \angle A &\approx 37° \end{aligned}$$

This is the same answer we got by solving with Method 1.



To view the animated solution, go and look at *Cosine Solution* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/ma10_cosine.html*). The media piece demonstrates two ways you could solve the problem.

My Notes

Activity 6
Self-Check

Practise finding angles given the cosine ratio.

1. Find ∠A given cos A = 0.6.

   To find the angle, use both methods shown in Example 5.

   Method 1

Method 2

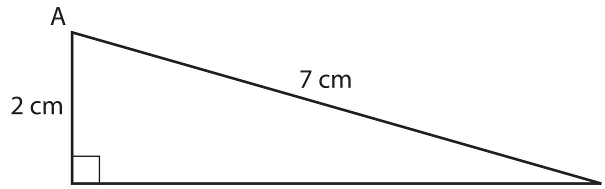
My Notes

2.  Use your calculator to find each angle from its cosine. Round your answers to the nearest tenth.

sin A	∠A
0.1257	
0.7826	
0.9000	
$\frac{2}{3}$	
$\frac{3}{4}$	

My Notes

3. Find ∠A to the nearest tenth of a degree.



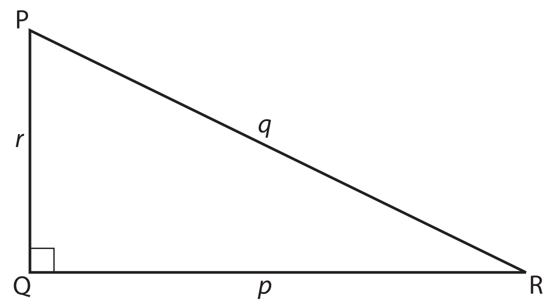
Turn to the solutions at the end of the section and mark your work.

Activity 7
Mastering Concepts

My Notes

Try this question. When you are finished, check your answer.

ΔPQR is a right triangle.



1. For ΔPQR write the ratios for sin P, cos P, sin R, and cos R.

2. Which ratios in Question 1 are equal to each other?

My Notes

3. How are ∠P and ∠R related?

Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes

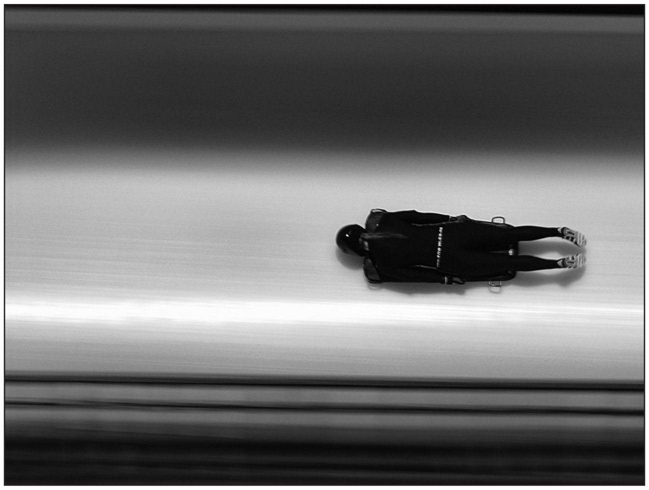


Photo by IPK Photography © 2010

The skeleton track built for the Vancouver 2010 Winter Olympics in Vancouver is 1450 metres long, and the steepest section is at corner 2. This corner has a 20% grade—this means the angle of elevation for the track is 11.3°.

Now that you have an understanding of trigonometry, you could have calculated this angle by measuring the height of the track and the distance along the ground. The engineers who built and designed this track would have had to make sure their trigonometry skills were sharp.

In this lesson, you explored the definition of the cosine function as the ratio of the side adjacent to the hypotenuse for a given acute angle in a right triangle. You used this ratio to find missing sides and missing angles.

Lesson B
Using Cosines to Solve Problems

To complete this lesson, you will need:

- an imperial tape measure
- a calculator

In this lesson, you will complete:

- 4 activities

Essential Questions

- How is the cosine ratio used to solve a variety of practical problems?

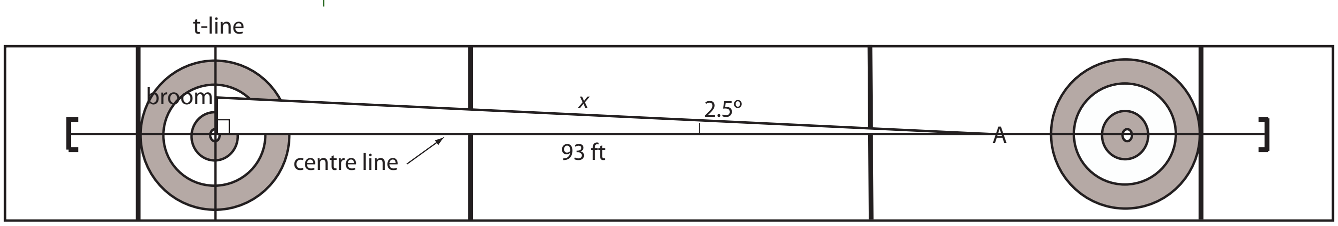
My Notes

Focus

Photo by corepics © 2010

Curling can be a game of inches. The difference between winning and losing can depend on the sweepers' ability to determine at what pressure and speed to sweep the rock. Success can also depend on how accurate the curler was when she released the stone.

From the point where a curler must release her rock to the centre of the rings at the other end of the rink is 93 ft. Suppose the skip positions her broom along the T-line in the house at 2.5° to the right of the centre line as seen from the curler who is throwing the stone.



How far is the broom, to the nearest inch, away from the point where the curler must release the stone?

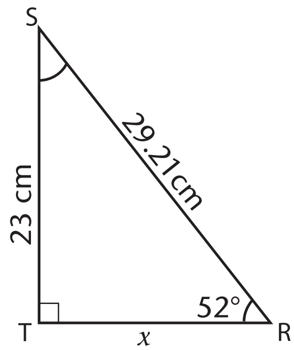
Get Started

In this activity you will review how the cosine ratio is used to find an unknown side or angle in a right triangle.

My Notes

Activity 1
Try This

1. Find the missing sidelength by filling in the blanks in the given calculations.



$$\cos R = \frac{\square}{\text{hypotenuse}}$$

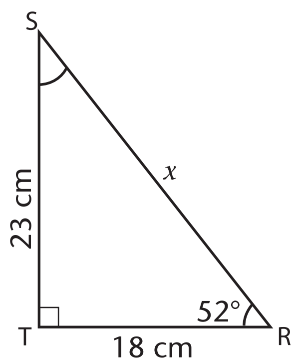
$$\cos 52° = \frac{x}{\square}$$

$$x = (\square)(\square)$$

$$x = 0.616(\square)$$

$$x = \square \text{ cm}$$

2. Find the missing sidelength by filling in the blanks in the given calculations.



$$\cos R = \frac{\text{adjacent}}{\square}$$

$$\text{hypotenuse} = \frac{\square}{\square}$$

$$y = \frac{\square}{\square}$$

$$y = \frac{\square}{0.616}$$

$$y = \square \text{ cm}$$

My Notes

3. Choose for your UNKNOWN the hypotenuse.

Record the drawing here:

Record the displayed calculations here:

4. Look at the steps for finding the two lengths in Questions 1 and 2. How are the calculations different?

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5. You may see the calculations for finding an angle are displayed as follows:

$$\cos R = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\cos R = \frac{12}{17.4}$$
$$\cos R = 0.69$$
$$\angle R = 46.4^\circ$$

Throughout this section, we have used the inverse of cosine to find angles. Add in the step that is missing that shows when to apply the inverse of cosine to find the angle.

My Notes

Turn to the solutions at the end of the section and mark your work.

My Notes

Explore

In this activity you will determine the angle between the diagonal of a rectangular room and one of its walls.

Activity 2
Try This

In this activity you will need an imperial tape measure and your calculator.

Step 1: Choose a rectangular room in your home or school. Choose a room that you can easily measure using your tape measure.

Step 2: Measure the length of one of the walls to the nearest inch. Then, measure the diagonal distance from one corner of the wall you measured, across the room, to the opposite corner. Again, measure to the nearest inch.

The measure of the length of the wall: ________ feet and

_________ inches

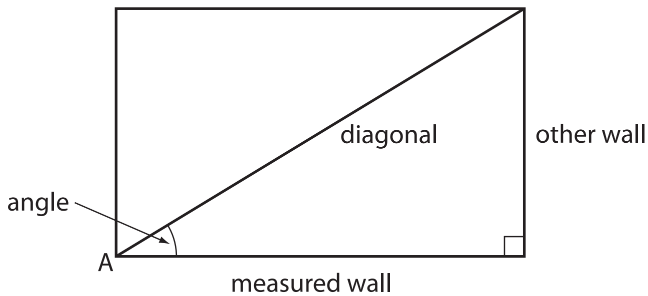
The measure of the diagonal distance: ________ feet and

_________ inches

My Notes

Questions

1. Label the diagram with your measurements. If the diagram does not quite model your situation, redraw the diagram and label your measurements on it. The angle you will try to find will be angle between the measured wall and the diagonal length.



2. Use the cosine ratio to calculate the angle between the measured wall and the diagonal. In the diagram, this angle has been labelled A. Show all steps.

My Notes

3. Because your measurements were in feet and inches, what did you have to do with these measurements before you could determine the angle?

____

____

____

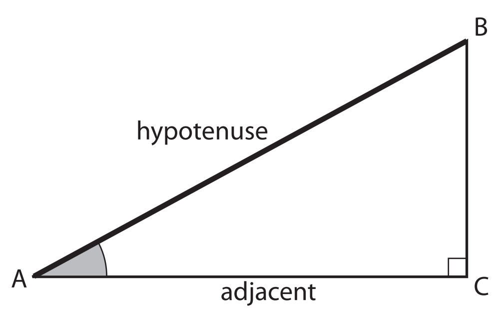
____

Turn to the solutions at the end of the section and mark your work.

Bringing Ideas Together

In the Explore, you worked with indirect measurement and the cosine ratio. You used the following relationship to find the angle between the adjacent side (the wall) and the hypotenuse (the diagonal).

$$\text{cosine } \angle A = \frac{\text{adjacent}}{\text{hypotenuse}} \text{ or } \cos A = \frac{\text{adj}}{\text{hyp}}$$



In the Explore, you used the relationship to find an unknown angle. However, this relationship can also be used in a variety of problem situations to determine the hypotenuse or the adjacent side if you are given the angle and one of these two sides.

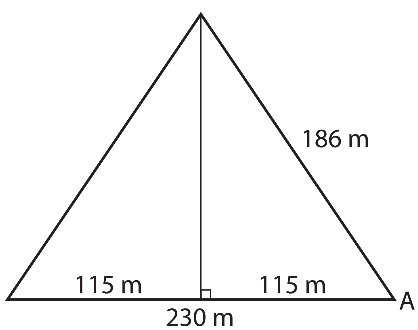
Example 1

Photo by Maksym Gorpenyuk © 2010

The Great Pyramid of Khufu at Giza was built more than 4500 years ago. Of the seven wonders of the ancient world, this pyramid is the only one still standing. It stands on a square base with a side length of approximately 230 m. The shortest distance from the apex of the pyramid to the ground along a slanted face of the pyramid is 186 m. That is, the slant height of the pyramid as measured along the middle of a face is 186 m. At what angle is the slanted face inclined from the horizontal? Round to the nearest tenth of a degree.

Solution

Draw a diagram of a cross-section of the pyramid.



The cross-sectional view of the pyramid can be split into two right triangles, each with a base of 115 m. Each base is 115 m since the larger triangle's base of 230 m was split in half (230 ÷ 2 = 115).

Let ∠A be the angle of elevation.

My Notes

My Notes

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{115}{186}$$

$$\angle A = \cos^{-1}\left(\frac{115}{186}\right)$$

$$\angle A = 51.80939189\ldots$$

The slanted side is inclined at approximately 51.8° from the horizontal.

Example 2

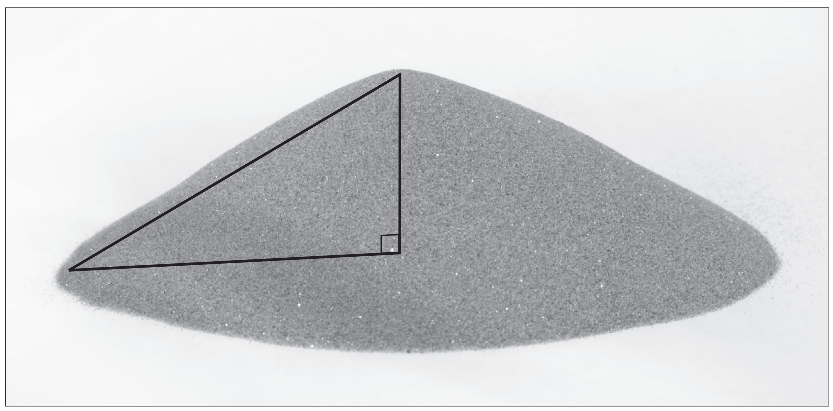
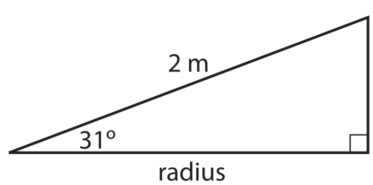


Photo by vita khorzheuska © 2010

A conical pile of dry sand has a slant side 2 m in length. If the angle the conical surface makes with the ground is 31°, what is the radius of the pile? Round to one decimal place.

Solution

Draw a diagram.



Let x be the length of the radius.

My Notes

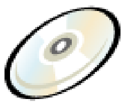
$$\cos 31^\circ = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 31^\circ = \frac{x}{2}$$

$$2(\cos 31^\circ) = x$$

$$1.714334601\ldots = x$$

The radius of the sand pile is approximately 1.7 m.



If you'd like to see another example, go and look at *Guy Wire Solution* (*http://media.openschool.bc.ca/osbcmedia/math/mathawm10/html/ma10_guywire.html*).

Activity 3
Self-Check

Apply the cosine ratio to find unknown sides and angles. Draw a diagram and show work whenever possible.

1. The Broadway Bridge in Portland, Oregon was constructed in 1913 over the Willamette River. Two spans, each 42 m in length, may be raised to an angle of 89° to the horizontal.

Photo by Charles Noble © 2010

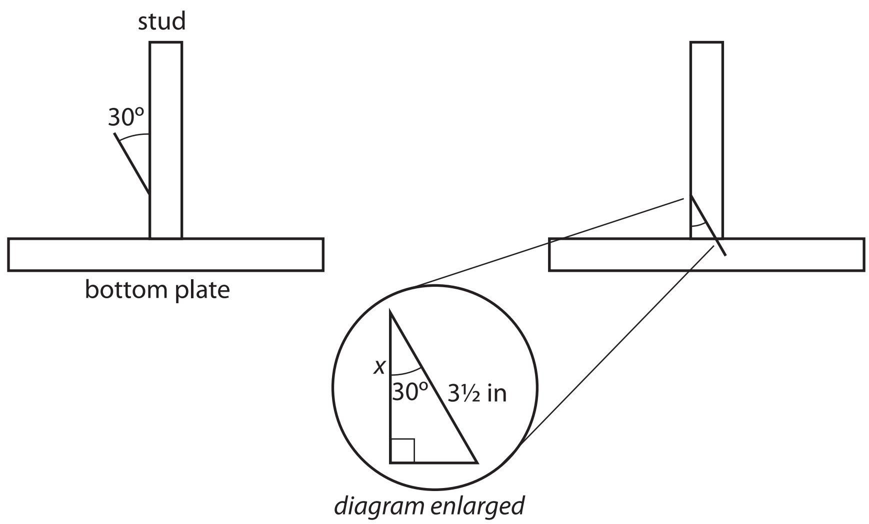
My Notes

In the photograph, one span is being raised. Suppose the span is raised to the full 89°. This scenario is represented by the following diagram.

Use the diagram to determine how far the upper end of the span would extend towards the river beyond the lower end. Calculate this distance, labeled *x* in the diagram, to one decimal place.

My Notes

2. Simon is framing his basement. To nail a vertical stud (two-by-four) to the horizontal bottom plate, Simon angles a $3\frac{1}{2}$ inch nail 30° to the vertical stud and 2 inches from the joint.



Simon says when the nail is driven in at 30°, the tip will be at least 1 inch below the surface of the bottom plate. Is he correct?

My Notes

3. Jim and Angus are part of a painting crew preparing to work on the exterior of an apartment building. They have just unloaded a 4-m ladder and are placing it against an exterior wall.

   Jim says, "I know that safety experts recommend that ladders be positioned at an angle of 75° with the ground. But how do I determine that angle?"

   Angus laughs, "Well, we don't have a clinometer in the truck, do we?"

   Can you help Jim and Angus by telling them how far to place the foot of the ladder from the wall? Show your solution.

4. If the foot of the ladder described in Question 3 were placed 1.5 m from the wall, what would the angle be between the ladder and the ground?

My Notes

My Notes

5. From the jungle canopy of Costa Rica to the Yukon Territory, zip lining is becoming a very popular activity. How long must a zip line be to carry people a horizontal distance of 250 m, if the angle of depression from the upper platform to the end of the line is 5°? Round to the nearest metre.

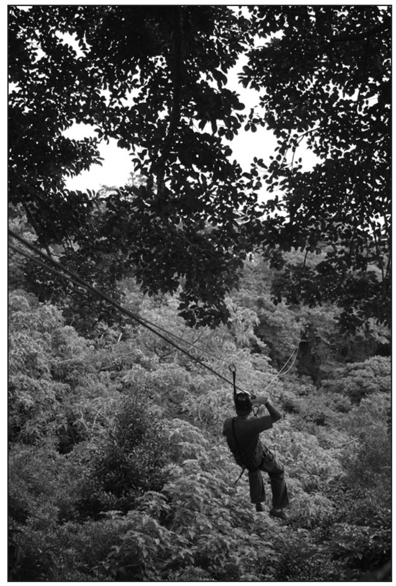
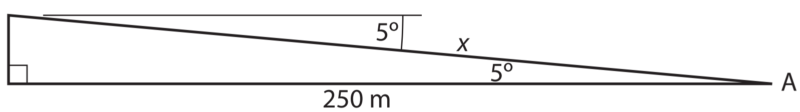


Photo by Brandon Stein © 2010



Turn to the solutions at the end of the section and mark your work.

My Notes

Activity 4
Mastering Concepts

As shown in the photograph, a lean-to is built against one wall of a barn.

Photo by Bellajay © 2010

The roof of the lean-to is 30 feet long. Its horizontal width is 12 feet. As shown in the diagram, the roof is inclined at 20° to the horizontal.

To the nearest square foot, what is the area of the lean-to's roof? Use the space on the next page for your calculations.

My Notes

Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes

Photo by Phillip Durand © 2010

Curling is a game of skill and strategy. Placing guards (putting rocks in the way of opposition rocks), deciding where to position stones in the house (the red, white, and blue circled area), or determining whether to remove an opponent's stone are some decisions to make while curling.

A key to winning or losing in curling hinges on the ability to carry out decisions. This includes placing the rocks accurately when they are thrown. Having an understanding of the physics and geometry of the path a rock takes, gives players a great advantage.

In this lesson, you reviewed the definition of the cosine function as the ratio of the side adjacent to the hypotenuse for a given acute angle in a right triangle. You used this ratio to find missing sides and missing angles in a variety of practical problems.

Lesson C
General Problems

To complete this lesson, you will need:

- a calculator
- Data Pages

In this lesson, you will complete:

- 4 activities

Essential Questions

- How are the appropriate trigonometric ratios identified and applied to a variety of practical problems?

My Notes

Focus

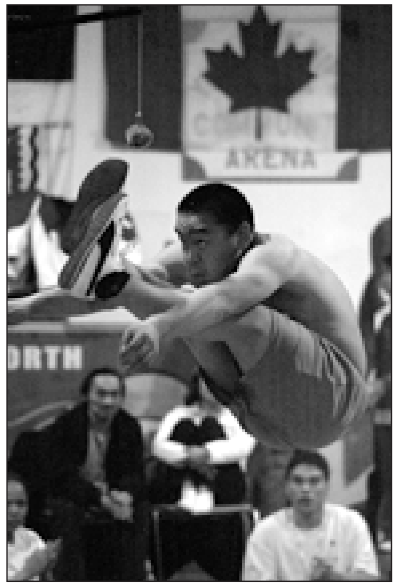


Photo by Xander © 2010

At the 2010 Winter Olympics in Vancouver, athletes from Canada's Arctic demonstrated traditional Inuit and Dene games. In the past, these games, involving strength and agility, helped prepare northerners to survive in their harsh environment.

In this photograph, the athlete is competing in the two-foot high kick (*Akratcheak*).

In this event, the participant can either launch himself from a running or standing position with his legs no more than shoulder-width apart. With his legs parallel, the athlete must strike the suspended object with both feet. He must then land on both feet positioned no more than a shoulder-width apart.

Through what angle from the vertical would you estimate this athlete has swung his legs to strike the suspended object? Would it be possible to draw a right triangle that contained this angle? Why, or why not?

Get Started

My Notes

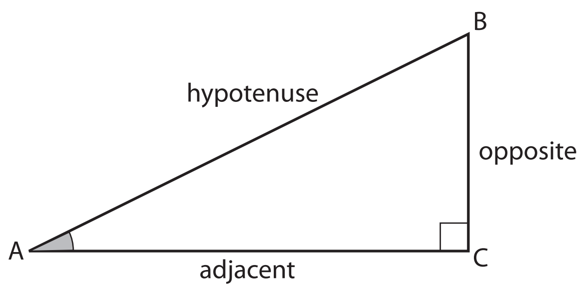
How can you keep track of the three trigonometric ratios and what sides belong in what ratios?

One way is to look at your Data Pages. On the first page you'll see a box labelled "Trigonometry." In it you'll find the formulas for each trig ratio as well as the formula for the Pythagorean Theorem.

While you can use the data pages on your test, it is helpful to be familiar with the formulas. Being able to automatically recall the formulas will help you solve problems more efficiently.

Let's look at some tricks you can use to remember the trig functions.

Remembering Trig Formulas



Do you remember how the sides of the right triangle are identified in relation to ∠A? What are the definitions for sin A, cos A, and tan A?

$\sin A = \frac{opp}{hyp}$ $\cos A = \frac{adj}{hyp}$ $\tan A = \frac{opp}{adj}$

The capitalized letters from the definitions above, spell **SOH-CAH-TOA**. This is a mnemonic to help you remember which sides are related to which trigonometric ratios.

This word is pronounced: "Soak a Toe! Ahhh."

My Notes

For some people, just the word SOH-CAH-TOA is enough to help them remember the trig ratios. Others find that attaching words to each letter of SOH-CAH-TOA helps them remember. No one method is best. You must find one that works for you!

A student named Randy created this mnemonic to help him remember SOH-CAH-TOA:

SOH	**S**askatoon **O**ur **H**ome	(Randy lives in Saskatoon.)
CAH	**C**algary **A**lberta **H**ockey	(The Calgary Flames are Randy's favourite team.)
TOA	**T**oronto **O**ntario **A**unt	(Randy's favourite aunt lives in Toronto.)

Randy's choices might not be the ones you would make. However, you can use Randy's method to pick words you will easily remember.

Create a mnemonic to help you remember SOH-CAH-TOA. Record your mnemonic here.

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Explore

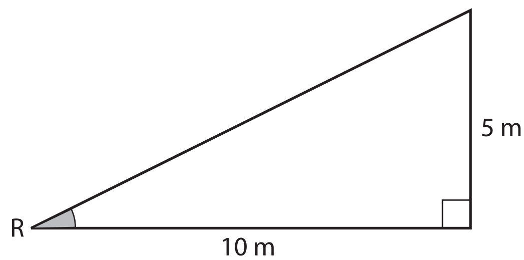
In previous lessons, questions were grouped by trigonometric ratio. But suppose you are given a general problem involving one or more trigonometric ratios. How do you decide which ratio to use?

In this activity you will review all three trigonometric ratios—sine, cosine, and tangent.

Activity 1
Try This

My Notes

1.



a. If ∠R is the reference angle:

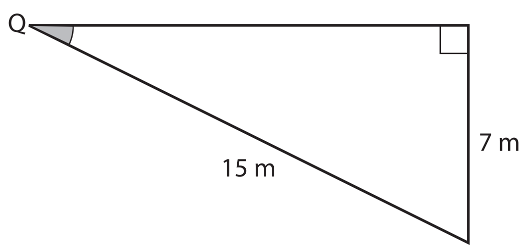
5 m is the ______________ side.

10 m is the ______________ side.

b. When calculating ∠R, the ratio to use when you are given the opposite and adjacent side lengths is:

$\boxed{} = \frac{\boxed{}}{\boxed{}}$

2.



a. If ∠Q is the reference angle:

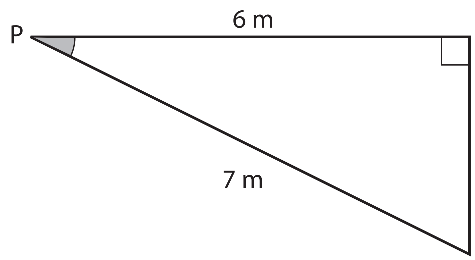
7 m is the ______________ side.

15 m is the ______________ side.

b. When calculating ∠Q, the ratio to use when you are given the length of the opposite side and the hypotenuse is:

$\boxed{} = \frac{\boxed{}}{\boxed{}}$

My Notes

3. 

a. If ∠P is the reference angle:

6 m is the ______________ side.

7 m is the ______________ side.

b. When calculating ∠P, the ratio to use when you are given the length of the adjacent side and the hypotenuse is:

$\boxed{} = \dfrac{\boxed{}}{\boxed{}}$

4. Which ratio do you find easiest to remember? Why? Did your mnemonic from the activity in the "Get Started" help you remember the ratios?

____

____

____

____

5. Which do you find easier: finding angles or finding sides? Why?

____

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____

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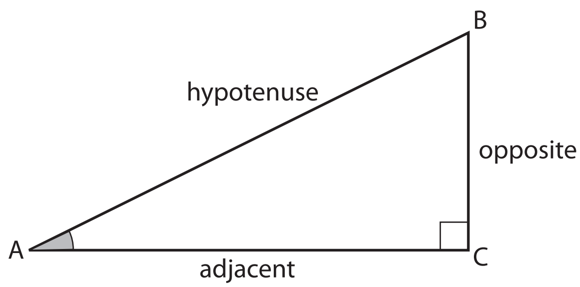
Turn to the solutions at the end of the section and mark your work.

Bringing Ideas Together

My Notes

In the Explore, you applied the definitions of sine, cosine, and tangent to find missing sides and angles in an interactive game of minigolf.



$\sin A = \frac{\text{opp}}{\text{hyp}} \quad \cos A = \frac{\text{adj}}{\text{hyp}} \quad \tan A = \frac{\text{opp}}{\text{adj}}$

Did you use SOH-CAH-TOA to help you recall these definitions?

You will now apply these definitions in problem situations where you have to find all the missing side lengths and angle measures in a triangle. This is called **solving a right triangle**.

Solving a Right Triangle

Let's work through some examples of how you can "solve a triangle."

In Example 1 you are given the measure of one side and one angle of a right triangle. From this information you will find the missing sides and angles in the triangle.

In Example 2 you are given the measure of two sides in a right triangle. From this information you will find the missing side and all the unknown angles in the triangle.

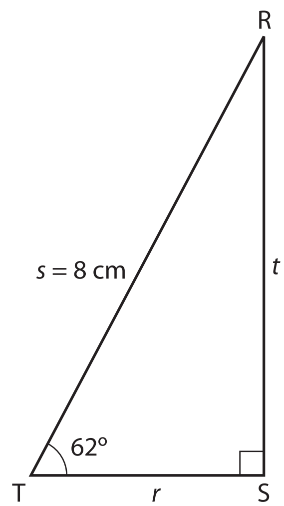
My Notes

Example 1

In ΔRST, ∠S = 90°, ∠T = 62°, and s = 8 cm. Solve ΔRST. Round to one decimal place when appropriate.

Solution

Draw a diagram and label it with the information you are given.



You could start this problem by creating a table to see what you know and what you don't know.

∠R	
∠T	62°
∠S	90°
Side s	8 cm
Side r	
Side t	

To solve this triangle, you must find one angle (∠R) and two sides (r and t).

My Notes

Step 1: Start by finding the missing angle.

Since you are given two angles, you can find the third angle by recalling that all the angles in a triangle add up to 180°.

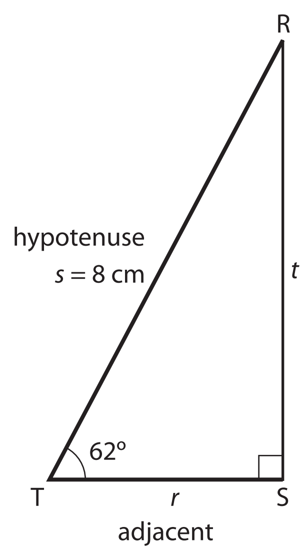
$$\angle R + \angle T + \angle S = 180°$$
$$\angle R + 62° + 90° = 180°$$
$$\angle R = 180° - 62° - 90°$$
$$\angle R = 28°$$

Step 2: Find the missing lengths.

You can find either length first, so let's start with side *r*.

Side *r* is adjacent to the given angle, ∠T. You know the hypotenuse, *s*. Don't label side *t*. You only want to have two sides labelled, so that you can determine which trigonometric ratio to use.

Since the "adjacent" side and the "hypotenuse" are labelled you use the cosine ratio. Or using the mnemonic SOH-CAH-TOA circle the part that has the letters A and H.



SOH-(CAH)-TOA

My Notes

Set up an equation using the cosine ratio.

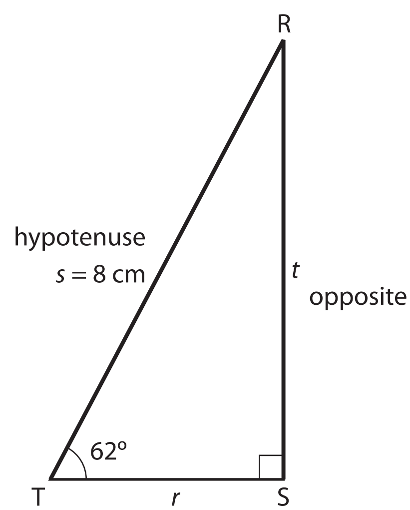
$$\cos T = \frac{\text{adj}}{\text{hyp}}$$
$$\cos 62^\circ = \frac{r}{8}$$
$$8(\cos 62^\circ) = r$$
$$3.755772502\ldots = r$$

Side r is approximately 3.8 cm.

Lastly, let's find side t.

Side t is opposite to the given angle, ∠T. You know the hypotenuse, s. Don't label side r. You only want to have two sides labelled, so that you can determine which trigonometric ratio to use.

Since the "opposite" side and the "hypotenuse" are labelled you use the sine ratio. Or using the mnemonic SOH-CAH-TOA circle the part that has the letters O and H.



SOH-CAH-TOA

My Notes

Set up an equation using the sine ratio.

$$\sin \text{T} = \frac{\text{opp}}{\text{hyp}}$$
$$\sin 62^\circ = \frac{r}{8}$$
$$8(\sin 62^\circ) = r$$
$$7.063587043\ldots = r$$

Side t is approximately 7.1 cm.

Note: The side t could have also been found by using the lengths of the two sides and the Pythagorean theorem.

Recall, that the Pythagorean Theorem states that, in a right triangle, the sum of the squares of the side lengths, equals the length of the hypotenuse squared.

For this scenario, this means:

$$r^2 + t^2 = s^2$$
$$3.8^2 + t^2 = 8^2$$
$$14.44 + t^2 = 64$$
$$t^2 = 64 - 14.44$$
$$t^2 = 49.56$$
$$t = \sqrt{49.56}$$
$$t = 7.039886362719\ .\ .\ .$$

Using this different method, side t is approximately 7.0 cm. You'll notice that using this method resulted in a slightly different answer. This is because we used the rounded value of 3.8 for side r. To get a more exact solution, we should have kept more decimal places for the value of r. Try the calculation using 3.755772502 instead of 3.8. Does it make a difference?

____

____

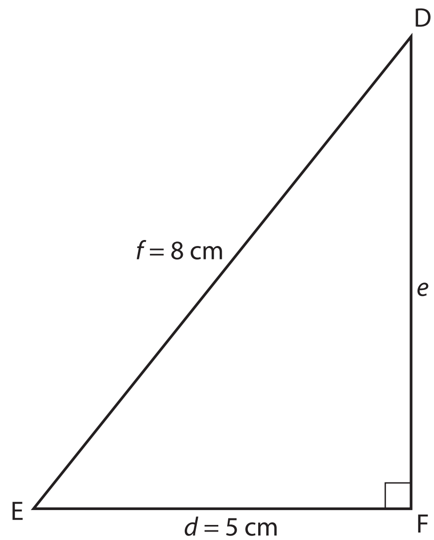
My Notes

Example 2

In ΔDEF, ∠F = 90°, d = 5 cm, and f = 8 cm. Solve ΔDEF. Round to one decimal place when appropriate.

Solution

Draw a diagram and label it with the information you are given.



You could start this problem by creating a table to see what you know and what you don't know.

∠D	
∠E	
∠F	90°
Side d	5 cm
Side e	
Side f	8 cm

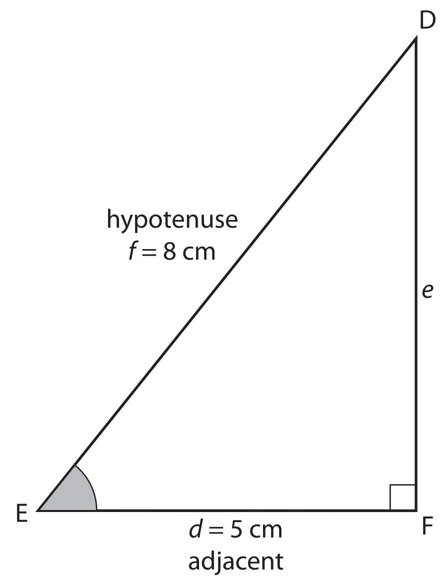
To solve this triangle, you must find two angles (∠D and ∠E) and one side (e).

Step 1: Start by finding the missing angles.

You can find either angle first, so let's start with ∠E.

To find an angle, you need to know two sides. The side, *d*, is adjacent to ∠E and the side, *f*, is the hypotenuse. Don't label side *e*. You only want to have two sides labelled, so that you can determine which trigonometric ratio to use.

Since the "adjacent" side and the "hypotenuse" are labelled you use the cosine ratio. Or using the mnemonic SOH-CAH-TOA circle the part that has the letters A and H.



SOH-(CAH)-TOA

Set up an equation using the cosine ratio.

$$\cos E = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos E = \frac{5}{8}$$
$$\angle E = \cos^{-1}\left(\frac{5}{8}\right)$$
$$\angle E = 51.317812546\ldots$$

My Notes

My Notes

Angle E is approximately 51.3°.

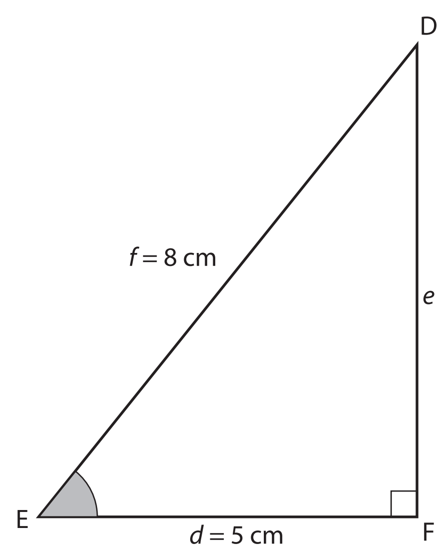
Now, that you know two angles in the triangle the third can be found by using the rule that all three angles in a triangle add to 180°.

$\angle D = 180° - 90° - 51.3° = 38.7°$

Step 2: Find the missing length.

Since two sides are given in this problem, you can use the Pythagorean Theorem to find side *e*.

Recall, that the Pythagorean Theorem states that in a right triangle the lengths of the sides squared and then added together, equals the length of the hypotenuse squared.



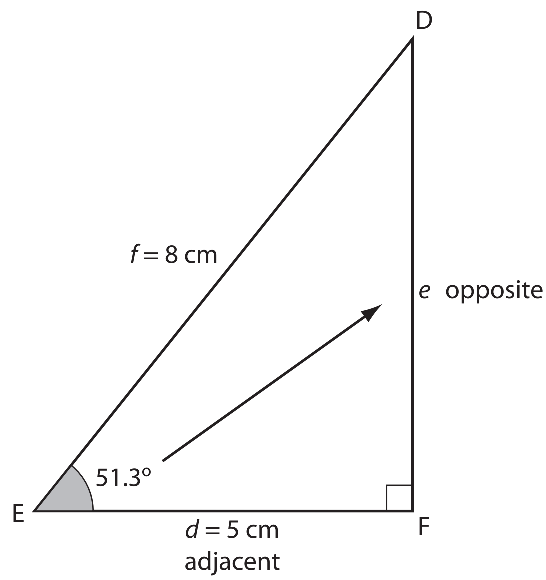
For this scenario, this means:

$$d^2 + e^2 = f^2$$
$$5^2 + e^2 = 8^2$$
$$25 + e^2 = 64$$
$$e^2 = 64 - 25$$
$$e^2 = 39$$
$$e = \sqrt{39}$$
$$e = 6.24499799\ldots$$

The length of *e* is approximately 6.2 cm.

My Notes

Note: The side *e* could have also been found by using a trigonometric ratio. You would need an angle and a length in the triangle to do this. Don't label the third side. You only want to have two sides labelled, so that you can determine which trigonometric ratio to use.



Since the "opposite" side and the "adjacent" side are labelled you use the tangent ratio. Or using the mnemonic SOH-CAH-TOA circle the part that has the letters O and A.

SOH-**CAH**-TOA

Set up an equation using the tangent ratio.

$$\begin{aligned} \tan 51.3° &= \frac{\text{opposite}}{\text{hypotenuse}} \\ \tan 51.3° &= \frac{e}{5} \\ 5(\tan 51.3°) &= e \\ 6.2410201817\ldots &= e \end{aligned}$$

Using this different method, the result is the same. The length of *e* is approximately 6.2 cm.

My Notes

As you may have noticed, there are many ways to solve these problems. Depending on the order you solved for the unknowns, you may have chosen different trigonometry ratios to find the unknowns. Ultimately, whatever methods are used, the final solutions for the unknowns will be the same.

Activity 2
Self-Check

Solve each right triangle.

1. In ΔPQR, ∠Q = 90°, p = 5 cm, and r = 12 cm. Solve ΔPQR. Round to one decimal place when appropriate.

2. In ΔABC, ∠C = 90°, ∠A = 30°, and a = 6 cm. Solve ΔABC. Round to one decimal place when appropriate.

My Notes

Turn to the solutions at the end of the section and mark your work.

My Notes

Deciding on a Trigonometric Ratio

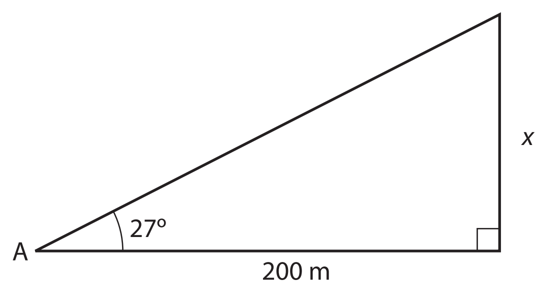
In most practical trigonometric problems you will not be trying to find all the missing side lengths and angle measure. You'll have to look at the problem and decide which trig ratio to use to solve it.

Example 3

Eldon is watching a hot air balloon drifting over a cell phone tower 200 m away. Using his clinometer, Eldon measures the angle of elevation of the balloon. Find the height, to the nearest metre, of the balloon if the angle of elevation is 27°.

Solution

Draw a diagram.

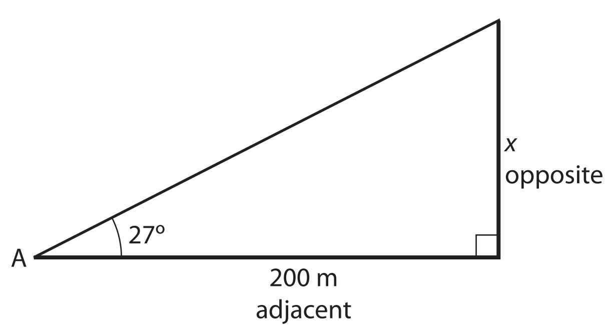


Let *x* be the height of the balloon.

The unknown side, *x*, is *opposite* the angle of elevation (27°).

The given side (the distance to the balloon) is the side *adjacent* to the angle of elevation.

Since the opposite side and the adjacent side will be used in this problem, you will use the tangent ratio to set up an equation.



SOH-CAH-**TOA**

$$\tan 27° = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 27° = \frac{x}{200}$$
$$200(\tan 27°) = x$$
$$101.9050899\ldots = x$$

The balloon is approximately 102 m above the ground.

My Notes

Activity 3
Self-Check

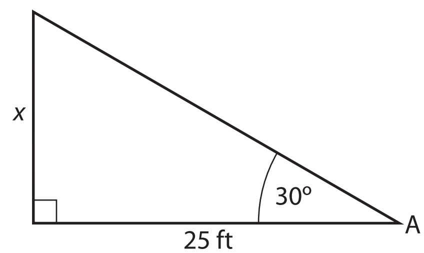
Solve each problem by first drawing a diagram.

1. The funicular railway (*Funiculaire du Vieux-Québec*) in Quebec City, built in 1879, carries passengers from the historic district of the lower town to the upper town. It is 64 m in length with a vertical height of 59 m. To the nearest degree, what is the angle at which the tracks are inclined to the horizontal? Use the next page to solve this problem.

Photo courtesy of Funiculaire du Vieux-Québec

My Notes

2. A garden plot in the corner of Iris's yard is in the shape of a right triangle as illustrated. What is the plot's area? Round your answer to the nearest square foot.



My Notes

My Notes

3. The transcontinental railroad in Canada was competed in 1885. Tunnels were constructed inside the mountains to lengthen the track and reduce the grade's steepness.

   A train travelling east and entering the Lower Tunnel travels 891 m before exiting the tunnel 15 m higher than when it entered. What is the Lower Tunnel's average slope to the nearest tenth of a degree?

Turn to the solutions at the end of the section and mark your work.

Activity 4
Mastering Concepts

My Notes

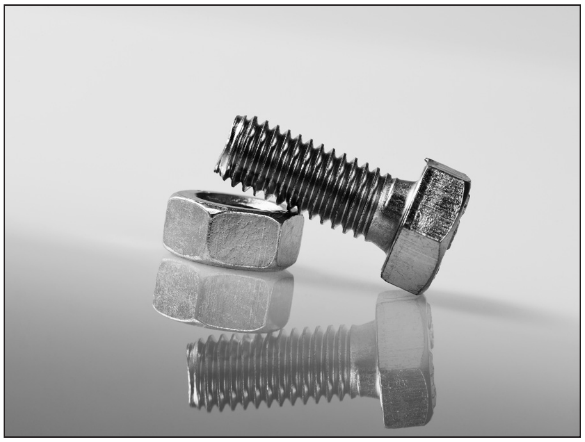
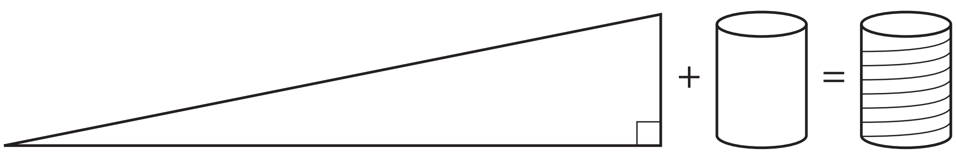


Photo by Diego Cervo © 2010

The spiral threads on a bolt may be modelled by wrapping a paper right triangle around a cylinder.



A bolt with a 1-inch diameter and a coarse thread, similar to the one in the photograph, must be turned all the way eight times around to advance the bolt 1 inch onto the nut. What is the slope to the nearest tenth of a degree of the threads? Use the next page to solve this problem.

My Notes

Turn to the solutions at the end of the section and mark your work.

Lesson Summary

My Notes

Photo by Greg Epperson © 2010

In the photograph the climber has reached the summit. If you have ever been rock climbing, competed successfully in sporting events, or achieved some other personal victory, you know the pride this young climber feels in his accomplishments.

Whether it is competing in sports, or successfully mastering new trigonometric skills, take pride in everything you do!

In this lesson, you reviewed the definition of the sine, cosine, and tangent functions. You used these ratios to solve for unknown sides and angles in right triangles and to investigate a variety of practical problems.

Trigonometry II— Appendix

Data Pages 81
Solutions 89
Glossary 107

TABLE OF CONVERSIONS

1 inch	$\approx$	2.54 centimetres
1 foot	$\approx$	30.5 centimetres
1 foot	$\approx$	0.305 metres
1 foot	=	12 inches
1 yard	=	3 feet
1 yard	$\approx$	0.915 metres
1 mile	=	1760 yards
1 mile	$\approx$	1.6 kilometres
1 kilogram	$\approx$	2.2 pounds
1 litre	$\approx$	1.06 US quarts
1 litre	$\approx$	0.26 US gallons
1 gallon	$\approx$	4 quarts
1 British gallon	$\approx$	$\frac{6}{5}$ US gallon

FORMULAE

Temperature
$C = \frac{5}{9}(F - 32)$

Trigonometry

(Put your calculator in Degree Mode)

- Right triangles

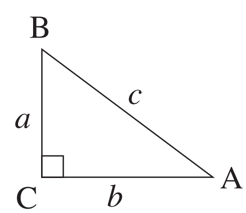
  Pythagorean Theorem

  $$a^2 + b^2 = c^2$$

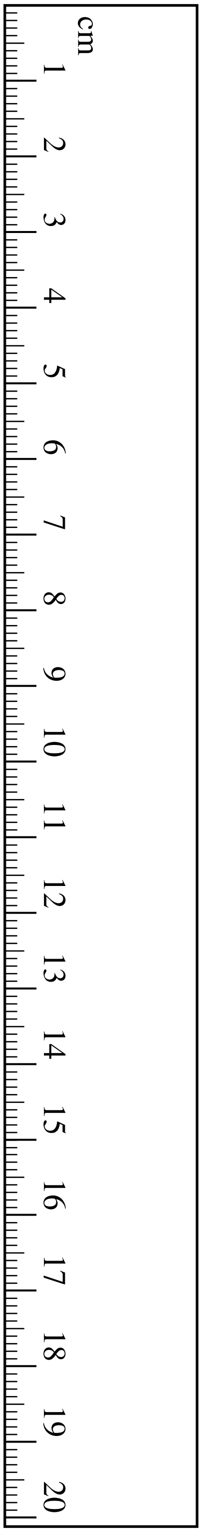
  $$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

  $$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

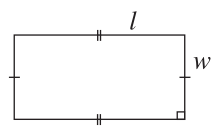
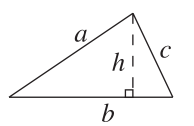
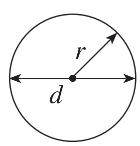
  $$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$



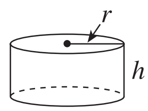
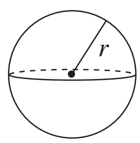
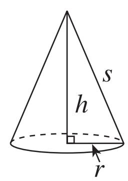
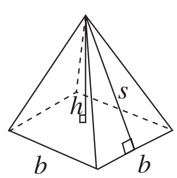
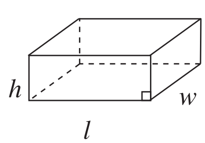
GEOMETRIC FORMULAE

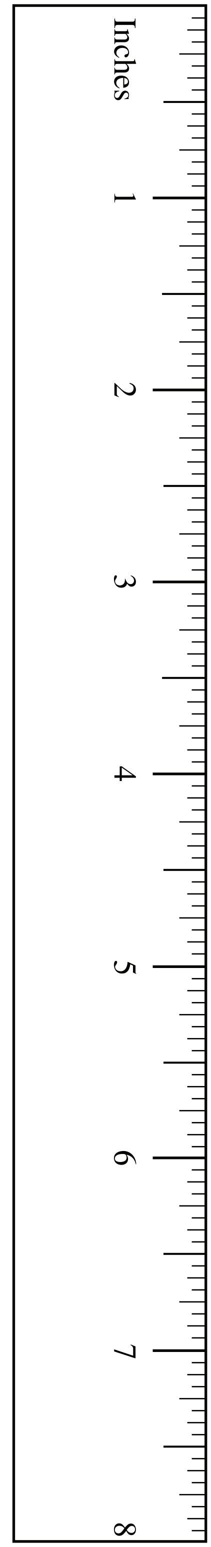


Key Legend	
l = length	P = perimeter
w = width	C = circumference
b = base	A = area
h = height	SA = surface area
s = slant height	V = volume
r = radius	
d = diameter	

Geometric Figure	Perimeter	Area
Rectangle 	$P = 2l + 2w$ **or** $P = 2(l + w)$	$A = lw$
Triangle 	$P = a + b + c$	$A = \frac{bh}{2}$
Circle 	$C = \pi d$ **or** $C = 2\pi r$	$A = \pi r^2$

Note: Use the value of π programmed in your calculator rather than the approximation of 3.14.

Geometric Figure	Surface Area
Cylinder 	$A_{top} = \pi r^2$ $A_{base} = \pi r^2$ $A_{side} = 2\pi rh$ $SA = 2\pi r^2 + 2\pi rh$
Sphere 	$SA = 4\pi r^2$ **or** $SA = \pi d^2$
Cone 	$A_{side} = \pi rs$ $A_{base} = \pi r^2$ $SA = \pi r^2 + \pi rs$
Square-Based Pyramid 	$A_{triangle} = \frac{1}{2}bs$ (for each triangle) $A_{base} = b^2$ $SA = 2bs + b^2$
Rectangular Prism 	$SA = wh + wh + lw + lw + lh + lh$ **or** $SA = 2(wh + lw + lh)$
General Right Prism	SA = the sum of the areas of all the faces
General Pyramid	SA = the sum of the areas of all the faces



Note: Use the value of **π** programmed in your calculator rather than the approximation of 3.14.

Canada Pension Plan Contributions | **Cotisations au Régime de pensions du Canada**

Weekly (52 pay periods a year) | Hebdomadaire (52 périodes de paie par année)

Pay Rémunération From - De	To - À	CPP RPC	Pay Rémunération From - De	To - À	CPP RPC	Pay Rémunération From - De	To - À	CPP RPC	Pay Rémunération From - De	To - À	CPP RPC
358.11	358.31	14.40	372.66	372.85	15.12	387.20	387.40	15.84	401.75	401.94	16.56
358.32	358.51	14.41	372.86	373.05	15.13	387.41	387.60	15.85	401.95	402.14	16.57
358.52	358.71	14.42	373.06	373.25	15.14	387.61	387.80	15.86	402.15	402.35	16.58
358.72	358.91	14.43	373.26	373.46	15.15	387.81	388.00	15.87	402.36	402.55	16.59
358.92	359.11	14.44	373.47	373.66	15.16	388.01	388.20	15.88	402.56	402.75	16.60
359.12	359.32	14.45	373.67	373.86	15.17	388.21	388.41	15.89	402.76	402.95	16.61
359.33	359.52	14.46	373.87	374.06	15.18	388.42	388.61	15.90	402.96	403.15	16.62
359.53	359.72	14.47	374.07	374.26	15.19	388.62	388.81	15.91	403.16	403.36	16.63
359.73	359.92	14.48	374.27	374.47	15.20	388.82	389.01	15.92	403.37	403.56	16.64
359.93	360.12	14.49	374.48	374.67	15.21	389.02	389.21	15.93	403.57	403.76	16.65
360.13	360.33	14.50	374.68	374.87	15.22	389.22	389.42	15.94	403.77	403.96	16.66
360.34	360.53	14.51	374.88	375.07	15.23	389.43	389.62	15.95	403.97	404.16	16.67
360.54	360.73	14.52	375.08	375.27	15.24	389.63	389.82	15.96	404.17	404.37	16.68
360.74	360.93	14.53	375.28	375.48	15.25	389.83	390.02	15.97	404.38	404.57	16.69
360.94	361.13	14.54	375.49	375.68	15.26	390.03	390.22	15.98	404.58	404.77	16.70
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361.95	362.14	14.59	376.50	376.69	15.31	391.04	391.23	16.03	405.59	405.78	16.75
362.15	362.35	14.60	376.70	376.89	15.32	391.24	391.44	16.04	405.79	405.98	16.76
362.36	362.55	14.61	376.90	377.09	15.33	391.45	391.64	16.05	405.99	406.18	16.77
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362.76	362.95	14.63	377.30	377.50	15.35	391.85	392.04	16.07	406.40	406.59	16.79
362.96	363.15	14.64	377.51	377.70	15.36	392.05	392.24	16.08	406.60	406.79	16.80
363.16	363.36	14.65	377.71	377.90	15.37	392.25	392.45	16.09	406.80	406.99	16.81
363.37	363.56	14.66	377.91	378.10	15.38	392.46	392.65	16.10	407.00	407.19	16.82
363.57	363.76	14.67	378.11	378.31	15.39	392.66	392.85	16.11	407.20	407.40	16.83
363.77	363.96	14.68	378.32	378.51	15.40	392.86	393.05	16.12	407.41	407.60	16.84
363.97	364.16	14.69	378.52	378.71	15.41	393.06	393.25	16.13	407.61	407.80	16.85
364.17	364.37	14.70	378.72	378.91	15.42	393.26	393.46	16.14	407.81	408.00	16.86
364.38	364.57	14.71	378.92	379.11	15.43	393.47	393.66	16.15	408.01	408.20	16.87
364.58	364.77	14.72	379.12	379.32	15.44	393.67	393.86	16.16	408.21	408.41	16.88
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364.98	365.17	14.74	379.53	379.72	15.46	394.07	394.26	16.18	408.62	408.81	16.90
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365.39	365.58	14.76	379.93	380.12	15.48	394.48	394.67	16.20	409.02	409.21	16.92
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365.99	366.18	14.79	380.54	380.73	15.51	395.08	395.27	16.23	409.63	409.82	16.95
366.19	366.39	14.80	380.74	380.93	15.52	395.28	395.48	16.24	409.83	410.02	16.96
366.40	366.59	14.81	380.94	381.13	15.53	395.49	395.68	16.25	410.03	410.22	16.97
366.60	366.79	14.82	381.14	381.34	15.54	395.69	395.88	16.26	410.23	410.43	16.98
366.80	366.99	14.83	381.35	381.54	15.55	395.89	396.08	16.27	410.44	410.63	16.99
367.00	367.19	14.84	381.55	381.74	15.56	396.09	396.28	16.28	410.64	410.83	17.00
367.20	367.40	14.85	381.75	381.94	15.57	396.29	396.49	16.29	410.84	411.03	17.01
367.41	367.60	14.86	381.95	382.14	15.58	396.50	396.69	16.30	411.04	411.23	17.02
367.61	367.80	14.87	382.15	382.35	15.59	396.70	396.89	16.31	411.24	411.44	17.03
367.81	368.00	14.88	382.36	382.55	15.60	396.90	397.09	16.32	411.45	411.64	17.04
368.01	368.20	14.89	382.56	382.75	15.61	397.10	397.29	16.33	411.65	411.84	17.05
368.21	368.41	14.90	382.76	382.95	15.62	397.30	397.50	16.34	411.85	412.04	17.06
368.42	368.61	14.91	382.96	383.15	15.63	397.51	397.70	16.35	412.05	412.24	17.07
368.62	368.81	14.92	383.16	383.36	15.64	397.71	397.90	16.36	412.25	412.45	17.08
368.82	369.01	14.93	383.37	383.56	15.65	397.91	398.10	16.37	412.46	412.65	17.09
369.02	369.21	14.94	383.57	383.76	15.66	398.11	398.31	16.38	412.66	412.85	17.10
369.22	369.42	14.95	383.77	383.96	15.67	398.32	398.51	16.39	412.86	413.05	17.11
369.43	369.62	14.96	383.97	384.16	15.68	398.52	398.71	16.40	413.06	413.25	17.12
369.63	369.82	14.97	384.17	384.37	15.69	398.72	398.91	16.41	413.26	413.46	17.13
369.83	370.02	14.98	384.38	384.57	15.70	398.92	399.11	16.42	413.47	413.66	17.14
370.03	370.22	14.99	384.58	384.77	15.71	399.12	399.32	16.43	413.67	413.86	17.15
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371.04	371.23	15.04	385.59	385.78	15.76	400.13	400.33	16.48	414.68	414.87	17.20
371.24	371.44	15.05	385.79	385.98	15.77	400.34	400.53	16.49	414.88	415.07	17.21
371.45	371.64	15.06	385.99	386.18	15.78	400.54	400.73	16.50	415.08	415.27	17.22
371.65	371.84	15.07	386.19	386.39	15.79	400.74	400.93	16.51	415.28	415.48	17.23
371.85	372.04	15.08	386.40	386.59	15.80	400.94	401.13	16.52	415.49	415.68	17.24
372.05	372.24	15.09	386.60	386.79	15.81	401.14	401.34	16.53	415.69	415.88	17.25
372.25	372.45	15.10	386.80	386.99	15.82	401.35	401.54	16.54	415.89	416.08	17.26
372.46	372.65	15.11	387.00	387.19	15.83	401.55	401.74	16.55	416.09	416.28	17.27

Employee's maximum CPP contribution for the year 2009 is $2,118.60 | B-6 | La cotisation maximale de l'employé au RPC pour l'année 2009 est de 2 118,60 $

Employment Insurance Premiums — Cotisations à l'assurance-emploi

Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE	Insurable Earnings Rémunération assurable From - De	To - À	EI premium Cotisation d'AE
333.24	333.81	5.77	374.86	375.43	6.49	416.48	417.05	7.21	458.10	458.67	7.93
333.82	334.39	5.78	375.44	376.01	6.50	417.06	417.63	7.22	458.68	459.24	7.94
334.40	334.97	5.79	376.02	376.58	6.51	417.64	418.20	7.23	459.25	459.82	7.95
334.98	335.54	5.80	376.59	377.16	6.52	418.21	418.78	7.24	459.83	460.40	7.96
335.55	336.12	5.81	377.17	377.74	6.53	418.79	419.36	7.25	460.41	460.98	7.97
336.13	336.70	5.82	377.75	378.32	6.54	419.37	419.94	7.26	460.99	461.56	7.98
336.71	337.28	5.83	378.33	378.90	6.55	419.95	420.52	7.27	461.57	462.13	7.99
337.29	337.86	5.84	378.91	379.47	6.56	420.53	421.09	7.28	462.14	462.71	8.00
337.87	338.43	5.85	379.48	380.05	6.57	421.10	421.67	7.29	462.72	463.29	8.01
338.44	339.01	5.86	380.06	380.63	6.58	421.68	422.25	7.30	463.30	463.87	8.02
339.02	339.59	5.87	380.64	381.21	6.59	422.26	422.83	7.31	463.88	464.45	8.03
339.60	340.17	5.88	381.22	381.79	6.60	422.84	423.41	7.32	464.46	465.02	8.04
340.18	340.75	5.89	381.80	382.36	6.61	423.42	423.98	7.33	465.03	465.60	8.05
340.76	341.32	5.90	382.37	382.94	6.62	423.99	424.56	7.34	465.61	466.18	8.06
341.33	341.90	5.91	382.95	383.52	6.63	424.57	425.14	7.35	466.19	466.76	8.07
341.91	342.48	5.92	383.53	384.10	6.64	425.15	425.72	7.36	466.77	467.34	8.08
342.49	343.06	5.93	384.11	384.68	6.65	425.73	426.30	7.37	467.35	467.91	8.09
343.07	343.64	5.94	384.69	385.26	6.66	426.31	426.87	7.38	467.92	468.49	8.10
343.65	344.21	5.95	385.27	385.83	6.67	426.88	427.45	7.39	468.50	469.07	8.11
344.22	344.79	5.96	385.84	386.41	6.68	427.46	428.03	7.40	469.08	469.65	8.12
344.80	345.37	5.97	386.42	386.99	6.69	428.04	428.61	7.41	469.66	470.23	8.13
345.38	345.95	5.98	387.00	387.57	6.70	428.62	429.19	7.42	470.24	470.80	8.14
345.96	346.53	5.99	387.58	388.15	6.71	429.20	429.76	7.43	470.81	471.38	8.15
346.54	347.10	6.00	388.16	388.72	6.72	429.77	430.34	7.44	471.39	471.96	8.16
347.11	347.68	6.01	388.73	389.30	6.73	430.35	430.92	7.45	471.97	472.54	8.17
347.69	348.26	6.02	389.31	389.88	6.74	430.93	431.50	7.46	472.55	473.12	8.18
348.27	348.84	6.03	389.89	390.46	6.75	431.51	432.08	7.47	473.13	473.69	8.19
348.85	349.42	6.04	390.47	391.04	6.76	432.09	432.65	7.48	473.70	474.27	8.20
349.43	349.99	6.05	391.05	391.61	6.77	432.66	433.23	7.49	474.28	474.85	8.21
350.00	350.57	6.06	391.62	392.19	6.78	433.24	433.81	7.50	474.86	475.43	8.22
350.58	351.15	6.07	392.20	392.77	6.79	433.82	434.39	7.51	475.44	476.01	8.23
351.16	351.73	6.08	392.78	393.35	6.80	434.40	434.97	7.52	476.02	476.58	8.24
351.74	352.31	6.09	393.36	393.93	6.81	434.98	435.54	7.53	476.59	477.16	8.25
352.32	352.89	6.10	393.94	394.50	6.82	435.55	436.12	7.54	477.17	477.74	8.26
352.90	353.46	6.11	394.51	395.08	6.83	436.13	436.70	7.55	477.75	478.32	8.27
353.47	354.04	6.12	395.09	395.66	6.84	436.71	437.28	7.56	478.33	478.90	8.28
354.05	354.62	6.13	395.67	396.24	6.85	437.29	437.86	7.57	478.91	479.47	8.29
354.63	355.20	6.14	396.25	396.82	6.86	437.87	438.43	7.58	479.48	480.05	8.30
355.21	355.78	6.15	396.83	397.39	6.87	438.44	439.01	7.59	480.06	480.63	8.31
355.79	356.35	6.16	397.40	397.97	6.88	439.02	439.59	7.60	480.64	481.21	8.32
356.36	356.93	6.17	397.98	398.55	6.89	439.60	440.17	7.61	481.22	481.79	8.33
356.94	357.51	6.18	398.56	399.13	6.90	440.18	440.75	7.62	481.80	482.36	8.34
357.52	358.09	6.19	399.14	399.71	6.91	440.76	441.32	7.63	482.37	482.94	8.35
358.10	358.67	6.20	399.72	400.28	6.92	441.33	441.90	7.64	482.95	483.52	8.36
358.68	359.24	6.21	400.29	400.86	6.93	441.91	442.48	7.65	483.53	484.10	8.37
359.25	359.82	6.22	400.87	401.44	6.94	442.49	443.06	7.66	484.11	484.68	8.38
359.83	360.40	6.23	401.45	402.02	6.95	443.07	443.64	7.67	484.69	485.26	8.39
360.41	360.98	6.24	402.03	402.60	6.96	443.65	444.21	7.68	485.27	485.83	8.40
360.99	361.56	6.25	402.61	403.17	6.97	444.22	444.79	7.69	485.84	486.41	8.41
361.57	362.13	6.26	403.18	403.75	6.98	444.80	445.37	7.70	486.42	486.99	8.42
362.14	362.71	6.27	403.76	404.33	6.99	445.38	445.95	7.71	487.00	487.57	8.43
362.72	363.29	6.28	404.34	404.91	7.00	445.96	446.53	7.72	487.58	488.15	8.44
363.30	363.87	6.29	404.92	405.49	7.01	446.54	447.10	7.73	488.16	488.72	8.45
363.88	364.45	6.30	405.50	406.06	7.02	447.11	447.68	7.74	488.73	489.30	8.46
364.46	365.02	6.31	406.07	406.64	7.03	447.69	448.26	7.75	489.31	489.88	8.47
365.03	365.60	6.32	406.65	407.22	7.04	448.27	448.84	7.76	489.89	490.46	8.48
365.61	366.18	6.33	407.23	407.80	7.05	448.85	449.42	7.77	490.47	491.04	8.49
366.19	366.76	6.34	407.81	408.38	7.06	449.43	449.99	7.78	491.05	491.61	8.50
366.77	367.34	6.35	408.39	408.95	7.07	450.00	450.57	7.79	491.62	492.19	8.51
367.35	367.91	6.36	408.96	409.53	7.08	450.58	451.15	7.80	492.20	492.77	8.52
367.92	368.49	6.37	409.54	410.11	7.09	451.16	451.73	7.81	492.78	493.35	8.53
368.50	369.07	6.38	410.12	410.69	7.10	451.74	452.31	7.82	493.36	493.93	8.54
369.08	369.65	6.39	410.70	411.27	7.11	452.32	452.89	7.83	493.94	494.50	8.55
369.66	370.23	6.40	411.28	411.84	7.12	452.90	453.46	7.84	494.51	495.08	8.56
370.24	370.80	6.41	411.85	412.42	7.13	453.47	454.04	7.85	495.09	495.66	8.57
370.81	371.38	6.42	412.43	413.00	7.14	454.05	454.62	7.86	495.67	496.24	8.58
371.39	371.96	6.43	413.01	413.58	7.15	454.63	455.20	7.87	496.25	496.82	8.59
371.97	372.54	6.44	413.59	414.16	7.16	455.21	455.78	7.88	496.83	497.39	8.60
372.55	373.12	6.45	414.17	414.73	7.17	455.79	456.35	7.89	497.40	497.97	8.61
373.13	373.69	6.46	414.74	415.31	7.18	456.36	456.93	7.90	497.98	498.55	8.62
373.70	374.27	6.47	415.32	415.89	7.19	456.94	457.51	7.91	498.56	499.13	8.63
374.28	374.85	6.48	415.90	416.47	7.20	457.52	458.09	7.92	499.14	499.71	8.64

Yearly maximum insurable earnings are $42,300
Yearly maximum employee premiums are $731.79
The premium rate for 2009 is 1.73 %

Le maximum annuel de la rémunération assurable est de 42 300 $
La cotisation maximale annuelle de l'employé est de 731,79 $
Le taux de cotisation pour 2009 est de 1,73 %

Federal tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the provincial table

Retenues d'impôt fédéral
En vigueur le 1er janvier 2009
Hebdomadaire (52 périodes de paie par année)
Cherchez aussi les retenues d'impôt dans la table provinciale

Pay / Rémunération		Federal claim codes/Codes de demande fédéraux										
		0	1	2	3	4	5	6	7	8	9	10
From / De	Less than / Moins de	Deduct from each pay / Retenez sur chaque paie										
335	339	44.65	15.55	12.70	7.00	1.30						
339	343	45.20	16.10	13.25	7.55	1.85						
343	347	45.80	16.65	13.80	8.10	2.45						
347	351	46.35	17.20	14.35	8.65	3.00						
351	355	46.90	17.75	14.90	9.25	3.55						
355	359	47.45	18.35	15.50	9.80	4.10						
359	363	48.00	18.90	16.05	10.35	4.65						
363	367	48.60	19.45	16.60	10.90	5.25						
367	371	49.15	20.00	17.15	11.45	5.80	.10					
371	375	49.70	20.55	17.70	12.05	6.35	.65					
375	379	50.25	21.15	18.30	12.60	6.90	1.20					
379	383	50.80	21.70	18.85	13.15	7.45	1.80					
383	387	51.40	22.25	19.40	13.70	8.00	2.35					
387	391	51.95	22.80	19.95	14.25	8.60	2.90					
391	395	52.50	23.35	20.50	14.85	9.15	3.45					
395	399	53.05	23.95	21.10	15.40	9.70	4.00					
399	403	53.60	24.50	21.65	15.95	10.25	4.60					
403	407	54.20	25.05	22.20	16.50	10.80	5.15					
407	411	54.75	25.60	22.75	17.05	11.40	5.70					
411	415	55.30	26.15	23.30	17.65	11.95	6.25	.55				
415	419	55.85	26.75	23.90	18.20	12.50	6.80	1.15				
419	423	56.40	27.30	24.45	18.75	13.05	7.40	1.70				
423	427	57.00	27.85	25.00	19.30	13.60	7.95	2.25				
427	431	57.55	28.40	25.55	19.85	14.20	8.50	2.80				
431	435	58.10	28.95	26.10	20.45	14.75	9.05	3.35				
435	439	58.65	29.50	26.70	21.00	15.30	9.60	3.95				
439	443	59.20	30.10	27.25	21.55	15.85	10.20	4.50				
443	447	59.80	30.65	27.80	22.10	16.40	10.75	5.05				
447	451	60.35	31.20	28.35	22.65	17.00	11.30	5.60				
451	455	60.90	31.75	28.90	23.25	17.55	11.85	6.15	.50			
455	459	61.45	32.30	29.50	23.80	18.10	12.40	6.75	1.05			
459	463	62.00	32.90	30.05	24.35	18.65	12.95	7.30	1.60			
463	467	62.60	33.45	30.60	24.90	19.20	13.55	7.85	2.15			
467	471	63.15	34.00	31.15	25.45	19.80	14.10	8.40	2.70			
471	475	63.70	34.55	31.70	26.05	20.35	14.65	8.95	3.30			
475	479	64.25	35.10	32.30	26.60	20.90	15.20	9.55	3.85			
479	483	64.80	35.70	32.85	27.15	21.45	15.75	10.10	4.40			
483	487	65.40	36.25	33.40	27.70	22.00	16.35	10.65	4.95			
487	491	65.95	36.80	33.95	28.25	22.60	16.90	11.20	5.50			
491	495	66.50	37.35	34.50	28.85	23.15	17.45	11.75	6.10	.40		
495	499	67.05	37.90	35.10	29.40	23.70	18.00	12.35	6.65	.95		
499	503	67.60	38.50	35.65	29.95	24.25	18.55	12.90	7.20	1.50		
503	507	68.20	39.05	36.20	30.50	24.80	19.15	13.45	7.75	2.05		
507	511	68.75	39.60	36.75	31.05	25.40	19.70	14.00	8.30	2.65		
511	515	69.30	40.15	37.30	31.65	25.95	20.25	14.55	8.90	3.20		
515	519	69.85	40.70	37.90	32.20	26.50	20.80	15.15	9.45	3.75		
519	523	70.40	41.30	38.45	32.75	27.05	21.35	15.70	10.00	4.30		
523	527	71.00	41.85	39.00	33.30	27.60	21.95	16.25	10.55	4.85		
527	531	71.55	42.40	39.55	33.85	28.20	22.50	16.80	11.10	5.45		
531	535	72.10	42.95	40.10	34.45	28.75	23.05	17.35	11.70	6.00	.30	
535	539	72.65	43.50	40.70	35.00	29.30	23.60	17.90	12.25	6.55	.85	
539	543	73.20	44.10	41.25	35.55	29.85	24.15	18.50	12.80	7.10	1.40	
543	547	73.80	44.65	41.80	36.10	30.40	24.75	19.05	13.35	7.65	2.00	
547	551	74.35	45.20	42.35	36.65	31.00	25.30	19.60	13.90	8.25	2.55	
551	555	74.90	45.75	42.90	37.25	31.55	25.85	20.15	14.50	8.80	3.10	

British Columbia provincial tax deductions
Effective January 1, 2009
Weekly (52 pay periods a year)
Also look up the tax deductions in the federal table

Retenues d'impôt provincial de la Colombie-Britannique
En vigueur le 1er janvier 2009
Hebdomadaire (52 périodes de paie par année)
Cherchez aussi les retenues d'impôt dans la table fédérale

Pay / Rémunération		Provincial claim codes/Codes de demande provinciaux										
		0	1	2	3	4	5	6	7	8	9	10
From / De	Less than / Moins de	Deduct from each pay / Retenez sur chaque paie										
	343	*	.00									
343 -	345	9.30	.20									
345 -	347	9.45	.35									
347 -	349	9.60	.50									
349 -	351	9.80	.65									
351 -	353	9.95	.80									
353 -	355	10.10	.95									
355 -	357	10.25	1.15	.10								
357 -	359	10.40	1.30	.25								
359 -	361	10.55	1.45	.40								
361 -	363	10.75	1.60	.60								
363 -	365	10.90	1.75	.75								
365 -	367	11.05	1.90	.90								
367 -	369	11.20	2.10	1.05								
369 -	371	11.35	2.25	1.20								
371 -	373	11.50	2.40	1.35								
373 -	375	11.70	2.55	1.55								
375 -	377	11.85	2.70	1.70								
377 -	379	12.00	2.90	1.85								
379 -	381	12.15	3.05	2.00								
381 -	383	12.30	3.20	2.15	.10							
383 -	385	12.45	3.35	2.30	.25							
385 -	387	12.65	3.50	2.50	.45							
387 -	389	12.80	3.65	2.65	.60							
389 -	391	12.95	3.85	2.80	.75							
391 -	393	13.10	4.00	2.95	.90							
393 -	395	13.25	4.15	3.10	1.05							
395 -	397	13.40	4.30	3.30	1.20							
397 -	399	13.60	4.45	3.45	1.40							
399 -	401	13.75	4.60	3.60	1.55							
401 -	403	13.90	4.80	3.75	1.70							
403 -	405	14.05	4.95	3.90	1.85							
405 -	407	14.20	5.10	4.05	2.00							
407 -	409	14.35	5.25	4.25	2.15	.10						
409 -	411	14.55	5.40	4.40	2.35	.30						
411 -	413	14.70	5.55	4.55	2.50	.45						
413 -	415	14.85	5.75	4.70	2.65	.60						
415 -	417	15.00	5.90	4.85	2.80	.75						
417 -	419	15.15	6.05	5.00	2.95	.90						
419 -	421	15.30	6.20	5.20	3.10	1.05						
421 -	423	15.50	6.35	5.35	3.30	1.25						
423 -	425	15.65	6.50	5.50	3.45	1.40						
425 -	427	15.80	6.70	5.65	3.60	1.55						
427 -	429	15.95	6.85	5.80	3.75	1.70						
429 -	431	16.10	7.00	5.95	3.90	1.85						
431 -	433	16.25	7.15	6.15	4.10	2.00						
433 -	435	16.45	7.30	6.30	4.25	2.20	.15					
435 -	437	16.60	7.45	6.45	4.40	2.35	.30					
437 -	439	16.75	7.65	6.60	4.55	2.50	.45					
439 -	441	16.90	7.80	6.75	4.70	2.65	.60					
441 -	443	17.05	7.95	6.90	4.85	2.80	.75					
443 -	445	17.20	8.10	7.10	5.05	2.95	.90					
445 -	447	17.40	8.25	7.25	5.20	3.15	1.10					
447 -	449	17.55	8.40	7.40	5.35	3.30	1.25					
449 -	451	17.70	8.60	7.55	5.50	3.45	1.40					

*You normally use claim code "0" only for non-resident employees. However, if you have non-resident employees who earn less than the minimum amount shown in the "Pay" column, you may not be able to use these tables. Instead, refer to the "Step-by-step calculation of tax deductions" in Section "A" of this publication.

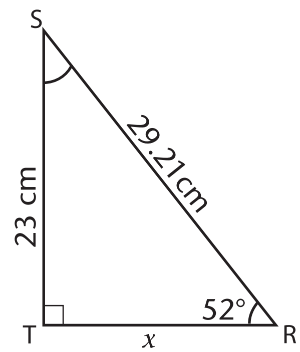
*Le code de demande «0» est normalement utilisé seulement pour les non-résidents. Cependant, si la rémunération de votre employé non résidant est inférieure au montant minimum indiqué dans la colonne «Rémunération», vous ne pourrez peut-être pas utiliser ces tables. Reportez-vous alors au «Calcul des retenues d'impôt, étape par étape» dans la section «A» de cette publication.

Solutions

Lesson A: The Cosine Ratio

Lesson A: Activity 1: Try This

1.



$$\cos R = \frac{\boxed{\text{adjacent}}}{\text{hypotenuse}}$$

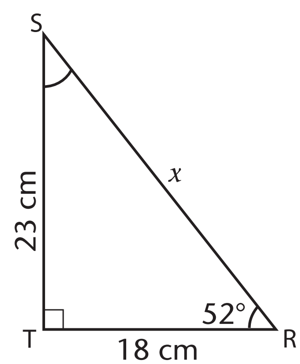
$$\cos 52° = \frac{x}{\boxed{29.21}}$$

$$x = \left(\boxed{\cos 52°}\right)\left(\boxed{29.21}\right)$$

$$x = 0.616\left(\boxed{29.21}\right)$$

$$x = \boxed{18}\text{ cm}$$

2.



$$\cos R = \frac{\text{adjacent}}{\boxed{\text{hypotenuse}}}$$

$$\text{hypotenuse} = \frac{\boxed{\text{adjacent}}}{\boxed{\cos R}}$$

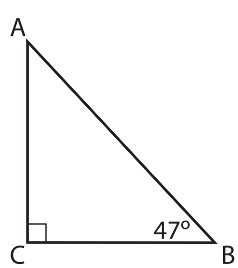
$$y = \frac{\boxed{18}}{\boxed{\cos 52°}}$$

$$y = \frac{\boxed{18}}{0.616}$$

$$y = \boxed{29.21}\text{ cm}$$

3. Two angles that add up to 90° are called complementary angles. Each angle is the complement of the other.

4. Triangles will vary. An example is given below.



$$\angle A + \angle B = 90°$$

$$\angle A + 47° = 90°$$

$$\angle A = 90° - 47°$$

$$\angle A = 43°$$

5. In other words, if one acute angle in a right triangle is 60°, what is the measure of the other acute angle? The complement of a 60° angle is 30°, since 60° + 30° = 90°.

Lesson A: Activity 2: Try This

Answers will vary based on the triangles in the diagram you drew. The sample answer below is based on the diagram in the activity.

Right triangle	Side adjacent ∠A (nearest mm)	Hypotenuse (nearest mm)	$\frac{\text{adjacent side}}{\text{hypotenuse}}$ (to 2 decimal places)
ΔAB_1C_1	AC_1 = **35 mm**	AB_1 = **39 mm**	$\frac{AC_1}{AB_1} = \mathbf{0.90}$
ΔAB_2C_2	AC_2 = **69 mm**	AB_2 = **77 mm**	$\frac{AC_2}{AB_2} = \mathbf{0.90}$
ΔAB_3C_3	AC_3 = **104 mm**	AB_3 = **115 mm**	$\frac{AC_3}{AB_3} = \mathbf{0.90}$

1. Answers will vary. The ratios of the adjacent side to the hypotenuse are the same/or very close in all three triangles.

2. Yes. Explanations will vary. Ratios of corresponding sides of similar triangles are equal.

   The right triangles are similar because they share a common acute angle.

Lesson A: Activity 3: Self-Check

Angle	Hypotenuse	Adjacent Side	Cosine of an Angle
10°	**10 cm**	**9.8 cm**	$\cos 10° = \frac{\mathbf{9.8}}{\mathbf{10}} = \mathbf{0.98}$
20°	**10 cm**	**9.4 cm**	$\cos 20° = \frac{\mathbf{9.4}}{\mathbf{10}} = \mathbf{0.94}$
30°	**10 cm**	**8.7 cm**	$\cos 30° = \frac{\mathbf{8.7}}{\mathbf{10}} = \mathbf{0.87}$
40°	**10 cm**	**7.7 cm**	$\cos 40° = \frac{\mathbf{7.7}}{\mathbf{10}} = \mathbf{0.77}$
45°	**10 cm**	7.1 cm	$\cos 45° = 0.71$
50°	**10 cm**	**6.4 cm**	$\cos 50° = \frac{\mathbf{6.4}}{\mathbf{10}} = \mathbf{0.64}$
60°	**10 cm**	5.0 cm	$\cos 60° = 0.50$
70°	**10 cm**	**3.4 cm**	$\cos 70° = \frac{\mathbf{3.4}}{\mathbf{10}} = \mathbf{0.34}$
80°	**10 cm**	**1.8 cm**	$\cos 80° = \frac{\mathbf{1.7}}{\mathbf{10}} = \mathbf{0.17}$

Lesson A: Activity 4: Self-Check

Angle	Cosine Ratio
10°	**0.9848**
20°	**0.9397**
30°	**0.8660**
40°	**0.7660**
45°	**0.7071**
50°	**0.6428**
60°	0.5000
70°	**0.3420**
80°	**0.1736**

Lesson A: Activity 5: Self-Check

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$
$$\cos 31^\circ = \frac{5.43}{x}$$
$$x(\cos 31^\circ) = 5.43$$
$$x = \frac{5.43}{\cos 31^\circ}$$
$$x = 6.334819347\ldots$$

Press

5 . 4 3 ÷ cos 3 1) =

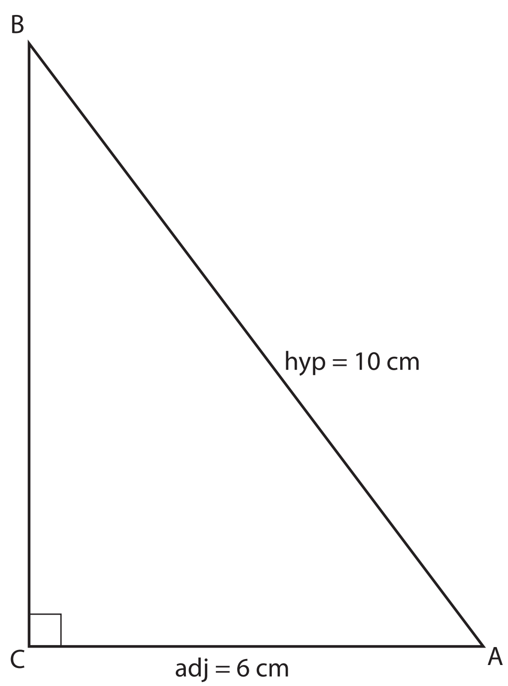
The value of x is approximately 6.33 cm.

Lesson A: Activity 6: Self-Check

1. **Method 1**. Draw a right triangle, and measure the angle using a protractor.

   Since cos A = 0.6 and cos A = $\frac{\text{adjacent}}{\text{hypotenuse}}$, we need to write 0.6 as a ratio in order to know how long to draw the two lengths in the right triangle.

   Since 0.6 = 6 tenths = $\frac{6}{10}$, then the adjacent side can be 6 cm long and the hypotenuse should be 10 cm in length.



Measure ∠A with your protractor.

∠A ≈ 53º

This answer makes sense, because cos 60° is 0.50. and cos 45° is about 0.71.

Method 2. Use your calculator.

To find an angle from its cosine, strike these keys.

INV or 2nd	$\cos^{-1}$	0	.	6	)	=

Your display should show 53.13010235...

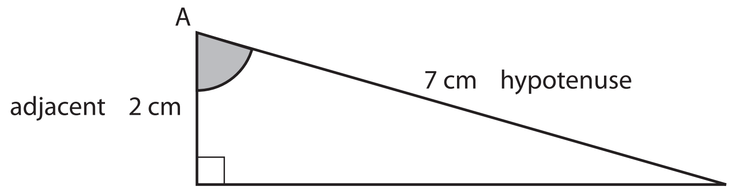
So, ∠A ≈ 53º.

You should write out your solution as follows.
cos A = 0.6
∠A = $\cos^{-1}(0.6)$
∠A ≈ 53º

2.

sin A	∠A
0.1257	**82.8°**
0.7826	**38.5°**
0.9000	**25.8°**
$\frac{2}{3}$	**48.2°**
$\frac{3}{4}$	**41.4°**

3. Shade in angle A, and name the sides with lengths.



Since the triangle involves "adjacent" and "hypotenuse," use the cosine ratio to set up an equation.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{2}{7}$$

$$\angle A = \cos^{-1}\left(\frac{2}{7}\right)$$

$$\angle A = 73.3984504\ldots$$

$$\angle A \approx 73.4°$$

Lesson A: Activity 7: Mastering Concepts

1. $\sin P = \frac{\text{opp}}{\text{hyp}} = \frac{p}{q}$ — Side *p* is opposite Angle P.

   $\sin R = \frac{\text{opp}}{\text{hyp}} = \frac{r}{q}$ — Side *r* is opposite Angle R.

   $\cos P = \frac{\text{adj}}{\text{hyp}} = \frac{r}{q}$ — Side *r* is opposite Angle P.

   $\cos R = \frac{\text{adj}}{\text{hyp}} = \frac{p}{q}$ — Side *p* is opposite Angle R.

2. sin P = cos R and cos P = sin R

3. ∠P and ∠R are complementary angles. The sum of their measures is 90.

Lesson B: Using Cosines to Solve Problems

Lesson B: Activity 1: Try This

1. Answers will vary. Diagram measurements should match the ones used in the calculations.

2. Answers will vary. Diagram measurements should match the ones used in the calculations.

3. When finding the adjacent side, the unknown is in the numerator and the calculations involve multiplication. When finding the hypotenuse, the unknown is in the denominator and the calculations involve division.

4. $\cos R = \dfrac{\text{opposite}}{\text{hypotenuse}}$

   $\cos R = \dfrac{12}{17.4}$

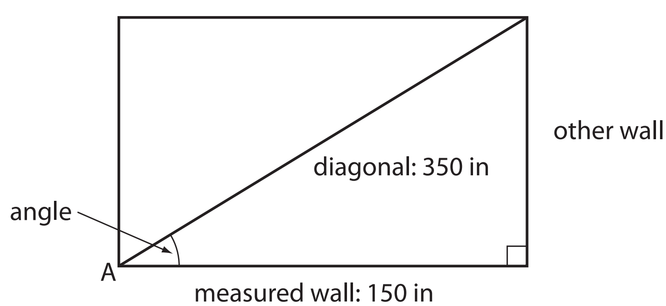
   $\cos R = 0.69$

   $\angle R = 46.4°$

   $\angle R = \cos^{-1}\left(\dfrac{12}{17.4}\right)$ *or* $\angle R = \cos^{-1}(0.69)$

Lesson B: Activity 2: Try This

1. Measures will vary. The diagram could look like this.



2. Answers will vary. A possible answer, based on the measurements for Question 1, follows.

   Substitute into the formula for cosine ratio and solve for angle A.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos A = \frac{150}{350}$$
$$\angle A = \cos^{-1}\left(\frac{150}{350}\right)$$
$$\angle A = 64.62306647\ldots$$

   The angle is approximately 65°.

3. Before the angle is determined, you must convert the measurements to the same units: either both in feet or both in inches.

Lesson B: Activity 3: Self-Check

1. Let "*x*" be the distance the top extends over the bottom.

$$\cos 89° = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 89° = \frac{x}{42}$$
$$42(\cos 89°) = x$$
$$0.7330010704\ldots = x$$

   The top end of the span extends 0.7 m beyond the bottom end.

2. Let "x" be the vertical depth of the nail.

   Set up an equation to solve. Rewrite $3\frac{1}{2}$ inches as 3.5 inches.

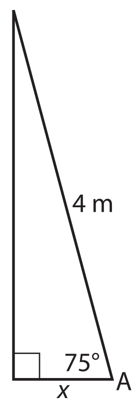
$$\cos 30° = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 30° = \frac{x}{3.5}$$
$$3.5(\cos 30°) = x$$
$$3.031088913\ldots = x$$

   This means the nail's vertical depth is approximately 3.03 inches. Since Simon starts 2 inches above the joint, the nail is just over one inch (1.03") below the surface of the bottom plate.

So, yes Simon is correct when he says that by driving the nail in at an angle of 30°, the tip will be at least 1 inch below the surface.

3. Let x be the distance the foot of the ladder should be placed from the wall to form a 75° angle with the ground.

   Draw a diagram.

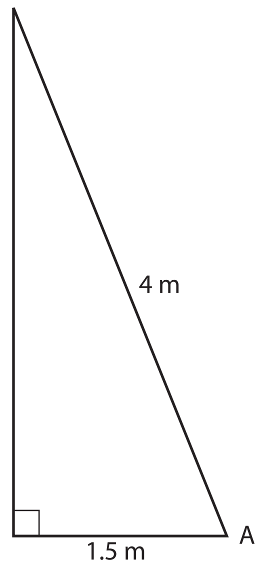


Set up a cosine equation to solve.

$$\cos 75° = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 75° = \frac{x}{4}$$
$$4(\cos 75°) = x$$
$$1.03527618\ldots = x$$

The foot of the ladder should be approximately 1.0 m from the foot of the wall.

4. Draw a diagram.



Set up a cosine equation to solve.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos A = \frac{1.5}{4}$$
$$\angle A = \cos^{-1}\left(\frac{1.5}{4}\right)$$
$$\angle A = 67.97568716\ldots$$

The ladder makes an angle of approximately 68° with the ground.

5. Let x metres be the length of the zip line.

Set up a cosine equation to solve.

$$\cos 5° = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 5° = \frac{250}{x}$$
$$x(\cos 5°) = 250$$
$$x = \frac{250}{\cos 5°}$$
$$x = 250.9549549\ldots$$

The zip line will be at least 251 m long.

Lesson B: Activity 4: Mastering Concepts

Draw a diagram, and label the unknown side "*x*." Let *x* be the slant width of the roof. This is the side to find, since to find the area of the shaded part of the roof, the length and width are needed.

Set up a cosine equation to solve.

The slant width of the roof is approximately 12.8 feet. Do not round this answer, but keep it in your calculator and then find the area of the shaded part of the roof.

Area = length × width

Area = 30 × 12.770 1332...

Area = 383.103 9981...

The area of the roof is approximately 383 ft².

Lesson C: General Problems

Lesson C: Activity 1: Try This

1. a. opposite
   adjacent

   b. $\tan R = \dfrac{\text{opposite}}{\text{adjacent}}$

2. a. opposite
   hypotenuse

   b. $\text{Sin}Q = \dfrac{\text{opposite}}{\text{hypotenuse}}$

3. a. adjacent
   hypotenuse

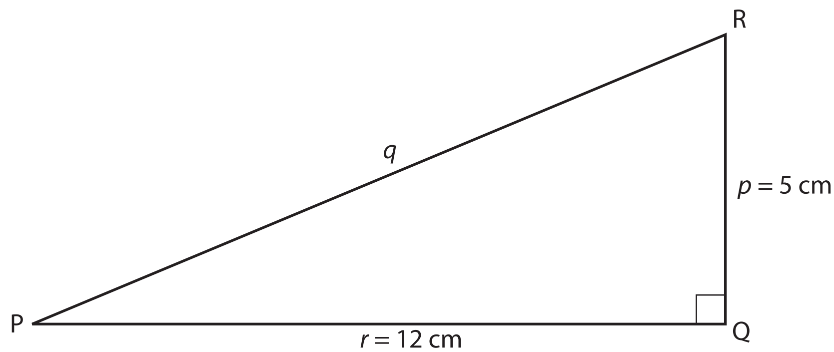
   b. $\text{Cos}P = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

4. Answers will vary. Sample answers are given. Many students find the tangent ratio the easiest to remember. The sine and cosine are more easily confused due to their similar names.

5. Answers will vary. Sample answers are given. Angles are easier to find than an unknown side, particularly if the unknown side is the denominator of the ratio.

Lesson C: Activity 2: Self-Check

1. Draw a diagram and label it with the information you are given.



The unknown angles and sides are: ∠P, ∠R ,and side q.

Step 1: Find the missing angles. (Methods may vary.)

To find ∠P, notice that side p is opposite ∠P and side r is adjacent to ∠P. Use the tangent ratio to find ∠P.

SOH-CAH-**TOA**

$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan P = \frac{5}{12}$$

$$\angle P = \tan^{-1}\left(\frac{5}{12}\right)$$

$$\angle P = 22.6198649\ldots$$

∠P is approximately 22.6°.

Use the two angles to find the third angle.

∠R = 180° – 90° – 22.6° = 67.4°

Therefore, ∠R is approximately 67.4°.

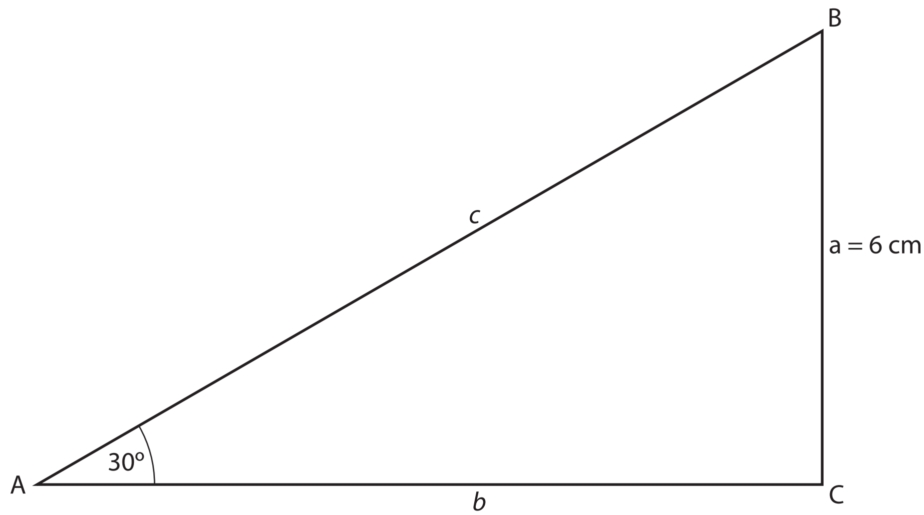
Step 2: Find the missing lengths.

Find side q. Because two sides are given, the third side can be found using the Pythagorean Theorem.

$$p^2 + r^2 = q^2$$
$$5^2 + 12^2 = q^2$$
$$25 + 144 = q^2$$
$$q^2 = 25 + 144$$
$$q^2 = 169$$
$$q = \sqrt{169}$$
$$q = 13$$

Therefore, side q is 13 cm.

2. Draw a diagram and label it with the information you are given.



The unknown angles and sides are: ∠B and sides b and c.

Step 1: Find the missing angles.

All three angles in a right triangle add up to 180°. But even more specifically in a right triangle, since one angle is 90°, the other two add to 90°. This means the two acute angles in a right triangle are complementary.

∠B = 90° – 30° = 60°

Step 2: Find the missing sides. (Methods may vary.)

Either side can be found first. Let's find side b.

Side a is opposite the angle of 30° and the side you want to find is adjacent to this angle. Use the tangent ratio to set up an equation.

SOH-CAH-**TOA**

$$\tan 30° = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 30° = \frac{6}{b}$$
$$b(\tan 30°) = 6$$
$$b = \frac{6}{\tan 30°}$$
$$b = 10.39230485\ldots$$

Side *b* is approximately 10.4 cm.

Lastly, find side *c*.

Side *a* is opposite the angle of 30° and the side you want to find is the hypotenuse. Use the sine ratio to set up an equation.

SOH-CAH-TOA

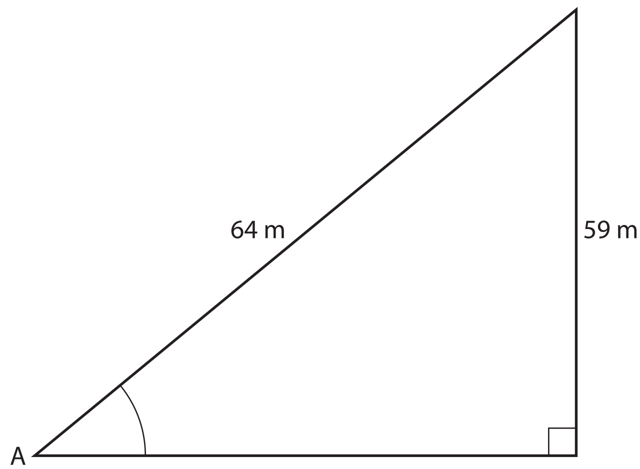
$$\sin 30° = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 30° = \frac{6}{c}$$
$$c(\sin 30°) = 6$$
$$c = \frac{6}{\sin 30°}$$
$$c = 12$$

Side *c* is 12 cm.

You could have used the Pythagorean Theorem here to solve for the third side, but here's a warning. If your calculation to find side *b* was incorrect, then you would not be able to get this question correct. If you use trig ratios, you'll only use information given in the question to find side *c*. This will ensure that you use correct information from the start.

Lesson C: Activity 3: Self-Check

1. Draw a diagram using the information given.



Find ∠A.

Since 59 m is the length of the side *opposite* ∠A. The *hypotenuse* is 64 m long, you will use the sine ratio to set up an equation.

SOH-CAH-TOA

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{59}{64}$$

$$\angle A = \sin^{-1}\left(\frac{59}{64}\right)$$

$$\angle A = 67.2017519\ldots$$

The funicular railroad is inclined at approximately 67°.

2. To find the area you have to find *x*.

Since the unknown side is the side *opposite* of 30° and the given side of 25 ft is adjacent to the angle, you will use the tangent ratio to set up an equation.

SOH-CAH-**TOA**

$$\tan 30° = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 30° = \frac{x}{25}$$

$$25(\tan 30°) = x$$

$$14.43375673\ldots = x$$

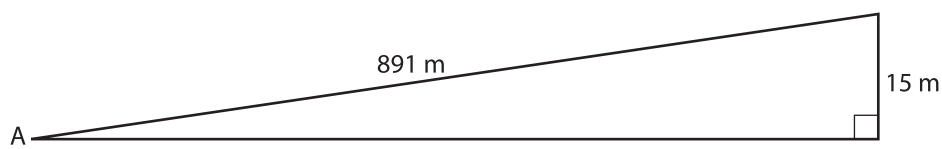
The unknown *x* is approximately 14.4 feet.

Use this to find the area of the triangular plot.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(25\ \text{ft})(14.4\ \text{ft}) \\ &= 180\ \text{ft}^2 \end{aligned}$$

The garden is about 180 square feet in area.

3. Draw a diagram.



Find ∠A.

Since you are given the side *opposite* ∠A and the *hypotenuse*, you will use the sine ratio to set up an equation.

SOH-CAH-TOA

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{15}{891}$$

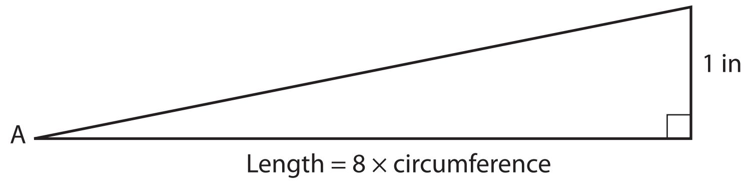
$$\angle A = \sin^{-1}\left(\frac{15}{891}\right)$$

$$\angle A = 0.9646209815\ldots$$

The average slope of the first spiral tunnel is approximately 1.0°.

Lesson C: Activity 4: Mastering Concepts

1. Draw a diagram using the information that is given.



Find ∠A.

The side *opposite* the angle is the 1-inch distance the bolt advances when turned though eight full rotations.

The side *adjacent* is eight times the circumference of the bolt.

Since you are working with the opposite and adjacent side, you will use the tangent ratio to set up an equation. However, you first need an approximate length of the adjacent side before you can continue.

Remember that the circumference of a circle is equal to pi multiplied by the diameter. The diameter is 1 inch, so use this measure and this formula to find the length of the adjacent side.

Length = 8 times the Circumference

Length = 8 × π × diameter

Length = 8 × π × 1

Length = 25.13274...

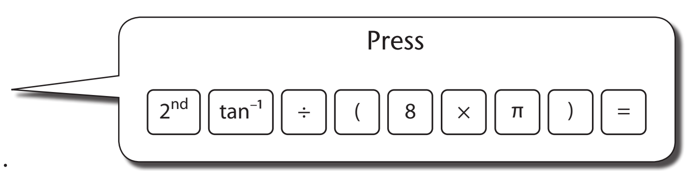
To avoid rounding, we will keep this length as 8π.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{1}{8\pi}$$

$$\angle A = \tan^{-1}\left(\frac{1}{8\pi}\right)$$

$$\angle A = 2.2785247\ldots$$

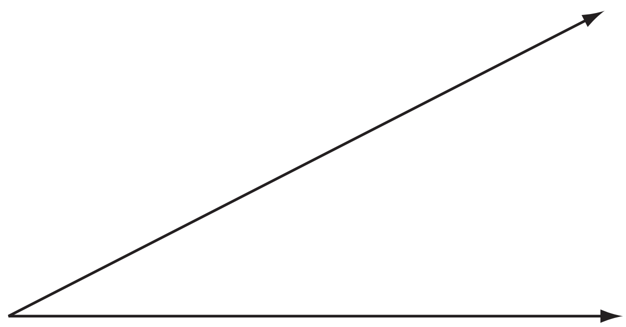


The slope of the threads is approximately 2.3°.

Glossary

acute angle
an angle greater than 0° but less than 90°

For example, this is an acute angle.



adjacent angles
angles which share a common vertex and lie on opposite sides of a common arm

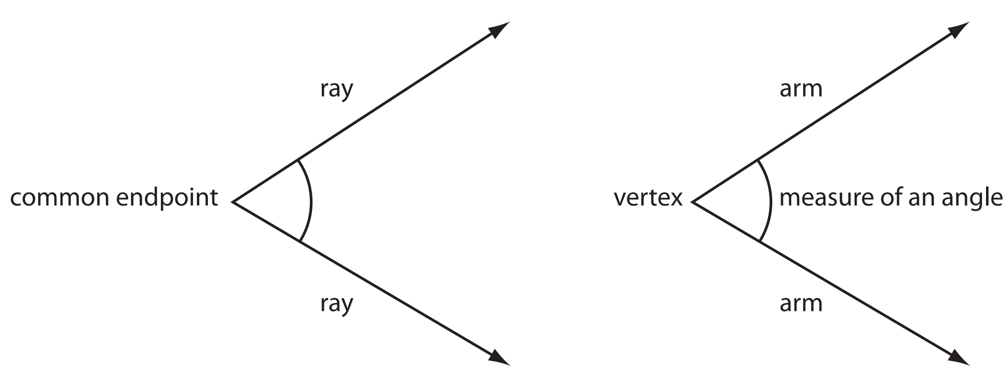
adjacent side
the side next to the reference angle in a right triangle. (The adjacent side cannot be the hypotenuse.)

alternate exterior angles
exterior angles lying on opposite sides of the transversal

alternate interior angles
interior angles lying on opposite sides of the transversal

angle
a geometric shape formed by two rays with a common endpoint

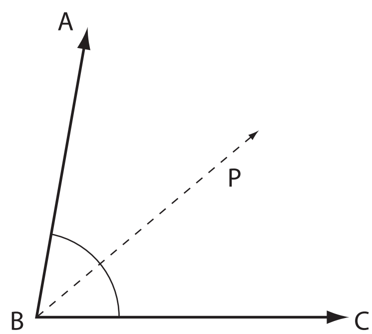
Each ray is called an *arm of the angle*. The common endpoint of the arms of the angle is the vertex of the angle.



angle of depression
an angle below the horizontal that an observer must look down to see an object that is below the observer

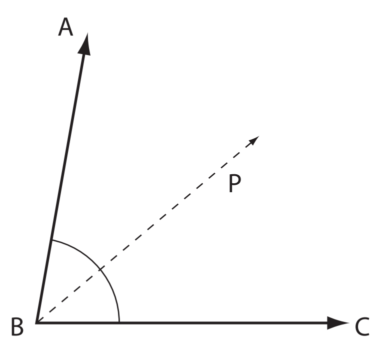
angle of elevation
the angle above the horizontal that an observer must look to see an object that is higher than the observer

bisect
divide into two congruent (equal in measure) halves



bisector
a line or ray which divides a geometric shape into congruent halves

Ray BP is a bisector of ∠ABC, since it bisects ∠ABC into two congruent halves.



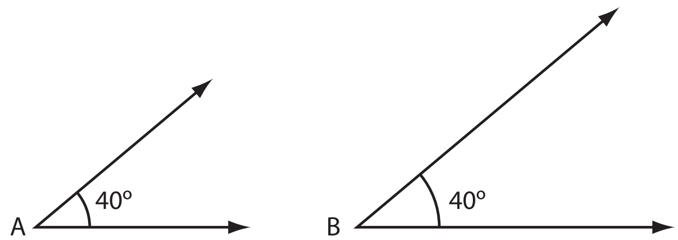
∠ABP ≅ ∠PBC

clinometer
a device for measuring angles to distant objects that are higher or lower than your position

complementary angles
two angles with measures that add up to 90°

One angle is called the *complement* to the other.

congruent angles
angles with the same measure



In the diagram ∠A = 40° and ∠B = 40°. So, ∠A and ∠B are congruent.

There is a special symbol for "is congruent to." The congruence symbol is ≅.

So, you can write ∠A ≅ ∠B.

corresponding angles
angles in the same relative positions when two lines are intersected by a transversal

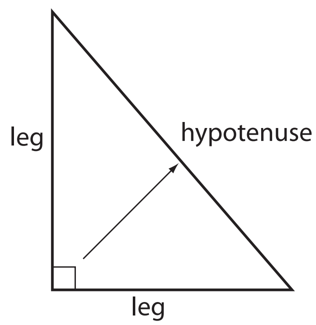
cosine ratio
the ratio of the length of the side adjacent to the reference angle, to the length of the hypotenuse of the right triangle

exterior angles
angles lying outside two lines cut by a transversal

full rotation
an angle having a measure of 360°

This is a full rotation angle.

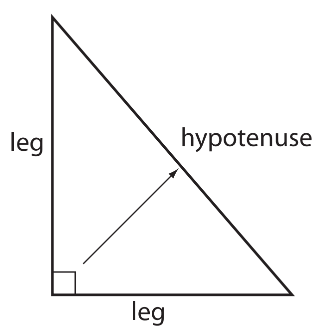
hypotenuse
in a right triangle, the side opposite the right angle; the longest side in a right triangle



indirect measurement
taking one measurement in order to calculate another measurement

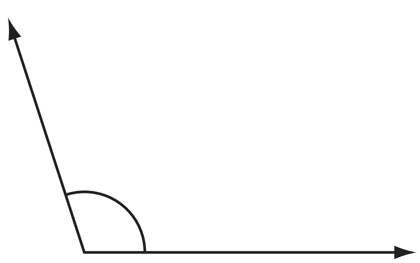
interior angles
angles lying between two lines cut by a transversal

leg
one of the two sides of a right triangle that forms the right angle



obtuse angle
an angle greater than 90° but less than 180°

For example, this is an obtuse angle.



opposite side
the side across from the reference angle in a right triangle

parallel
lines that are the same distance apart everywhere: they never meet

perpendicular
lines that meet at right angles

polygon
a many-sided figure

A triangle is a polygon with three sides, a quadrilateral is a polygon with four sides, and so on.

proportion
the statement showing two ratios are equal

Pythagorean Theorem
for any right triangle, the square of the hypotenuse is equal to the sum of the squares of the two legs

Pythagorean triple
three whole numbers, which represent the lengths of the sides of a right triangle

There are an infinite number of such triples.

reference angle
an acute angle that is specified (example, shaded) in a right triangle

referent
an object or part of the human body you can refer to when estimating length or distance

reflex angle
an angle having a measure greater than 180° but less than 360°

This is an example of a reflex angle.

regular polygon
a polygon with all its angles equal in measure and all its sides equal in measure

right angle
one quarter of a complete rotation. It is 90° in measure.

scale factor
the number by which the length and the width of a figure is multiplied to form a larger or smaller similar figure

similar figures
figures with the same shape but not necessarily the same size

A figure similar to another may be larger or smaller

sine ratio
the ratio of the length of the side opposite to the reference angle, over the hypotenuse of the right triangle

solve a right triangle
to find all the missing sides and angles in a right triangle

straight angle
one half a rotation; an angle 180°

This is a straight angle.

straightedge
a rigid strip of wood, metal, or plastic having a straightedge used for drawing lines

When a ruler is used without reference to its measuring scale, it is considered to be a straightedge.

supplementary angles
two angles, which add up to 180°

In a pair of supplementary angles, one angle is the supplement to the other.

symmetry
the property of being the same in size and shape on both sides of a central dividing line

tangent ratio
the ratio of the length of the side opposite to the selected acute angle, to the length of the side adjacent to the selected acute angle in a right triangle

transversal
a line that cuts across two or more lines

trigonometry
the branch of mathematics based originally on determining sides and angles of triangles, particularly right triangles

vertically opposite angles
angles lying across from each other at the point where two lines intersect

Vertically opposite angles are also referred to as *opposite angles*.