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Chapter 2: Percent, Division with Fractions, and Measurement Conversion

Utah Core Standard(s):

- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (6.RP.3)
  c) Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
  d) Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Academic Vocabulary: percent, rational number, fraction, decimal, partial table, ratio, equivalent ratios, equivalent fractions, factor, multiple, greatest common factor (GCF), unit fraction, dividend, divisor, quotient, factor, product, metric system of measurement, customary system of measurement, conversion

Chapter Overview:
In this chapter, students use their work with ratio to understand a percent as a part to whole ratio with a whole equal to one hundred. They learn how to express parts of a whole using fraction, decimal, and percent notation and they convert fluently between these different but equivalent forms. Next, students learn about three different types of percent problems: 1) Finding a percent given a part and the whole. 2) Finding a part of a quantity given a percent and the whole. 3) Finding the whole given a part and a percent. While reasoning about and solving percent problems, students use a variety of models and strategies such as tape diagrams, double number lines, partial tables, unit rate, equations, etc. In Section two, students apply and extend previous understandings of multiplication and division to divide fractions by fractions. Using a variety of strategies and models, students solve mathematical and real-world problems that require an understanding of how to divide with fractions. They come to understand that dividing by a number is the same as multiplying by the number's reciprocal. In section three, students study measurement conversion. Measurement conversion provides another opportunity for students to apply their understanding of ratio and unit rate.

Connections to Content:
Prior Knowledge: In this chapter students draw on their work with ratio from Chapter 1 as they explore the meaning of percent – a part to whole ratio with a whole equal to 100. They express parts of a whole using fraction, decimal, and percent notation. To do this, they construct models learned previously (area models, including hundred grids, tape diagrams, double number lines, tables, etc.). Students know how to express a fraction as a decimal by creating an equivalent fraction with a denominator of 10 or 100 (4.NF). Students will rely on their ability to operate fluently with rational numbers (5.NF and 6.NS). They will also use their understanding of a rational number, $\frac{a}{b}$, as both a groups of $\frac{1}{b}$ (3.NF and 4.NF) and $a \div b$ (5.NF). Students have also used models to divide whole numbers by unit fractions and unit fractions by whole numbers (5.NF). They will build on this knowledge to divide fractions by fractions. Lastly, students have converted measurements within a single system of measurement (customary and metric) using ideas about multiplication and division (4.MD and 5.MD). They will connect these ideas to the ideas of ratio and use them to convert between systems of measurement.

Future Knowledge: In Chapter 6 of this text, students will learn how to write and solve equations to represent the different types of percent problems studied in this chapter. In 7th grade, students will continue to focus on proportional relationships, learning how to set up and solve a proportion to solve percent problems, including problems involving discounts, interest, taxes, tips, and percent increase and decrease. They will also apply these skills and understandings to solve problems involving other types of proportional relationships (e.g., similar figures, scale drawings, probability and statistics, etc.). Students will examine the representations of a proportional relationship, a subset of linear relationships. In 8th grade, students transition to linear relationships in general and proportions form the basis for understanding the concept of constant rate of change (slope). As they progress through high school coursework, they use ratios in algebra (functions), trigonometry (the basic trigonometric functions), and calculus (average and instantaneous rate of change of a function).

MATHEMATICAL PRACTICE STANDARDS

<table>
<tr>
<td></td>
<td>Make sense of problems and persevere in solving them.</td>
<td>Flora's new baby has a birth weight of 8 pounds exactly. Her mother calls from London, England to ask about the baby and wants to know the baby's weight in kilograms.
Throughout the chapter, students will reason through the size of their answer. For this problem, students will use a conversion chart to find the conversion between pounds and kilograms. From here, students need to think about the size of their answer – should it be smaller or bigger than eight? This will help them determine a solution pathway and determine whether their answer makes sense.</td>
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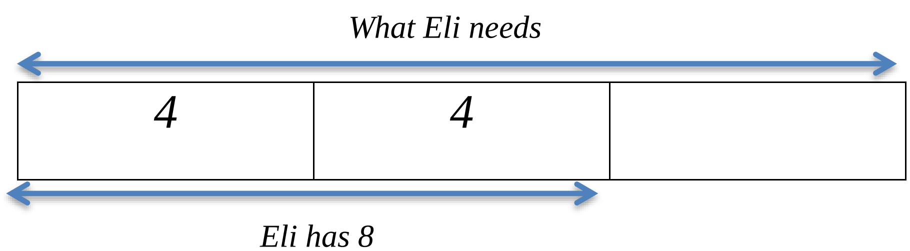
<tr>
<td></td>
<td>Reason abstractly and quantitatively.</td>
<td>Eli has 8 pints of ice cream. It's $\frac{2}{3}$ of what he needs. How much does he need? Draw a model of your choice to answer this question. Then, write number sentence to represent the problem.
Students start by drawing a model to represent this problem.


The model shows that Eli needs 8 pints of ice cream. Students then connect this model to a division sentence: $8 \div \frac{2}{3} = 12$. The model reveals how to perform the division. First, eight must be distributed evenly into $\frac{2}{3}$ of the total (take half of 8), therefore each part must contain 4. There are 3 parts in the total, each with 4, so the total is 12 (multiply by 3):
$8 \div \frac{2}{3} \rightarrow 8 \times \frac{1}{2} \times 3 = 4 \times 3 = 12.$
We see that dividing by $\frac{2}{3}$ is the same as multiplying by $\frac{3}{2}$.

Students can also write a multiplication problem to solve this problem:
$\frac{2}{3}$ of what equals 8
$\frac{2}{3} \times ? = 8$
$? = 8 \div \frac{2}{3}$
This type of thinking forms the foundation for solving equations which students will study later in the year.</td>
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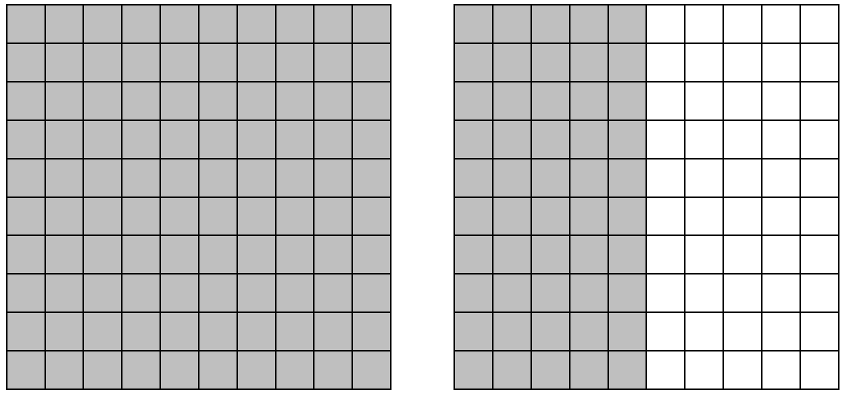
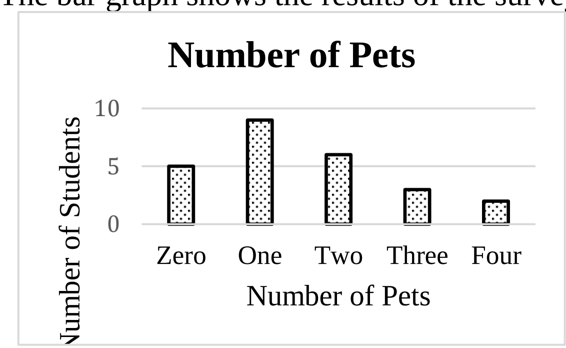
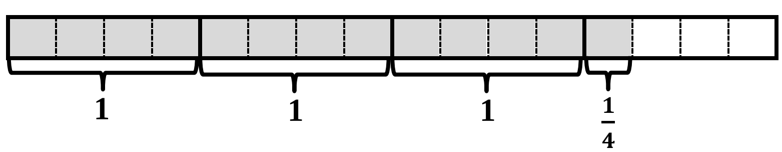
	Construct viable arguments and critique the reasoning of others.	Express the shaded portion as a fraction, decimal, and percent.  *Students may give either* $1\frac{1}{2}, 1.5, 150\%$ *OR* $\frac{3}{4}, 0.75, 75\%$ *as correct answers. The correctness depends on the "whole":* $1\frac{1}{2}, 1.5, 150\%$ *is correct if the student is interpreting the whole as ONE square, while* $\frac{3}{4}, 0.75, 75\%$ *is correct if the student is interpreting the whole as the TWO squares. Ask students to explain "150% of* ***what whole****"? Or "75% of* ***what whole****?"*
	Model with mathematics.	Renee surveyed the students in her class to see how many pets they have. The bar graph shows the results of the survey:  What percent of the students in Renee's class own two or more pets? What percent of the students in Renee's class own less than two pets? *Lesson 2.1d focuses on students solving real world problems involving percents. Students analyze and interpret a variety of graphs to draw conclusions about the data shown.*
	Use appropriate tools strategically.	Use the model below to solve the problem. $3\frac{1}{4} \div \frac{3}{4} =$ _$4\frac{1}{3}$__  *Students use a variety of models throughout the chapter to solve percent problems, divide fractions, and perform measurement conversions. They use these models to make sense of why dividing by a number is the same as multiplying by the number's reciprocal.*

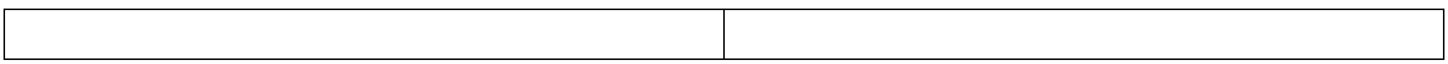
	Attend to precision.	Tia is putting red and blue marbles into different bags. In which of the bags are 25% of the marbles red? Justify your answers. **Bag 1:** One out of every four marbles in the bag is red. **Bag 2:** There are a total of 40 marbles in the bag. Of the 40 marbles, 10 are red. **Bag 3:** There are 25 red marbles and 75 blue marbles in the bag. **Bag 4:** The ratio of red marbles to blue marbles is 1:4. **Bag 5:** There are 400 marbles in the bag. Of the 400 marbles, 100 are red. **Bag 6:** For every three blue marbles Tia puts in the bag, she puts 1 red marble. **Bag 7:** The number of blue marbles is three times the number of red marbles. *A percent is a part to whole ratio with a whole equal to 100. As students convert ratios to percents throughout the chapter, they will need to make sense of what they are given, what they are looking for, and how the two are related. Models will be extremely useful in making sense of problems.*
	Look for and make use of structure.	Eli has 8 pints of ice cream. If a serving size is $\frac{2}{3}$ of a pint of ice cream, how many servings does he have? Estimate the answer. Draw a model to solve the problem. Then, write a number sentence to represent the problem. *Students construct a variety of models to divide with fractions. They connect these models to the number sense. Viewing the model and number sentence simultaneously helps students to understand why* $8 \div \frac{2}{3}$ *is equivalent to* $8 \times \frac{3}{2} = \frac{24}{2} = 12$. *We can create twenty-four thirds from 8 and then we pull them out two at a time.*
	Look for and express regularity in repeated reasoning.	Find 22% of 54. *There are a variety of strategies students can use to solve percent problems. One strategy is to use repeated reasoning. For the problem above, students may think that 10% of 54 is 5.4 so 20% is 10.8 and 1% is 0.54 so 2% is 1.08; therefore 22% of 54 is 10.8 + 1.08 or 11.88.* If 10% of a number is 8, what is… 20% of the number? 50% of the number? 100% of the number? *The students can apply this reasoning to a second type of percent problem they will encounter in the chapter - finding the whole given a part and a percent. If we are given a part and a percent, we can iterate that percent up and down until we reach 100% which represents the whole.*

2.0 Anchor Problem:

Part 1:
Calvin's grandma, Maggie, is a math professor who loves to play math games with her grandson. One day, she said to him, "I am going to ask you some questions involving money. If you answer a question correctly, I will give you the amount of money equal to the answer."

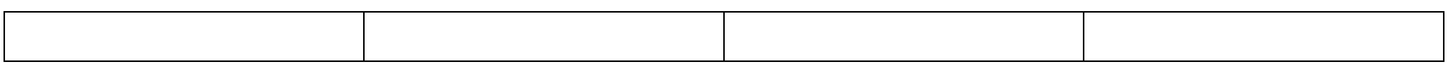
Determine the amount of money Calvin can earn in each question.

Directions: Use the tape diagram shown below to answer questions #1 and 2.



1. If the entire bar has a value of $1, what is the value of each box?
2. If the entire bar has a value of $100, what is the value of each box?

Directions: Use the tape diagram shown below to answer questions #3 and 4.

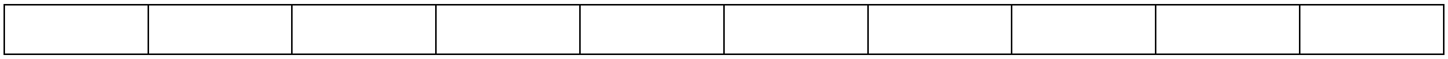


3. If the entire bar has a value of $1, what is the value of each box?
4. If the entire bar has a value of $100, what is the value of each box?

Directions: Use the tape diagram shown below to answer questions #5 and 6.

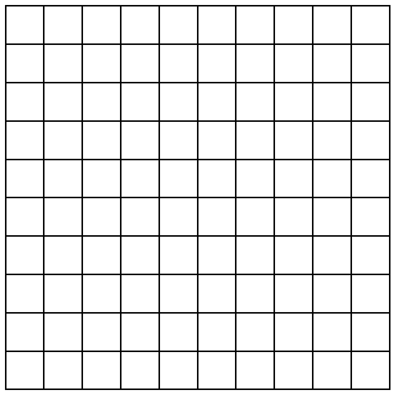
5. If the entire bar has a value of $1, what is the value of each box?
6. If the entire bar has a value of $100, what is the value of each box?

Directions: Use the tape diagram shown below to answer questions #7 and 8.



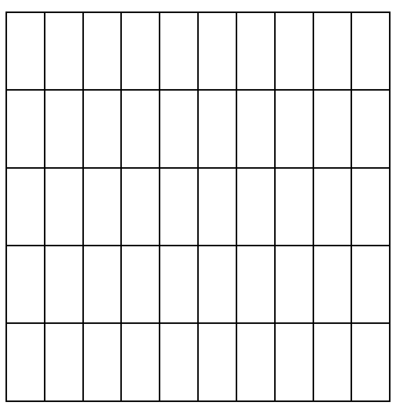
7. If the entire bar has a value of $1, what is the value of each box?
8. If the entire bar has a value of $100, what is the value of each box?

Directions: The grid below is a 10 by 10 grid (100 total squares). Use the grid to answer questions #9 and 10.



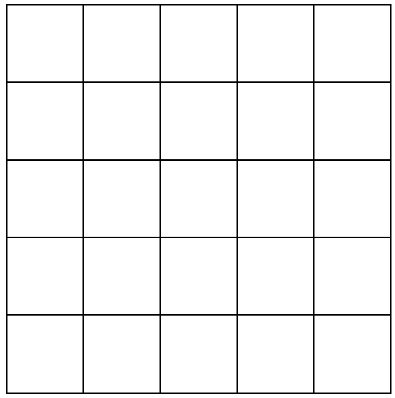
9. If the entire grid has a value of $1, what is the value of each box?
10. If the entire grid has a value of $100, what is the value of each box?

Directions: The grid below is a 10 by 5 grid (50 total squares). Use the grid to answer questions #11 and 12.



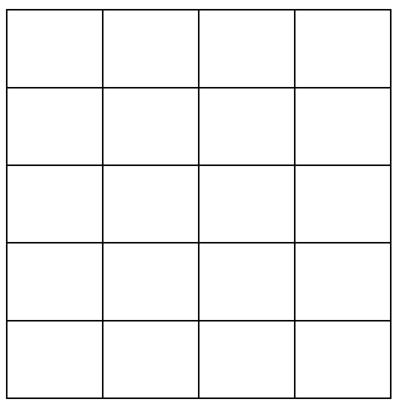
11. If the entire grid has a value of $1, what is the value of each small square in the grid?
12. If the entire grid has a value of $100, what is the value of each small square in the grid?

Directions: The grid below is a 5 by 5 grid (25 total squares). Use the grid to answer questions #13 and 14.



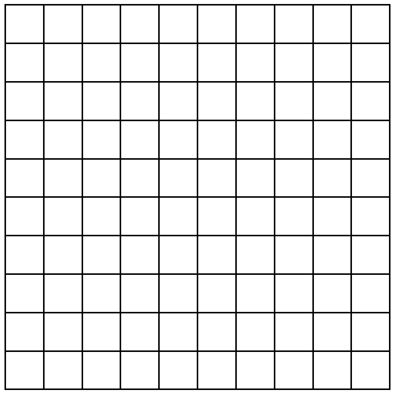
13. If the entire grid has a value of $1, what is the value of each small square in the grid?
14. If the entire grid has a value of $100, what is the value of each small square in the grid?

Directions: The grid below is a 4 by 5 grid (20 total squares). Use the grid to answer questions #15 and 16.



15. If the entire grid has a value of $100, what is the value of each small square in the grid?
16. If the entire grid has a value of $1, what is the value of each small square in the grid?

Part 2:



1. If the entire grid has a value of $100, what is the value of each small square in the grid?

2. If the entire grid has a value of $100, what is the value of 10 small squares in the grid?

3. If the entire grid has a value of $200, what is the value of each small square in the grid?

4. If the entire grid has a value of $200, what is the value of 10 small squares in the grid?

5. If the entire grid has a value of $80, what is the value of each small square in the grid?

6. If the entire grid has a value of $80, what is the value of 10 small squares in the grid?

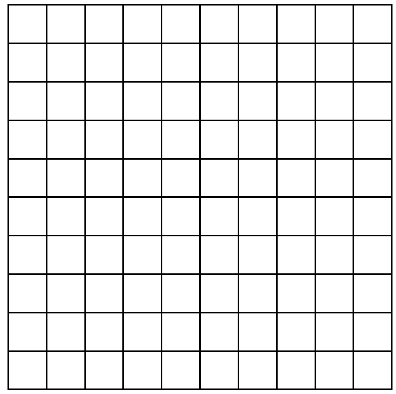
7. If the entire grid has a value of $120, what is the value of each small square in the grid?

8. If the entire grid has a value of $120, what is the value of 10 small squares in the grid?

9. If the entire grid has a value of $24, what is the value of each small square in the grid?

10. If the entire grid has a value of $24, what is the value of 10 small squares in the grid?

Part 3:



1. If 1 small square has a value of $0.3, what is the value of the entire grid?

2. If 1 small square has a value of $0.78, what is the value of the entire grid?

3. If 1 small square has a value of $2.1, what is the value of the entire grid?

4. If 2 small squares have a value of $5, what is the value of the entire grid?

5. If 2 small squares have a value of $1.1, what is the value of the entire grid?

6. If 10 small squares have a value of $9, what is the value of the entire grid?

7. If 10 small squares have a value of $6.4, what is the value of the entire grid?

8. If 80 small squares have a value of $88, what is the value of the entire grid?

9. If 16 small squares have a value of $32, what is the value of the entire grid?

10. If 75 small squares have a value of $30, what is the value of the entire grid?

Section 2.1: Ratios out of 100

Section Overview:
In this section, students will learn to fluently transition between fractions, decimals, ratios, and percents. The section begins by introducing students to percent – a rate per 100. They explore problems that highlight the value of percent as a tool that can be used to compare ratios and fractional amounts of different quantities. The skills learned in Chapter 1 of building rates up to, or down to, rates out of 100 will be used continuously in this section. Students will be encouraged to use tables, tape models, and double line models to build their understanding. In the end, students will use the strategy they like best. The goal is that students make sense of problem situations rather than memorize algorithmic strategies.

In 2.1a through 2.1d the focus is exclusively on part to whole relationships—transitioning between fractions, decimals and percents. A primary goal will be on helping students convert representations to equivalent rates out of 100 using a model or partial table. For example, students will learn to see $\frac{13}{25}$ as $\frac{52}{100}$ or $\frac{17}{200}$ as $\frac{8.5}{100}$ and that $\frac{3}{5}$ of 200 is the same as 120 out of 200 or $\frac{60}{100}$, thus turning conversions to equivalences.

In 2.1e, students transition to converting part-to-part ratios to a percent. In other words, they will note that if a part to part relationship is 2 to 3, then two different part to whole relationships may be derived: $\frac{2}{5}$ or $\frac{3}{5}$. This means there is 40% of one quantity and 60% of the other.

Then in 2.1f – 2.1i, students build on these understandings to find a part, percent, or whole given the other two. Foundational in these sections is the relationship between quantities. Students will be encouraged to use models or tables to solve problems rather than algorithms.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Understand a percent as a part to total ratio with a whole equal to 100.
2. Represent fractional amounts of a quantity as a percent.
3. Fluidly transition between quantities represented as a percent, fraction, decimal or ratio.
4. Find a part of a quantity given a percent and the whole.
5. Find the whole given a part and a percent.
6. Solve real-world percent problems.

2.1a Class Activity: Introduction to Percent as Rate Per 100

Activity 1:

a. Justin, Ariana, and Longar all have summer jobs and are saving part of the money they earn:
   - Justin saves $4 for every $10 he earns.
   - Ariana saves $9 for every $25 she earns.
   - Longar saves $7 for every $20 he earns.

If they all earn the same amount of money over the summer, who will save the most? Who will save the least? Justify your answer.

One way students may approach this problem is to create equivalent ratios with a common value. In this case, students may change each of the ratios to a ratio with a total equal to 100:

- Justin saves $40 for every $100 he earns.
- Ariana saves $36 for every $100 she earns.
- Longar saves $35 for every $100 he earns.

When we do this, we can see that Justin will save the most and Longar will save the least.

These problems are aimed at helping students to see why changing a part to whole ratio (fraction) to a percent can be a very useful tool in mathematics and everyday life. By using a common denominator of 100, we can easily compare fractional amounts of different quantities.

b. Stefan took three math tests last quarter. These are the scores he got on the tests:
   - Test 1: 23 correct out of 25
   - Test 2: 48 correct out of 50
   - Test 3: 18 correct out of 20

Which test did Stefan do the best on?

Using the same line of thinking as in the previous problem, Stefan did the best on Test 2.

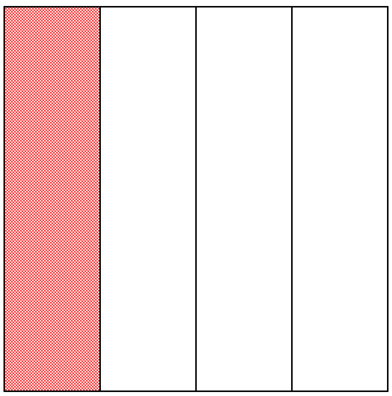
Activity 2: Each of the large squares below are the same size.

a. Shade each of the following models to represent $\frac{1}{4}$. Then write the fraction that represents the part that is shaded under the model. There are many ways to shade the models. Sample answers are shown.

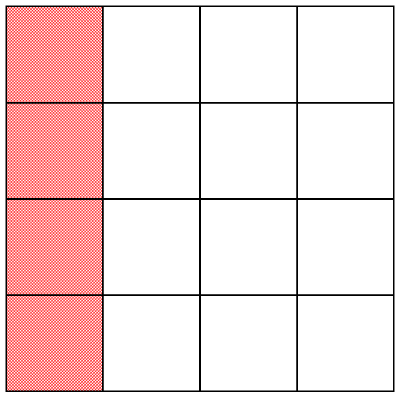
Students will be familiar with these representations of $\frac{1}{4}$. The instructional goal is the equivalence of $\frac{1}{4}$, $\frac{4}{16}$, $\frac{25}{100}$, etc. all representing the same portion of the whole.

Be sure to use, and encourage students to use, precise language in describing what they see (e.g., one section out of four is shaded, thus we can say $\frac{1}{4}$ of the whole is shaded). Some students may say that out of the whole, 1 section is shaded and 3 are not. Both statements are true, but when we talk about a fractional portion of the whole we are talking about a part of the whole.

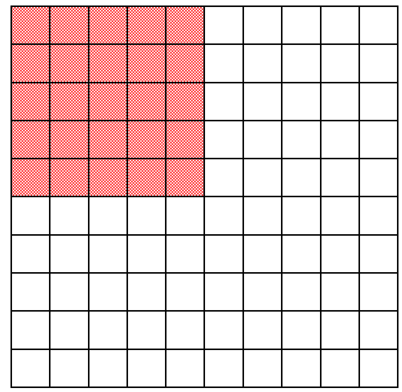
Emphasize to students the importance of making sense of the quantities they use to represent a situation.



Fraction: ___$\frac{1}{4}$_____

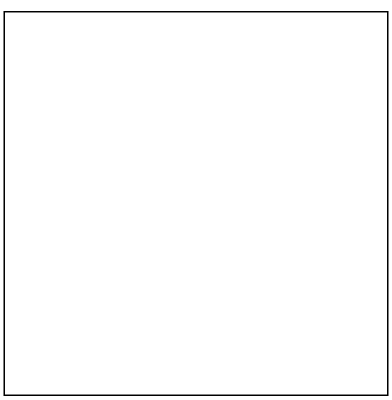


Fraction: ___$\frac{4}{16}$_____



Fraction: ___$\frac{25}{100}$_____

b. Cut up the square below in a way that is different than the three above and shade it to show $\frac{1}{4}$. Write the fraction that represents the shaded part under the model.



Encourage students to cut the square up into a different *total* number of parts than the ones given above. For example, they may cut the square into eight equal parts and shade 2 of them or into 10 equal parts and shade 2.5 of them.

Fraction: ________

c. What percent of each of the grids in parts a. and b. above is shaded? Explain. 25%; Students should notice that the model cut into 100 equal parts (a ratio out of 100) gives us the value of the percent.

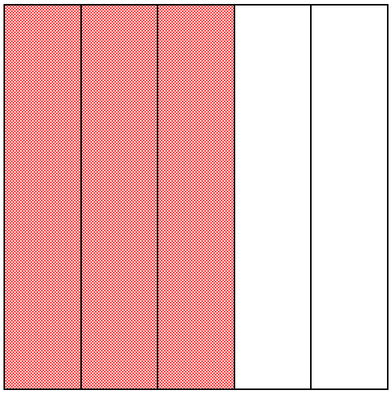
This is a good time to define percent. A percent is a part to whole ratio with a whole equal to 100. It is also defined as a rate per 100. Ask students which model shows 100 equal parts. This model will tell us the percent.

Activity 3: Each of the large squares shown below are the same size.

a. Shade $\frac{3}{5}$ of each model.

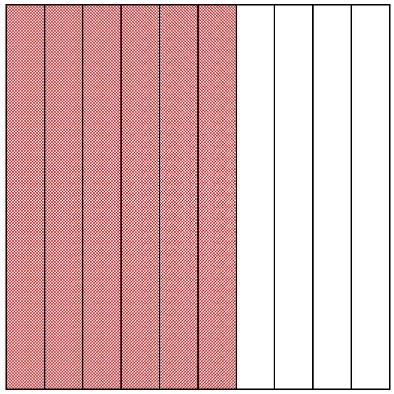
b. Express the shaded part of each model as a fraction and a percent.

Converting fluently between fractions and percents is a useful tool. If we want the whole to be 100, we need to think about the value of each box that will make the whole equal to 100 and then consider the value of the shaded portion when the whole is 100. The models below provide a structure for students – they can see that 1 part out of 5, corresponds to 2 parts out of 10 and 20 parts out of 100. When they consider three parts, they use repeated reasoning – if 1 part is equal to 20%, then 3 parts are equal to 60%.



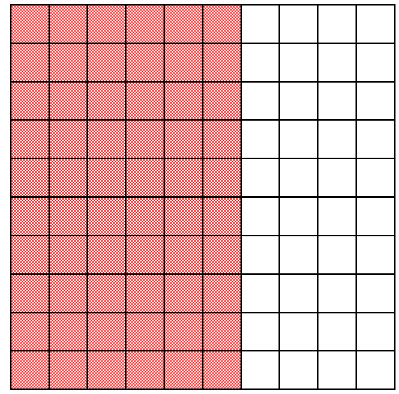
Fraction: ___$\frac{3}{5}$_____

Percent: ___60%_____



Fraction: ___$\frac{6}{10}$_____

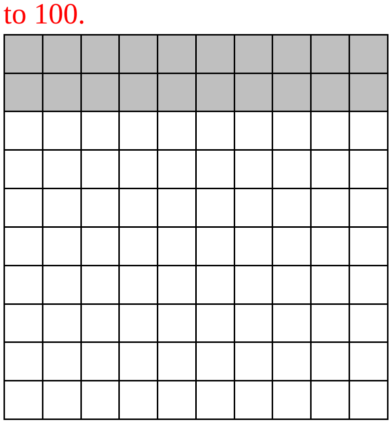
Percent: ___60%_____



Fraction: ___$\frac{60}{100}$_____

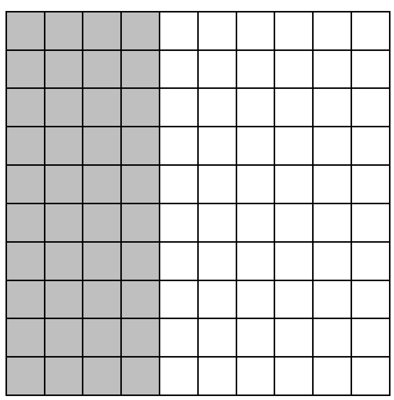
Percent: ___60%_____

Activity 4: Write the fraction and percent of each figure that is shaded. Students may or may not reduce the fraction for the portion; highlight the equivalence of the fractions. Three discussions should result here: 1) The location of the "shaded" portion does not affect the percent of the total. 2) The whole in each of the examples is ONE large square. Later in the section, there will be more squares (See 2.1b Class Activity #7); students will again be asked for the portion shaded. The answer always depends on *what we are considering the WHOLE to be*. 3) How "much" is in each shaded area. Remind students that a percent is a rate out of 100, thus to convert to a percent, the "whole" they are looking at is related to 100.



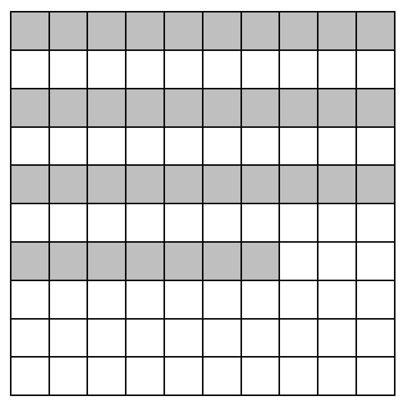
Fraction: ___$\frac{20}{100}$_____

Percent: ___20%_____



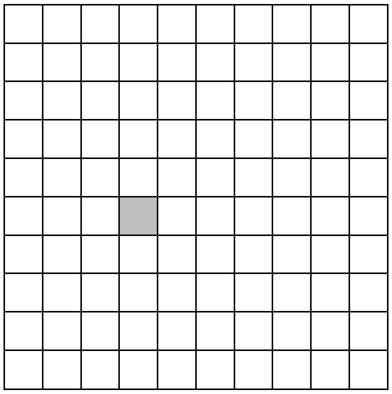
Fraction: ___$\frac{40}{100}$_____

Percent: ____40%____



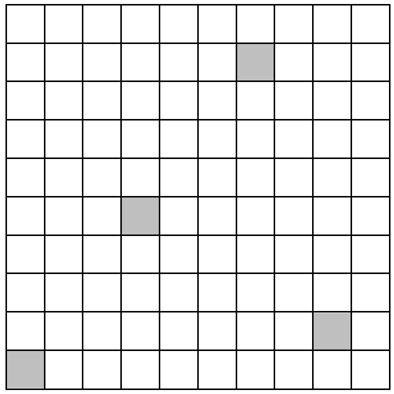
Fraction: ____$\frac{37}{100}$____

Percent: ___37%_____



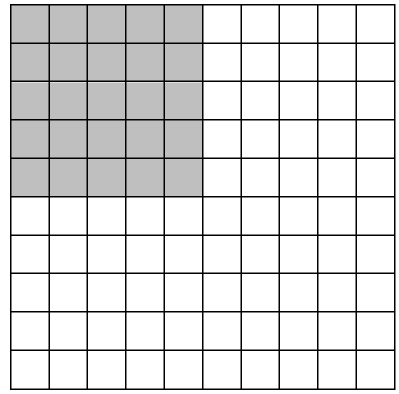
Fraction: ___$\frac{1}{100}$_____

Percent: ___1%_____



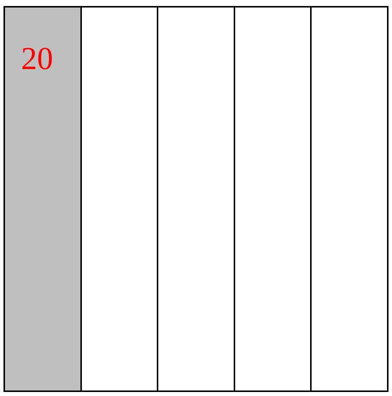
Fraction: ____$\frac{4}{100}$____

Percent: ___4%_____



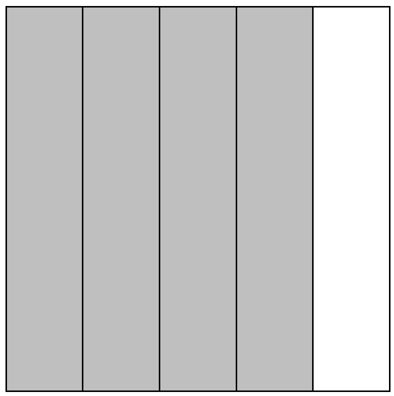
Fraction: ____$\frac{25}{100}$_____

Percent: ____25%_____



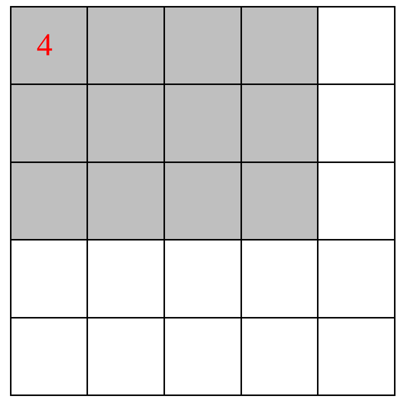
Fraction: ___$\frac{1}{5}$_____

Percent: __20%______



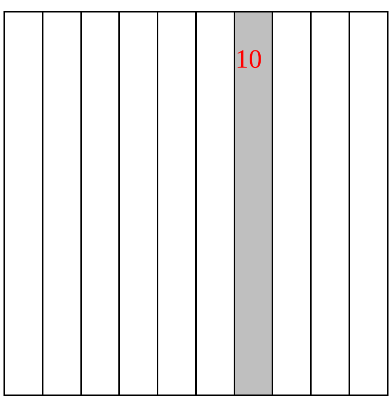
Fraction: ___$\frac{4}{5}$_____

Percent: ___80%_____



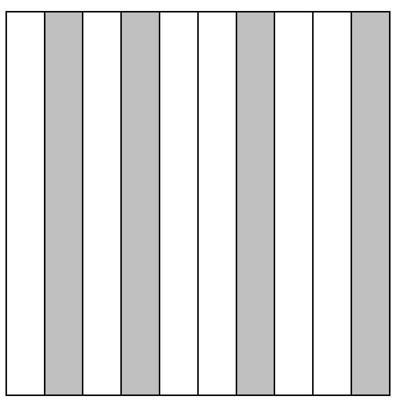
Fraction: ____$\frac{12}{25}$_____

Percent: ___48%_____



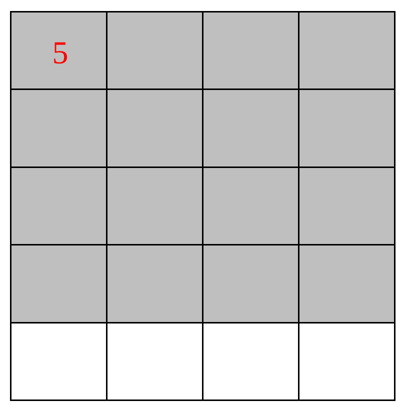
Fraction: ___$\frac{1}{10}$_____

Percent: ___10%_____



Fraction: ___$\frac{4}{10}$_____

Percent: ___40%_____

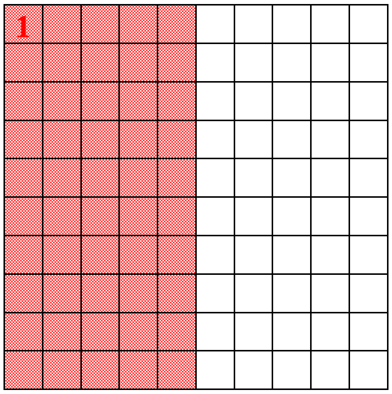


Fraction: _____$\frac{16}{20}$____

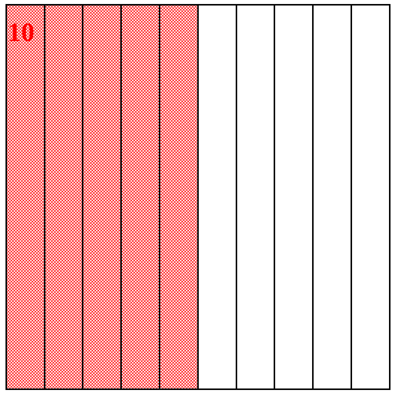
Percent: ____80%____

Activity 5:

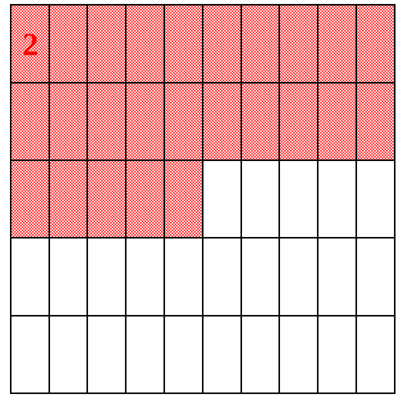
a. Shade 50% of each model. Then, write the fraction of the model that is shaded. For these problems, ask students what the value of each part needs to be for the whole to be 100. In this first model, the value of each part would be 1 ($100 \cdot 1 = 100$). In the second one, the value of each part would be 10 because there are 10 parts; $10 \cdot 10 = 100$. For the third one, the value of each part would be 2 because there are 50 parts; $50 \cdot 2 = 100$. From here, ask, "How many boxes do we need to shade in each model to have a total of 50?"



Fraction: $\frac{50}{100}, \frac{1}{2}$

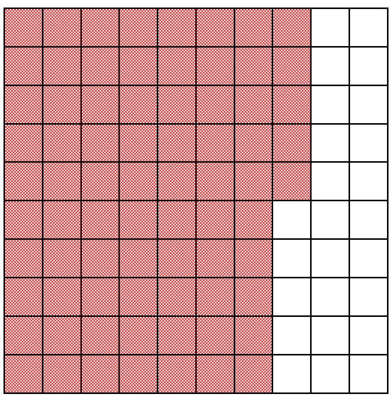


Fraction: $\frac{5}{10}, \frac{1}{2}, \frac{50}{100}$

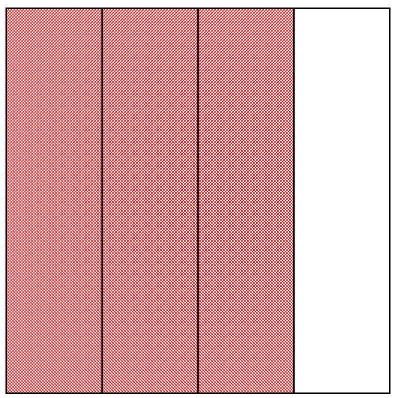


Fraction: $\frac{25}{50}, \frac{1}{2}, \frac{50}{100}$

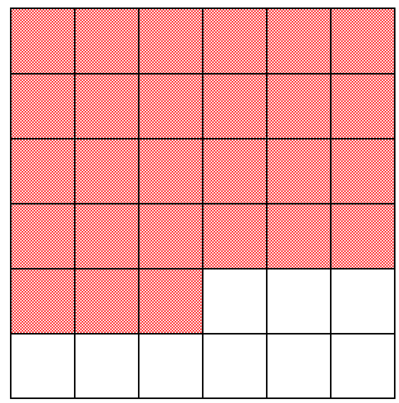
b. Shade 75% of each model. Then, write the fraction of the model that is shaded.



Fraction: $\frac{75}{100}$



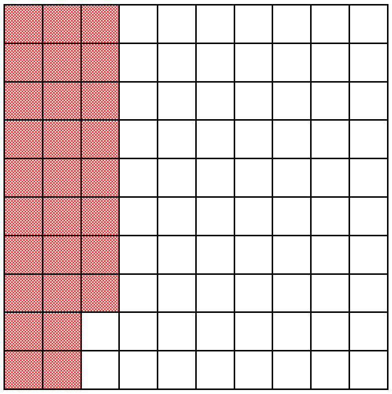
Fraction: $\frac{3}{4}$



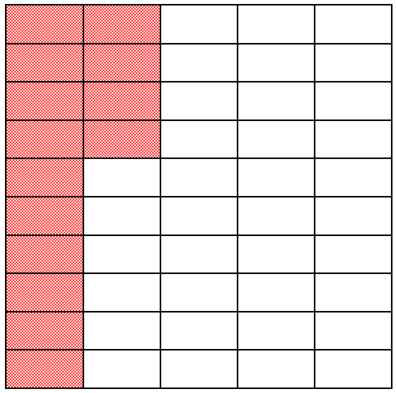
Fraction: $\frac{27}{36}$

It may be difficult for students to determine the value of each box for the third model on part b. They can use ideas from Chapter 1 to simplify $\frac{75}{100}$ to $\frac{3}{4}$ and then iterate up to 27 out of 36. It would be interesting to have students determine the value of each box. Have them start by determine the number of wholes they can fit into the 36 boxes to get to 100. They can fit 2. $2 \cdot 36 = 72$. They have $100 - 72$ or 28 remaining to divide between the 36 boxes. We can think of this as $\frac{28}{36}$ or $\frac{7}{9}$. Each box has a value of $2\frac{7}{9}$. Does $2\frac{7}{9} \cdot 36 = 100$? Yes. Does $2\frac{7}{9} \cdot 27 = 75$? Yes. Alternatively, they can take 100 and divide it equally into the 36 parts: $\frac{100}{36} = \frac{25}{9} = 2\frac{7}{9}$.

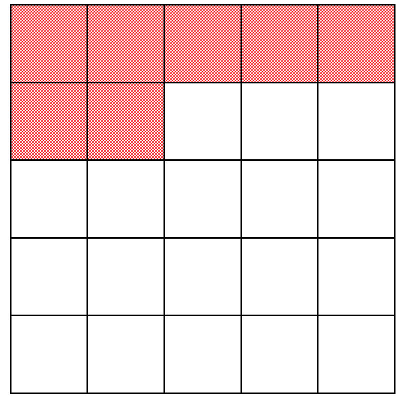
c. Shade 28% of each model. Then, write the fraction of the model that is shaded.



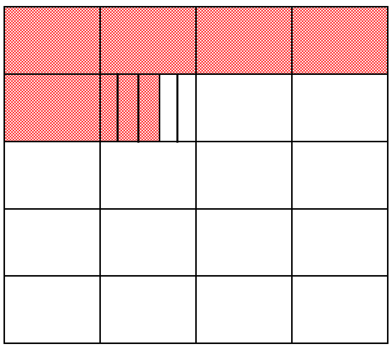
Fraction: $\frac{28}{100}$



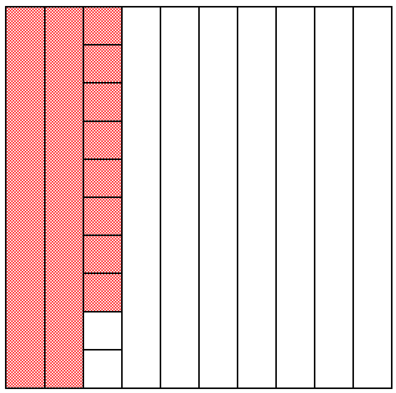
Fraction: $\frac{14}{50}$



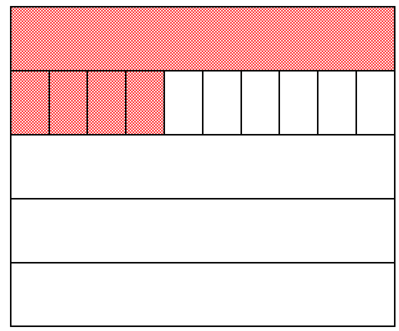
Fraction: $\frac{7}{25}$



Fraction: $\frac{5\frac{3}{5}}{20}$



Fraction: $\frac{2\frac{8}{10}}{10}$



Fraction: $\frac{7}{25}$

The last three models on part c. may be challenging for students. In the first model, there are 20 parts; therefore, each part would need to have a value of 5 for the total to be 100. Students can shade 5 parts giving a total of 25. Now, students need 3 more. If we partition a part into 5 equal smaller parts, each smaller part will have a value of 1. So, we need to shade 3 of these small parts or $\frac{3}{5}$ of the original part. Similar logic can be used for the second and third models. In the third model, each part has a value of 20. So, we shade 1 part and then need 8 more. We could partition a part into 20 equal pieces and shade 8 of them; however, it is easier to partition the part into 10 equal pieces (each with a value of 2) and shade 4 of them.

Spiral Review

1. Write three fractions that are equivalent to $\frac{1}{2}$.

2. What number is $\frac{1}{2}$ of 30?

3. What number is $\frac{1}{2}$ of 50?

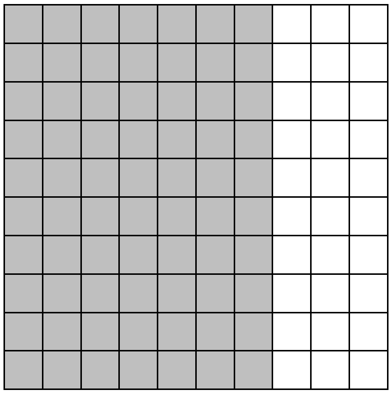
4. Express the following fractions as decimals.

a. $\frac{3}{10}$	b. $\frac{80}{100}$	c. $1\frac{9}{10}$
d. $\frac{45}{100}$	e. $\frac{150}{100}$	f. $\frac{2}{5}$
g. $\frac{1}{4}$	h. $\frac{9}{20}$	i. $1\frac{1}{2}$

2.1a Homework: Introduction to Percent as Rate Per 100

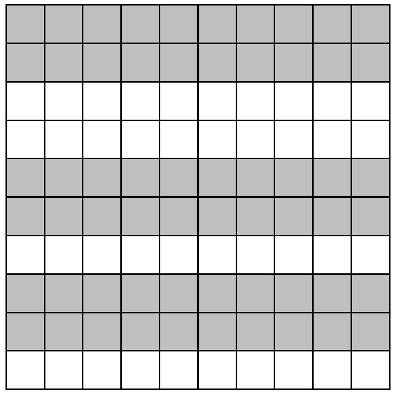
Note that a fraction equivalent to the one shown in the answers is correct.

1. Write the fraction and percent of each figure that is shaded.



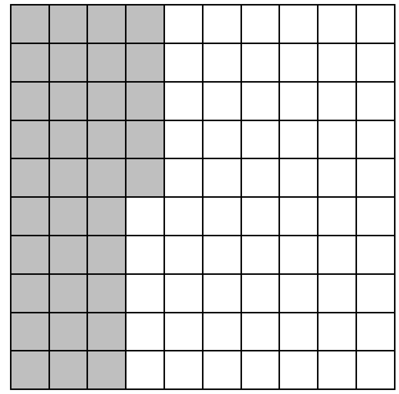
Fraction: ________

Percent: ________



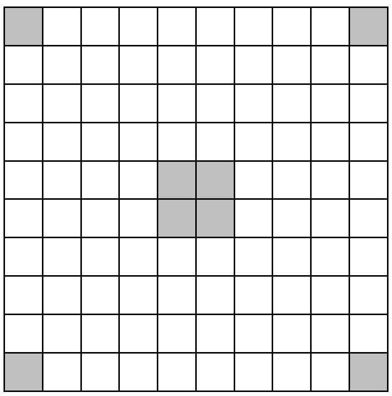
Fraction: ________

Percent: ________



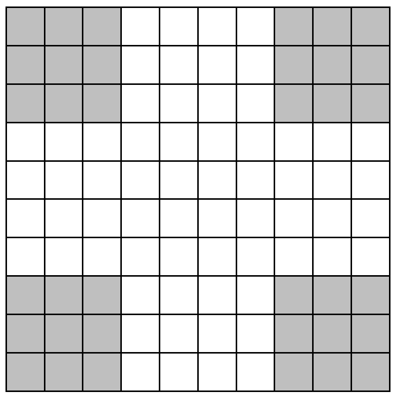
Fraction: ___$\frac{35}{100}$_____

Percent: ___35%_____



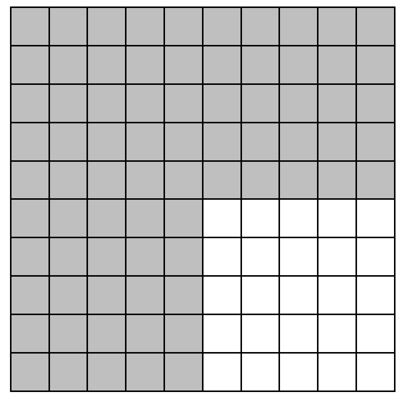
Fraction: ________

Percent: ________



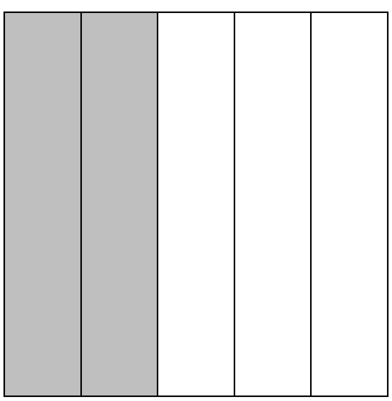
Fraction: ________

Percent: ________



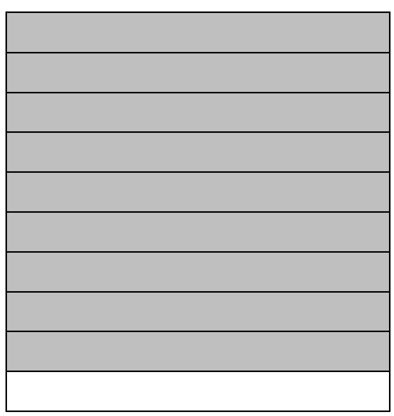
Fraction: ________

Percent: ________



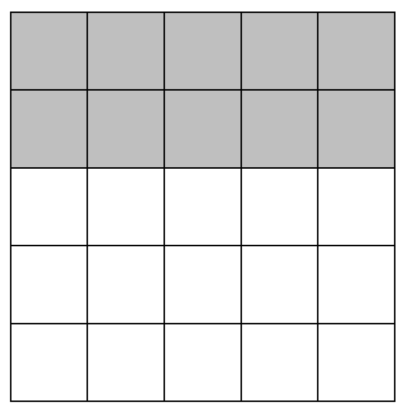
Fraction: ___$\frac{2}{5}$_____

Percent: ___40%_____



Fraction: ________

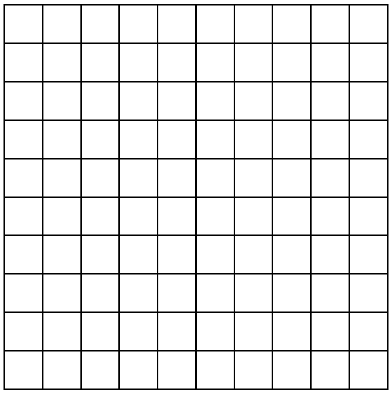
Percent: ________



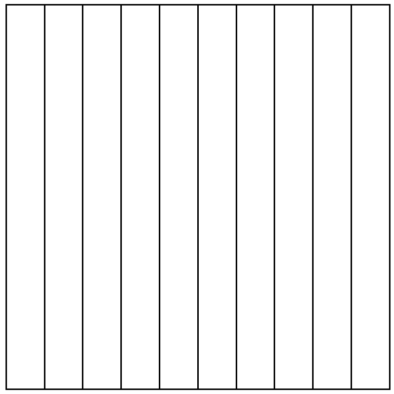
Fraction: ________

Percent: ________

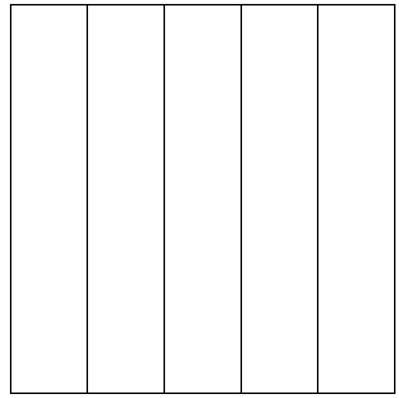
2. Shade 80% of each model. Then, write the fraction of the model that is shaded.



Fraction: ________

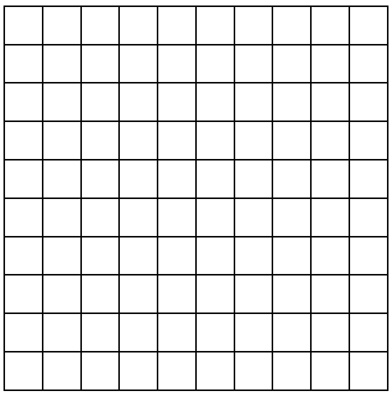


Fraction: ________

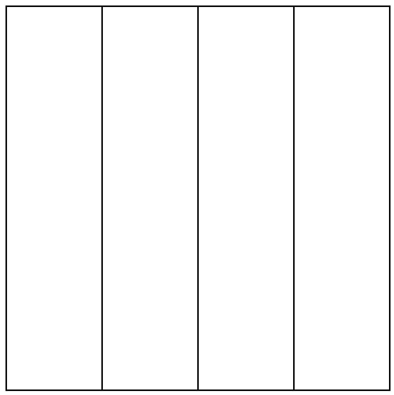


Fraction: ________

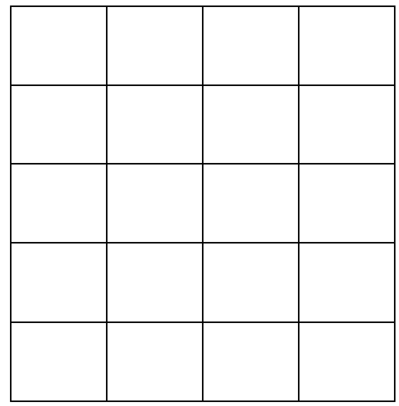
3. Shade 25% of each model. Then, write the fraction of the model that is shaded.



Fraction: ________

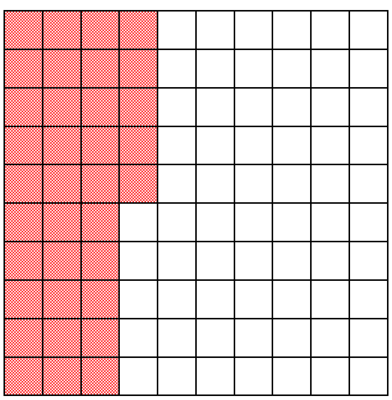


Fraction: ________

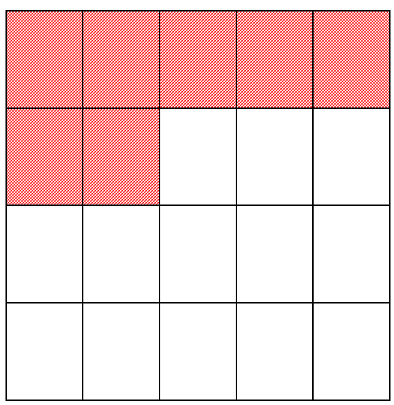


Fraction: ________

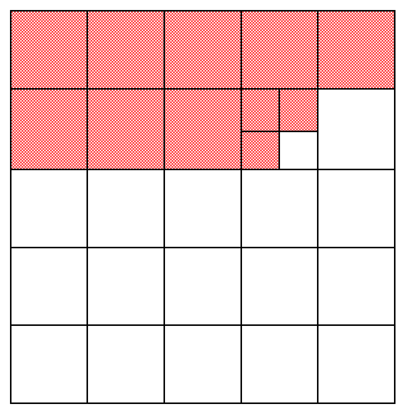
4. Shade 35% of each model. Then, write the fraction of the model that is shaded.



Fraction: ___$\frac{35}{100}$_____

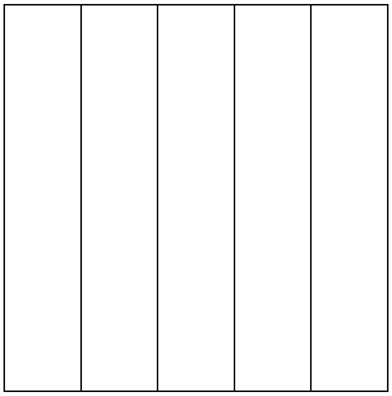


Fraction: ___$\frac{7}{20}$_____

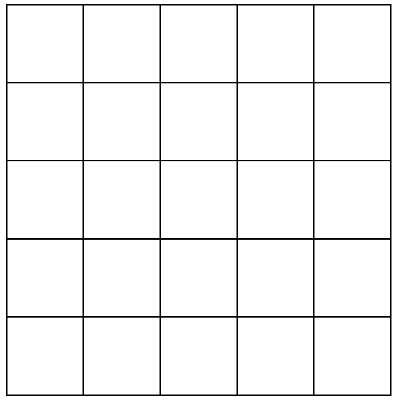


Fraction: ___$\frac{8\frac{3}{4}}{25}$_____

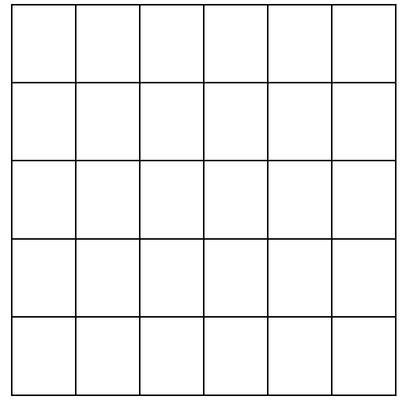
5. Shade 50% of each model. Then, write the fraction of the model that is shaded.



Fraction: ________



Fraction: ________



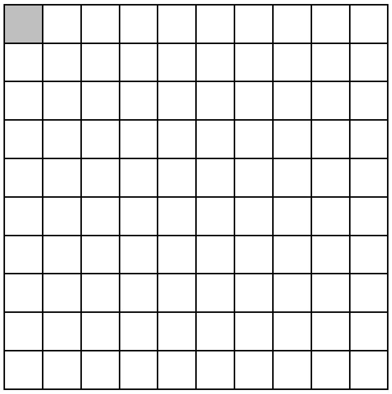
Fraction: ________

6. Draw three different models that represent 12%. Use grid or graph paper if needed.

2.1b Class Activity: Fraction, Decimal, Percent Equivalences

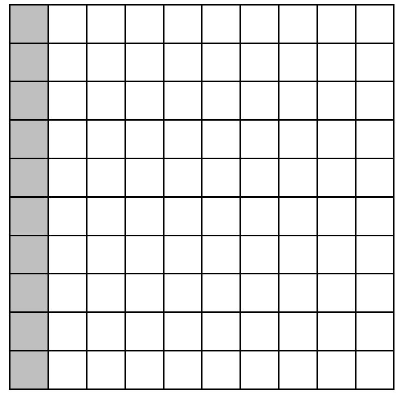
Directions: Express the shaded portion of each grid as a fraction, decimal, and percent. One large square represents the whole. Refer to the Anchor Problem Part 1 to help students to make sense of what they are being asked to do. If they are trying to express a part as a decimal, the whole is equal to 1. If they are trying to express the part as a percent, the whole is 100. Once they have identified the value of each box, they can use repeated reasoning to determine the value of the shaded portion of the grid. The models are an excellent tool for helping students to understand the relationship between fractions, decimals, and percents and for developing fluency converting between the different forms.

1.



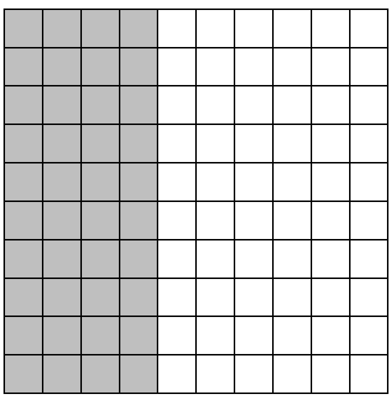
Fraction: $\frac{1}{100}$ Decimal: 0.01 Percent: 1%

2.



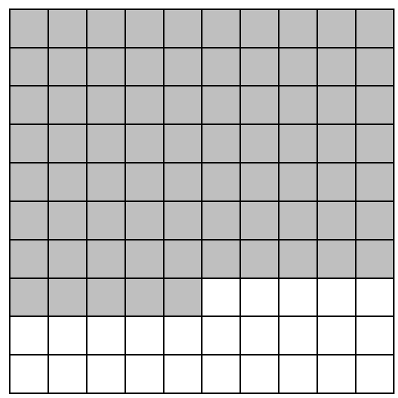
Fraction: $\frac{10}{100}$ Decimal: 0.1 Percent: 10%

3.



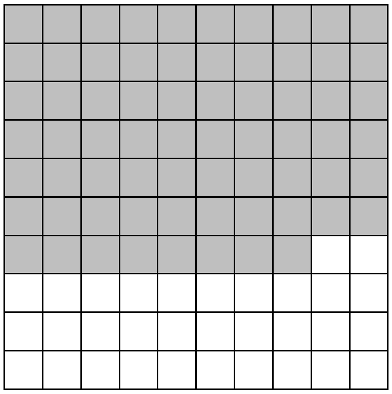
Fraction: $\frac{40}{100}$ Decimal: 0.4 Percent: 40%

4.



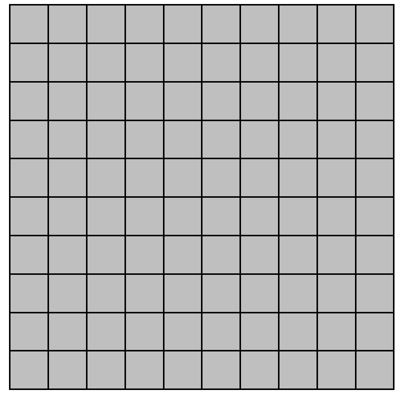
Fraction: $\frac{75}{100}$ Decimal: 0.75 Percent: 75%

5.



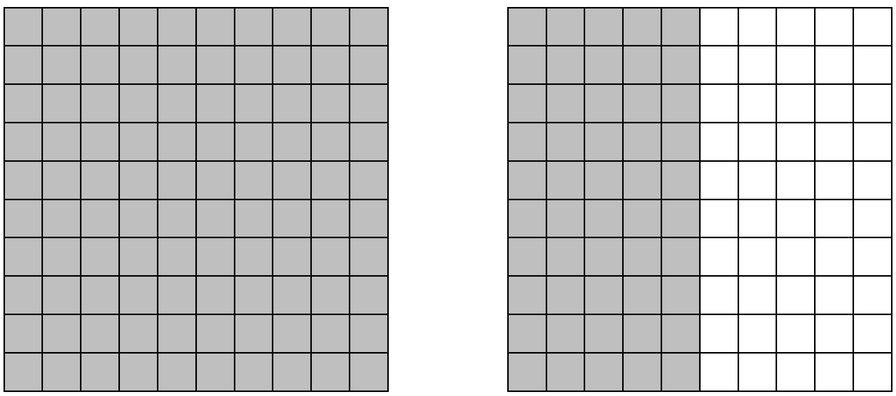
Fraction: __$\frac{68}{100}$__ Decimal: _0.68_ Percent: _68%_

6.



Fraction: _$\frac{100}{100} = 1$ Decimal: 1.0 Percent: _100%__

7.

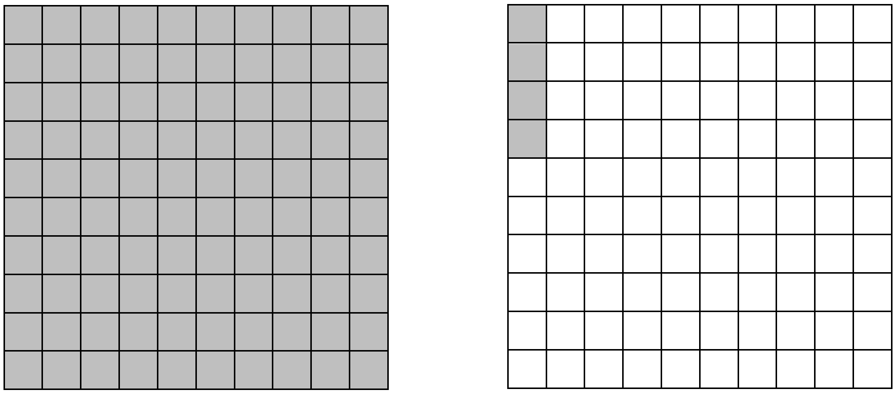


Fraction: __$\frac{150}{100}$ or $1\frac{1}{2}$ ____ Decimal: __1.5____ Percent: __150%____

Remind students to always identify the WHOLE. If the student is interpreting the whole as one large square, then #6 shows 100% or 1 and #7 shows 150%, 1.5, $1\frac{1}{2}$. However, if the student is interpreting the whole as two large squares, then #6 shows 50%, 0.5, $\frac{1}{2}$ and #7 shows 75%, 0.75, $\frac{3}{4}$. Both sets of answers are true for DIFFERENT WHOLES.

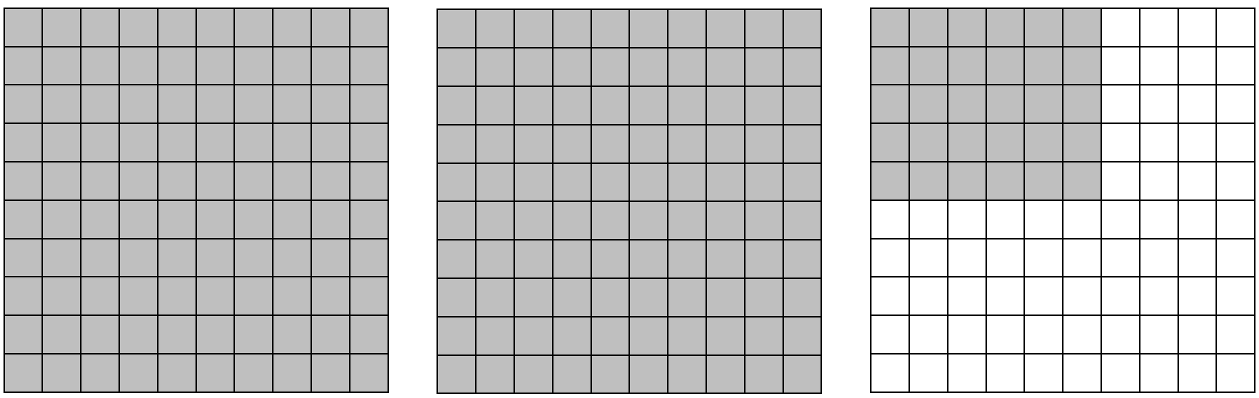
As students give their answers for #7, have them answer "150% of ***what***"? Or "75% of ***what***?" In the text, we have chosen to consider ONE square as the whole. If students answered 100% for #6, then they have also defined the whole as one large square.

8.



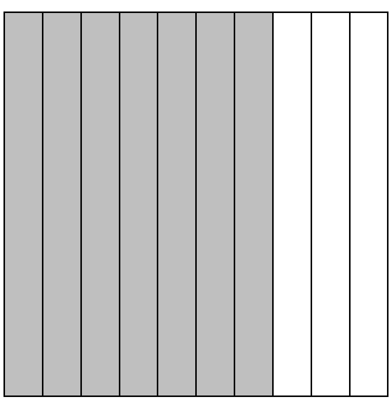
Fraction: __$\frac{104}{100}$ or $1\frac{4}{100}$____ Decimal: __1.04____ Percent: __104%____

9.



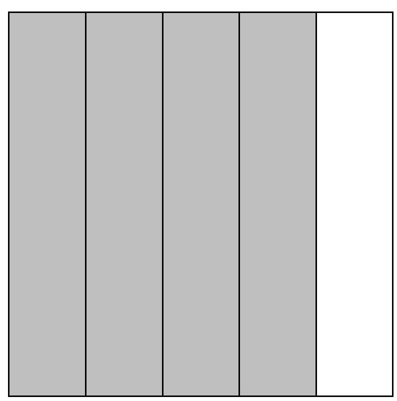
Fraction: ___$\frac{230}{100}$ or $2\frac{30}{100}$___ Decimal: __2.3____ Percent: ___230%___

10.



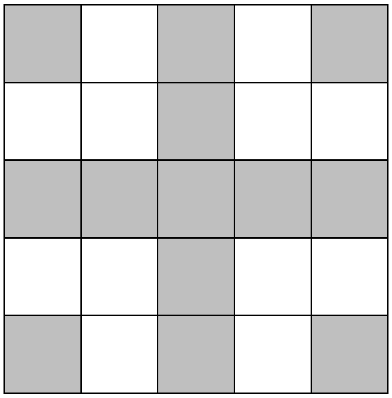
Fraction: _$\frac{7}{10}$__ Decimal: __0.7__ Percent: _70%__

11.



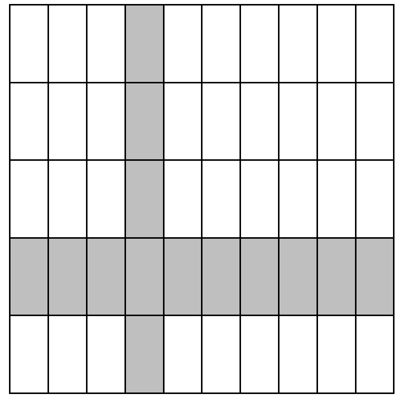
Fraction: _$\frac{4}{5}$__ Decimal: __0.8__ Percent: _80%__

12.



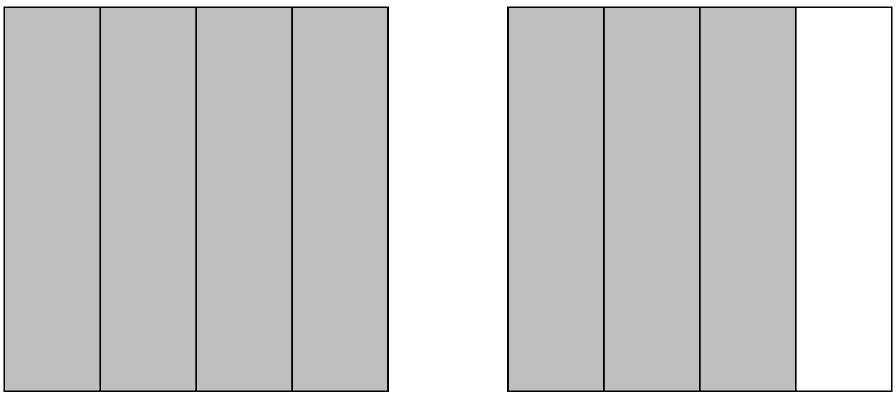
Fraction: _$\frac{13}{25}$ Decimal: _0.52__ Percent: __52%___

13.



Fraction: _$\frac{14}{50}$__ Decimal: __0.28__ Percent: _28%_

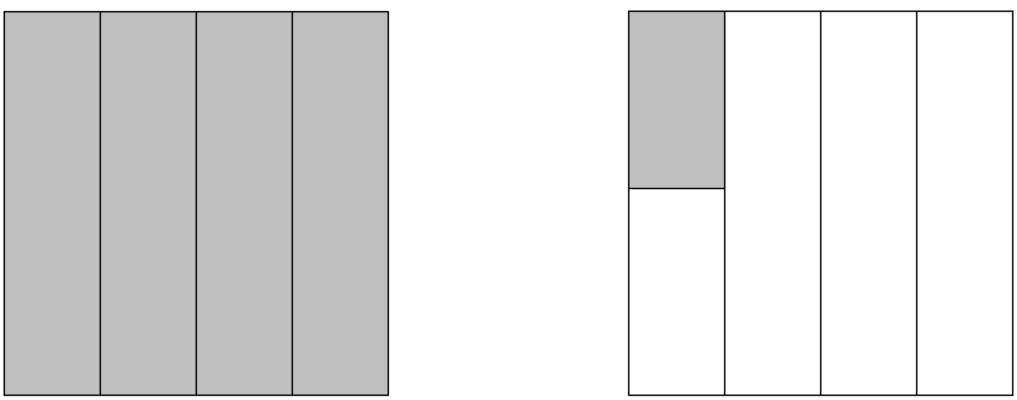
14.



Again, we will consider ONE square to be a whole, so 1 ¾, 1.75, 175%

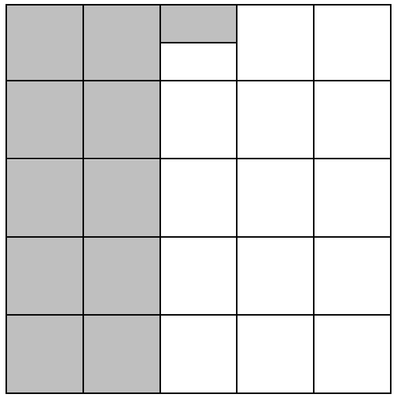
Fraction: _$\frac{7}{4}$ or_1 $\frac{3}{4}$_ Decimal: __1.75__ Percent: _175%__

15.



Fraction: _$\frac{9}{8}$ or_1 $\frac{1}{8}$_ Decimal: __1.125__ Percent: _112.5%__

16.



Fraction: _$\frac{10.5}{25}$_ Decimal: __0.42__ Percent: _42%__

Directions: Complete the table. Accept equivalent fractions and decimals.

Simplifying fractions is not essential in these exercises, nor is it an emphasis in the core. What is essential is that students recognize that numbers can be represented in different but equivalent forms and that sometimes one form of a number can be more useful than another depending on what you are trying to do. Students should recognize and understand the relationship between these different forms. (3/5 is the same portion of a whole as 6/10, 60/100, 0.6, 60%, etc.).

To convert to a percent, students will be looking for denominators that are factors OR multiples of 100. Thus, if the denominator is 35 for example (#38), dividing it and the numerator by 7 will produce a denominator that is easily converted to 100 OR if the denominator is 300, dividing both the numerator and denominator by 3 immediately produces a denominator of 100. Other problems may take more than one step; #38 21/35 is the same as 3/5, which is the same 60/100.

In the problems that follow, the models are not shown; therefore, students may turn to more numeric and abstract strategies such as partial tables and equations if they have not done so already. Students will continue to move toward a more abstract solving method in future coursework.

For #26

2	5	
20	50	>multiply by 10
40	100	>multiply by 2

$\frac{2}{5} = \frac{?}{100}$

For #29

22	25	
44	50	>multiply by 2
88	100	>multiply by 2

$\frac{22}{25} = \frac{?}{100}$

For #34

20	500	
4	100	>divide by 5

$\frac{20}{500} = \frac{?}{100}$

For #36

3	12	
1	4	>divide by3
25	100	>multiply by 25

$\frac{3}{12} = \frac{?}{100}$

	Fraction	Decimal	Percent
17.	$\frac{35}{100}$	0.35	35%
18.	$\frac{80}{100}$ or $\frac{4}{5}$	0.8	80%
19.	$\frac{75}{100}$ or $\frac{3}{4}$	0.75	75%
20.	$\frac{72}{100}$	0.72	72%
21.	$\frac{6}{100}$ or $\frac{3}{50}$	0.06	6%
22.	$\frac{25}{100}$ or $\frac{1}{4}$	0.25	25%
23.	$\frac{9}{10}$	0.90	90%
24.	$\frac{50}{100}$ or $\frac{1}{2}$	0.5	50%
25.	$\frac{36}{100}$ or $\frac{9}{25}$	0.36	36%
26.	$\frac{2}{5}$	0.4	40%
27.	$\frac{92}{100}$ or $\frac{23}{25}$	0.92	92%
28.	$\frac{8}{100}$ or $\frac{2}{25}$	0.08	8%
29.	$\frac{22}{25}$	0.88	88%
30.	$\frac{130}{100}$	1.3	130%
31.	$\frac{145}{100}$ or $1\frac{9}{20}$	1.45	145%
32.	$\frac{150}{100}$ or $1\frac{1}{2}$	1.5	150%
33.	$\frac{14}{200}$	0.07	7%
34.	$\frac{20}{500}$	0.04	4%
35.	$\frac{4}{400}$	0.01	1%
36.	$\frac{3}{12}$	0.25	25%
37.	$\frac{6}{8}$	0.75	75%
38.	$\frac{21}{35}$	0.60	60%

Directions: How do the portions of the whole relate? Compare using $<, >$, or $=$.

39. $\frac{9}{10}$ ___ = ___ 90% 40. $\frac{6}{200}$ ___ < ___ 6% 41. 5% ___ < ___ 0.5 42. $\frac{14}{25}$ ___ < ___ 60%

43. $\frac{2}{3}$ ___ < ___ 75% 44. $\frac{3}{12}$ ___ = ___ 25% 45. 50% ___ > ___ $\frac{2}{5}$ 46. $\frac{48}{50}$ ___ > ___ 48%

47. 125% ___ < ___ 1.3 48. $\frac{7}{4}$ ___ = ___ 175% 49. 1 ___ = ___ 100% 50. 1.01 ___ > ___ 99%

Directions: Put the following portions of a whole in order from smallest to largest. Justify your answer.

Encourage students to think about how they compared ratios in the previous chapter. When they compare ratios, it is generally easiest to use the same form (e.g., write both as percent or decimal). Students may use an algorithmic approach, changing all numbers to a percent or all numbers to a fraction with a common denominator (e.g., 20 or 100 for #51). Alternatively, students may reason through the problem. For example, in #51, students may think of 0.05 as 5%. From here, they may reason that $\frac{9}{20}$ is smaller than $\frac{1}{2}$ or 50%, and $\frac{11}{20}$ is greater than $\frac{1}{2}$ or 50%.

51. $\frac{9}{20}$, 0.05, 50%, $\frac{11}{20}$
 0.05, $\frac{9}{20}$, 50%, $\frac{11}{20}$
52. 118%, $1\frac{1}{5}$, 11.9%, $\frac{5}{4}$
 11.9%, 118%, $1\frac{1}{5}$, $\frac{5}{4}$

Spiral Review

1. Find $\frac{1}{4}$ of 24.

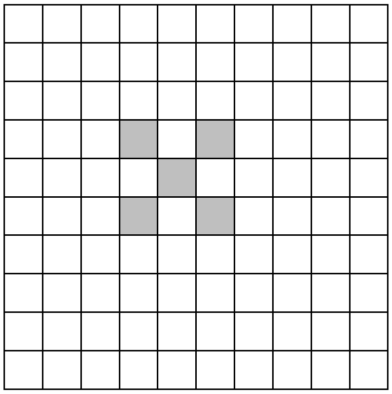
2. Find $\frac{3}{4}$ of 24.

3. Simplify. Look for patterns.
   a. 100×420
   b. 10×420
   c. 1×420
   d. 0.1×420
   e. 0.01×420
4. Make a double number line to show the relationship between feet and inches.

2.1b Homework: Fraction, Decimal, Percent Equivalences

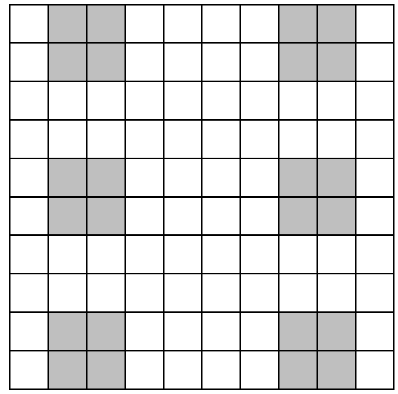
Directions: Express the shaded portion of each grid as a fraction, decimal, and percent. One large square represents the whole.

1.



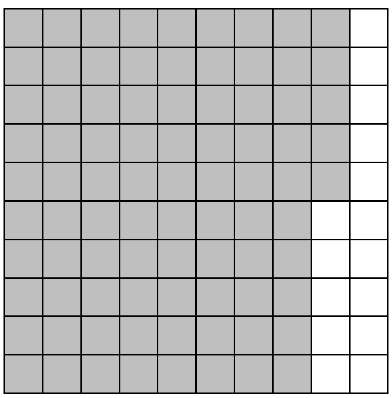
Fraction: _____ Decimal: ______ Percent: ______

2.



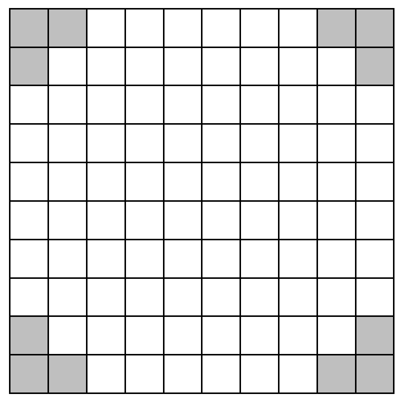
Fraction: _____ Decimal: ______ Percent: ______

3.



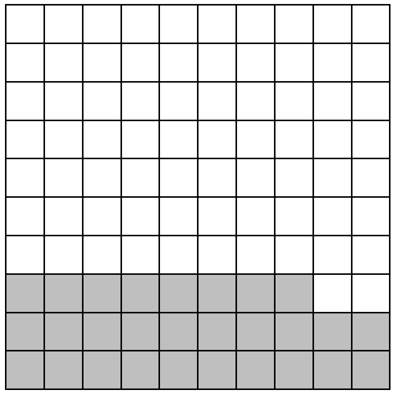
Fraction: ____ Decimal: ______ Percent: ______

4.



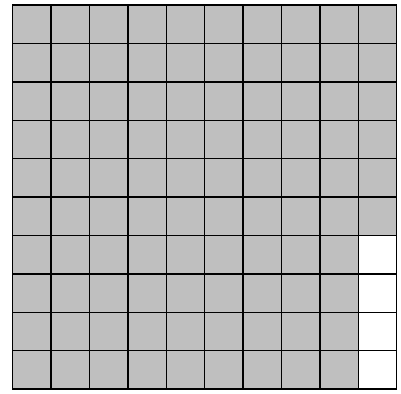
Fraction: _____ Decimal: _____ Percent: ______

5.



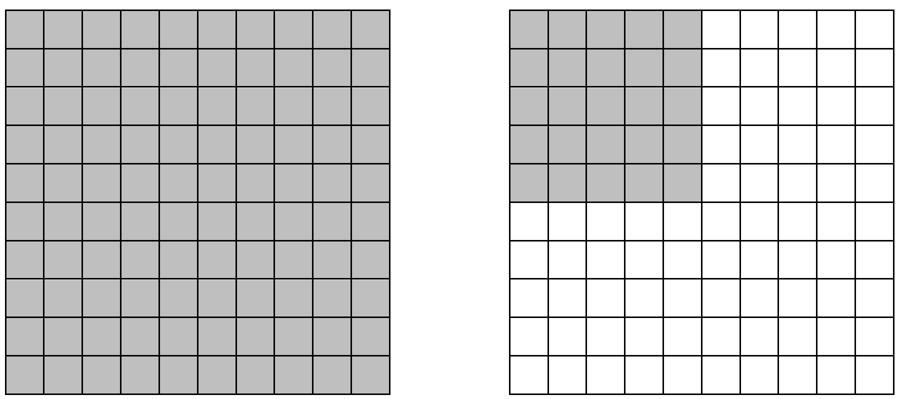
Fraction: ___ Decimal: _____ Percent: ______

6.



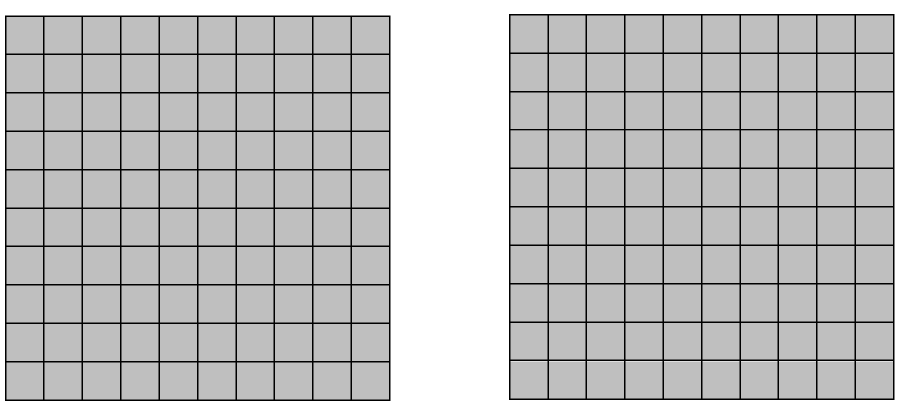
Fraction: ___ Decimal: _____ Percent: _____

7.



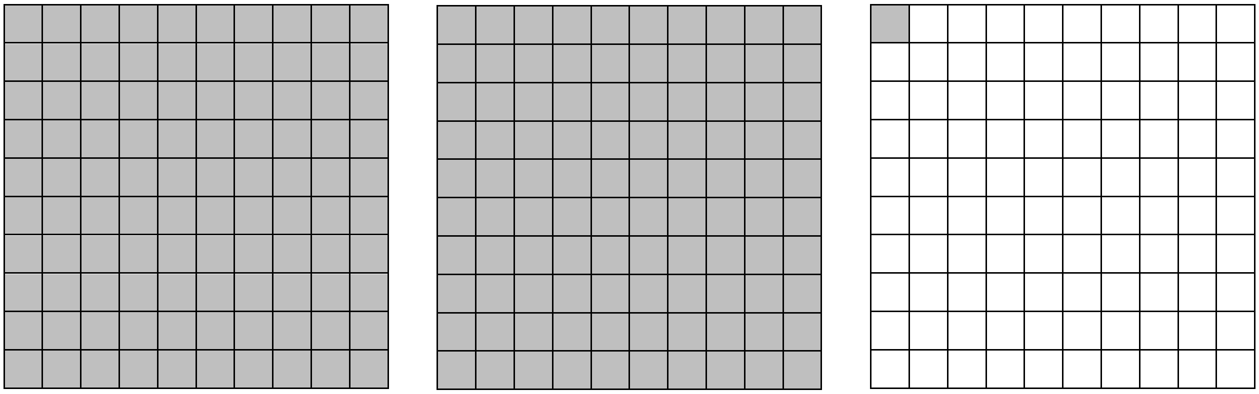
Fraction: ____ Decimal: ______ Percent: ______

8.



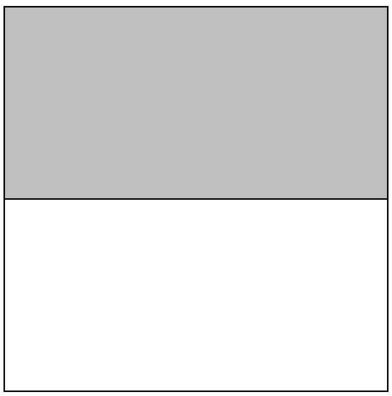
Fraction: _$\frac{2}{1}$_ Decimal: _2__ Percent: __200%__

9.



Fraction: ____ Decimal: _____ Percent: ______

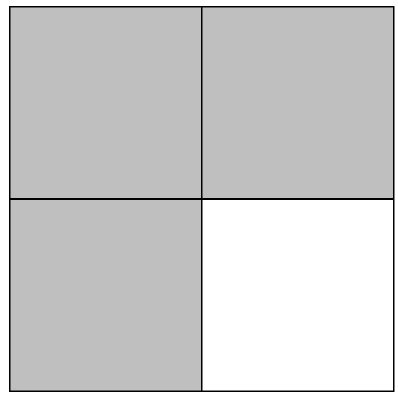
10.



The square is cut into two equal parts, so each part is 50%.

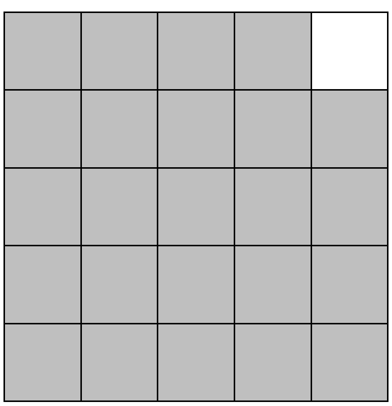
Fraction: _$\frac{1}{2}$_ Decimal: _0.5__ Percent: __50%_

11.



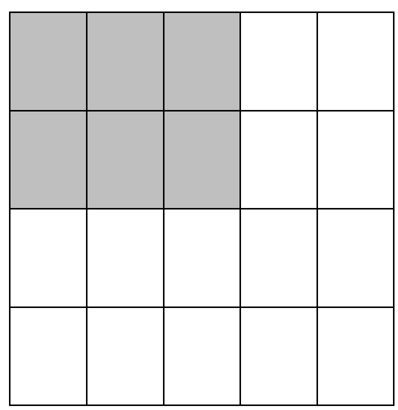
Fraction: ____ Decimal: _____ Percent: _____

12.



Fraction: ____ Decimal: _____ Percent: _____

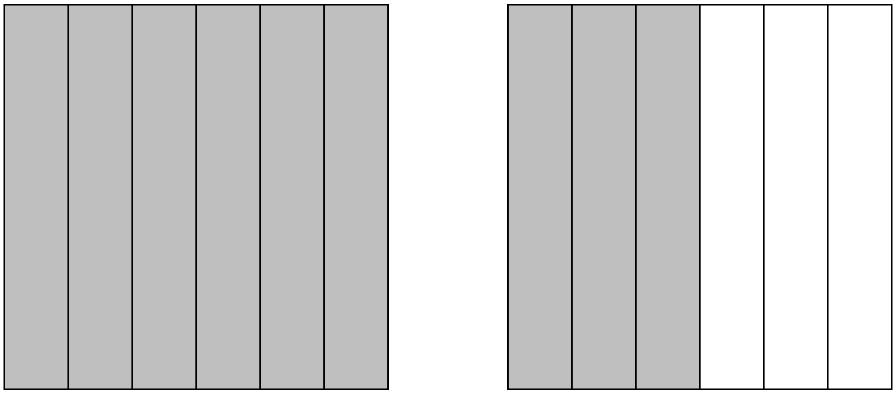
13.



Fraction: _$\frac{6}{20}$_ Decimal: _0.3__ Percent: __30%_

Some students may have a hard time with 12 and 13. Remind them that the MORE parts something is cut into, the smaller each part is.

14.



Fraction: ____ Decimal: _____ Percent: _____

Directions: Complete the table.

	Fraction	Decimal	Percent
15.	$\frac{9}{100}$		
16.			88%
17.	$\frac{9}{20}$	0.45	45%
18.	$\frac{92}{100}$		
19.		0.01	
20.			75%
21.	$\frac{14}{20}$		
22.		0.84	
23.	$\frac{14}{25}$	0.56	56%
24.	$\frac{38}{50}$		
25.		0.2	
26.			2%
27.	$\frac{1}{10}$		
28.	$\frac{175}{100}$	1.75	175%

29.	$\frac{5}{2}$	2.5	250%
30.		1.01	
31.	$\frac{100}{200}$		
32.	$\frac{33}{300}$		
33.	$\frac{100}{1000}$		
34.	$\frac{9}{12}$		
35.	$\frac{28}{40}$		
36.	$\frac{36}{90}$		

Directions: How do the portions of the whole relate? Compare using $<, >$, or $=$.

37. $\frac{8}{10}$ __ > __ 8%

38. $\frac{1}{5}$ __ = __20%

39. 45%_ > _0.045

40. 0.3_____3%

41. $\frac{3}{20}$_____16%

42. 0.39____40%

43. 33%_____$\frac{8}{25}$

44. $\frac{1}{50}$_____2%

45. 150% ____150

46. $\frac{5}{2}$_____200%

47. 1.2 ____120%

48. $1\frac{1}{10}$____110%

Directions: Put the following portions of a whole in order from smallest to largest.

49. $\frac{7}{10}$, 68%, 0.08, $\frac{36}{50}$

50. $\frac{9}{10}$, 95%, $\frac{24}{25}$, 0.099

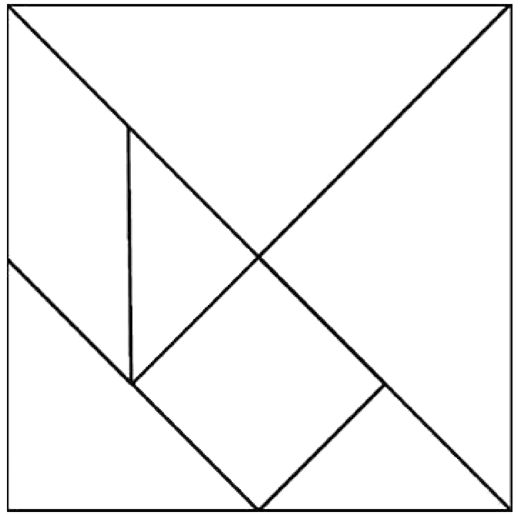
0.099, $\frac{9}{10}$, 95%, $\frac{24}{25}$

51. $\frac{13}{25}$, 0.5, 49%, $\frac{50}{200}$

52. 199%, $\frac{200}{100}$, $2\frac{1}{4}$, 2.3

2.1c Class Activity: Tangrams and Percentages

Directions: Find the value of each shape relative to the entire square. Remember the large (entire) square represents 1 whole. Record your findings in the table below.



Name of Shape	Fraction	Decimal	Percent
Large triangle	$\frac{1}{4}$	0.25	25%
Medium triangle	$\frac{1}{8}$	0.125	12.5%
Small triangle	$\frac{1}{16}$	0.0625	6.25%
Small Square	$\frac{1}{8}$	0.125	12.5%
Parallelogram	$\frac{1}{8}$	0.125	12.5%

This activity is a review of the previous lesson. It will be helpful to provide students with tangrams or a black-line master so they might manipulate pieces. Help students see relationships between objects. For example, students should notice that two middle-sized triangles form one larger triangle and two of the smallest triangles make the middle-size triangle or four of the smallest triangles make the largest. Then orchestrate a discussion about the numeric representations of the portions.

Make sure students verify that the total is in fact 100%. They can make another column to show the total area (e.g., 2 large triangles = $2 \cdot 25\% = 50\%$; 1 medium triangle = $1 \cdot 12.5\% = 12.5\%$; 2 small triangles = $2 \cdot 6.25\% = 12.5\%$; 1 small square = $1 \cdot 12.5\% = 12.5\%$; 1 parallelogram = $1 \cdot 12.5\% = 12.5\%$.)

2.1c Homework: Benchmark Percentages

Directions: The bar models shown represent common percentages. Write the fraction, decimal, and percent that corresponds to each bar model. The first bar shown represents 1 whole or 100%. The first problem has been done for you. This exercise is designed to help students recognize and memorize benchmark percentages and their equivalent fraction and decimal representations. Fluency with these benchmark percentages will serve as a valuable tool for students both in math class and every day life. It may be useful to create a poster for students to display in the classroom:

Students should also keep this as a reference in their binder/notebook.

A few notes:

- Help students see relationships between quantities. In the unit divided into 8 equal parts, students should notice that it can be thought of as a unit first divided into 4 equal parts and then each of those parts divided. Thus, if one part of a unit divided into 4 parts is 25%, half of 25% is 12.5%. Each of the 1/8 units is 12.5%. The same relationship exists between the unit divided into 5 parts and 10 parts.
- The unit divided into 10 parts may be the model students use most often. This model can be used to find values such as 30%, 70%, etc. And it is easily used to find 5% (half of 10%) or 1% (10% divided into 10 parts—a matter of moving the decimal) and then iterating it for 2%, 3%...
- The unit divided into 3 parts may bring up the questions, "why does 0.999 repeating = 1?" The convention at this level would be to say, we agree that it's equal to 1 and there are a few ways we justify it. The answer is not as straight forward as this video suggests: https://www.youtube.com/watch?v=G_gUE74YVos&t=4s but, this video may make things more confusing for students: https://www.youtube.com/watch?v=x-fUDqXlmHM. It's a great discussion that may help students begin to see the true nature of mathematics as a field of study.

One Part: Fraction: $\frac{1}{1}$ or 1 Decimal: 1.0 Percent: 100%

One Part: Fraction: ___$\frac{1}{2}$____ Decimal: ___0.5____ Percent: ___50%____

Two Parts: Fraction: ___$\frac{2}{2}$____ Decimal: ___1.0____ Percent: ___100%____

One Part: Fraction: ___$\frac{1}{4}$____ Decimal: ___0.25____ Percent: ___25%____

Two Parts: Fraction: ___$\frac{1}{2}$____ Decimal: ___0.5____ Percent: ___50%____

Three Parts: Fraction: ___$\frac{3}{4}$____ Decimal: ___0.75____ Percent: ___75%____

Four Parts: Fraction: ___$\frac{4}{4}$____ Decimal: ___1.0____ Percent: ___100%____

One Part: Fraction: ___$\frac{1}{8}$____ Decimal: ___0.125____ Percent: ___12.5%____

Two Parts: Fraction: ___$\frac{1}{4}$____ Decimal: ___0.25____ Percent: ___25%____

Three Parts: Fraction: ___$\frac{3}{8}$____ Decimal: ___0.375____ Percent: ___37.5%____

Four Parts: Fraction: ___$\frac{1}{2}$____ Decimal: ___0.5____ Percent: ___50%____

Five Parts: Fraction: ___$\frac{5}{8}$____ Decimal: ___0.625____ Percent: ___62.5%____

Six Parts: Fraction: ___$\frac{3}{4}$____ Decimal: ___0.75____ Percent: ___75%____

Seven Parts: Fraction: ___$\frac{7}{8}$____ Decimal: ___0.875____ Percent: ___87.5%____

Eight Parts: Fraction: ___$\frac{8}{8}$____ Decimal: ___1.0____ Percent: __100%_____

One Part: Fraction: ___$\frac{1}{5}$____ Decimal: ___0.2____ Percent: __20%_____

Two Parts: Fraction: ___$\frac{2}{5}$____ Decimal: ___0.4____ Percent: __40%_____

Three Parts: Fraction: ___$\frac{3}{5}$____ Decimal: ___0.6____ Percent: __60%_____

Four Parts: Fraction: ___$\frac{4}{5}$____ Decimal: ___0.8____ Percent: __80%_____

Five Parts: Fraction: ___$\frac{5}{5}$____ Decimal: ___1____ Percent: __100%_____

One Part: Fraction: ___$\frac{1}{10}$____ Decimal: ___0.1____ Percent: __10%_____

Two Parts: Fraction: ___$\frac{2}{10}$____ Decimal: ___0.2____ Percent: __20%_____

Three Parts: Fraction: ___$\frac{3}{10}$____ Decimal: ___0.3____ Percent: __30%_____

Four Parts: Fraction: ___$\frac{4}{10}$____ Decimal: ___0.4____ Percent: __40%_____

Five Parts: Fraction: ___$\frac{5}{10}$____ Decimal: ___0.5____ Percent: __50%_____

Six Parts: Fraction: ___$\frac{6}{10}$____ Decimal: ___0.6____ Percent: __60%_____

Seven Parts: Fraction: ___$\frac{7}{10}$____ Decimal: ___0.7____ Percent: __70%_____

Eight Parts: Fraction: ___$\frac{8}{10}$____ Decimal: ___0.8____ Percent: __80%_____

Nine Parts: Fraction: ___$\frac{9}{10}$____ Decimal: ___0.9____ Percent: __90%_____

Ten Parts: Fraction: ___$\frac{10}{10}$____ Decimal: ___1.0____ Percent: __100%_____

One Part: Fraction: ___$\frac{1}{3}$____ Decimal: ___$0.\overline{3}$____ Percent: __$33\frac{1}{3}\%$_____

Two Parts: Fraction: ___$\frac{2}{3}$____ Decimal: ___$0.\overline{6}$____ Percent: __$66\frac{2}{3}\%$_____

Three Parts: Fraction: ___$\frac{3}{3}$____ Decimal: ___1.0____ Percent: __100%_____

2.1d Class Activity: Fractions, Decimals, Percents in the Real World

In this section students are working with percents in contextual situations. You will notice that the focus is still on "portions" of wholes. What is different is the idea of the "whole." In the previous sections, the whole was often a shape of some sort, now students will be thinking of the "whole" as a collection of objects – a group of students, a bag of marbles, etc. These objects can be discrete objects like people or "continuous" like a liquid. Students will be required to read and interpret real world models such as graphs to answer the questions.

Directions: Solve the following problems.

1. $\frac{3}{5}$ of the 6th grade class at a certain school own a cell phone.
   a. Make a tape diagram to represent this situation.

      ← 6^{th} grade class →

20%	20%	20%	20%	20%

      Owns a Cell Phone

   b. What percent of the students own a cell phone? ____60%_______

   c. What percent of the students do not own a cell phone? ____40%_______

   d. What is the ratio of students who own a cell phone to the ratio of students who do not own a cell phone? 3: 2
   e. If there are seventy-five 6^{th} graders at this school, how many own a cell phone? ____45_______

      ← 75 6^{th} graders →

15	15	15	15	15

      $75 \div 5 = 15$

2. 25% of the marbles in a bag are red.
   a. Make a tape diagram to represent this situation.

Red	Not Red	Not Red	Not Red

   b. What fraction of the marbles are red? ______$\frac{1}{4}$_______
   c. What fraction of the marbles are not red? ______$\frac{3}{4}$_______
   d. What is the ratio of marbles that are red to marbles that are not red? 1: 3
   e. Give some possible pairs of values for the number of marbles that are red and the number of marbles that are not red. Organize your results in a table.

Red	Not Red
1	3
10	30
30	90

3. Students at a certain high school can choose from three different language classes. $\frac{9}{20}$ of the students choose Spanish, 37% choose French, and $\frac{9}{50}$ choose Chinese.
   a. Order the languages from the one the most number of students take to the one the least number of students take. Justify your answer.

Spanish 9/20 = 45/100
French 37% = 37/100
Chinese 9/50 = 18/100

Spanish, French, Chinese

Throughout the exercises, emphasize total in the group and its relationship to "out of 100."

4. Owen has put several different colored marbles into a bag. The table below shows the different color marbles and how many of each color are in the bag:

Color of Marble	**Number of Marbles**
Red	40
Orange	10
Yellow	20
Green	10
Blue	20

   a. Make a tape diagram to represent this situation.
   There are 100 marbles in the bag. Students will likely use a 10-frame partition:

R	R	R	R	O	Y	Y	G	B	B

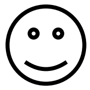
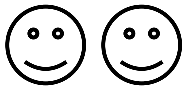
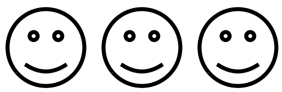
   b. What percent of the marbles in the bag are red? 40%

   c. What percent of the marbles in the bag are not red? 60%

   d. What percent of the marbles are yellow or green? 30%

5. Hannah surveyed students at her school and asked what their favorite vegetable was. The pictograph shows the results of the survey.

 **= 25 students**

Favorite Vegetable	**Number of Students**	**Percent of Students**
Tomatoes		10%
Green Beans		10%
Corn		20%
Carrots		30%
Broccoli		30%

a. Make a tape diagram to represent this situation.
There are a total of 10 smiley faces each with a value of 25.

T	GB	C	C	CT	CT	CT	BR	BR	BR

b. Complete the table to show the percent of students who chose each vegetable.
Although it is given that each smiley face represents 25 students, the value of the smiley faces can be anything and it will not change the *percentage* of students who chose each vegetable. Each smiley face represents a box in the tape diagram or 10%. We can change the value of each box and the ratio to the total (or percent) does not change.

c. What percent of students chose tomatoes or corn as their favorite vegetable? 30%

6. Noah surveyed the students in his class and asked how they got to school. Here are the results of the survey:

Mode of Transportation	Number of Students	Percent of Students
Car	16	50%
Bike	4	12.5%
Bus	8	25%
Walk	4	12.5%

a. Make a tape diagram to represent this situation.

C	C	C	C	Bus	Bus	BK	W

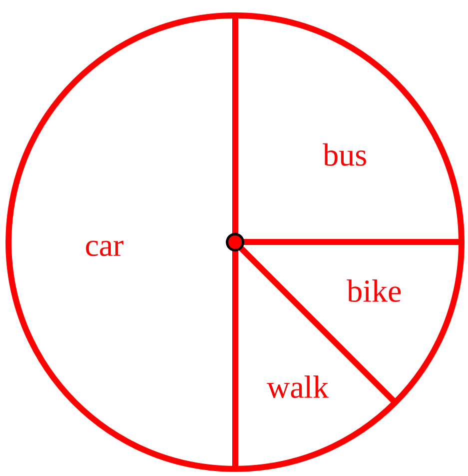
There are a few different ways students may approach making this tape diagram. They may first find the total (32) and then see that the car (16) is half of the total. From here, students have half the diagram left (with a value of 16). Half of that half (8) goes to bus. Now, one quarter of the diagram is left (with a value of 8). Half of this goes to bike (4) and half goes to walking (4). From here, students will see that to create equal size pieces, there should be a total of 8 boxes in the tape diagram.

Students may also notice that each category is divisible by 4 (or a multiple of 4). If we give each box a value of 4, we will need 8 total boxes (4 for car, 2 for bus, 1 for bike, and 1 for walk).

$16 \div 4 = 4$ parts car or $4 \cdot 4 = 16$
$8 \div 4 = 2$ parts bus or $4 \cdot 2 = 8$
$4 \div 4 = 1$ part bike or $4 \cdot 1 = 4$
$4 \div 4 = 1$ part walk or $4 \cdot 1 = 4$

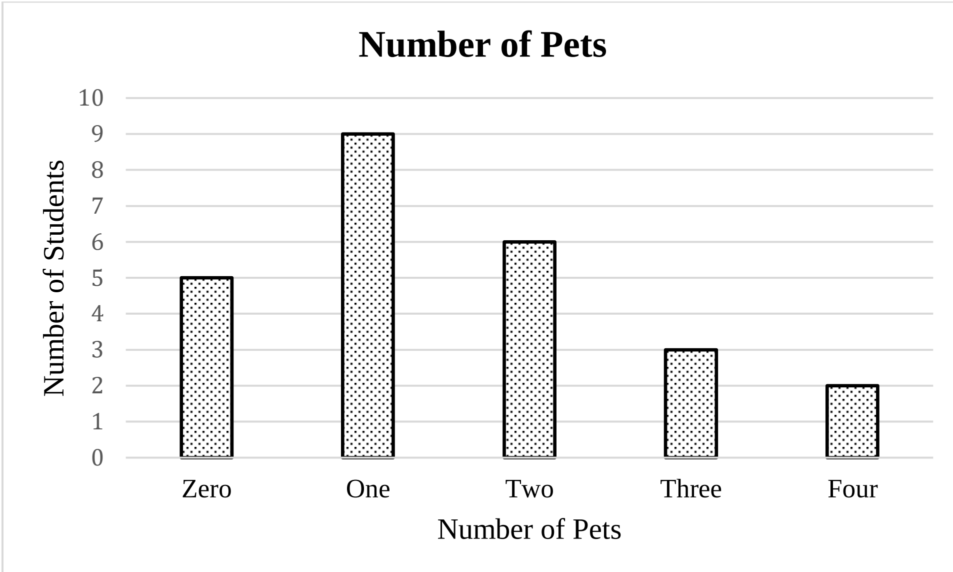
b. Complete the table to show the percent of students who take each mode of transportation.

c. Use the results of the survey to create a circle graph below. Be sure to include a key for your circle graph.



Discuss the relationship between the tape diagram and circle graph.

7. Renee surveyed the students in her class to see how many pets they have. The bar graph shows the results of the survey: Anticipate: Students may not know what the total represents in this case; they may think it's the total number of pets.



a. What percent of the students in Renee's class own two or more pets? 44%

b. What percent of the students in Renee's class own fewer than two pets? 56%

There are a total of 25 students responding to the survey. Of the students, we see:

5 have Zero pets e.g. 5/25
9 have One pet e.g. 9/25
6 have Two pets e.g. 6/25
3 have Three pets e.g. 3/25
2 have Four pets e.g. 2/25

We can multiply each by 4/4 to find the rate out of 100.

a. Two or more 6/25 + 3/25 + 2/25 = 11/25 = 44/100 = 44%
b. 56%

Spiral Review

1. Carina is drawing circles and squares on her paper. The ratio of circles to squares in Carina's pattern is 2:5. Create Carina's pattern below.

2. Miguel is also drawing circles and squares on his paper. The ratio of circles to total shapes on Miguel's pattern is 2:5. Create Miguel's pattern below.

3. Simplify.

a. $\frac{1}{2}\times 20$	b. $\frac{1}{4}\times 20$	c. $\frac{1}{5}\times 20$	d. $\frac{1}{10}\times 20$

4. Complete the sentences.
   a. Multiplying by $\frac{1}{2}$ is the same as ________________________________.

   b. Multiplying by $\frac{1}{4}$ is the same as ________________________________.

   c. Multiplying by $\frac{1}{5}$ is the same as ________________________________.

   d. Multiplying by $\frac{1}{10}$ is the same as ________________________________.

2.1d Homework: Fractions, Decimals, Percents in the Real World

Directions: Solve the following problems.

1. $\frac{1}{2}$ of the items in a bake sale are cookies.

   a. Draw a tape diagram to represent this situation.

   b. What percent of the items in the bake sale are cookies? ___________

   c. What percent of the items in the bake sale are not cookies? ___________

   d. What is the ratio of items that are cookies to items that are not cookies? ___________

   e. If there are 150 cookies in the bake sale, how many items total are being sold? ___________

2. $\frac{1}{5}$ of the students at Washington Middle School participate in the school band.

   a. Draw a tape diagram to represent this situation.

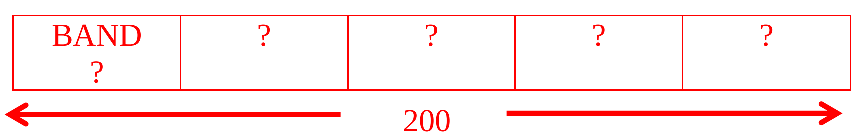
   b. What percent of the students participate in band? _____20%______

   c. What percent of the students do not participate in band? ____80%_______

   d. What is the ratio of students who participate in band to students who do not participate in band? ___1: 4___

   e. What is the ratio of students who participate in band to total students? __1: 5___

   f. If there are 200 students at Washington Middle School, how many participate in band? __40____



If there are 200 students, we divide them equally into the five groups, thus there are 40 in each group; 40 who participate in band, 160 that do not.

3. Sixty percent of the students in Ms. Serr's class are boys.

    a. Draw a tape diagram to represent this situation.

    b. What fraction of the class is boys? ___________

    c. What fraction of the class is girls? ___________

    d. If there are 30 students in Ms. Serr's class, how many are girls and how many are boys?

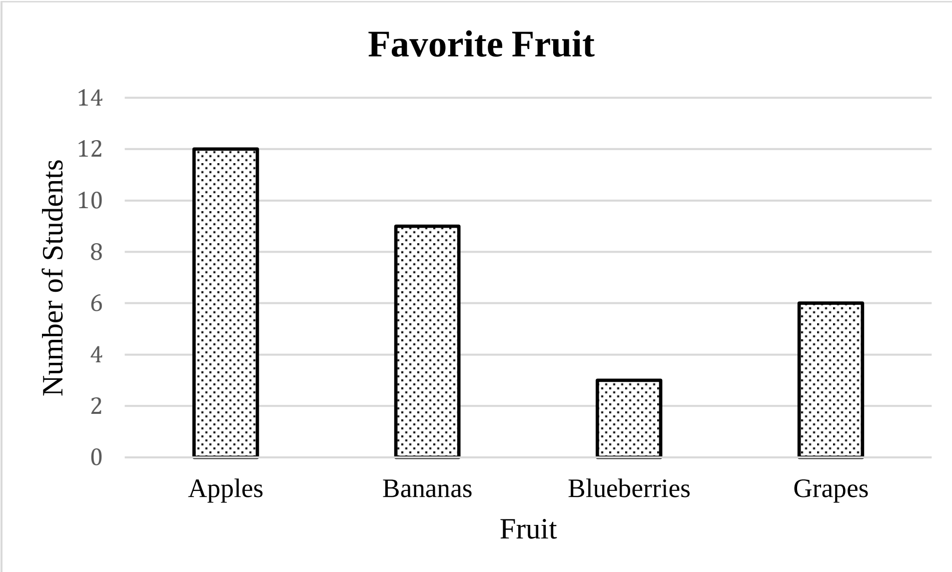
4. Seventy percent of iPhone users use the calculator application on their phone.
    a. Draw a tape diagram to represent this situation.

    b. What fraction of iPhone users use the calculator application on their phone? ___________

    c. In a group of 60 people, how many would you expect use the calculator application on their phone?

5. Eli puts 75% of the money he earns working for his grandfather in the bank. Lucy puts $\frac{36}{50}$ of the money she earns working for her grandfather in the bank. If they earn the same amount, who puts more money in the bank? Justify your answer.

6. Jennifer surveyed the students in her class and asked what their favorite fruit is. The bar graph shows the results of the survey.



a. What percent of the students in Jennifer's class chose each type of fruit?

Apples: ______ Bananas: ________ Blueberries: _______ Grapes: ______

7. Zoe is on a competitive soccer team. The table below shows her team's record in the regular season:

Outcome	Number of Games
Wins	12
Losses	2
Ties	1

a. What percent of the soccer games did Zoe's team win? (A tie is not considered a win.)
80%

Students may choose to use a tape model or work with equivalent fractions.

There are a total of 15 games played. Zoe's team won 12/15. It is easiest to work with denominators that are factors of 100, so if we divide the numerator and denominator by 3 we get 4/5, which is 80%

2.1e Class Activity: Percent as a Part to Total Ratio

In this section students need to attend to precision and make sense of what they are given: Are they given two parts? A part and a whole? How can they translate the ratio they are given whether it is a part to part or a part to whole to a percent which is a part to whole ratio? Encourage student to draw models to justify their answers.

Directions: Solve the following problems.

1. Tia is putting red and blue marbles into different bags. In which of the bags are 25% of the marbles red? Justify your answers.

   a. **Bag 1:** One out of every four marbles in the bag is red. 25%

1	1	1	1

   b. **Bag 2:** There are a total of 40 marbles in the bag. Of the 40 marbles, 10 are red. 25%

10	10	10	10

← 40 marbles →

   c. **Bag 3:** There are 25 red marbles and 75 blue marbles in the bag. 25%

25	25	25	25

← 100 marbles →

   d. **Bag 4:** The ratio of red marbles to blue marbles is 1:4. Not 25%

Red	Blue	Blue	Blue	Blue

← 1R : 4 B marbles →

   e. **Bag 5:** There are 400 marbles in the bag. Of the 400 marbles, 100 are red. 25%

100	100	100	100

← 400 marbles →

   f. **Bag 6:** For every three blue marbles Tia puts in the bag, she puts 1 red marble. 25%

Red	Blue	Blue	Blue

← 1R : 3B marbles →

   g. **Bag 7:** The number of blue marbles is three times the number of red marbles. 25%
   We can see in the model above that blue is three times red.

2. Consider the following situations about five different basketball teams. Circle the letters of the teams that have the same winning percentage. Justify your answer.
   a. Team A wins four out of every 5 games it plays.

W	W	W	W	L

   Winning Percentage: 80%

   b. For Team B, the ratio of games won to games lost is 4 to 5.

W	W	W	W	L	L	L	L	L

   Students can estimate, each box represents about 11% so the winning percentage is about 44%.

   c. For Team C, the ratio of games won to games lost is 4 to 1.

W	W	W	W	L

   Winning percentage 80%, same as Team A.

   d. Team D wins 24 out of 30 games.
   $\frac{24}{30} = \frac{4}{5} = \frac{80}{100}$ Team D wins 80% of its games, same percentage as Teams A and C.

   e. Team E wins three times as many games as it loses.

W	W	W	L

   Team E wins 75% of its games, not the same as any of the other teams.

Teams A, C, and D have the same winning percentage at 80%.

3. Cheryl is mixing red and white paint to make pink. To make the correct shade of pink, 60% of the mixture needs to be red. Which of the batches below will make the correct shade of pink?
Each of these may be answered in a number of ways. Numeric, including creating a common unit – fractions with a common denominator, percents. Students may also create models such as tape diagrams.

   a. **Batch 1:** 40% of the mixture is white.
   Correct shade

   b. **Batch 2:** The ratio of red paint to white paint is 6:10.
   Not the correct shade; the ratio of red to *total* needs to be 6:10

   c. **Batch 3:** Cheryl mixes 3 cups of red paint for every 2 cups of white paint.
   Correct shade

   d. **Batch 4:** The amount of red paint is 1.5 times the amount of white paint.
   Correct shade

4. The following situations show the amount of money several different people save based on what they make. Find the percentage that each person saves. Again, students can solve using models (tape diagrams, partial tables) or numeric approaches.
   a. Jen saves \$20 for every \$100 she earns.
   $\frac{\text{part saved}}{\text{total}} : \frac{20}{100} = 20\%$

   b. For Brian, the ratio of dollars saved to dollars spent is 100 to 200.
   $\frac{\text{part saved}}{\text{total}} : \frac{100}{300} = 33\frac{1}{3}\%$

   c. For every \$9 Penelope spends, she saves \$1.
   $\frac{\text{part saved}}{\text{total}} : \frac{1}{10} = 10\%$

   d. Each time Tiffany earns \$50, she saves \$25 of it.
   $\frac{\text{part saved}}{\text{total}} : \frac{25}{50} = 50\%$

   e. Drew spends three times more than he saves.
   $\frac{\text{part saved}}{\text{total}} : \frac{1}{4} = 25\%$

Spiral Review

1. What number is $\frac{1}{10}$ of 400? What number is $\frac{3}{10}$ of 400?

2. Find $\frac{4}{5}$ of 55.

3. Simplify. Look for patterns.
   a. 100×90
   b. 10×90
   c. 1×90
   d. 0.1×90
   e. 0.01×90

4. Simplify. Look for patterns.
   a. 100×24
   b. 10×24
   c. 1×24
   d. 0.1×24
   e. 0.01×24

2.1e Homework: Percent as a Part to Total Ratio

1. Ricky is putting red and blue marbles into different bags. In which of the bags are 20% of the marbles red? Justify your answers.
   a. **Bag 1:** One out of every five marbles in the bag is red.

Red	Other	Other	Other	Other

   20% of the marbles are red.

   b. **Bag 2:** There are 40 marbles in the bag. Of the 40 marbles, 8 are red.

Red 8	Other 8	Other 8	Other 8	Other 8

   20% of the marbles are red.
   There are a total of 40 marbles, 8 are red, the rest are not. We may also think about this as a ratio of 8:32, red to not red OR 8 to 40 red to total.

   c. **Bag 3:** There are 20 red marbles and 80 blue marbles in the bag.

20 RED	20 Blue	20 Blue	20 Blue	20 Blue

   20% of the marbles are red.

   d. **Bag 4:** The ratio of red marbles to blue marbles is 2 to 10.

The ratio of red to blue is 2 to 10, so the ratio of red to total is 2 to 12; this is not 20%.

   e. **Bag 5:** There are 400 marbles in the bag. Of the 400 marbles, 80 are red.

The ratio of red to total is 80 to 400 or 80/400 or 20/100; this is 20%.

   f. **Bag 6:** There are five times as many blue marbles as red marbles.

The ratio of blue to red is 5 to 1, so the ratio of red to total is 1 to 6; this is not 20%

2. Consider the following situations about four different people shooting free throws. Circle the letters of the people that have the same free throw percentage. Justify your answer.
   a. Piper makes 75% of her free throws.

   b. Mia makes 3 free throws for every 4 that she attempts.

   c. Evelyn makes 4 shots for every 1 that she misses.

   d. Fran makes 3 shots for every 1 that she misses.

   e. Harper makes two times more shots than she misses.

3. Consider the following situations about how four different students did on an exam. Determine each student's score as a percentage if each problem is worth the same number of points.
   a. Erik missed 4 out of the 50 questions.

   b. For every 4 questions that Jon answered correctly, he missed 1.

   c. Malorie answered 4 out of every 5 questions correctly.

   d. For Dave, the ratio of correct answers to incorrect answers was 23 to 2.

   e. Trevor got three times more questions correct than incorrect.

4. Xander is mixing red and yellow paint to make orange paint. To make the correct shade of orange, 80% of the mixture needs to be red. Which of the batches below will make the correct shade of orange?

   a. **Batch 1:** 4 parts red to 1 part yellow

Red	Red	Red	Red	Yellow

80% red--correct

   b. **Batch 2:** $\frac{4}{10}$ of the mixture is red

R	R	R	R	Y	Y	Y	Y	Y	Y

40% red—not correct

   c. **Batch 3:** 1 out of every 4 cups is yellow

R	R	R	Y

75% red—not correct

   d. **Batch 4:** For every 5 cups of paint, 4 are red.

R	R	R	R	Y

80% red--correct

   e. **Batch 5:** There is four times more red than yellow.
   Ratio of red to yellow is 4:1; ratio of red to total is 4 to 5, 80%--correct.

   f. **Batch 6:** There is five times more red than yellow.
   Ratio of red to yellow is 5:1; ratio of red to total is 4:6, NOT 80%.

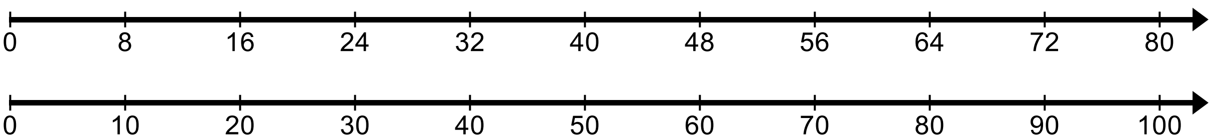
2.1f Class Activity: Types of Percent Problems

This lesson is intended to introduce students to three different types of percent problems: 1) Find a percent given a part and the whole; 2) Find a part of a quantity given a percent and the whole; 3) Find the whole given a part and a percent. Students have already practiced changing a fraction to a percent in the previous lessons (e.g., Express 15/20 as a percent). These types of problems are included again here so that students can compare the different types of percent problems. These problems may seem different to students because they have not seen this type of problem written in this language (e.g., 40 out of 80 is what percent vs. Express $\frac{40}{80}$ as a percent). The language that is used to express these different types of problems can be challenging for students to interpret.

n# Reason abstractly and quantitatively plays a big role in the following lessons. Students use models (double number lines, tape diagrams, partial tables, etc.) to solve the different types of percent problems. At the same time, they start thinking about more abstract solving methods, such as writing and solving equations.

Activity 1:

a. Write three statements that are true based on the vertical double number line shown.



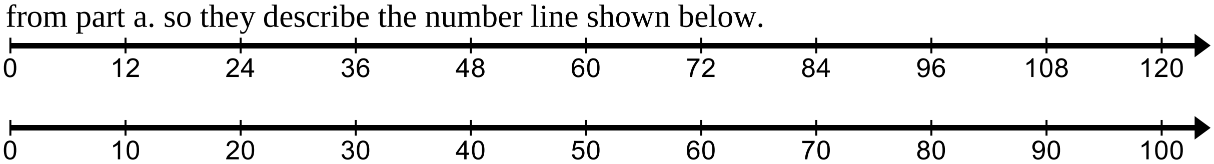
Answers will vary. Possible answers include: The total (or 100%) is equal to 80. 50% of the total (80) is equal to 40. 10% of the total (80) is equal to 8. 32 is 40% of the total (80). 72 is 90% of 80. 48 out of 80 is equal to 60%. You can ask students extension questions such as: 1) What number would correspond to 110%? 2) What number is 25% of 80?

b. Complete the following statements using the double number line shown in part a.

$\frac{50}{100} = \frac{?}{80}$ $\frac{30}{100} = \frac{?}{80}$ $\frac{?}{100} = \frac{72}{80}$ $\frac{16}{?} = \frac{20}{100}$

This is a preview of the percent proportion which students will study formally in 7th grade. The proportions help them to see the three different types of percent problems they will encounter in these lessons (see above). For example, if whole is 100 and we are considering 50 parts, how many parts do we need to consider when we change the whole to 80? If we consider 72 out of 80 parts, how many parts do we need to consider if we change the whole to 100 to talk about the same portion? If 20 out of 100 parts is equal to 16, what is the total?

c. Compare the double number line below with the double number line from part a. Revise your statements from part a. so they describe the number line shown below.



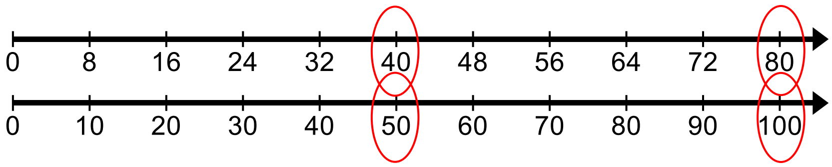
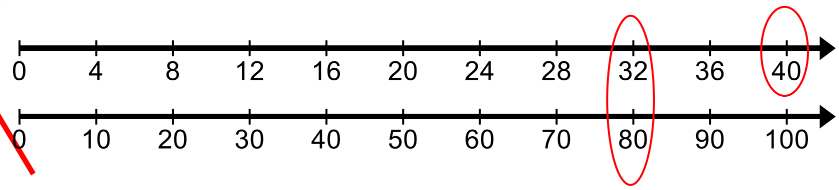
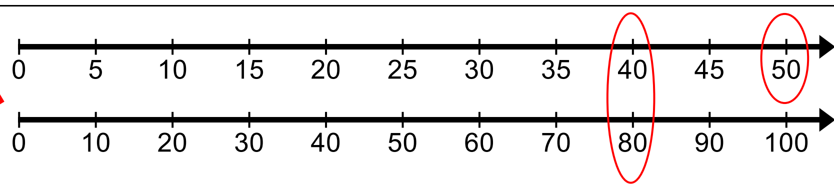
Ask students how their statements from part a. change. 1) The total (or 100%) is equal to 120. 50% of the total (120) is equal to 60. 10% of the total (120) is equal to 12. 48 is 40% of the total (120). 108 is 90% of 120. 72 out of 120 is equal to 60%. Again, consider extension questions: 1) What number would correspond to 110%? 2) What number is 25% of 120?

Ask students what type of percent problem their statements fall into (see teacher notes at beginning of lesson). Challenge students to come up with additional statements based on the number lines above that fit into each category.

d. Complete the following statements using the double number line shown in part c.

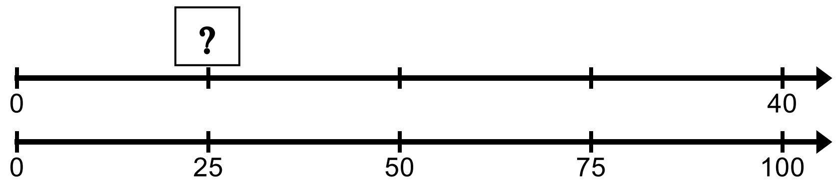
$\frac{50}{100} = \frac{?}{120}$ $\frac{30}{100} = \frac{?}{120}$ $\frac{?}{100} = \frac{72}{120}$ $\frac{20}{100} = \frac{24}{?}$

Activity 2: Draw lines to match each question to the double number line that can be used to solve the problem. Then, solve the problem.

What is 80% of 40? 32 This double number line shows that the total of 40 corresponds to a total of 100 (a rate per 100 or percent). 80% of 40 corresponds to 32. Find a part of a quantity given a percent and the whole.	
40 is 80% of what number. 50 In this problem, we know that 40 corresponds to 80 out of 100. We are being asked to find the total. Find the whole given a part and a percent.	
40 out of 80 is what percent? In this one, we are given a part and total. We need to change this to an equivalent ratio with a total of 100. Find a percent given a part and the whole.	

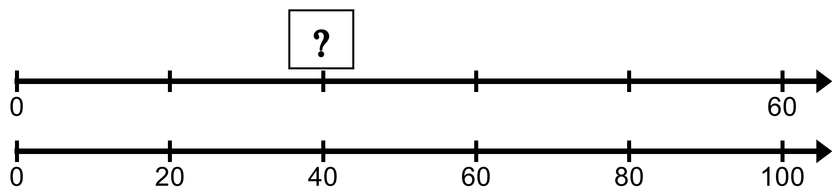
Activity 3: Write the question being asked by each model. Then, answer the question.

a. Question: What is 25% of 40?



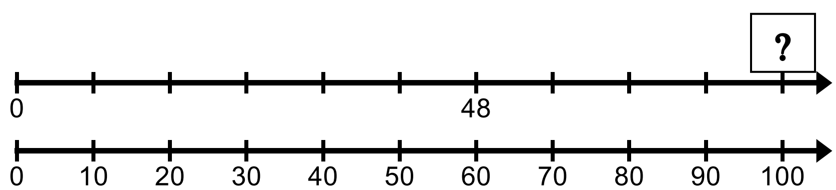
Answer: 10; To solve this problem, students may just look at the question and take $\frac{1}{4}$ of 40. Other students may use the double number line. To get from 100 to 25 on the bottom number line, we divide by 4. 40 divided by 4 is equal to 10.

b. Question: What is 40% of 60?

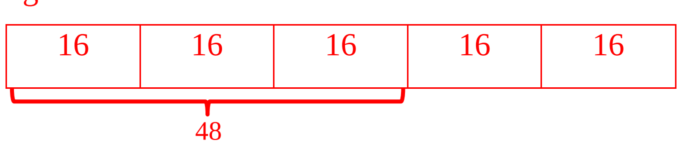


Answer: 24; Again, students may use a numeric approach, changing the percent to a fraction and taking 4/10 or 2/5 of 60. They may set up equivalent fractions: $\frac{?}{60} = \frac{40}{100}$. Students may not be able to solve this at first glance but if they reduce 40/100 to 2/5, it will be easier to find an equivalent fraction with a denominator of 60 (multiply by 12/12). Other students may use the double number line. To get from 100 to 40 on the bottom number line is not very easy to determine. What if we go from 100 to 20? Divide by 5. If we divide 60 by 5, we get 12 for each increment on the top number line. So, if 20% is 12, 40% is 24.

c. Question: 48 is 60% of what number? or 60% of a number is 48. What is the number? Or 60% of what number is 48?

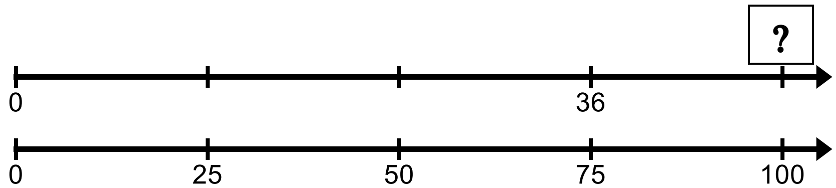


Answer: 80; Again, students may use a numeric approach: $\frac{60}{100} \times ? = 48$ or $\frac{3}{5} \times ? = 48$. Students can create a tape diagram from here:

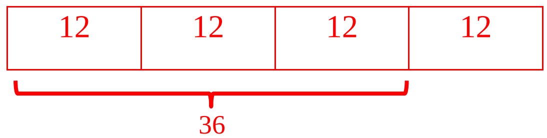


Each box has a value of 16 so the total is 80. Students may also use the double number line provided. To get from 60 to 10 on the bottom number line, we divide by 6. 48 divided by 6 on the top number line gives 8, showing that the scale on the top number line is 8. 8 multiplied by 10 is 80.

d. Question: 36 is 75% of what number? Or 75% of a number is 36. What is the number? Or 75% of what number is 36?

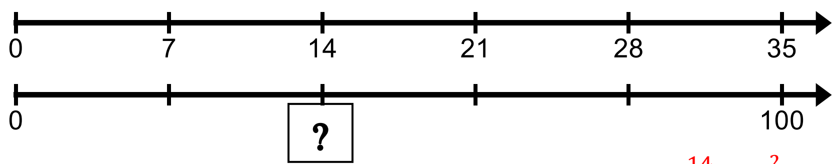


Answer: 48; Again, students may use a numeric approach: $\frac{75}{100} \times ? = 36$ or $\frac{3}{4} \times ? = 36$. Students can create a tape diagram from here:

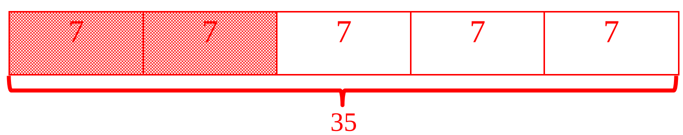


Each box has a value of 12 so the total is 48. Students may also use the double number line provided. To get from 75 to 25 on the bottom number line, we divide by 3. 36 divided by 3 on the top number line gives 12, showing that the scale on the top number line is 12. 12 multiplied by 4 is 48.

e. Question: 14 out of 35 is what percent (what number out of 100)? What percent of 35 is 14? 14 is what percent of 35?



Answer: 40%; Again, students may use a numeric approach: $\frac{14}{35} = \frac{?}{100}$ or $\frac{2}{5} = \frac{?}{100}$, multiply 2/5 by 20/20. Tape diagram:



Spiral Review

1. What is $\frac{1}{10}$ of 30?

2. What is $\frac{1}{100}$ of 30?

3. Simplify. Look for patterns.

   a. $2{,}400 \div 10$

   b. $240 \div 10$

   c. $24 \div 10$

   d. $2.4 \div 10$

4. Simplify. Look for patterns.

   a. $8{,}000 \div 100$

   b. $800 \div 100$

   c. $80 \div 100$

   d. $80 \div 1{,}000$

2.1f Homework: Getting Ready – Review Concepts

These are review problems from earlier grades that will help students with the upcoming lessons.

Directions: For each set of problems, draw a model. Then, answer the questions using the model and numeric strategies.

Set 1 Model:

4	4	4	4	4

1. $\frac{1}{5}$ of 20 4	2. $\frac{3}{5}$ of 20 12	3. $\frac{4}{5}$ of 20 16

Set 2 Model:

4. $\frac{1}{10}$ of 80	5. $\frac{5}{10}$ of 80	6. $\frac{9}{10}$ of 80

Set 3 Model:

7. $\frac{1}{4}$ of 28	8. $\frac{2}{4}$ of 28	9. $\frac{3}{4}$ of 28

Directions: Simplify. Look for patterns.

10.

a. 100×30 3,000

b. 10×30 300

c. 1×30 30

d. 0.1×30 3

e. 0.01×30 0.3

11.

a. 100×78

b. 10×78

c. 1×78

d. 0.1×78

e. 0.01×78

12.

a. 0.1×80 8

b. 0.2×80 16

c. 0.3×80 24

d. 0.05×80 4

e. 0.01×80 0.8

13.

a. 0.1×50

b. 0.2×50

c. 0.3×50

d. 0.05×50

e. 0.01×50

14.

a. $5000 \div 100$ 50

b. $500 \div 100$ 5

c. $50 \div 100$ 0.5

d. $5 \div 100$ 0.05

15.

a. $32{,}000 \div 100$

b. $3{,}200 \div 100$

c. $320 \div 100$

d. $32 \div 100$

Directions: Simplify each problem in a set. Then, think about the relationship between the problems in the set. The goal of these problems is for students to see that the problems in each set are equivalent.

Set 1:

16. $40 \div 10$	17. $\frac{40}{10}$	18. $\frac{1}{10}$ of 40	19. 0.1×40

Set 2:

20. $40 \div 100$ 0.4	21. $\frac{40}{100}$ 0.4	22. $\frac{1}{100}$ of 40 0.4	23. 0.01×40 0.4

Set 3:

24. $12 \div 10$	25. $\frac{12}{10}$	26. $\frac{1}{10}$ of 12	27. 0.1 of 12

Set 4:

28. $32 \div 4$	29. $\frac{32}{4}$	30. $\frac{1}{4}$ of 32	31. 0.25 of 32

Directions: Simplify.

32. 0.5×16	33. 0.6×30 18	34. 0.75×28
35. 0.05×22	36. 0.34×200	37. 0.26×80 20.8
38. 0.99×50	39. 0.50×24	40. 0.05×24
41. 1.5×36	42. 2.1×60	43. 0.31×75 23.25

2.1g Class Activity: Finding a Percent of a Quantity

Students should always start by estimating their answer. Encourage multiply strategies: numeric (equivalent fractions, partial tables), mental math, models (tape diagrams, double number lines), abstract (equations), etc. Have students explain their strategies and consider strategies used by others.

Activity 1:

a. What number is 10% of 120? 12

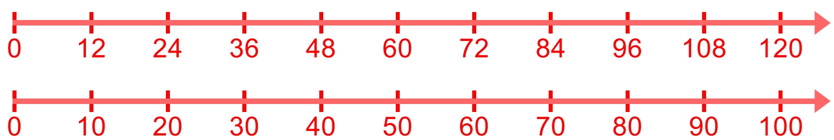
Have students estimate first, 10% of 100 is 10 so we know our answer should be greater than 10. Students may first convert the 10% to a fraction $\frac{1}{10}$. Taking $\frac{1}{10}$ of 120 is the same as dividing by 10. Students may also convert the percent to a decimal (0.1) and multiply by 120.

They may draw a model:

12	12	12	12	12	12	12	12	12	12

We want 10% of 120, so we divide 120 into 10 equal parts and take one.

Students may also create a double number line, one showing 100 as the total (percent) and one showing 120 as the total:



Or set up equivalent ratios:

$\frac{10}{100} = \frac{?}{120}$ The easiest way to solve this is to simplify the 10/100 to 1/10 and then multiply it by 12/12 to get a denominator of 120.

×12

÷ 10

Part	1	10	?
Whole	10	100	120

The equivalent ratios and table are a preview of the percent proportion which will be studied in 7th grade.

b. What number is 30% of 120? 36

If we know that 10% of 120 is 12, then 30% of 120 is 36. We can think of this as 3 copies of 10% of 120: $10\%(120) + 10\%(120) + 10\%(120) \rightarrow 12 + 12 + 12 = 36$

Students can also use any of the models from above to solve this problem. Consider 3 parts of the tape diagram, look at the number that corresponds to 30/100 on the double number line, the proportion changes to $\frac{30}{100} = \frac{?}{120}$, and the table:

Part	3	30	? = 36
Whole	10	100	120

c. What number is 5% of 120? 6

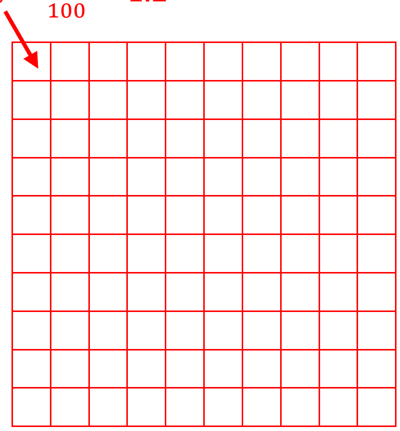
6	12	12	12	12	12	12	12	12	12
6									

If we know that 10% is 12, then 5% is 6. Again, connect to the models above. We want 5% of 120, so we divide 120 into 10 equal parts and take HALF of one.

d. What number is 25% of 120. 30
$10\% + 10\% + 5\% = 12 + 12 + 6 = 30$
5 copies of $5\% = 5(6) = 30$

e. What number is 1% of 120. 1.2
$\frac{1}{100} \times 120 \rightarrow 120$ divided into 100 equal parts or $0.01 \times 120 = 1.2$. If 5% is 6, then 1% is $\frac{6}{5} = 1\frac{1}{5}$ or 1.2.
If 10% is 12, then 1% is $\frac{12}{10}$ or 1.2.

$1\% = \frac{120}{100} = 1.2$



f. What number is 2% of 120. 2.4
If 1% is 1.2, then 2% is $1.2 + 1.2 = 2.4$ or 2 copies of 1.2: $2(1.2) = 2.4$
2 parts of the hundred grid shown above.

g. What number is 26% of 120. 31.2
$26\% = 25\% + 1\% = 30 + 1.2 = 31.2$ or
$26\% = 20\% + 2\% + 2\% + 2\% = 24 + 2.4 + 2.4 + 2.4 = 31.2$

h. What number is 29% of 120.
$25\% + 2\% + 2\% = 30 + 2.4 + 2.4 = 34.8$ or
$30\% - 1\% = 36 - 1.2 = 34.8$.

If students need more practice multiplying with decimals, you can have them apply the algorithm 0.29×120 or have them verify that this algorithm works on a calculator. If students have access to a calculator, the algorithm of changing the percent to a decimal and multiplying by 120 is a quick method. In this lesson, we are emphasizing models and mental math strategies; however, students should also be familiar with this algorithmic method.

Activity 2:

a. If 10% of a number is 18, what is…

A possible model:

18	18	18	18	18	18	18	18	18	18

20% of that number? 36

Consider two parts of the model or $10\% + 10\% = 18 + 18 = 36$

5% of that number? 9

Consider half of one part of the model or $10\% \div 2 = 18 \div 2 = 9$

1% of that number? 1.8

$10\% \div 10 = 18 \div 10 = 1.8$

2% of that number? 3.6

$1\% + 1\% = 1.8 + 1.8$

100% of that number? 180

$10\% \times 10 = 18 \times 10 = 180$ or $20\% \times 5 = 36 \times 5 = 180$

This is what students will be doing in the next lesson – if we know the percent and a part, what is the whole or 100%?

99% of that number? 178.2

$100\% - 1\% = 180 - 1.8 = 178.2$

200% of that number? 360

$2(100\%) = 2(180) = 360$

b. If 25% of a number is 8, what is…Use strategies like those in part a.

8	8	8	8

50% of that number? 16

10% of that number? Students may find it easiest to find 5% and then double it: $5\% = 50\% \div 10 = 16 \div 10 = 1.6 \rightarrow 10\% = 5\% + 5\% = 1.6 + 1.6 = 3.2$. Students may also find the total using the tape diagram (100% = 32) and then divide that by 10 by moving the decimal one place to the left.

20% of that number? 6.4

Double 10%

1% of that number? 0.32

Students may find it easier to find 2% of the number and then half it. If 20% = 6.4 then 2% = 0.64 and 1% = 0.32.

100% of that number? 32

Double 50%

150% of that number? 50% + 100% = 16 + 32 = 48

Activity 3: For the following problems: 1) Estimate the answer. 2) Explain or show a strategy for finding the answer. 3) Find the answer. 4) Check the answer with a calculator. Numeric strategies are shown but students can always make a model.

a. 60% of 30
   Estimate: close to but greater than 15
   Strategy: $50\% + 10\% = 15 + 3 = 18$
   Strategy: $10\% + 10\% + 10\% + 10\% + 10\% + 10\%$ or $6(10\%) = 6(3) = 18$
   Calculator Check: $0.6 \times 30 = 180$

b. 15% of 40
   Estimate, somewhere between 4 (10% of 40) and 8 (20% of 40).
   Strategy: $10\% + 5\% = 4 + 2 = 6$
   Calculator Check: $0.15 \times 40 = 6$

c. 76% of 32
   Estimate: around 24 (3/4 of 32)
   Strategy: $25\% + 25\% + 25\% + 1\% = 8 + 8 + 8 + 0.32 = 24.32$
   Calculator Check: $0.76 \times 32 = 24.32$

d. 52% of 180
   Estimate: around 90
   Strategy: $50\% + 1\% + 1\% = 90 + 1.8 + 1.8 = 93.6$
   Calculator Check: $0.52 \times 180 = 93.6$

e. 90% of 48
   Estimate: close to but smaller than 48
   Strategy: 9 copies of 10% of $48 = 9(4.8) = 43.2$
   Strategy: $100\% - 10\% = 48 - 4.8 = 43.2$
   Calculator Check: $0.9 \times 48 = 43.2$

f. 21% of 25
   Estimate: This may be difficult for students to estimate but even saying smaller than 12.5 (50%) or about 6 (about 25%) is a good start to give them a sense of how big the answer should be.
   Strategy: $10\% + 10\% + 1\% = 2.5 + 2.5 + 0.25 = 5.25$
   Calculator Check: $0.21 \times 25 = 5.25$

g. 200% of 75
   Estimate: Many students will know the exact answer right away.
   Strategy: $100\% + 100\% = 75 + 75 = 150$
   Calculator Check: $2 \times 75 = 150$

h. 120% of 40
Estimate: Greater than 40.
Strategy: $100\% + 10\% + 10\% = 40 + 4 + 4 = 48$
Calculator Check: $1.2 \times 40 = 48$

i. 175% of 120
Estimate: Between 120 and 240, closer to 240.
Strategy: $100\% + 25\% + 25\% + 25\% = 120 + 30 + 30 + 30 = 210$
Strategy: $200\% - 25\% = 240 - 30 = 210$
Calculator Check: $1.75 \times 120 = 210$

j. 11% of 16
Estimate: around 1.6 (10% of 16)
Strategy: $10\% + 1\% = 1.6 + 0.16 = 1.76$
Calculator Check: $0.11 \times 16 = 1.76$

k. 97% of 65
Estimate: very close to 65 but smaller
Strategy: $100\% - 1\% - 1\% - 1\% = 65 - 0.65 - 0.65 - 0.65 = 63.05$
Calculator Check: $0.97 \times 65 = 63.05$

Activity 4: Real-world Application – Statistics

Toby recently read an article and discovered the following facts:

- 15% of teens get the recommended amount of sleep each night (8 – 10 hours).
- 91% of teens have owned a pet
- 70% of teens own a cell phone
- 68% of teens enjoy cooking
- 58% of teens play an organized sport
- 29% of teens choose Summer as their favorite season
- 86% of teens say they enjoy school
- 63% of teens watch YouTube daily
- 41% of teens go Black Friday shopping

There are 400 students at Toby's high school. Based on the statistics above, how many students at Toby's school would you expect…

a. Get the recommended amount of sleep each night? ___60________

b. Have owned a pet? ____364_______

c. Own a cell phone? ____280_______

d. Enjoy cooking? ____272_______

e. Play an organized sport? ___232________

f. Would choose Summer as their favorite season? ___116________

g. Would say they enjoy school? ____344_______

h. Watch YouTube daily? ___252________

i. Go Black Friday shopping? ___164________

Again there are a number of ways a student may answer the questions. We continue to recommend that students build fluency with the concept of percent by using models or other conceptual strategies to answer these.

For each, we find the percent out of 100, and then multiply by 4 to find the amount out of 400. Additionally, we will show basic mental math strategies to be supported (e.g., Distributive Property):

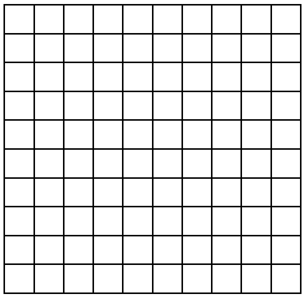
a. 15 * 4 = 4(10 + 5) = 40 + 20 = 60
b. 91 * 4 = 4(90 + 1) = 360 + 4 = 364
c. 70 * 4 = 280
d. 68 * 4 = 4(60 + 8) = 240 + 32 = 272
e. 58 * 4 = 4(50 + 8) = 200 + 32 = 232
f. 29 * 4 = 4(30 – 1) = 120 – 4 =116

Spiral Review

1. Simplify.

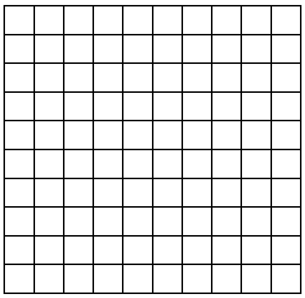
a. $50 \div 1$	b. $50 \div 0.1$	c. $50 \div 0.01$
d. $64 \div 32$	e. $64 \div 3.2$	f. $64 \div 0.32$
g. $8 \div 5$	h. $8 \div 0.5$	i. $8 \div 0.05$
j. $12 \div 15$	k. $12 \div 1.5$	l. $12 \div 0.15$

2. Give the value of each small square in the hundred grid below if the entire grid has a value of…



a. 100	b. 1	c. 80	d. 25	e. 120

3. Give the value of the *entire* grid in the hundred grid below if each small square has a value of…



a. 0.02	b. 0.3	c. 1.5	d. 0.24	e. 6

4. If one box on the tape diagram has a value of 7.9, what is the value of the entire tape diagram?

7.9									

2.1g Homework: Finding a Percent of a Quantity

Directions: Draw a model to represent each set of problems. Use the model to answer the questions. Check your answers with a calculator.

Set 1 Model: 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5		
1. 10% of 50 5	2. 60% of 50 30	3. 90% of 50 45

Set 2 Model:		
4. 25% of 36	5. 50% of 36	6. 75% of 36

3. Use mental math to find 10%, 15%, and 20% of 240.
   $10\% = \frac{1}{10} \cdot 240 = 24;\ 20\% \text{ is twice } 10\% \text{ so } 48;\ 15\% = 10\% + 5\% \text{ so } 24 + 12 = 36$

4. If 1% of a number is 2, what is…

   a. 2% of the number?

   b. 10% of the number?

   c. 32% of the number?

   d. 5% of the number?

   e. 25% of the number?

   f. 100% of the number?

   g. 200% of the number.

5. Explain how you can find 19% of 450 using the following information. Check your answer with a calculator.
   - 20% of 450 is 90.
   - 1% of 450 is 4.5.

Directions: For the following problems: 1) Estimate the answer. 2) Explain or show a strategy for finding the answer. 3) Find the answer. 4) Check with a calculator.

6. 50% of 21

10.5	10.5

Estimate: around 10
Strategy: $\frac{1}{2} \times 21 = 10.5$ (model)
Calculator Check: $0.5 \times 21 = 10.5$

7. 61% of 20

8. 22% of 54

9. 99% of 75
Estimate: less than 75
Strategy: $100\% - 1\% = 75 - 0.75 = 74.25$
Calculator Check: $0.99 \times 75 = 74.25$

10. 26% of 44.

11. 150% of 16
Estimate: more than 16
Strategy: $100\% + 50\% = 16 + 8 = 24$
Calculator Check: $1.5 \times 16 = 24$

12. 210% of 60

13. Lavinia is friends with Toby (from the classroom activity) but goes to a different high school in town. She is also interested in determining the number of students at her school that would fit into each of the following categories based on the statistics below:

- 15% of teens get the recommended amount of sleep each night (8 – 10 hours).
- 91% of teens have owned a pet
- 70% of teens own a cell phone
- 68% of teens enjoy cooking
- 58% of teens play an organized sport
- 29% of teens choose Summer as their favorite season
- 86% of teens say they enjoy school
- 63% of teens watch YouTube daily
- 41% of teens go Black Friday shopping

There are 500 students at Lavinia's high school. Based on the statistics above, how many students at Lavinia's school would you expect…

a. Get the recommended amount of sleep each night? ___________

b. Have owned a pet? ___________

c. Own a cell phone? ___________

d. Enjoy cooking? ___________

e. Play an organized sport? ___________

f. Would choose Summer as their favorite season? ___________

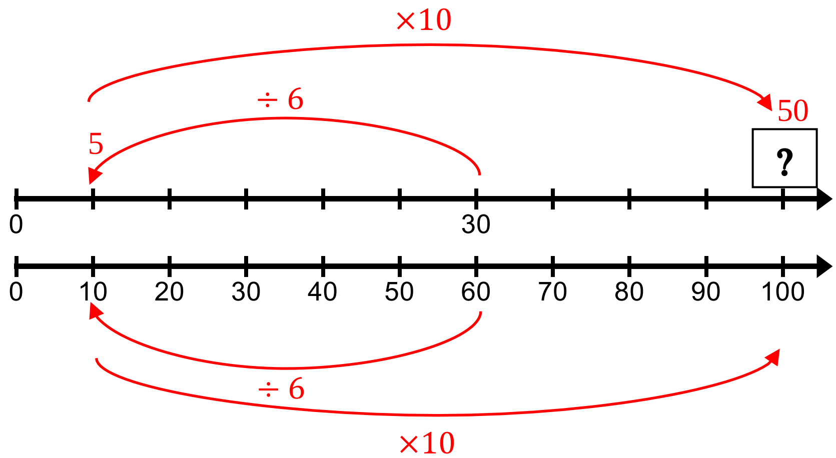
g. Would say they enjoy school? ___________

h. Watch YouTube daily? ___________

i. Go Black Friday shopping? ___________

2.1h Class Activity: Finding the Whole Given the Percent and a Part

Activity 1: Use the double number line below for this activity.



a. Describe what the double number line shows. What question is being asked?

The double number line shows that 60% of a number is 30. The question being asked is, “If 60% of a number is 30, what is the number?” or “60 out of 100 is equivalent to 30 out of what number?”

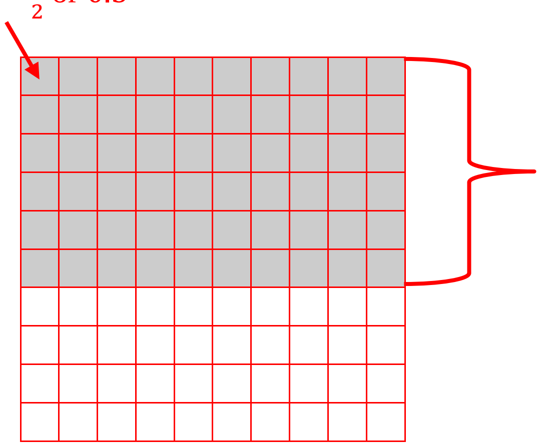
Help students to see the equations:

$60\% \times n = 30$

$$\frac{60}{100} = \frac{30}{n}$$

A hundred grid may be helpful as well. To solve any of these problems, we can always find the value of 1% or 1 box on the hundred grid. Then we can iterate up to find 100%.

$1\% = \frac{1}{2}$ or 0.5



$60\% = 30$ which means that each part in the 100 grid has a value of 0.5, a row has a value of $10 \times 0.5 = 5$ and the entire chart has a value of $10 \times 5 = 50$.

b. What is the answer to the question in part a.

50; Students can solve this problem by finding equivalent ratios on the double number line (iterate down to find 10% and then iterate up to find 100%) – see model. Students can also use the strategy shown in the hundred chart. They may also write equivalent fractions for the proportion that is set up. We will focus on how to solve the equation $60\% \times n = 30$ in Chapter 6 but students may be able to use guess and check to solve the equation or write a related division problem. You will probably want to change the percent to a decimal as students may not know how to divide by a fraction (they will learn that later in this chapter).

c. Check your answer using ideas from the previous lesson.
To check, students can take 60% of 50 and verify that it equals 30. Students may also verify that $\frac{60}{100} = \frac{30}{50}$.

Activity 2: Encourage students to estimate first and draw models.

a. If 10% of a number is 8, what is…

8	8	8	8	8	8	8	8	8	8

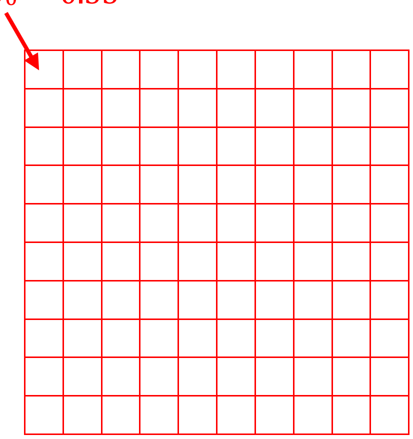
20% of the number?
16

50% of the number?
40

100% of the number?
80

b. If 1% of a number is 0.35, what is…

$1\% = 0.35$



10% of the number?
3.5

20% of the number?
7

100% of the number?
35

c. If 80% of a number is 64, what is…

16	16	16	16	16

64

Students may also make a tape diagram with 10 boxes.

50% of the number?

40 (2.5 of the boxes in the tape diagram)

Or simplify to 10% is 8 and then iterate up to 50%

25% of the number?

20 (1/2 of 50%)

100% of the number?

80 (5 boxes in the tape diagram or 2(50%))

Activity 3: Solve the problems using multiple strategies (models, equivalent ratios, tables, equations, numeric reasoning, etc.). Then, check your answer. To check, students can use mental math for many of the problems. Consider the use of calculators for solving the equations, checking answers, etc.

a. 25% of a number is 4. What is the number? 16

4	4	4	4

If 25% is 4, 5% is $\frac{4}{5}$, and 100% is $\frac{4}{5} \times 20 = 16$

Equations:

$25\% \times n = 4$

$0.25 \times n = 4$ Related Division Sentence: $n = 4 \div 0.25$ or $4 \div \frac{1}{4} = 16$

$$\frac{25}{100} = \frac{4}{n} \to \frac{1}{4} = \frac{4}{n} \to \frac{1}{4} = \frac{4}{16}$$

Part	1	25	4
Whole	4	100	? = 16

Iterate down to $\frac{1}{4}$ and then up to $\frac{4}{16}$.

Check answer: Is 25% of 16 = 4? Yes

b. 50% of a number is 11. What is the number?

11	11

22: If 50% is 11 then 100% is $2(11) = 22$

Check answer: Is 50% of 22 = 11? Yes

c. 10% of a number is 4.2. What is the number?

4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2

42; Check answer: Is 10% of 42 = 4.2? Yes

d. 40% of a number is 10. What is the number?

5	5	5	5	5

25; Students may also create a tape diagram with 10 equal parts which would give each part a value of $\frac{10}{4} = \frac{5}{2} = 2.5$. The total would be $10(2.5) = 25$. They may also find 1% of the number which is $\frac{10}{40} = \frac{1}{4} = 0.25$. Then total would be $100(0.25) = 25$.

Part	40	10
Whole	100	? = 25

In the table, divide by 4/4 to create an equivalent ratio.
Check answer: Is 40% of 25 = 10? Yes

e. 120% of a number is 48. What is the number? Talk to students about why we know our answer is going to be less than 48. If 120% of our number is 48, we know 100% of our number is going to be less than 48.

8	8	8	8	8	8

$120\% = 48$

$100\% = 40$

Part	6	120	48
Whole	5	100	? = 40

Iterate down to 6/5 by dividing 120/100 by 20/20, then iterate up to 48/40 by multiplying by 8/8. Check: Is 120% of 40 = 48? Yes

f. 1% of a number is 0.54. What is the number?
54: If 1% is 0.54 then 100% is $100(0.54) = 54$

g. 5% of a number is 7. What is the number?
140: If 5% is 7 then 100% is $20(7) = 140$

h. 30% of a number is 6.3. What is the number?
21: If 30% is 6.3 then 10% is $\frac{6.3}{3} = 2.1$ and 100% is $10(2.1) = 21$

i. 11% of a number is 5.5. What is the number?
50: If 11% is 5.5 then 1% is $\frac{5.5}{11} = 0.5$ and 100% is $100(0.5) = 50$

j. 15% of a number is 12. What is the number?
80: If 15% is 12 then 5% is $\frac{12}{3} = 4$ and 100% is $20(4) = 80$

k. 68% of a number is 34. What is the number?

For problems like these, it may not be as obvious to students what they iterate the percent down to. Ask, "What are easy numbers to change into 100%?" Answer: 1%, 2%, 4%, 5%, 10%, 20%, and 25%. Examine how to get from 68% to each of these percents (e.g., $68\% \div 68 = 1\%, 68\% \div 34 = 2\%, 68\% \div 17 = 4\%$, etc.) Looking at these, which of these numbers goes into 34 evenly? Students will likely say 34 but they may also say 17.

50: If 68% is 34 then 2% is $\frac{34}{34} = 1$ and 100% is $50(1) = 50$

If 68% is 34 then 1%: $\frac{34}{68} = 0.5$ and 100% is $100(0.5) = 50$

Writing an equation:

$0.68 \times n = 34$ Related division sentence: $n = 34 \div 0.68$

Table of Equivalent Ratios:

Part	68	34
Whole	100	? = 50

l. 99% of a number is 79.2. What is the number?

80; This may be challenging for students. They may see that we can iterate 99% down to 3% or 33% but that does not really help us to iterate back up to 100%. We could iterate down to 3%, then up to 12%, then down to 4% and then up to 25%:

If 99% = 79.2, then 3% = 2.4, 12% = 9.6, 4% = 3.2, and 100% = 80.

We could also determine the value of 1%: If 99% = 79.2, then 1% = $\frac{79.2}{99}$ or 0.8. 100% = 0.8(100) = 80.

Writing an equation:

$0.99 \times n = 79.2$ Related division sentence: $n = 79.2 \div 0.99$

m. 8% of a number is 6. What is the number?

75: If 8% is 6 then 2% is $\frac{6}{4} = 1.5$ and 100% is $50(1.5) = 75$

Spiral Review

1. Make a double number line to show the relationship between cups and pints.

2. Complete the table to show the relationship between meters and kilometers.

Kilometers	**Meters**
1	
2	
3	
10	

3. Tell whether the simplified form of the expression $50 \times a$ will be greater than 50 or less than 50 depending on the value of a.

a. $a = 2$	b. $a = \frac{1}{2}$	c. $a = 0.01$	d. $a = 1.01$

4. Tell whether the simplified form of the expression $50 \div a$ will be greater than 50 or less than 50 depending on the value of a.

a. $a = 2$	b. $a = \frac{1}{2}$	c. $a = 0.01$	d. $a = 1.01$

2.1h Homework: Finding the Whole Given the Percent and a Part

Directions: Solve the problems using multiple strategies (models, equivalent ratios, tables, equations, numeric reasoning, etc.). Then, check your answer.

1. 10% of a number is 13. What is the number?
2. 20% of a number is 21. What is the number?
3. 4% of a number is 3. What is the number? 75
4. 75% of a number is 30. What is the number?
5. 80% of a number is 120. What is the number? 150
6. 150% of a number is 90. What is the number?
7. 1% of a number is 2. What is the number?
8. 65% of a number is 13. What is the number?
9. 20% of a number is 14.2. What is the number? 71
10. 26% of a number is 28.6. What is the number?
11. 12% of a number is 3. What is the number?
12. 40% of a number is 16. What is the number?

2.1i Class Activity: Types of Percent Problems Mixed Review

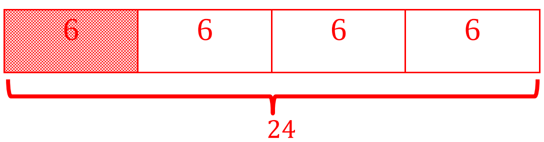
Directions: The three problems below are examples of the three different types of percent problems we have studied in this chapter. One of the most challenging aspects of the problems in this lesson is interpreting the language. Help students to decode the statements, identifying what they are given (a percent, a part, or a whole) and what they are trying to find (a percent, a part, or a whole). Have them re-state the problems in their own words. The whole can generally be found after the word "of" in the statements.

For each problem:

- Identify the type of percent problem. Are you…
  1) Finding a percent given a part and the whole?
  2) Finding a part of a quantity given a percent and the whole?
  3) Finding the whole given a part and a percent?
- Create a model to represent the problem.
- Solve the problem using a variety of methods (models, equivalent ratios, tables, equations, mental math, etc.).

1. 25% of 24 is what number?

Finding a part of a quantity given a percent and the whole



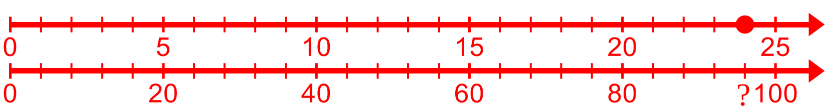
$$\frac{25}{100} = \frac{n}{24}$$

Equation: $0.25 \times 24 = 6$

Answer: 6

2. 24 is what percent of 25?

Finding a percent given a part and the whole

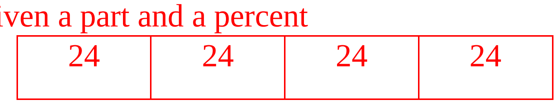


$$\frac{24}{25} = \frac{n}{100}$$

Answer: 96

3. 24 is 25% of what number?

Finding the whole given a part and a percent



$$\frac{25}{100} = \frac{24}{n}$$

Writing an equation:

$0.25 \times n = 24$ Related division sentence: $n = 24 \div 0.25$

Answer: 96

Directions: Solve the following problems using a variety of methods (models, equivalent ratios, tables, equations, mental math, etc.). See previous lessons for strategies.

1. What is 25% of 32? 8	2. 32 is 25% of what number? 128
3. Find 75% of 16. 12	4. 7 is what percent of 20? 35%
5. 25 is what percent of 40? 62.5%	6. 30 is 120% of what number? 25
7. 35 is what percent of 100? 35%	8. What percent of 16 is 8? 50%
9. What number is 80% of 90? 72	10. 4.5 is 15% of what number? 30

11. 1.2 is what percent of 10? 12%	12. What number is 20% of 60? 12
13. Find 18% of 50. 9	14. 20 is what percent of 5? 400%
15. Find 49% of 300. 147	16. Find 125% of 40. 50
17. 30 is what percent of 40? 75%	18. What is 150% of 32? 48
19. Shelley got 22 questions correct on her math test. She got an 88%. How many questions were on Shelley's math test? 25	20. Charlie got 48 points out of 50 on his Science test. What percent did Charlie get on his Science test? 96%

Spiral Review

1. Evan has 4 cups of lemonade.
   a. How many 2-cup servings can he make?

   b. How many $\frac{1}{2}$-cup servings can he make?

2. At Theo's birthday party, $\frac{3}{4}$ of the cake was eaten. The next day, Theo's family of five shared the leftover cake. What part of the original cake did each family member get on the second day? Draw a picture and write an equation to model this situation. Then, solve the problem.

3. Solve for a in the equation $4 \div a = 20$.

4. Complete the table to show the relationship between centimeters and meters.

Meters	**Centimeters**
1	
2	
3	
8	

2.1i Homework: Types of Percent Problems Mixed Review

1. What number is 50% of 80?	2. What number is 40% of 90?
3. 4 is what percent of 20?	4. Find 25% of 180.
5. What number is 15% of 200? 30	6. What number is 75% of 140?
7. 4.8 is 10% of what number? 48	8. What percent of 50 is 75? 150%
9. What number is 51% of 128?	10. What number is 250% of 50?

11. 15 is 60% of what number? 25	12. 16 is 32% of what number?
13. 20% of what number is 6?	14. 2.5 is what percent of 20? 12.5%
15. What number is 110% of 150?	16. What number is 20% of 60?
17. Thirty-five percent of the students at Parker Junior High take the bus to school. If there are 200 students at Parker Junior High, how many take the bus?	18. Nine children on the swim team don't like ice cream. If this represents 18% of the children on the swim team, how many children are on the swim team? 50 children
19. Jennifer went to lunch. Her bill was $12. If she tips 20%, how much did she leave for tip?	20. A realtor makes 4% commission on the sale of a home. If she sold a home for $250,000, how much will she make in commission?

2.1j Self-Assessment: Section 2.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Understand a percent as a part to total ratio with a whole equal to 100.				
2. Represent fractional amounts of a quantity as a percent.				
3. Fluidly transition between quantities represented as a percent, fraction, decimal or ratio.				
4. Find a part of a quantity given a percent and the whole.				
5. Find the whole given a part and a percent.				
6. Solve real-world percent problems.				

Sample Problems for Section 2.1
Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. Define percent in your own words. Use examples and or/models to support your definition. [1]

2. Eileen is making lemonade with water and lemon concentrate. What percent of each mixture is lemon concentrate? [1] [2]
   a. The ratio of water to lemon concentrate is 3 to 1
   b. Eileen mixes 4 parts water for each part of lemon concentrate
   c. 4 out of every 5 cups of the mixture is water
   d. There is twice as much water as lemon concentrate in the mixture
   e. The ratio of cups of water to cups of lemonade is 6:10.

3. The table below shows Miguel's score on several different math quizzes he took this quarter. Complete the table by determining the letter grade that Miguel earned on each quiz. The grading scale is shown. [1] [2]

Points Earned by Miguel	Total Possible Points	Percentage	Letter Grade
4	5		
9	10		
23	25		
16	20		
44	50		
32	40		

Percentage	Letter Grade
90 – 100%	A
80 – 89%	B
70 – 79%	C
60 – 69%	D
59% and below	F

4. Complete the table below. [1][2][3]

Fraction	**Decimal**	**Percent**
$\frac{70}{100}$		
		25%
	0.32	
$\frac{5}{10}$		
	0.01	
$\frac{11}{20}$		
		180%
	1.25	
$\frac{41}{200}$		
$\frac{24}{40}$		

5. $\frac{4}{25}$ of the students at Clayton Middle School have food allergies. [6]
   a. What percent of the students at Clayton Middle School have food allergies? ___________

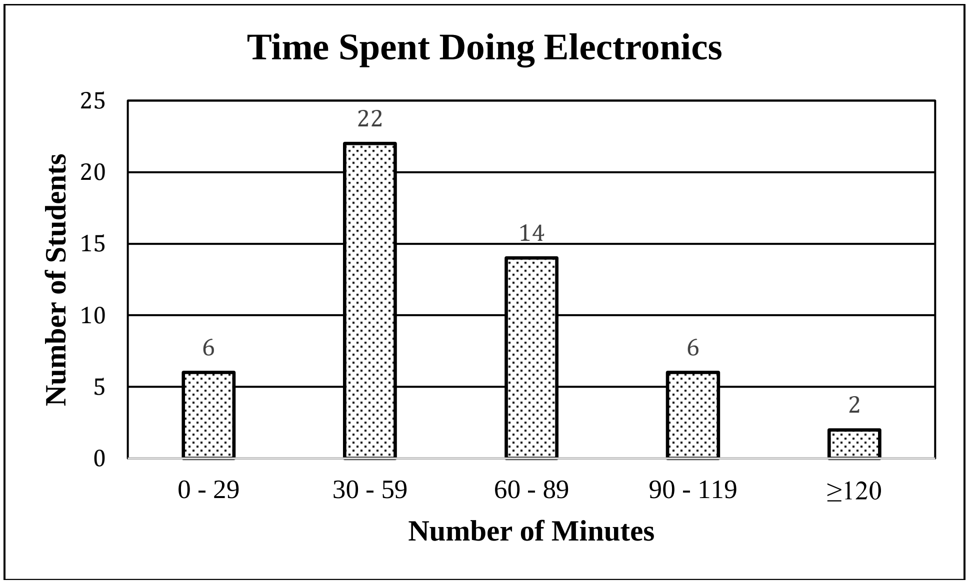
   b. What percent of the students at Clayton Middle School do not have food allergies? _______

   c. If there are 36 students with food allergies at Clayton Middle School, how many total students are there? ____________

6. Talen earns allowance each week. The pictograph below shows what he does with his money. Complete the table to show the percent of money that Talen spends, saves, and donates. [6]

Where Talen's Money Goes	**Amount of Money**	**Percent of Money**
Spends		
Saves		
Donates		

7. Monroe surveyed the students in his class and asked how much time they spend doing electronics each day. The bar graph shows the results of the survey: [6]



   a. What percentage of students in Monroe's class spend an hour or more each day doing electronics?

   b. What percentage of students in Monroe's class spend less than an hour each day doing electronics?

8. Use models to show the difference between the two problems below. Then, solve both problems. [4][5]

   42 is 60% of what number

   What is 60% of 42?

Directions: Solve the following problems. [4][5]

9. 30% of what number is 120?

10. 30 is what percent of 120

11. What is 30% of 120?

12. What number is 120% of 30?

13. 120 is what percent of 30?

14. 30 is 120% of what number?

15. 25% of the students on the mock trial team are in 8th grade. If there are 28 students on the mock trial team, how many are in 8th grade? [6]

16. Shelly got 60 questions correct on her math test. If she got 75%, how many questions were on the test? [6]

17. There are 40 questions on a math test. Calvin got an 90%. How many questions did Calvin answer correctly? [6]

18. Maddie is buying a pair of jeans that cost $40. There is a 6% tax rate on the jeans. How much will she pay in tax? [6]

Section 2.2: Division of Fractions

Section Overview:
The section begins with an optional review of the multiplication and division operation as well as a review lesson of standards in 5^{th} grade (5.NF 3 – 7). It then moves to division of fractions in context (6NS.1). Students are encouraged to use various models (tape, double line, partial tables, etc.) to build conceptual understanding and connect ideas to ratio thinking. Students should make sense of problem contexts and the structure of the operations. For example, students should understand dividing by a fraction between 0 and 1 will produce a quotient larger than the dividend; or that multiplying by a fraction between 0 and 1 will produce a product smaller than the other factor. Making sense of problem structures and products and quotients with fractions will be very helpful for Section 3 (unit conversion). The section wraps up with a lesson to help students understand how $a \div \frac{b}{c}$ and $a \times \frac{c}{b}$ are related conceptually and that they produce the same answer.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Understand and explain the relationship between multiplication and division with fractions.
2. Solve division problems involving fractions using a variety of strategies, including models, related multiplication sentences, and the algorithm.
3. Create contexts for division of a fraction by a fraction.

Students will arrive at the "invert and multiply" algorithm for dividing with fractions at different points during the section. Continue to emphasize the models and help students to connect the models to the algorithm.

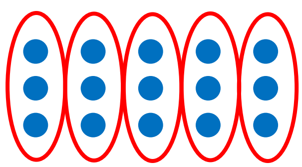
2.2a Class Activity: Division with Whole Numbers and Unit Fractions

In elementary school, **multiplication** is modeled in several ways: arrays, skip counting, number lines, blocks, etc. Each model emphasizes the idea that one factor tells you how many groups, the other how many items in a whole group, and the product is the total number of items:

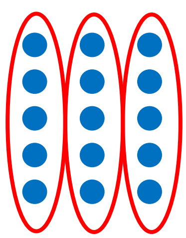
One factor tells you: GROUPS	X	**The other factor** tells you: ITEMS per group or the SIZE of the group	=	**Product** describes: TOTAL ITEMS or WHOLE SIZE

For example, 3×5 means three groups of 5 units in each group. The "product" is the total number of objects. Three representations for 3×5 and 5×3 are:

GROUPS and ITEMS MODEL

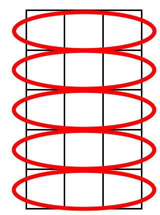


5 groups of 3

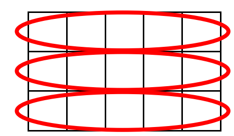


3 groups of 5

ARRAY MODEL

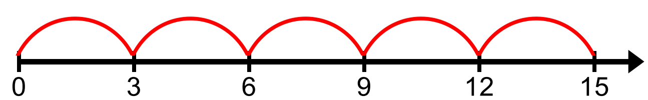


5 groups of 3

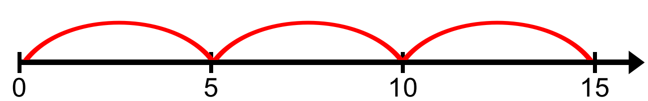


3 groups of 5

NUMBER LINE MODEL



5 groups of 3



3 groups of 5

Multiplication is commutative (order of the factors does not matter) and the product is always a total number of items or the whole size. In other word, for 3×5, we may have 3 bags (groups) with 5 candy bars (items) in each bag OR 3 candy bars (items) in each of 5 bags (groups) and in both cases, we get a total of 15 candy bars.

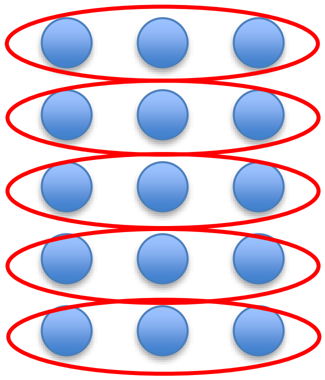
Division is the inverse of multiplication. We are looking for a missing factor, either the **number of groups** or number of **items per group/size of the group**. So, 15 ÷ 3 may mean either:

15 TOTAL ITEMS ÷ 3 ITEMS per GROUP = how many GROUPS? (Example 1)

OR

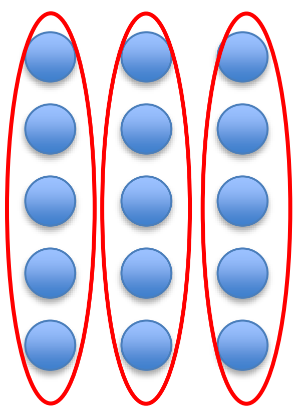
15 TOTAL ITEMS ÷ 3 GROUPS = how many ITEMS per GROUP? (Example 2)

<u>Example 1:</u>
Cora has 15 cookies, if she gives each friend 3 cookies, how many friends get cookies?



This is 15 ÷ 3 where we are looking for the "number of groups" factor. Thus, we are counting the number of groups with 3 items in each group. For the context, 5 friends (groups) get cookies. We can think 15 TOTAL ITEMS ÷ 3 ITEMS per GROUP = 5 GROUPS

<u>Example 2:</u>
Cora has 15 cookies to fair share among 3 friends. How many cookies can she give each friend?

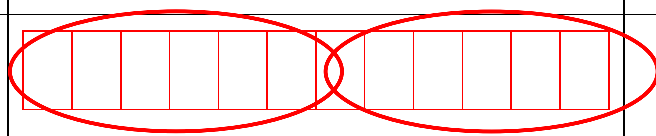
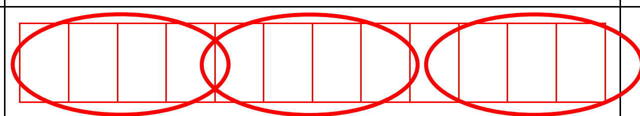
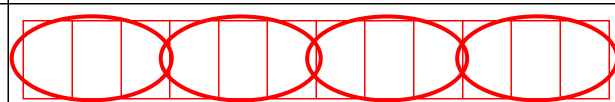
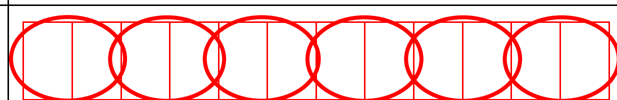


Again, we have 15 ÷ 3, but now we are looking for the "number of items per group" factor. There are 3 groups with each group having 5 items in a whole group. So, for the context, each friend gets 5 cookies. We can think 15 TOTAL ITEMS ÷ 3 GROUPS = 5 ITEMS/GROUP

We can also think about the expression 15 ÷ 3 where 15 represents a WHOLE SIZE (for example, it might represent 15 feet). The 3 can represent either 1) the number of groups of 3 feet we can make OR 2) the size of a group if 3 groups are made.

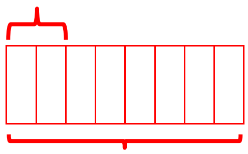
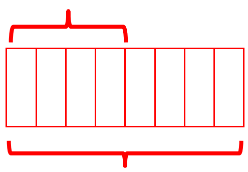
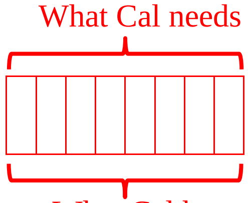
In 5th grade we extended these ideas to divide whole number by a fraction or a fraction by a whole number both conceptually and as the inverse of multiplication.

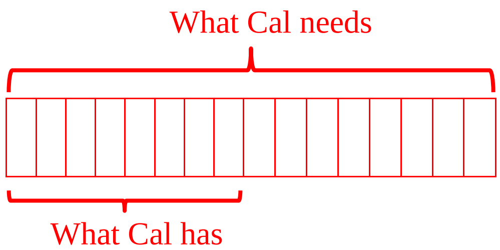
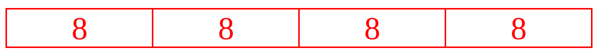
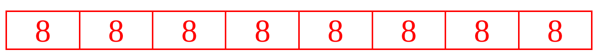
Activity 1: Cristina has 12 gallons of water to take on a hike. Complete the table below to answer each question. This problem illustrates **quotative (measurement) division**, that is, we are looking for the number of groups based on the number of items or size in each group.

Question	Model	Division Sentence	Related Multiplication Sentence
If she rations 6 gallons of water per person, how many people can go hiking?		$12 \div 6 = 2$	$2 \times 6 = 12$
If she rations 4 gallons of water per person, how many people can go hiking?		$12 \div 4 = 3$	$3 \times 4 = 12$
If she rations 3 gallons of water per person, how many people can go hiking?		$12 \div 3 = 4$	$4 \times 3 = 12$
If she rations 2 gallons of water per person, then how many people can go hiking?		$12 \div 2 = 6$	$6 \times 2 = 12$
If she rations 1 gallon of water per person, how many people can go hiking?	1 1 1 1 1 1 1 1 1 1 1 1 The 1 in each box represents 1 person per gallon.	$12 \div 1 = 12$	$12 \times 1 = 12$
If she rations $\frac{1}{2}$ a gallon of water per person, how many people can go hiking?	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 We can cut each gallon in half and 24 people can go hiking. You can also think it of as 2 people sharing each gallon.	$12 \div \frac{1}{2} = 24$	$24 \times \frac{1}{2} = 12$
If she rations $\frac{1}{3}$ of a gallon of water per person, how many people can go hiking?	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$12 \div \frac{1}{3} = 36$	$36 \times \frac{1}{3} = 12$

Discuss with students that the smaller the amount rationed, the more people can go hiking. They should notice that the smaller the positive divisor, the bigger the quotient. Talk explicitly with students about the fact that the quotient may be bigger than either the divisor or dividend. When we divide by a number greater than 1, the quotient is smaller than the dividend. When we divide by a number between 0 and 1, the quotient is greater than the dividend. The last column is a way we can check our answer using the related multiplication sentence.

Activity 2: Cal is having a party and serving pizza. Complete the table below to determine how much pizza Cal needs based on the different activities. This is an example of **partitive division**, that is, the number of groups is known and we are trying to find the number of items in each group or the size of the group.

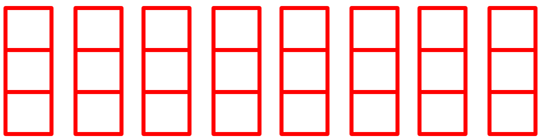
Question	Model	Division Sentence	Related Multiplication Sentence
Cal has 8 pizzas, it's four times as much as what he needs for his party. How many pizzas does he need? 2 pizzas	What Cal needs  What Cal has 8 represents 4 groups, what is the size of 1 group?	$8 \div 4 = 2$	$4 \times ? = 8$ $4 \times 2 = 8$
Cal has 8 pizzas, it's twice as much as what he needs for his party. How many pizzas does he need? 4 pizzas	What Cal needs  What Cal has 8 represents 2 groups, what is the size of 1 group?	$8 \div 2 = 4$	$2 \times ? = 8$ $2 \times 4 = 8$
Cal has 8 pizzas, it's exactly what he needs for his party. How many pizzas does he need? 8 pizzas	What Cal needs  What Cal has 8 represents 1 group, what is the size of 1 group?	$8 \div 1 = 8$	$1 \times ? = 8$ $1 \times 8 = 8$

Cal has 8 pizzas, it's $\frac{1}{2}$ of what he needs for his party. How many pizzas does he need?	 8 represents $\frac{1}{2}$ a group, what is the size of 1 group?	$8 \div \frac{1}{2} = 16$	$\frac{1}{2} \times ? = 8$ $\frac{1}{2} \times 16 = 8$
Cal has 8 pizzas, it's $\frac{1}{4}$ of what he needs for his party. How many pizzas does he need?	At this point, it makes sense to simplify the model:  8 represents $\frac{1}{4}$ a group, what is the size of 1 group?	$8 \div \frac{1}{4} = 32$	$\frac{1}{4} \times ? = 8$ $\frac{1}{4} \times 32 = 8$
Cal has 8 pizzas, it's $\frac{1}{8}$ of what he needs for his party. How many pizzas does he need?	 8 represents $\frac{1}{8}$ a group, what is the size of 1 group?	$8 \div \frac{1}{8} = 64$	$\frac{1}{8} \times ? = 8$ $\frac{1}{8} \times 64 = 8$

Activity 3: Below are two contexts involving 8 and $\frac{1}{3}$. Solve both using models and equations and explain how the problems are similar and how they are different.

Both contexts below can be represented by the expression $8 \div \frac{1}{3}$. Both have an answer of 24, but the contexts mean the models look very different. Part a. is an example of **quotative division**. Relate back to Activity 1 with Cristina and the water. In both, we are finding the number of groups we can create if we know the size of the group. Part b. is an example of **partitive division**. Relate back to Activity 2 with Cal and the pizzas.

a. Eli has 8 pints of ice cream. If a serving size is $\frac{1}{3}$ a pint of ice cream, how many servings are in the 8 pints he has? START by asking students if their answer should be bigger or smaller than 8 and why.
Solution:



8 pints of ice cream; each pint is cut up into 3 parts

This context is the division problem $8 \div \frac{1}{3}$. In this problem, **we are counting the number of $\frac{1}{3}$'s (of a unit) in 8 whole units,** e.g. we are counting the NUMBER of GROUPS of $\frac{1}{3}$ there are in 8. The model reveals why we invert and multiply to find the solution:

$8 \div \frac{1}{3} = 8 \times \frac{3}{1} = \frac{24}{1} = 24$

Notice that we start by multiplying 8×3 to show there are 24 parts (24 groups of $\frac{1}{3}$), then we divide by 1 because each serving consists of "1" part (counting the number of $\frac{1}{3}$'s).

b. Eli has 8 pints of ice cream; he believes this is $\frac{1}{3}$ of what he needs. How much ice cream does he think he needs? START by asking students if their answer should be bigger or smaller than 8 and why.
Solution:

Again, algorithmically, the context is the division problem $8 \div \frac{1}{3}$. In this problem, **we are looking for the size of the whole group**. We know 8 is $\frac{1}{3}$ of the whole group. Notice, in the algorithmic solution, we show that we are first dividing the 8 by 1, and then multiplying be 3. Dividing 8 by 1 is because 8 is in one part of the whole, multiplying by 3 is because there are 3 parts in the whole.

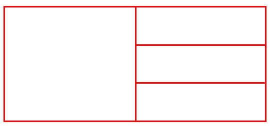
You can also connect this to work done in Section 2.1. The question is $\frac{1}{3}$ of what number is 8? This can be represented by the equation or $\frac{1}{3} \times n = 8$. The related division problem is $n = 8 \div \frac{1}{3}$.

$8 \div \frac{1}{3} = \frac{8}{1} \times 3 = \frac{24}{1} = 24$

Activity 4: You have $\frac{1}{2}$ a pizza left over from dinner last night. You invite two friends over, and the 3 of you will share the leftover pizza. What portion of a whole pizza will you each be getting? Solve using a model and an equation.

START by asking students if their answer should be bigger or smaller than $\frac{1}{2}$ and why.

Solution:



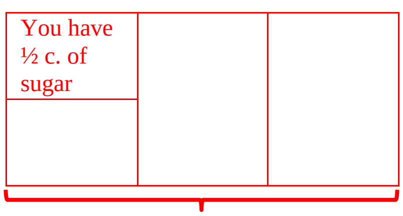
3 people will
share ½ pizza

The half gets cut up into 3 parts which means each person will get $\frac{1}{6}$ of the whole original pizza. Here we are fair sharing the $\frac{1}{2}$ among 3 groups, so there is $\frac{1}{6}$ of the original amount in each group.

Algorithmically, we have $\frac{1}{2} \div 3 = \frac{1}{6}$ or

Students may also think of this as taking $\frac{1}{3}$ of $\frac{1}{2}$ or $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$. We can see the relationship between the division problem and the multiplication problem: $\frac{1}{2} \div 3 = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Activity 5: You have $\frac{1}{2}$ a cup of sugar but you need 3 cups to make a recipe. How much of the recipe can you make? If you have ½ cup and you are trying to make groups with 3 cups in each group, you cannot even make one group. You can only make part of the group.



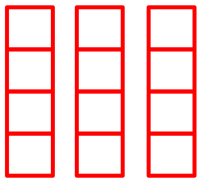
You need 3 cups of
sugar

In the model above, the total needed is 3 cups, you have $\frac{1}{2}$ of a cup. Each cup has two $\frac{1}{2}$ cups (students can create six equal parts), so the $\frac{1}{2}$ cup is $\frac{1}{6}$ of what is needed.

Algorithmically, we have $\frac{\frac{1}{2}}{3}$ or $\frac{1}{2} \div 3 = \frac{1}{6}$

Directions: For each context, a) draw a model of the context, b) write a number sentence showing the answer, and c) write your answer in a complete sentence. There are a variety of models a student might use.

1. Cora has 3 cups of sugar. The cookies she's making call for $\frac{1}{4}$ of a cup of sugar per batch. How many batches of cookies can Cora make with her 3 cups of sugar?



Cora can make 12 batches of cookies.

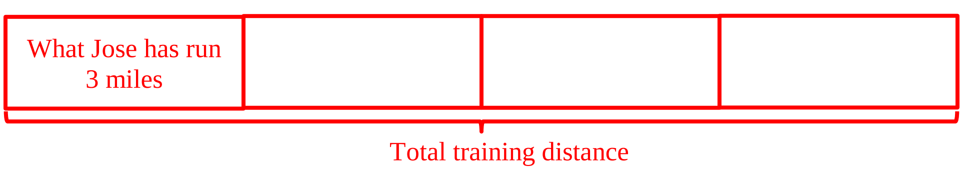
$3 \div \frac{1}{4} = 12$

2. On average, Lucy needs a drink of water every $\frac{1}{2}$ mile. If Lucy runs 5 miles, how often will she take a drink of water?

Drinks of water	Miles run
0	0
1	1/2
2	1
4	2
8	4
10	5

Models could also include a tape diagram or a number line from 0 – 5 with jumps every $\frac{1}{2}$ mile for 10 jumps.
Lucy will need to drink 10 times in five miles of running.
$5 \div \frac{1}{2} = 10$

3. Jose has run 3 miles, which is $\frac{1}{4}$ of his training for the day. How far is he planning on running? 3 is $\frac{1}{4}$ of what?

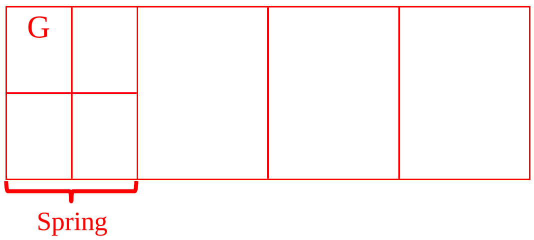


Distance completed	Portion of training
3	1/4
6	1/2
12	1

Jose plans to run 12 miles.

$3 \div \frac{1}{4} = 12$

4. Eduardo is painting a mural depicting the beauty of the four seasons in his neighborhood. He wants to divide the spring portion ($\frac{1}{4}$ of the mural) into 4 equal parts showing graduation activities, planting of gardens, melting of snow, and children playing outside. What portion of the mural will depict graduation activities?



$\frac{1}{16}$ of the mural will depict graduation

$\frac{1}{4} \div 4 = \frac{1}{16}$

Spiral Review

1. Simplify.

a. $20 \times \frac{3}{5}$	b. $\frac{3}{8} \times \frac{1}{8}$	c. $\frac{4}{5} \times \frac{10}{3}$
d. $6 \times 1\frac{1}{2}$	e. $2\frac{1}{2} \times \frac{2}{11}$	f. $\frac{3}{8} \times 8$

2. Owen is making pancakes. The recipe calls for $\frac{2}{3}$ cups of water for each cup of pancake mix. If Owen only has $\frac{1}{2}$ cup of pancake mix, how much water should he use to follow the recipe?

3. Make a double number line to show the comparison between pounds and ounces.

4. Complete the table to show the relationship between days and hours.

Days	**Hours**
1	
2	
3	
7	

2.2a Homework: Division with Whole Numbers and Unit Fractions

Directions: Solve each problem using a model of your choice. Check your work with the related multiplication sentence.

1. $5 \div \frac{1}{4}$	2. $3 \div \frac{1}{5}$
3. $\frac{1}{3} \div 4$	4. $\frac{1}{4} \div 2$

5. Write a context for either 1 or 2.
   Answers will vary. Possible context for #1: Roberto has 5 licorice ropes. He is cutting them into pieces that are each $\frac{1}{4}$ of the rope in size. How many pieces can he make?

6. Write a context for either 3 or 4.

2.2b Class Activity: Division with Rational Numbers - How Many Groups?

Activity 1: Estimating is a valuable tool that can help students determine a solution pathway and assess the reasonableness of their answer.

a. Will the quotient $13 \div \frac{4}{5}$ be bigger or smaller than 13? Justify your answer. MORE than 13. Some students may try a simpler problem and look for patterns to work toward the problem given: $13 \div 2 = 6.5$; $13 \div 1 = 13$; $13 \div \frac{1}{2} = 26$; $13 \div \frac{1}{4} = 52$. As the divisor gets smaller, the quotient gets bigger. Also, when the divisor is less than 1, the quotient is bigger than 13. $\frac{4}{5}$ is less than 1, so the quotient is bigger than 13.

**Anticipate: Some students may want to count the number of groups of $\frac{4}{5}$ in 13. Thus, you're counting the number of times you can pull out $\frac{4}{5}$. Because you're pulling out less than 1 each time, you're going to get a quotient bigger than 13.

**Anticipate: Some students may understand 13 as $\frac{4}{5}$ of a whole and then recognize that the whole must be bigger than 13.

**Encourage students to draw a model. Highlight both measurement and partitive models.

b. Will the quotient $13 \div \frac{5}{4}$ be bigger or smaller than 13? Justify your answer.

LESS than 13. 5/4 is more than 1; using similar reasoning from the above explanation (i.e., we have a number bigger than 1, so the quotient will be less than 13).

**Encourage students to draw a model. Highlight both measurement and partitive models. Compare models for a and b.

c. Which quotient will be bigger $13 \div \frac{4}{5}$ or $13 \div \frac{2}{5}$? Justify your answer.

13 ÷ 2/5 will be larger than 13 ÷ 4/5 because 2/5 is smaller than 4/5. Help students understand that the smaller the divisor the bigger the quotient. We can make more groups of 2/5 out of 13 than 4/5 out of 13.

**Encourage models.

Throughout the chapter when dividing or multiplying, encourage students to estimate first.

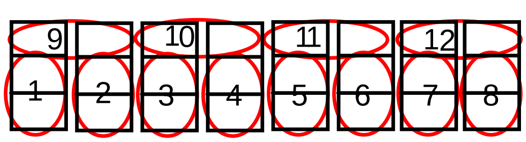
Activity 2: Eli has 8 pints of ice cream. If a serving size is $\frac{2}{3}$ of a pint of ice cream, how many servings does he have?

a. Estimate first.

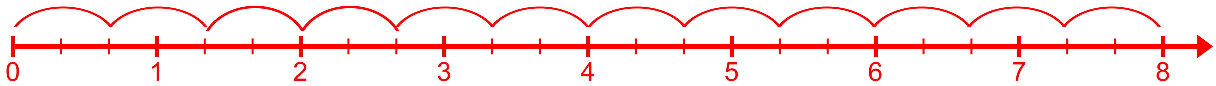
Estimate First: More than 8 servings. We are dividing 8 by a number between 0 and 1 so our answer will be greater than 1.

b. Draw a model of your choice to answer this question. We are counting the number of groups of $\frac{2}{3}$ there are in 8. There are 12 groups of $\frac{2}{3}$ in 8, thus there are 12 servings (remember a serving is $\frac{2}{3}$ of a point and you're counting ***servings***.)

Tape Diagram Model:



Students can also use a number line model:



Servings	Pints of ice cream
1	2/3
2	4/3
3	6/3 = 2
6	4
12	8

Another way to solve is to use a partial table or equivalent fractions. We know that 1 serving is $\frac{2}{3}$ pints, so we can continue writing equivalent ratios until we get to 8 pints.

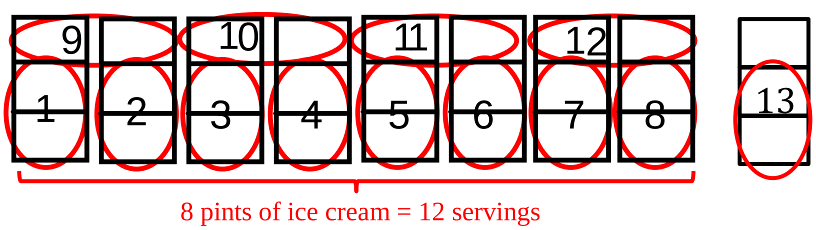
c. Write a division number sentence to represent the problem.

$8 \div \frac{2}{3} = 12$

d. Check your work with the related multiplication sentence.

$12 \times \frac{2}{3} = 8$

e. What if Eli has 9 pints of ice cream? How many servings would he have?



The 9th pint of ice cream can make 1 more full serving for a total of 13 full servings. There is one more section left. This is one out of the two we need to make another serving. In other words, it is half of what we need to make another serving. So, we can create another ½ serving (remember, we are counting ***servings***). Thus, $9 \div \frac{2}{3} = 13\frac{1}{2}$.

Table of Equivalent Ratios:

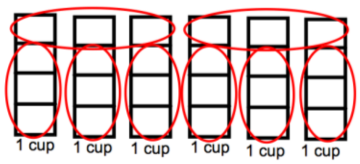
Servings	Pints of ice cream
1	$\frac{2}{3}$
2	$\frac{4}{3}$
3	$\frac{6}{3} = 2$
6	4
12	8

13	$8\frac{2}{3}$
13 ½	9
14	$9\frac{1}{3}$

With the table, we see 13 servings is $8\frac{2}{3}$ pints of ice cream and for 14 servings, we need $9\frac{1}{3}$ pints of ice cream - a difference of $\frac{2}{3}$ of a pint; in the middle is 9 and in the middle of 13 and 14 is $13\frac{1}{2}$.

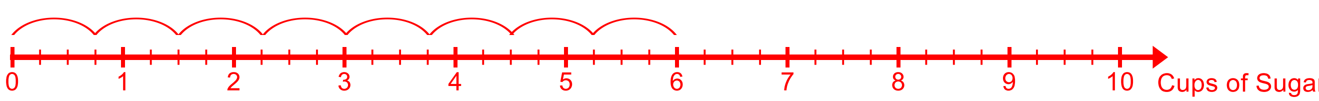
Activity 3: Calvin wants to make as many batches of chocolate chip cookies as he can for a fundraiser bake sale. He has 6 cups of brown sugar. Each batch of cookies takes $\frac{3}{4}$ of a cup of brown sugar.

a. How many batches of cookies can he make with the 6 cups of brown sugar? Draw a model of your choice to answer this question. Then, write and solve a number sentence.



We cut the 6 cups into 4 parts, giving a total of 24 parts. Three parts makes the amount of brown sugar we need for one batch. There are a total of 8 groups of $\frac{3}{4}$ in 6 cups of sugar.

Number Line Model:

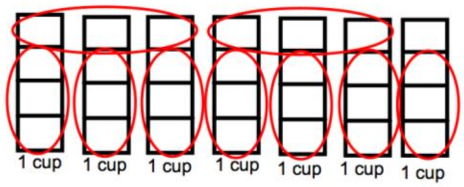


Cups of Sugar	Batches of Cookies
$\frac{3}{4}$	1
$\frac{6}{4}$	2
$\frac{9}{4}$	3
$\frac{12}{4} = 3$	4
6	8

Students can use repeated reasoning. If 3 cups will make 4 batches, then 6 cups will make 8 batches.

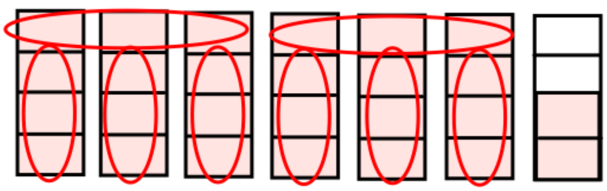
Equation: $6 \div \frac{3}{4} \rightarrow 6 \times \frac{4}{3} = \frac{24}{3} = 8$. The algorithm shows that we create 24 parts and then take 3 of them to make each batch of cookies.

b. Eli comes over to help Calvin. He brings one more cups of brown sugar. Now how many batches of cookies can they make?



We do the same process. Now there is one more group of $\frac{3}{4}$ of a cup. The extra piece is 1 out of the 3 we need to make the $\frac{3}{4}$ cup. Thus it's $\frac{1}{3}$ of what we need. We can make $9\frac{1}{3}$ batches of cookies.

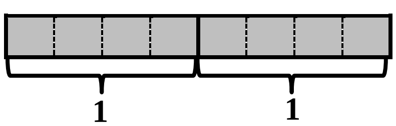
Activity 4: Mateo has $6\frac{1}{2}$ cups of sugar. How many batches of cookies can he make if each batch requires $\frac{3}{4}$ cups of sugar?



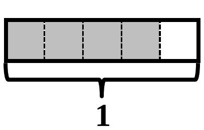
He can make 8 full recipes and $\frac{2}{3}$ of another recipe. Notice that while $\frac{1}{2}$ of a cup of sugar is left, it's $\frac{2}{3}$ of what's needed to make a recipe.

Activity 5: For each division problem, 1) Use the model to solve the problem and 2) Write a related multiplication sentence. The first one has been done for you.

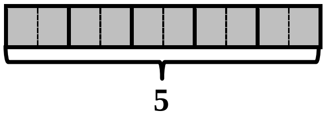
a. Students may need help interpreting the model. For example, ask, 1) What do we have? "2 wholes" 2) What size groups are we creating? "quarters" "How many quarters can we make?" Students can circle the quarters to show we can make 8 quarters. Again, connect to the multiplication sentence.

$2 \div \frac{1}{4} =$ __8__  Again, point out that the divisor is less than 1 so our quotient should be bigger than the number we start with (dividend).	Related Multiplication Sentence: $? \times \frac{1}{4} = 2$ $\mathbf{8} \times \frac{1}{4} = 2$

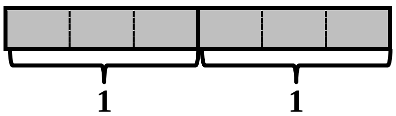
b.

$\frac{4}{5} \div \frac{1}{5} =$ __4__ 	Related Multiplication Sentence: $? \times \frac{1}{5} = \frac{4}{5}$ $\mathbf{4} \times \frac{1}{5} = \frac{4}{5}$

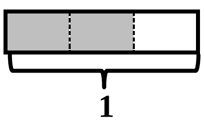
c.

$5 \div \frac{5}{2} =$ __2__ 	<u>Related Multiplication Sentence:</u> $? \times \frac{5}{2} = 5$ $\mathbf{2} \times \frac{5}{2} = 5$

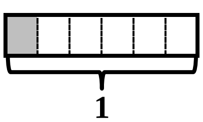
d.

$2 \div \frac{2}{3} =$ _3_ 	<u>Related Multiplication Sentence:</u> $? \times \frac{2}{3} = 2$ $\mathbf{3} \times \frac{2}{3} = 2$

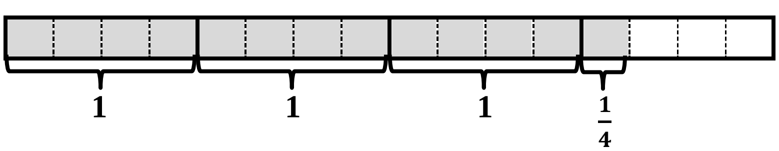
e.

$\frac{2}{3} \div \frac{1}{6} =$ _4__ 	<u>Related Multiplication Sentence:</u> $? \times \frac{1}{6} = \frac{2}{3}$ $\mathbf{4} \times \frac{1}{6} = \frac{2}{3}$

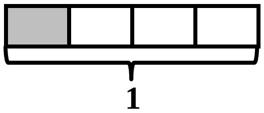
f. Again, students may need help annotating the model. If you start with $\frac{1}{6}$ and try to create a group of $\frac{2}{3}$, you cannot even create one group of $\frac{2}{3}$. Circle the group you are trying to create $\frac{2}{3} = \frac{4}{6}$ of the model. You have $\frac{1}{4}$ of what you need to create a group of $\frac{1}{6}$.

$\frac{1}{6} \div \frac{2}{3} =$ _ $\frac{1}{4}$ __ 	<u>Related Multiplication Sentence:</u> $? \times \frac{2}{3} = \frac{1}{6}$ $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$

g.

$3\frac{1}{4} \div \frac{3}{4} =$ _ $4\frac{1}{3}$ __ 	<u>Related Multiplication Sentence:</u> $? \times \frac{3}{4} = \frac{13}{4}$ $\frac{13}{3} \times \frac{3}{4} = \frac{13}{4}$

h. Help students to annotate the model. If we start with $\frac{1}{4}$ and cut it in half, we have $\frac{1}{8}$ of the total.

$\frac{1}{4} \div 2 =$ ___ $\frac{1}{8}$ ___ 	Related Multiplication Sentence: $? \times 2 = \frac{1}{4}$ $\frac{1}{8} \times 2 = \frac{1}{4}$

Spiral Review

1. Simplify.

a. $1\frac{1}{2} \times \frac{1}{9}$	b. $\frac{3}{8} \times 2\frac{3}{4}$	c. $6\frac{2}{5} \times \frac{5}{8}$

2. Estimate whether the quotient will be bigger than, smaller than, or equal to the dividend.

a. $20 \div 2$	b. $20 \div 1$	c. $20 \div \frac{1}{2}$
d. $20 \div \frac{1}{4}$	e. $5 \div \frac{3}{4}$	f. $5 \div \frac{4}{3}$
g. $6\frac{1}{2} \div \frac{1}{2}$	h. $8 \div 1\frac{3}{4}$	i. $3 \div \frac{1}{100}$

3. Simplify.

a. $5 \div 0.1$	b. $5 \div 0.01$	c. $62 \div 0.1$
d. $3 \div 0.25$	e. $12 \div 0.3$	f. $22 \div 0.11$

4. Use your answers from #3 to answer the following problems.

a. $5 \div \frac{1}{10}$	b. $5 \div \frac{1}{100}$	c. $62 \div \frac{1}{10}$
d. $3 \div \frac{1}{4}$	e. $12 \div \frac{3}{10}$	f. $22 \div \frac{11}{100}$

What is the relationship between the problems in #3 and #4?

2.2b Homework: Division with Rational Numbers - How Many Groups?

Directions: Solve each problem with a model of your choice. Check your answer using multiplication.

1. $9 \div \frac{3}{4}$	2. $4 \div \frac{2}{5}$
3. $5 \div \frac{5}{2}$	4. $10 \div 1\frac{1}{3}$

5. Write a context for either 1 or 2.
Answers will vary. Possible context for #1: Kara has a piece of ribbon that is 9 feet long. If she is making ribbons that are ¾ of a foot long for a cheerleading competition, how many ribbons can she make?

6. Write a context for either 3 or 4.

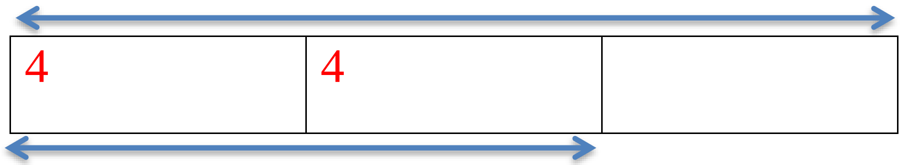
2.2c Class Activity: Division with Rational Numbers - How Big is the Whole?

Activity 1: Review of 2.2a Activity 2

a. Eli has 8 pints of ice cream. It's $\frac{2}{3}$ of what he needs. How much does he need? Draw a model of your choice to answer this question. Then, write number sentence to represent the problem.

Using a tape model:

What Eli needs



Eli has 8

Eli has 8 pints of ice cream which is $\frac{2}{3}$ of what he needs. This means, you are looking for the size of one whole group (how much he needs). Eight must be distributed evenly into $\frac{2}{3}$ of the total, therefore each part must contain 4. There are 3 parts in the total, each with 4, so the total is 12. We can think about this as 8 total pints ÷ $\frac{2}{3}$ of a group = 12 pints per ONE whole group. Connecting back to work done in chapter 1 – we just found the unit rate of pints for one whole.

If we connect back to work done in 2.1, we can think about the following:

$\frac{2}{3}$ of what equals 8

$\frac{2}{3} \times n = 8$

$n = 8 \div \frac{2}{3}$

Using a table:

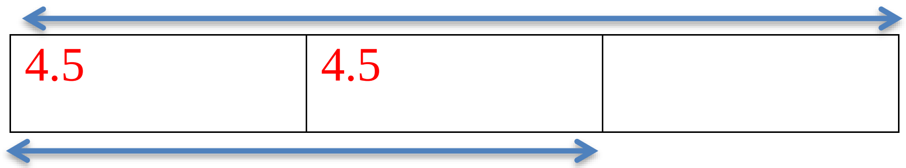
Portion of what's needed	Pints
2/3	8
1/3	4
3/3	12

With a partial table, we are looking for how big one part is so that we can multiply the one part by how many parts are in the whole. Students are finding the unit rate of pints/1 whole.

How does the "invert and multiply" algorithm connect to the models above? First, we took $\frac{1}{2}$ of 8 to determine the amount in $\frac{1}{3}$. Then, we multiplied by 3 to find the amount in $\frac{3}{3}$. $8 \div \frac{2}{3} \rightarrow \left(8 \times \frac{1}{2}\right) \times 3 = 4 \times 3 = 12$. $\left(8 \times \frac{1}{2}\right) \times 3$ is the same as $8 \times \frac{3}{2}$.

b. What if 9 pints of ice cream is $\frac{2}{3}$ of what Eli needs? Then, how much does he need?

What Eli needs



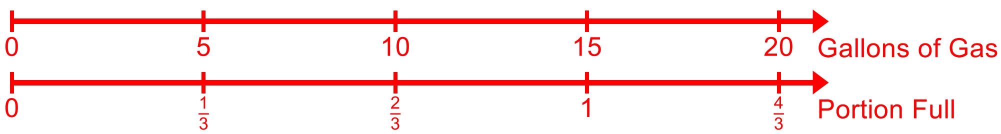
Eli has 9

Now 9, rather than 8, must be distributed between two parts, giving each part 4.5. So, the whole is 13.5.

Activity 2:

a. It takes 10 gallons of gas to fill Talen's gas tank $\frac{2}{3}$ of the way. How many gallons of gas does Talen's car hold? Draw a model of your choice to answer this question. Then, write and solve a number sentence.

It holds 15 gallons.



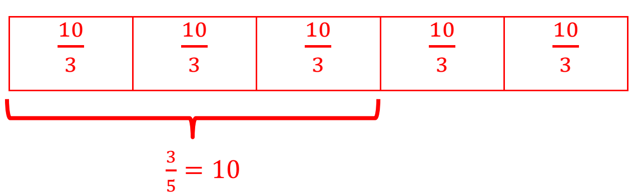
Students can start at 10 gallons of gas for a tank that is $\frac{2}{3}$ full and then iterate down to 5 gallons in $\frac{1}{3}$ of a tank by dividing by 2. Then they can iterate up by multiplying by 3 to get to 15 gallons of gas for 1 full tank.

b. Owen and Lucy also put 10 gallons of gas into their tanks. With ten gallons of gas, Owen's tank is $\frac{2}{5}$ of the way full. Ten gallons of gas fills Lucy's tank $\frac{3}{5}$ of the way. How many gallons of gas does each of their tanks hold?

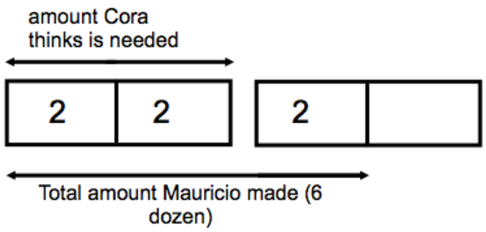
Owen:

Portion of Tank	Gallons of Gas
1/5	5
2/5	10
5/5	25

Lucy's tank holds $\frac{50}{3}$ or $16\frac{2}{3}$ gallons. This one may be more difficult for students. Help them see that if $\frac{3}{5}$ of the tank holds 10 gallons, each $\frac{1}{5}$ must hold $\frac{10}{3}$. In 5th grade, students learned that $10 \div 3$ can be expressed as $\frac{10}{3}$. Five groups of $\frac{10}{3}$ is $\frac{50}{3}$.



Activity 3:
Mauricio made 6 dozen cookies. Cora thinks it's one and a half times what he needs. How much does Cora think Mauricio should have made? Draw a model of your choice to answer this question. Then, write a number sentence to represent the problem.



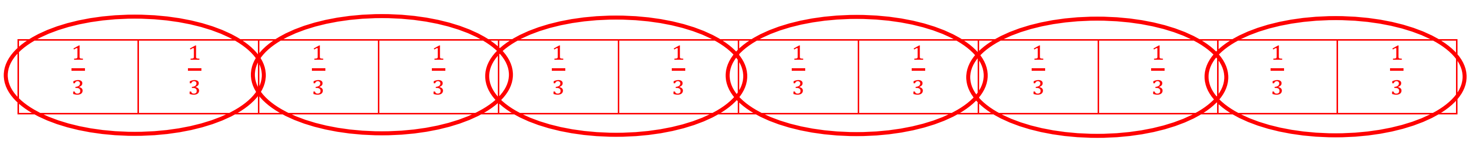
Cora thinks Maurico should have made 4 dozen cookies. Mauricio made 6 dozen cookies, which was one and a half times what was needed, so the 6 had to be evenly spaced over one and a half. Two wholes are shown, 6 is spread over 1.5 of the two wholes, putting 2 into each half. Thus, a whole is 4.
Partial table/equivalent fraction:

Amount made in dozens	part of the whole
6	1.5
12	3
4	1

Students may start by thinking, 6 is 1.5 of what I need, so doubling both is 12 and 3, divide both by 3, 4 and 1.

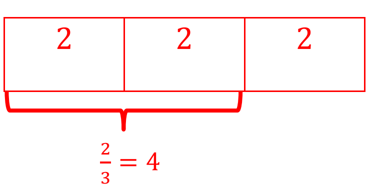
Activity 4:
Write two different contexts for $4 \div \frac{2}{3}$: one where you are counting the number of $\frac{2}{3}$'s there are in 4 and the other where 4 is $\frac{2}{3}$ of a whole and you're looking for the whole. For each of the two contexts, a) draw a model showing how to solve the problem, and b) write a number sentence showing how the solution is related to the context.

How many $\frac{2}{3}$'s in 4? How many strings that are $\frac{2}{3}$ of a foot can you cut from a string that is 4 feet long?



$4 \div \frac{2}{3} \to 4 \times \frac{3}{2} \to \frac{12}{2} = 6$ We can connect the model to the algorithm by looking at 1) How many thirds can we make out of 4 wholes? "$3 \cdot 4 = 12$" 2) If we take out 2 of the $\frac{1}{3}$'s at a time, how many groups can we make? "6"

4 is $\frac{2}{3}$ of the whole. What is the whole? Maddie has 4 cups of Rice Krispies. This is $\frac{2}{3}$ of what she needs to make a batch of Rice Krispies treats. How many cups of Rice Krispies do you need to make a batch?



The whole is 6.

Spiral Review

1. At a school carnival, there is a dunk tank. Teachers will take turns sitting in the dunk tank for $\frac{1}{2}$ an hour at a time. If the carnival is 3 hours long, how many teachers will take a turn sitting in the dunk tank?

2. Write the reciprocal of each number.

a. $\frac{2}{3}$	b. $\frac{3}{4}$	c. $\frac{5}{2}$
d. $\frac{1}{2}$	e. 4	f. $2\frac{2}{3}$

3. There are 12 inches in a foot. How many inches are in…
   a. 3 feet?

   b. 4 feet?

   c. $3\frac{1}{2}$ feet?

4. There are 16 cups in a gallon. How many cups are in…

   a. $\frac{1}{2}$ gallon?

   b. 2 gallons?

   c. $2\frac{1}{2}$ gallons?

   d. 10 gallons?

2.2c Homework: Division with Rational Numbers - How Big is the Whole?

Directions: Solve each problem using a model of your choice. Then, write a number sentence to represent the problem.

1. Lucy has 9 yards of string for her kite, but it's only $\frac{3}{4}$ of what she needs. How much string does she need?	2. Dina has 15 yards of string for her kite, but it's only $\frac{2}{3}$ of what she needs. How much string does she need? 22.5 or $\frac{45}{2}$ yards
3. Sasha has 18 yards of string for her kite; it's one and a half times what she needs. How much string does she need?	4. Kira has 25 yards of string for her kite; it's two and a half times what she needs. How much string does she need? 10 yards

5. Write two different contexts for $6 \div \frac{3}{4}$: one where you are counting the number of $\frac{3}{4}$'s there are in 6 and the other where 6 is $\frac{3}{4}$ of a whole and you're looking for the whole. For each of the two contexts, a) draw a model showing how to solve the problem, and b) write a number sentence showing how the solution is related to the context.

2.2d Class Activity: Mixed Division of Fractions

Activity 1: Explain your estimation for the following:

a. Will the quotient: $\frac{1}{2} \div \frac{1}{4}$ be bigger or smaller than $\frac{1}{2}$? Explain.

Bigger. You are dividing $\frac{1}{2}$ by a number between 0 and 1, so the quotient will be bigger. Refer back to Activity 1 in 2.2b as a reference.

**Encourage students to draw a model. Highlight both measurement and partitive models.

b. Will the quotient: $\frac{1}{4} \div \frac{1}{2}$ be bigger or smaller than $\frac{1}{4}$? Explain.

Bigger, again the divisor is less than 1.

**Encourage students to draw a model. Highlight both measurement and partitive models. Compare models for a. and b.

c. Will the quotient: $\frac{1}{2} \div \frac{5}{4}$ be bigger or smaller than $\frac{1}{2}$? Explain.

Smaller. $\frac{5}{4}$ is more than 1; using similar reasoning from the above explanation (e.g., because we have a number bigger than 1, the quotient will be less than $\frac{1}{2}$).

d. Which quotient will be bigger $\frac{1}{2} \div \frac{4}{5}$ or $\frac{1}{2} \div \frac{2}{5}$?

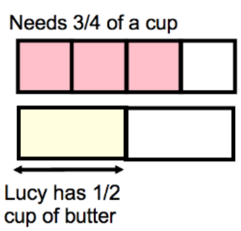
$\frac{1}{2} \div \frac{2}{5}$ will be larger than $\frac{1}{2} \div \frac{4}{5}$ because $\frac{2}{5}$ is smaller than $\frac{4}{5}$. Help students understand that the smaller the divisor the bigger the quotient. Again, refer back to Activity 1 from 2.2a

**Encourage models.

Activity 2:

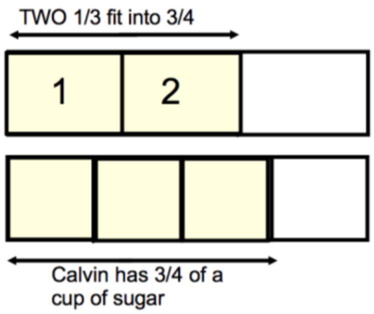
Calvin, Eli, Lusy and Cora are making cookies. *Estimate* the number of *whole* batches of cookies each can make. There are a variety of models/strategies that might be used – tape diagram, double number line, partial tables, equations, etc. In this activity, we are highlighting tape models, in the next we will highlight partial tables. Take time to talk about how your students think about the problems and how they see the division.

a. Lucy has $\frac{1}{2}$ of a cup of butter. She needs $\frac{3}{4}$ of a cup of butter to make a batch of cookies. How many batches of cookies can she make?



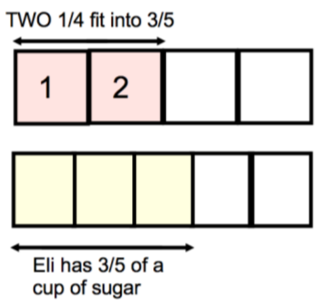
Lucy does not have enough for even one recipe. The amount she needs is MORE than the amount she has. You may want to discuss that she has $\frac{2}{3}$ of what she needs which is the answer to the question: If Lucy has $\frac{1}{2}$ a cup of butter, and needs $\frac{3}{4}$ of a cup, what portion of what she needs does she have?

b. Calvin has $\frac{3}{4}$ of a cup of sugar. He needs $\frac{1}{3}$ of a cup of sugar to make a batch of cookies. How many batches of cookies can he make?



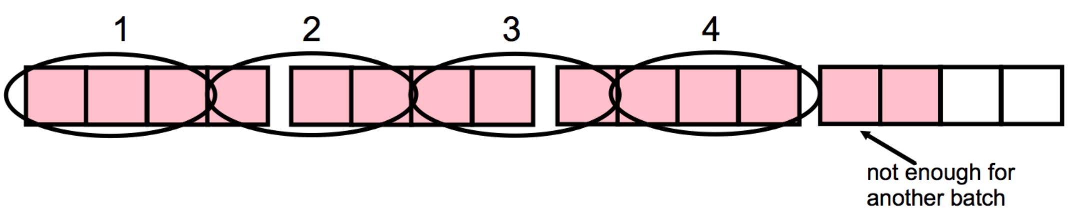
Two groups of $\frac{1}{3}$ fit into $\frac{3}{4}$. You can see that a third would be a whole, which is more than 1. So, the most he can make is 2 and he will have some sugar left over.

c. Eli has $\frac{3}{5}$ of a cup of sugar. He needs $\frac{1}{4}$ of a cup of sugar to make a batch of cookies. How many batches of cookies can he make?



The most he can make is 2. Two groups of $\frac{1}{4}$ is a $\frac{1}{2}$, 3 is $\frac{3}{4}$. Remind students that $\frac{3}{5}$ is 60% and $\frac{3}{4}$ is 75%.

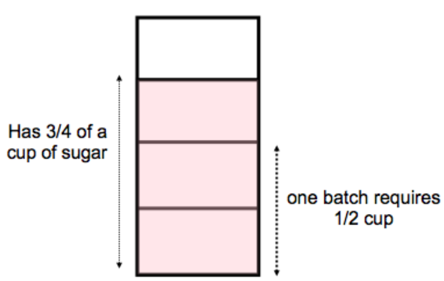
d. Cora has $3\frac{1}{2}$ cups of flour. She needs $\frac{3}{4}$ of a cup of flour to make a batch of cookies. How many batches of cookies can she make?



Every two $\frac{3}{4}$ cups is 1.5 cups, so 4 groups of $\frac{3}{4}$ is 3 cups; thus she can make 4 whole batches. She will have 2 groups of $\frac{1}{4}$ left over.

Activity 3:

Bernardo has $\frac{3}{4}$ of a cup of sugar and wants to make cookies that require $\frac{1}{2}$ of a cup of sugar. How many batches of cookies can he make?



Bernardo can make $1\frac{1}{2}$ batches of cookies because there are 1 and $\frac{1}{2}$ groups of $\frac{1}{2}$ in $\frac{3}{4}$.

Activity 4: Create a model of your choice to answer this question. Then, write a number sentence to represent the problem.

a. Eva ran $\frac{2}{3}$ of a mile which is $\frac{1}{5}$ of her total route. How far is her route?

Portion of the route	Distance traveled
$\frac{1}{5}$	$\frac{2}{3}$
$\frac{2}{5}$	$\frac{4}{3}$
$\frac{3}{5}$	$\frac{6}{3} = 2$
$\frac{4}{5}$	$\frac{8}{3}$
$\frac{5}{5}$	$\frac{10}{3} = 3\frac{1}{3}$

There are $\frac{5}{5}$ in the total route. Each of the fifths is $\frac{2}{3}$, so five of them is $\frac{10}{3}$ or $3\frac{1}{3}$. Note that students may set up their table the other way (distance traveled, portion of the route).

Equation: $m = \frac{2}{3} \div \frac{1}{5} \rightarrow \frac{2}{3} \times 5 = \frac{10}{3}$. Connect the division algorithm to the model. The model shows us that to solve the problem we multiply by 5 (to get from $\frac{1}{5}$ to $\frac{5}{5}$). If we connect that to the equation, we see that we are multiplying by the reciprocal of the divisor.

b. Penny has $4\frac{1}{2}$ quarts of ice cream. It's $\frac{3}{4}$ of what she needs for her party. How much ice cream does she need?

Portion of what Penny needs	Amount of ice cream
$\frac{3}{4}$	$4\frac{1}{2} = \frac{9}{2}$
$\frac{1}{4}$	$\frac{3}{2}$
$\frac{2}{4} = \frac{1}{2}$	$\frac{6}{2} = 3$
1	$\frac{12}{2} = 6$

We know that $\frac{3}{4}$ of the amount needed is $4\frac{1}{2}$ or $\frac{9}{2}$ (remind students $\frac{9}{2}$ means 9 groups of $\frac{1}{2}$). We can divide $\frac{9}{2}$ into three equal parts, each part will have $\frac{3}{2}$. That means $\frac{1}{4}$ of what's needed is $\frac{3}{2}$ and 4 groups of $\frac{3}{2}$ is $\frac{12}{2}$ or 6.

Equation: $\frac{9}{2} \div \frac{3}{4} \rightarrow \frac{9}{2} \times \frac{4}{3} = \frac{3}{2} \times 4 = \frac{12}{2} = 6$. The table shows that we first divide the $\frac{9}{2}$ by 3 (or take $\frac{1}{3}$ of it) which allows us to see that $\frac{1}{4}$ of what is needed is equal to $\frac{3}{2}$ gallons of ice cream. To find, $\frac{4}{4}$, we then multiply both quantities by 4.

c. Josephine has run 7.5 miles, which is $\frac{2}{5}$ of her training distance for the day. How far was she planning on running today?

Portion of Training	Distance
2/5	7.5
1/5	3.75
3/5	7.5 + 3.75 = 11.25
4/5	15
5/5	18.75

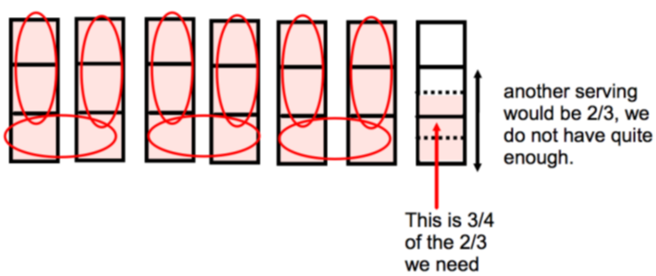
There are a total of 5/5 in a whole. Two groups of 1/5 is 7.5 so four groups is 15. To make up the last 1/5 of the route, use the 3.75 miles and add it to the 15 to get 18.75 miles.

d. Ben swam $\frac{3}{5}$ of a mile. It's $\frac{3}{5}$ of the distance he will swim today. How far will he swim today?

Portion of Distance	Distance
3/5	3/5
1/5	1/5
2/5	2/5
4/5	4/5
5/5	5/5

This may seem strange to students, but if 3/5 of a mile is 3/5 of the distance, the total distance must be 1 mile.

Activity 5: Cora has $6\frac{1}{2}$ cups of pancake syrup left. She estimates each person uses $\frac{2}{3}$ of a cup of syrup per serving. How many servings of syrup does she have?

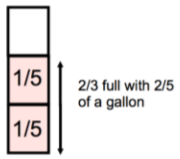


There are $6\frac{1}{2}$ cups of syrup. This means there are a total of $19\frac{1}{2}$ groups of $\frac{1}{3}$ (what we have is shaded in). We pull 2 groups of 1/3 (2*1/3 = 2/3) out at a time, we get 9 whole servings. There is not enough in what's left for a whole serving; rather, it's $\frac{3}{4}$ of the $\frac{2}{3}$ of a cup for a serving. The algorithm:

$$6\frac{1}{2} \div \frac{2}{3} = \frac{13}{2} \div \frac{2}{3} = \frac{13}{2} \times \frac{3}{2} = \frac{39}{4} = 9\frac{3}{4}$$

Activity 6:

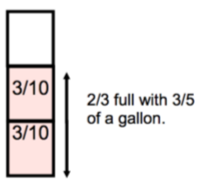
a. Lucy put $\frac{2}{5}$ of a gallon of gas in the lawn mower. It's $\frac{2}{3}$ full. How much gas does the lawn mower hold?



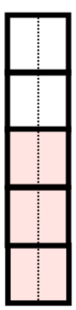
The $\frac{2}{5}$ of a gallon must be evenly split into the two portions, so $\frac{1}{5}$ is in each portion. There are 3 portions, so the tank holds a total of $\frac{3}{5}$ of a gallon. Algorithm:

$\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{3}{2} = \frac{6}{10} = \frac{3}{5}$

b. Lucy double checked her measurements and realized she'd actually put $\frac{3}{5}$ of a gallon of gas in the lawn mower, which still filled it to $\frac{2}{3}$ full. How much gas does the lawn mower hold?



Now we need to split $\frac{3}{5}$ into 2 parts and it's not even. This model may help students:

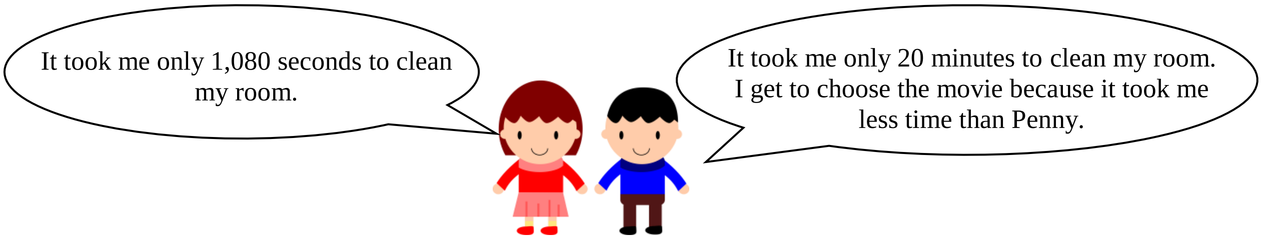


This model shows $\frac{3}{5}$ divided into to two parts, each part is $\frac{3}{10}$.

So, from the above model, if $\frac{3}{10}$ is in each of the three parts, the whole is $\frac{9}{10}$.

Spiral Review

1. Ben and Penny's dad has asked them to each clean their own rooms. He told them, whoever cleans their room the fastest gets to choose the movie they get to watch that night. After cleaning their rooms, they make the following statements:



   Do you agree with Ben's claim? Justify your answer.

2. There are 3 feet in 1 yard.
   a. How many feet are in 100 yards?

   b. How many yards are in 1,500 feet?

3. There are 1000 meters in 1 kilometer.
   a. How meters are in 5 km?

   b. How many kilometers are in 3,500 meters?

4. One pound is equal to 16 ounces.

   a. How many pounds are in 96 ounces?

   b. How ounces are in 12.5 pounds?

2.2d Homework: Mixed Division of Fractions

Directions: Create a model of your choice to answer the questions below. Then, write a number sentence to represent the problem. In the answers below, we show the "invert and multiply" algorithm. Students may show a different equation.

1. How many halves fit into three-fourths? $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2}$	2. How many halves fit into two-fifths?
3. How many three-fifths fit into two-thirds?	4. Six is two-thirds of a group of what? $6 \div \frac{2}{3} = 6 \times \frac{3}{2} = \frac{18}{2} = 9$
5. Two-thirds is six of a group of what?	6. Lisa has $\frac{3}{4}$ of a cup of sugar; it's $\frac{1}{2}$ of what she needs. How much sugar does she need?
7. Yolanda has $\frac{3}{4}$ of a cup of sugar; it's $\frac{3}{5}$ of what she needs. How much sugar does she need? $\frac{3}{4} \div \frac{3}{5} = \frac{3}{4} \times \frac{5}{3} = \frac{15}{12} = 1\frac{1}{4}$ cups of sugar	8. Gina has $\frac{3}{4}$ of a cup of sugar; it's $\frac{3}{2}$ of what she needs. How much sugar does Gina need?

2.2e Class Activity: Dividing by Two or Multiplying by One-Half?

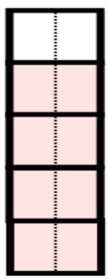
Activity 1: Lucy and Cora want to share $\frac{4}{5}$ of a pizza left over from a party the night before. They have the following conversation:

Lucy says, "That means we'll each get $\frac{4}{5}$ divided by 2 of the pizza."

Cora says, "Wait, doesn't it mean that we get $\frac{1}{2}$ of $\frac{4}{5}$ of a pizza?"

Lucy responds, "I'm confused, I think it's $\frac{4}{5} \div 2$, and you think it's $\frac{4}{5} \times \frac{1}{2}$. Aren't those two different expressions? Won't they give us two different answers?"

Answer Lucy's question with a model and a mathematical sentence.
Dividing a number by 2 is the same as multiplying it by $\frac{1}{2}$.
Ask students why both $\frac{4}{5} \div 2$ and $\frac{4}{5} \times \frac{1}{2}$ give an answer smaller than $\frac{4}{5}$.



The model shows both division by 2 and multiplication by $\frac{1}{2}$.
Division by 2: $\frac{4}{5}$ or $\frac{8}{10}$ of the whole is shaded region. If we divide it into two parts, each part is $\frac{4}{10}$ or $\frac{2}{5}$. Thus, $\frac{4}{5} \div 2$ is $\frac{4}{10}$ or $\frac{2}{5}$.
Multiplication by $\frac{1}{2}$: $\frac{4}{5}$ or $\frac{8}{10}$ of the whole is shaded region. Multiplying by $\frac{1}{2}$ means we want $\frac{1}{2}$ of the group, so we want $\frac{4}{10}$ or $\frac{2}{5}$. Thus, $\frac{4}{5} \times \frac{1}{2} = \frac{2}{5}$. Remind students that multiplication is commutative, but division is not. You are moving towards landing the idea that $a \div b = a \times \frac{1}{b}$.

Activity 2:
Cal and Eli have raised \$450 to put a new educational app on the school's personal devices. Their teacher tells them it's 80% of what they need to buy the app for the school. Which of the following expressions can be used to determine how much Cal and Eli need to raise for the app? Justify your answer.

a. $450 \div \frac{5}{4}$	b. $450 \times \frac{4}{5}$
c. $450 \div \frac{4}{5}$	d. $450 \times \frac{5}{4}$

First, students should realize that the answer needs to be greater than 450 (if 450 is only a part of what they need, the whole must be bigger). For a., dividing by a number greater than 1 will produce a result that is smaller than 450. For b., multiplying by a number smaller than 1 will produce a result that is smaller than 450. This should allow students to automatically eliminate parts a. and b.

For c., dividing by a number less than 1 will produce a result larger than 450 - you're counting the number of groups of 4/5 in 450, clearly more than 450 because the 4/5 < 1 and there are 450 1's in 450. For d., multiplying by a number greater than 1 will produce a result greater than 450. So, both c. and d. are correct expressions.

Activity 3:

For each context below 1) State a number you know the answer is either greater or less than, 2) Write a division and multiplication sentence to solve the problem, and 3) Solve the problem.

a. Talen's paper airplane flew 10 feet which was $\frac{3}{5}$ as far as Owen's plane flew. How far did Owen's plane fly?

We know the answer is greater than 10 feet. To get a number greater than 10 feet, we can either 1) multiply by a number greater than 1 OR 2) divide by a number between 0 and 1.

$10 \div \frac{3}{5} = 10 \times \frac{5}{3} = \frac{50}{3} = 16\frac{2}{3}$

Owen's plane flew $16\frac{2}{3}$ feet.

b. Geraldo has already run 4.5 miles. That's $\frac{8}{9}$ of the total distance he needs to run. How far is he planning to run in total?

We know the answer is greater than 4.5 miles.

$4.5 \div \frac{8}{9} = 4\frac{1}{2} \div \frac{8}{9} = \frac{9}{2} \div \frac{8}{9} = \frac{9}{2} \times \frac{9}{8} = \frac{81}{16} = 5\frac{1}{16}$

He is planning to run $5\frac{1}{16}$ miles.

c. Mario is selling artwork at a summer fair. He needs to pay the organizers 20% of his profit. If he made $220 at the fair, how much will he need to pay the organizers?

We know the answer is smaller than $220.

$\frac{1}{5} \times 220$ or $220 \div 5$

He will pay the organizers $44.

d. How many pieces of rope that are $\frac{3}{4}$ of a foot can be made from a rope that is 6 feet long?

We know the answer is bigger than 6 feet.

$6 \div \frac{3}{4} = 6 \times \frac{4}{3} = \frac{24}{3} = 8$

You can make 8 pieces of rope that are ¾ of a foot long.

Activity 4:

Select all the contexts that can be solved with the number sentence $\frac{3}{4} \times \frac{2}{5} = ?$. If the context cannot be solved with $\frac{3}{4} \times \frac{2}{5} = ?$, write a number sentence that would work.

a. Aaron has $\frac{3}{4}$ of a cup of sugar. It's $\frac{2}{5}$ of what he needs. How much does he need? No, $\frac{3}{4} \div \frac{2}{5} = ?$

b. Nina has $\frac{2}{5}$ of a pizza; her brother eats $\frac{3}{4}$ of it. How much of the pizza did he eat? Yes

c. Patricio has $\frac{3}{4}$ of a cup of oil. It's $\frac{5}{2}$ as much as he needs. How much does he need? Yes

Activity 5:

Select all the contexts that can be solved with the number sentence $\frac{3}{4} \div \frac{2}{3} = ?$. If the context cannot be solved with $\frac{3}{4} \div \frac{2}{3} = ?$, write a number sentence that would work.

a. Tasha has $\frac{3}{4}$ of a cup of sugar. It's $\frac{2}{3}$ of what he needs. How much does he need? Yes

b. Tony has $\frac{3}{4}$ of a cup of sugar. If he spills $\frac{2}{3}$ of it, how much sugar is left? No, $\frac{3}{4} \times \frac{2}{3}$.

c. Juan has $\frac{3}{4}$ cup of sugar, but he needs one and a half times that amount. How much does he need? Yes, $\frac{3}{4} \div \frac{2}{3}$ is the same as $\frac{3}{4} \times \frac{3}{2}$

Spiral Review

1. There are 1000 milliliters in 1 liter.
   a. How many liters are in 1,500 ml?
   b. How many mL are in 13 liters?
2. What unit would you use to measure the length of a pencil?
3. What unit would you use to measure the length of your classroom?
4. What unit would you use to measure the amount of water a sink can hold?

2.2e Homework: Dividing by Two or Multiplying by One-Half?

Directions: Show both a multiplication and division number statement for each problem. Then, state the solution.

1. Julia and Mateo want to split $\frac{3}{5}$ of a pizza. How much of the pizza will they each get?

2. Lincoln has 12 gallons of ice cream for a party. It's $\frac{3}{4}$ of what he needs. How much does he need?

   $12 \div \frac{3}{4}$ or $12 \times \frac{4}{3}$. Both equal 16 gallons of ice cream.

3. Cora has $\frac{2}{3}$ of a gallon of gas in her tank. She needs double that amount to get to the end of the trail. How much gas does she need to get to the end of the trail?

4. Curtis has $4\frac{1}{2}$ cups of brown sugar. He needs $\frac{3}{4}$ of a cup brown sugar to make one batch of cookies. How many batches of cookies can he make?

5. There is $\frac{3}{4}$ of a gallon of milk to split among 3 people. How much milk will each person get?

   $\frac{3}{4} \div 3$ or $\frac{3}{4} \times \frac{1}{3}$. Both equal $\frac{1}{4}$ of a gallon.

6. Write a context for $3 \times \frac{3}{4}$ Answers will vary

   Sample answer: Teo and his mom are running a 3-mile race. Teo runs $\frac{3}{4}$ of the race. How many miles does Teo run?

7. Write a context for $3 \div \frac{3}{4}$

8. Write a multiplication and division statement for: 8 students in Ms. Garcia's are doing biology science fair projects. That's $\frac{4}{5}$ as many as are doing physics projects. How many students are doing physics projects?

9. Write a multiplication and division statement for: 8 students in Ms. Garcia's are going to space camp this summer. $\frac{3}{4}$ of them went last year. How many went last year?

Directions: Find the quotient by multiplying by the reciprocal. Students should be practicing the "invert and multiply" algorithm here.

10. $\frac{2}{5} \div \frac{2}{3}$	11. $\frac{6}{7} \div \frac{3}{5}$ $\frac{10}{7}$ or $1\frac{3}{7}$	12. $\frac{2}{3} \div \frac{4}{5}$
13. $\frac{1}{4} \div 5$ $\frac{1}{20}$	14. $5 \div \frac{1}{4}$	15. $6\frac{2}{3} \div \frac{2}{3}$
16. $\frac{8}{9} \div 9$	17. $2\frac{1}{2} \div \frac{5}{2}$ 1	18. $\frac{3}{4} \div \frac{4}{3}$
19. $3 \div \frac{2}{5}$	20. $\frac{5}{8} \div \frac{3}{4}$	21. $5\frac{3}{5} \div 2$ $\frac{14}{5}$ or $2\frac{4}{5}$
22. $\frac{1}{2} \div \frac{1}{3}$	23. $\frac{1}{3} \div \frac{1}{2}$	24. $22\frac{1}{2} \div 11\frac{1}{4}$ 2

2.2f Self-Assessment: Section 2.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Understand and explain the relationship between multiplication and division with fractions.				
2. Solve division problems involving fractions using a variety of strategies, including models, related multiplication sentences, and the algorithm.				
3. Create contexts for division of a fraction by a fraction.				

Sample Problems for Section 2.2

Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. How much chocolate will each person get if 3 people share $\frac{1}{2}$ a pound of chocolate equally? Complete the following to answer this question. [1]

   $\frac{1}{2} \div 3 =$ ____ because ____ $\times 3 \ = \frac{1}{2}$

2. Carlos has run 10 miles. It is $\frac{2}{3}$ of the distance he is planning to run. How far is Carlos planning to run? [1]

   $10 \div \frac{2}{3} =$ ____ because ____ $\times \frac{2}{3} \ = 10$

3. Solve each problem using a method of your choice. [2]
   a. Peter bought $2\frac{1}{2}$ pounds of hamburger meat. He is planning to make $\frac{1}{4}$-pound hamburgers. How many $\frac{1}{4}$-pound hamburgers can he make?

   b. Christina needs $8\frac{3}{4}$ cups of tomato sauce to make her grandma's famous spaghetti sauce. She has 5 cups of tomato sauce. What portion of the tomato sauce that Christina needs does she have?

   c. How many $\frac{3}{4}$-cup servings are in $\frac{2}{3}$ of a cup of yogurt?

   d. A science teacher has 5 cups of solution for an experiment. Each lab table needs $\frac{2}{3}$ of a cup of solution for the experiment. How many $\frac{2}{3}$-cups of the solution can the science teacher make?

   e. Bo has raised $450 for a tablet. He realizes it is $1\frac{1}{2}$ times what he needs for the tablet. How much is the tablet?

4. Write a story for the equation $4 \div \frac{2}{3} = ?$. Then, answer the question. [3]

5. Write a story for the equation $2\frac{1}{3} \div \frac{1}{6} = ?$. Then, answer the question. [3]

Section 2.3: Ratio Reasoning and Measurement Conversion

Section Overview:
Students have been converting between units of measurement throughout elementary school. The emphasis in 6th grade is to use ratio reasoning to convert between units of measurement. Students will continually be making sense of problems and using number sense, asking the questions, "Should my answer be bigger or smaller than the number I start with and what operations can I perform to produce the desired result?" Students will also apply their fluency with operations with rational numbers to solve many of the problems in this section. The section starts with a lesson that helps students to reason through the size of their answers and the operations that will get them there. The lessons proceed with conversion of units in the same measurement system and then conversion across the customary and metric systems of measurement. Students will use a variety of models from Chapter 1, including double number lines, tape diagrams, partial tables, equations, etc.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Reason about the size of an answer when performing a measurement conversion.
2. Use ratio reasoning to convert units in the same measurement system (metric to metric and customary to customary).
3. Use ratio reasoning to convert across measurement systems (metric to customary and customary to metric).

Before beginning this section you may want to review:

- Converting decimals to fractions or fractions to decimals (Section 2.1)
- Different types of measurements (length, weight, time, capacity, mass) and the units within those measurements.
- The difference between customary and metric units.
- Converting unit rates to the "other" rate, e.g. Unit rate for cups of concentrate may be 4/5 cups of water per 1 cup of concentrate but to write it as a unit rate for water it's 5/4 cups of concentrate per 1 cup of water. We can also write it as 0.8 cups of water to 1 cup of concentrate OR 1.25 cups of concentrate for 1 cup of water.

2.3a Class Activity: Reasoning About Measurement Conversion

Many students understand that to perform conversions they need to multiply or divide but they are not always sure which to do. Understanding the size of their answer and how to get there is a valuable tool in conversion.

Activity 1: Without doing any calculations, tell whether the expression will simplify to a number that is bigger or smaller than the first number in the expression. One of the first things students should do when converting between units is to think about the size of their answer and how it should compare with the original value. Should the value be bigger or smaller? What should I multiply (or divide) by to make my answer bigger? What should I divide (or multiply) by to make my answer smaller?

a. $48 \div 12$ Smaller than	b. $8 \times \frac{1}{4}$ Smaller than	c. $500 \div 0.1$ Bigger than
d. 0.8×100 Bigger than	e. $2{,}000 \div 1{,}000$ Smaller than	f. $12 \div 2.54$ Smaller than
g. 5×0.66 Smaller than	h. $80 \div 0.454$ Bigger than	i. $360 \times \frac{1}{60}$ Smaller than

Activity 2: Complete the following. Activities 2 and 3 are intended to review concepts learned in previous grades with measurement conversion (unit equivalences, abbreviations, customary vs. metric, etc.).

a. 2 ft = __24__ in.

b. 5 yd = __15__ ft

c. 5 gal = __20__ qt

d. 3 T = _6,000___ lb

e. 2 m = __200__ cm

f. 5 L = _5,000___ mL

Complete the following statement: When you convert from a larger unit of measure to a smaller unit of measure…

The number should be bigger. You can accomplish this by multiplying by a number greater than 1 or dividing by a number between 0 and 1.

Activity 3: Complete the following.

a. 9 ft = __3__ yd

a. 10 c = __5__ pt

b. 180 min = __3__ hr

c. 32 oz = __2__ lb

d. 5,000 m = __5__ km

b. 5,280 ft = _1___ mi

Complete the following statement: When you convert from a smaller unit of measure to a larger unit of measure…

The number should be smaller. You can accomplish this by dividing by a number greater than 1 or dividing by a number between 0 and 1.

Activity 4: Jenny was asked to change 42 quarts into gallons. Circle all the expressions that can be used to do this conversion? Students should start by thinking about, "How many quarts are in a gallon?" "Should my answer be bigger or smaller than 42?" "Which expressions will produce the desired result?"

a. $42 \div 4$ Yes	b. $42 \div 0.25$ No
c. $42 \times \frac{1}{4}$ Yes	d. $\frac{42}{4}$ Yes

Activity 5: Lloyd was asked to change 6 minutes to hours. Circle all the expressions that can be used to do this conversion.

a. 6×60 No	b. $6 \times \frac{1}{60}$ Yes
c. $\frac{6}{60}$ Yes	d. $6 \div \frac{1}{60}$ No

Activity 6: Owen was asked to change 500 meters to centimeters. Circle all the expressions that can be used to do this conversion.

a. $500 \div 100$ No	b. 500×100 Yes
c. $500 \times \frac{1}{100}$ No	d. $500 \div 0.01$ Yes

Spiral Review

1. Create a number line to show $75, 150$, and 125.

2. Create a number line to show $\frac{3}{4}, 1\frac{1}{2}$, and 1.25.

3. Find the area of the rectangle with a base of 5 inches and a height of 3 inches.

4. Find the area of the rectangle with a height of $4\frac{1}{2}$ feet and a base of $1\frac{2}{3}$ feet.

2.3a Homework: Reasoning About Measurement Conversion

Directions: Answer the following questions.

1. Naoli is 13 years old. She wants to figure out how old she is in days. Will her answer be bigger or smaller than 13? Bigger than

2. It took Tomaso 14 months to train for a marathon. If he wants to express his training time in years, will the number of years be bigger of smaller than 14? Smaller than

3. A restaurant made 56 cups of salsa. If the restaurant wants to determine the number of gallons of salsa they made, will the number of gallons be bigger or smaller than 56?

4. Justin swam 2.5 km. If Justin wants to express the distance he swam in meters, will the number be bigger or smaller than 2.5?

5. A hockey puck weights approximately 150 grams. If Mike wants to express the weight of the hockey puck in kilograms, will the answer be bigger or smaller than 150?

6. Use a model of your choice to convert between feet and yards. It will help students to have an idea of the size of a foot and the size of a yard. Also, ask students what types of objects we measure in feet and what kinds of object we measure in yards.

a. 6 yd = __18____ ft

b. 24 ft = ______ yd

c. 10 yd = ______ ft

d. 1 yd = ______ ft

e. 1 ft = ___$\frac{1}{3}$___ yd

f. 10 ft = ______ yd

g. $4\frac{1}{2}$ yd = ______ ft

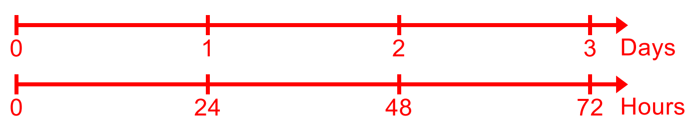
h. 20 ft = ______ yd

i. $2\frac{1}{3}$ yd = ______ ft

2.3b Class Activity: Converting Within the Same System of Measurement

Activity 1: Create a model of your choice to convert between the units given. Then, write a numeric statement for the conversion. Students have a variety of tools they can use from Chapter 1. Encourage students to use strategies and models they feel comfortable with from Chapter 1. Students should reason through whether the answer should be bigger or smaller. Provide physical measuring tools for students so that they can make sense of the size of the units.

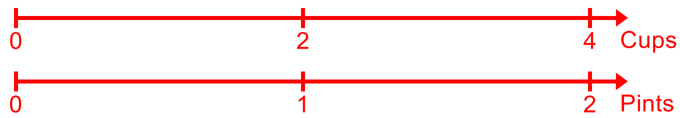
<table>
<tr><th>Problem</th><th>Model</th><th>Numeric Statement</th></tr>
<tr><td>a. 6 in. = ______ ft</td><td>First, if I am changing inches to feet, should my answer be bigger or smaller than 6?
Partial Table of Equivalent Ratios:
<table><tr><th>Inches</th><th>Feet</th></tr><tr><td>12</td><td>1</td></tr><tr><td>6</td><td>$? = \frac{1}{2}$</td></tr></table></td><td>$6 \div 12 = \frac{1}{2}$

$6 \times \frac{1}{12} = \frac{1}{2}$</td></tr>
<tr><td>b. 2.5 days = _____ hr</td><td>I am changing days to hours so my answer should be bigger than 2.5.
Double Number Line:


Half-way between 2 and 3. A half-unit day interval is equal to 12 hours.</td><td>$2.5 \times 24 = 60$

Use Distributive Property to quickly simplify this expression:
$2.5 \times 24 =$
$(2\times24) + (0.5\times24)$</td></tr>
<tr><td>c. 72 yd = ______ in.</td><td><table><tr><th>Yards</th><th>Inches</th></tr><tr><td>1</td><td>36</td></tr><tr><td>2</td><td>72</td></tr><tr><td>3</td><td>108</td></tr><tr><td>72</td><td>?</td></tr></table>
Students may start making a table or double number line and realize that it would take a long time. They may look for an equation to relate the two quantities (e.g. to get from yards to inches, multiply by 36):

$72 \text{ yards} \times \frac{36 \text{ inches}}{\text{yard}} = 2{,}592 \text{ inches}$</td><td>$72 \times 36 = 2{,}592$</td></tr>
</table>

<table>
<tr><td>d. $2\frac{1}{4}$ T = _______ lb</td><td>Unit Rate: There are 2,000 pounds in 1 ton:
$\frac{2000 \text{ pounds}}{1 \text{ ton}} \times 2\frac{1}{4} \text{tons} = 2(2{,}000) + \frac{1}{4}(2{,}000) = 4{,}000 + 500 = 4{,}500$</td><td>$2\frac{1}{4} \times 2000 = 4{,}500$</td></tr>
<tr><td>e. $\frac{1}{2}$ c = _______ pt</td><td>To find $\frac{1}{2}$ cup, partition the first interval into quarters. One pint cut into quarters = $1 \div 4 = \frac{1}{4}$

</td><td>$\frac{1}{2} \div 2 = \frac{1}{4}$</td></tr>
<tr><td>f. 55 km = _______ m</td><td><table><tr><th>km</th><th>m</th></tr><tr><td>1</td><td>1,000</td></tr><tr><td>55</td><td>? = 55,000</td></tr></table></td><td>$55 \times 1{,}000 = 55{,}000$</td></tr>
<tr><td>g. 2.3 m = _______ cm</td><td><table><tr><td colspan="10">100</td></tr><tr><td colspan="10">100</td></tr><tr><td>10</td><td>10</td><td>10</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table></td><td>$2.3 \times 100 = 230$</td></tr>
<tr><td>h. 50 mL = _______ L</td><td><table><tr><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td><td>100</td></tr></table>
50 mL is half of one of the boxes above so it is 1/20 of the total (1 L)</td><td>$50 \div 1000 = \frac{50}{1000} = \frac{1}{20}$</td></tr>
</table>

Activity 2: Compare the two measurements using $<, >,$ or $=$.

a. 2.1 ft ___>__ 24 in.	b. $\frac{4}{3}$ yd __=___ 4 ft	c. 3 mo __=___ $\frac{1}{4}$ yr
d. 3.5 c __<___ 30 fl. oz.	e. 40 hr ___>___ $\frac{1}{5}$ wk	f. 299 mL ___<___ 0.3 L
g. 78 mm __<___ 780 cm	h. 10 km __=____ 10,000 m	i. 10 m ___>___ 1,000 mm

Activity 3: Find the sum or difference. Express your answer in the unit given.

1. 5 ft + 2 in. = ___$5\frac{1}{6}$_____ ft	2. $1\frac{1}{2}$ min + 15 sec = ___105_____ sec
3. 500 mL + 1 L = __1,500______ mL	4. 3 km + 750 m = ___$3\frac{3}{4}$_____ km

Activity 4: A manager of a football team needs one quart of water for each of her 22 players. Her large orange cooler holds 5 gallons of water. Will it be big enough to hold all the water for her team? Justify your answer.

There are a variety of strategies/models students can use to solve this problem. Refer to Activity 1 for ideas. For 22 players, the manager will need 22 quarts of water. To convert from quarts to gallons, divide by 4: $22 \div 4 = 5.5$ gallons of water. The container is not big enough.

Activity 5: A catering business plans for a 3-oz serving of chicken for each guest. There are 25 people attending the party. If the catering business purchases $4\frac{1}{2}$ pounds of chicken, will they have enough chicken? Justify your answer.

There are a variety of strategies/models students can use to solve this problem. Refer to Activity 1 for ideas. The catering business needs 75 ounces of chicken. There are only $4\frac{1}{2} \times 16 = 4(16) + \frac{1}{2}(16) = 72$ ounces of chicken in 4.5 pounds so they do not have enough chicken.

Activity 6: There are 8 ounces in 1 cup, 2 cups in a pint, and 2 pints in a quart.

a. How many cups are in 64 ounces?

8 cups

b. How many ounces are in 5 quarts?

160 ounces

Activity 7:

Bethany and Beatrice are stacking dominos on end in rows to eventually knock over. In 40 seconds Bethany can stack 60 dominos and in 15 seconds Beatrice can stack 30 dominos.

Who is faster at stacking the dominos?

Students will use a variety of methods. Connect ideas here to ideas in Chapter 1 about unit rate. If you students still struggle at finding the unit rate you may want to provide a partial table for them with some of the units filled in or a tape or double number line model.

Bethany

Seconds	Dominos
1	2
2	4
3	6
4	8
15	30

Beatrice

Seconds	Dominos
1	1.5
2	3
3	4.5
4	6
40	60

Bethany's unit rate is 1.5 dominos per second and Beatrice's unit rate is 2 dominos per second. This means that Beatrice is faster at stacking dominos.

How many dominos can each girl stack in one minute?

Bethany can stack 90 dominoes per minute; Beatrice can stack 120 dominos per minute.

How many dominos can each girl stack in one hour?

Bethany can stack 5400 dominoes per hour; Beatrice can stack 7200 dominos per hour.

Spiral Review

1. Create a number line to show $10, 50$, and 85.

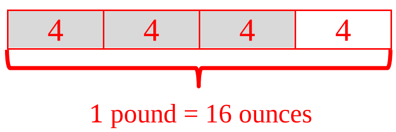
2. Create a number line to show $\frac{1}{3}$, $1\frac{2}{3}$, and $\frac{1}{6}$.

3. The area of a rectangle is 50 square centimeters. If the base of the rectangle measures 10 centimeters, what is the height of the rectangle.

4. The area of a rectangle is $\frac{1}{8}$ of a square meter. If the base of the rectangle measures $\frac{1}{2}$ of a meter, what is the height of the rectangle?

2.3b Homework: Converting Within the Same System of Measurement

Directions: Create a model of your choice to convert between the units given. Then, write a numeric statement for the conversion.

Problem	Model	Numeric Statement
1. 12 oz = _____ lb	There are 16 ounces in a pound so students should reason that they do not even have enough ounces to make one pound. Tape Diagram: 16 ounces = 1 pound  12 ounces is $\frac{3}{4}$ of a pound.	$\frac{12}{16} = \frac{3}{4}$
2. $5\frac{1}{2}$ pt = ______ c		
3. $\frac{2}{3}$ mi = ______ ft		
4. 500 sec = _____ min		

5. 1.5 mi = ______ ft		
6. 500 mL = ______ L		
7. 100 g = ______ kg	<table><tr><th>g</th><th>kg</th></tr><tr><td>1,000</td><td>1</td></tr><tr><td>100</td><td>$? = \frac{1}{10}$</td></tr></table>	$100 \div 1000 = \frac{1}{10}$ $100 \times \frac{1}{1000} = \frac{1}{10}$
8. $5\frac{1}{2}$ L = ______ mL		

Directions: Compare the two measurements using $<, >,$ or $=$.

9. $\frac{3}{4}$ mi ____ 1,500 yd	10. 15 qt __ 3.25 gal	11. 120 oz _____ 7.5 lb
12. $5\frac{1}{3}$ hr __<___ 330 min	13. 240 cm _____ $2\frac{1}{2}$ m	14. 50 g ______ 0.5 kg

Directions: Find the sum or difference. Express your answer in the unit given.

15. $4\frac{1}{2}$ hr + 45 min = ____$5\frac{1}{4}$____ hr	16. 50 cm + 1 m = ________ m
17. $2\frac{1}{4}$ gal + 12 c = ________ gal	18. 1 kg + 1 g = ________ g

Directions: Solve the following problems.

19. A science teacher needs 50 mL of a solution for each lab table for a lab experiment. There are 15 lab tables in the room. The science teacher has 1 L of the solution. Does he have enough for all the lab tables?

20. Penny has 3 gallons of punch for a class party. She wants to have $1\frac{1}{2}$ cups of punch for each student in her class. There are 20 students in her class. Does she have enough punch?
Yes, Penny does have enough. She needs 30 cups of punch. There are 48 cups in 3 gallons of punch.

21. There are 7 days a week, 24 hours in a day, and 60 minutes in an hour.
    a. How many minutes are in 2 weeks?

    b. How many days in 4,320 minutes?

22. Calvin, Eli, Lucy and Cora are preparing hygiene kits for the Red Cross. Calvin fills 4 in 3 minutes, Eli fills 75 per hour, it takes Lucy 20 minutes to fill 30, and Cora fills 140 every two hours. Who fills kits the fastest?

2.3c Class Activity: Converting Across Systems of Measurement

Activity 1: Tell which unit of measure is larger. Again, measuring tools and benchmarks will be very helpful for students. Additionally, students should have a reference sheet available for the conversions.
Depending on what conversion chart a student looks at, they may only be given the conversion in one direction (e.g., 1 cm is equal to 0.394 inches). To help students to figure out how to convert when going in the other direction, they need to find the "other" unit rate associated with the conversion. If one unit rate is $\frac{0.394 \text{ inches}}{1 \text{ cm}}$, the other unit rate is the reciprocal (e.g., the unit rate of centimeters to inches is $\frac{1}{0.394} = \frac{2.54 \text{ cm}}{1 \text{ in}}$).

1. Centimeters or inches Show students a ruler with centimeters vs. inches. There are approximately 2.54 centimeters in 1 inch (or approximately $\frac{1}{2.54} = 0.394$ inches in 1 centimeter).
   An inch is a larger unit of measure. Ask students, "If we are converting from centimeters to inches, should our answer be bigger or small? If we are converting from inches to centimeters, should our answer be bigger or smaller?"

2. Miles or kilometers Many students are familiar with the number of miles in a 5-k race (about 3.1). A mile is a larger unit of measure. 1 mile is approximately 1.61 kilometers. 1 kilometer is approximately $\frac{1}{1.61} = 0.621$ miles.

3. Pounds or kilograms
   There are approximately 2.205 pounds in 1 kilogram. There are approximately 0.45 kilograms in 1 pound. A kilogram is a larger unit of measure.

4. Liters or gallons
   1 liter is approximately 0.264 gallons. 1 gallon is approximately 3.79 liters. A gallon is a larger unit of measure.

Activity 2:

a. A pencil is 8 inches long. About, how long is the pencil in centimeters? Use the ideas from Activity 1 to ask students whether their answer should be bigger or smaller than 8. We are going from a bigger unit to a smaller unit so our answer should be bigger.
   Some students may realize, if my answer needs to be bigger, I need to multiply by 2.54 (If I multiply by 0.394, I will get a smaller answer).
   If students are unsure, have them create a model. Students may question which unit rate to use. Depending on the direction of the conversion, one rate will be much easier to use. In the first table shown below, determining what to multiply by to get from 0.394 to 8 is not an easy; however, if we use the second table, we can easily see that to get from 1 to 8 we multiply by 8. Thus, our equation is $2.54 \times 8 \approx 20.32$.

in.	cm
0.394	1
8	?

in.	cm
1	2.54
8	?

b. If you have 2 gallons of milk, what is its approximate measure in liters? Use similar reasoning as in part a.
$3.79 \times 2 \approx 7.58$ liters

c. Flora's new baby has a birth weight of 8 pounds exactly. Her mother calls from London, England to ask about the baby and wants to know the baby's weight in kilograms. Again, using ideas from Activity 1, we know that our answer should be smaller than 8. There are 2.205 pounds in 1 kilogram. We can divide 8 by 2.205 to get approximately 3.6 kilograms. We also know that there are approximately 0.45 kilograms in 1 pound. We can also multiply 8 by 0.45 to get 3.6. Again, with a partial table:

pounds	Kilograms
2.205	1
8	?

pounds	Kilograms
1	0.45
8	?

We can see that the second table is easier for the direction of the conversion. The baby's approximate weight in kilograms is 3.6 kilograms.

Activity 3: Compare the following rates to tell which is traveling faster. Sample answers are shown but there are several different ways to approach these problems.

a. Car A travels 40 miles per hour   Car B travels 60 kilometers per hour
There are approximately 0.62 miles in a kilometer; therefore 60 kilometers ≈ 37.2 miles. Car A travels faster.

b. Snail A crawls 10 cm per minute   Snail B crawls 4 inches per minute
There are approximately 2.54 centimeters in an inch; therefore, Snail A crawls approximately 3.9 inches per minute. Snail B crawls faster.

c. Biker A is riding 22 feet per second   Biker B is riding travels 400 meters per minute
Biker A is faster. There are approximately 3.28 feet in a meter; therefore, Biker B travels approximately 1,312 feet per minute. We can then divide this number by 60 to find Biker B's rate in feet per second (≈ 21.9 feet per second). Alternatively, we can find Biker A's rate in feet per minute by multiplying by 60 (1,320 feet per minute).

d. Bird A flies 4,400 feet per minute   Bird B flies 100 kilometers per hour
Bird A flies 264,000 feet per hour which is 50 miles per hour. Bird B flies approximately 62 miles per hour. Bird B is faster.

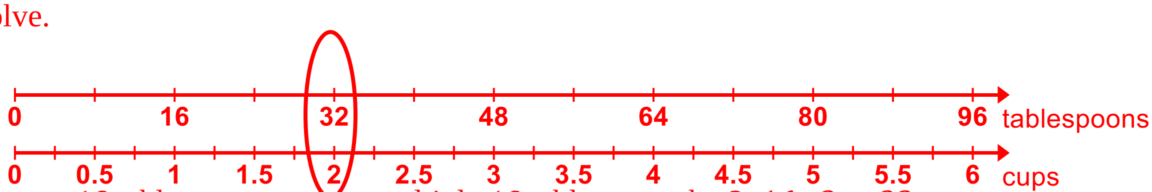
Use Activities 4 and 5 as additional practice activities/extensions as needed.

Activity 4: Eli is using a cookbook to make a recipe, but he cannot find his measuring cups. He has, however, found a tablespoon. Inside the back cover of the cookbook, it says that 1 cup = 16 tablespoons. **This is an Illustrative Mathematics Task*

Use this task to solidify how you can identify a scale factor that relates the two measurements given and convert measurements by either multiplying or dividing by this scale factor.

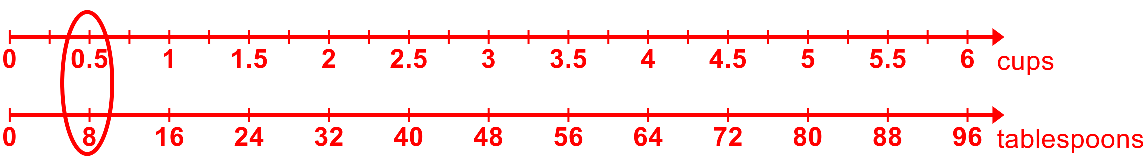
Explain how he could use the tablespoon to measure the following ingredients:

a. 2 cups of flour A double number line is used below to find the solution, students might choose other representations as well. Also students might use the multiplicative relationship between the two numbers to solve.



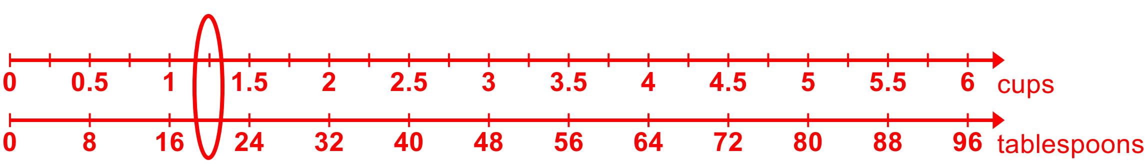
Since 1 cup = 16 tablespoons we can multiply 16 tablespoons by 2. $16 \times 2 = 32$
There are 32 tablespoons in 2 cups.

b. $\frac{1}{2}$ cup sunflower seeds



$16 \times \frac{1}{2} = 8$
There 8 tablespoons in ½ cup

c. $1\frac{1}{4}$ cup of oatmeal



$16 \times 1\frac{1}{4} = 20$
There are 20 tablespoons in 1¼ cups

Activity 5:
Lucy and Cora are comparing the effects of a new plant food on Heirloom tomato plants. To run their experiment, they each had a total of 10 plants; 5 received the new plant food every week, and 5 others did not. Both groups of plants were planted in the same soil and got the same amount of water and sunlight. At the end of 8 weeks, Lucy and Cora averaged the height of the 5 plants in each of their two groups and recorded the following data:

Lucy's Data				Cora's Data			
Tomato plants that got plant food		Tomato plants that did NOT get plant food		Tomato plants that got plant food		Tomato plants that did NOT get plant food	
Week	Average Height of 5 plants	Week	Average Height of 5 plants	Week	Average Height of 5 plants	Week	Average Height of 5 plants
0	5 inches	0	6 inches	0	8 cm	0	7 cm
8	16 inches	8	14 inches	8	45 cm	8	40 cm

Lucy and Cora want to combine their data into one report, how would you advise them to put the data together?
Students should notice that Lucy's height data is in inches, while Cora's is in centimeters. They will need to convert to all inches or centimeters.
Notice also, the conversion rate is not given. Students will need to look it up.
1 inch = 2.54 centimeters.
Lucy's data:
Plant food group grew 11 inches per 8 weeks or an average of 1.375 inches per week.
No plant food group grew 8 inches per 8 weeks or an average of 1 inch per week.
Cora's data:
Plant food group grew 37 cm per 8 weeks or an average of 4.625 cm per week.
No plant food group grew 33 cm per 8 weeks or an average of 4.125 cm per week.

	INCHES		CENTIMETERS	
	Received Plant Food	Did NOT receive Plant Food	Received Plant Food	Did NOT receive Plant Food
Lucy	1.375 inches/1 week	1 inch/1 week	3.4925 cm/1 week	2.54 cm/ 1 week
Cora	≈1.82 inches/ 1 week	≈1.62 inches/ 1 week	4.625 cm/1 week	4.125 cm/1 week

Conversation 1: In both cases, plants receiving plant food (treatment group) grew at a faster rate than those that did not get plant food. To compare the rates, students must convert both to the same units.
The samples are the same size, so one might *first* add Lucy's 11 inches/8 weeks and Cora's ≈12.99 inches/8 weeks (33 cm/8 weeks converts to ≈12.99 inches per / 8 weeks) and then divide by 2 for the average over 8 weeks, and then by 8 for the rate per week OR first divide each group inches per 8 weeks by 8 and then add the two and divide by 2. Either way, the average for the two groups is:
Treatment: ≈1.6 inches/ week; ≈4.06 cm/week
No plant food: ≈1.31 inches/week; ≈3.3325 cm/week
Conversation 2: Even though both Lucy and Cora's data seem to indicate plants receiving the food grew faster than those that did not, there is quite a bit of difference between their results. Students should wonder what else is happening.

Spiral Review

1. Simplify $5 - 2 + 1$ and $5 - (2 + 1)$. Compare the problems.

2. Simplify $3 + \frac{1}{5} \times 10$ and $\left(3 + \frac{1}{5}\right) \times 10$. Compare the problems.

3. Simplify $8 \div \frac{1}{4} \times \frac{1}{2}$ and $8 \div \left(\frac{1}{4} \times \frac{1}{2}\right)$. Compare the problems.

4. Simplify $8 - 2 \times 0.5$ and $(8 - 2) \times 0.5$. Compare the problems.

2.3c Homework: Converting Across Systems of Measurement

Directions: Answer the following questions using any method you wish. You may need to reference a conversion chart. Justify your answers.

1. How many kilometers are in 10 miles?

2. How many meters are in 100 yards? There are approximately 0.91 meters in 1 yard; therefore 100 yards is approximately 91 meters.

3. How many feet are in 10 meters?

4. How many quarts are in 2 liters?

5. How many liters are in 5 quarts?

6. Which dog weighs more, a dog that weighs 10 kilograms or a dog that weighs 16 pounds? There are approximately 2.2 pounds in 1 kilogram so a 10-kilogram dog would weigh approximately 22 pounds; therefore, a dog that weighs 10 kilograms weighs more.

7. Which running track is longer, a track that is 400 meters long or one that is $\frac{1}{3}$ a mile long?

8. Which container holds more soda, a container with a capacity of 2.5 liters or a container with a capacity of 0.75 gallons?

9. Which is heavier: 30 ounces or 1 kilogram?

10. Which hose pumps water at a faster rate, a hose that pumps water at a rate of 4 gallons per minute or a hose that pumps water at a rate of 0.25 liters per second? A hose that pumps 0.25 liters per second pumps 15 liters per minute which is approximately 3.96 gallons per minute.

2.3d Self-Assessment: Section 2.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Reason about the size of an answer when performing a measurement conversion.				
2. Use ratio reasoning to convert units in the same measurement system (metric to metric and customary to customary).				
3. Use ratio reasoning to convert across measurement systems (metric to customary and customary to metric).				

Sample Problems for Section 2.3

Square brackets indicate which skill/concept the problem (or parts of the problem) align to.

1. There are 3 feet in 1 yard. Laurence is converting 125 feet into yards. [1]
   a. Should his answer be bigger or smaller than 125?

   b. To do this conversion, Laurence can…

2. There are 100 cm in 1 meter. Stephanie is converting 10 meters into centimeters. [1]
   a. Should her answer be bigger or smaller than 10?

   b. To do this conversion, Stephanie can…

3. There are approximately 0.62 miles in 1 kilometer. Peter is converting 20 miles to kilometers. [1]
   a. Should his answer be bigger or smaller than 20?

   b. To do this conversion, Peter can…

4. Perform the following conversions. [2]

a. 12 ft = _____ in.	b. 4 yd = _____ ft	c. 3 yr = _____ mo
d. 22 c = _____ qt	e. 3.25 c = _____ pt	f. 55 g = _____ mg
g. 1 mm = _____ cm	h. $5\frac{1}{2}$ km = _____ m	i. 60.2 kg = _____ g

5. Perform the following conversions. [3]

a. 20 cm ≈ _____ in.	b. 10 m ≈_____ ft	c. 60 mi ≈ _____ km
d. 80 lb ≈ _____ kg	e. 10 gal ≈ _____ L	f. 1,500 m ≈ _____ mi