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Chapter 5: Geometry (3 Weeks)

Utah Core Standard(s)

- Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. (6.G.1)
- Find the volume of a right rectangular prism with appropriate unit fraction edge lengths by packing it with cubes of the appropriate unit fraction edge lengths (for example, $3\frac{1}{2} \times 2 \times 6$), and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (Note: Model the packing using drawings and diagrams.) (6.G.2)
- Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. (Standard 6.G.3)
- Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. (6.G.4)

Academic Vocabulary: area, square unit, polygon, quadrilateral, parallelogram, rectangle, square, kite, Rhombus, Trapezoid, length, width, triangle, right triangle, acute triangle, obtuse triangle, scalene triangle, isosceles triangle, equilateral triangle, base, height, irregular figure, horizontal, vertical, coordinate plane, ordered pair, vertices, volume, right rectangular prism, unit cube, cubic inch/cubic centimeter, fractional edge length, net, prism, pyramid, face, lateral face, edge, vertex, surface area

Chapter Overview:

In this chapter students extend previous work done with area and volume. Using shape composition and decomposition skills learned in previous grades, students develop and use formulas for the area of parallelograms and triangles. As they work with these polygons they investigate how they can choose any side to be the base of the parallelogram or triangle and that this choice determines the height. In turn, students learn that they can find the area of special quadrilaterals and other polygons by subdividing them into rectangles and triangles.

Next the focus is turned to developing their understanding of properties of two-dimensional shapes within the coordinate system. They graph and connect points to create polygons. They find vertical and horizontal side lengths of these polygons by counting out the distance between two points or by analyzing the difference between a corresponding x and y coordinates. Finally they use these techniques to find the area of polygons in the coordinate plane within the context of the real-world application.

Building on the knowledge of volume and spatial structuring abilities developed in previous grades students learn to find the volume of right rectangular prisms with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. They apply the formulas $V = lwh$ and $V = Bh$ to find the volumes of right rectangular prisms and other polyhedral solids by decomposition of their parts.

As students continue to work with polyhedral solids they learn to describe the shape of the faces, and identify the edges and vertices on a solid. This leads them to make and use drawings of nets to represent the surface area of the solid. They find the surface area of solids by decomposing their associate nets into rectangles and triangles of which they can find the area. As student's knowledge of surface area and volume evolve they solve problems that require them to apply strategies and formulas to find solutions of real-world and mathematical problems.

Connections to Content:

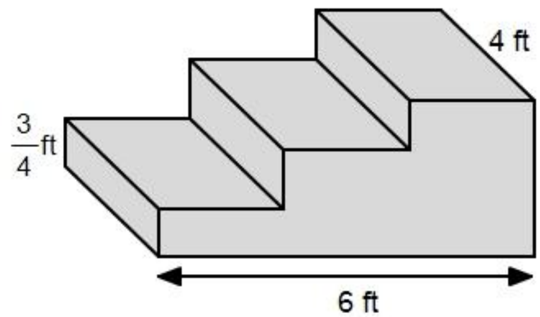
Prior Knowledge:

In previous grades students have investigated writing and solving simple equations. Working with area and volume provides a context for developing and using these equations. Students have also classified triangles and quadrilaterals and have developed an understanding of their properties and relationships. They have also learned how to graph points in the coordinate plane. In 3rd grade they recognize area as an attribute of plane figures and investigate concepts of area measurement. In 4th grade they apply area formulas to real-world and mathematical problems. Volume is studied in 5th grade where students learn to recognize it as an attribute of solid figures and investigate concepts of volume measurement. This prior work with area and volume help them to develop competencies in shape composition and decomposition and form a foundation for understanding the formulas for area and volume and the coordinate plane.

Future Knowledge:

Composition and decomposition of shapes is used throughout geometry in middle school and high school. In 7th grade will solve problems involving scale drawings of geometric figures. A good understanding and strong ability to calculate area will come to bear as they compute lengths and areas from scale drawings and reproduce a scale drawing at a different scale. Students will use their knowledge of finding distance of line segments in the coordinate plane when they study the distance formula. They will also work with three-dimensional figures as they learn about in investigate their cross sections. Lastly investigations involving area, volume, and surface area continue in 7th and 8th grade as students tackle more complex problems related to real-world applications.

MATHEMATICAL PRACTICE STANDARDS

	Make sense of problems and persevere in solving them.	Gloria is planning on pouring a set of concrete cement steps on the side of her front porch. She has drawn out a diagram of the steps below where the "rise" and "run" of each step is equal. a. Determine the total amount of cement she will need for the steps. Assume that angles that appear to be right angles are right angles.  *Students use the understanding that you need to subdivide the steps into composite figures that you can find the volume of as an entry point for solving this problem. The total volume is the sum of the composite volumes. This explanation acts as a "road map" for solving the problem, rather than just jumping right into making calculations that they do not understand.*
	Reason abstractly and quantitatively.	Describe in words and write a formula about how to the find area of any parallelogram. *Throughout this chapter students generalize methods for finding area and volume of polygons and 3D objects into formulas. As students develop these formulas they decontextulize as the move from finding the area or volume of one specific shape or object to finding the area or volume any of these given shapes or objects.*
	Construct viable arguments and critique the reasoning of others.	Olivia claims that $V = s^3$, where *s* is the side length of a cube is the formula you should use to find the volume of a cube. Harrison claims that the correct formula is $V = lwh$, where *l* is the length, *w* is the width, and *h* is the height of the cube. *This problem requires students to think critically about the claims made; they must draw upon previous knowledge about the composition of an algebraic expression and what the variables in the formulas represent. They use this knowledge to construct an argument as to who they think is correct in their reasoning.*

Look for and express regularity in repeated reasoning.

Find the length of each vertical segment, record your answers in the table below.

From Point	To Point	Length
(2, 5)	(2, 3)	
(6, 5)	(6, 3)	
(8, 1)	(8, 6)	

What do you notice about the ordered pairs that line up vertically?
Is there are way that you could determine the length of these segments without plotting the points.
As students repeatedly observe the length of vertical line segments they begin to see a pattern emerge about the relationship between the difference between the x and y coordinates and the length of the line. Through these observations they can generalize a method or "shortcut" for finding the length without counting out the distance on a graph.

Model with mathematics.

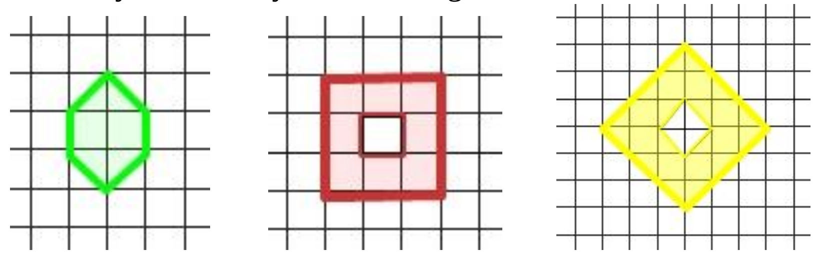
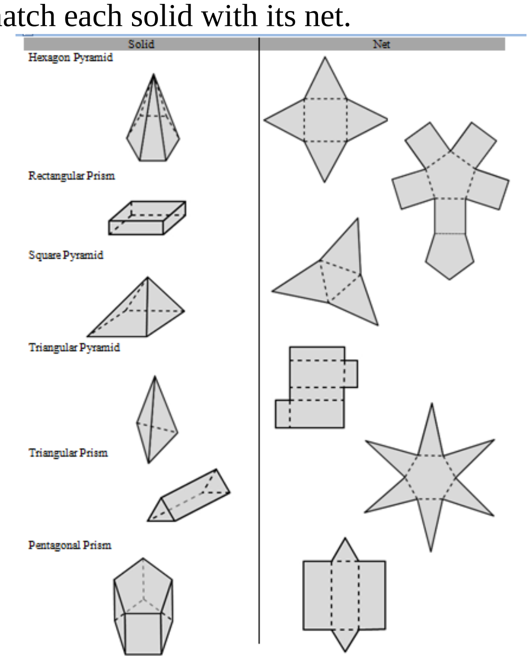
A new park is being designed for your city. The plans for the design are being drawn on the coordinate plane below. The vertices given form a polygon that represents the location for 5 different features in the park. Plot each set of points in the order they are given to form a polygon. Label the feature that the polygon represents on the graph.
In the problem above students use the coordinate plane to model real world objects and their locations. By assigning these objects a location on the coordinate plane they can find lengths of different dimensions within an object and distances between objects. In turn they will use these measurements to find area and use this information to answer questions and make informed decisions about real-world applications.

Attend to precision.

A company that manufactures dice packages the dice in rectangular boxes that measure 7 inches by 2 inches by 5 inches as shown.

a. How many 1-inch by 1-inch dice can fit into one box?
b. What is the volume of the box in cubic inches?
c. The company also makes mini dice that measure $\frac{1}{2}$-inch by $\frac{1}{2}$-inch. How many $\frac{1}{2}$-inch dice can fit into a rectangular box that has the same dimensions? Use the figure above to help you answer.
d. What is the volume of the box when measured in ½-inch cubes?
e. Why do you get two different numeric values for the volume for parts b. and d. on the previous page? Does the volume of the box the change depending on whether it is packed with 1-inch dice or ½-inch mini dice?
f. How can you show or prove that the number of 1-inch dice that fit into the box takes up the same amount of space or has the same volume as the number of ½-inch mini dice that fit into the box.

Two different units of measure are being compared and contrasted in the problems above. It is imperative for students to attend to precision as they communicate about these problems and their solutions, because if they neglect units of measure while speaking, this is likely to lead to confusion.

	Look for and make use of structure.	Find the area of each colored figure if each square represents one square unit. Be ready to discuss your reasoning.  *As students try to find the area of each figure, they must recognize that they can decompose different parts of the figure in order to recompose them into whole square units that they can count. As they do this they must step back and shift their perspective of how to view a square unit of area.*
	Use appropriate tools strategically.	Draw a line to match each solid with its net.  You can explore nets with online interactive manipulatives as you investigate the problem above. *Online tools for viewing 3D objects and their nets can really help students that have difficulty visualizing how an object and its net are related. These tools bring this relationship to life as students can see how the faces, edges, vertices, etc correspond to each other between a 3D object and its net.*

Section 5.1: Area of Polygons

Section Overview:
This section revolves around finding the area of right triangles, other triangles, special quadrilateral, and polygons. Students begin their work with area by finding area of figures on a graph and counting out unit squares. They learn that when confronted with irregular figures they can decompose and rearrange the figures into shapes that they can easily find the area of. A lot of work is done in this section with developing methods and formulas for finding the area of parallelograms, triangles, and trapezoids. Students do this by finding the area of several different examples of these shapes through transformations and then generalize their methods. Once students are familiar with finding area they turn to finding missing measurements. They use the relationship between a polygon's dimensions and its area to do so. They use their knowledge of finding area to answer question related to real-world applications and finally work with finding the area of irregular figures by decomposing these figures into triangles, rectangles, and/or trapezoids.

Concepts and Skills to Master in this Section:

By the end of this section, students should be able to:

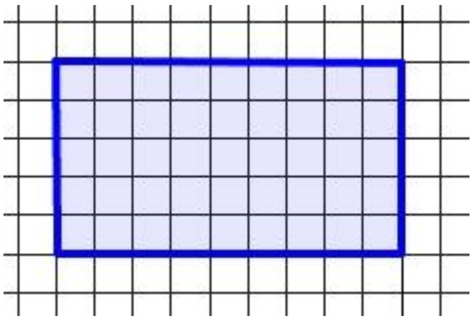
1. Find the area of a parallelogram including a rectangle and square.
2. Find the area of a triangle.
3. Find the area of a trapezoid.
4. Find the area of irregular figures by decomposing them into triangles, rectangles, and/or trapezoids.
5. Solve real-world and mathematical problems that involve finding the area of polygons.

5.1a Class Activity: Finding Area

Find the area of each colored figure if each square represents one square unit. Be ready to discuss your reasoning.

Encourage your student to share their reasoning and different methods for finding the area. As they try to find the area of each figure, they must recognize that they can decompose different parts of the figure in order to recompose them into whole square units that they can count.

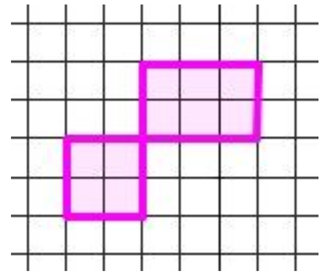
1.



45 square units-Students might simply count the number of squares or multiply the length (9) by the width (5). Use this problem as an opportunity to review the formula for finding the area of any rectangle. Be sure to discuss different notations for expressing the formula. $Area = length \times width,\ A = lw$

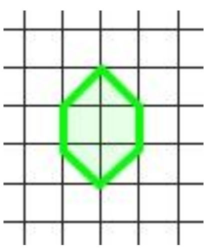
$Area = base \times height, \quad A = bh$

2.



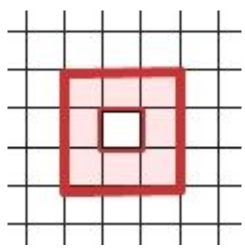
10 square units-Students can count the number of square units in each rectangle and then add them together. They can also find the area of each rectangle by multiplying its length and width and then add the two areas together. They also might reason that they can slide the square up two units to create one large rectangle.

3.



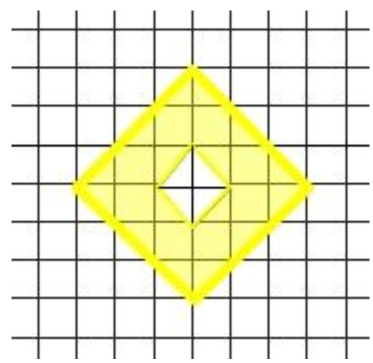
4 square units-Encourage students to rearrange this shape into a square with four square units in it.

4.



8 square units-Students can count the number of square units or find the area of the larger square and subtract off the one square unit in the middle.

5.

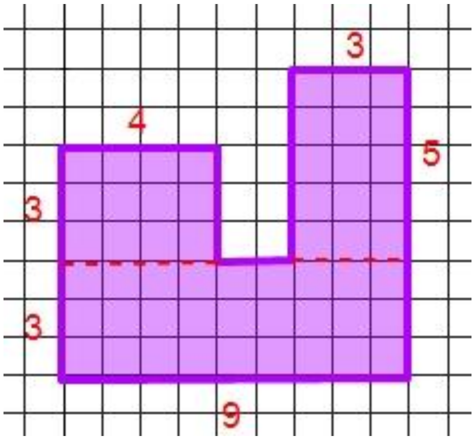


16 square units-Encourage students to rearrange this shape into a rectangle or combine the half square units into whole square units and count the number of total square units.

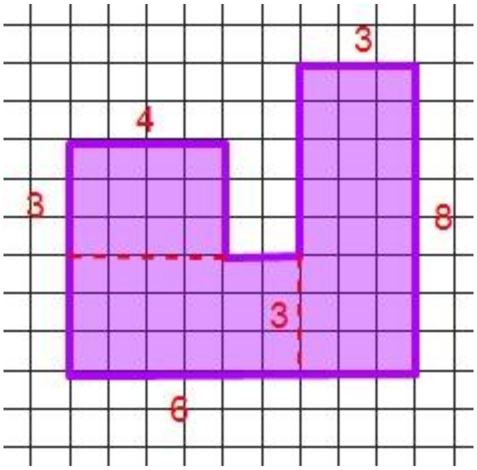
Discuss with students that area can be found by decomposing a figure and then recomposing it into a figure with non-overlapping regions. Students could find the area of the triangular pieces in numbers 3 and 5.

6. Three copies of the same figure are shown below. Each square represents one square unit. There are many ways to find the area of this figure.

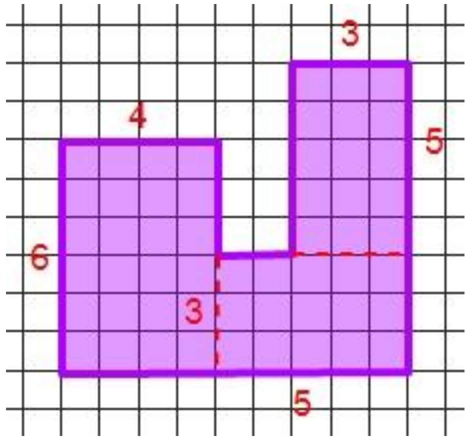
1



2



3



a. Split this figure into different non-overlapping regions in at least three ways. Show your work above.
Sample answers are shown.

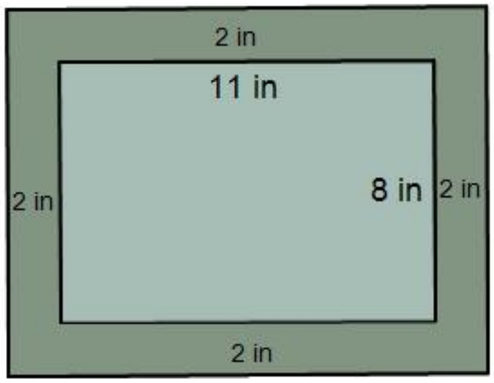
b. For every figure that you split in part a write an expression that represents the area of the sum of the regions.
Example 1: $(3 \times 4) + (3 \times 9) + (3 \times 5) = 12 + 27 + 15 = 54$ square units
Example 2: $(3 \times 4) + (3 \times 6) + (3 \times 8) = 12 + 18 + 24 = 54$ square units
Example 3: $(6 \times 4) + (3 \times 5) + (3 \times 5) = 24 + 15 + 15 = 54$ square units

c. Find the total area of the figure.
The total area is 54 square units

Ask your student to share their different methods with each other. Also talk about why it makes sense that we use a power of 2 to indicate area. For example in the problem below the units are feet and the area is represented as ft^2 or square feet.

7. Gloria is painting a feature wall in her bedroom. The dimensions of the wall measure 14 feet by 11 feet. The gallon of paint that she purchased will cover 400 square feet. Does she have enough paint to do two coats on the wall? Justify your answer.
Yes, she will have enough paint; the area of the wall is 154 ft^2, for two coats she will need to double this. $154 \times 2 = 308$. This is less than 400.

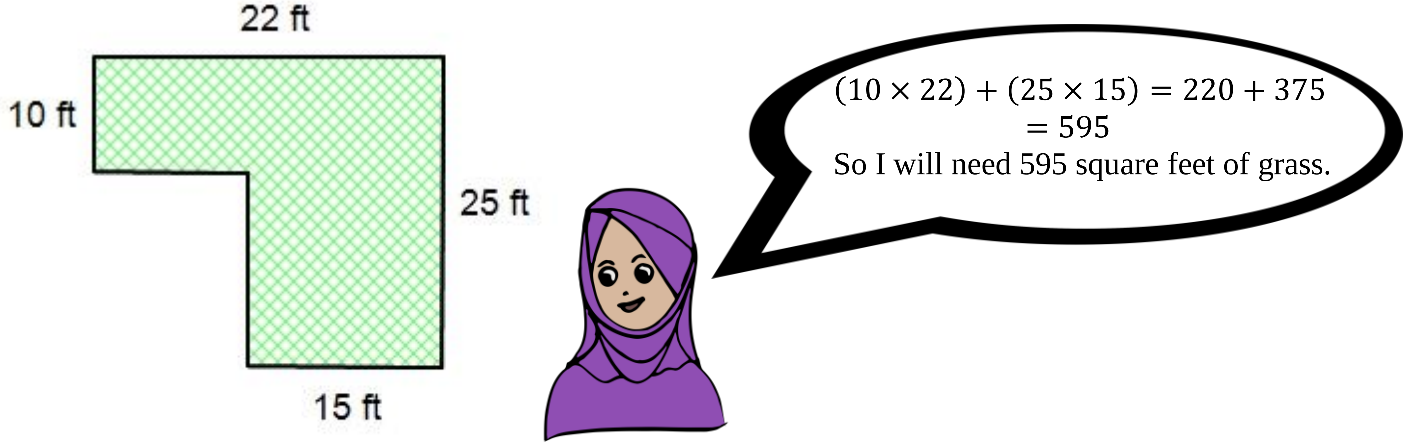
8. Steven is designing a rectangular frame to go around a drawing that measures 11 inches by 8 inches, he has budgeted $15.00 to cover the cost of the material for the frame. He would like the frame to be 2 inches wide around the perimeter of the drawing. The diagram below shows the dimensions of the drawing and the frame. The material for the frame costs $0.20 per square inch. Will Steven have enough money to cover the cost of the frame?



The area of the frame is 92 square inches. At $0.20 an inch it will cost Steven $18.40 to make the frame. He does not have enough money.

Find, Fix, and Justify

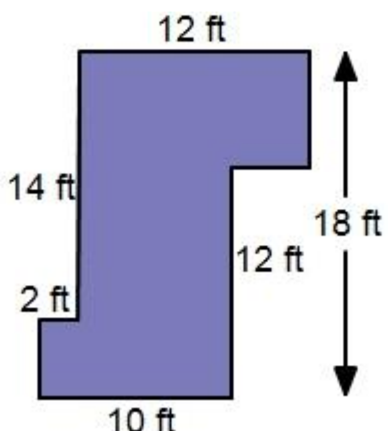
9. Antonia is laying grass sod in her back yard. She has drawn a diagram of where she would like to put the grass. Antonia's calculation to find the area of the yard that will need grass sod is shown. She has made a mistake. Find Antonia's mistake, explain what she did wrong, and then find the correct area. Assume that all angles that appear to be right angles are right angles.



Antonia has overlapped the two rectangles within the figure. If she multiplies 10 by 22, then she will need to subtract 10 feet from 25 feet to get the length of the other rectangle. The correct calculation is $(10 \times 22) + (15 \times 15) = 220 + 225 = 445$. Antonia will need 445 square feet of grass sod.

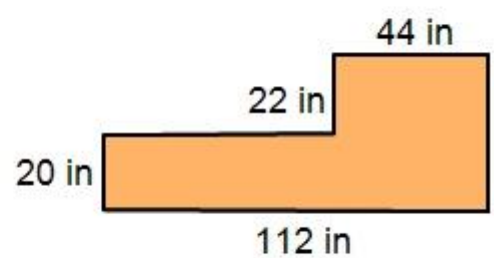
Directions: For numbers 9-12 find the total area of each figure by decomposing it into non-overlapping regions. Assume that all angles that appear to be right angles are right angles.

9.



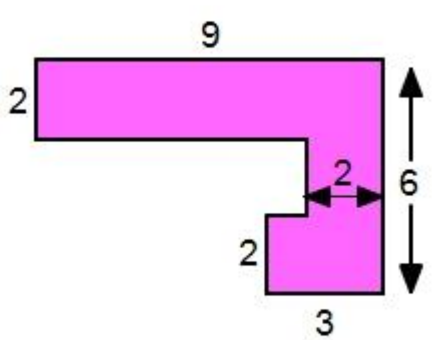
176 ft^2

10.



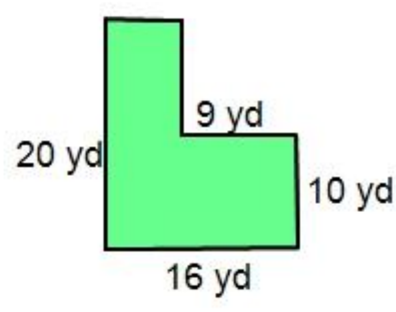
3,208 in^2

11.



28 square units

12.

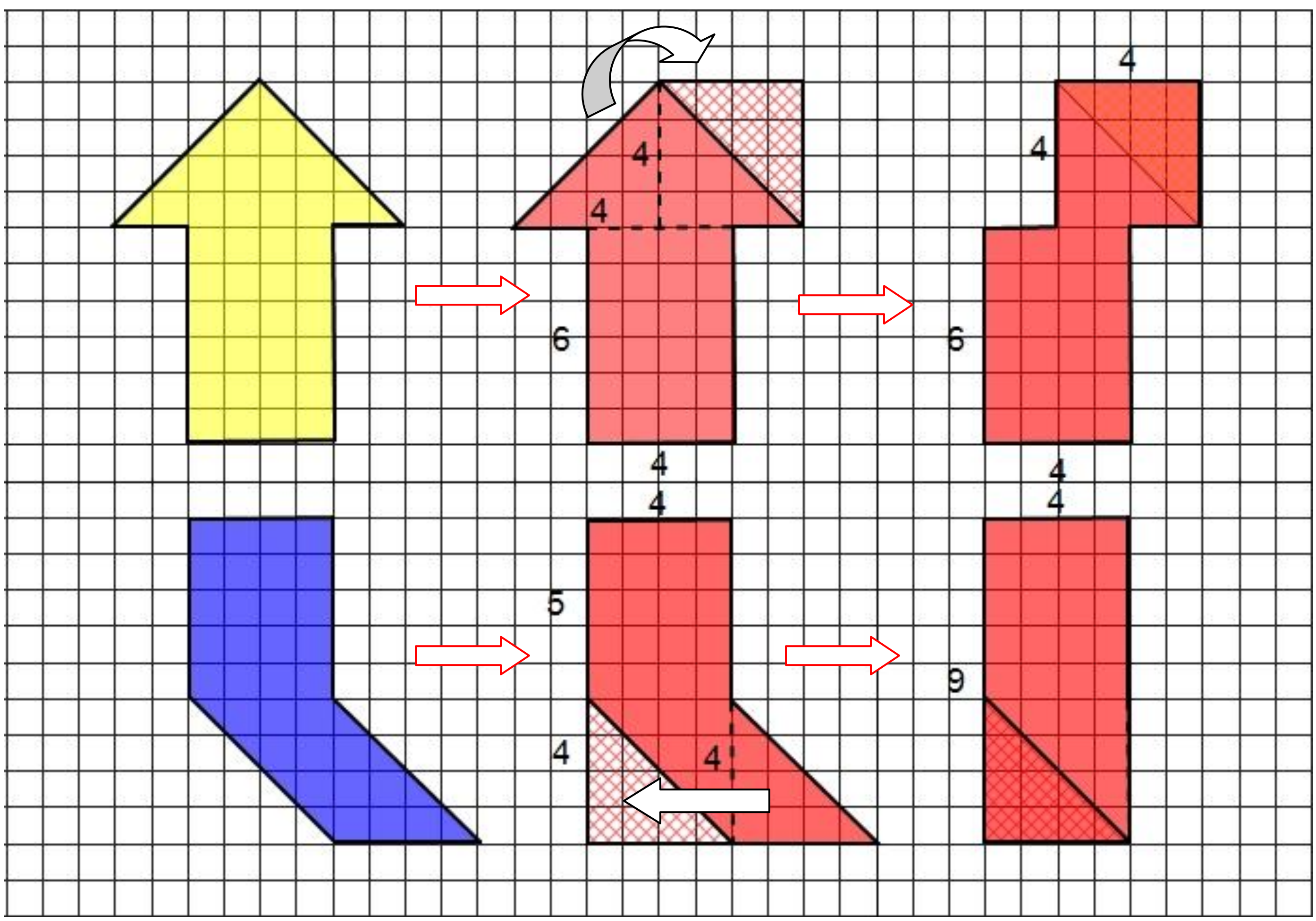


230 yds^2

13. The area of a flat screen TV is 864 square inches. The width of the TV is 36 inches. What is the height of the TV? Draw and label a picture to help you solve.
The height of the TV is 24 inches

14. Find the area of the shapes given below if each square represents one unit. Be ready to discuss your reasoning.

These problems are meant to act as a preface for the next few lessons. It is intended for students to decompose these figures and then recompose them into rectangles by transforming the triangular parts and not the formula for the area of a triangle. Discuss ideas for recomposing these figures into rectangles. Sample answers are given. If desired ask students to trace these figures onto graph paper and cut the shapes out. They can then manually decompose the figures by cutting and then recompose by sliding, rotating, and/or reflecting the pieces.

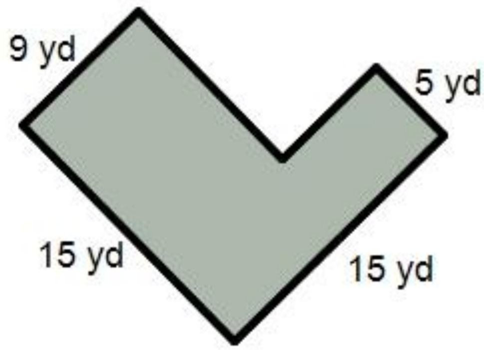


Area of Yellow Area is 40 square units
Area of Blue Figure is 36 square units

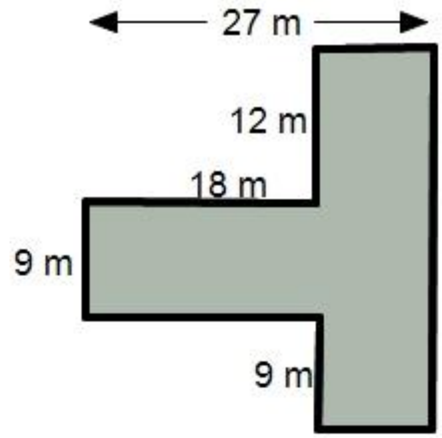
5.1a Homework: Finding Area

Directions: Find the area of each figure. Assume that all angles that appear to be right angles are right angles.

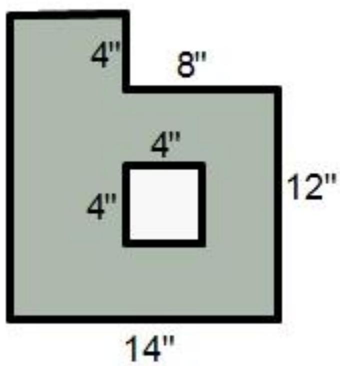
1.



2.

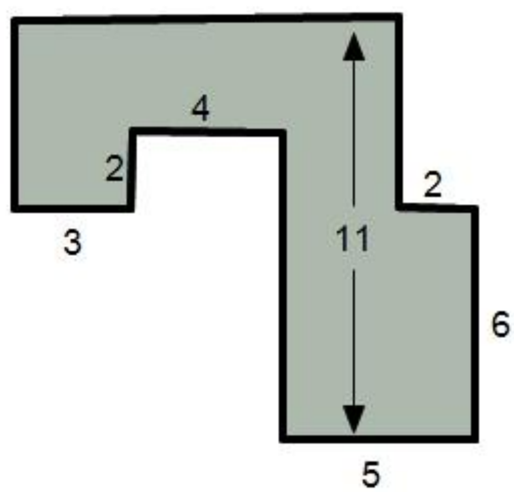


3.



176 in^2

4.



5. A painter is painting a wall in an office building. The wall measures 4 yards by 3.5 yards. The wall has two windows on it each that measure 1 yard by 1.5 yards.
   a. Draw a label a diagram that represents the wall.

   b. Find the amount of paint that is needed to do one coat of paint on the wall.

   c. One can of paint will cover 8 square yards. How many cans of paint does the painter need to paint the wall?

6. How many 4-inch square tiles are needed to cover a table that measures 24 inches by 40 inches? Draw and label a picture if needed.
   60 tiles are needed

7. Sandra is making a quilt that she wants to have an area of 51 square feet. The length of the quilt is 8.5 feet. What is the width of the quilt? Draw and label a picture if needed.

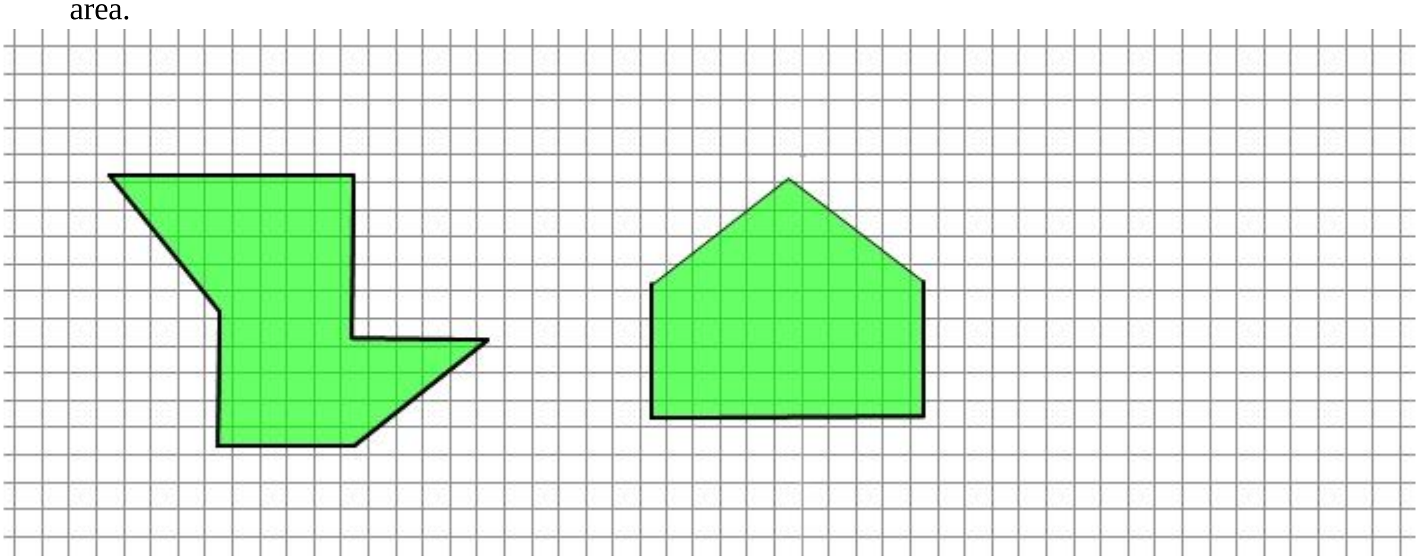
8. You are surrounding your backyard pool will patio pavers. The pavers come in two sizes, the 2 feet by 2 feet pavers costs $6 each and the 1 foot by 2 feet pavers cost $3 each. Your swimming pool measures 20 feet by 30 feet. You would like the patio to surround the pool extending 3 feet in all directions.
   a. Draw a diagram that represents the pool, patio, and the pavers.
      See student answer

   b. Find the total area that you want to cover in pavers?
      336 square feet

   c. Your brother claims that it will cost less to use the $6 pavers because they are bigger. Do you agree with him? Why or why not. What pavers do you think you should use?
      It does not matter which pavers you use. They cost the same per square foot.

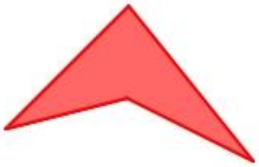
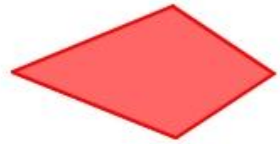
   d. How much money will is cost to buy the pavers?
      It will cost $504 for the pavers.

8. Prove that the figures below have the same area. Then sketch a figure of your own that has the same area.



5.1b Class Activity: Area of Parallelograms

1. Draw a picture of each quadrilateral under its name. Then write a sentence that describes the quadrilateral.

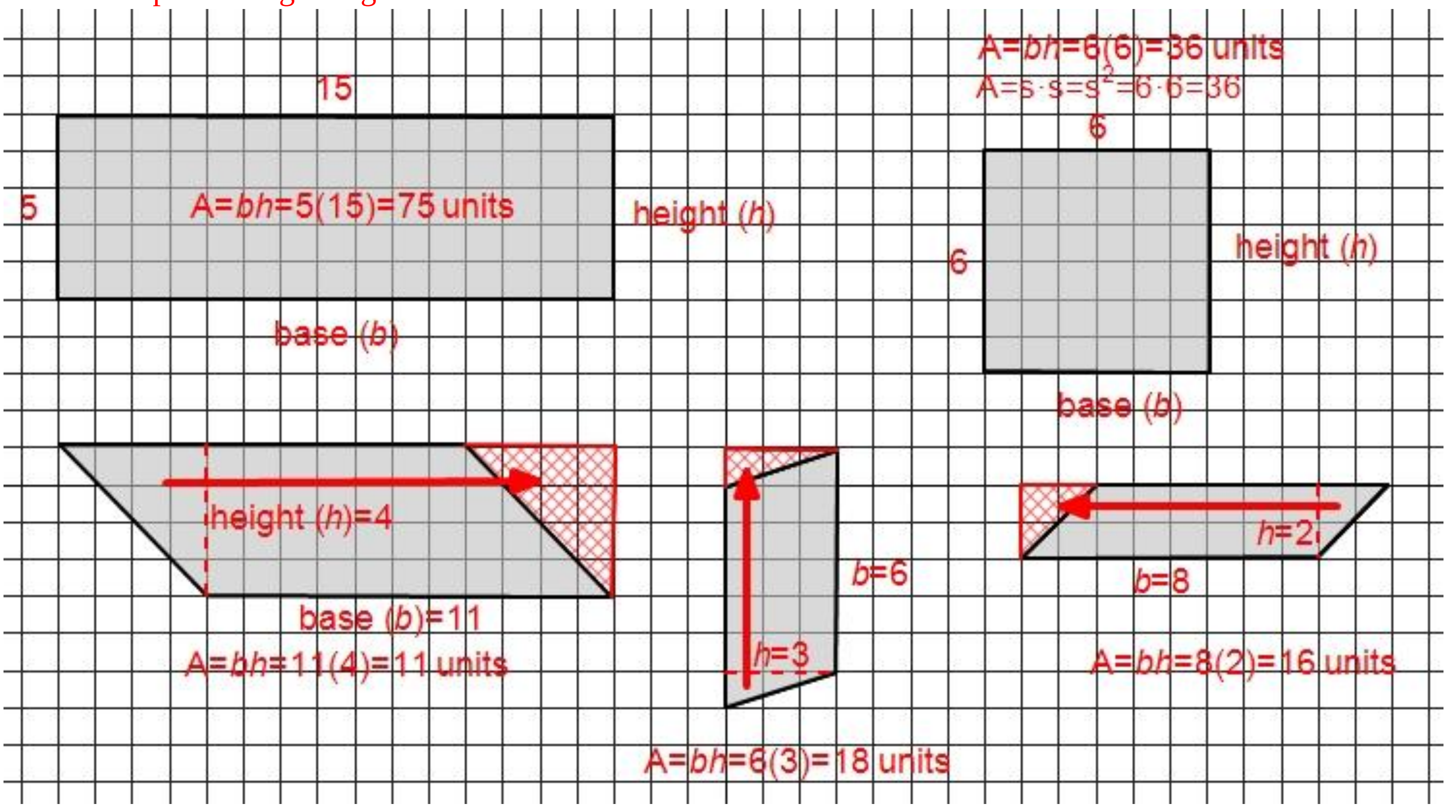
Quadrilateral:	Trapezoid:	Kite:
A polygon with four sides 	A quadrilateral with at least one pair of parallel side. 	A quadrilateral with two pairs of adjacent sides of the same length 

Parallelogram:	Rectangle:	Square:	Rhombus:
A quadrilateral with both pairs of opposite sides parallel 	A parallelogram for which all angles are right angles 	A rectangle with all sides of the same length 	A parallelogram with all sides of the same length 

As you review the different types of quadrilaterals, focus on the properties of parallelograms. You might ask questions like; Is every square a rectangle? Is every rectangle a square? Is every parallelogram a rhombus? Are all squares rhombi? Are all rhombi squares?

2. Find the area of each parallelogram below. Be ready to discuss how you found the area.

Encourage your student to articulate their methods for finding the area of the parallelograms especially the "slanted ones". If needed, provide several copies of the figures below and have students cut out the parallelograms and manually slice and move the pieces into a rectangle. Have them label the base and height; this will help them begin to generalize a formula.

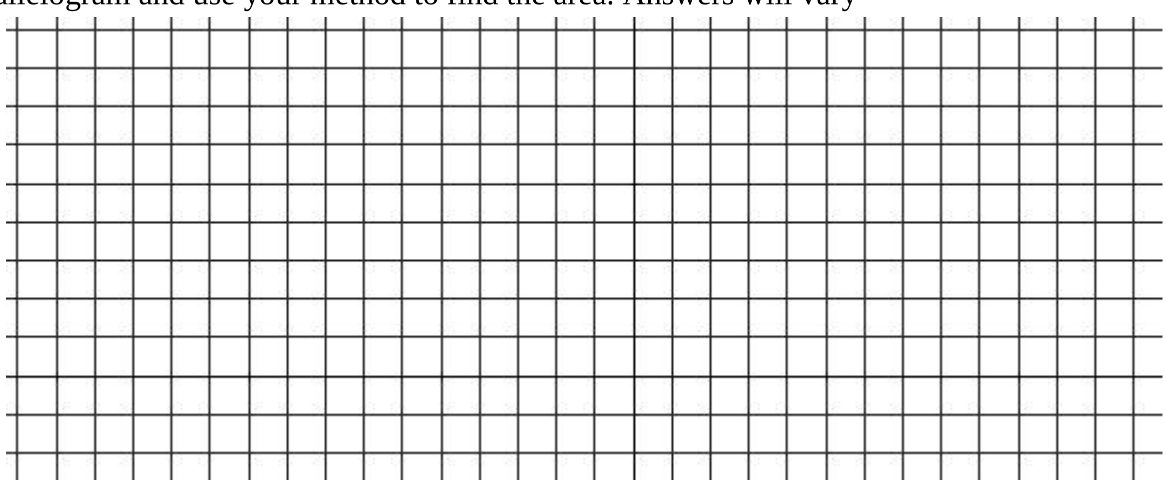


There are many online interactive tools that show how to find the area of any parallelogram. Consider using the following resources.

http://illuminations.nctm.org/Activity.aspx?id=4206 http://illuminations.nctm.org/Activity.aspx?id=4166
https://illuminations.nctm.org/Activity.aspx?id=3587 https://illuminations.nctm.org/Activity.aspx?id=3567
http://illuminations.nctm.org/Activity.aspx?id=6385

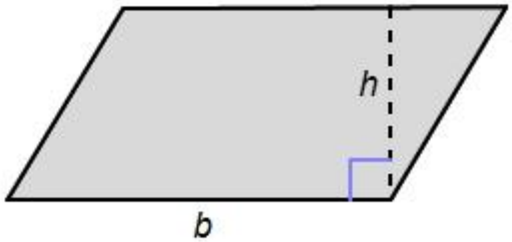
Discuss the formula for the area of a square and how it is the same as any parallelogram. However, sometimes it will be written as $\boldsymbol{A = s \cdot s = s^2}$ where s is the side length.

3. Will your method for finding the area work for any parallelogram? In the space provided draw any parallelogram and use your method to find the area. Answers will vary



The area of the parallelogram does not change by moving pieces around to make a rectangle. It may be helpful to cut a corner off of a rectangle and slide it to make a slanted parallelogram. This will help students see that the manipulation goes both ways and the area does not change. Develop a formula for the area of any parallelogram. Talk about the base and the height; be sure that students understand that the height is the perpendicular distance between the base and its opposite side.

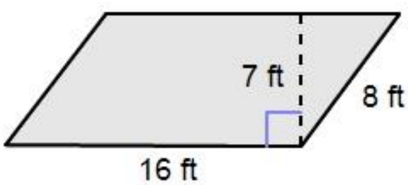
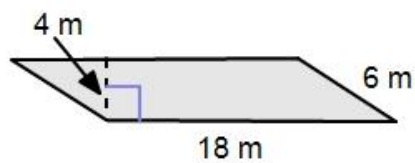
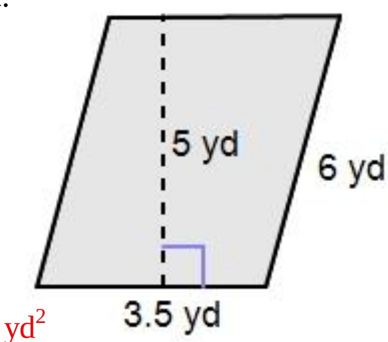
4. Describe in words and write a formula about how to the find area of any parallelogram.



The area of a parallelogram is found by multiplying the length of the base by the length of the height. The height is the perpendicular distance between the base and its opposite side.

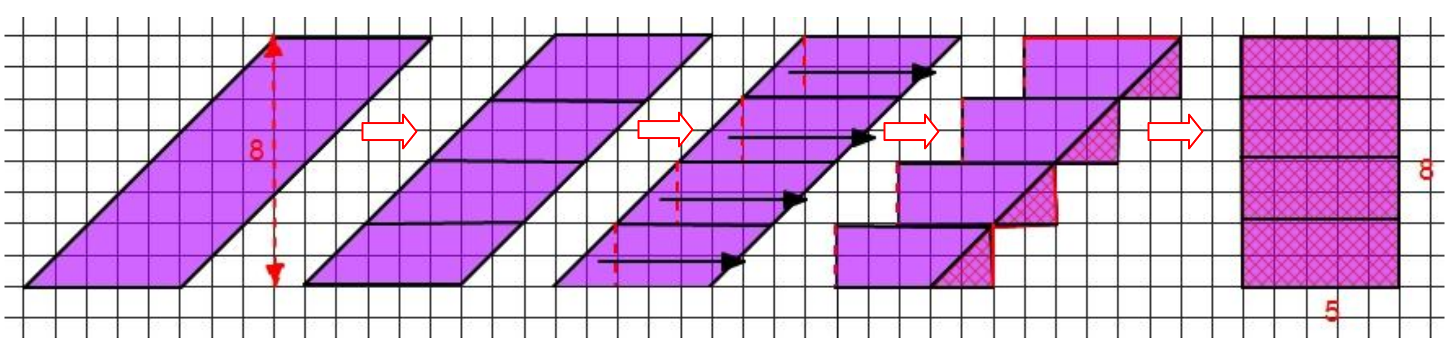
$$\boldsymbol{Area = base \cdot height = bh}$$

5. Find the area of each parallelogram.

a.	b.	c.
		
112 ft^2	72 m^2	17.5 yd^2

6. Find the area of the parallelogram below.

For this parallelogram students may struggle to find the height because they may not know that it can extend outside of the parallelogram.

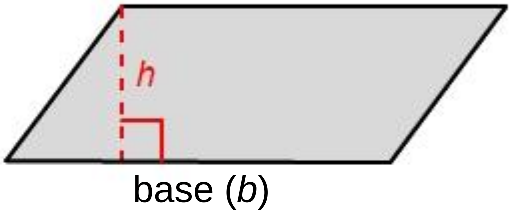


One method is shown above, where the parallelogram is split into 4 smaller parallelograms. Each of these parallelograms are transformed into rectangles by sliding the right triangles to the other side of each parallelogram. This forms 4 rectangles that can be translated together to form one rectangle. You can then see that the height of the rectangle is the perpendicular distance from the base of the parallelogram to the opposite side.

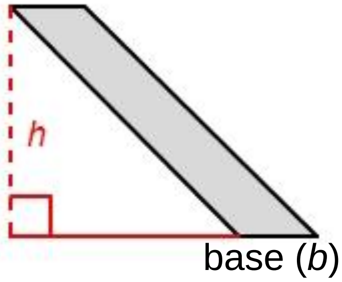
7. Draw and label the height for each parallelogram given the base.

Sample answers are given

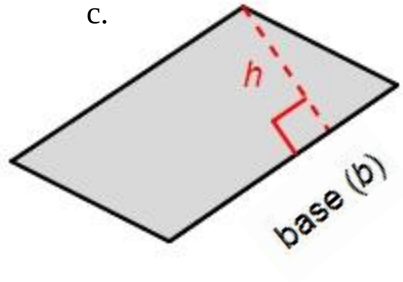
a.



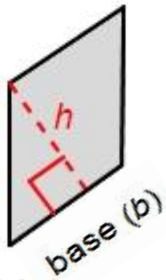
b.



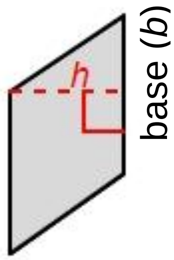
c.



d.

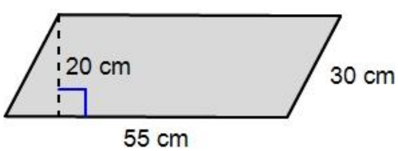


e.



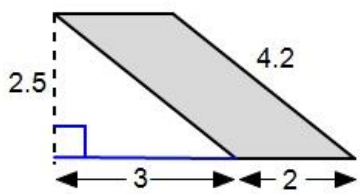
8. Find the area of each parallelogram.
*Figures are not drawn to scale

a.



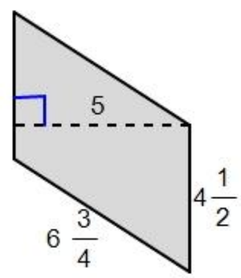
$1{,}100\ cm^2$

b.



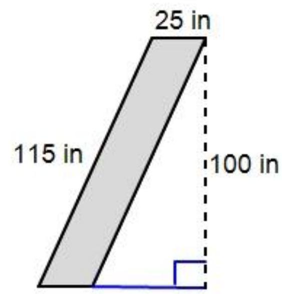
5 square units

c.



22.5 units

d.

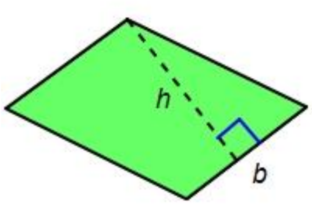


$2500\ in^2$

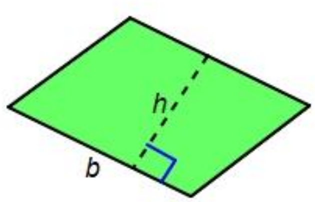
These problems provide opportunities to discuss the relationship between the height and the base.

9. Chris, Susan, Tyler, and Harper have all labeled the height and base for the parallelogram below. Are they all correct? If not, who is correct and why?

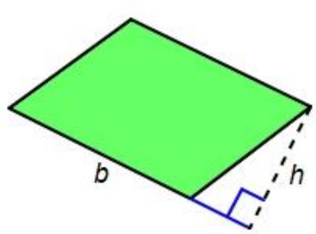
Chris



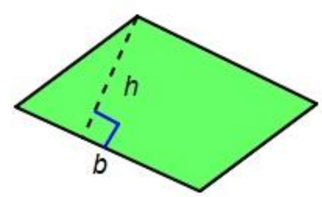
Susan



Tyler



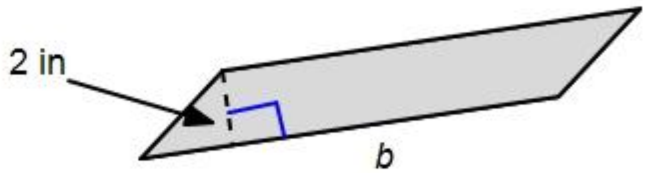
Harper



They are all correct; the heights they have labeled are the perpendicular distances from the base to the opposite side.

The height is usually drawn from a vertex that is opposite the base but does not have to be, as is the case with Susan above. You can choose any side of the parallelogram as the base and the height is found based off of the chosen base. Above Chris choose a different base than everybody else.

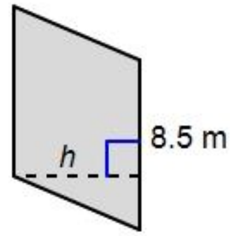
10. The area of the parallelogram below is 24 square inches, what is the length of the base?



For problems 10 and 11 use the formula to write an equation that relates the given dimension with the area. Then solve the equation for the unknown dimension.

The length of the base is 12 inches

11. The area of the parallelogram below is 59.5 m^2, what is the height?



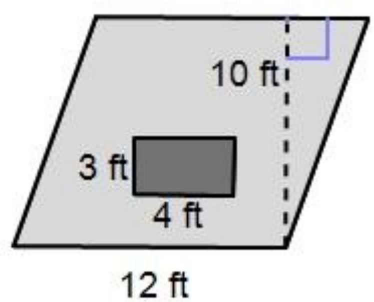
The height is 7 meters

12. A parallelogram has a base length of 4.5 centimeters and a height of 2.75 centimeters. What is the area of the parallelogram? If needed draw and label a picture.
The area is 12.375 cm^2

13. A staircase has 4 parallelogram shaped panels on the wall. The base of each panel is 2 ft and the height 1.5 ft. How much area do the 4 panels take up on the wall altogether?
The panels take up 12 square feet of space.

14. Olivia is drawing up blue print designs for the floor space in a tree house. Her drawing is shown; the rectangular hole in the floor of the tree house is the opening to climb down the ladder. How much floor space does the tree house have?

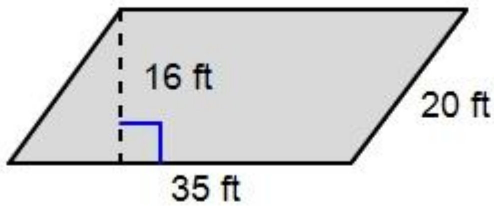
The floor space takes up 108 ft^2



5.1b Homework: Area of Parallelograms

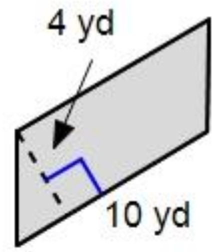
Find the area of each parallelogram
*Figures are not drawn to scale

1.

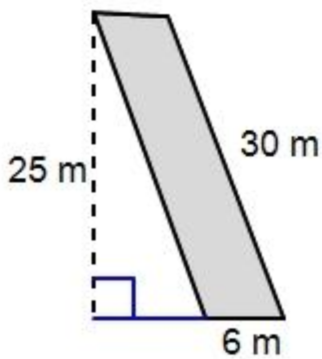


560 ft^2

2.

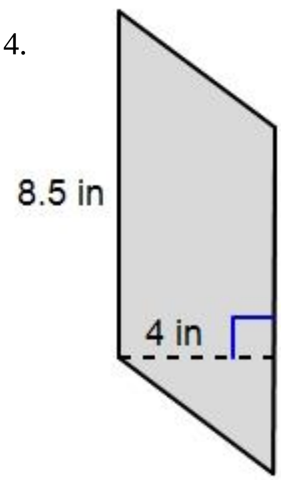


3.



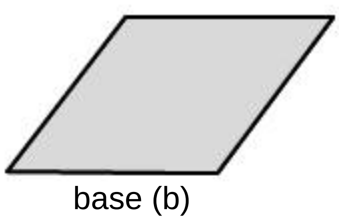
150 m^2

4.

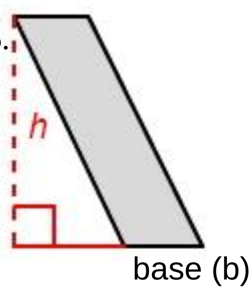


For each parallelogram given below the base is given, find and label the height.
*Figures are not drawn to scale

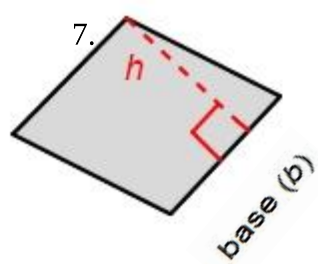
5.



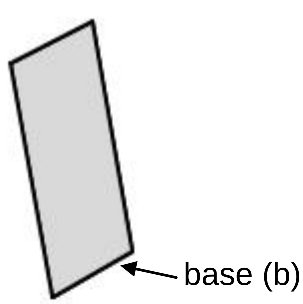
6.



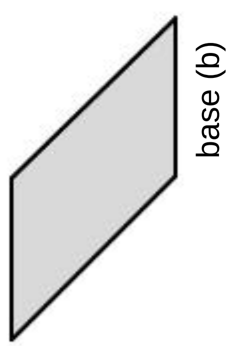
7.



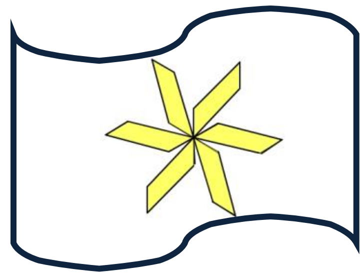
8.



9.



10. You are making a design for a flag. The design is created by rotating a parallelogram around a fixed point 6 times as shown. Find the total area that the design will take up on the flag if the parallelogram has a height of 3 inches and a base length of 8 inches.
The total area that the design covers on the flag is 144 square inches.



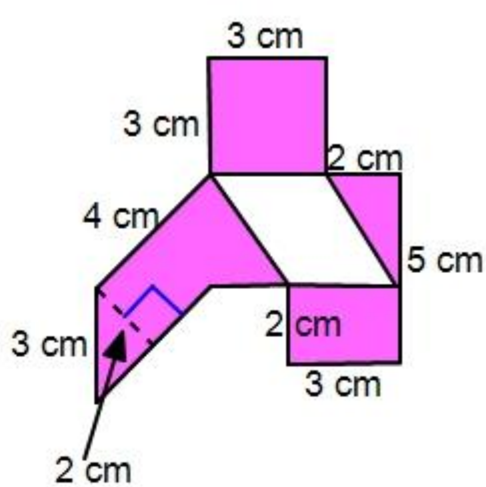
11. Bonnie is making a sign that is a parallelogram. Its base measures 14 feet and its height is 10 feet. She cuts a rectangular piece out of the middle of the sign that measures 6 feet by 2 feet.
    a. Draw and label a picture of Bonnie's sign.

    b. Find the total area of the sign after she cuts out the rectangle.

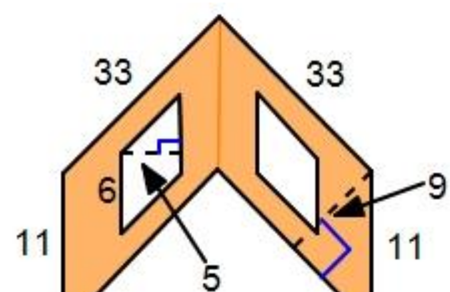
12. A tile that is the shape of a parallelogram has a base of 5 inches and a total area of 17.5 in^2. What is the height of the tile?

13. Find the area of each shaded region. Assume that angles that appear to be right angles are right angles.

a.



b. The two parallelograms inside the figure are the same size.

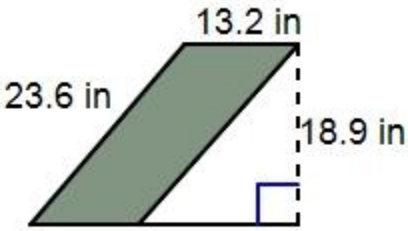


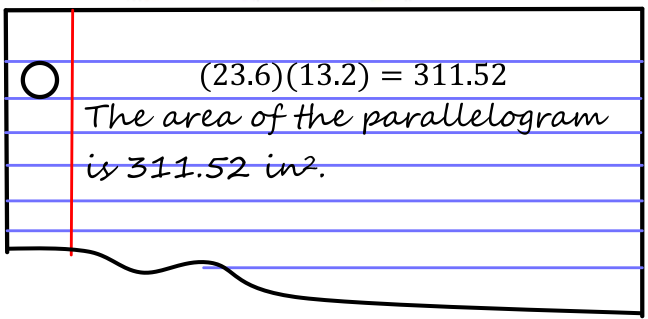
534 square units

Find, Fix, and Justify

14. Zara has made a mistake on the two problems below, her work is shown. Find her mistake, explain what she did wrong, and then solve the problem correctly.

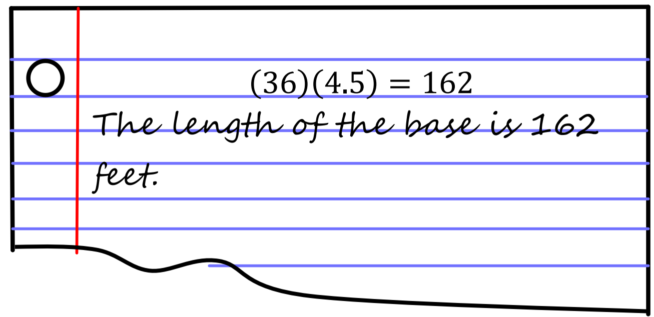
Find the area of the parallelogram below.



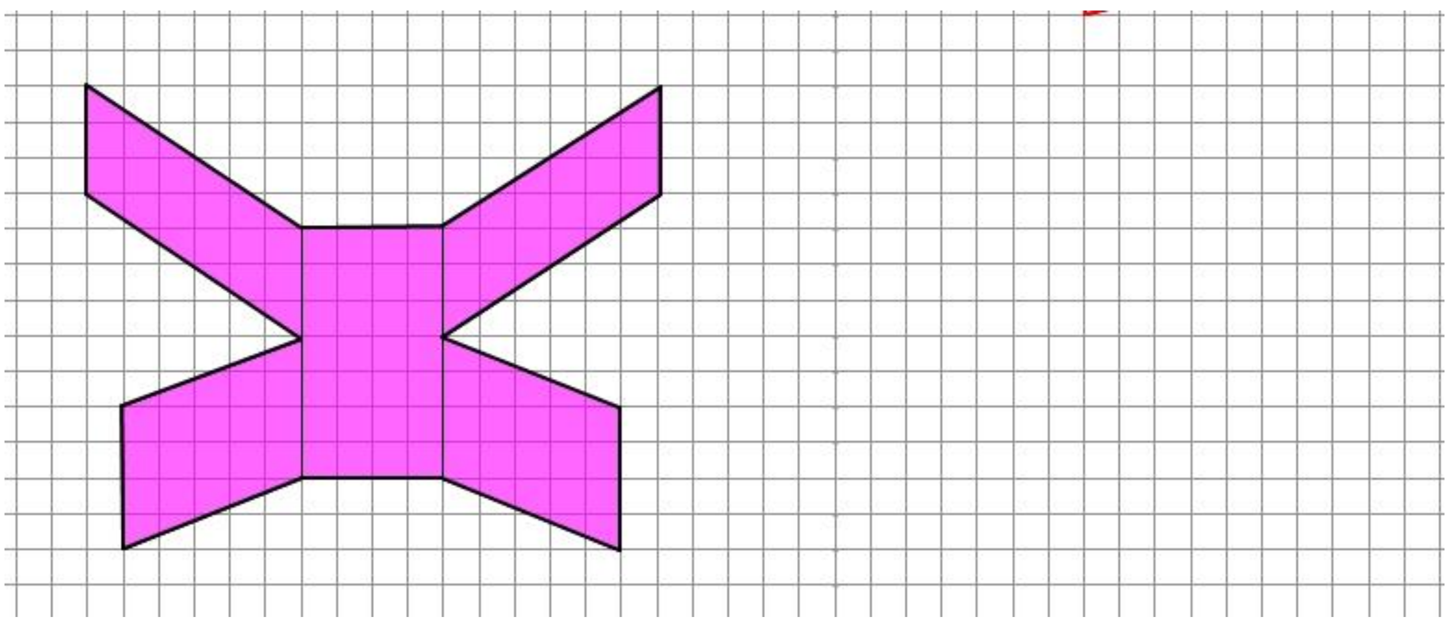


Zara multiplied the slant height by the length of the base. She should have multiplied13.2 by 18.9. This gives an area of 249.48 in^2.

The area of a parallelogram is 36 square feet, the height is 4.5 feet. What is the length of the base of the parallelogram?

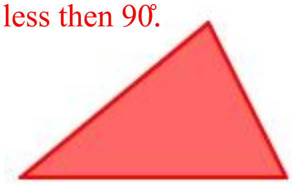
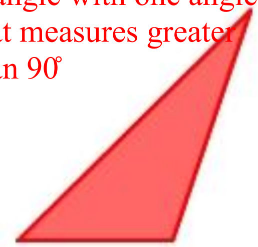
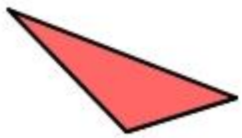
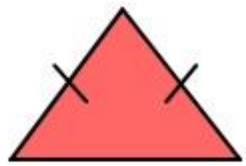
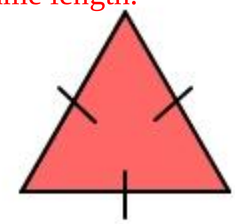


15. Find the area of the figure below and then draw a figure that has the same area.



5.1c Class Activity: Area of Triangles

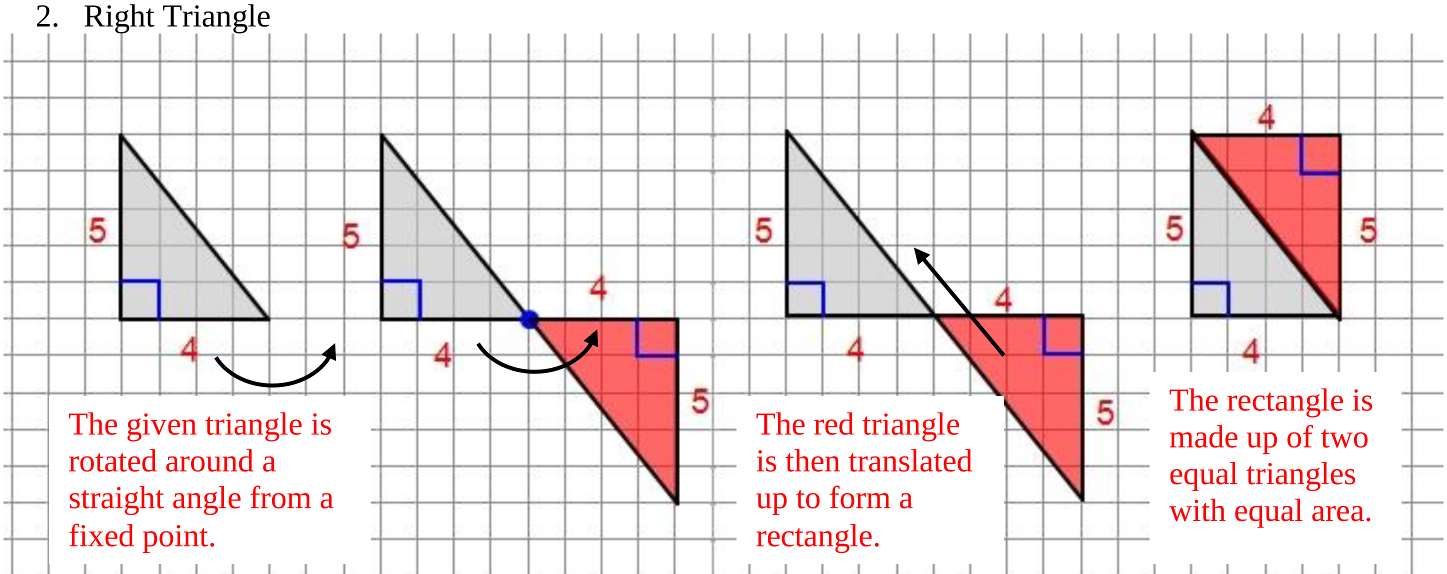
1. Draw a picture of each triangle under its name. Then write a sentence that describes the triangle.

Triangle: A polygon with three sides 	Right Triangle: A triangle with one angle that is a right angle (measures 90°) 	Acute Triangle: A triangle with all angles that measure less then 90°. 	Obtuse Triangle: A triangle with one angle that measures greater than 90° 
	Scalene: A triangle where all three sides are of different lengths. 	Isosceles: A triangle with two or more sides the same length. 	Equilateral: A triangle with all three sides the same length. 

Directions: Find the area of each triangle if each square represents one square unit. Be ready to discuss how you found the area.

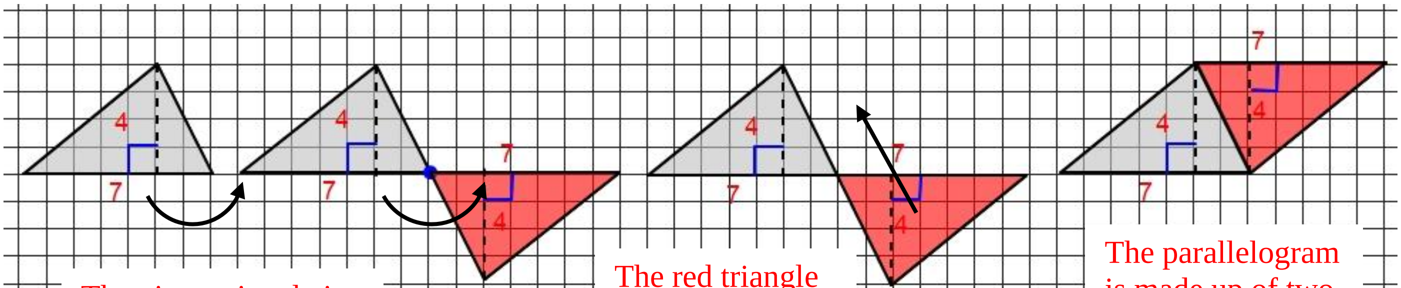
Encourage your student to articulate their methods for finding the area of the triangles. If needed provide several copies of the figures below and have them cut out the triangles and manually slice and move the pieces around. Have them label the base and height; this will help them begin to generalize a formula. One method is shown below.

2. Right Triangle



The area of the original triangle is half the area of the rectangle. The area of the rectangle is $5(4) = 20$ units. Thus the area of the original triangle is $20 \div 10 = 2$ units.

3. Acute Triangle



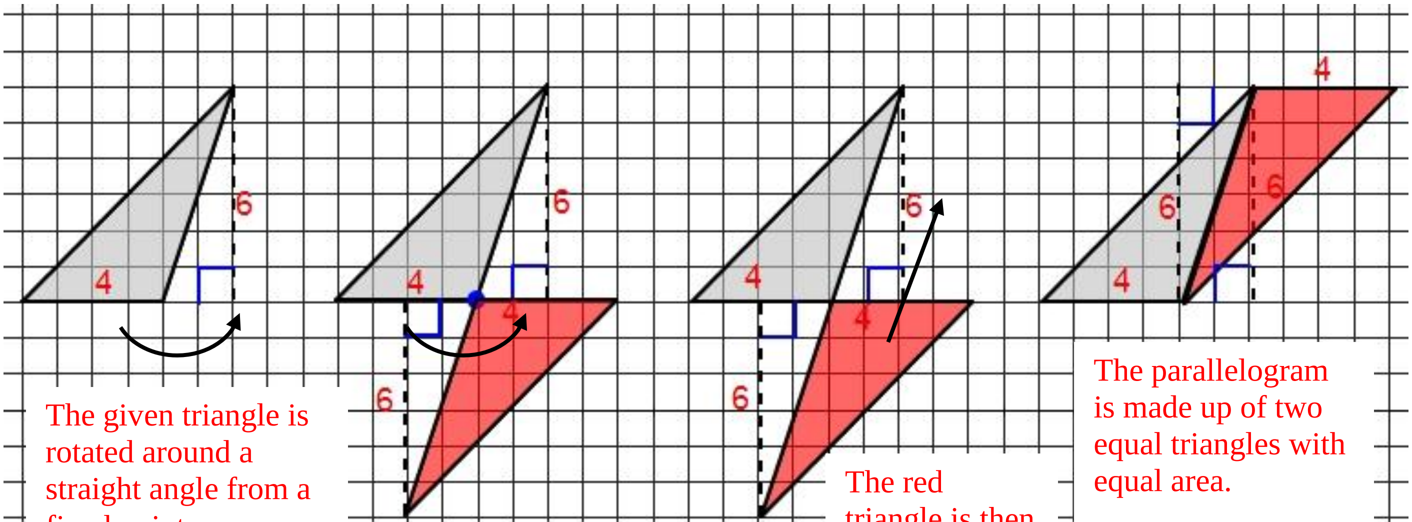
The given triangle is rotated around a straight angle from a fixed point.

The red triangle is then translated up to form a parallelogram.

The parallelogram is made up of two equal triangles with equal area.

The area of the original triangle is half the area of the parallelogram. The area of the parallelogram is $7(4) = 28$ square units. Thus the area of the original triangle is $28 \div 2 = 14$ square units.

4. Obtuse Triangle



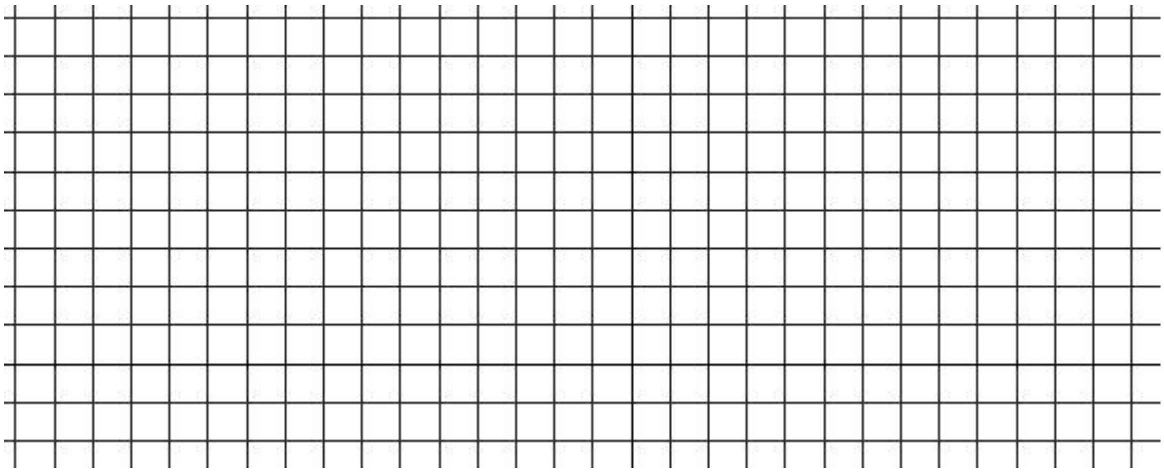
The given triangle is rotated around a straight angle from a fixed point.

The red triangle is then translated up to form a parallelogram.

The parallelogram is made up of two equal triangles with equal area.

The area of the original triangle is half the area of the parallelogram. The area of the parallelogram is $6(4) = 24$ square units. Thus the area of the original triangle is 12 square units.

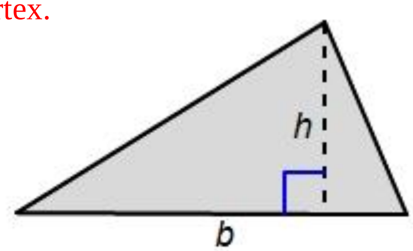
5. Will your method for finding the area work for any triangle? In the space provided draw any triangle and use your method to find the area. Answers will vary



Discuss why the parallelogram that is formed from the rotation and translation of the original triangle is double the area of the original triangle. Thus the area of the triangle will be half the area of the parallelogram. Develop a formula for the area of any triangle together as a class. Talk about how the base and height of the parallelogram is the exact same as the base and height of the original triangle. Since the formula for a parallelogram is $\boldsymbol{A = bh}$ and the triangle is half of this area, then the formula for the area of a triangle is $\boldsymbol{A = \frac{1}{2}bh}$ or $\boldsymbol{= \frac{bh}{2}}$. Talk about why both notations of this formula will yield the same result, i.e., multiplying by ½ is the same as dividing by 2. You may have to show some examples of how and why $\boldsymbol{A = \frac{1}{2}bh}$ and $\boldsymbol{A = \frac{bh}{2}}$ are the same formula.

6. Describe in words and write a formula about how to find the area of any triangle. n#

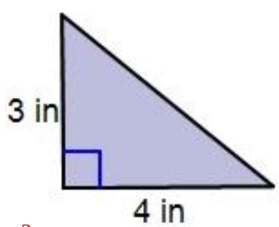
The area of any triangle is half of its base length times its height length. The height is the perpendicular distance between the base and its opposing vertex.



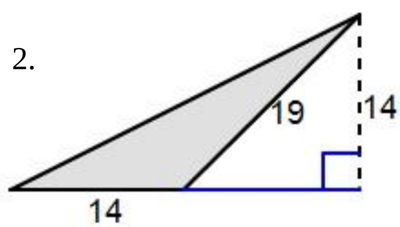
$$\boldsymbol{A = \frac{1}{2}bh} \text{ or } \boldsymbol{A = \frac{bh}{2}}$$

7. Find the area of each triangle.

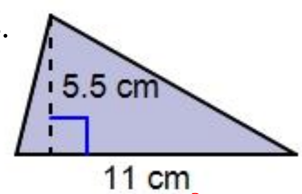
*Figures are not drawn to scale

1.


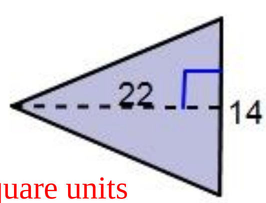
6 in^2

2.


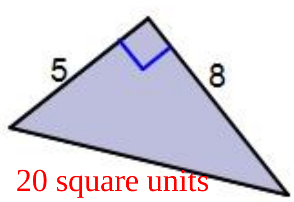
98 square units

3.


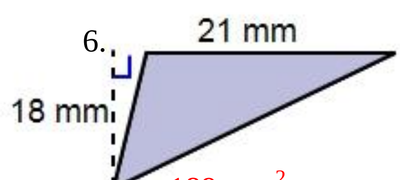
30.25 cm^2

4.


154 square units

5.


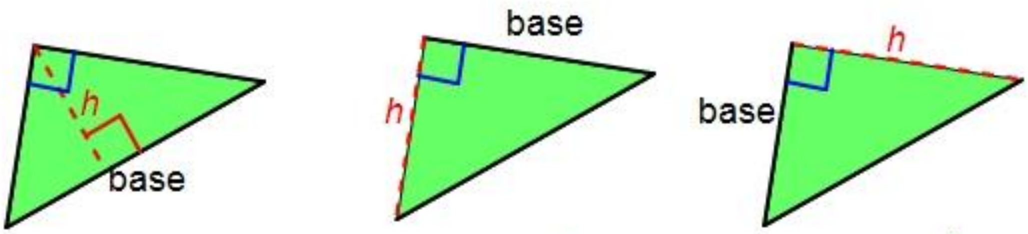
20 square units

6.


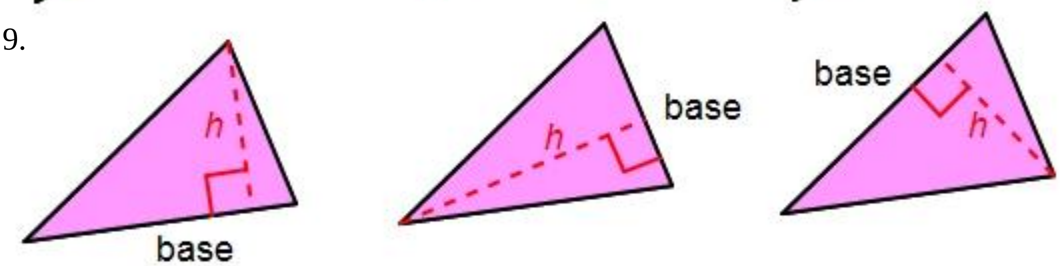
189 mm^2

The same triangle is given three times. Draw and label the height from the given base.

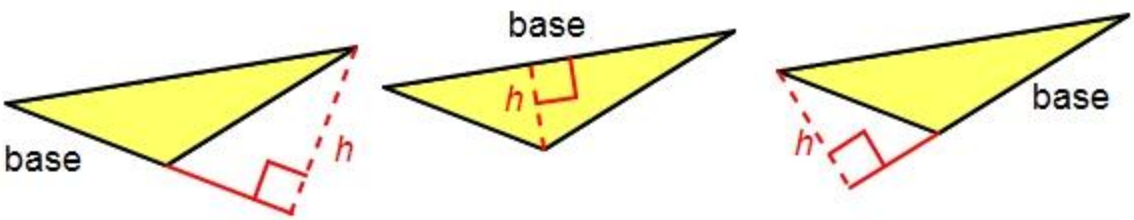
8.



9.

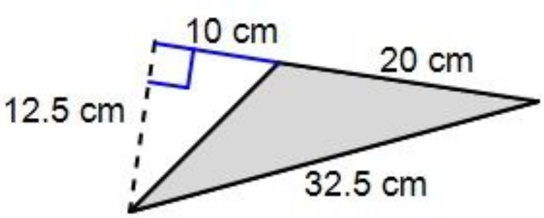


10.



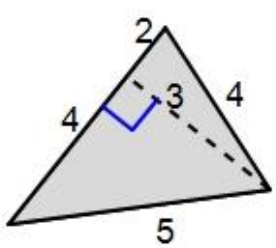
Directions: Find the area of each figure
*Figures are not drawn to scale

11.



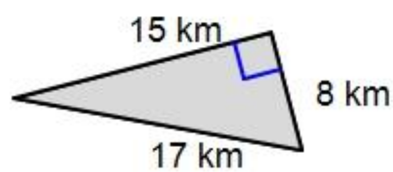
125 cm^2

12.



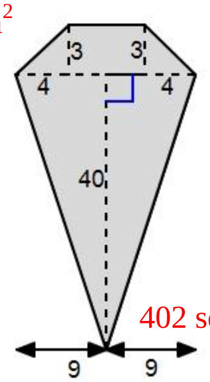
9 square units

13.



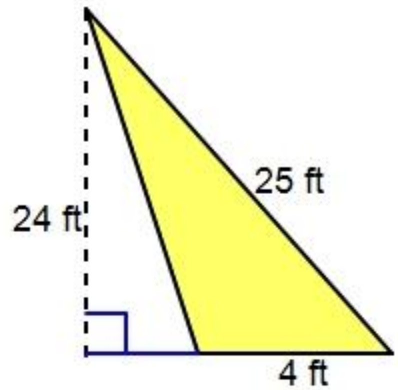
60 km^2

14.

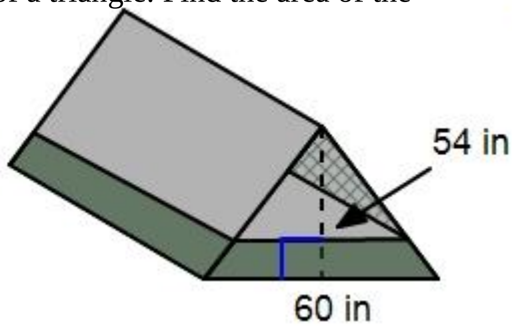


402 square units

15. Olivia is replacing the material for the sail on her sail boat. The dimensions of the sail are given. How much money will Olivia spend on replacing the material if it costs $4 per square foot?
It will cost Olivia $192 to replace the sail.

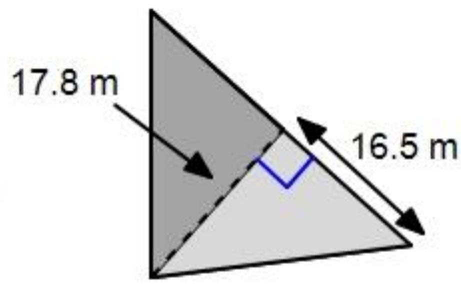


16. The door of a tent is the shape of a triangle. Find the area of the door.



The door's area is 1620 in^2

17. A large triangle is made up of two small triangles as shown. The area of the small dark grey triangle is 124.6 m^2. Find the area of the large triangle.
The area of the large triangle is 271.45 m^2

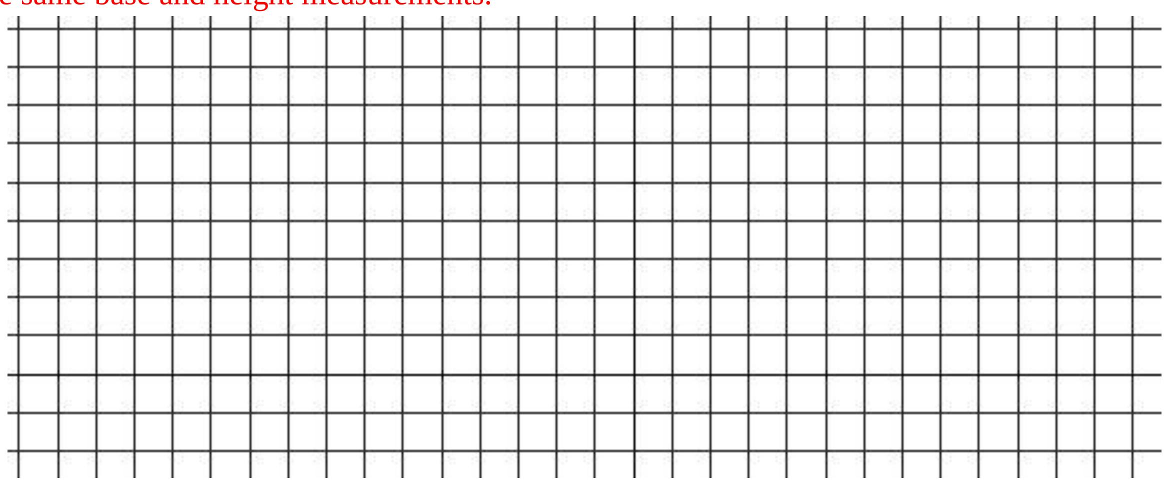


18. The area of a triangle is 23 yd^2. Its base measures 8 yards. What is the height of the triangle?
The height is 5.75 yards

n# For problems 17-19 encourage students to use the formula to write an equation that relates the given dimension with the area. They can then solve the equation for the unknown dimension.

19. The area of a triangle is 72 ft^2. Its height measures 12 ft. What is the length of the triangle's base?
The base is 12 feet long

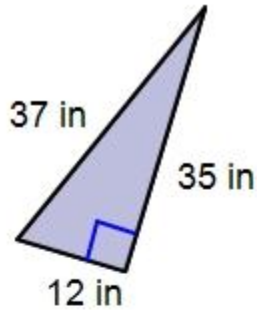
20. Create a right triangle, an acute triangle, and an obtuse triangle that all have the same area. Explain how you know that their areas are the same. Answers will vary; one option is for all of the triangles to have the same base and height measurements.



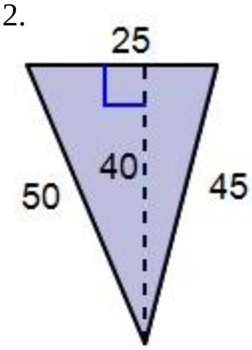
5.1c Homework: Area of Triangles

Directions: Find the area of each triangle

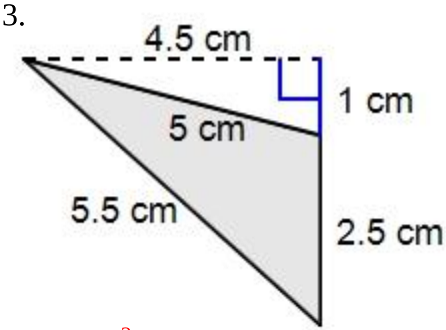
1.



2.

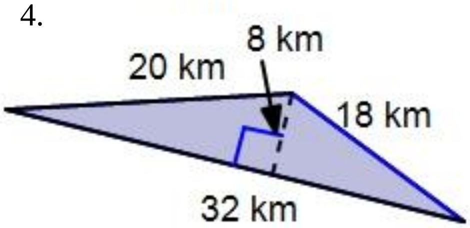


3.

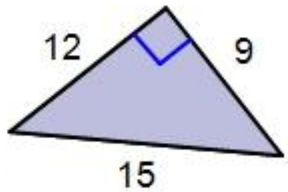


5.625 cm^2

4.

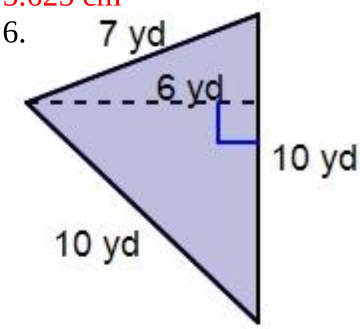


5.



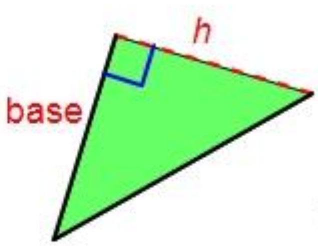
54 square units

6.



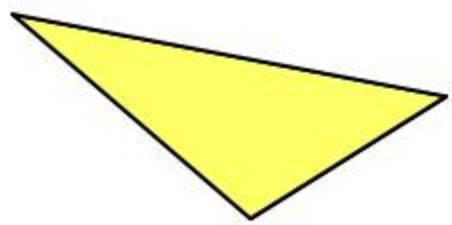
For each triangle below label the base, then draw and label the height based off of your chosen base.

7.



8.

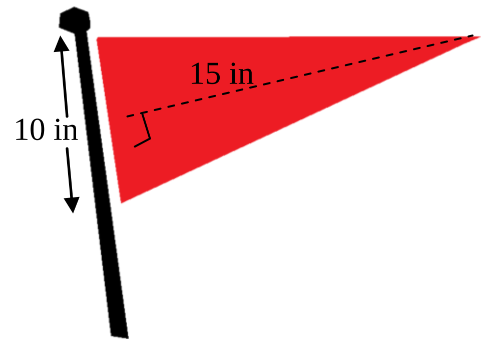
9.



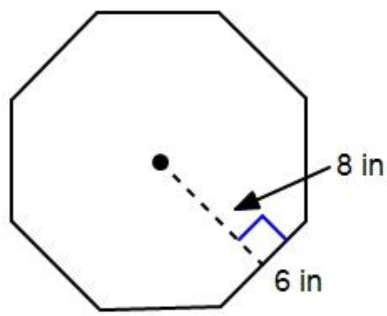
10. The wingspan of a triangular kite is 26 inches at its base, the height is 14 inches.
    a. Draw a label a picture of the kite.

    b. Find the area of the kite?

11. Find the area of the flag.

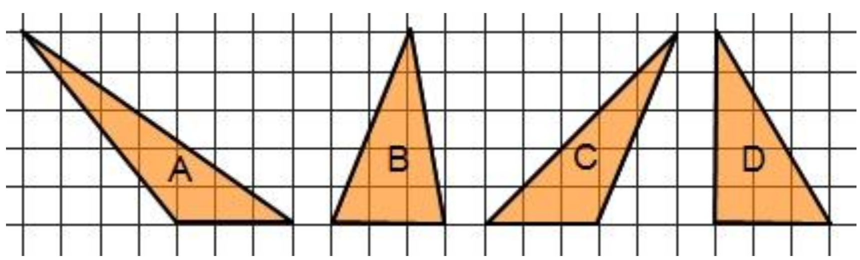


12. Find the area of the regular octagon below. A regular octagon has all sides that are equal. Review with students that all sides and angles are equal in a regular polygon.



192 in^2

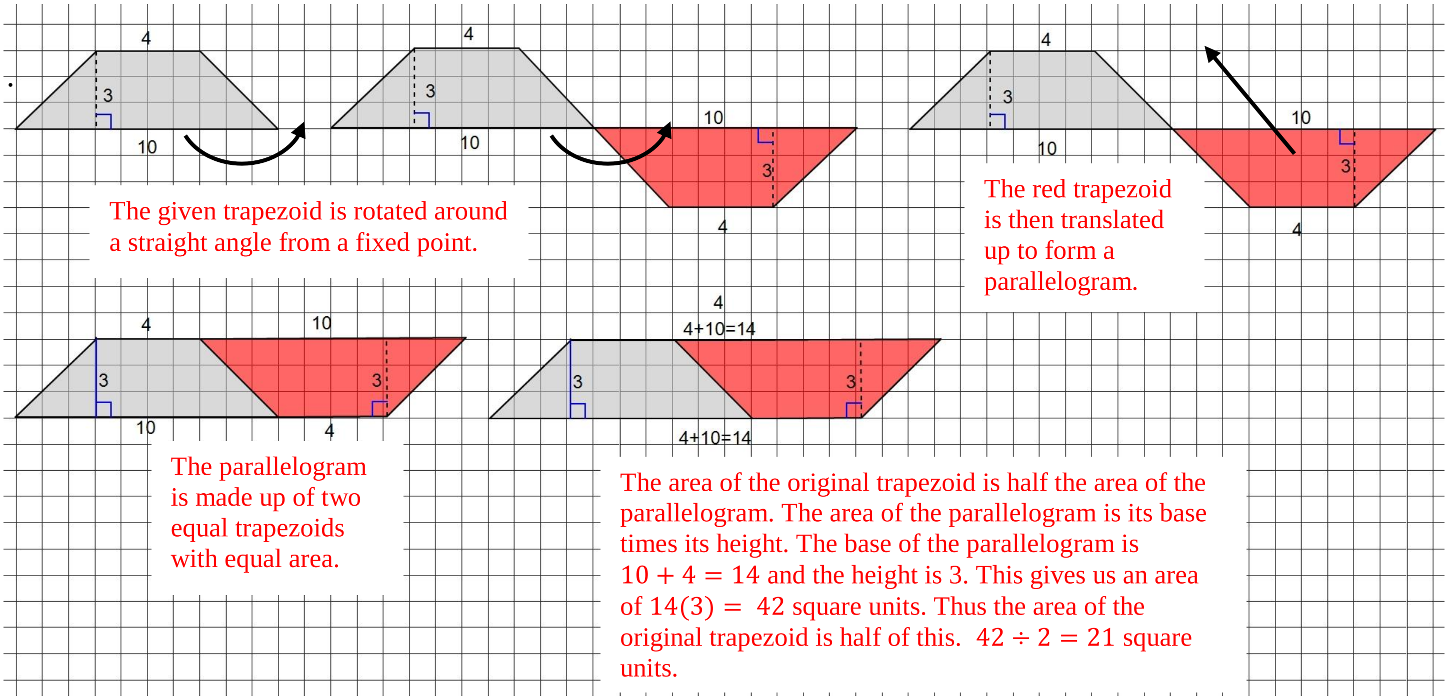
13. Without doing any calculations determine which triangle below has the largest area if each square represents one square unit? Explain how you know.



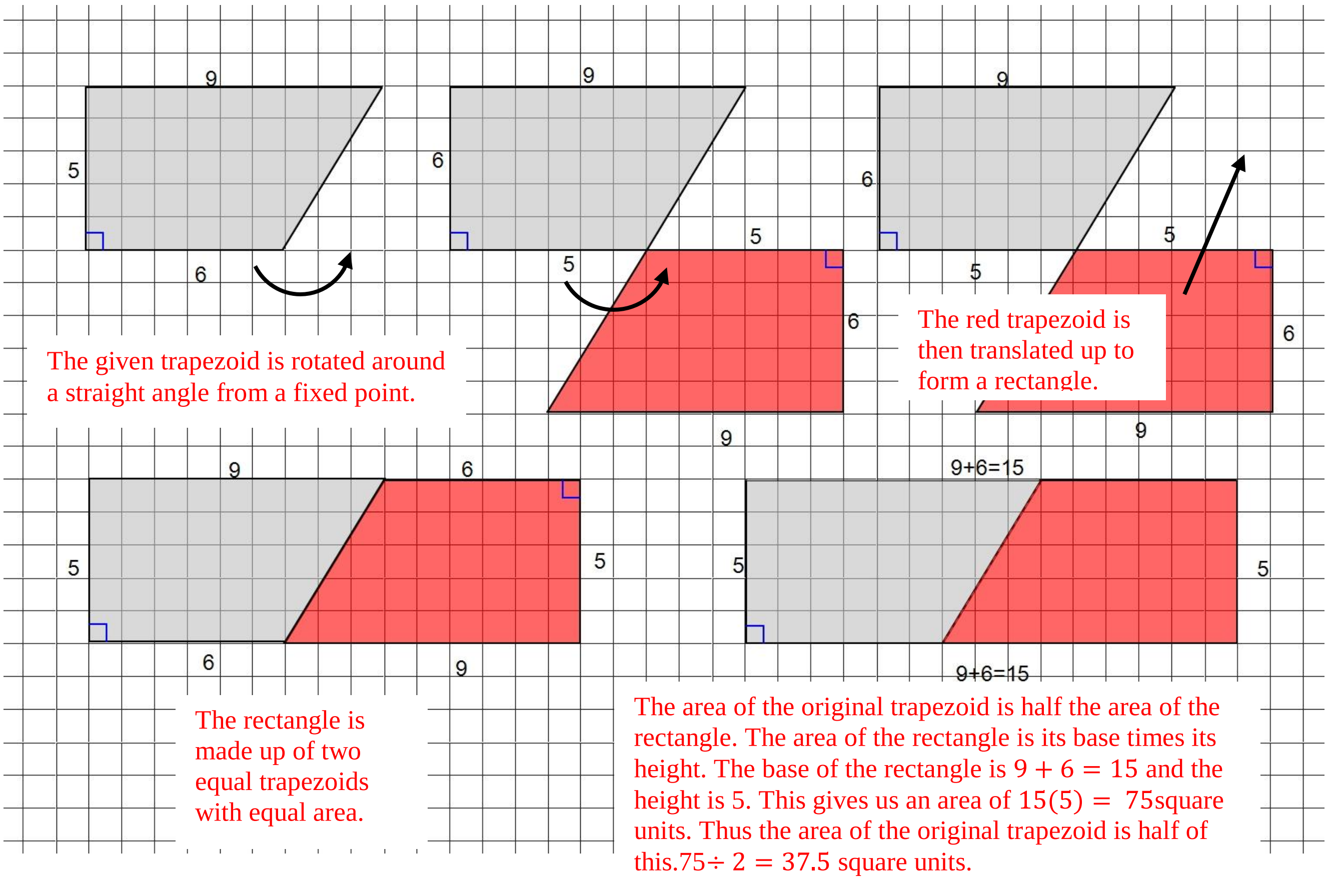
5.1d Class Activity: Area of Trapezoids

1. Find the area of each trapezoid if each square represents one square unit. Be ready to discuss how you found the area.

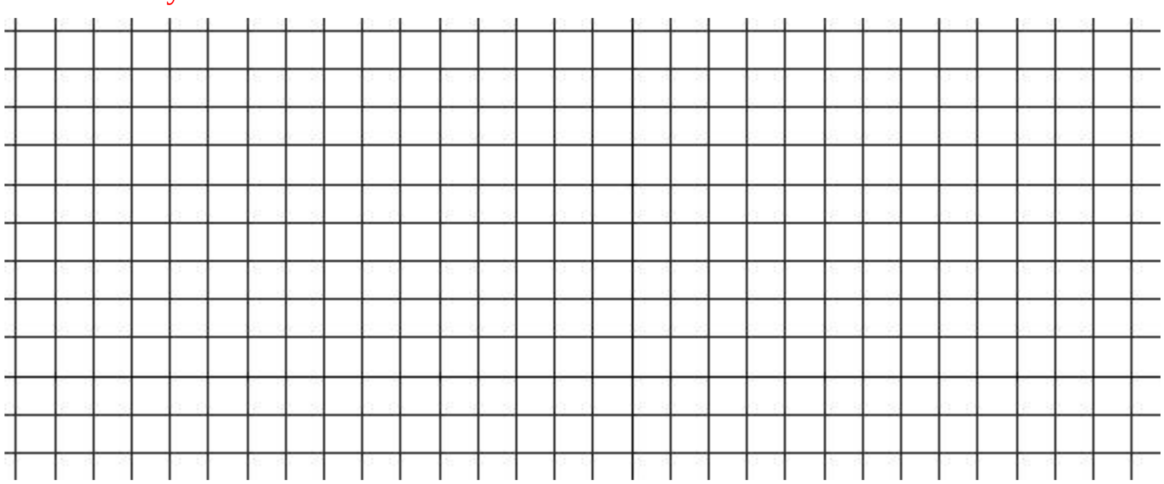
Encourage your student to articulate their methods for finding the area of the trapezoids. If needed, provide several copies of the figures below and have students cut out the trapezoids and manually slice and move the pieces around. Have them label the bases and height; this will help them begin to generalize a formula. Summarize their methods together as a class. There are many methods for finding the area of a trapezoid and deriving a formula.



2.

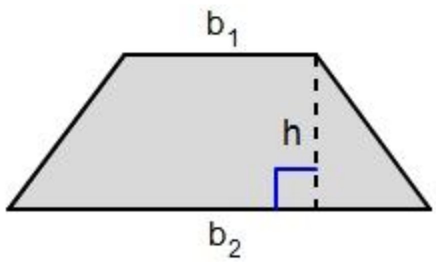


3. Will your method for finding the area work for any trapezoid? In the space provided, draw any trapezoid and use your method to find the area.
Answers will vary



n# Discuss why the parallelogram that is formed from the rotation and translation of the original trapezoid is double the area of the original trapezoid. Thus the area of the trapezoid will be half the area of the parallelogram. Develop a formula for the area of any trapezoid together as a class. Talk about how the height of the parallelogram is the exact same as the height of the original trapezoid and the base of the parallelogram is the sum of the two bases of the original trapezoid. Since the formula for a parallelogram is $\boldsymbol{A = bh}$ and the trapezoid is half of this area, then the formula for the area of a trapezoid is $\boldsymbol{A = \frac{1}{2}(b_1 + b_2)h}$ or $= \frac{(\boldsymbol{b_1+b_2})\boldsymbol{h}}{\boldsymbol{2}}$. Remind students about the role that the parentheses play in this formula; you must add the base lengths together before you multiply by the height or divided by 2.

4. Describe in words and write a formula about how to find the area of any trapezoid.
   The area of any trapezoid is half of the sum of its two bases times its height.

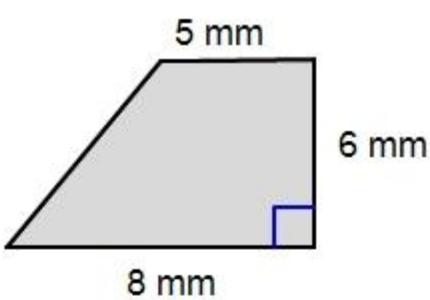


$$\boldsymbol{A = \frac{1}{2}(b_1 + b_2)h}$$

5. Directions: Find the area of each trapezoid.

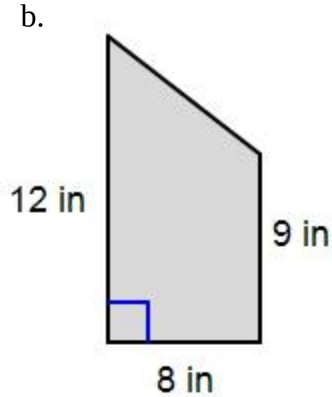
*Figures are not drawn to scale.

a.



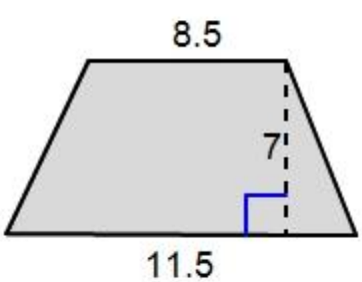
39 mm^2

b.



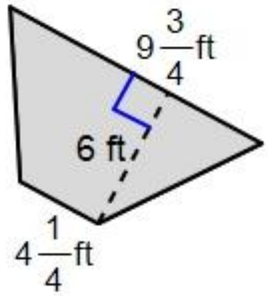
84 in^2

c.



70 square units

d.



42 ft^2

Find the area of each trapezoid. If needed draw and label a picture.

6. A trapezoid with base lengths of 4 meters and 8 meters and a height of 2 meters.

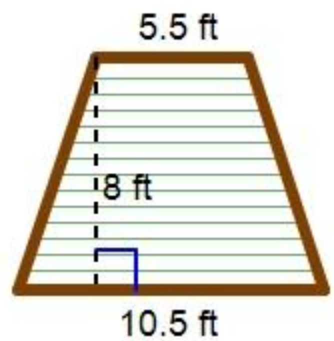
   12 square meters

7. A trapezoid with base lengths of 12 inches and 6 inches and a height of 3 inches.

   27 square inches

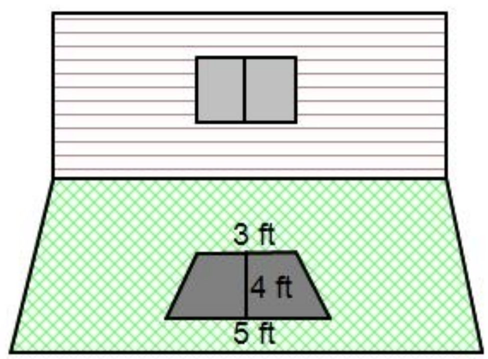
8. A garden plot is the shape of a trapezoid as shown. Find the total area that the garden plot takes up.

   The area of the garden plot is 64 square feet.



9. A window casts a shadow on the ground that is the shape of a trapezoid. Find the area that the shadow takes up.

   The area of the shadow is 16 square feet.



10. The area of a trapezoid is 600 square inches. One base of the trapezoid measures 10 inches and the other base measures 15 inches. Find the height of the trapezoid. Draw a label a picture if needed.

Encourage students to use the formula to write an equation that relates the given dimension with the area. They can then solve the equation for the unknown dimension.

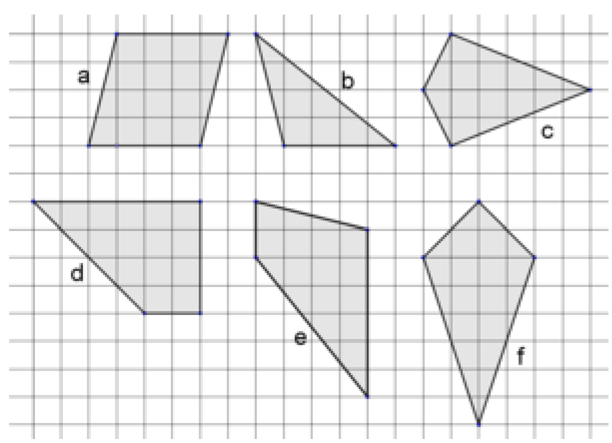
The height of the trapezoid is 48 inches.

11. A traffic sign is the shape of a trapezoid; it has a total area of 7 square feet. The height of the sign is 2 feet. Find two possible values for the measurements of each base. Draw and label a picture to help you find the base measurements.

For this problem students must reasoning abstractly as they think about the relationship between the height, bases, and area of a trapezoid. It may be helpful for them to use the formula for the trapezoid and solve for the unknown base values.
Answers will vary

The values for the base measurements can be any two numbers that have a sum of 7.

12. Of the polygons shown, which have equal areas? Explain how you know.
Polygons a, d, e, and f each has an area of 16 square units.
(The area of polygon b is 8 square units, and the area of polygon c is 12 square units.)

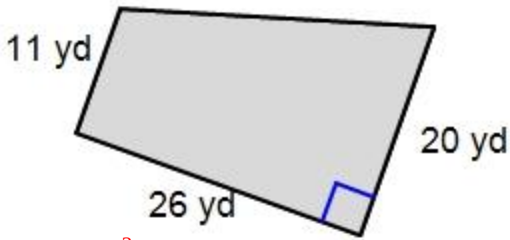


This is an Illustrative Mathematics Task

5.1d Homework: Area of Trapezoids

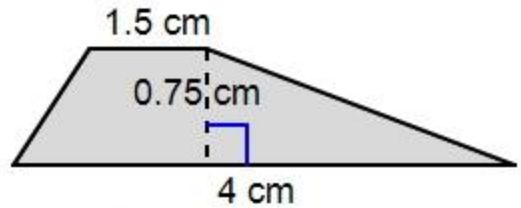
Directions: Find the area of each trapezoid.
*Figures are not drawn to scale.

1.

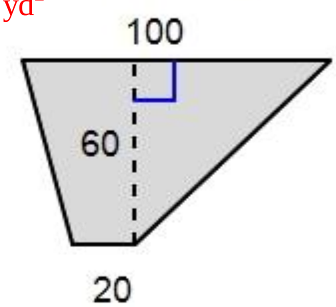


404 yd^2

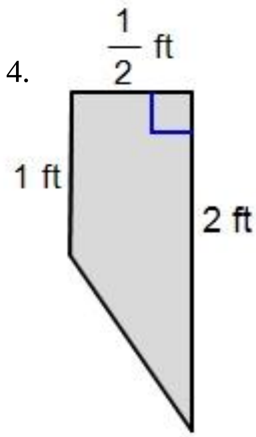
2.



3.



4.



0.75 ft^2

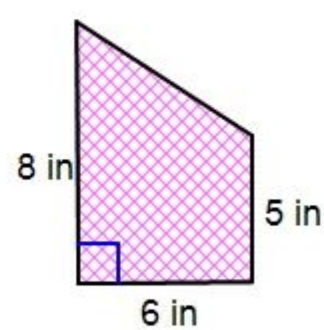
Find the area of each trapezoid, if needed draw and label a picture.

5. A trapezoid with base lengths of 14.8 yards and 20.3 yards and a height of 23.5 yards.

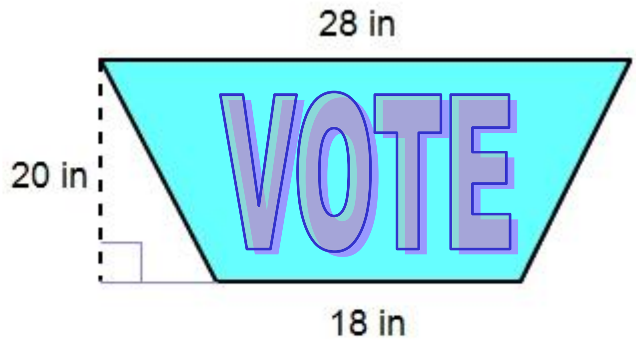
6. A trapezoid with base lengths of 105 cm and 80 cm and a height of 65 cm.

   6012.5 cm^2

7. Lucy is cutting out fabric into trapezoids for a quilt that she is piecing together. She needs 8 trapezoids like the one pictured. How many square inches of fabric will she use to cut out all 8 trapezoids?



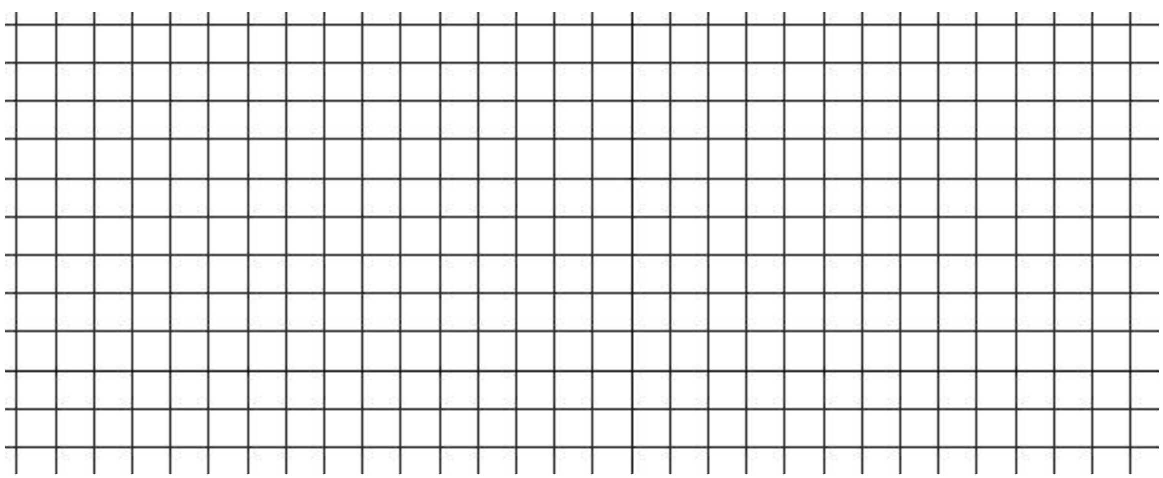
8. The sign for a student election campaign is shown. Find the total area of the sign.



9. The area of a trapezoid is 1,125 square centimeters. One base of the trapezoid measures 35 cm and the other base measures 55 cm, find the height of the trapezoid. Draw and label a picture if needed.

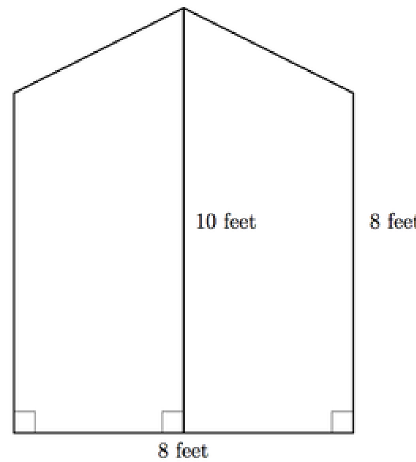
The height is 25 cm.

10. The area of a trapezoid is 20 square meters. The height of the trapezoid is 10 meters. Find two possible values for the measurements of each base. Draw and label a picture to help you find the base measurements.

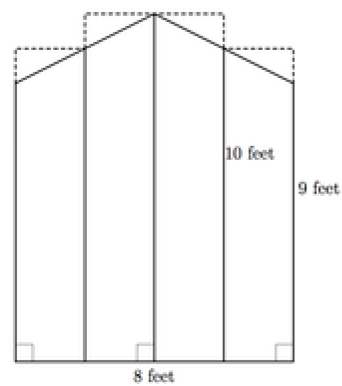


11. Jamie is planning to cover a wall with red wallpaper. The dimensions of the wall are shown.

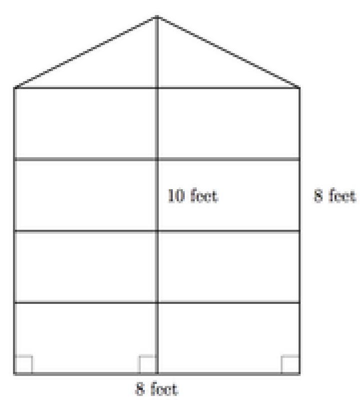
a. How many square feet of wallpaper are required to cover the wall?



b. Wallpaper comes in long rectangular strips which are 24 inches wide. If Jamie lays the strips of wallpaper vertically, can she cover the wall without wasting any wallpaper? Explain.



c. If Jamie lays the strips of wallpaper horizontally, can she cover the wall without wasting any wallpaper? Explain.

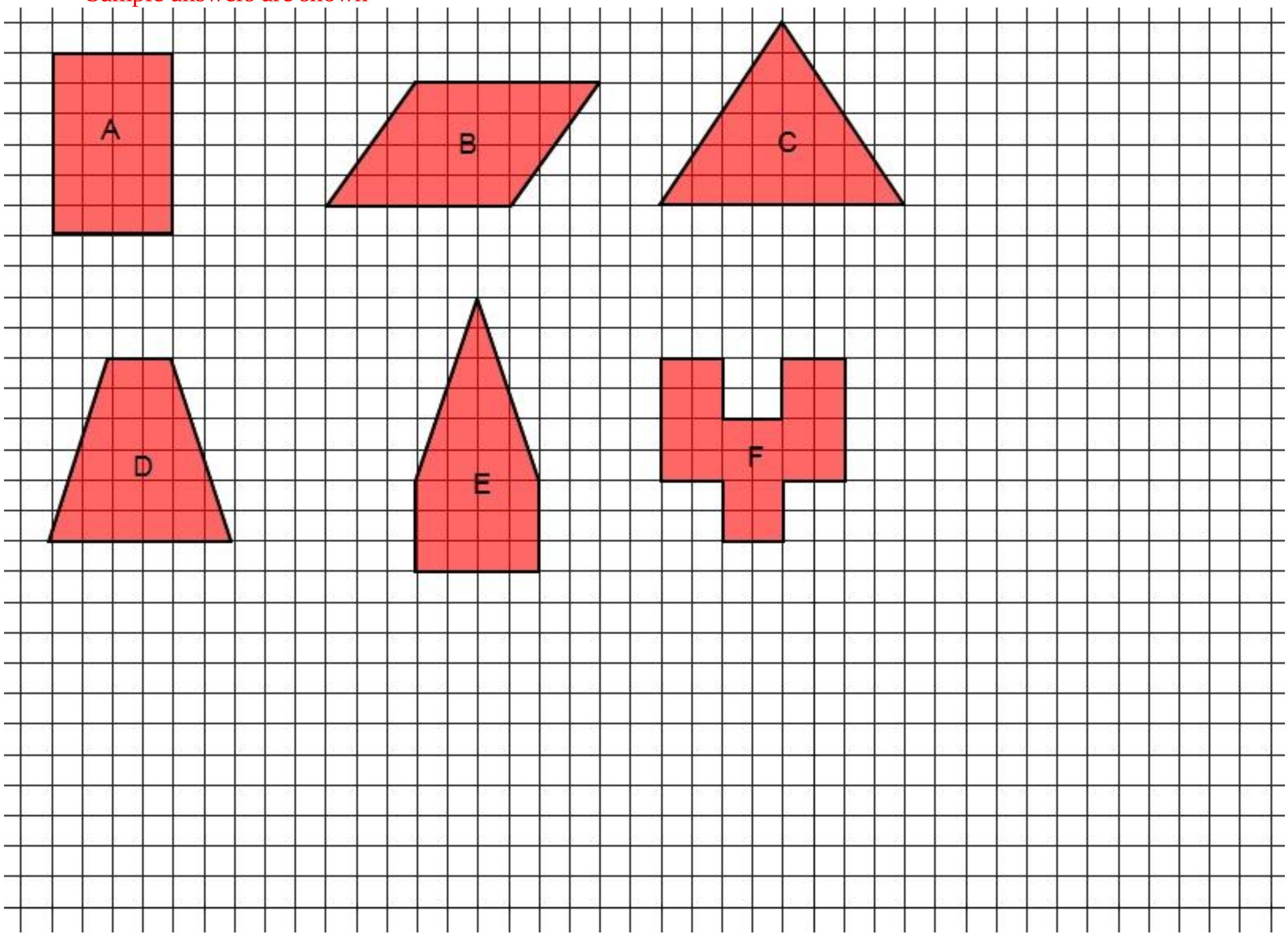


This is an Illustrative Mathematics Task

5.1e Class Activity: Area of Irregular Figures

1. On the graph below one square represents one square unit. Draw one of each of the following;
   a. A rectangle with an area of 24 square units
   b. A parallelogram with an area of 24 square units
   c. A triangle with an area of 24 square units
   d. A trapezoid with an area of 24 square units
   e. A 5 sided polygon with an area of 24 square units
   f. A polygon with more than 5 sides with an area of 24 square units.

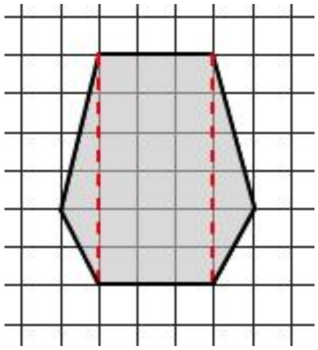
   Be ready to explain why each polygon has an area of 24 square units.
   Sample answers are shown



Discuss with your student their different methods for constructing their polygons. Some students might use the area formulas and think about the dimensions algebraically while others might use reasoning related to how the polygons relate to one another. For example they might reason that a triangle will have half of the area of a parallelogram with the same dimensions. They might also think about surrounding their figures with a rectangle and subtract out or rearrange pieces to construct an area of 24 square units.

2. Find the area of each shaded figure. Show all your work.
Sample answers are shown

a.

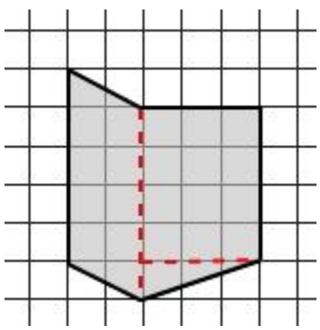


Area of the rectangle is $3(6) = 18$
Area of the two triangles is

$$2 \times \frac{1}{2}(6 \times 1) = 2 \times 3 = 6$$

Total area is $18 + 6 = 24$ units

b.



Area of the rectangle is $3(4) = 12$
Area of the parallelogram is $5(2) = 10$
Area of the triangle is $\frac{1}{2}(1)(3) = 1.5$
Total area is $12 + 10 + 1.5 = 23.5$ units

c.

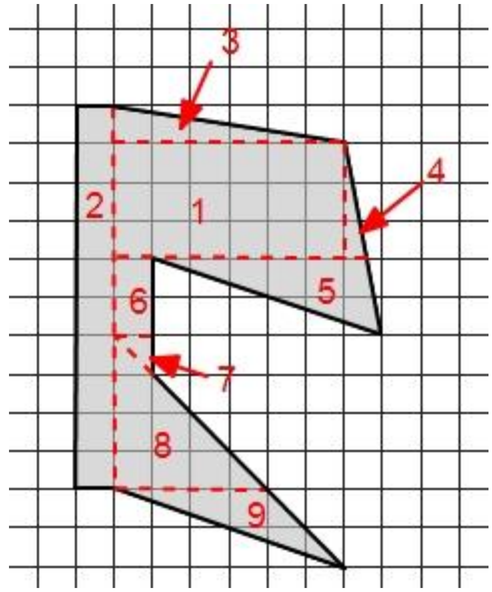
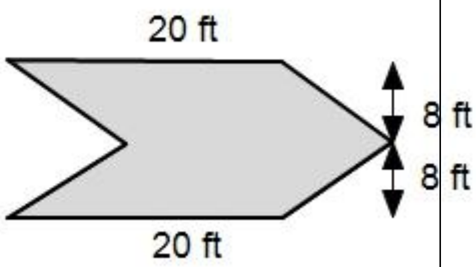
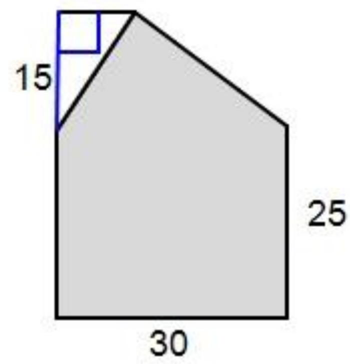
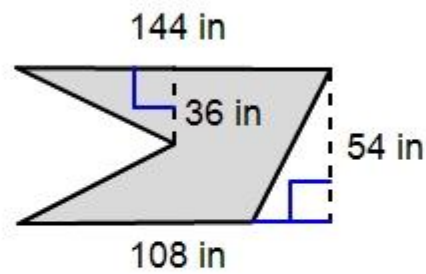


Figure	Area
1	$6(3) = 18$
2	$1(10) = 10$
3	$\frac{1}{2}(6)(1) = 3$
4	$\frac{1}{2}(3)(1) = 1\frac{1}{2}$
5	$\frac{1}{2}(6)(2) = 6$
6	$(2)(1) = 2$
7	$\frac{1}{2}(1)(1) = \frac{1}{2}$
8	$\frac{1}{2}(4)(4) = 8$
9	$\frac{1}{2}(4)(2) = 4$

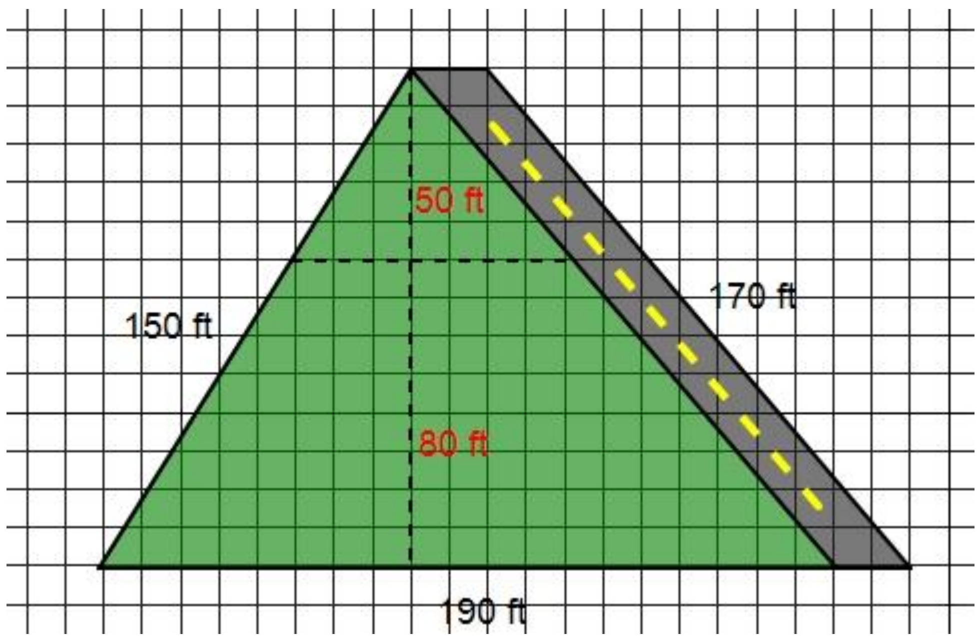
Total Area is $18 + 10 + 3 + 1\frac{1}{2} + 6 + 2 + \frac{1}{2} + 8 + 4 = 53$ square units

Emphasize that any polygon can be subdivided into parallelograms and/or triangles, and thus the area can be computed. As you break a figure apart always be looking for the perpendicular height and base of the sub-figures. This can be tricky with acute and obtuse triangles. This is the case with triangles 5 and 9 in part c above.
It is also acceptable for students to break the polygon into pieces that include a trapezoid if desired.

3. Find the area of each shaded figure. Assume that angles that appear to be right angles are right angles.

a.	b.	c.
		
320 ft^2	975 square units	3564 in^2

4. The Suderman family bought a triangular piece of property, with 170 feet of roadside frontage as shown below. The distance between grid lines is 10 feet, and thus each grid square has area 100 square feet. The lengths of the other two sides are (roughly) 190 feet (the bottom side of the figure) and 150 feet (the side to the left). Mr. Suderman decides to divide the property into two pieces with the dashed line as shown; on the top piece he will put his house and on the bottom piece he will create a garden.



a. What is the approximate area of the house lot?

1750 ft^2

b. What is the approximate area of the garden?

10600 ft^2

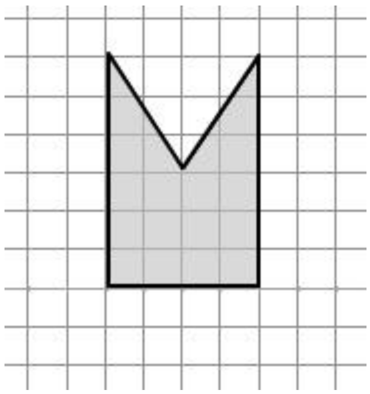
5.1e Homework: Area of Irregular Figures

1. On the graph below one square represents one square unit. Draw one of each of the following;
   a. A blue 5 sided polygon with an area of 30 square units
   b. A green 6 sided polygon with an area of 40 square units.
   c. A yellow7 sided polygon with an area of 50 square units.

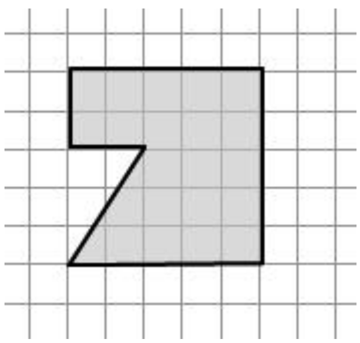
Show how you know that each polygon has the given area.

2. Find the area of each shaded figure. Show all your work.

a.



b.



c.

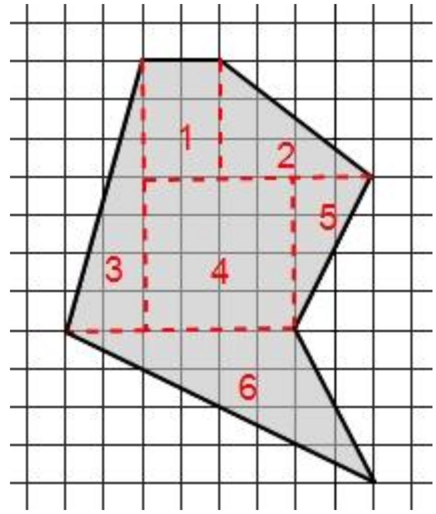
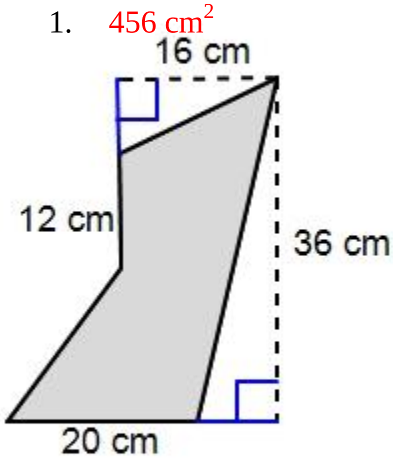


Figure	Area
1	$2(3) = 6$
2	$\frac{1}{2}(4)(3) = 6$
3	$\frac{1}{2}(2)(7) = 7$
4	$4(4) = 16$
5	$\frac{1}{2}(2)(4) = 4$
6	$\frac{1}{2}(6)(4) = 12$

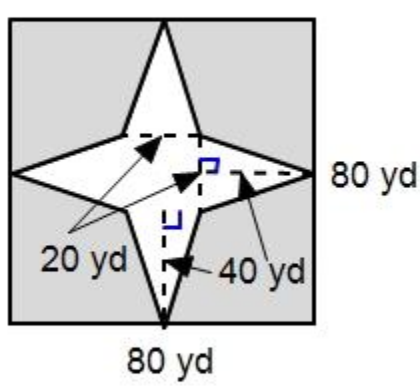
Total Area is $6 + 6 + 7 + 16 + 4 + 12 =$ 51 units

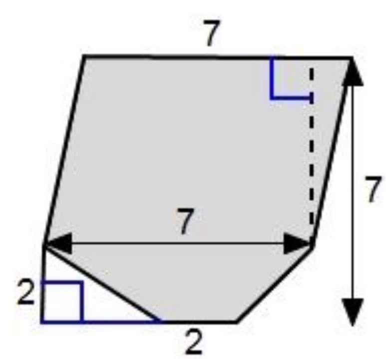
3. Find the area of each shaded figure. Assume that angles that appear to be right angles are right angle.

1. 456 cm^2



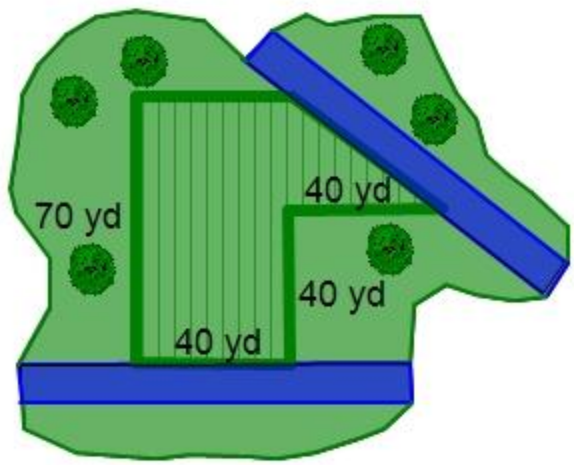
2.



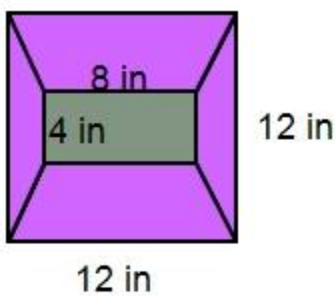


3. A fairway on a golf course is enclosed by two streams as shown. Find the area of the fairway.

The area of the fairway is 3400 yd^2



4. Ruby is designing a quilt block like the one below.



a. What is the area of the purple section of the block?

b. How much purple fabric will Ruby need if she would like 8 of these blocks in a quilt?

Section 5.2: Polygons in the Coordinate Plane

Section Overview:

This section begins with students graphing and connecting coordinates to form polygons in the coordinate plane. Then they turn to finding the length of vertical and horizontal lines in the coordinate plane. They do this either by graphing points and then counting the number of spaces between the points or examining the difference between the set of *x*-coordinates or the set of *y*-coordinates. Once students are comfortable with graphing polygons and finding their vertical and horizontal side lengths they turn to using these measurements to find area. Often this includes shifting their perspectives and decomposing given polygons into triangles and rectangles of which they can easily find the area of. Finally students use the coordinate plane to represent real-world objects and locations and use their developed techniques of finding distance and area to answer questions and make conjectures about the relationship and attributes of these objects.

Concepts and Skills to Master in this Section:

By the end of this section, students should be able to:

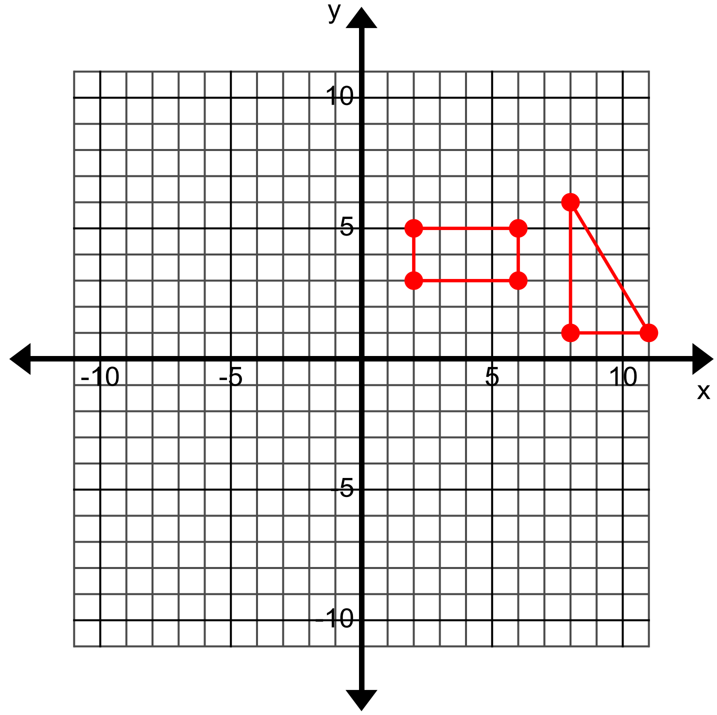
1. Draw a polygon in the coordinate plane from a given set of coordinates for the vertices.
2. Find the length of vertical and horizontal lines in the coordinate plane.
3. Find the area of a polygon in the coordinate plane.
4. Use polygons in the coordinate plant to solve real-world and mathematical problems.

5.2a Class Activity: Graphing Polygons in the Coordinate Plane

1. Plot the points on the coordinate plane, then connect the points in the order they are listed to form a polygon.

   Polygon 1: (2, 5), (6, 5), (6, 3), (2, 3)

   Polygon 2: (8, 6), (11, 1), (8, 1)



2. What polygons are formed by the points?
   Rectangle and Triangle

3. Find the length of each horizontal segment, record your answers in the table below

Be sure to review the difference between the words horizontal and vertical.

From Point	To Point	Length
(2, 5)	(6, 5)	4
(6, 3)	(2, 3)	4
(8, 1)	(11, 1)	3

Watch out for a common misconception when counting out the length of a segment on the coordinate plane. Often students will try to count the number of lines and not the number of spaces.

4. What do you notice about the ordered pairs that line up horizontally?
   They have the same *y*-coordinate.

Ask your student why it makes sense that ordered pairs that create horizontal line segments have the same *y*-coordinate.

5. Find the length of each vertical segment, record your answers in the table below

From Point	To Point	Length
(2, 5)	(2, 3)	2
(6, 5)	(6, 3)	2
(8, 1)	(8, 6)	5

6. What do you notice about the ordered pairs that line up vertically?
   They have the same *x*-coordinate

7. Is there are way that you could determine the length of these segments without plotting the points.
   The length of each horizontal segment is found by taking the difference of *y*-coordinates taking the larger number first.
   The length of each vertical segment is found by taking the difference of the *x*-coordinates taking the larger number first.

8. Find the area of each polygon if one square represents one square unit
   Rectangle: 8 square units
   Triangle: 7.5 square units

9. Use your method on the previous page to find the length of the segments created by connecting the following points. First determine if the segment formed is horizontal or vertical.

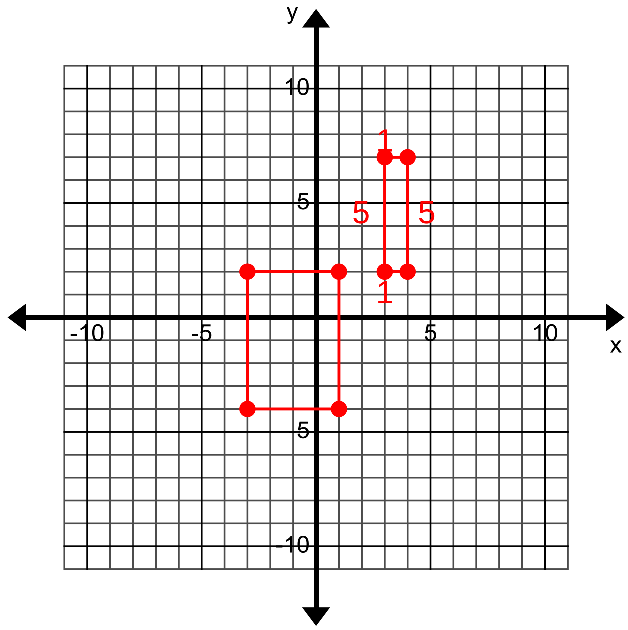
Polygon 3: (3, 2), (4, 2), (4, 7), (3, 7)

If your student struggles to determine whether the segment is horizontal or vertical remind students to look at what ordered pairs are the same.

Horizontal line segment coordinates: (3, 2), (4, 2) and (4,7),(3, 7)
$4 - 3 = 1$ and $4 - 3 = 1$, each horizontal segment has a length of 1

Vertical line segment coordinates: (3, 2), (3, 7) and (4, 2) (4, 7)
$7 - 2 = 5$ and $7 - 2 = 5$, each vertical segment has a length of 5

10. Plot the points on the coordinate plane, connect the points in the order they are listed. Check your answers above by counting out the distance for each line segment.
See graph



11. Will the subtraction method work for segments that extend into other quadrants? Try it by plotting the points below. Connect the points in the order they are listed. Count out the distance for each horizontal and vertical line segment. Use your method from above to find the distance and compare.

Polygon 4: $(1, 2), (-3, 2), (-3, -4)$, and $(1, -4)$

Students should see that when your line extends into another quadrant you get coordinates that include negative numbers. In order to find the distance between the points they will have to consider the absolute value of the negative coordinates. Students do not yet know how to subtract integers; it is acceptable at this point for them to reason that since we are looking for a distance which is positive we look at the absolute value of the coordinates. For example, the distance between the points $(1, 2)$ and $(-3, 2)$ is 4 because distance between 1 and 0 (the y-axis) 1 and the distance between -3 and 0 is 3. The total distance is $1 + 3 = 4$. It is also acceptable to encourage students to simply count the number of spaces between the two points when the line extends between quadrants.

You can also ask your student if the subtraction method will work for lines that are not horizontal or vertical

12. Find the area of each polygon
Polygon 3: 5 square units
Polygon 4: 24 square units

13. Without plotting determine whether the line segment that joins each pair of points is horizontal, vertical, or neither. Justify your answer.

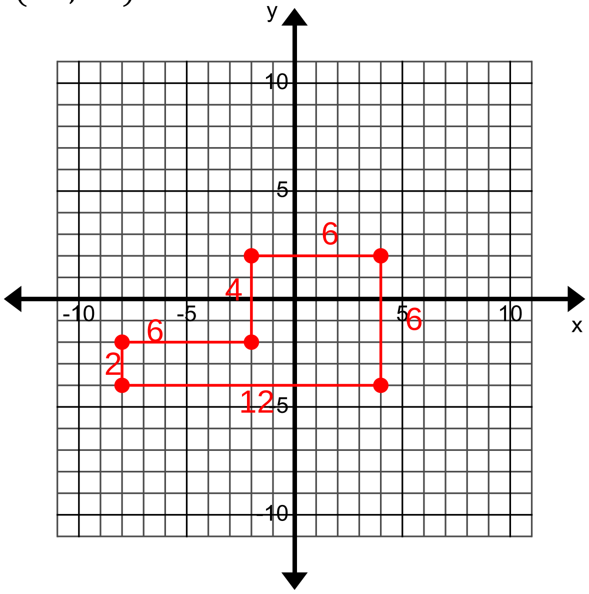
a. $(-10, 3)$ $(4, 3)$
Horizontal, same y-coordinate

b. $(0, 7)$ $(4, 0)$
Neither

c. $(-3, 4)$ $(-3, 5)$
Vertical, same x-coordinate

14. Draw a polygon on the coordinate plane by plotting each set of vertices and connecting them in the order they are listed. Label the length of each vertical and horizontal line segment and find the area of the polygon if each square represents one square unit.

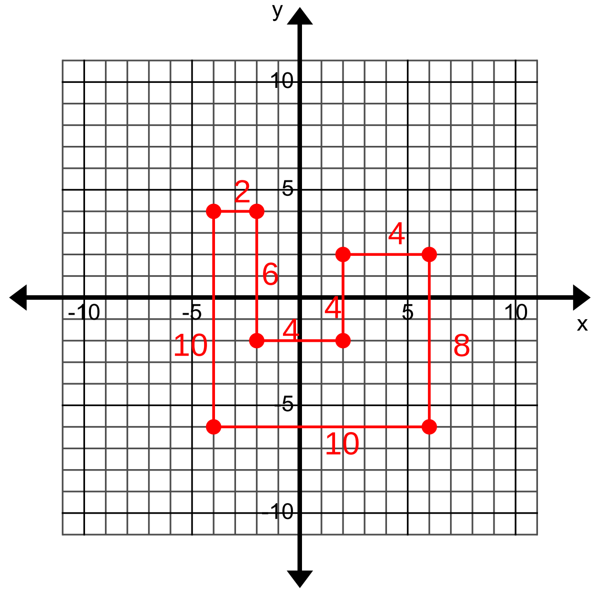
a. $(-2,2), (4,2), (4,-4), (-8,-4), (-8,-2), (-2,-2)$



Area = 48 square units

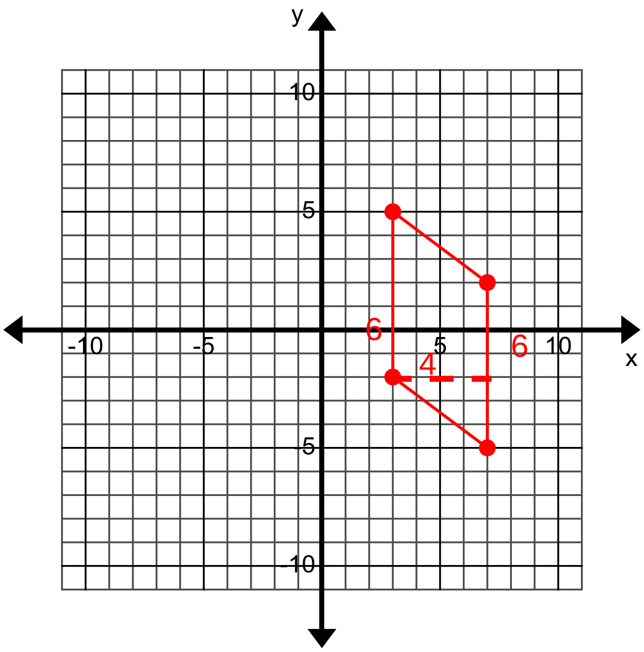
Encourage your student to think about the distance a point is away from zero as they find the lengths of the segments that extend into a different quadrant

b. $(-2,-2)(2,-2), (2,2), (6,2), (6,-6), (-4,-6), (-4,-4), (-2,4)$



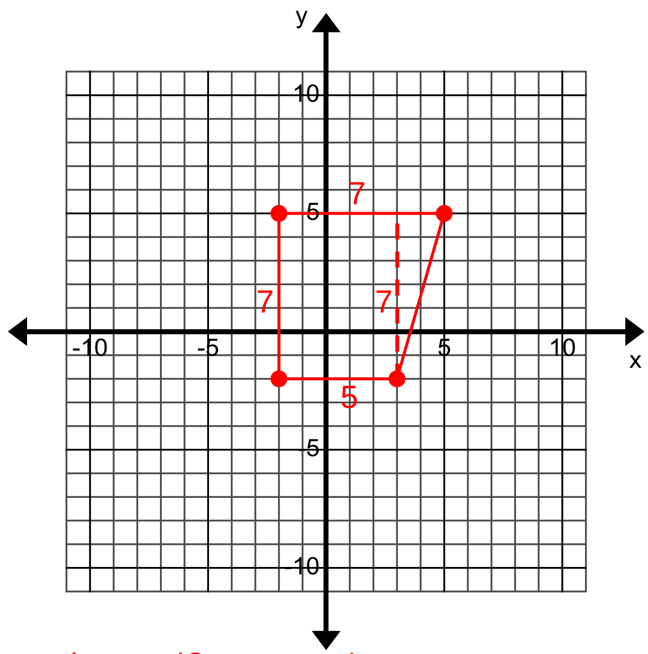
Area = 68 square units

c. $(3,5), (7,2), (7,\text{-}5)\ (3,\text{-}2)$



Area = 24 square units

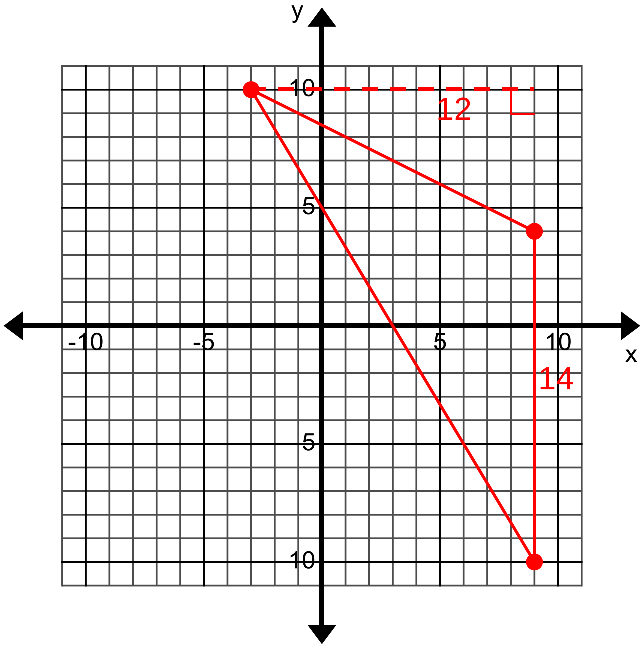
d. $(-2,5)(-2,-2), (3,-2), (5,5),$



Area = 42 square units

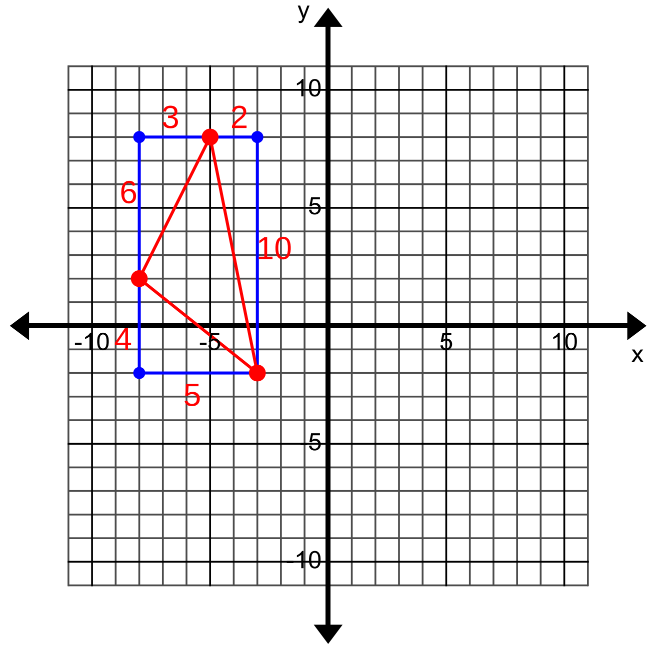
Students may find the area of the trapezoid by using either the trapezoid formula or by breaking the figure into a rectangle and triangle.

e. $(-3, 10), (9, 4), (9, -10),$



Area = 84 square units

f. $(-5, 8), (-8, 2), (-3, -2),$



Area = 21 square units

Finding the vertical and horizontal segments that will help you find the area is not so obvious in problems e. and f. For problem e. students must choose a base that lies horizontally or vertically in the coordinate plane. Then they must find the height that corresponds to that base and find its length.

In problem f. the triangle does not have a base that lies vertically or horizontally on the coordinate plane. See if students can use another method to find the area. As shown above a rectangle is circumscribed around the triangle and three small right triangles are formed. You can find the area of the given triangle by subtracting the combined area of three small right triangles from the area of the circumscribed rectangle

5.2a Homework: Graphing Polygons in the Coordinate Plane

1. Without plotting determine whether the line segment that joins each pair of points is horizontal, vertical, or neither. Justify your answer.

   a. $(-4,-6)\ (-4,3)$
   Vertical, same *x*-coordinate

   b. $(0,5)\ (0,10)$

   c. $(-2,4)\ (-2,5)$

   d. $(3,-6)\ (-4,-6)$
   Horizontal, same *y*-coordinate

   e. $(14,4)\ (-14,7)$

   f. $(8,8)\ (0,8)$

2. Name any two points that form a vertical line segment.
   Any two points with the same *x*-coordinate

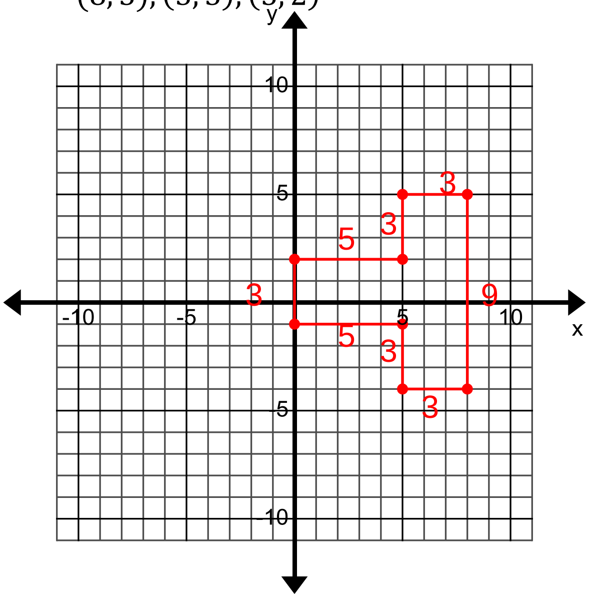
3. Name any two point that form a horizontal line segment

4. Name two points that form a vertical line segment that has a length of 8

5. Name two points that form a horizontal line segment that has a length of 4.
   Any two points with the same *y*-coordinate that have a difference whose absolute value is 4 between the *x*-coordinates.

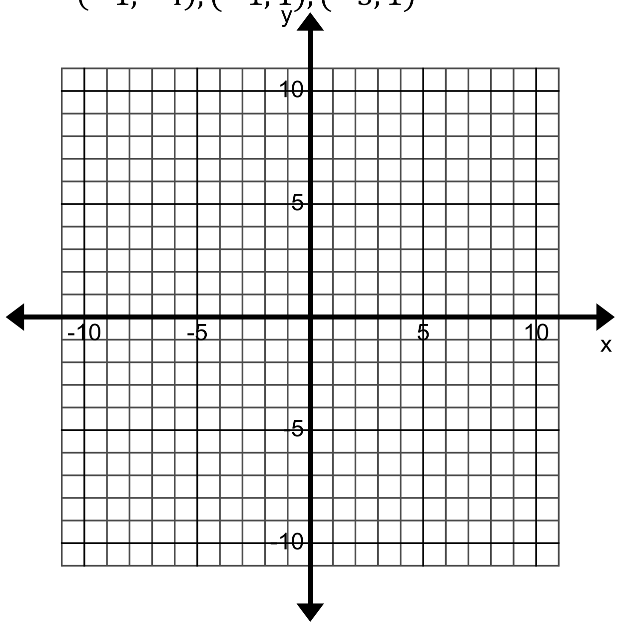
Draw a polygon on the coordinate plane by plotting each set of vertices and connecting them in the order they are listed. Label the length of each horizontal and vertical line segment and find the area of the polygon if each square represents one square unit.

6. $(0,2), (0,-1), (5,-1), (5,-4), (8,-4), (8,5), (5,5), (5,2)$

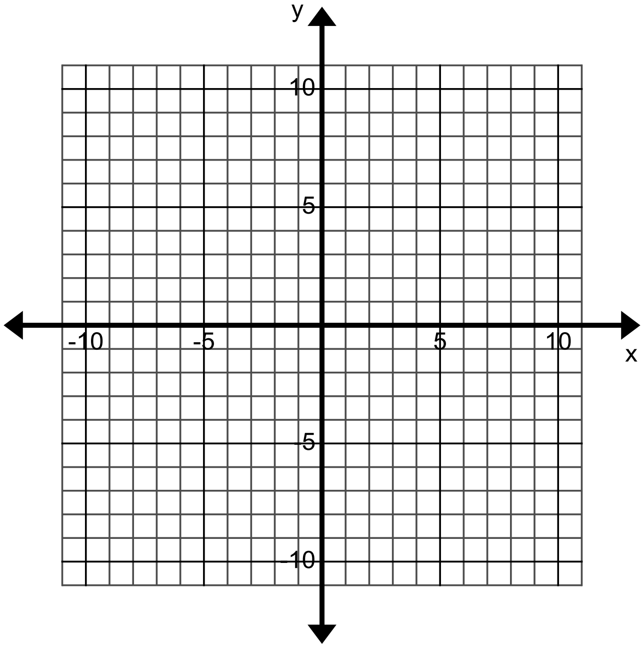


= 33 square units

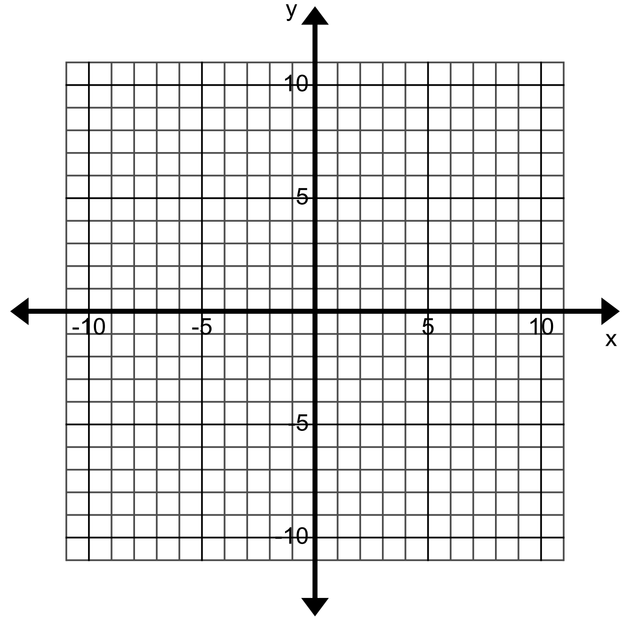
7. $(-5,7)(1,7), (1,2), (5,2), (5,-4), (-1,-4), (-1,1), (-5,1)$



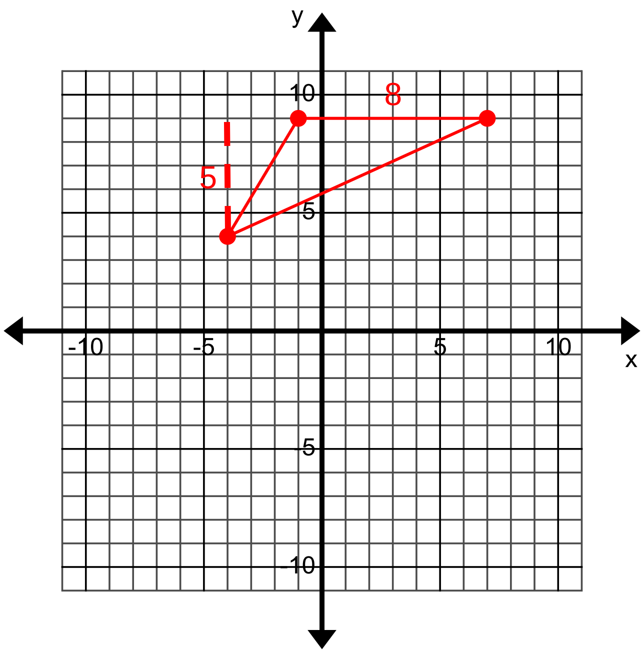
8. $(-3,5),(1,2),(-3,-3)$



9. $(-5,-3),(4,-3),(5,-5),(-4,-5)$

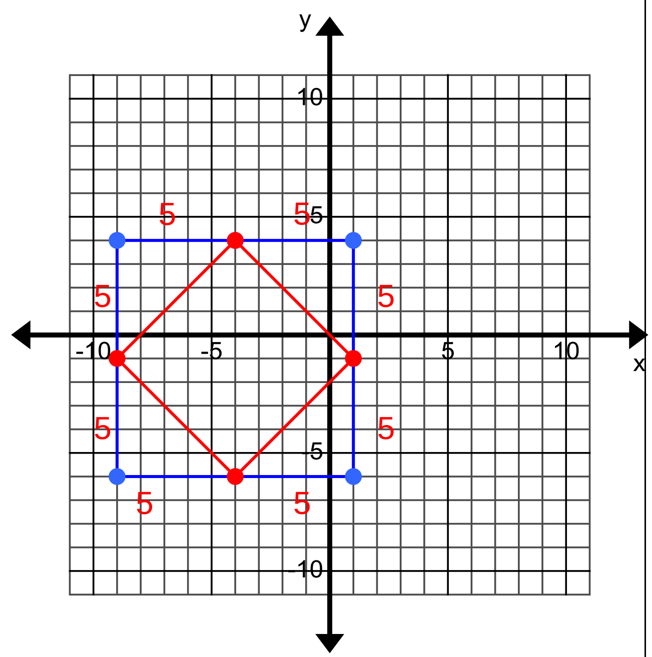


10. $(7,9),(-1,9),(-4,4)$



Area $=$ 20 square units

11. $(-4,4),(-9,-1),(-4,-6),(1,-1)$



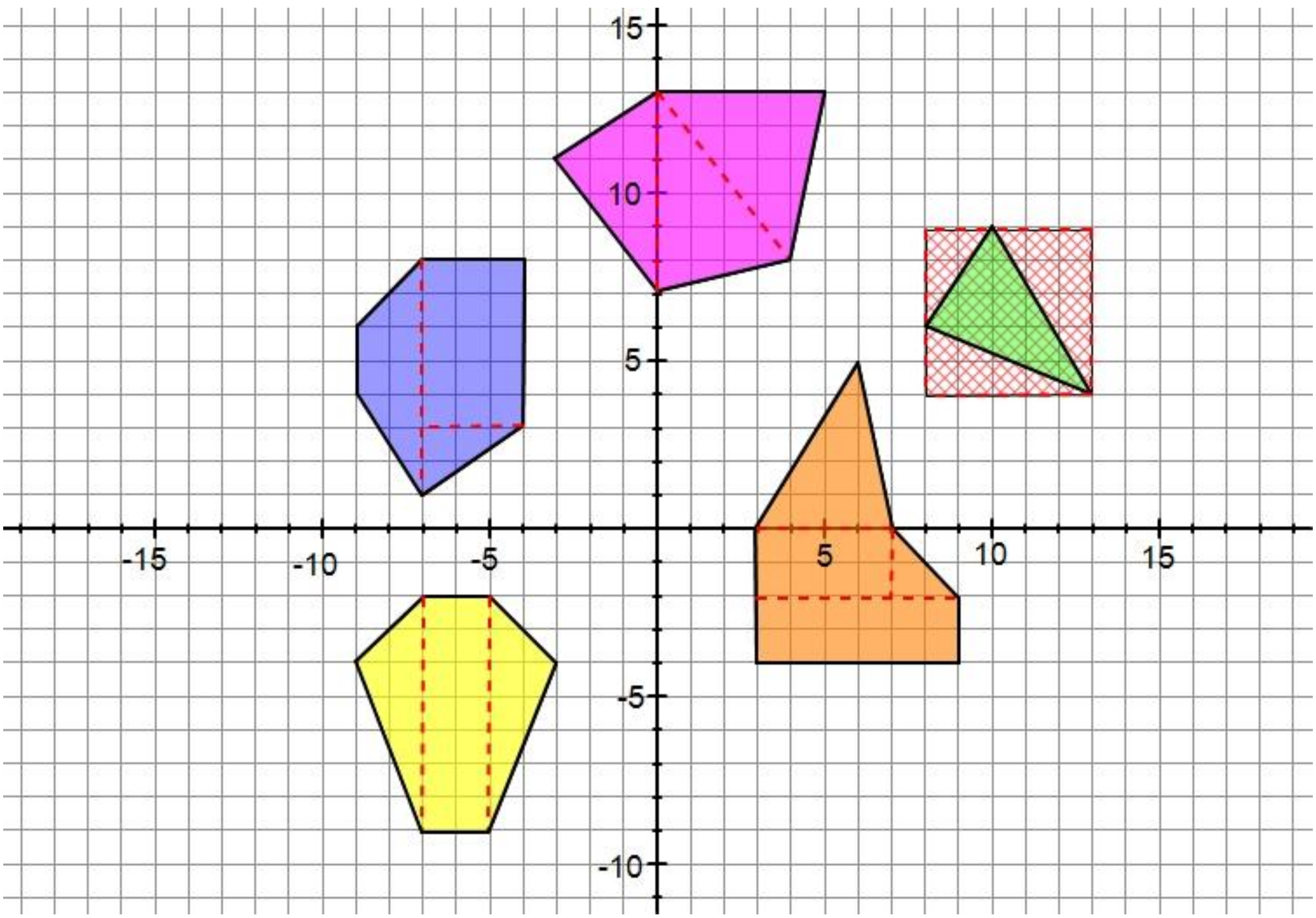
Area $=$ 50 square units

5.2b Class Activity: Finding Area of Polygons in the Coordinate Plane

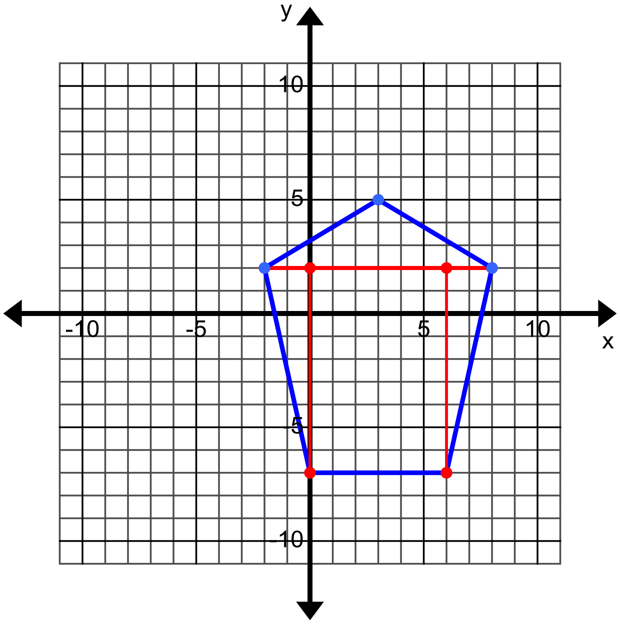
The vertices of five polygons are given below. For each polygon:

- Plot the points in the coordinate plane and connect the points in the order that they are listed.
- Color the shape the indicated color.
- Find the area of the polygon if one square represents one square unit. Be sure to show all your work

1. Blue Polygon: $(-4,8),(-4,3),(-7,1),(-9,4),(-9,6),(-7,8)$ 27 square units
2. Pink Polygon: $(-3,11),(0,13),(5,13),(4,8),(0,7)$ 33.5 square units
3. Green Polygon: $(10,9),(13,4),(8,6)$ 9.5 square units
4. Orange Polygon: $(6,5),(7,0),(9,-2),(9,-4),(3,-4),(3,0)$ 32 square units
5. Yellow Polygon: $(-7,-2),(-5,-2),(-3,-4),(-5,-9),(-7,-9),(-9,-4)$ 28 square units



6. Show two different methods for finding the area of the polygon. Two copies are provided.

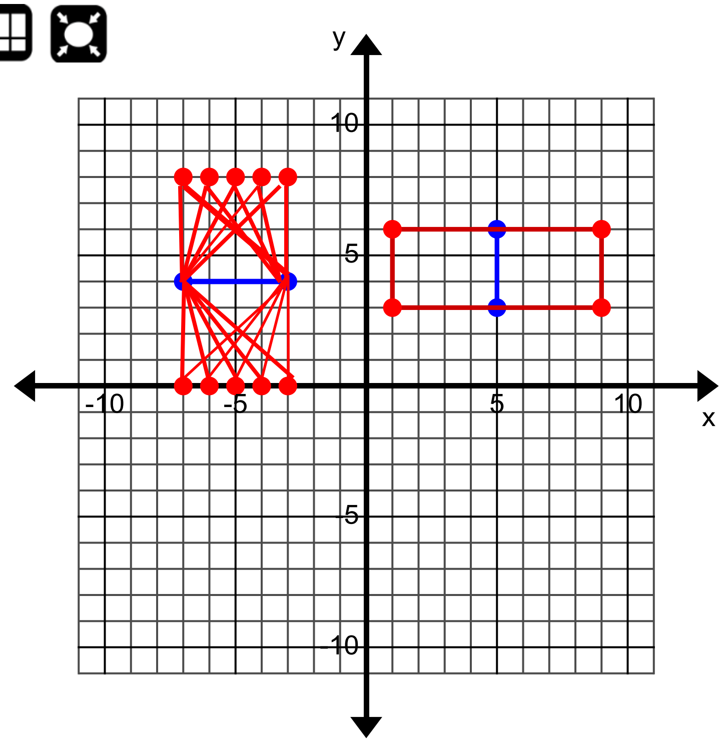


Area = 87 square units

Two different methods are shown. The first one includes finding the area of three triangles and a rectangle. The second method circumscribes a rectangle around the pentagon. The area of the pentagon is equal to the difference of the area of the rectangle and the 4 triangles formed.

7. Two consecutive vertices (vertices that are next two each other) of a rectangle are $(5, 3)$ and $(5, 6)$, the area of the rectangle is 12. Name two possible locations of the other two vertices of the rectangle. Use the coordinate plane if needed.

Possible locations could be $(1, 6)$, and $(1, 3)$ or $(9, 3)$ and $(9, 6)$



8. Two vertices of a triangle are $(-3, 4)$ and $(-7, 4)$, the area of the triangle is 8. Name 3 possible locations of the other vertex of the triangle. Use the coordinate plane if needed.

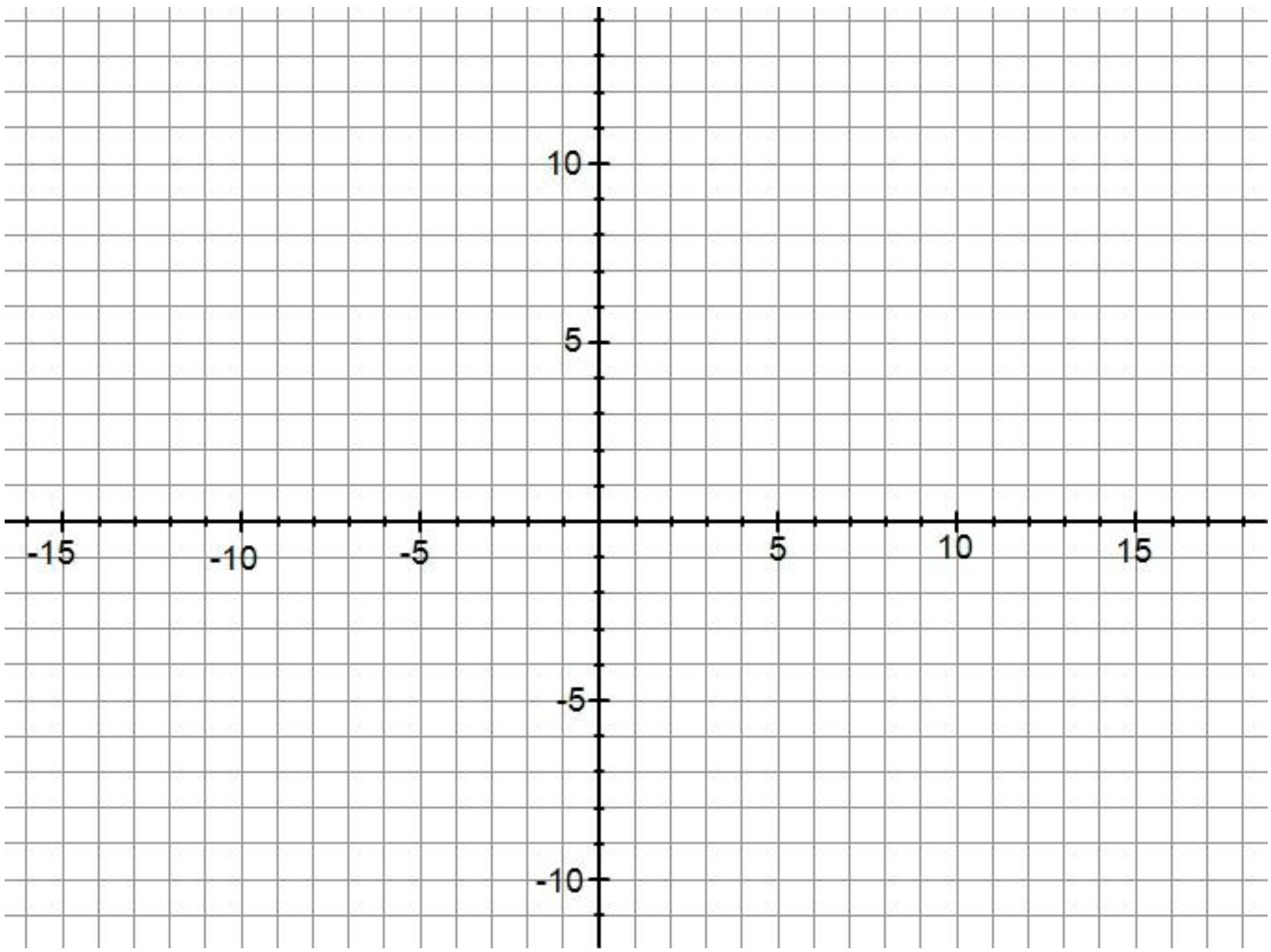
Possible locations include any vertex that is 4 square units above or below the line segment formed by the given vertices.
$(-3, 8), (-4, 8), (-5,8), (-6, 8), (-7, 8)(-3, -8)$, $(-4, -8), (-5, -8), (-6, -8), (-7, -8)$

5.2b Homework: Finding Area of Polygons in the Coordinate Plane

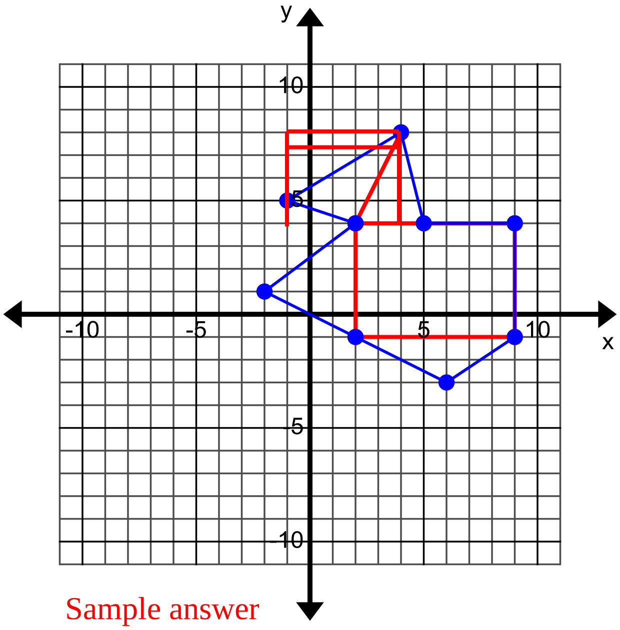
The vertices of four polygons are given below. For each polygon:

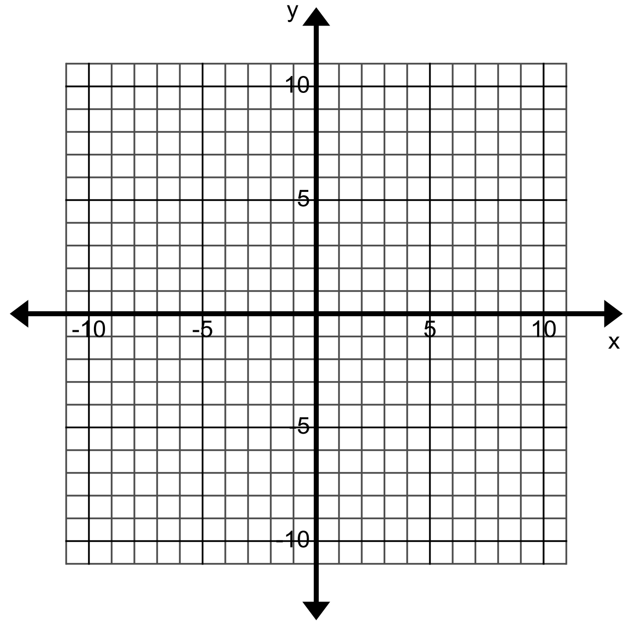
- Plot the points in the coordinate plane and connect the points in the order that they are listed.
- Color the shape the indicated color.
- Find the area of the polygon if one square represents one square unit. Be sure to show all your work.

1. Blue Polygon: $(0, 6), (4, 10), (6, 5), (6,4), (5, 4)$
2. Green Polygon: $(-7, 5), (-8, 8), (-5, 11), (-2, 8), (-6, 8)$
3. Orange Polygon: $(-5, 2), (-2, -2), (-2, -6), (-6, -6), (-8, -7), (-8, -2)$
4. Yellow Polygon: $(6,1,), (9, 1), (11, -2), (9, -4), (9, -8), (6, -8), (6, -4), (2, -2)$



5. Show two different methods for finding the area of the polygon. Two copies are provided.





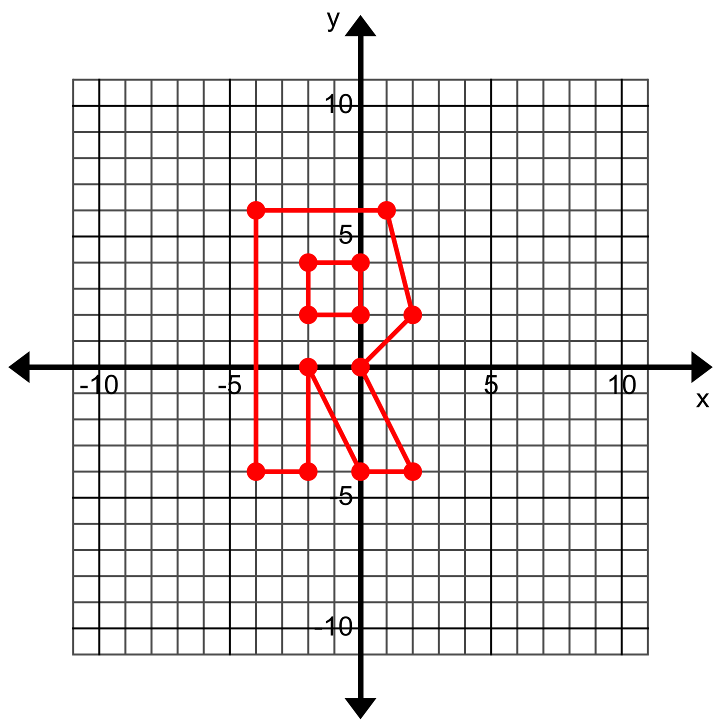
Sample answer

Area = 65 square units

7. Draw one of the letters in your name using rectangles and/or triangles on the coordinate plane below. Then find the area of the letter.

Sample answer is given.

Area is 39 square units



5.2c Class Activity: Using Polygons in the Coordinate Plane to Solve Real World Probems

1. A new park is being designed for your city. The plans for the design are being drawn on the coordinate plane below. The vertices given form a polygon that represents the location for 5 different features in the park. 

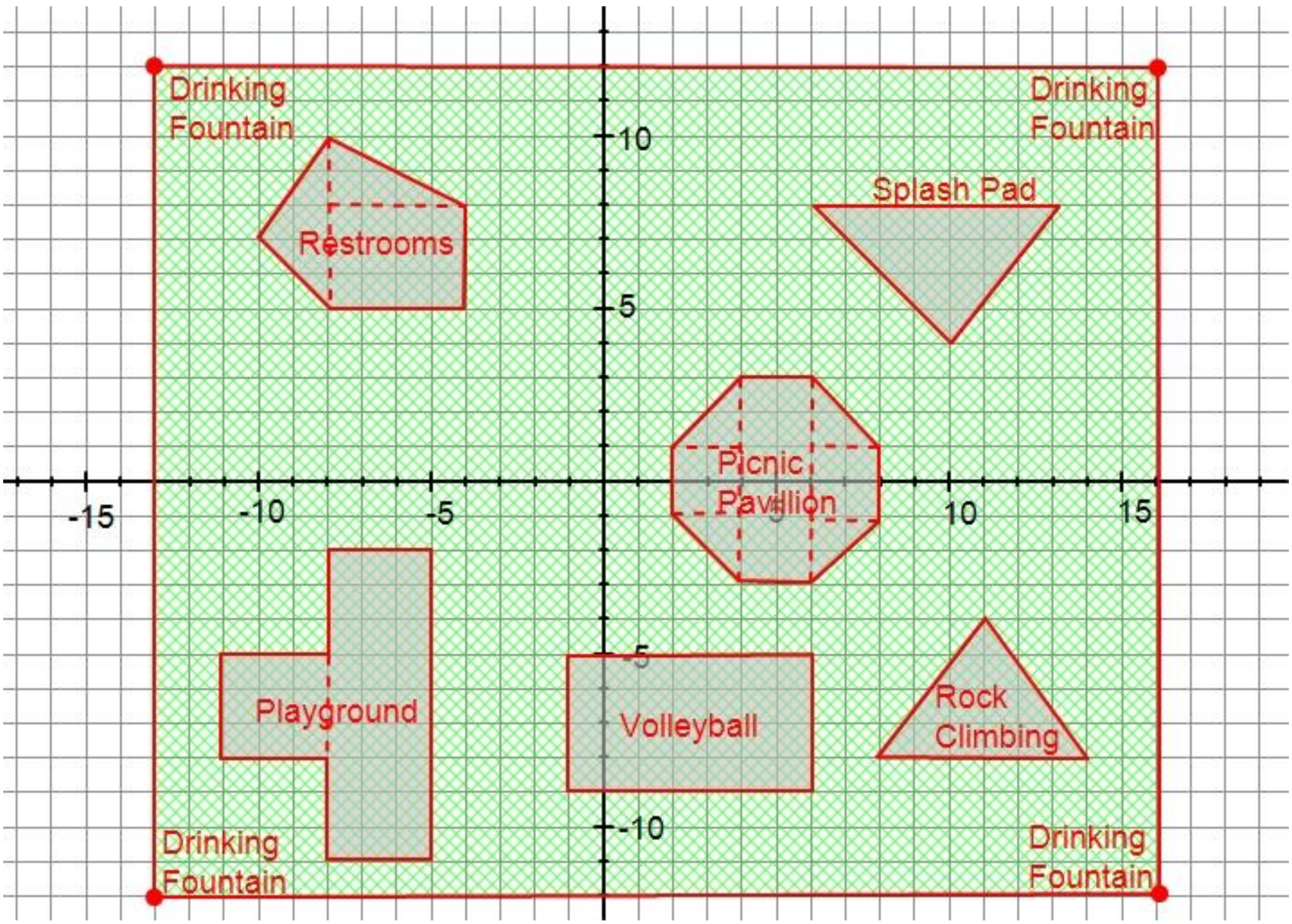
   a. Plot each set of points in the order they are given to form a polygon. Label the feature that the polygon represents on the graph.

Playground: $(-5,-2),(-8,-2),(-8,-5),(-11,-5),(-11,-8),(-8,-8),(-8,-11),(-5,-11)$

Splash Pad: $(10,4),(13,8),(6,8)$

Restrooms: $(-4,5),(-4,8),(-7,10),(-10,7),(-7,5)$

Picnic Pavilion: $(8,-1),(6,-3),(4,-3),(2,-1),(2,1),(4,3),(6,3),(8,1)$



If each square on the graph represents 1 square meter, answer the questions that follow.

b. Cement needs to be poured to lay the foundation for the bathroom and the splash pad. How many square meters of cement will the city need for the foundation of these two features?
The city will need 14 m^2 for the splash pad and 26 m^2 for the restrooms. This makes 40 m^2 of total cement.

c. The playground is going to be covered with wood chips, how many square meters of wood chips will the city need for the playground?
The city will need 36 m^2 of bark to cover the playground area.

d. The citizens of the city have asked that the playground have a fence around its perimeter. How many meters of fencing will they need for the playground?
The city will need 30 m of fencing for the playground.

e. The foundation of the pavilion picnic area is going to be covered with special pavers. How many square meters of pavers will they need for the pavilion?
The city will need 28m^2 of pavers for the pavilion.

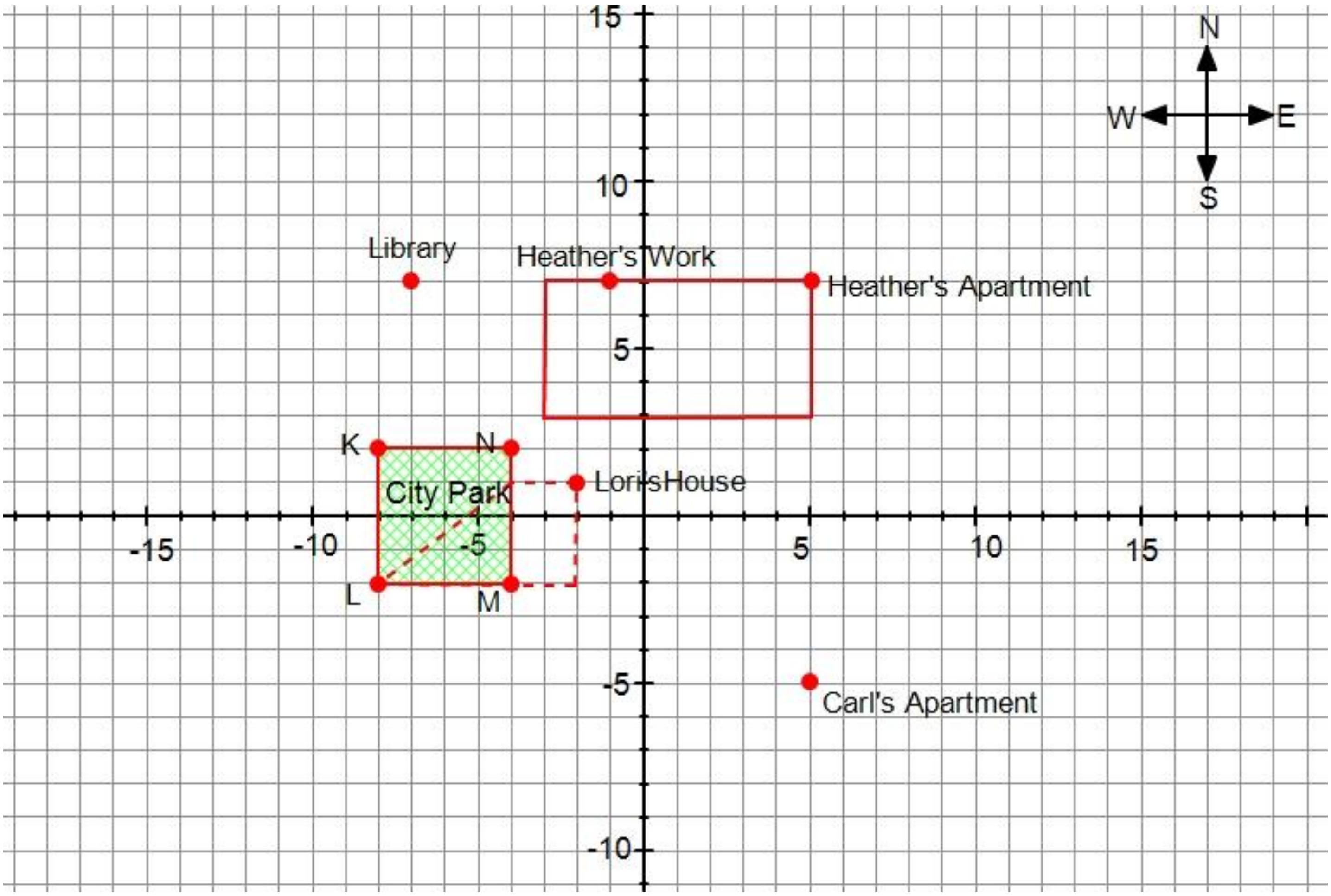
f. There are drinking fountains at the four corners of the walking path. The drinking fountains are at the points $(16, 12), (16, -12), (-13 - 12), (-13, -12)$. Draw these drinking fountains on the graph and find the distance a person will walk if they do three laps around the walking path.
One lap is meters 106, so a person that walks 3 laps will go 318 meters.

g. The city council has received word that they have enough money in the budget to build a rectangular sand volleyball court. They would like the dimensions of the court to be 4 meters by 7 meters. If two coordinates that connect to form one side of the court are $(-1, -5)$ and $(-1, -9)$ what are the other two coordinates that form the court? Keep in mind that you do not want the court to run into any other feature.
$(6, -5), (6, -9)$

h. The youth city council would like to install a rock climbing wall in the park. They have been allotted 12 square meters to use for their wall. Draw a polygon that could represent the location of the rock climbing wall.
Sample answer is given, students may draw any polygon with an area of 12 square meters.

i. The remainder of the park is to be covered with grass. How many square meters of grass will the city need to plant?
They will need to plant 557 m^2 of grass.

2. A coordinate grid represents the map of a city. Each square on the grid represents one city block.



a. Heather's apartment is at the point$(5, 7)$. She walks 4 blocks south, then 8 blocks west, then 4 blocks north, and then finally 8 blocks east back to her apartment. How many blocks did she walk total? Describe the shape of her path. Mark and label her apartment and highlight her walk.
Heather walked 24 blocks. Her path made a rectangle.

b. Draw and describe in words least two different ways you could walk exactly 20 blocks and end up back where you started.
Any polygon with a perimeter of 20.

c. Carl lives at the point $(\mathbf{5}, -\mathbf{5})$. Find the distance between Heather's house and Carl's house, and then mark and label Carl's house.
The distance between Carl and Heather's house is 12 blocks.

d. Heather's work is directly due west of her house and is half the distance between Heather's house and Carl's house. Mark where Heather works on the graph and write the order pair.
Heather works at $(\mathbf{0}, \mathbf{7})$

e. Four corners of the city park are located at the points $(-8, -2)$, $(-8, 2)$, $(-4, 2)$, and $(-4, -2)$. Without graphing determine what kind of quadrilateral is made by the boundaries of the park? Explain how you know. Then plot and connect the ordered pairs to check your answer. Label the park on the graph.
The boundaries of the city park make a rectangle because two of the ordered pairs share the same *x*-coordinate and two of the ordered pairs share the same *y*-coordinate.

f. What is the area of the city park?
The park is 16 square blocks.

g. The library is exactly 12 blocks west of Heather's apartment and 5 blocks north of the park. Mark the library on the graph and write the order pair that represents its location.
See graph (-7,7)

h. Lori lives at the point $(-2, 1)$. She goes on a walk that starts and ends at her house. She states that the path that she traveled enclosed a polygon with an area of 12 square blocks and she cut through the park. Draw two possible shapes with a dotted line that her walk could have taken.
Any polygon with an area of 12 square blocks that cuts through the park. Sample answer is given.

5.2c Homework: Using Polygons in the Coordinate Plane to Solve Real World Probems

1. You are drawing up plans for a map that is featured in a video game. The vertices given below form a polygon that represents the location for 5 different features on the map.

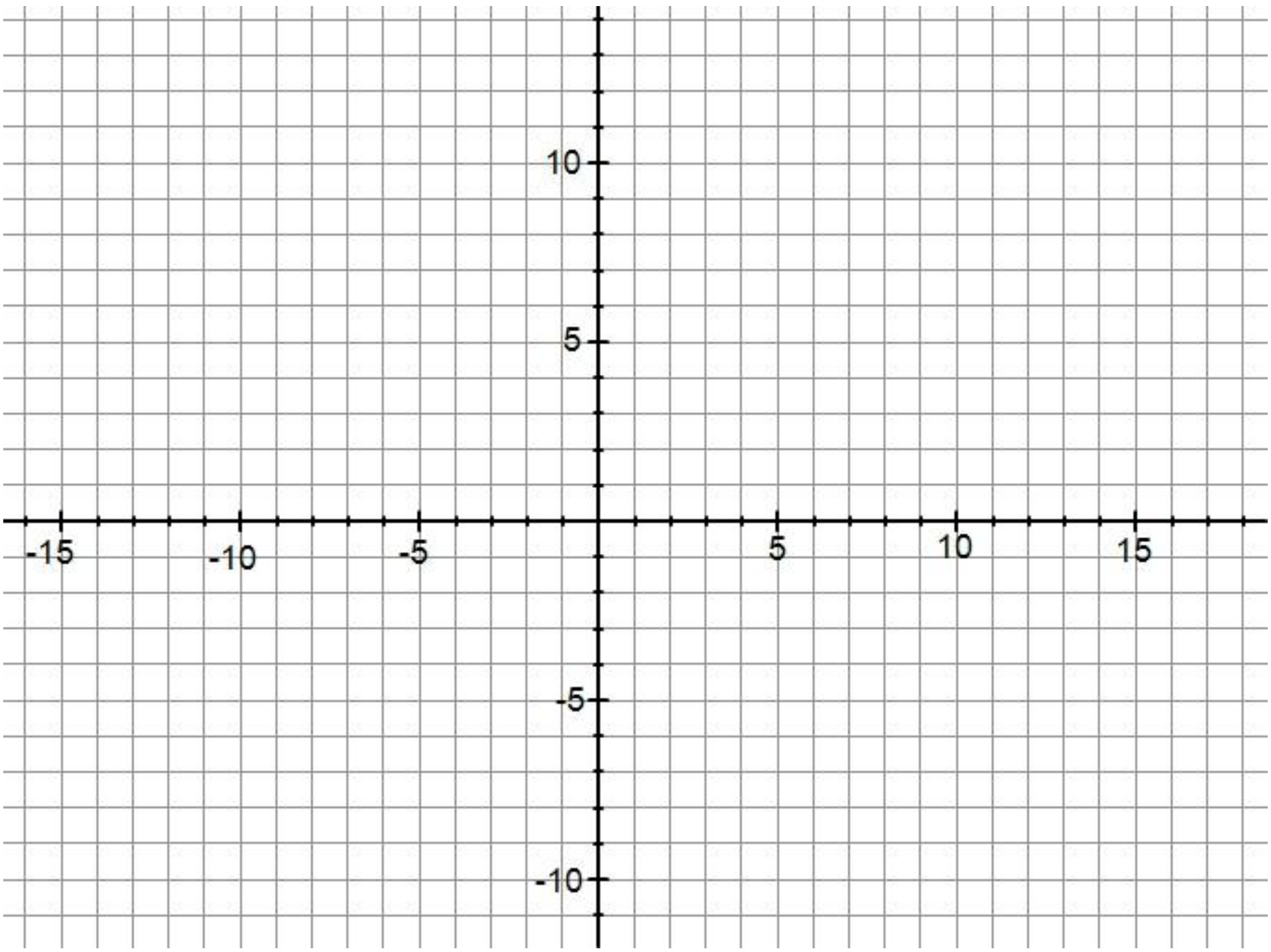
   a. Plot each set of points in the order they are given to form a polygon. Label the feature that the polygon represents on the graph.

      Crystal Palace: $(-5, 3), (-5, 6), (-7, 8), (-11, 9), (-10, 5), (-8, 3)$

      Battlefield Bunker: $(6,0), (10, 0), (10, 7), (7, 7), (7, 8), (5, 8), (5, 5), (8, 5), (8, 2), (6, 2)$

      Captain's Fortress: $(4, -9), (8, -7), (12, -7), (12, -2), (10, -5), (6, -3), (6, -7)$

      Lava Lake: $(1, 1), (3, -1), (0, -3), (-7, -1), (-9, -1), (-9, 1)$



Each square on the grid represents one square pixel unit.

b. You want the area of the Crystal Palace to be blue, how many square pixels of blue will you need?

c. You want the area of the Lava Lake to be red, how many square pixels of red will you need?

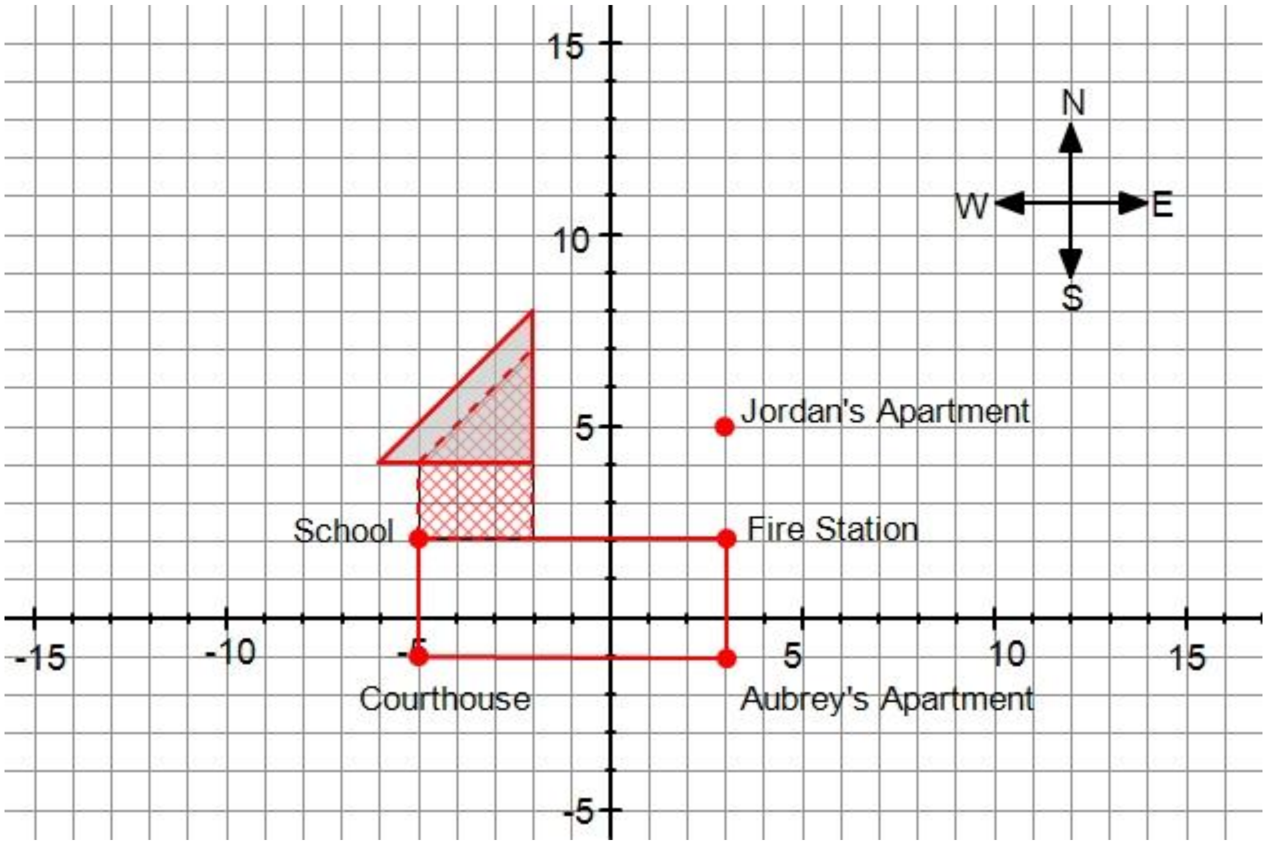
d. You want the area of the Battlefield Bunker and the Captain's Fortress to be brown, how many square pixels of brown will you need?

e. The Battlefield Bunker needs a steel wall built around it, how many pixels long will it take to show the wall?

f. There are four checkpoints that surround the Crystal Palace. They are at the points $(-4, 10), (-4, 2), (-13, 2), (-13, 10)$. Draw these checkpoint on the graph and find the distance in pixels a guard will walk if they do 2 laps around the checkpoints.

g. Finally you would like to add the Pit of Pythons. You want it to be triangular in shape and have an area of 27 square pixels that are green. Draw and color a possible triangle that could represent the Pit of Pythons.

2. A coordinate grid represents the map of a city. Each square on the grid represents one city block.



a. The fire station is located at the point $(3, 2)$. The school is exactly 8 blocks west of the fire station. What ordered pair represents the location of the school? Mark the locations of the fire station and school on the graph.

b. Aubrey's apartment is located at the point $(3, -1)$. Without, graphing determine the distance in blocks between Aubrey's house and the fire station. Then mark the location of Aubrey's apartment to check your answer.

c. If you were to connect the school, fire station, Aubrey's house, and the courthouse to form a polygon, a rectangle would be formed. What ordered pair represents the location of the courthouse? Draw the rectangle on the graph to check your answer.

d. Jordan lives the same distance away from the fire station as Aubrey but in the opposite direction. What ordered pair represents the location of Jordan's apartment? Mark Jordan's apartment on the graph to check your answer.

e. Draw and describe in words least two different ways you could walk exactly 16 blocks and end up back where you started.

f. Three corners of the city plaza are located at the points $(-2, 4)$, $(-6, 4)$, and $(-2, 8)$. Without graphing determine what kind of polygon is made by the boundaries of the plaza? Explain how you know. Then plot and connect the ordered pairs to check your answer. Label the plaza on the graph.

g. What is the area of the city plaza?

h. Larry goes on a walk that starts and ends at school. He states that the path that he traveled enclosed a polygon with an area of 10.5 square blocks and he cut through the plaza. Draw two possible shapes with a dotted line that his walk could have taken.

Section 5.3: Volume of Three-Dimensional Shapes

Section Overview:
The first lesson of this sections reviews concepts of volume measurement. Such as a cube with side length of 1 unit is said to have “one cubit unit” of volume. A right rectangular prism can be packed without gaps or overlaps using unit cubes. Volume can be measured by counting unit cubes within the rectangular prism and is the same as multiplying the prisms edge lengths or equivalently the prisms height by the area of its base. Attention is then turned to right rectangular prisms with fractional edge lengths. Students use drawings and diagrams to pack these prisms with unit cubes of appropriate fractional edge lengths and find the volume by counting fractional unit cubes. They count these cubes by multiplying the number of fraction cubes along the length, width, and height of the prism. By changing the unit of measure from a fractional cube to a whole cube students see that they can obtain the same answer by multiplying the edge lengths of the prism. Once they have this understanding they apply the formulas for finding volume to find the volume of right rectangular prisms in a variety of mathematical problems and real-world applications.

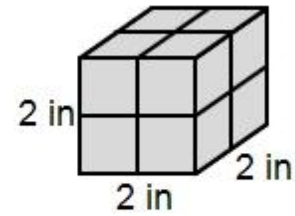
Concepts and Skills to Master in this Section:

By the end of this section, students should be able to:

1. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that it would be the same as would be found by multiplying the edge lengths of the prism.
2. Find the volume of right rectangular prisms with fractional edge lengths by applying the formulas $V = lwh$ and $V = Bh$.
3. Solve real-world and mathematical problems involving volume.

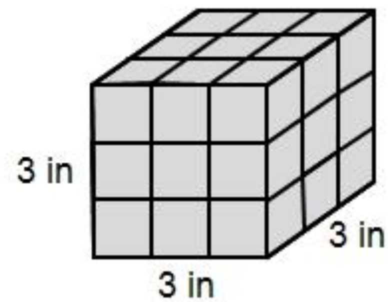
5.3a Class Activity: Finding Volume of Rectangular Prisms

1. Skyler wants to build a cube with sides that are 2 inches long using 1-inch cubes. How many 1-inch cubes will he need? Explain your reasoning.
   Each single layer of the cube contains $2 \times 2 = 4$ cubes. There are two layers so Skyler needs $4 \times 2 = 8$ one-inch cubes in all. Or Skyler needs $2 \times 2 \times 2 = 8$ in^3.

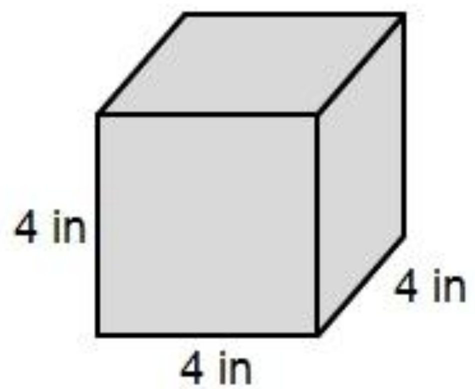


The purpose of these questions is to help students review volume and how you can find the volume of any rectangular prism by multiplying the area of the base of the prism by the number of layers or the height. In turn, volume is calculated by multiplying all the side lengths together.
If your student struggles encourage them to shade the base layer of the cube pictured and mark or count out the "layers".

2. How many 1-inch cubes will he need to build a cube with 3-inch sides? Explain your reasoning.
   Each single layer of the cube contains $3 \times 3 = 9$ cubes. There are three layers so Skyler needs $9 \times 3 = 27$ one-inch cubes in all. Or Skyler needs $3 \times 3 \times 3 = 27$ in^3



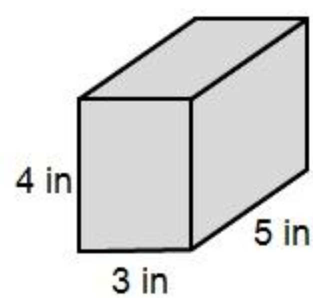
3. How many 1-inch cubes will he need to build a cube with 4-inch sides? Explain your reasoning.
   Each single layer of the cube contains $4 \times 4 = 16$ cubes. There are four layers so Skyler needs $16 \times 4 = 64$ one-inch cubes in all. Or Skyler needs $4 \times 4 \times 4 = 64$ in^3



Notice that the lines that indicate individual unit cubes are removed in the picture; it is intended to slowly provide less scaffolding to push students to more abstract representations. In problem 4 below, no picture is provided. However, if your student needs to draw the lines to indicate square unit cubes encourage them to do so.

4. How many 1-inch cubes will he need to build a cube with 6-inch sides? Explain your reasoning.
   Each single layer of the cube contains $6 \times 6 = 36$ cubes. There are six layers so Skyler needs $36 \times 6 = 216$ one-inch cubes in all. Or Skyler needs $6 \times 6 \times 6 = 216$ in^3

5. How many 1-inch cubes will he need to build a rectangular prism that measures 3 inches by 5 inches by 4 inches? Explain your reasoning.
   Each single layer of the prism contains $3 \times 5 = 15$ cubes. There are four layers so Skyler needs $15 \times 4 = 60$ one-inch cubes in all. Or Skyler needs $3 \times 5 \times 4 = 60$ in^3



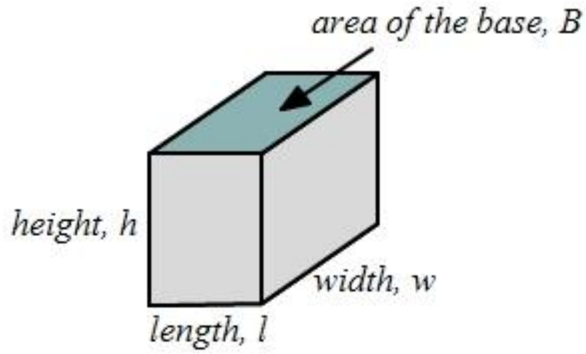
In this problem we extend volume to rectangular prisms. Since a cube is a rectangular prism, the process or method for finding the volume is the same.

6. Explain how to find the volume of any rectangular prism. Relate your method to a formula for volume.

The volume of a rectangular prism is found by multiplying the area of the base of the prism by the height. Since the area of the base of a rectangular prism is the length times the width, then the volume is found by multiplying the length times the width times the height.

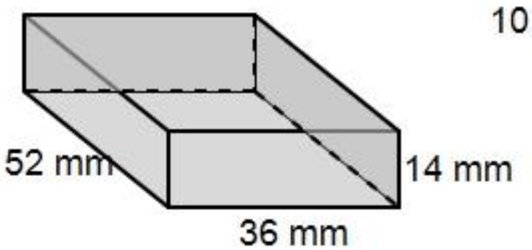
$V = Bh$ where B is the area of the base of the prism and h is the height of the prism

$V = lwh$ where l is the length of the prism, w is the width of the prism, and h is the height of the prism.



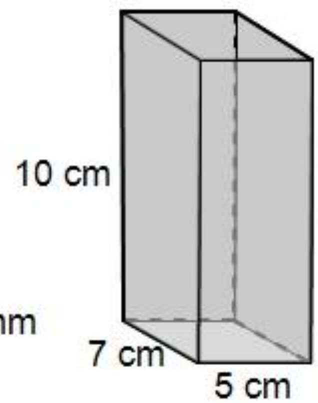
7. Find the volume of each rectangular prism given below.

a.



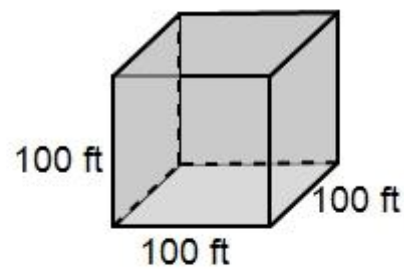
26208 mm^3

b.



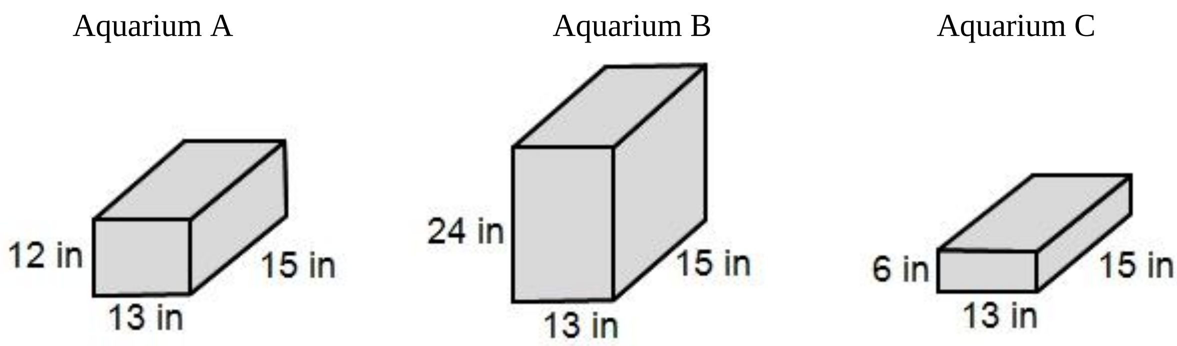
350 cm^3

c.



$1{,}000{,}000 \text{ ft}^3$

8. Three glass aquariums are rectangular prisms as shown below.



$V = 2340 \text{ in}^3$ $V = 4680 \text{ in}^3$ $V = 1170 \text{ in}^3$

a. Find the volume of each aquarium.
See above

b. Order the aquariums from least to greatest according to the amount of water they will hold.
Aquarium C, Aquarium A, Aquarium B

c. Is there any way to order the aquariums without calculating the volume?
Since the aquariums all have the same base area of 195 in^2, you can order them according to their heights.

d. Without using their volumes, determine the number of times you would have to fill the smallest aquarium with water and pour it into the largest aquarium to completely fill it.
Since the areas of the bases are the same and aquarium B has a height of 24 inches and aquarium C has a height of 6 inches, then the volume of aquarium B is 4 times the volume of aquarium C so you would have to fill aquarium C four times.

e. What are the dimensions of an aquarium whose volume is 6 times that of the middle sized aquarium and takes up the same amount of floor space?
Since you want the base area to be the same then you would need an aquarium whose height is 6 times the height of the middle aquarium: $6 \cdot 12 = 72$ inches. The dimensions would be 15 inches by 13 inches, by 72 inches. It is true that the dimensions of the length and the width can be any two numbers whose product is 195 in^2.

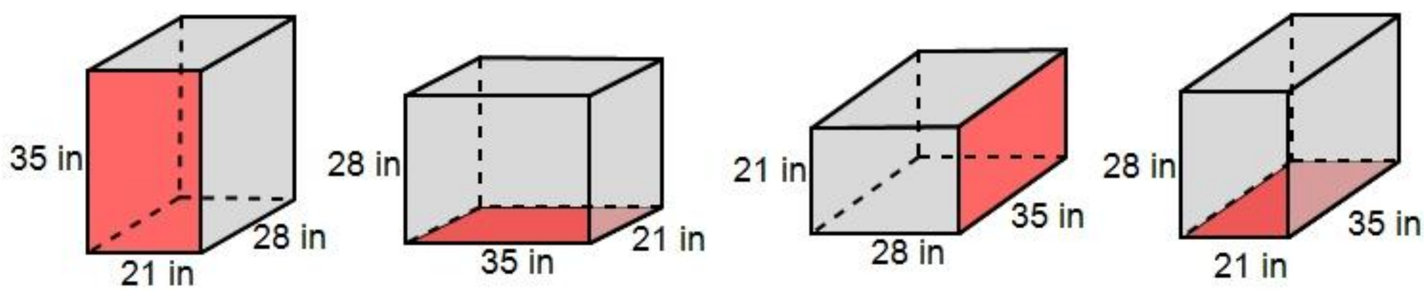
9. Use the table below to answer the questions that follow.

	Length	Width	Height
aquarium A	10 inches	4 inches	6 inches
aquarium B	8 inches	5 inches	12 inches

a. Without calculating the volumes determine which aquarium below will hold more water. Justify your answer.
Aquarium B will hold more water. The base areas of both aquariums are 40 square inches, even though they have different dimensions for the length and width. Since the height of aquarium B is bigger than the height of aquarium A, aquarium B will hold more water.

b. How many more cubic inches will the larger aquarium hold than the smaller aquarium?
Since the height of aquarium B is double the height of aquarium A and they have the same base areas, than the volume of aquarium B will be double the volume of aquarium A. Aquarium A has a volume of 240 in^3, using the reasoning above aquarium B will have 240 more cubic inches than aquarium A, $240\text{ in}^3 + 240\text{ in}^3 = 480\text{ in}^3$.

10. Make a prediction about which rectangular prism below has the greatest volume, which one has the smallest volume? Find the volume of each prism to check your predictions.



$V = 21(28)(35) = 20580\text{ in}^3$

$V = 35(21)(28) = 20580\text{ in}^3$

$V = 28(35)(21) = 20580\text{ in}^3$

$V = 21(35)(28) = 20580\text{ in}^3$

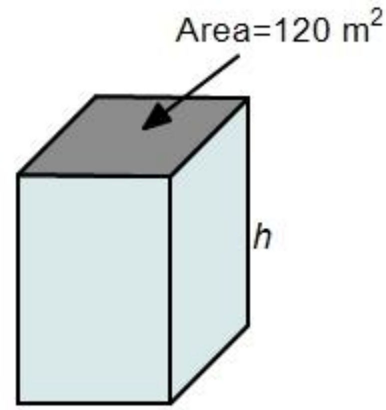
They all have a volume of 20580 in^3 because they are the same prism just turned or rotated different ways. They have the same dimensions.

Lead a discussion about the bases of the prisms above, some talking points are given below.

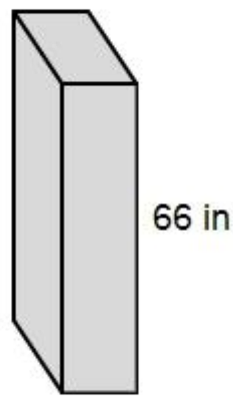
- The "base" of the rectangular prism is usually the face of the prism that is on the bottom or would touch the ground if the rectangular prism were set down. However, just like the base and height of a triangle you can choose any face to be the base of the rectangular prism.
- The dimensions are interchangeable; this can be seen by writing out the expressions used to calculate the volume of each prism. The commutative property of multiplication allows us to rearrange the dimensions in any order and we will get the same volume.
- Ask them to shade one of the corresponding bases for each prism as shown above. It may be helpful for you to use a model (textbook, tissue box, cereal box, etc.) of a prism for this problem that your student can rotate and touch.

11. The base area of a rectangular prism is 56 cm^2 and its height is 9 cm, what is the rectangular prism's volume?
The volume is 504 cm^3

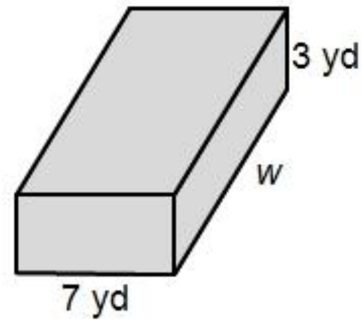
12. The volume of the rectangular prism given is 2,640 m^3 find the height.
The height is 22 m



13. The volume of the rectangular prism given is 23,958 in^3 find the area of its base
The area of the base is 363 in^2



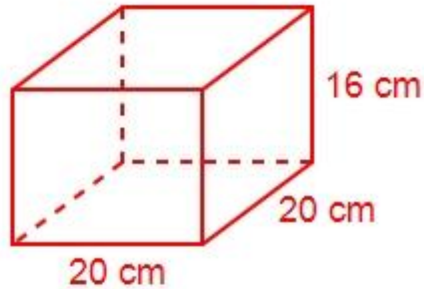
14. The volume of a rectangular prism given is 378 yd^3, find the width.
The width is 18 yards



The following three problems are [Illustrative Mathematics Tasks]

15. Amy has a fish tank shaped like a rectangular prism that is 20 cm by 20 cm by 16 cm.

a. Draw and label a picture of the tank.



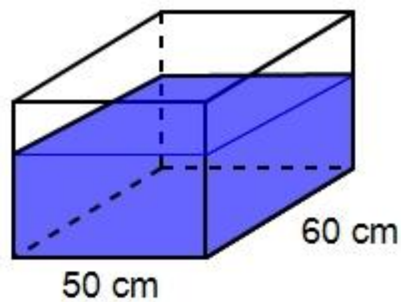
b. What is the volume of the tank?

$V = l \times w \times h = 20 \times 20 \times 16 = 6400 \text{ cm}^3$

c. If Amy only fills the tank $\frac{3}{4}$ of the way, what will be the volume of the water in the tank?

If Amy fills the tank $\frac{3}{4}$ of the way, the height of the water in the tank will be $\frac{3}{4} \times 16 = 12\ cm$, while the width and the length remain unchanged. So the volume of the water will be: $V = lwh = 20 \times 20 \times 12 = 4800 \text{ cm}^3$.

16. A rectangular tank is 50 cm wide and 60 cm long. It can hold up to 126 liters of water when full. If Amy fills $\frac{2}{3}$ of the tank as shown, find the height of the water in centimeters. (Recall that 1 liter =1000 cm^3.)



First, find the volume of tank in cubic centimeters:

$$126\ liter \times \frac{1000\ cm^3}{1\ liter} = 126000 \text{ cm}^3.$$

The height of tank is the volume divided by the length and the width or the volume divided by the area of the base:

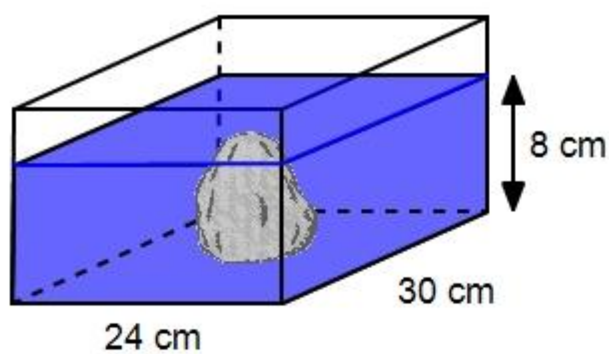
$$\frac{126000\ cm^3}{50 \times 60} = \frac{126000}{3000} = 42 \text{ cm}.$$

The height of water is $\frac{2}{3}$ the height of the tank:

$$\frac{2}{3} \times 42 = 28\ cm.$$

So the height of the water is 28 cm.

17. A rectangular tank is 24 cm wide and 30 cm long. It contains a stone and is filled with water to a height of 8 cm. When Amy pulls the stone out of the tank, the height of the water drops to 6 cm. Find the volume of the stone.

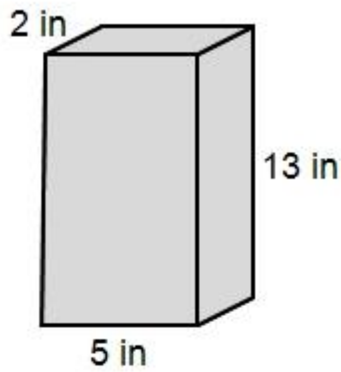


The change in water height is 8 cm − 6 cm = 2 cm. The volume of the displaced water is the product of the length, width, and change in the height of the water, $V = 24 \times 30 \times 2 = 1440$.
The volume of the stone is the same as the volume of the displaced water; we know the stone has volume 1440 cm^3.

5.3a Homework: Finding Volume of Rectangular Prisms

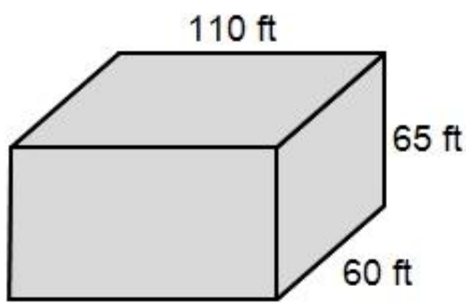
1. Find the volume of each rectangular prism given below.

a.

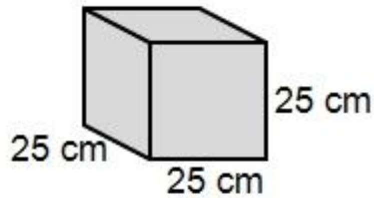


130 in^3

b.



c.



2. Use the table below to answer the questions that follow.

	Length	Width	Height
aquarium A	90 inches	40 inches	45 inches
aquarium B	60 inches	60 inches	15 inches

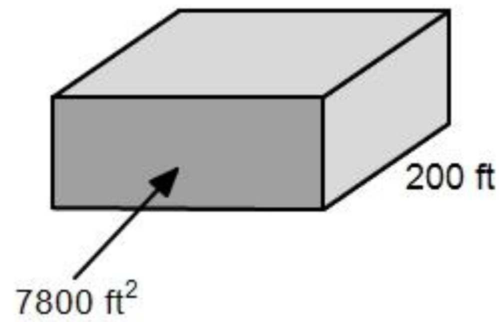
a. Without calculating the volumes determine which aquarium below will hold more water. Justify your answer.
Aquarium A will hold more water. The base areas of both aquariums are 3,600 square inches, even though they have different dimensions for the length and width. Since the height of aquarium A is bigger than the height of aquarium B, aquarium A will hold more water.

b. How do the volumes of the aquarium compare? How many times bigger is the larger aquarium than the smaller aquarium?
Since the height of aquarium A is three times the height of aquarium B and they have the same base areas, the volume of aquarium A will be three times the volume of aquarium B. Aquarium B has a volume of 54,000 in^3, using the reasoning above aquarium A will have a volume of $3 \cdot 54{,}000 =$ 1,620,000 in^3.

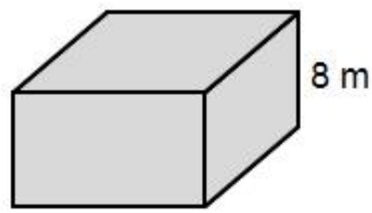
c. List a different set of dimensions that an aquarium could have if it has the same height and the same volume as aquarium A.
An aquarium could have any length and width whose product is 3600 in^2 and a height of 45 inches. Samples answers include 30 inches by 120 inches by 45 inches or 20 inches by 180 inches by 45 inches.

3. The base area of a rectangular prism is 280 in^2 and its height is 26 in, what is the rectangular prism's volume?

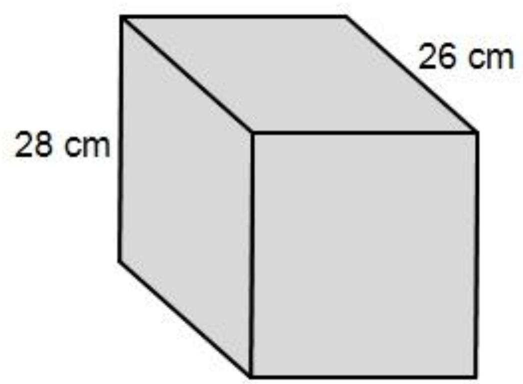
4. Find the volume of the rectangular prism given.



5. The volume of the rectangular prism given is 616 m^3 find the area of its base.



6. The volume of a rectangular prism given is 16,744 cm^3, find the length of the rectangular prism.
The length is 28 cm.



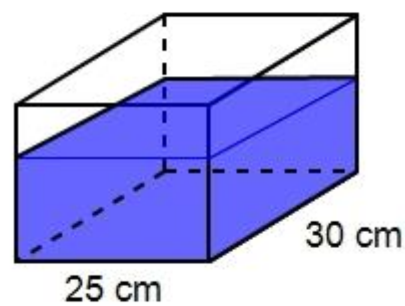
7. Andrew has a fish tank shaped like a rectangular prism that is 30 cm by 35 cm by 20 cm.

   a. Draw and label a picture of the tank.

   b. What is the volume of the tank?

   c. If Andrew only fills the tank $\frac{1}{4}$ of the way, what will be the volume of the water in the tank?

8. A rectangular tank is 25 cm wide and 30 cm long. It can hold up to 18.75 liters of water when full. If Andrew fills $\frac{3}{5}$ of the tank as shown, find the height of the water in centimeters. (Recall that $1\ liter = 1000\ \text{cm}^3$).



First, find the volume of tank in cubic centimeters:

$$18.75\ liter \times \frac{1000\ cm^3}{1\ liter} = 18{,}750\ \text{cm}^3.$$

The height of tank is the volume divided by the length and the width or the volume divided by the area of the base:

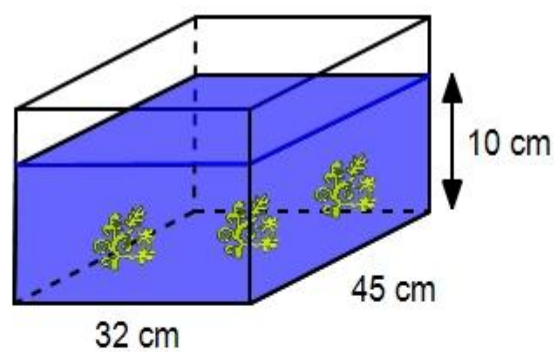
$$\frac{18{,}750\ cm^3}{25 \times 30} = \frac{18{,}750}{750} = 25\ \text{cm}.$$

The height of water is $\frac{3}{5}$ the height of the tank:

$$\frac{3}{5} \times 25 = 15\ cm.$$

So the height of the water is 15 cm.

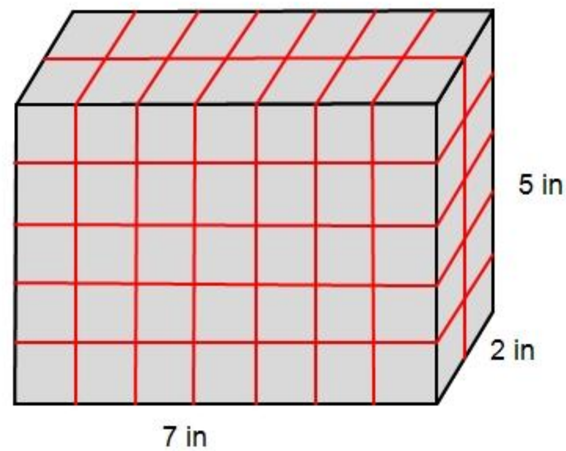
9. A rectangular tank is 32 cm wide and 45 cm long. It contains three identical plastic water plants and is filled with water to a height of 10 cm. When Amy pulls all three plastic water plants out of the tank, the height of the water drops to 8 cm. Find the volume of one plastic water plant.



5.3b Class Activity: Finding Volume of Rectangular Prisms with Fractional Edge Lengths

1. A company that manufactures dice packages the dice in rectangular boxes that measure 7 inches by 2 inches by 5 inches as shown.

Consider watching the video below as your student works through the tasks in this section.
https://www.youtube.com/watch?v=2yPv6xqu1Z8&t=443s

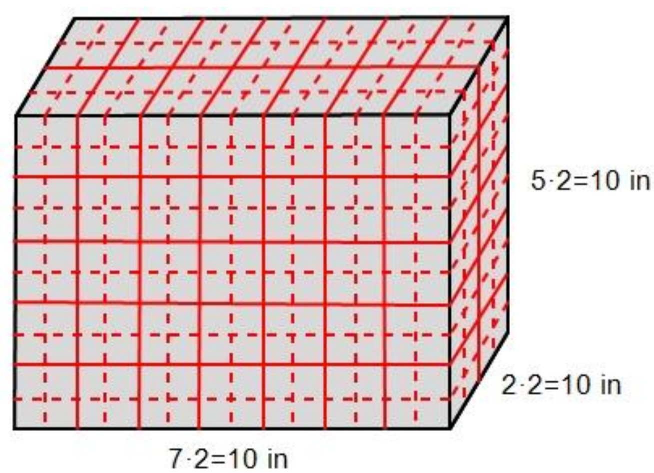


a. How many 1-inch by 1-inch dice can fit into one box?
Students may find the number of dice in the box similarly to how we derived the volume formula in lesson 5.3a. Using this method we can reason that each single layer of the prism contains 7 dice by 2 dice. This is a total of $7 \times 2 = 14$ dice. There are 5 of these layers so altogether there are $14 \times 5 = 70$ dice. Be sure to review this method for finding volume as it will help students to solve problems later on in this lesson. It is also acceptable for students to answer this question by finding the volume of the box if they can infer that a 1-inch die is one cubic inch. This can be found with the formula for volume.
$V = 7 \times 2 \times 5 = 14 \times 570 \text{ in}^3$.

b. What is the volume of the box in cubic inches?
The box is 70 in^3.

c. The company also makes mini dice that measure $\frac{1}{2}$-inch by $\frac{1}{2}$-inch. How many $\frac{1}{2}$-inch dice can fit into a rectangular box that has the same dimensions? Use the figure above to help you answer.

Ask your students to make a prediction before answering this question. Many of them might predict that there will be double the number of ½-inch dice as 1-inch dice, which is not correct. Ask them to justify their answer.



Students could reason about how many ½-inch dice will fit along the length, width, and height of the prism. For the rectangular package above, you can fit two ½-inch dice along a 1-inch length so you would need to double the given dimension to get the number of ½ inch dice along each side of the prism. $7 \times 2 = 14$ mini dice will fit along the length, $2 \times 2 = 4$ mini dice will fit across the width, and $5 \times 2 = 10$ mini dice will fit along the height. Using the same reasoning for finding volume as above, each layer of the prism now contains $14 \times 4 = 56$ mini dice. There are 10 of these layers so altogether there are $56 \times 10 = 560$ mini dice or 560 ½-inch dice.

d. What is the volume of the box when measured in ½-inch cubes?
The volume of the box is 560 ½-inch cubes.

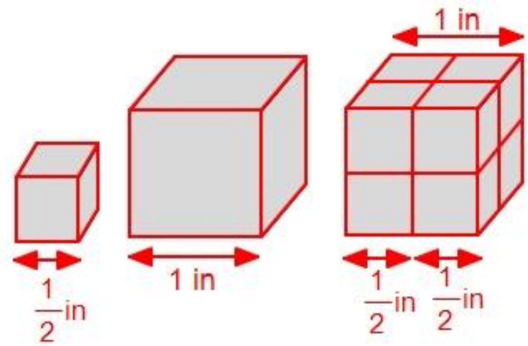
e. Why do you get two different numeric values for the volume for parts b. and d. on the previous page? Does the volume of the box the change depending on whether it is packed with 1-inch dice or ½-inch mini dice?

You get different answers because you are expressing the volume with different units, a 1-inch cubic unit and a ½-inch cubic unit. Since a ½-inch mini die is smaller than a 1-inch die, more of them will fit into the rectangular prism. However, the volume does not change regardless of what type of dice is used to fill it.

f. How can you show or prove that the number of 1-inch dice that fit into the box takes up the same amount of space or has the same volume as the number of ½-inch mini dice that fit into the box.

A sample argument is given here.

Consider how many ½-inch mini dice will fit into a single 1-inch die. The drawings given may help. You can fit two ½-in mini die along the length, width, and height of each 1-inch die. This means you can fit $2 \times 2 \times 2 = 8$ mini die into one 1-inch die or a 1-inch die takes up 8 times the amount of space that a mini die does. Its volume is 8 times bigger.



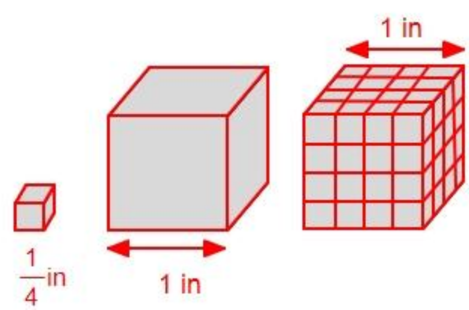
This means that there are 8 times as many ½-inch mini die that can fit into the rectangular prism than 1-inch die. If you multiply the number of 1-inch dice by 8 you get the number of ½-inch mini dice. $70 \times 8 = 560$. Thus 70 one-inch dice takes up the same amount of space or has the same volume as 560 ½-in mini dice.

g. What if you were to fill the rectangular package with ¼ inch "mini-mini" dice? How many mini-mini dice will fit into the box?

Students could reason about how many $\frac{1}{4}$ inch dice will fit along the length, width, and height of the prism. You can fit four $\frac{1}{4}$ inch dice along a 1-inch length so you would need to multiply the given dimension by four to get the number of $\frac{1}{4}$ inch dice along each side. $7 \times 4 = 28$ mini-mini dice will fit along the length, $2 \times 4 = 8$ mini-mini dice will fit across the width, and $5 \times 4 = 20$ mini-mini dice will fit along the height. Using the same reasoning for finding volume as above each layer of the prism now contains $28 \times 8 = 224$ mini-mini dice. There are 20 of these layers so altogether there are $224 \times 20 = 4{,}480$ ¼ inch "mini-mini" dice that will fit in the box.

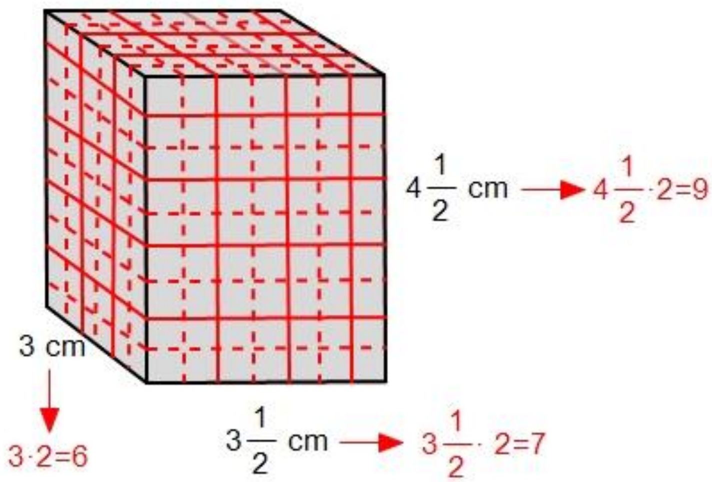
h. Show how 4,480 ¼-cubic inches is the same volume as 70 cubic inches.

You can fit $4 \times 4 \times 4 = 64$ ¼ in mini-mini die into a single 1-inch die. Thus the volume of a 1-inch dice is 64 times larger than a mini-mini die. If you multiply the volume of the rectangular prism in cubic inches by 64 you get the number of mini-mini cubes that will fit into the box. $70 \times 64 = 4480$. Thus 4,480 ¼ cubic inches is the same volume as 70 cubic inches.



Why do we care about how many ½-inch mini dice and ¼-inch mini-mini dice will fit into a rectangular prism? Sometimes the edges of a rectangular prism will not have whole number lengths but rather fractional edge lengths. Will the volume formula work for finding the volume rectangular prisms with fractional edge lengths?

2. Find the volume of the rectangular prism below by packing it with fractional unit cubes.



You must first decide what fractional unit cubes would be most appropriate to pack this prism with. Since the dimensions of the prism have fractional edge lengths with halves we can determine how many ½ in unit cubes will fit in the prism and then convert our units to 1-inch unit cubes.

To find the number of ½ inch unit cubes along each edge multiply each edge length by 2.

Length: $3\frac{1}{2} \times 2 = 7$ half inch cubes

Width: $3 \times 2 = 6$ half inch cubes

Height: $4\frac{1}{2} \times 2 = 9$ half inch cubes

Find the volume by multiplying the length, width, and height.

$V = lwh = 7 \times 6 \times 9 = 378$ half inch cubes.

Recall that there are eight 1/2-inch cubes in a single 1-inch cube. So we must divide the number of ½-inch cubes by 8 in order to determine the volume in 1-inch cubes.

$$V = 378 \div 8 = 47.25 \text{ in}^3$$

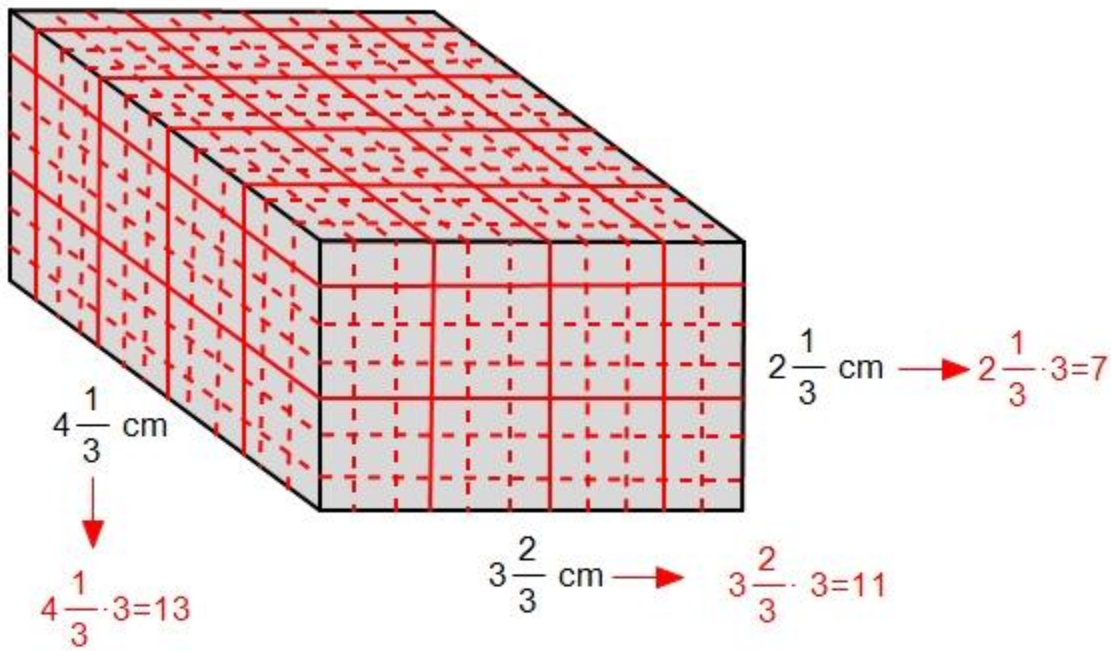
3. Find the volume of the rectangular prism by multiplying the edge lengths together; do you get the same answer as above?

$$V = lwh = 3\frac{1}{2} \times 3 \times 4\frac{1}{2} = \frac{7}{2} \times 3 \times \frac{9}{2} = \frac{189}{4} = 47.25 \text{ in}^3$$

Yes, you get the same volume by multiplying the length, width, and height.

You may want to point out that when you write the edge lengths as improper fractions, you are essentially calculating the number of ½-inch cubes that lie along the edge. For example, $3\frac{1}{2} = \frac{7}{2}$, there are 7 half inch cubes along this edge, similarly, $3 = \frac{6}{2}$ or six halves and $4\frac{1}{2} = \frac{9}{2}$ or nine halves.

4. Find the volume of the rectangular prism given by both packing it with fractional unit cubes and by using the formula. Then compare your answers.



Packing Fraction Unit Cubes Method: For this prism it would be most appropriate to pack it with $\frac{1}{3}$-inch cubes. To find the number of $\frac{1}{3}$-inch unit cubes along each edge multiply each edge length by 3.

Length: $3\frac{2}{3} \times 3 = 11$ one-third inch cubes

Width: $4\frac{1}{3} \times 3 = 13$ one-third inch cubes

Height: $2\frac{1}{3} \times 3 = 7$ one-third inch cubes

Find the volume by multiplying the length, width, and height in $\frac{1}{3}$-inch units.

$V = lwh = 11 \times 13 \times 7 = 1001$ one-third inch cubes.

In order to convert to 1 cubic inch units, we must determine the number of $\frac{1}{3}$-inch cubes there are in a single 1-inch cube. You can fit three $\frac{1}{3}$-in cubes along the length, width, and height of each 1-inch cube. This means you can fit $3 \times 3 \times 3 = 27$ one-third inch cubes into one 1-inch cube. So we must divide the number of $\frac{1}{3}$ inch cubes by 27 in order to determine the volume in 1-inch cubes.

$$V = 1001 \div 27 = 37.\overline{074}\ in^3$$

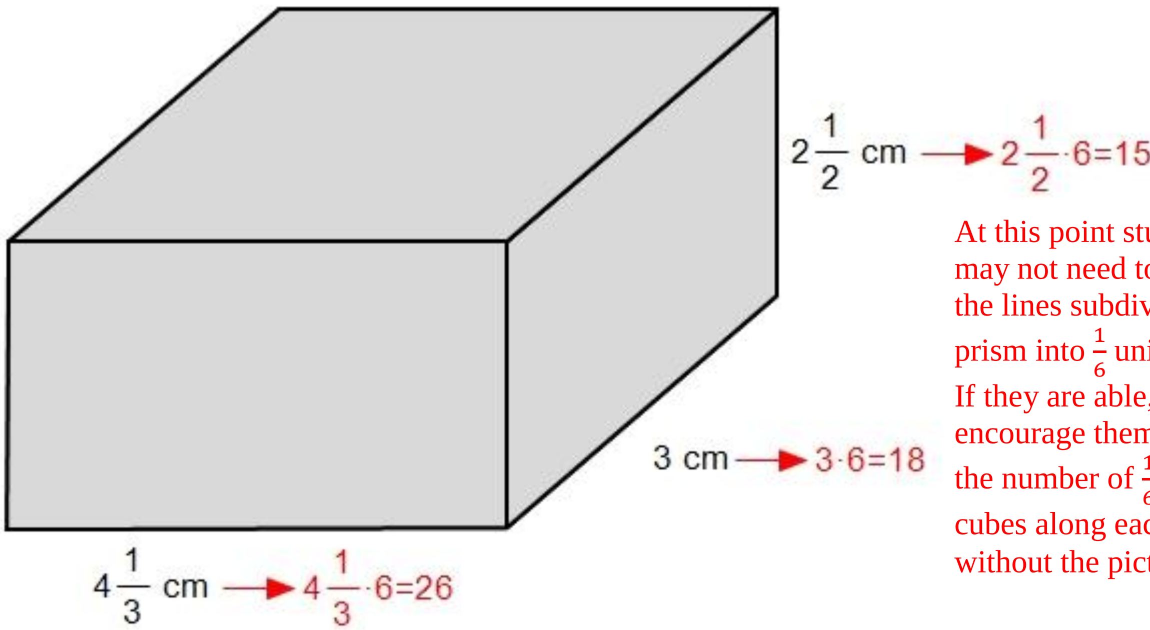
Volume Formula Method:

$$V = lwh = 3\frac{2}{3} \times 4\frac{1}{3} \times 2\frac{1}{3} = \frac{11}{3} \times \frac{13}{3} \times \frac{7}{3} = \frac{1001}{27} = 37.\overline{074}\ \text{in}^3$$

5. Are there any other fractional unit cubes that would fit inside the prism perfectly without any space leftover?

Yes, you could use $\frac{1}{6}, \frac{1}{12}, \frac{1}{24}$,... However a $\frac{1}{2}, \frac{1}{8}, \frac{1}{16}$, ... would not work.

6. Find the volume of the rectangular prism given by both packing it with fractional unit cubes and by using the formula. Then compare your answers.



At this point students may not need to draw the lines subdividing the prism into $\frac{1}{6}$ unit cubes. If they are able, encourage them to find the number of $\frac{1}{6}$ unit cubes along each edge without the picture.

Packing Fraction Unit Cubes Method: For this prism, it would be most appropriate to pack it with $\frac{1}{6}$ inch cubes since the prism has edges lengths with halves and thirds. To find the number of $\frac{1}{6}$ inch unit cubes along each edge, multiply each edge length by 6.

Length: $4\frac{1}{3} \times 6 = 26$ one-third cm cubes

Width: $3 \times 6 = 18$ one-third cm cubes

Height: $2\frac{1}{2} \times 6 = 15$ one-third cm cubes

Find the volume by multiplying the length, width, and height in $\frac{1}{6}$ inch units.

$V = lwh = 26 \times 18 \times 15 = 7020$ one-third inch cubes.

In order to convert to 1 cubic inch units we must determine the number of $\frac{1}{6}$ inch cubes there are in a single 1-inch cube. You can fit six $\frac{1}{6}$ in cubes along the length, width, and height of each 1-inch cube. This means you can fit $6 \times 6 \times 6 = 216$ one-sixth inch cubes into one 1-inch cube. So we must divide the number of $\frac{1}{6}$ inch cubes by 216 in order to determine the volume in 1-inch cubes.

$$V = 7020 \div 216 = 32.5 \text{ in}^3$$

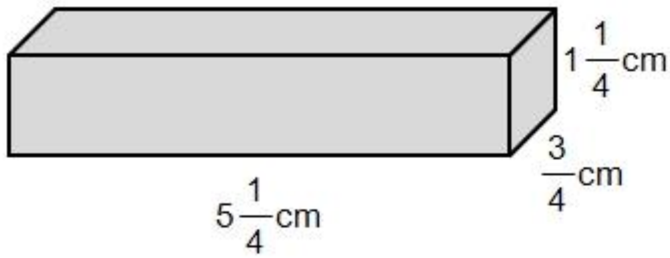
Volume Formula Method:

$$V = lwh = 4\frac{1}{3} \times 3 \times 2\frac{1}{2} = \frac{13}{3} \times 3 \times \frac{5}{2} = \frac{195}{6} = 32.5 \text{ in}^3$$

7. Which method do you prefer to use to find the volume of a rectangular prism with fraction edge lengths? Answers will vary.

5.3b Homework: Finding Volume of Rectangular Prisms with Fractional Edge Lengths

1. Use the rectangular prism given to answer the questions that follow.



a. Find the volume of the rectangular prism by using the volume formula.

b. If you fill the prism with cubes whose sides lengths are less than 1 cm what size would be best?

c. How many of these smaller cubes will fit into a single 1 cm cube?

d. List the number of these smaller cubes that will fit along each edge length of the rectangular prism.

length:

width:

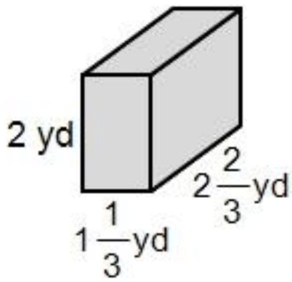
height:

e. How many of these smaller cubes will fill the prism?

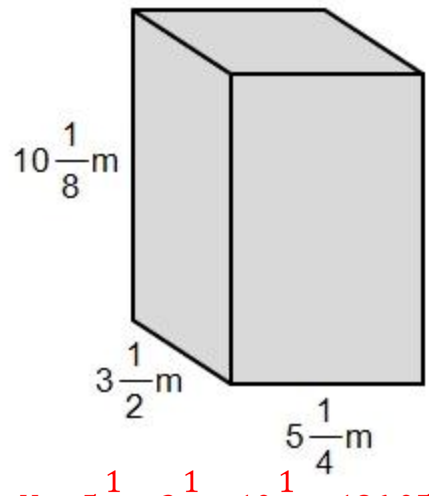
f. Use the relationship between the number of cubes and the volume to prove that the volume that you found in part a. is correct.

2. Find the volume of each right prism using any method you want. Round your answer to the nearest hundredth.

a.



b.



$V = 5\frac{1}{4} \times 3\frac{1}{2} \times 10\frac{1}{8} = 186.05\text{ m}^3$

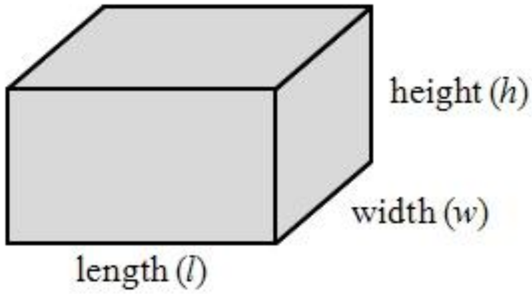
3. A rectangular prism is $\frac{5}{8}$ ft by $1\frac{1}{8}$ ft by $2\frac{1}{4}$ ft. Find the volume of the given prism using both methods. If needed, draw a picture. Round your answer to the nearest hundredth.

4. The volume of a rectangular prism is $109\ \frac{3}{8}\ m^3$, it has a width of $5\frac{1}{4}$ meters and a length of $6\frac{2}{3}$ meters. What is the height of the prism?

The height is $3\frac{1}{8}$ meters.

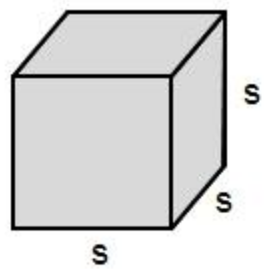
5.3c Class Activity: Finding Volume of Rectanuglar Prisms with Fraction Edge Lengths using Formulas

1. Write and explain the formulas for the volume of a rectangular prism.



$V = lwh$ where l is the length of the prism, w is the width of the prism, and h is the height of the prism. The volume of a prism is the product of its length, width, and height.
$V = Bh$, where B is the area of the base of the prism and h is the height of the prism.
The volume of a prism is the product its base area and its height.

2. Write and explain the formula for the volume of a cube. Explain how this formula relates to the formula for the volume of a rectangular prism.
$V = s^3$ where s is the length of a cube.
A cube is a rectangular prism where the length, width, and height are all the same measurement. This means that you only need one variable to represent each of the dimensions algebraically. If we choose s to represent the distance of the length, width, and height then the volume for a cube is $V = s \cdot s \cdot s = s^3$.



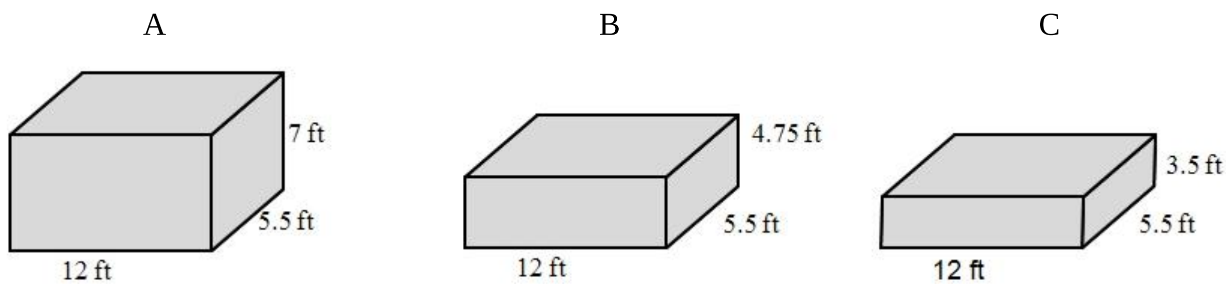
3. Find the volume of each rectangular prism with the given dimensions. Be sure to show your work.

1. A rectangular prism that measures 12 meters by 2 ½ meters, by $3\frac{3}{5}$ meters. $V = 12 \times 2\frac{1}{2} \times 3\frac{3}{5} = 108\ \text{m}^3$ Dicuss why it does not matter if the given dimensions are not specified as the length, width, and height because the commutative property of multiplication allows us to multiply in any order.	2. A rectangular prism with a length of 1 ft, a width of $1\frac{1}{3}$ ft, and a height of $\frac{1}{2}$ ft. $V = 1 \times 1\frac{1}{3} \times \frac{1}{2} = \frac{2}{3}\text{ft}^3$	3. A cube with a side length of 0.75 yards. $V = 0.75 \times 0.75 \times 0.75 = 0.421875\ \text{yd}^3$

4. A rectangular prism has a base area of 170.5 ft^2. It has a height of $20\frac{1}{4}$ ft.
   a. Draw and label a picture of the rectangular prism.
   See student answer.

   b. Find the volume of the prism. Round your answer to the nearest hundredth.
   $V = Bh = 170.5 \times 20\frac{1}{4} = 3{,}452.63\text{ft}^3$

5. Use the figures given to answer the questions that follow.



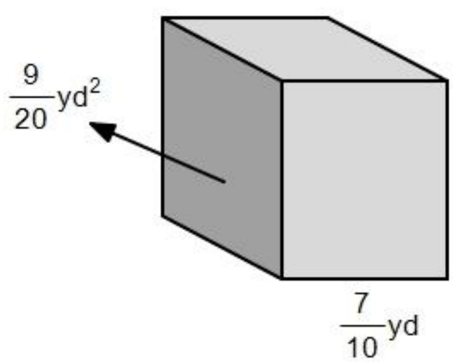
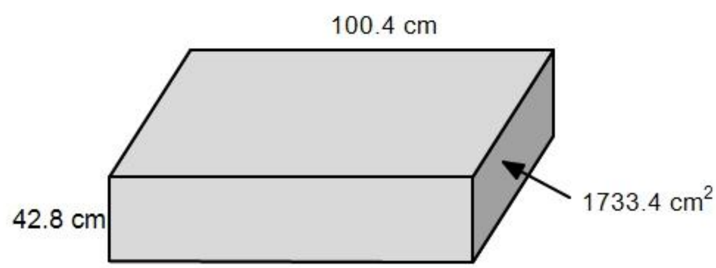
   a. Without calculating the volume, order the rectangular prisms from least to greatest based off of how many cubic centimeters they will hold. Explain your reasoning.
   Prism C, Prism B, Prism A; all of the prisms have the same base area so you can order the prisms based off of their heights. Prism C has a height of 7 ft, Prism B has a height of 4.75 ft, and Prism A has a height of 3.5 ft.

   b. How many times larger is the biggest prism than the smallest prism. Justify your answer.
   Prism C is 2 times larger than Prism A because their base areas are the same and Prism C's height of 7 is two times Prism A's height of $3\frac{1}{2}$. So Prism C will hold 2 times as many centimeter cubes.

5. A rectangular prism has a width that is $2\frac{1}{2}$ times its height and a length that is $\frac{3}{4}$ of its height. The height of the prism is 4 feet.
   a. Draw and label a picture with the information that you know.
   See student answer.

   b. Find the width and length of the prism.
   width: $4 \times 2\frac{1}{2} = 10$ ft
   length: $\frac{3}{4} \times 4 = 3$ ft
   c. What is the volume of the prism?
   $V = 3 \times 10 \times 4 = 120\ \text{ft}^3$

6. Find the volume of each rectangular prism given.

a.	b.
 $V = \frac{63}{200} = 0.315\ \text{yd}^3$	 $V = 174{,}033.36\ \text{cm}^3$

7. You are building a box to hold firewood in your shed. You have 2 ½ square feet of floor space in your shed for the box and would like the box to hold 6 cubic feet of wood.

   a. How high must the box be?
   The box must have a height of $2\frac{2}{5}$ ft.

   b. List two possible sets of measurements you use for the length and width of the box.
   Students may list any two numbers whose product is 2 ½ square feet.

8. Olivia claims that $V = s^3$, where s is the side length of a cube is the formula you should use to find the volume of a cube. Harrison claims that the correct formula is $V = lwh$, where l is the length, w is the width, and h is the height of the cube.

   a. Who is correct? Why or why not.
   They are both correct because they are essentially the same formula. Since the length, width, and height of a cube are all the same measurement, you can use one variable to represent them as is the case with Olivia's formula. When you find the volume, you multiply this same number by itself three times.

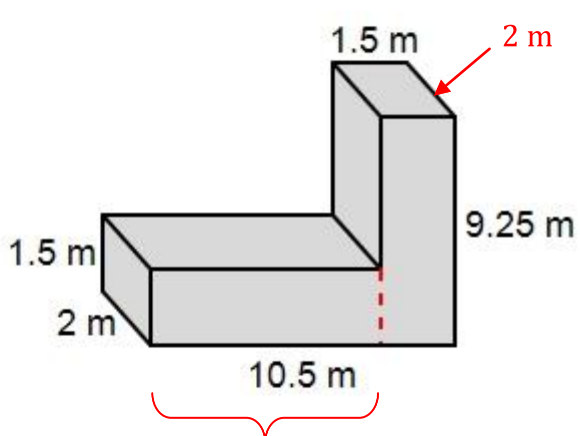
   b. Find the volume of a cube with a side length of $4\frac{3}{4}$ in. Round your answer to the nearest hundredth.
   $V = \left(4\frac{3}{4}\right)^3 = 107.17\text{ in}^3$

Problems 9, 10, and 11 may be challenging for your students; you might consider making them bonus problems.

9. Find the volume of the composite figures given. Assume that angles that appear to be right angles are right angles.

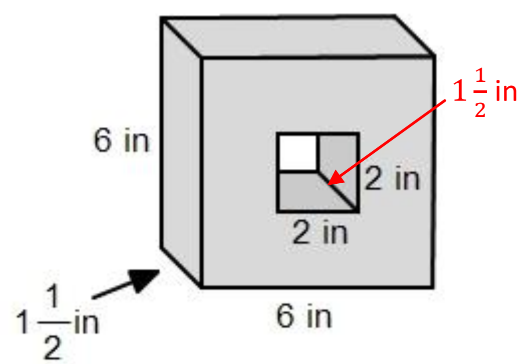
Encourage students to come up with a strategy for finding the volume before they begin to make calculations. This may begin by labeling missing dimensions on the figures and marking how they would break the solid up into composite solids.

a.



You can find the volume of the whole solid by adding the volumes of each composite figure together.
The volume of the rectangular prism on the left is $V = 9 \times 2 \times 1.5 = 27\text{ m}^3$.
The volume of the rectangular prism on the right is $V = 1.5 \times 9.25 \times 2 = 27.75\text{ m}^3$.
The volume of the entire figure is $27 + 27.75 = 54.75\text{ m}^3$.

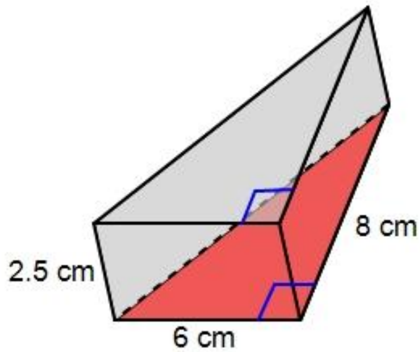
b.



You can find the volume of this solid by subtracting the volume of the small rectangular prism in the middle from the volume of the larger rectangular prism.
The volume of the larger rectangular prism is $V = 6 \times 6 \times 1\frac{1}{2} = 54\text{ in}^3$.
The volume of the smaller rectangular prism is $V = 2 \times 2 \times 1\frac{1}{2} = 6\text{ in}^2$.
The difference between them and volume of the solid given is $54 - 6 = 48\text{ in}^3$.

For number 10 below, some students might benefit from being able to manually touch a triangular prism in order to identify the parallel bases. You can make one out of cardstock or there are some packages that are the shape of a triangular prism.

10. Consider the triangular prism with a base in the shape of a right triangle. The triangular base has a base length of 6 cm and a height of 8 cm. The height of the prism is 2.5 cm. Use what you know about how to find the volume of a rectangular prism to find the volume of the triangular prism.



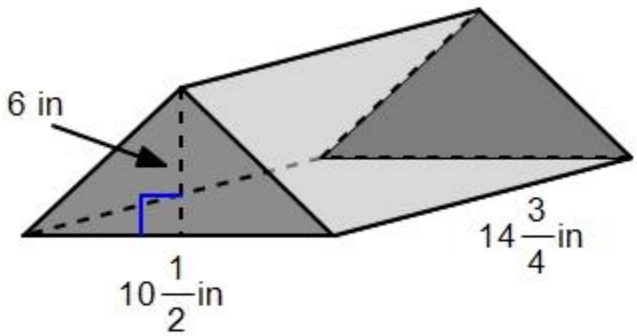
$$V = Bh = \left(\frac{1}{2} \times 6 \times 8\right) \times 2.5 = 120 \text{ cm}^3$$

Have a discussion about how to find the volume of the triangular prism. Some talking points are given.

- We know that you can find the volume of a rectangular prism by finding the area of the base layer and then by multiplying by the number of layers or the height. This will work for a triangular prism as well.
- Shade the base of the prism. What is the area of the base of this prism?
- What is the height of the prism or how many triangular layers are there?
- What if the triangular prism was turned on its side? Can you use any face of the prism as the base? Why or why not?
- Which volume formula works for finding the volume of a triangular prism? $V = lwh$ or $V = Bh$? Why?

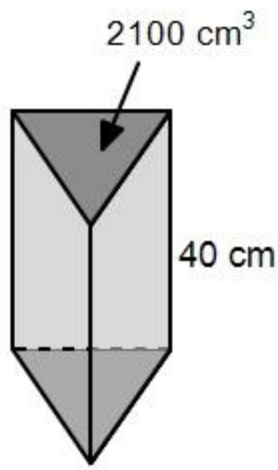
11. Find the volume of each right prism given; their bases have been shaded.

a.



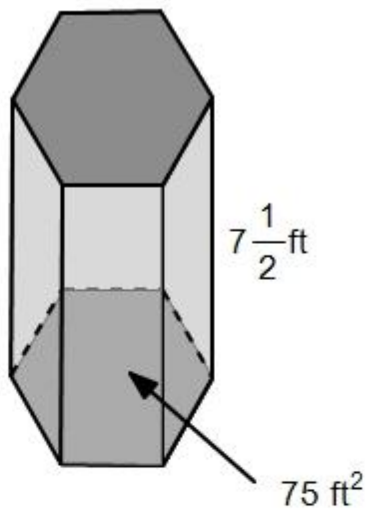
$V = Bh = \left(\frac{1}{2} \times 10\frac{1}{2} \times 6\right) \times 14\frac{3}{4} = 464.625 \text{ in}^3$

b.



$V = Bh = 2100 \times 40 = 84{,}000 \text{ cm}^3$

c.



$V = Bh = 75 \times 7\frac{1}{2} = 562.5 \text{ ft}^3$

5.3c Homework: Finding Volume of Rectanuglar Prisms with Fraction Edge Lengths using Formulas

1. Find the volume of each rectangular prism with the given dimensions. Round your answers to the nearest hundredth.

a. A rectangular prism that measures 10 meters by 3 ½ meters, by $5\frac{1}{4}$ meters.	b. A rectangular prism with a length of 2 ft, a width of $3\frac{1}{3}$ ft, and a height of $\frac{2}{3}$ ft. $V = 2 \times 3\frac{1}{3} \times \frac{2}{3} = 4.44\text{ft}^3$	c. A cube with a side length of 1.45 yards.

2. A rectangular prism has a base area of 70 ft^2. It has a height of $13\frac{1}{3}$ ft.
   a. Draw and label a picture of the rectangular prism.
   See student answer.

   b. Find the volume of the prism.
   $V = Bh = 70 \times 13\frac{1}{3} = 933\frac{1}{3}\text{ft}^3$

3. A rectangular prism has a width that is $3\frac{1}{2}$ times its height and a length that is $\frac{1}{3}$ of its height. The height of the prism is 9 feet.
   a. Draw and label a picture with the information that you know.

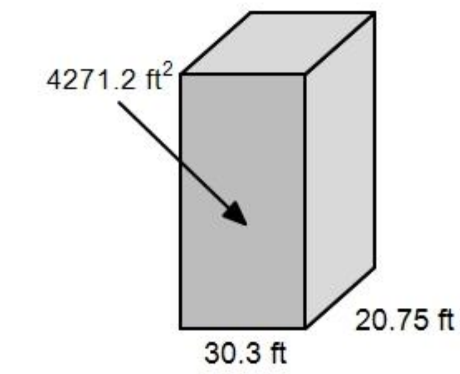
   b. Find the width and length of the prism.
   width:

   length:

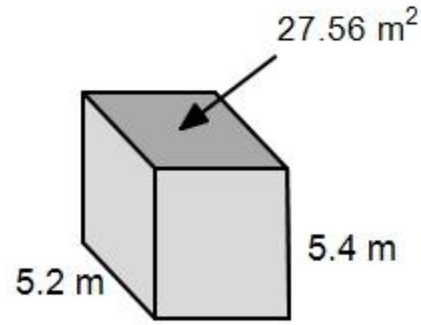
   c. What is the volume of the prism?

4. Find the volume of each rectangular prism given.

a.



b.



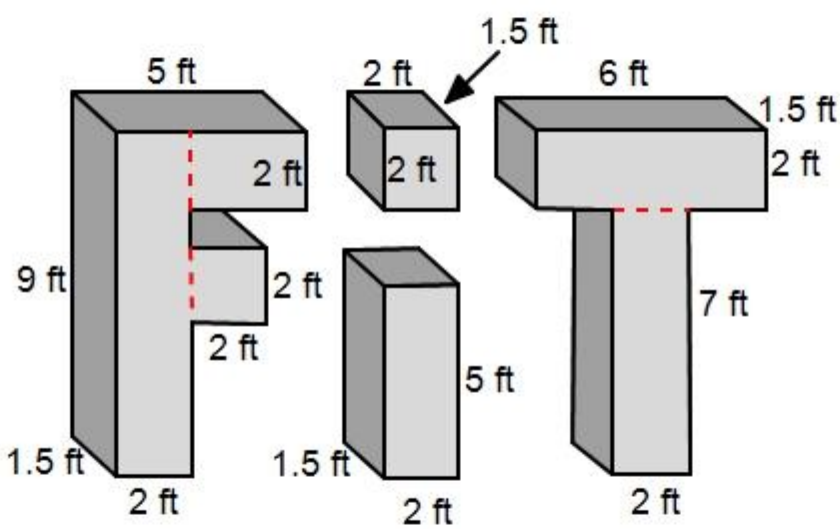
$V = 27.56 \times 5.4 = 148.824 \text{ m}^3$

5. You have a garden box in your backyard that covers 28 square feet and has a height of $\frac{3}{4}$ of a foot.

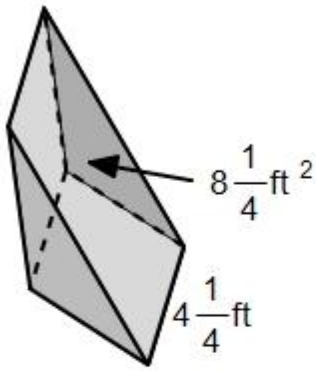
   a. How much soil will you need to fill the box up completely?
   You will need 21 ft^3 of soil to fill the box.

   b. List two possible sets of measurements that could be the length and width of the garden box.
   Students may list any two numbers whose product is 28 square feet.

6. Find the volume of the block word. Assume that angles that appear to be right angles are right angles.

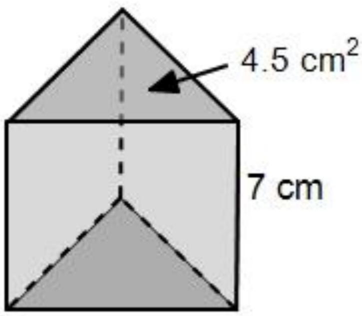


7. Find the volume of the right prism; its bases have been shaded.

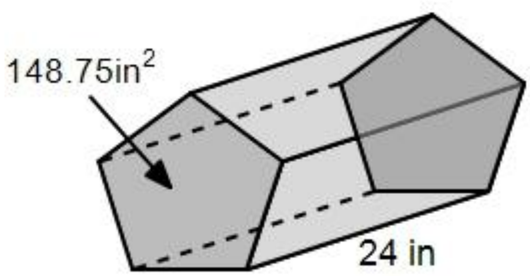


$V = Bh = 8\frac{1}{4} \times 4\frac{1}{4} = 35.0625 \text{ ft}^3$

8. Find the volume of the right prism; its bases have been shaded.

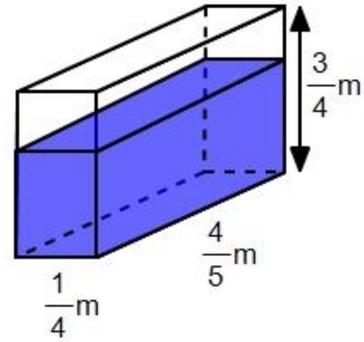


9. Find the volume of the right prism; its bases have been shaded.



5.3d Class Activity: Finding Volume in Real World Applications

1. You have just purchase a fish tank that is a rectangular prism.



   a. How much water can the fish tank hold?

   The tank can hold $\frac{3}{20}$ m^3.

   b. If you fill the tank $\frac{2}{3}$ of the way full, how much water will be in the tank?

   For this problem students must determine what the height of the water will be if the tank is $\frac{2}{3}$ full. This means they need to find $\frac{2}{3}$ of ¾ of a meter. $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$. Two thirds of the tank's height is ½ a meter. You can then multiply the tank's length, width, and water height to find out how much water is in the tank.

   There is $\frac{1}{10} m^3$ of water in the tank.

   c. You would like to add a rock feature to the tank. This feature has a volume of $\frac{1}{30} m^3$, do you have enough room in the tank to add the feature without any water spilling over?

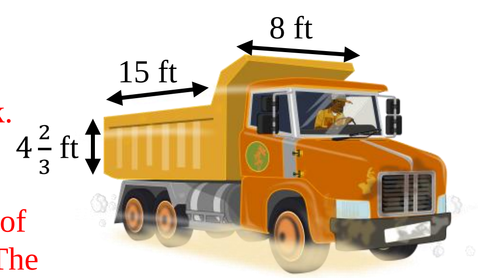
   In order to determine if the rock feature will fit you must find out how much space is left in the tank. You can do this by either finding the volume of the empty space or by subtracting the volume of the water from the volume of the tank.

   The volume of the tank minus the volume of the water is $\frac{3}{20} - \frac{1}{10} = \frac{1}{20}$; this means that there is $\frac{1}{20}$ of a cubic meter of space left over. The rock feature has a volume of $\frac{1}{30}$ which is smaller than $\frac{1}{20}$ so the feature will fit into the tank without any water spilling over.

2. A box of tissues has a volume of 1625 cm^3. Its length is 20 centimeters and its height is 6.5 centimeters. Find the width of the box of tissues.

   The width of the tissue box is 12.5 cm.

3. One cubic foot of gravel weighs 70 pounds.



   a. How many pounds of gravel can the dump truck haul?

   You must first find the volume of the bed of the dump truck.

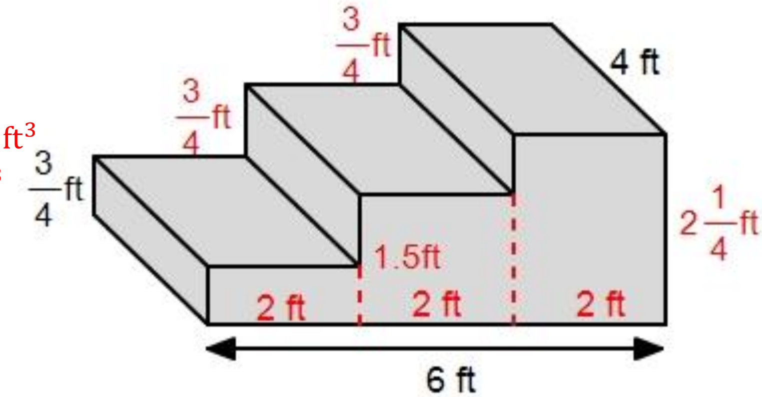
   $$V = 15 \times 8 \times 4\frac{2}{3} = 560 \text{ ft}^3$$

   Then you can multiply the volume by 70 to get the number of pounds of gravel the truck will hold. $560 \times 70 = 39{,}200$. The truck will hold 39,200 pounds of gravel.

   b. A sign posted in front of a bridge states that the bridge's weight capacity is 20 tons. One ton is equivalent to 2000 pounds. Can the dump truck safely cross the bridge with a full load of gravel?

   You must take into account the weight of the truck as well as the weight of the dirt. Since the weight of the dirt is 39,200 pounds, or $39200 \div 2000 = 19.6$ tons, which is less than 20 tons, but not by much, it is probably not safe to cross the bridge.

4. Gloria is planning on pouring a set of concrete cement steps on the side of her front porch. She has drawn out a diagram of the steps below where the "rise" and "run" of each step is equal.
   a. Determine the total amount of cement she will need for the steps. Assume that angles that appear to be right angles are right angles.



Volume of the first step: $V = \frac{3}{4} \times 4 \times 2 = 6 \text{ ft}^3$
Volume of the second step: $V = 1.5 \times 4 \times 2 = 12 \text{ ft}^3$
Volume of the third step: $V = 2\frac{1}{4} \times 4 \times 2 = 18 \text{ ft}^3$
Total volume: $V = 6 + 12 + 18 = 36 \text{ ft}^3$

Students use the understanding that you need to subdivide the steps into composite figures that you can find the volume of as an entry point for solving this problem. The total volume is the sum of the composite volumes. This explanation acts as a "road map" for solving the problem, rather than just jumping right into making calculations that they do not understand.

   b. The cement costs $85 per cubic yard plus $60 delivery fee. Determine how many cubic yards of cement you will need and then find the total cost.
   There are 27 cubic feet in one cubic yard. This means that Gloria has a total of $36 \div 27 = \frac{4}{3}\text{yd}^3$. At $85 per cubic yard it will cost her $85 \times \frac{4}{3} = 113.33$ dollars to purchase the cement plus $60 for the delivery fee. This is a total of $113.33 + 60 = \$173.33$.

5. Leo's recipe for banana bread won't fit in his favorite pan. The batter fills the 8.5 inch by 11-inch by 1.75 inch pan to the very top, but when it bakes it spills over the side. He has another pan that is 9 inches by 9 inches by 3 inches, and from past experience he thinks he needs about an inch between the top of the batter and the rim of the pan. Should he use this pan?

Once again, explicitly outlining a "road map" for solving this problem helps students understand why and how certain calculations will help them reach the intended goal. Ask them to share their "road maps" before solving.

In order to find out how high the batter will be in the second pan, we must first find out the total volume of the batter that the recipe makes. We know that the recipe fills a pan that is 8.5 inch by 11-inch by 1.75 inches. We can calculate the volume of the batter multiplying the length, the width, and the height:

$$V = 8.5 \text{ in} \times 11 \text{ in} \times 1.75 \text{ in} = 163.625 \text{ in}^3$$

We know that the batter will have the same volume when we pour it into the new pan. When the batter is poured into the new pan, we know that the volume will be $9 \times 9 \times h$ where h is the height of the batter in the pan. We already know that $V = 163.625\ in^3$, so:

$$V = l \times w \times h$$
$$163.625 \text{ in}^3 = 9 \text{ in} \times 9 \text{ in} \times h$$
$$163.625 \text{ in}^3 = 81 \text{ in}^2 \times h$$
$$\frac{163.625 \text{ in}^3}{81 \text{ in}^2} = h$$
$$2.02 \text{ in} \approx h$$

Therefore, the batter will fill the second pan about 2 inches high. Since the pan is 3 inches high, there is nearly an inch between the top of the batter and the rim of the pan, so it will probably work for the banana bread (assuming that Leo is right that that an inch of space is enough).
**This is an Illustrative Mathematics Task*

5.3d Homework: Finding Volume in Real World Applications

1. A sandbox is to be built in a children's museum exhibit to replicate an archeological digging site. The space allocated for the sandbox measures $6\frac{1}{2}$ ft by $5\frac{3}{4}$ ft . The museum has ordered $28\frac{1}{32}$ cubic feet of sand to fill the box.

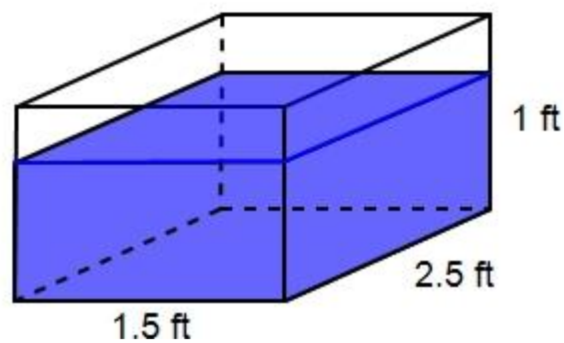
   a. How high should the walls of the sandbox be so that the sand will fit into the box perfectly?
   The walls need to be at least ¾ of a foot high.

   b. After the first day of children digging in the sandbox, the sand is measured and it only goes up to a height of ½ ft. How much sand fell out of the box?
   $9\ \frac{11}{32}\ \text{ft}^3$ of sand fell out of the box.

   c. What kind of adjustments, if any, do you think the museum should make to the archeological dig site exhibit?
   If the museum does not want to clean up over 9 cubic feet of sand off the floor each day, they should make the walls of the sandbox higher. They might also need to think about a drainage system to easily wash the sand off the floor and the children's hands as well.

2. A computer tower has a volume of 1728 in^3. Its width is 16 inches and its length is 6 inches. What is the computer tower's height?

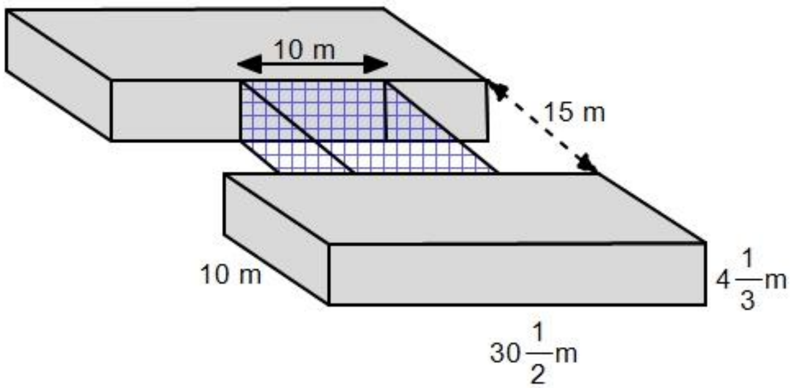
3. A fish tank in the shape of a rectangular prism is set on a folding table. The dimensions of the tank are shown.



   a. How much water can the tank hold?

   b. One cubic foot of water weights 64.4 pounds. The folding table is rated to hold 200 pounds. Can the table safely hold the fish tank when it is full of water?

4. The shape of a small office building is made up of two identical rectangular prisms connected by an enclosed glass walkway as shown.



a. Find the total amount of volume that the building takes up including the glass walkway.
The total volume is founding by summing the volumes of each composite part of the building.
Volume of the rectangular prism on the left: $V = 4\frac{1}{3} \times 30\frac{1}{2} \times 10 = 1321\frac{2}{3}\text{m}^3$.
Volume of the rectangular prism on the right: $1321\frac{2}{3}\text{m}^3$. This is the same as the one on the left since they are identical.
Volume of the glass walkway: $V = 10 \times 15 \times 4\frac{1}{3} = 650\text{m}^3$
Total volume of the building: $V = 650 + 1321\frac{2}{3} + 1321\frac{2}{3} = 3293\frac{1}{3}\text{m}^3$

b. On average it costs \$0.45 to air condition one cubic meter of space per month. How much will it cost the owner of the building to cool the building each month?
Multiply the number of cubic meters by 0.45 to get the total cost. $3293\frac{1}{3} \times 0.45 = 1482$
It will cost the owner \$1482 to air condition the building each month.

5. Roberto is making Jello to take to a family dinner. He has a 9 in $\times$ 13 in $\times$ 3 in glass dish with a layer of pineapple chunks in the bottom. He pours the Jello into the dish to a height of 2.5 inches. He then remembers that his mom does not like pineapple so he decides to spoon the pineapple out of the Jello before he puts it into the refrigerator to set up. The Jello drops to a height of 2 inches.

a. How much pineapple did he take out of the Jello?

b. He had 100 in^3 of whipped cream that he would like to spread across the top of the Jello after it has set up. Will the whipped cream fit inside the dish in order for Roberto to put a lid on top of it to transport to his mother's house without any spilling over?

Section 5.4: Surface Area

Section Overview:
In this section students investigate surface area and how nets can help you find surface area. They begin by constructing a net for a right rectangular prism. They learn how to decompose a three dimensional figure and represent it on a two dimensional surface by drawing its net composed of rectangles and triangles. Students learn that nets can also be used to identify the name of right prisms and pyramids by identifying the shape of the base. Once students are comfortable constructing nets they use them to find the surface area of its corresponding three-dimensional figure. Finally in the last lesson of this section students solve real-world and mathematical problems that relate to surface area and volume.

Concepts and Skills to Master in this Section:

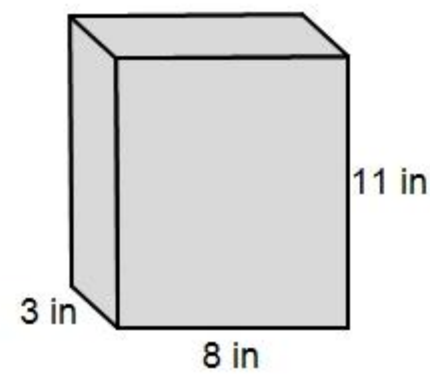
By the end of this section, students should be able to:

1. Identify and name right prisms and pyramids
2. Represent a three-dimensional figure using a net made up of rectangles and triangles.
3. Use a net to find the surface area of a three-dimensional figure.
4. Solve real-world and mathematical problems relating to surface area and volume.

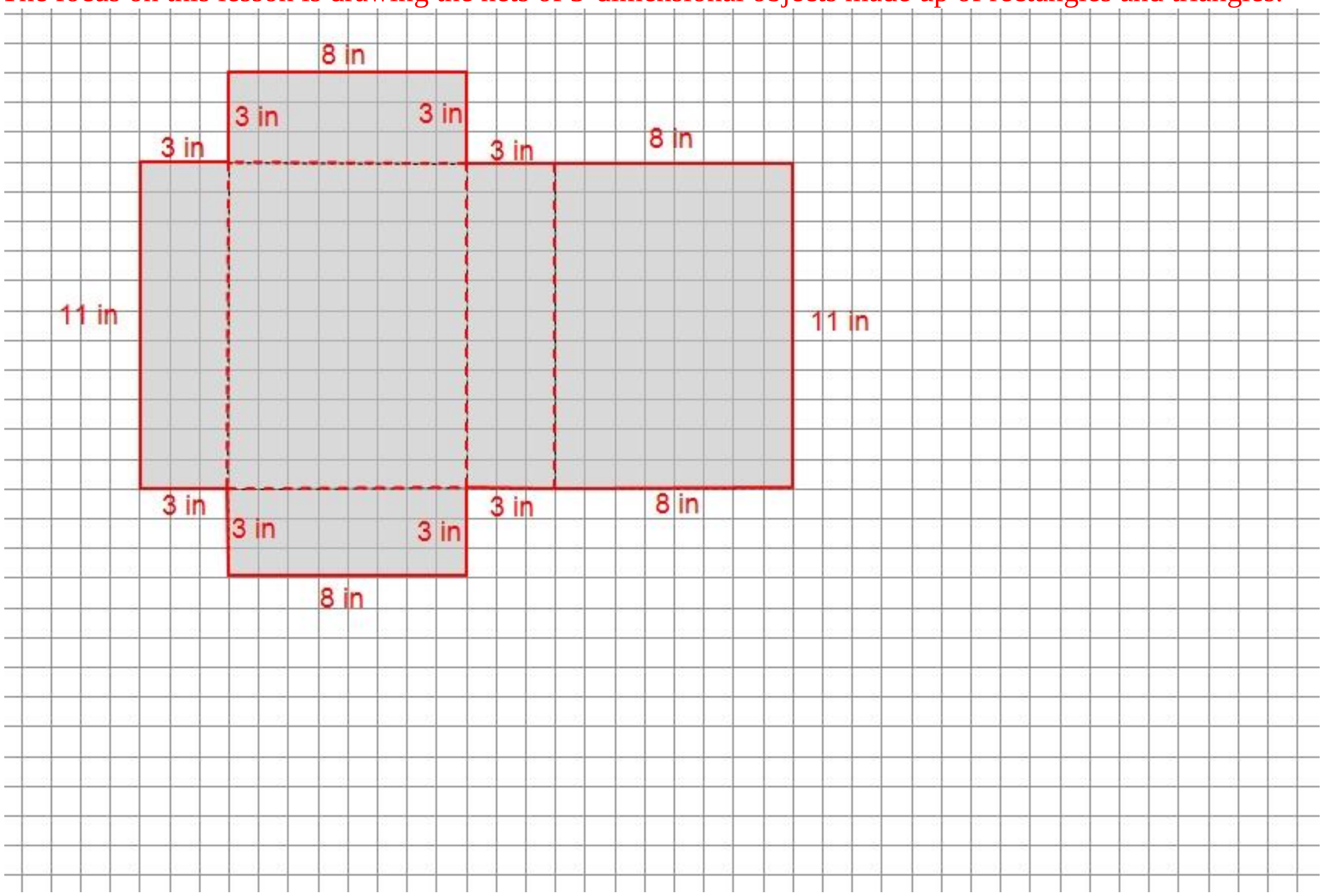
5.4a Class Activity: Nets of 3-Dimensional Figures

1. You have been asked by a popular breakfast cereal company to redesign the outside surface of their top selling cereal box. The dimensions of the box are shown.

   You need to send your ideas of what to put on each face of the box back to the company via email. Use the graph paper below and work with your group to figure out a way that you can show each side of the box on a flat two-dimensional surface.

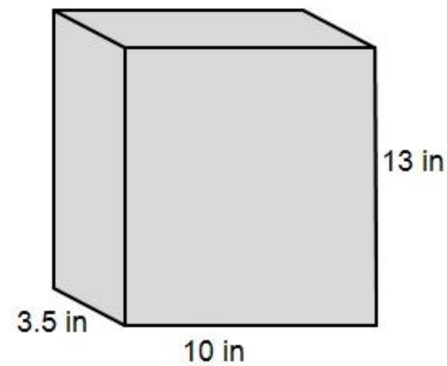


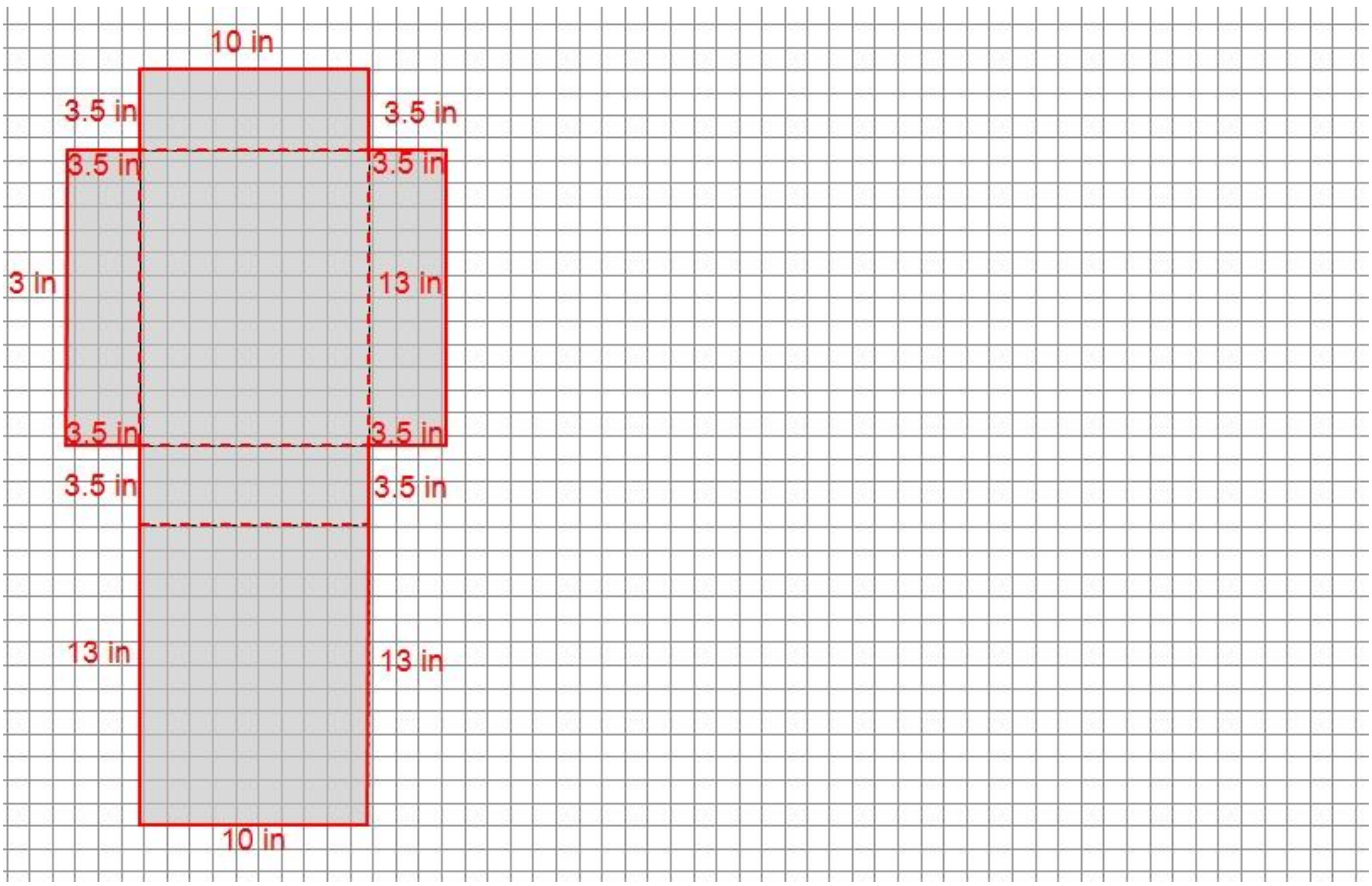
Talk about how there are many nets that will work for a rectangular prism as long as the dimensions are labeled appropriately. Finding the surface area of the cereal box will be discussed in the next lesson.
The focus on this lesson is drawing the nets of 3-dimensional objects made up of rectangles and triangles.



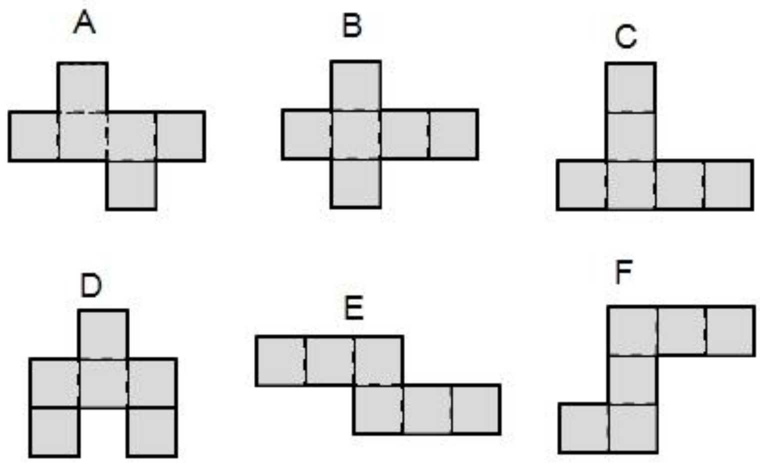
A two-dimensional drawing of a three-dimensional figure is called a **Net**. A net allows you to see all of the faces of a three-dimensional object on a two-dimensional surface.

2. Draw a net for the "Family Size" cereal box shown. Try to draw the net differently than you did for the cereal box described in number 1. Be sure to label all the dimensions.





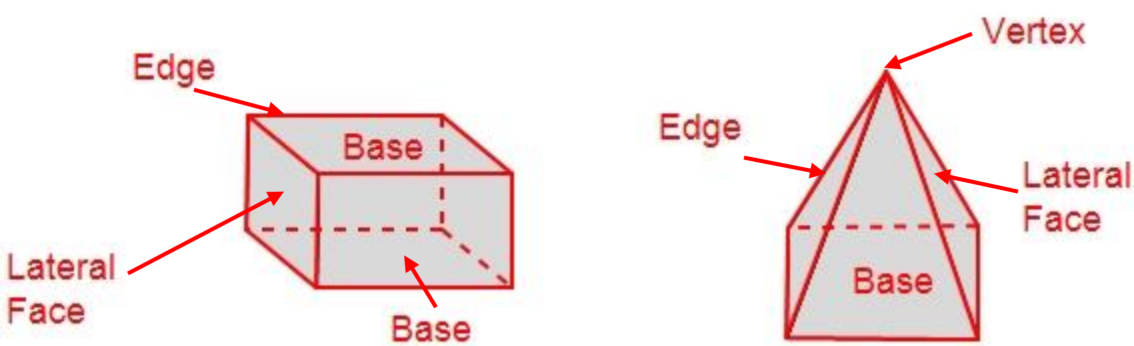
3. Which of the following Nets can be folded up to form a cube?
Nets A, B, and E will form cubes



Explore different prisms and pyramids using actual models.

- Discuss key features about the solids including faces, lateral faces (sides), edges, vertices, and bases.
- Compare and contrast prisms and pyramids.
- Talk about how a prism has two parallel faces that are the same shape. These are called the bases of the prism. The lateral faces of a prism are parallelograms.
- Pyramids are composed of a base shape, and triangles (lateral faces) that meet at a common vertex.
- Pyramids have only one base and prisms have two bases.
- The name of a prism or pyramid is determined by the shape of its base.

4. Draw an example of a prism and pyramid. Identify and label key features such as faces, edges, vertices, and bases based off of your discussion.

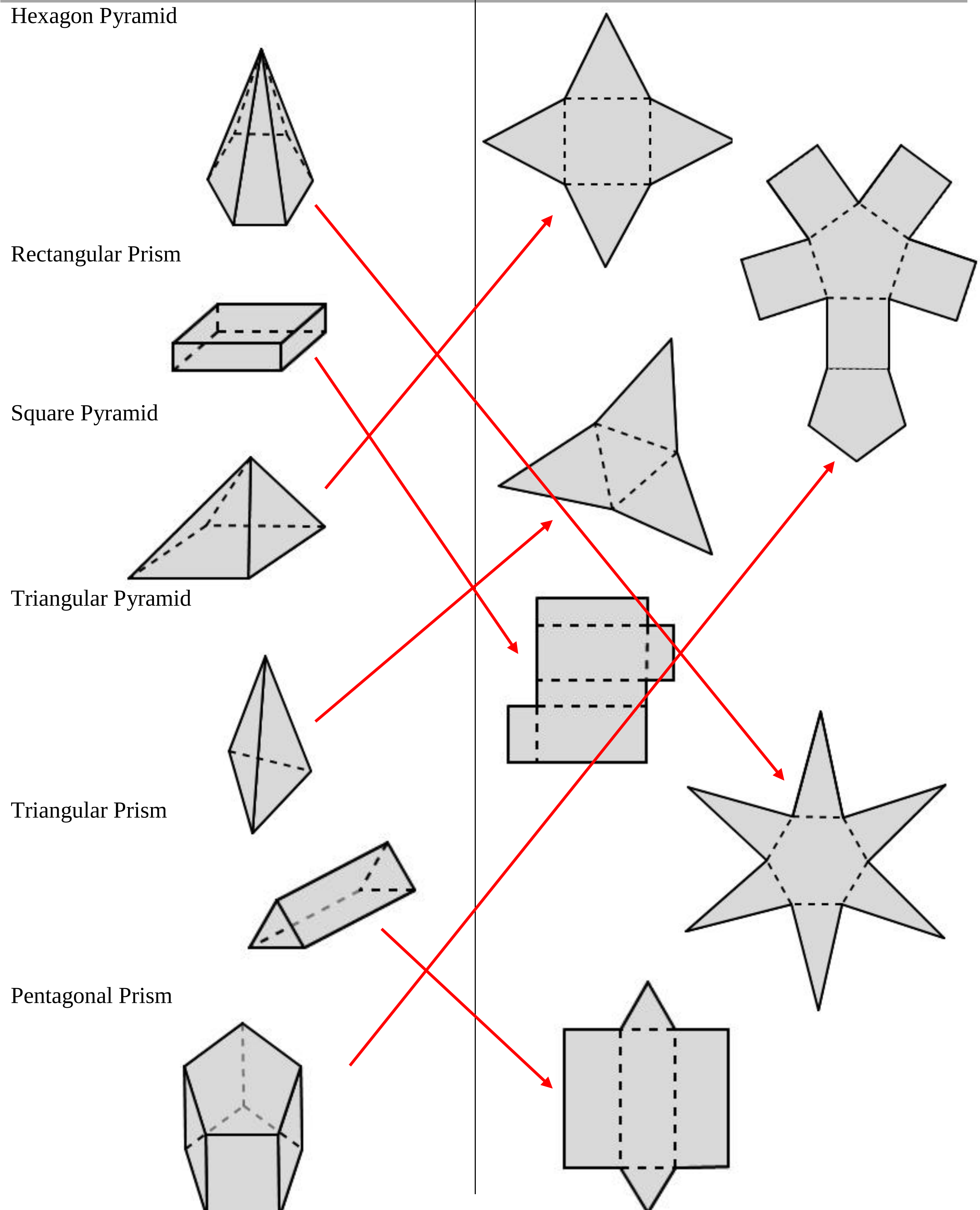


You can explore nets with online interactive manipulatives as you investigate the problems below. Some online interactives can be found at the links given.

http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.SURF&lesson=html/object_interactives/surfaceArea/use_it.html
https://www.learner.org/interactives/geometry/3d_prisms.html
https://www.turtlediary.com/game/nets-of-3d-shapes.html (note: this site calls right rectangular prisms cuboids)

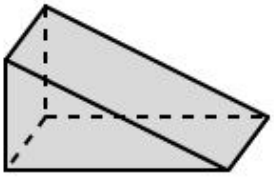
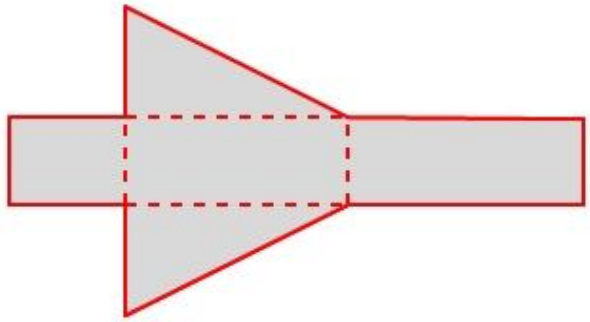
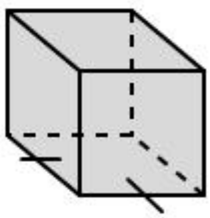
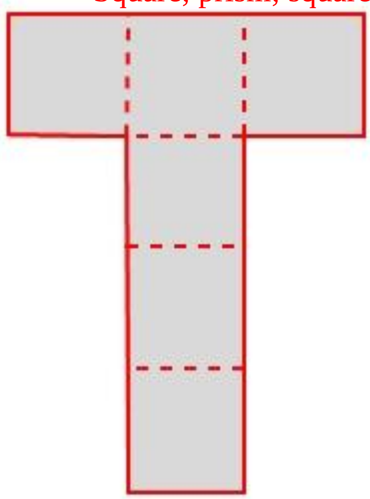
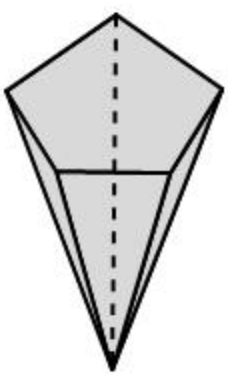
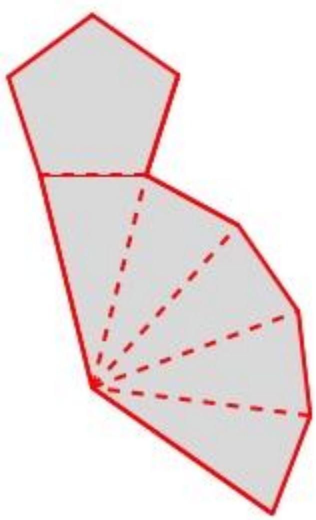
5. Draw a line to match each solid with its net. If students struggle ask them to shade the base(s) on the solids and the nets.

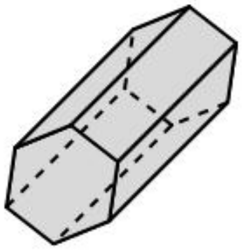
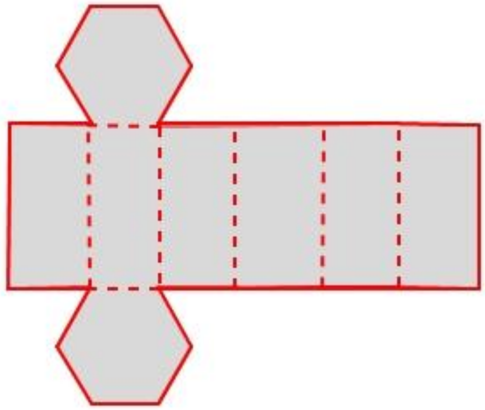
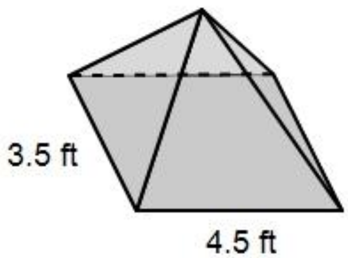
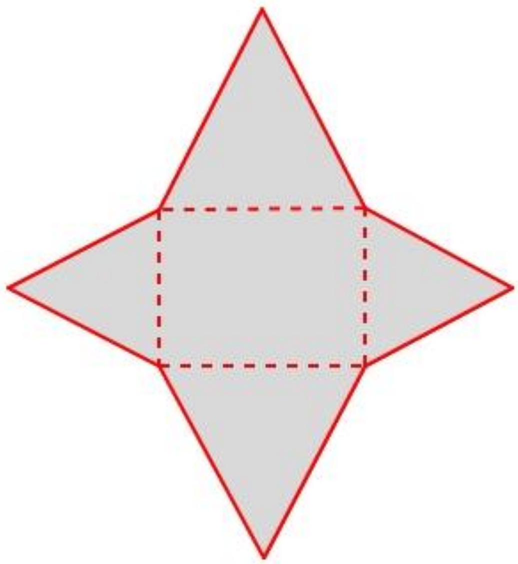
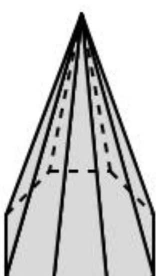
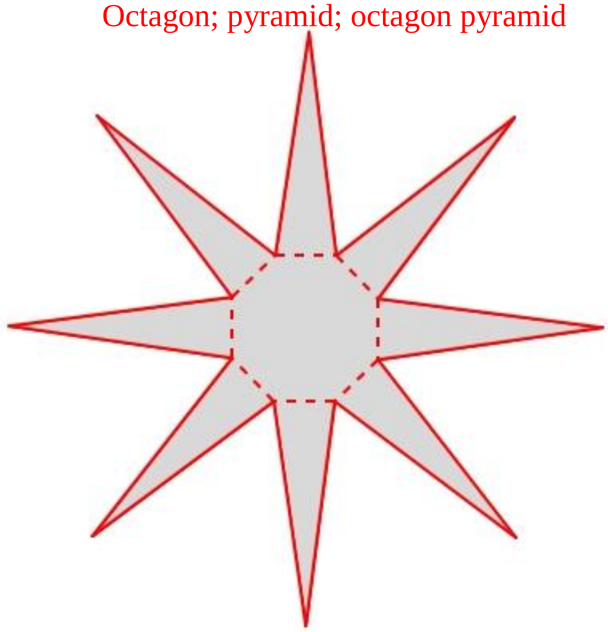
Solid	Net



Sketch the net for each solid given. Then identify the shape of the base(s), whether it is a prism or pyramid, and write the name of the solid.

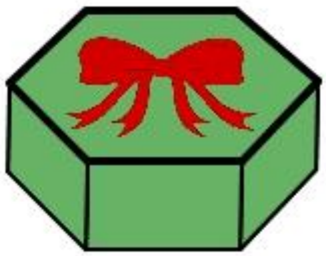
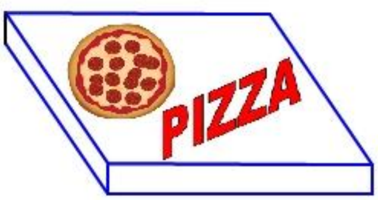
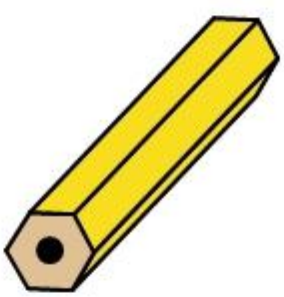
Answers will vary for nets, sample answers are given.

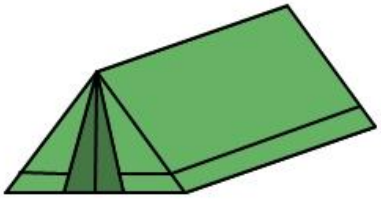
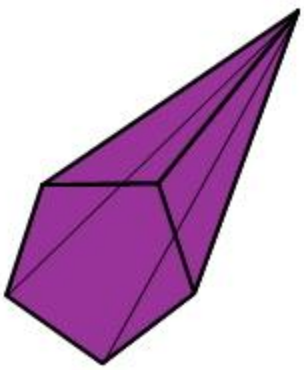
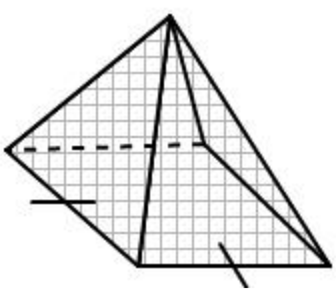
6. 	Triangle; prism; triangular prism 
7.  Remind students that the markings on the base of this prism indicate that these sides have the same length.	Square; prism; square prism/rectangular prism/cube 
8. 	Pentagon, pyramid, pentagonal pyramid 

9. 	Hexagon; prism; hexagonal prism 
10. 	Rectangle, pyramid, rectangular pyramid 
11. 	Octagon; pyramid; octagon pyramid 

5.4a Homework: Nets of 3-Dimensional Figures

Sketch the net for each solid given. Then identify the shape of the base(s), whether it is a prism or pyramid, and write the name of the solid.

1. Holiday Chocolate Tin 	
2. Pizza Box 	
3. Unsharpened pencil 	See student answer for net; hexagon, prism, hexagonal prism

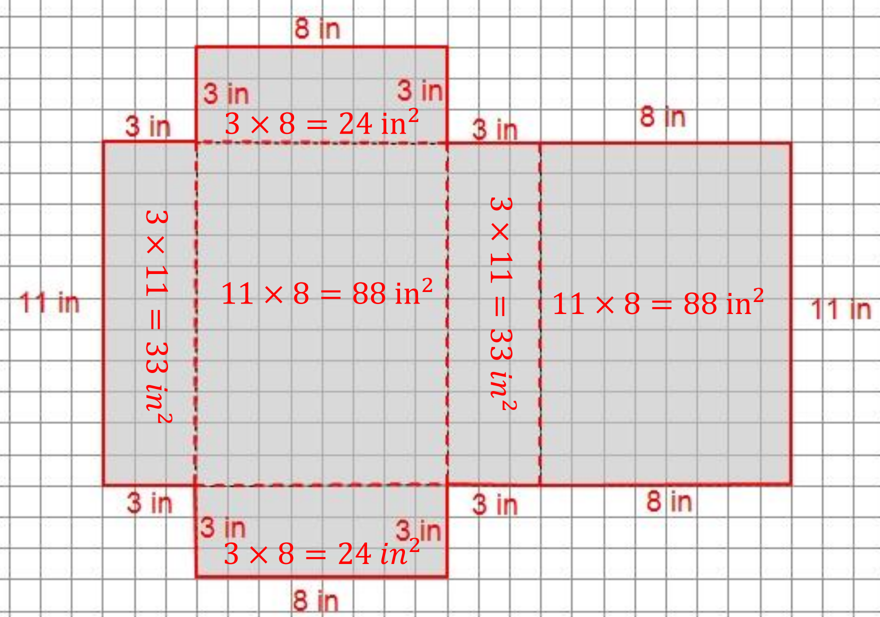
4. Tent 	
5. Crystal 	
6. The top of a stone monument. 	See student answer for net, square, pyramid, square pyramid

5.4b Class Activity: Finding Surface Area

1. Once again you are working on redesigning the packaging of a cereal box for a cereal company. Now they would like to know how much ink they will need to print your new designs on the outside of the box. In order to know how much ink they need they would like to know how much area the outside of the box takes up. Use the work that you did with the cereal box in the previous lesson to answer this question.

Ask your student how the net that they made of the cereal box in the previous lesson can help them solve this problem.

Have students share their ideas for finding how much area the outside of the box takes up. Talk about how you can find the total area by adding up the area of the individual parts of the box as seen on the net. Some of these composite parts of the net are the same size, what does this mean about their areas? When you find the space that the outside of a figure takes up it is called Surface Area. Although we are dealing with a 3D shape, we are finding the AREA of the surface, not the volume, so the units of measure will be in^2, cm^2, etc. instead of in^3, cm^3, etc.



Area of the "tall skinny" rectangle (side of the cereal box): $3 \times 11 = 33\ in^2$
Area of the other "tall skinny" rectangle (other side of the box) is the same as the other "tall skinny" rectangle (we know this because they have the same length and width): $3 \times 11 = 33\ in^2$

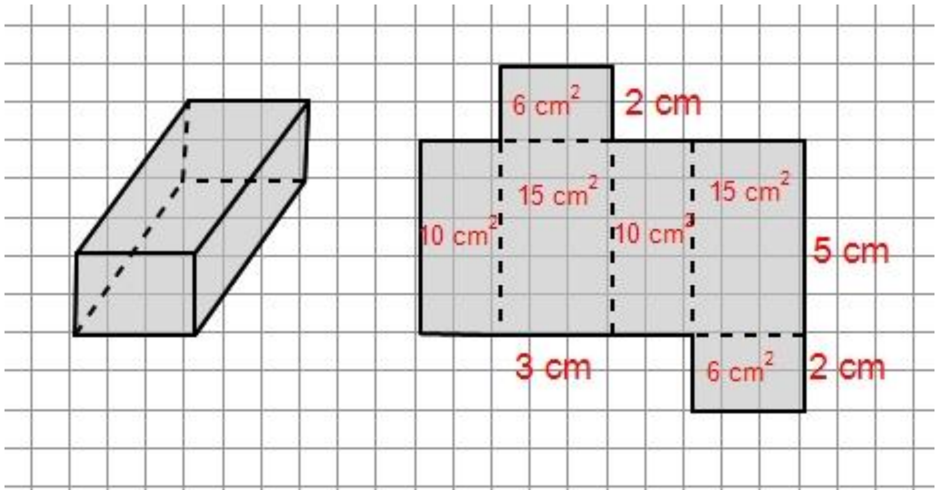
Area of the top rectangle (top flap of the cereal box): $3 \times 8 = 24\ in^2$
Area of the bottom rectangle (bottom flap) is the same as the top flap (we know this because they have the same length and width): $3 \times 8 = 24\ in^2$

Area of the front rectangle (front of the cereal box): $11 \times 8 = 88\ in^2$
Area of the other big rectangle (back of the box) is the same as the front rectangle (we know this because they have the same length and width): $11 \times 8 = 88\ in^2$

Sum of the individual areas: $24 + 24 + 88 + 88 + 33 + 33 = 290\ in^2$
The total area of the entire box is 290 in^2

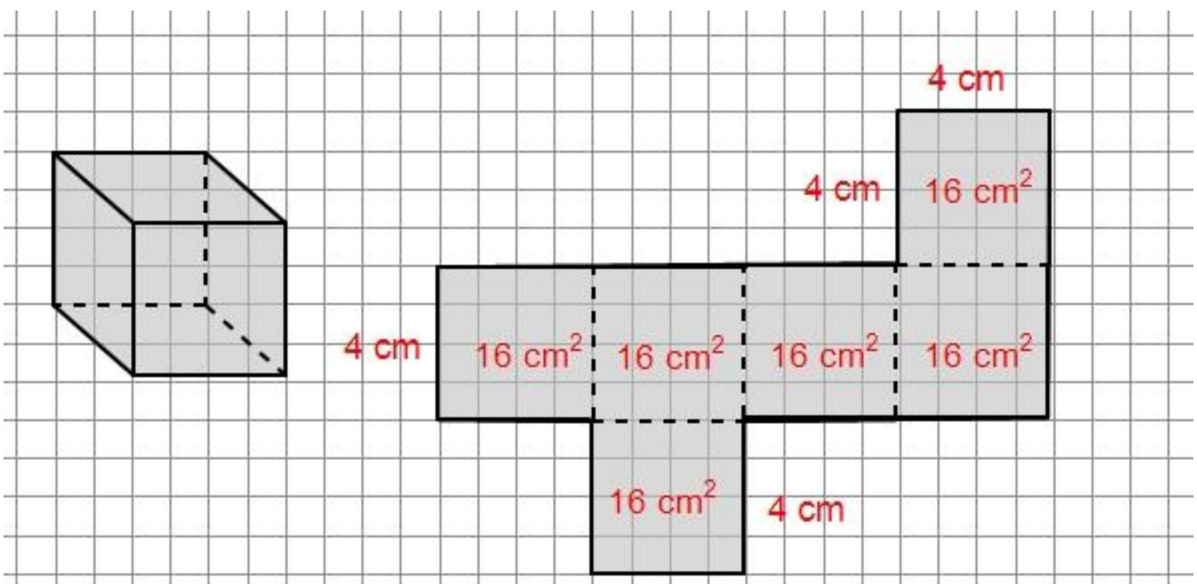
2. The nets for three different solids are shown. One square on the graph represents one square centimeter. Label any useful dimensions on each net, state the name of each solid, and find its surface area.
Sample labels are shown; students may choose to label the side lengths of the composite figures on the net any way they find to be helpful.

a.



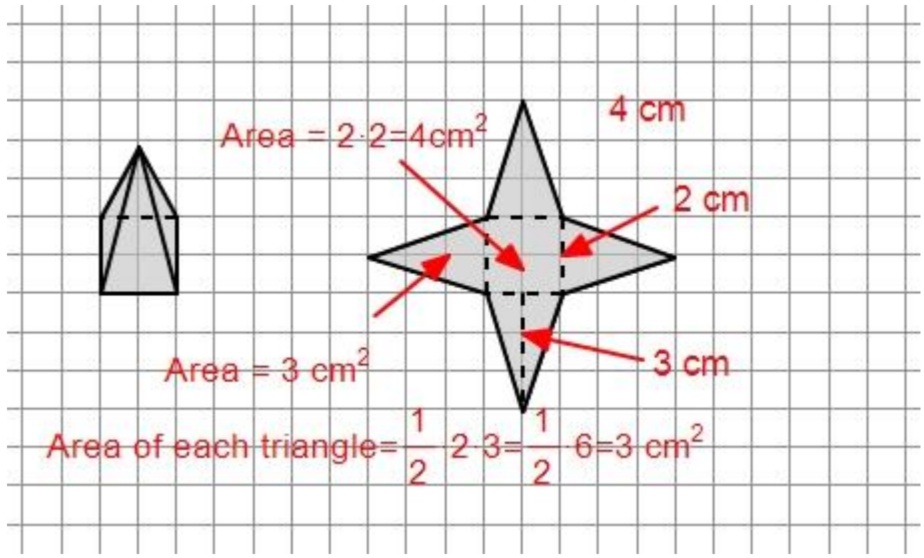
Rectangular prism; the surface area is 62 cm^2. Ask students to share their ideas for finding the surface area; it is the sum of the individual areas of the composite parts of the net.
$SA = 6 + 10 + 15 + 10 + 15 + 6 = 62 \text{ cm}^2$ or $SA = 2(6) + 2(10) + 2(15) = 12 + 20 + 30 = 62 \text{ cm}^2$.

b.



Cube/rectangular prism; the surface area is 96 cm^2.
Discuss how you can find the surface area of the cube by either summing the individual square areas or by multiplying the area of one 4×4 square by 6 since there are 6 faces that are all the same size. Talk about the expressions that arise from each method.
$SA = 16 + 16 + 16 + 16 + 16 + 16 = 6(16) = 96$
If you feel that your students are ready, ask them if they can develop a formula for finding the surface area of a cube. It is not required that students know the formula for the surface area of a cube.
$SA = s^2 + s^2 + s^2 + s^2 + s^2 + s^2 = 6s^2$, where s is the side length of the cube.

c.



Square Pyramid; the surface area is 16 cm^2

Discuss how you can find the surface area of the square pyramid by summing the areas of the composite shapes within its net. Students may initially struggle to find the area of the triangles. Ask them to share their ideas and how you can find the area of the triangles by finding their height and base lengths.

$SA = 3 + 3 + 3 + 3 + 4 = 12 + 4 = 16\ cm^2$ or $SA = 3(4) + 4 = 12 + 4 = 16\ \text{cm}^2$.

Students should begin to pick up on a process for finding surface area that will work for any three-dimensional solid. For students that struggle, it may be helpful if you ask them to articulate their process. For example, ask them to explain their process verbally. They might say that "to find the surface area of this pyramid you find the area of all the triangles and the area of the square and add them together." Ask them to do this for each different solid, and through repetition they will pick up on the "process' for finding the surface area of any solid as described in number 3 below.

3. Explain in your own words what surface area is and how to find the surface area of a three-dimensional object.

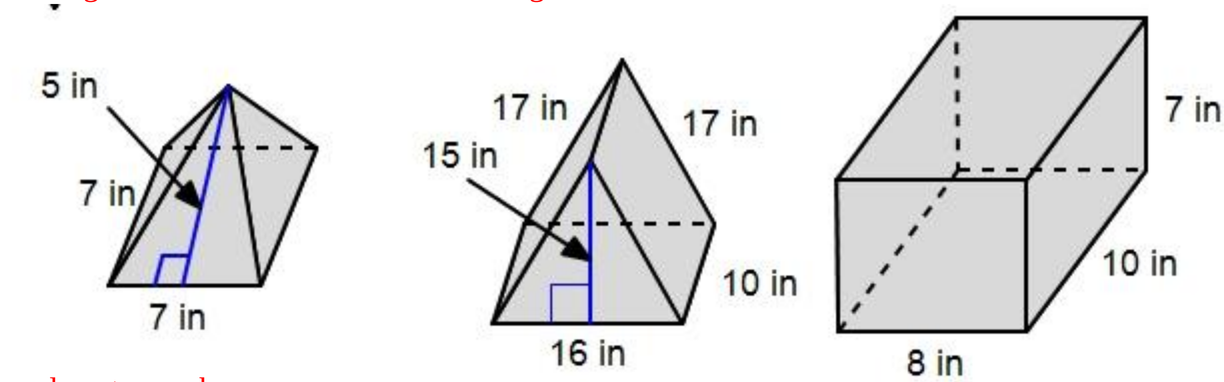
   Surface area is the area of the surface of a three-dimensional solid. It is found by finding the sum of individual face areas of the three-dimensional solid.

It is not required in 6th grade that students know or use the formulas to find the surface area of rectangular prisms, cubes, and other polyhedra. However, some of these formulas may evolve naturally as your students find the surface area of these objects repeatedly. For example it may be that students point out that for every rectangular prism they can double the area of each unique face (because each face has another face that is exactly the same size and shape as itself) and then add these areas together. This process can be represented algebraically with the surface area formula for a rectangular prism.

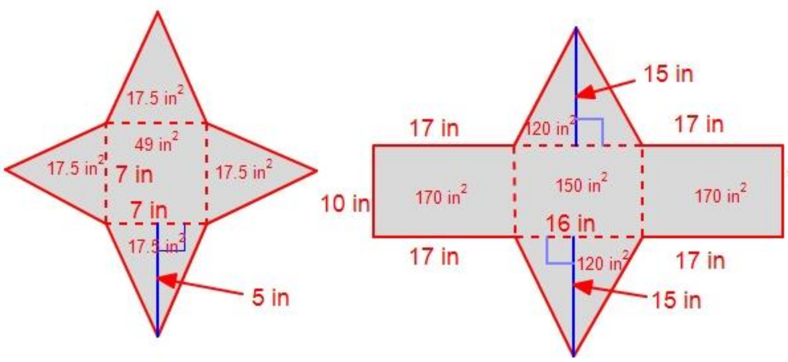
$SA = 2lw + 2lh + 2wh,$ Where l is the length of the prism, w is the width of the prism, and h is the height of the prism.

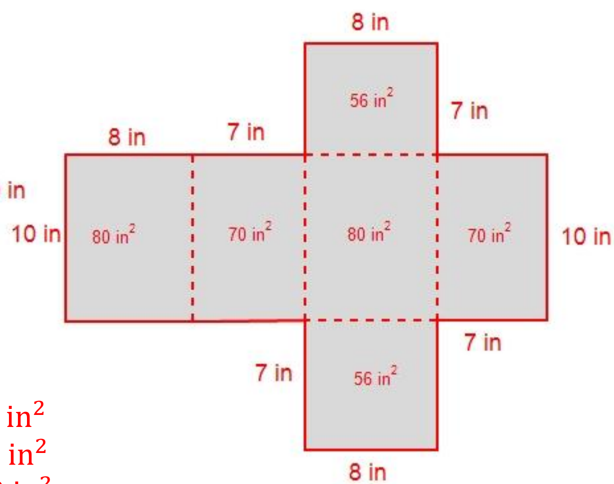
In the problems above, students were able to determine the dimensions of the composite parts of the nets by counting out the lengths on the graph paper. In the problems on the next page, students must transfer dimensions from the given solid to the net in order to calculate the surface area. Be mindful that this adds another level of difficulty for students.

4. Julio and Peggy's teacher has asked them to make the 3D figures below out of one sheet of poster board. The poster board measures 36 inches by 24 inches, will they have enough paper to make one of each 3D figure? If so how much paper will be left over? If not how much more paper will they need?
Encourage students to draw a net of each figure to find its surface area.



Sample nets are shown.





Square Pyramid: $SA = 49 + 17.5 + 17.5 + 17.5 + 17.5 = 119 \text{ in}^2$
Triangular Prism: $\text{SA} = 170 + 170 + 120 + 120 + 150 = 730 \text{ in}^2$
Rectangular Prism: $\text{SA} = 80 + 80 + 70 + 70 + 56 + 56 = 412 \text{ in}^2$
Combined Surface Area of all three figures: $SA = 119 + 730 + 412 = 1261 \text{ in}^2$.
The total area of the poster board: $26 \times 24 = 864\ in^2$
Julio and Peggy will not have enough poster paper to make all three figures. They will need 397 in^2 more paper in order to make all of the figures.

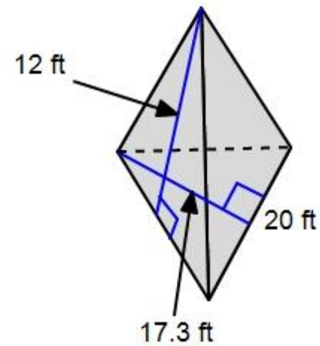
At this point some students are more comfortable finding the surface area with a net. However, many students may not need to draw the net to find the surface area. Ask them to share their methods, such as making a table that lists each composite shape and their area, or drawing each composite shape and labeling its corresponding area next to it. Encourage students to use whichever method they are most comfortable with to keep track of the areas of the subfigures.

5. The top of a stone monument is the shape of a triangular pyramid; it is removed to be repainted.
   a. How much paint is needed to cover all of the faces of the pyramid? The side lengths of the base of the pyramid are all equal.

      $SA = 3\left(\frac{1}{2} \cdot 20 \cdot 12\right) + \left(\frac{1}{2} \cdot 20 \cdot 17.3\right) = 3(60) + 173$

      $= 360 + 173 = 533 \text{ ft}^2$

      You will need 533 ft^2 of paint.
   b. Paint costs $2.50 per square foot, how much will it cost to paint the pyramid?
      It will cost $1332.50 to paint the pyramid.



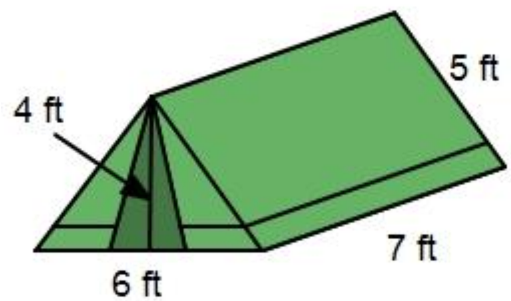
6. How much canvas fabric is needed to replace the canvas on the tent shown?

   $SA = 2\left(\frac{1}{2} \cdot 6 \cdot 4\right) + 6 \cdot 7 + 2(5 \cdot 7)$

   $= 2(12) + 42 + 2(35) = 24 + 42 + 70$

   $= 136 \text{ ft}^2$

   You will need 136 ft^2 of canvas for the tent.



7. A small gift box measures 5 inches by 5 inches by 6 ½ inches. What is the least amount of paper needed to wrap the box? If needed draw and label a picture.

This problem allows students to reason about finding surface area without the aid of a picture. See if your student can explain how to find the surface area by using only words to describe their process.

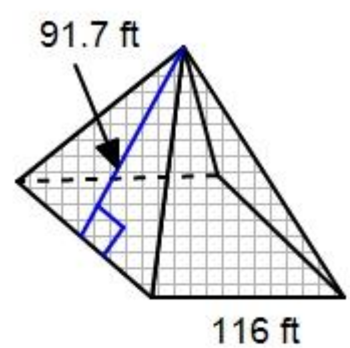
   $2(5 \times 5) + 2\left(5 \times 6\frac{1}{2}\right) + 2\left(6\frac{1}{2} \times 5\right) = 2(25) \times 2(32.5) \times 2(32.5) = 50 + 65 + 65 = 180\ in^2$

   You will need at least 180 in^2 of paper to cover the box. However, if you wrap the box using the traditional method by folding the paper over on itself of the ends of the box you will need much more paper.

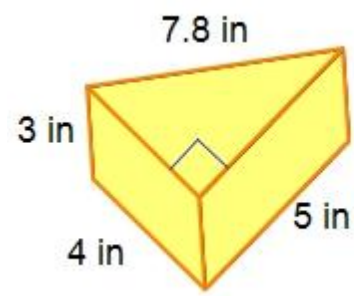
8. The entrance to the Louvre museum in Paris, France is a square pyramid made of glass. The side length of the base of pyramid is 116 feet and the height of one of the triangular faces is 91.7 feet. Find the surface area of it four triangular faces.

   $SA = 4\left(\frac{1}{2} \cdot 116 \cdot 91.7\right) = 4(5{,}318.6) = 21{,}274.4\text{ft}^2$

   The surface area of the entrance to the Louvre Museum is 21, 274.4 ft^2



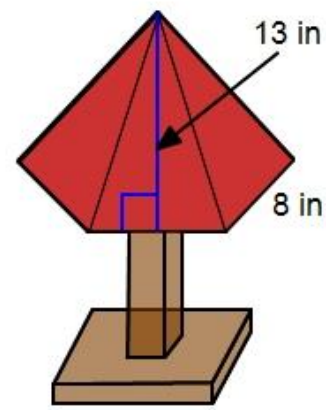
9. A cheese company produces wedges of cheese that are the shape of a triangular prism. Each wedge of cheese needs to be covered with a wax coating for preservation. How much wax coating will be needed to cover 100 wedges of cheese?



$$SA = 2\left(\frac{1}{2} \cdot 4 \cdot 5\right) + 3 \cdot 4 + 3 \cdot 5 + 3 \cdot 7.8$$
$$= 20 + 12 + 15 + 23.4 = 70.4 \text{ in}^2$$

Each wedge of cheese needs 70.4 in^2 of wax so 100 wedges will need $70.4 \times 100 = 7040$ in^2 of wax.

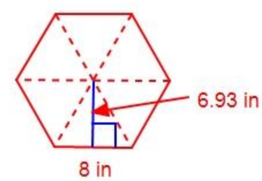
10. A lamp shade is shaped like a hexagonal pyramid. It does not have a bottom, how much fabric is needed to replace the fabric on the shade?



The surface area of the shade is 312 in^2

Talk to students about how this is an example of lateral surface area.

Challenge Extension: What is the surface area of a hexagonal pyramid with the same dimensions of the shade, if the perpendicular distance from the edge of the base hexagon to its center is 6.93 in?
This would require finding the area of the hexagonal base. This can be done by dividing the hexagon into 6 congruent triangles and finding the area of each triangle. The surface area would be $312 + 166.32 = 478.32 \text{ in}^2$



11. Toni and Gardner are finding the surface area of a cube with a side measure of 14 inches. Toni writes the expression; $14^2 + 14^2 + 14^2 + 14^2 + 14^2 + 14^2$ to represent the surface area of the cube. Gardner claims that Toni's expression is not correct but rather that the surface area is represented by the expression; $6(14^2)$. Whose expression is correct? Explain your reasoning.
They are both correct because their expressions are equivalent to each other.

12. A juice box measures 4 inches by 2 inches by 5 inches.
   a. How much juice will the box hold?
      The juice box will hold 40 in^3 of juice.

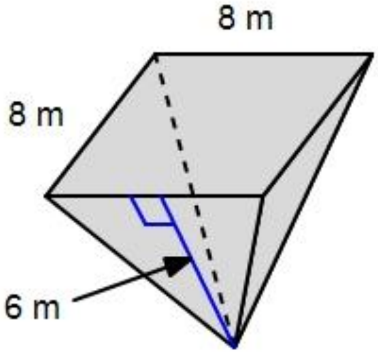
   b. What is the least amount of material needed to make one juice box?
      The least amount of material needed to make one box is 76 in^2.

   c. Explain the difference between volume and surface area.
      Surface area is the amount of space that covers the surface of an object and volume is the amount of space that the object takes up.

5.4b Homework: Finding Surface Area

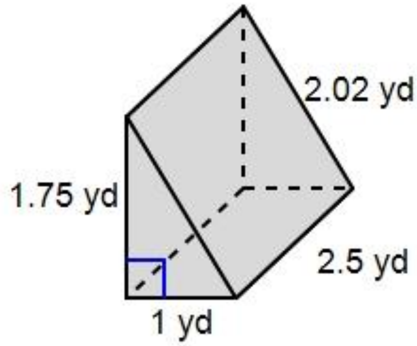
Find the surface area of each 3D figure below. Be sure to show all calculations.

1.



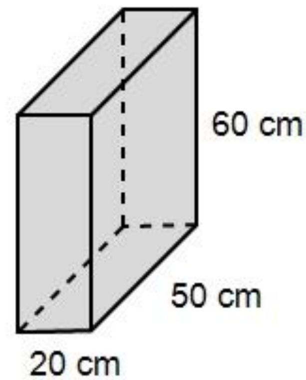
It is acceptable for students to find the surface area using any method they choose. They might draw and label a net or organize their calculations for finding the sub areas with a table, list, formula, etc.

2.

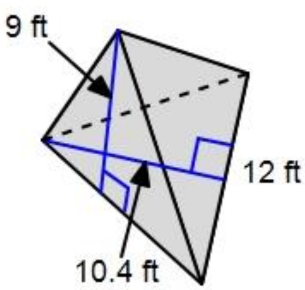


Surface Area = 13.675 yd^2

3.

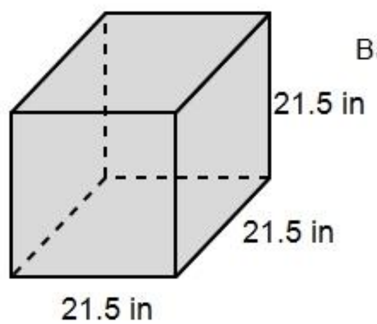


4.

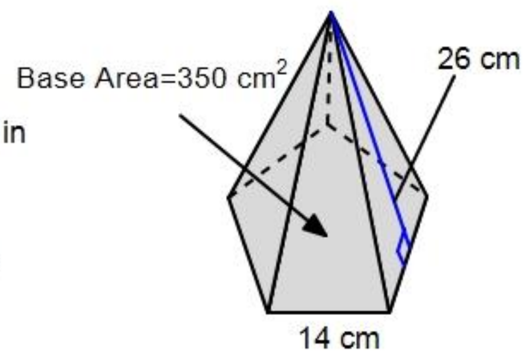


Surface Area = 224.4 ft^2

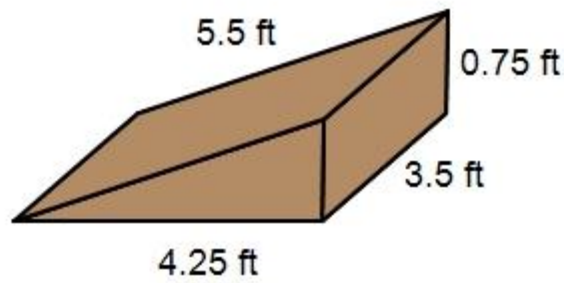
5.



6.



7. Carly and Nadia are painting their bike ramp. They would like to put two coats of paint on the entire ramp. They have one quart of paint which will cover 100 ft^2. Do they have enough paint to do the two coats? Justify your answer.

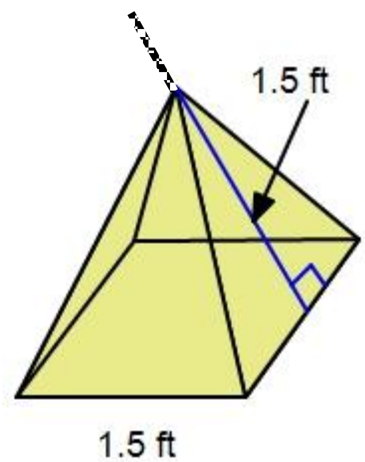


8. The material used to make a storage box costs $1.50 per square foot. The boxes have the same volume. How much money will a company save by choosing to make 100 of Box 1 over 100 of Box 2? Draw and label a net for each box if needed.

	Length	Width	Height
Box 1	20 in	5in	8 in
Box 2	25 in	4 in	8 in

Surface area of Box 1: $SA = 2 \cdot 20 \cdot 5 + 2 \cdot 20 \cdot 8 + 2 \cdot 5 \cdot 8 = 200 + 320 + 80 = 600 \text{ in}^2$
Surface area of Box 2: $SA = 2 \cdot 25 \cdot 4 + 2 \cdot 25 \cdot 8 + 2 \cdot 4 \cdot 8 = 200 + 400 + 64 = 664 \text{ in}^2$
Cost of making Box 1: $640 \cdot 1.5 = \$900$
Cost of make Box 2 $664 \cdot 1.5 = \$996$
Cost of making 100 copies of Box 1: $960 \cdot 100 = \$90{,}000$
Cost of making 100 copies of Box 2 $996 \cdot 100 = \$99{,}600$
Difference between costs: $99{,}600 - 90{,}000 = \$9{,}600$
The company will save $9,600 if they make 100 copies of Box 1 rather than 100 copies of Box 2.

9. A hanging light shade is the shape of a square pyramid that does not have a base or bottom. It is made of glass and hung by a chain that can hold 20 pounds of weight. One square foot of glass weighs 2.45 pounds. Can the chain support the light shade?



10. Chester is building a pool in his backyard. His only requirement is that the pool holds no more 3,000 ft^3 of water.

a. Which pool option below should he choose based off of his requirement. Justify your answer.

Option A: $20 \text{ ft} \times 30 \text{ ft} \times 6 \text{ ft}$
Option B: $15 \text{ ft} \times 25 \text{ ft} \times 8 \text{ ft}$
Option C: $20 \text{ ft} \times 20 \text{ ft} \times 8 \text{ ft}$
Option D: $16 \text{ ft} \times 31.25 \text{ ft} \times 6 \text{ ft}$

Chester should choose either Option B or Option D; they are the only pools that have a volume that is no more than 3000 ft^3.

b. One gallon of paint covers 400 ft^2 and costs $30. Based off of this information which pool, option B or option D, will cost more to paint and by how much?

Be sure that students understand that there is not a top to paint for the pool. They need to take this into consideration when finding the surface area of the pool.

Option B is has a surface area of 1015 ft^2; it will take 2.5375 gallons of paint to paint it. Chester will have to buy 3 gallons, this will cost in $90. Option D has a surface area of 1067 ft^2. It will take 2.6675 cans of paint to pain this pool, and so he will need to buy 3 cans of paint. This will cost him $90. It does not matter which option he chooses since he will have to buy 3 cans of paint regardless of which pool he choose.

11. Helen is ordering popcorn bags for her movie theater. The bags cost $0.03 per square inch of paper used to make the bags. She can choose between a bag that measures $4 \times 9 \times 10$ inches or a bag that measures 6 x 6 x 10 inches. Which bag do you think she should choose if she wants to be able to fill the bag with the most amount of popcorn as possible and spend the least amount of money? Justify your answer.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