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Chapter 1: Probability, Percent, Rational Number Equivalence (3-4 weeks)

UTAH CORE Standard(s)

Number Sense:

1. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.2d**

2. Solve real-world and mathematical problems involving the four operations with rational numbers. **7.NS.3**

Probability and Statistics:

1. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **7.SP.5**

2. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. **7.SP.6**

Equations and Expressions:

1. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **7.EE.3**

Chapter 1 Summary:

Chapter 1 begins with a brief introduction to probability as a means of reviewing and applying arithmetic with whole numbers and fractions. In addition to covering basic counting techniques and listing outcomes in a sample space, students distinguish theoretical probabilities from experimental approaches to estimate probabilities. One reason for starting the year with probability activities is to develop a culture of thinking about mathematics as a way to investigation real world situations. A second reason is that activities at the beginning of the year can help foster a classroom culture of discussion and collaboration.

Throughout the chapter students are provided with opportunities to review and build fluency with fractions, percents, and decimals from previous grades. Students should understand that fractions, percents and decimals are all relative to a whole. Students will also compare and order fractions (both positive and negative.) This chapter concludes with a section specifically about solving percent and fraction problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease.

VOCABULARY: chance, decimal, experimental probability, fraction, frequency, outcome, percent, probability, ratio, theoretical probability

CONNECTIONS TO CONTENT:

Prior Knowledge

In previous coursework, students developed the concept of a ratio. Though students should be familiar with the idea of part:whole and part:part relationships, they have likely not completely solidified it yet. It is important to emphasize that in this chapter only part:whole relationships are discussed (probability is a part:whole relationship as are fractions, decimals and percents.) In Chapter 4 students will practice differentiating between part:part and part:whole relationships and then examine "odds" which are part:part relationships.

Students have used all four operations (addition, subtraction, multiplication, and division) when working with fractions and decimals in prior grades. They should have used both number line and bar/tape models to represent fractions, percents, and decimals.

In 6th grade students placed both positive and negative numbers on a number line, however they do not operate with negative numbers until 7th grade (this will take place in Chapter 2).

Future Knowledge

As students move through this chapter, they will begin by studying probability (this chapter is only an introduction to probability, students will work more with probability in Chapter 7). The concepts learned in 7th grade regarding chance processes as well as theoretical and experimental probabilities will be extended in later courses when students study conditional probability, compound events, evaluate outcomes of decisions, use probabilities to make fair decisions, etc.

While studying probability students will continue their study of rational numbers. They will convert rational numbers to decimals and percents and will look at their placement on the number line. This lays the foundation for 8th grade where students study irrational numbers to complete the Real Number system.

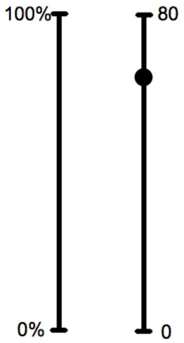
Another important concept that will be extended is the notion of "unit." Throughout the year students will: a) clearly distinguish between kinds of units (e.g. linear, square or cubic) and b) consider units of quantities. For example a "unit" of consideration might be a distance of 5 miles as in walking 5 miles per hour. So, if they want to know how far one walks in 20 minutes, 1/3 of an hour, then one simply divides the 5 mile unit into 3 parts. Thus in 20 minutes one has walked 5/3 miles.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

	Make sense of problems and persevere in solving them	Students explain and demonstrate rational number operations by using symbols, models, words, and real life contexts. Students demonstrate perseverance while using a variety of strategies (number lines, manipulatives, drawings, etc.). Students make sense of probability situations by creating visual models to represent situations.
n#	**Reason abstractly and quantitatively**	Students connect ideas of models to ideas of numbers. For example, students should reason that one can partition a whole into any number *n* equal pieces and then represent a portion of the whole as *m/n.* The number *m/n* can then be located on the real line.
	Construct viable arguments and critique the reasoning of others	Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, or models. Students discuss rules for operations with rational numbers using appropriate terminology and tools/visuals. Students approximate probabilities, create probability models and explain reasoning for their approximations. They also question each other about the representations they create to represent probabilities.
	Model with mathematics	Students model understanding of rational number operations using tools such as tiles, counters, visuals, tape/bar models, and number lines and connect these models to solve problems involving real-world situations. Students model problem situations symbolically, graphically, and contextually. Students use experiments or simulations to create probability models.
	Attend to precision	Students should use appropriate terminology when referring to ratios, probability models, rational numbers, and equations. Students also use appropriate forms of a number to fit a context, e.g. one would not say $3/2, rather they would say $1.50.
	Look for and make use of structure	Students look for structure in rational numbers when they place them appropriately on the number line. Students should connect the structure of fractions, percents, and decimals to the idea of part:whole relationships to missing finding values (e.g. students should use structure, not algorithms, to solve problems). Students recognize that probability can be represented in tables, visual models, or as a rational number.
	Use appropriate tools strategically	Students demonstrate their ability to select and use the most appropriate tool (paper/pencil, manipulatives, pictorial models, and calculators) while solving problems with rational numbers. Students might use physical objects or applets to generate probability.
	Look for and express regularity in repeated reasoning	Students use repeated reasoning to understand algorithms and make generalizations about patterns. They extend their thinking to include complex fractions and rational numbers. They create, explain, evaluate, and modify probability models to describe simple events.

1.0 Anchor Problem: Percent Estimation Game

Below are two vertical lines on which you will make a guess for both a number and a percent.



Estimate the numeric location of the point on the vertical line on the right.__________

Estimate its equivalent percent of 80 using the line on the left.

____

Student answers will vary. The point is located at *approximately* 60 (the point is approximately ¾ of the way up the line on the right.) Ask students to justify their answers. Let them know that there is no absolute right answer, they're just estimating. The corresponding point on the vertical line on the left should be at the same height e.g. about ¾ of the way up. Have them guess where they think it is on that line—it's at approximately 75%. Don't find the exact percent yet. Your goal is to help students develop an intuitive sense of percentages. You want students to talk about portions of each whole. For example a student might say, "the point is about ¾ of the way up on 80, so the percent is about ¾ of 100, so it's about 75%."

2) You and five of your friends like to go to McDonalds once a week and get Happy Meals for the prize inside. McDonalds has just started a new "dinosaur toy" promotion for their Happy Meals with six different dinosaurs you can collect: Brachiosaurus, Brontosaurus, Diplodocus, Tyrannosaurus, Plesiosaurus, and Allosaurus. You each want to collect at least one of all six dinosaurs, but the prizes are randomly placed in Happy Meals, and there is no way to know which dinosaur you're getting until you open the Happy Meal bag.

How might you design a simulation experiment to find the likelihood (experimental probability) of getting all six toys after one, two, three, etc. weeks?

Section 1.1: Investigate Chance Processes. Develop/Use Probability Models.

Section Overview:

This is students' first formal introduction to probability. In this section students will study chance processes, which concern experiments or situations where they know which outcomes are possible, but they do not know precisely which outcome will occur at a given time. They will look at probabilities as part:whole ratios expressed as fractions, decimals, or percents. Probabilities will be determined by considering the results or outcomes of experiments. Students will learn that the set of all possible outcomes for an experiment is a sample space. They will recognize that the probability of any single event can be expressed in terms of impossible, unlikely, equally likely, likely, or certain or as a number between 0 and 1, inclusive. Students will focus on two concepts in probability of an event: *experimental* and *theoretical*. They will understand the commonalities and differences between experimental and theoretical probability in given situations.

Concepts and Skills to be Mastered (from standards)

1. Understand and apply likelihood of a chance event as between 0 and 1.
2. Approximate probability by collecting data on a chance process (experimental probability).
3. Calculate theoretical probabilities on a chance process for simple events.
4. Given the probabilities (different scenarios in a chance process), predict the approximate frequencies for those scenarios (if experimenting on a chance process).
5. Use appropriate fractions, decimals and percents to express the probabilities.

TEACHER NOTES for Activity 1.1a:

Materials: (Note: If you do not have tiles you can use pieces of square paper or other objects that are different colors but the same shape and size. All bags should have 12 items.)

- 2 bags with 2 green tiles and 10 blue tiles
- 2 bags with 4 green tiles and 8 blue tiles
- 2 bags with 6 green tiles and 6 blue tiles
- 2 bags with 8 green tiles and 4 blue tiles
- 2 bags with 10 green tiles and 2 blue tiles
- Calculators

The primary objective of this lesson is to develop the idea of theoretical v. experimental probability. However, an important secondary objective is to review the concept that a fraction is relative to a whole.

During this activity students will use probability to try to determine the number of green tiles in a bag.

Tell students: "I am giving each group a bag of tiles. Each bag has exactly 12 tiles—some green and some blue. Each groups has a different number of green and blue tiles in their bags. Without looking in the bag, you're going to pull out a tile and record the color and put the tile back into the bag. You're going to do that a certain number of times as indicated on the activity paper. Then based on the outcomes of your draws, you're going to try to figure out how many of the tiles in your bag are green and blue. Do NOT look in your bag until I tell you to!"

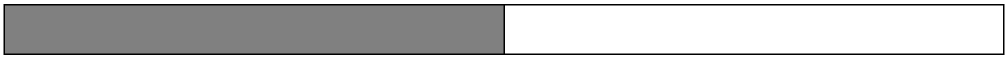
Discussion Ideas:

- During the activity have students discuss the difference between theoretical probability and experimental probability. Have students discuss the similarities and difference between the results of each group. Discuss which outcome is the most likely and least likely to occur.
- After the groups have counted the actual number of tiles in their bags, discuss (either as a group or in smaller groups) how experimental probabilities were the same and different for groups with the same number of green and blue tiles and for those with different numbers.

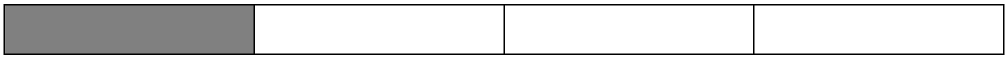
REVIEW FROM EARLIER GRADES:

$1 = 1.0 = 100\%$

$\frac{1}{2} = 0.5 = 50\%$



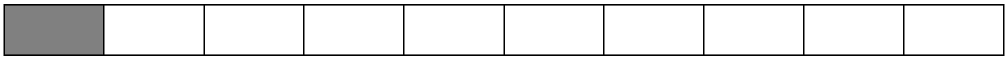
$\frac{1}{4} = 0.25 = 25\%$



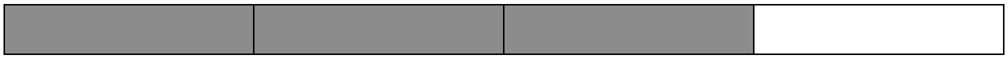
$\frac{1}{3} = 0.3\overline{3} = 33\frac{1}{3}\%\ or\ 33.\overline{3}\%$

$\frac{1}{5} = 0.2 = 20\%$

$\frac{1}{10} = 0.1 = 10\%$

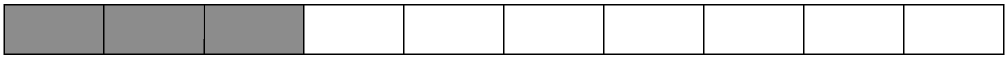


1. Use a bar model to represent 3/4 of a whole.



2. Use a bar model to represent 3/5 of a whole.

3. Use a bar model to represent 3/10 of a whole.



4. What do you notice about the fractions in #1-3?

The numerators of the fractions are all the same, so we are always taking 3 of the portions. As the denominator increases so do the number of equal sized pieces in our whole, so the size of our 3 sections is getting smaller with each fraction. (i.e. As the denominator increases the size of the fraction decreases.)

1.1a Class Activity: Using Data to Make Predictions (Probability)

1. In your own words, what do you think these terms mean?
“Experimental” probability: “Theoretical” probability:

HOW MANY GREEN TILES ARE IN YOUR BAG?
We will examine *experimental* and *theoretical* probability in this activity. You have been given a bag with a total of 12 tiles, some are green and the rest are blue. You will do the experiment described below 6, 12, 18, 24 and then 30 times to try to figure out how many green tiles are in your bag. DO NOT LOOK IN YOUR BAG! Each group has a different number of green tiles in their bag.

Instructions: a) without looking in the bag, draw ONE tile record the color (G or B) in the table, b) put the tile back into the bag and shake it to mix up the tiles, c) repeat the indicated number of trials (notice that there are a – e experiments), d) based on each experiment guess how many GREEN tiles are in the bag.

2. a. Draw a marble/tile (without looking in the bag), record the color, replace it, mix them by shaking, redraw, record, replace. Do this 6 times.

Draw #	1	2	3	4	5	6
Color						

Based on your experiment, how many of the 6 draws were GREEN? ________
Based on these 6 draws, how many of the 12 marbles/tiles in the bag do you think are green? _____

b. Repeat the experiment in “a” but now do it 12 times.

Draw #												
Color												

Based on your experiment, how many of the 12 draws were GREEN? ________
Based on these 12 draws, how many of the 12 marbles/tiles in the bag do you think are green? _____

c. Repeat the experiment in “a” but now do it 18 times.

Draw #																		
Color																		

Based on your experiment, how many of the 18 draws were GREEN? ________
Based on these 18 draws, how many of the 12 marbles/tiles in the bag do you think are green? _____

d. Repeat the experiment in “a” but now do it 24 times.

Draw #												
Color												
Draw #												
Color												

Based on your experiment, how many of the 24 draws were GREEN? ________
Based on these 24 draws, how many of the 12 marbles/tiles in the bag do you think are green? _____

e. Repeat the experiment in “a” but now do it 30 times.

Draw #																		
Color																		
Draw #																		
Color																		

Based on your experiment, how many of the 30 draws were GREEN? ________
Based on these 30 draws, how many of the 12 marbles/tiles in the bag do you think are green? _____

Probability has standard notation. We write $P(G)$ to means the probability of drawing a green tile. To write $P(G)$ we need to know the “observed frequency” and the “total number of trials.” Probability is the ratio of the observed frequency to the total number of trials:

$$P(G)=\frac{\textit{observed frequency}}{\textit{total number of trials}}$$

Note: probability is a part:whole relationship.

Write your group’s $P(G)$ as both a fraction and decimal for each of the experiments you did on the previous page.

a. $P(G)$ = __________ __________

b. $P(G)$ = __________ __________

c. $P(G)$ = __________ __________

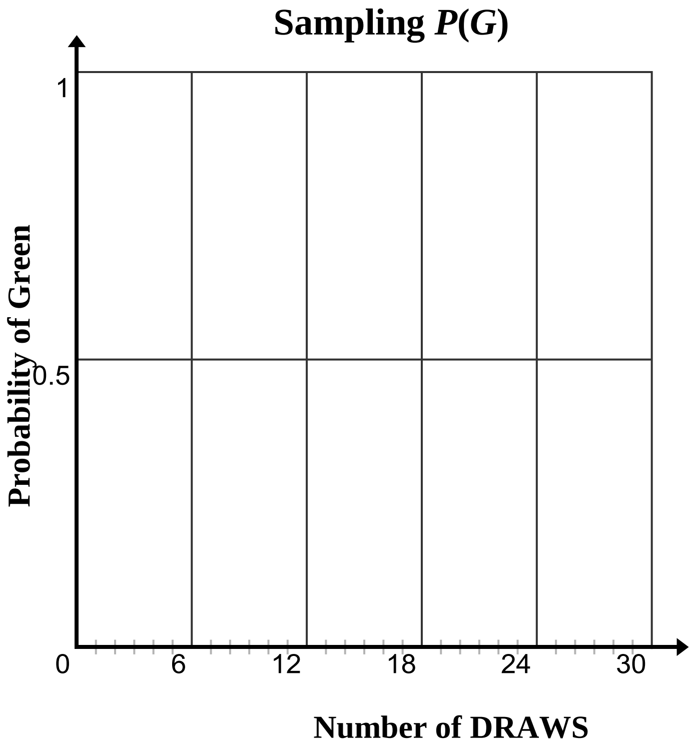
d. $P(G)$ = __________ __________

e. $P(G)$ = __________ __________

Use calculators to compute decimals at this point.

In the table below you are creating a dot plot. Ask students why the “probability of green” axis goes from 0 to 1. This will be an important point to emphasize throughout the unit. Students will have five data points (they should not connect the points), one each for 6, 12, 18, 24, and 36 draws. Each group will have different graphs. Have groups show their graphs as they present their findings for problems 3 - 6. Also emphasize with students that the graph is a visual representation of the observed probability of GREEN tiles for each number of draws.

3. Make a graph of your group’s samplings.



Most likely graphs will “flatten out” as the number of trials in an experiment increases. Ask students why this might be happening.

4. Explain what your graph shows about the probability of GREEN tiles.

   Experimental probability can vary for a group even though the number of tiles does not, *different experimental probabilities may tell us something about the actual number of tiles in the bags *probability is always between 0 and 1

5. Explain why your probability of GREEN tiles is a fraction (between 0 and 1).

   Probability is the ratio between observed frequency and total trials—observed frequency can never be smaller than 0 or bigger than total trials.

6. How does knowing the probability of GREEN tiles help you know the probability of BLUE tiles?

   The total outcome will be 1. Thus if the probability of GREEN is 1/3, the probability of BLUE must be 2/3.

7. Make a conjecture about how many GREEN tiles are in your bag if the bag contains 12 total tiles.

   Have all groups record their predictions.

8. Count how many blue and green marbles/tiles are actually in your bag. Based on this information, what is the "theoretical" probability of drawing a green tile from your bag?

9. How did your group's "experimental" $P(G)$ compare with the "theoretical" $P(G)$?

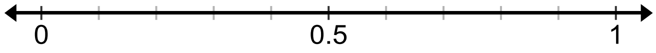
   It is likely that the more trials in the experiment, the closer the experimental probability came to the theoretical probability.

10. In your own words, what do you think:

    "Experimental" probability means?

    "Theoretical" probability means?

11. Place the theoretical probability of drawing a green for each group in the class on the number line below.



Do this as a class and discuss what students see.

12. Which groups are **most** and **least** likely to have an outcome of drawing a green out of the bag? Justify your answer.

Have students justify their arguments. Ask them for evidence to support their claims.

As groups discuss:

13. Suppose you had a bag of 1000 blue and green tiles, how many times do you think you would need to draw tiles to make an <u>accurate</u> prediction of the number of blue and green tiles are actually in the bag? Explain.

Note: students are making a conjecture here. The important word is "accurate." We cannot be 100% sure that our guess is correct. The "mathematical" answer would be something like: after n number of draws, with p% probability we are within q% of the correct amount.

14. You're a teacher in a 7th grade math class and you want to create an experiment for your class with red, yellow and purple marbles in a bag. You want the theoretical probability of drawing a red marble to be $\frac{1}{4}$, the theoretical probability of drawing a yellow to be $\frac{1}{4}$ and the theoretical probability of drawing a purple to also be $\frac{1}{2}$. If you want a total of 120 marbles in the bag:

a. How many red marbles should you put in the bag? _____30_____

b. How many yellow marbles should you put in the bag? ____30_____

c. How many purple marbles should you put in the bag? ____60______

Review exercises:

15. Without using a calculator, determine which fraction is larger. Justify your answer with an explanation and model.

a. $\frac{1}{8}$___ $\frac{1}{9}$ b. $\frac{7}{8}$___ $\frac{8}{9}$ c. $\frac{8}{15}$____ $\frac{8}{17}$ d. $\frac{9}{21}$____ $\frac{13}{22}$

There are a number of ways to determine which fraction is larger. Ask students to share their thinking. **a. 1/8**: 8th are larger than 9th s; if you cut something up into 8 pieces each piece will be larger than if you cut the whole into 9 pieces. **b. 8/9.** The same logic for "b" except now you're removing either an 8th or a 9th, removing a smaller portion of the whole leaves you with a bigger result. **c. 8/15** is larger because it's a little more than ½, while 8/17 is a little less. OR one might argue that for both fractions we're talking about 8 pieces. For 8/15 we have eight pieces that are 1/15 but for 8/17 we have the same number of pieces, but they are 1/17 of the whole. 1/15>1/17, therefore 8/15>8/17. d. 9/21<1/2; 13/22>1/2; therefore, **9/21<13/22.**

1.1a Homework: Probability Predictions

1. You flipped a coin 50 times and got 23 heads. What is the experimental probability of getting a head? Write your answer as a fraction, decimal and percent.

$$\frac{23}{50} = 0.46 = 46\%$$

2. If you flipped the coin 100 times, how many heads would you expect to get? Explain your answer.
   Answers will vary a little. They may say 46 times since $50(2) = 100$ so multiply 23 by 2.
   Or they may say 50 times since they would expect to get a head about ½ of the time.

3. A coin is tossed 20 times. It lands on heads 9 times. What is P(H) according to your experiment? Write your answer as a fraction, decimal and percent.

4. You're a teacher in a 7th grade math class and you want to create an experiment for your class with red, yellow and purple marbles in a bag. You want the theoretical probability of drawing a red marble to be $\frac{1}{2}$, The theoretical probability of drawing a yellow to be $\frac{1}{3}$ and the theoretical probability of drawing a purple To also be $\frac{1}{6}$, If you want a total of 1260 marbles in the bag:
   a. How many red marbles are you going to put in the bag? Why?

   b. How many yellow marbles are you going to put in the bag? Why?
   420 yellow marbles, that is 1/3 the total in the bag.

   c. How many purple marbles are you going to put in the bag? Why?

4. **Challenge:** You've decided you want to make the marble experiment a little more difficult. You want to use 400 marbles and you want six different colors—blue, red, green, yellow, purple, and pink. You also do not want more than two colors to have the same probability. State the number of each color you are going to put in the bag and what the theoretical probability of drawing the color will be (answers will vary.)

   (Answers will vary.)Here is one possibility.
   a. Blue: $P(B)$__1/2_____ and actual number of blue _____200______

   b. Red: $P(R)$________ and actual number of red ______100______

   c. Green: $P(G)$__1/8____ and actual number of green ____________

   d. Yellow: $P(Y)$________ and actual number of yellow ____4________

   e. Purple: $P(Purple)$__1/10___ and actual number of purple ___________

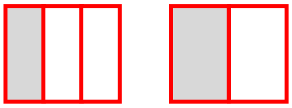
   f. Pink: $P(Pink)$________ and actual number of pink ______6______

What should the sum of all the probabilities be?
They should sum to 1.

Answer the following:

5. Without using a calculator, determine which fraction is bigger in each pair. Justify your answer with picture and words.

a. $\frac{1}{3}$ or [✓ $\frac{1}{2}$]

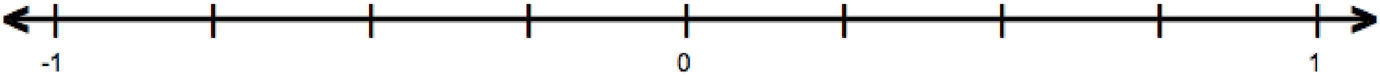


1 piece out of a pie cut into 2 will be bigger than1 piece out of a pie cut into 3 pieces

b. $\frac{3}{7}$ or $\frac{3}{5}$

6. Place the fractions on the number line below.

$\frac{1}{2}$, $\frac{3}{5}$, $\frac{1}{3}$, $\frac{2}{5}$



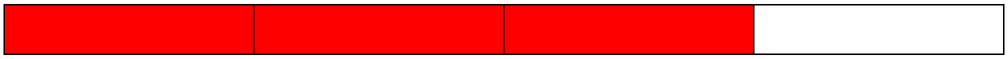
7. Order the following fractions from *least to greatest.*

$\frac{1}{2}$, $\frac{3}{5}$, $\frac{1}{3}$, $\frac{2}{5}$

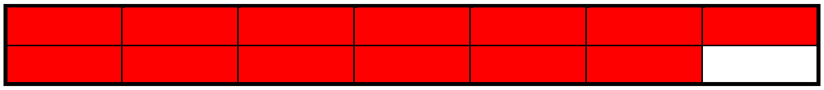
$\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{5}$

Spiral Review

1. Use a bar model to represent ¾ of a whole.



2. Solve using bar model $\frac{1}{2}+\frac{3}{7}=$ ______



1.1b Class Activity: Probability—Race to the Top

1. For the horse race experiment you will need two dice. For each roll of the dice record the sum in the appropriate column below by shading in the box. For example, a roll of 2,5 means you will shade one box in the 7 column, a roll of 1,4 means you will shade one box in the 5 column. Do this 30 times. ***BEFORE you start, predict which horse will win (2 through 12).***

2	3	4	5	6	7	8	9	10	11	12

2. Which horse won (had the most rolls) in your group?______
3. List your group's experimental probability for each outcome:
   a. $P(2)$________
   b. $P(3)$________
   c. $P(4)$________
   d. $P(5)$________
   e. $P(6)$________
   f. $P(7)$________
   g. $P(8)$________
   h. $P(9)$________
   i. $P(10)$_______
   j. $P(11)$_______
   k. $P(12)$_______

4. List the horse that won for each of the groups in your class. Which horse won the most often?
   Have each group report which horse won in their experiment. Circle the horse that won the most times.

5. Create a class histogram that combines the data from all of the groups' histograms. What do you notice about the histogram? The class histogram should include the outcomes for all the groups.

2	3	4	5	6	7	8	9	10	11	12

6. Which horse won the most often for all the groups? Why?

Have students justify their arguments. Ask them for evidence to support their claims.

7. Do you think that this game is fair? Why or why not?

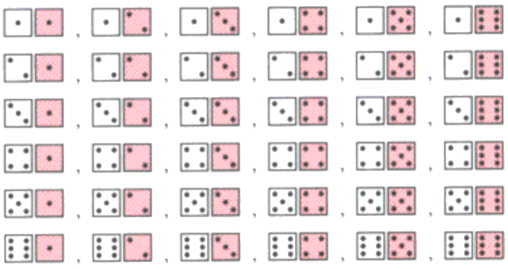
Note: you are developing the idea of "***fair***" in a game. The term will be formalized in Chapter 7.

8. What are all the possible outcomes when you roll two dice? In your group, organize these possible outcomes on a chart of your choosing. See examples below. Students will try a number of techniques, look for students that use a systematic one to share with the class. This is a good time to talk about "equally likely" outcomes; e.g. a roll with a sum of 3 and a rolled sum of 11 are equally likely. Talk about why this is true.

					4, 3					
				3, 3	3, 4	4, 4				
			3, 2	4, 2	5, 2	5, 3	5, 4			
		2, 2	2, 3	2, 4	2, 5	3, 5	4, 5	5, 5		
	2, 1	3, 1	4, 1	5, 1	6, 1	6, 2	6, 3	6, 4	6, 5	
1, 1	1, 2	1, 3	1, 4	1, 5	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6
2	3	4	5	6	7	8	9	10	11	12

Notice that the dice shown below are different colors, this is to help students distinguish between 6, 1 and 1,6. These are two different outcomes.

		Second Die					
		1	**2**	**3**	**4**	**5**	**6**
First Die	**1**	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6



9. How many total outcomes did you get? Explain the system you used to get all those outcomes.
36 possible outcomes; have students present their strategies. Look for systematic approaches to highlight.

10. Use the above information to determine the probability for each outcome:

$P(1)$____0____

$P(2)$____$\frac{1}{36}$____

$P(3)$____$\frac{2}{36} = \frac{1}{18}$____

$P(4)$____$\frac{3}{36} = \frac{1}{12}$____

$P(5)$____$\frac{4}{36} = \frac{1}{9}$____

$P(6)$____$\frac{5}{36}$____

Discuss the "likelihood of events with students e.g. a roll of 1 with two die is impossible. A roll of a 7 is more likely than a roll of either a 6 or 8. A roll of 6 or 8 are equally likely, etc.

$P(7)$____$\frac{6}{36} = \frac{1}{6}$____

$P(8)$____$\frac{5}{36}$____

$P(9)$____$\frac{4}{36} = \frac{1}{9}$____

$P(10)$____$\frac{3}{36} = \frac{1}{12}$____

$P(11)$____$\frac{2}{36} = \frac{1}{18}$____

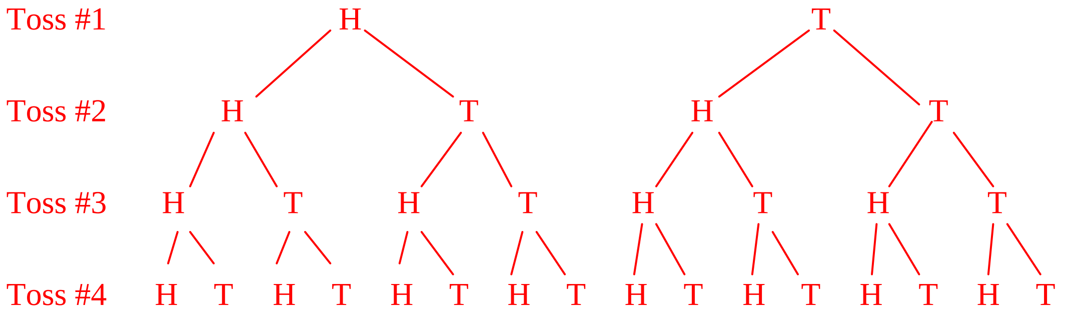
$P(12)$____$\frac{1}{36}$____

11. How do you think the individual probabilities relate to the probabilities of the sums? For example, $P(1, 6) = \frac{1}{36}$ but $P(7) = \frac{1}{6}$, why is this true?
Students should recognize that there are 6 ways to roll a 7, but only one way to roll a 1,6. Discuss how 1,6 is a different outcome from 6,1.

1.1b Homework: Probability

Write all probabilities as a fraction, decimal and percent.

1. Find <u>all</u> the possible outcomes of flipping a coin (heads or tails) FOUR times. In other words, how many different ways are there to get heads and/or tails in four flips?



Encourage students to use whatever model they choose to compute. Below is an example of a tree diagram that shows 16 possible outcomes. Have students display their models to the class and explain how they systematically ensured they found all possible outcomes. Connect the outcomes from the model to the idea of "theoretical probability."

2. Based on #1, what is the theoretical probability that you:

   a. Get HEADS for *all* four flips?
   $\frac{1}{16} = 0.0625 = 6.25\%$. There is only one strand on the tree diagram that shows HHHH.

   b. Get HEADS *at least* once in four flips?
   $\frac{15}{16} = 0.9375 = 93.75\%$. There are 15 strands on the tree diagram that contain an H.

   c. Get HEADS *exactly* three times in four flips?

   $\frac{4}{16} = \frac{1}{4} = 0.25 = 25\%$ There are only 4 strands that contain three heads(HHH) exactly three times.

3. Without using a calculator, determine which fraction is bigger in each pair. Justify your answer.

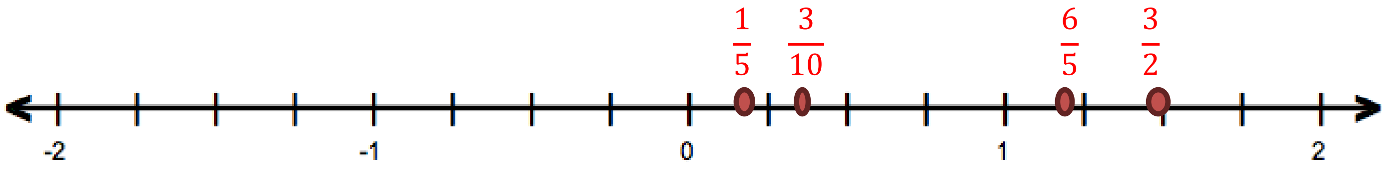
   a. [✓ $\frac{3}{7}$] or $\frac{3}{8}$

   For each we have 3 parts of a whole that was cut into either 7 or 8. 1/7 is larger than 1/8, so 3 groups of 1/7 is more than three groups of 1/8.

   b. $\frac{7}{13}$ or $\frac{9}{20}$

4. Place the fractions on the number line below.

$$\frac{6}{5}, \quad \frac{3}{10}, \quad \frac{1}{5}, \quad \frac{3}{2}$$



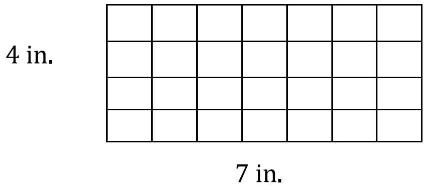
5. Order the following fractions from *least to greatest.*

$\frac{6}{5}, \quad \frac{3}{10}, \frac{1}{5}, \frac{3}{2}$

Spiral Review

1. Write $\frac{1}{3}$ as a percent. $33.\bar{3}\%$

2. What is the area of the following figure?



28 in. sq.

3. Write one fourth as a fraction, decimal, and percent. 1/4 = 0.25 = 25%

1.1c Class Activity: Probability Continued and Fair Game?

Use the data from problem 1 on Homework 1.1.b to answer the following:

1. What is the probability of getting exactly one HEAD on the first flip? _____ $\frac{1}{2}$ _______

2. What is the probability of getting one HEAD on the first *and* second flips? _____ $\frac{1}{4}$ _______

3. What is the probability of getting one HEAD on the first, second *and* third flips? _____ $\frac{1}{8}$ _______

4. What is the probability of getting one HEAD on *all four* flips? _____ $\frac{1}{16}$ _______

5. What pattern do you see emerging?
   Possible answers:
   - The denominator doubles each time. Be clear that doubling the denominator does NOT mean doubling the fraction.
   - You multiply by $\frac{1}{2}$ each time.

6. After two flips of the coin, what is the probability of getting *at least* one HEAD? ___ $\frac{3}{4}$ ___

7. After three flips of the coin, what is the probability of getting *at least* one HEAD? ___ $\frac{7}{8}$ ___

8. After four flips of the coin, what is the probability of getting *at least* one HEAD? ___ $\frac{15}{16}$ ___

9. What do you notice about the probabilities in questions 6-8?

   Students should notice that the probability of getting *at least* one HEAD is always 1 – 1/*n* .

10. Explain why the numerator is always one less than the denominator when finding the probability for <u>at least one</u> HEAD.

Fair Games:

Games 1 and 2: Use two dice OR an electronic random number generator on a calculator (on the TI-73 go to Math, PRB, dice, enter, type in 2 for two dice, then hit enter to roll the dice.) Play the game in pairs. One person will be "even" the other will be "odd". For the "Addition Game" the "even" person earns a point if the sum of the two dice is even and the "odd" person earns a point if the sum is odd. For the "Multiplication Game" the "even" person gets a point if the product of the dice is even and the "odd" person gets a point if the product is odd.

Game 1: The Addition Game

1. Do you think the Addition Game will be fair—do "odd" and "even" have the same chance at winning? Explain.

2. Play the game—36 rolls of the dice. Based on your data, what is the experimental probability of rolling an odd sum? Probability of rolling an even sum?
   P(odd) = ________ *P*(even) = ________

3. Look back at the data you gathered from the probability experiment in 1.1b Class Activity (Race to the Top game). What was the theoretical probability of rolling an odd sum or even sum from that data?
   P(odd) = ____ $\frac{1}{2}$ ______ *P*(even) = ____ $\frac{1}{2}$ ______
   P(odd) and P(even) are equally likely.

4. Do you think the addition game is a fair game? Explain.
   This is a fair game, because there is an equal likelihood of rolling an even sum or an odd sum.

Game 2: The Multiplication Game

1. Do you think the multiplication game will be fair—do "odd" and "even" have the same chance at winning? Explain.

2. Play the game—36 rolls of the dice. Based on your data, what is the experimental probability of rolling an odd product? Probability of rolling an even product?
   P(odd) = ________ *P*(even) = ___$\frac{27}{36} = \frac{3}{4}$_____

3. How will the sample space for the multiplication game be the same and/or different from the addition game?
   One could use the same sample space as created in 1.1b Class Activity, but would need to analyze it differently to account for the products.

4. Find the theoretical probability of rolling an odd product and even product?
   Discuss with students the P(even) is more likely that P(odd).
   P(odd) = _____ $\frac{9}{36} = \frac{1}{4}$ _______ *P*(even) = _____ $\frac{27}{36} = \frac{3}{4}$ ____

5. Do you think the multiplication game is a fair game? Explain why or why not.
   This is not a fair game. You might choose to discuss ideas in number theory further with your class.

1.1c Homework: Probability Continued

Write all probabilities as fractions, decimals and percents.

1. Without using a calculator, for each pair, determine which fraction is bigger. Justify your answer.

   a. $\frac{1}{9}$ or $\frac{1}{10}$

   b. $\frac{8}{9}$ or [✓ $\frac{9}{10}$]

   c. $\frac{5}{9}$ or $\frac{5}{11}$

   d. [✓ $\frac{5}{12}$] or $\frac{5}{14}$

2. A bag of marbles contains 3 red marbles, 5 blue marbles, and 2 yellow marbles.

   a. What is *P*(red)? $\frac{3}{10} = 0.3 = 30\%$

   b. What is *P*(blue)?

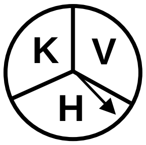
   c. What is *P*(yellow)?

   d. What is the most likely outcome when drawing a marble out of the bag? Explain.
   Blue, $P(B)$ is greater than either $P(R)$ or $P(Y)$

   e. What is the least likely outcome when drawing a marble out of the bag? Explain.
   Yellow, $P(Y)$ is less than either $P(R)$ or $P(B)$

   f. Have you been computing theoretical or experimental probabilities? Explain.

3. A spinner contains three letters of the alphabet.



   a. How many outcomes are possible if the spinner is spun three times?

   b. List all of the outcomes for spinning three times.
   KKK, KKV, KKH, KVK, KVV, KVH, KHK, KHV, KHH, VKK, VKV, VKH, VVK, VVV, VVH, VHK, VHV, VHH, HKK, HKV, HKH, HVL, HVV, HVH, HHK, HHV, HHH. Remember how KKV is different than VKK when order matters. In this context it does. If one were to draw 3 marbles from a bag at once, then red, red, blue is the same as blue, red, red. This will be an important distinction for secondary mathematics.

   c. What is the probability of getting exactly one H in three spins?

d. What is the probability of getting two V's on three spins?

$$\frac{6}{27} = \frac{2}{9} = 0.\bar{2} = 22.\bar{2}\%$$

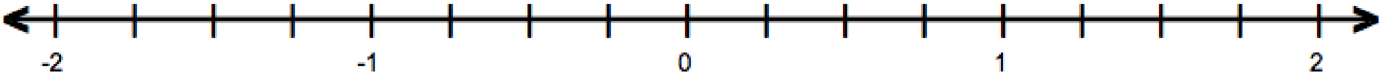
e. What is the probability of getting three K's on three spins?

f. What outcome(s) is/are most likely for three spins?
All individual outcomes are equally likely with a 1/27 probability.

g. What is the probability of getting a consonant in three spins?

4. Place the fractions on the number line below.

$$\frac{2}{3}, \quad \frac{3}{10}, \quad \frac{1}{2}, \quad \frac{3}{2}, \quad \frac{4}{3}$$

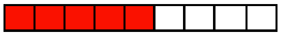


5. Order the following fractions from least to greatest.

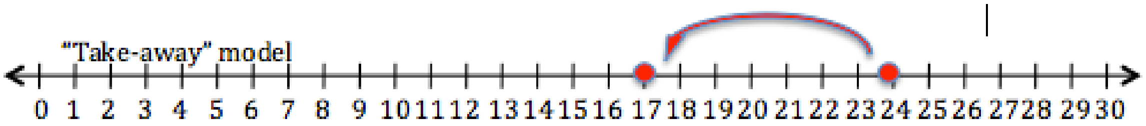
$$\frac{2}{3}, \frac{3}{10}, \frac{1}{2}, \frac{3}{2}, \frac{4}{3}$$

Spiral Review

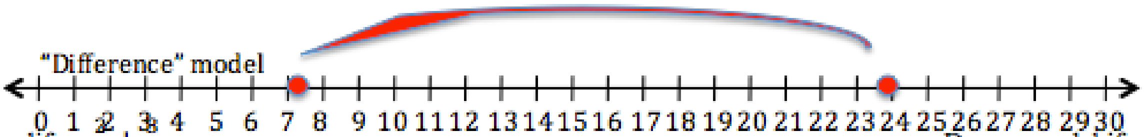
1. Model $\frac{5}{9}$



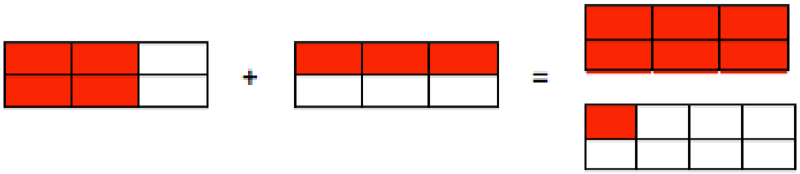
2. Use a number line to solve 24 – 7.



OR



3. Simplify: $\frac{2}{3} + \frac{3}{6}$ Draw a model if necessary.



1.1d Self-Assessment: Section 1.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Understand and apply likelihood of a chance event as between 0 and 1. [1a, 2a]	I'm not sure how to express a probability.	I know a probability is expressed as the relationship between possible outcomes and total outcomes, but I'm not sure how exactly to write that number.	I know that a probability is expressed as the quotient between possible outcomes and total outcomes, but I struggle with transitioning between fraction, decimal and percent forms.	I can easily express the probability of an event as a decimal, fraction, and percent.
2. Approximate probability by collecting data on a chance process (experimental probability).	I don't understand how to collect and record data in a probability experiment.	I can collect data on a chance process, but I don't know how to use that data to calculate experimental probability.	I can collect data on a chance process and calculate experimental probability using the data. However, I have a hard time explaining how the data relates to the theoretical probability of the event.	I can collect data on a chance process and calculate the experimental probability using the data. I can also explain how the experimental probability is different from theoretical probability.
3. Calculate theoretical probabilities on a chance process for simple events. [2a]	I don't understand the difference between a theoretical probability and an experimental probability so I'm not sure how to find it.	I can calculate theoretical probabilities for simple events like those in Problem 2a. However, I have a hard time explaining how to find it or why a theoretical probability isn't always the same as the experimental probability.	I can calculate theoretical probabilities for simple events like those in Problem 2a. However I sometimes struggle with either explaining how to find it and/or why an experimental probability may not have the same result as the theoretical probability.	I can calculate theoretical probabilities for simple events as in Problem 2a. I can easily explain why the process works and why a theoretical probability may not be the same as the experimental probability.
4. Given the theoretical probabilities of a chance event, predict the approximate frequencies for a given number of trials in an experiment of the event. [3a, 3b]	I don't understand how a theoretical probability helps me predict the approximate frequencies of an experiment.	When given a theoretical probability, I know the process for predicting the frequency of a scenario such as Problem 3a on the following page, but sometimes I struggle.	When given a theoretical probability, I know the process for predicting the frequency of a scenario such as Problem 3a on the following page and can usually find an approximation for the frequencies.	When given a theoretical probability, I know and can explain the process for predicting the frequency of a scenario such as Problem 3a on the following page.
5. Use appropriate fractions, decimals and percents to express the probabilities. [1a, 2a]	Sometimes I can express probabilities as either a fraction, decimal, or percent, but not all three.	I can express probabilities as two of the following: fraction, decimal, or percent.	I can express probabilities as a fraction, decimal, and percent.	I can express and understand probabilities as a fraction, decimal, and percent. I can also explain when each is most appropriate.

Sample Problems for Section 1.1

Express all probabilities as fractions, decimals, and percents.

1. Scarlet had a bag with red, green, and blue marbles. The following table shows what color she drew each time.

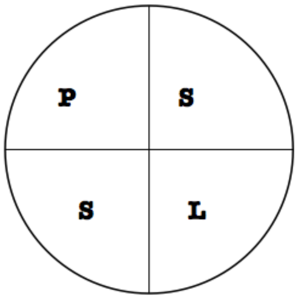
Draw	1	2	3	4	5	6	7	8	9	10
Result	red	green	blue	blue	red	blue	red	green	green	blue

a) Find the experimental probability of drawing a red marble.

b) If there are 100 marbles in the bag, how many of them do you think are red? Justify your answer.

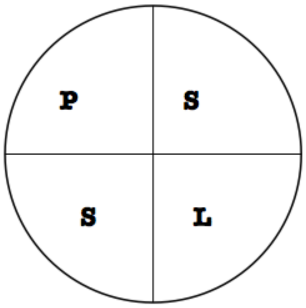
2. Find the theoretical probabilities for each of the following:

a.

If you flip a penny once, what is the probability of getting heads?	If you spin the following spinner once, what is the probability of spinning an S?  	If you roll a six-sided die once, what is the probability of rolling a 5?

Explain how you know your answer to each of the above is correct.

b.

If you flip a penny three times, what is the probability of getting heads for all three flips?	If you spin the following spinner twice, what is the probability of spinning a P and an S in any order?  	If you roll a six-sided die twice, what is the probability of rolling a 5 (on either roll)?

Explain how you know your answer to each of the above is correct.

3. Tommy has 30 marbles in a bag. If $\frac{1}{2}$ are blue, $\frac{1}{3}$ are red, and $\frac{1}{6}$ are yellow and he draws out a marble, records the color, returns it to the bag and repeats the process 10 times,

a) approximately how many times should he expect to draw a blue marble?

b) Suppose Tommy did the same experiment 1000 times, approximately how many times would Tommy expect to draw a blue marble?

c) Which of the two approximations is likely to be the most accurate? Justify your answer.

Section 1.2: Understand/Apply Equivalence in Rational Number Forms. Convert Between Forms (Fraction, Decimal, Percent).

Section Overview:

In this section students solidify and practice rational number sense through the careful *review* of fractions, decimals and percents in this section. The two key objectives of this section are: a) students should be confidently able to articulate with words, models and symbols the relationship among equivalent fractions, decimals, and percents and b) students should understand and use models to find portions of different wholes.

The concept of equivalent fractions naturally leads students to the issues of ordering and estimation. Students will represent order of fractions on the real number line. It is important that students develop estimation skills in conjunction with both ordering and operating on positive and negative rational numbers.

Lastly, students look at percent as being a fraction with a denominator of 100. Percent and fraction contexts in this section should be approached intuitively with models. In section 1.3 students will begin to transition to writing numeric expressions.

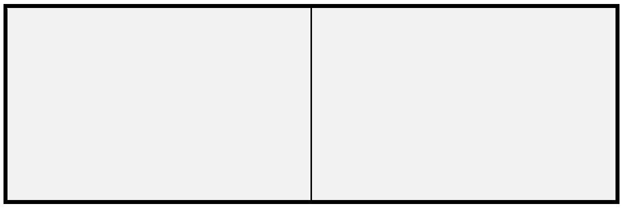
Concepts and Skills to be Mastered (from standards)

1. Express probability using appropriate fractions, decimals, and percents.
2. Express and convert between rational numbers in different forms.
3. Draw models to show equivalence among fractions and rational numbers.
4. Compare rational numbers in different forms.
5. Find the percent of a quantity using a model.
6. Solve problems with rational numbers using models.

It is important in this unit to realize that a 25% decrease is equal to 75% of the original whole while a 25% increase is equal to 125% of the original amount.

1.2a Class Activity: “10 × 10 Grids” & Conversion

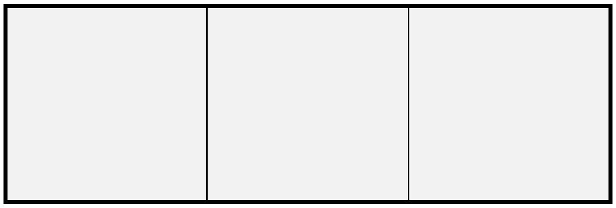
Write the equivalent values for the following parts of a candy bar.



The bar on the left is divided into two parts.

1 part = $\frac{1}{2}$ (fraction) = 0.5 (decimal) = 50% (percent)

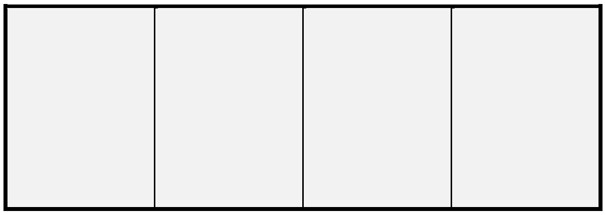
2 parts = $\frac{2}{2}$ (fraction) = 1.0 (decimal) = 100% (percent)



The bar on the left is divided into three parts.

1 part = $\frac{1}{3}$ (fraction) = $0.\overline{3}$ (decimal) = $33.\overline{3}\%$ (percent)

2 parts = $\frac{2}{3}$ (fraction) = $0.\overline{6}$ (decimal) = $66.\overline{6}\%$ (percent)

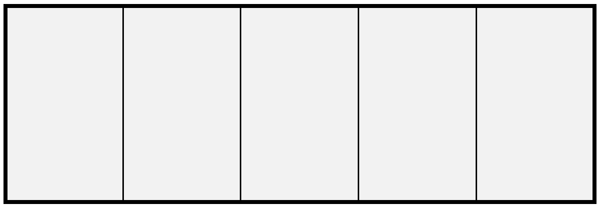


The bar on the left is divided into four parts.

1 part = $\frac{1}{4}$ (fraction) = 0.25 (decimal) = 25% (percent)

2 parts = $\frac{1}{2}$ (fraction) = 0.5 (decimal) = 50% (percent)

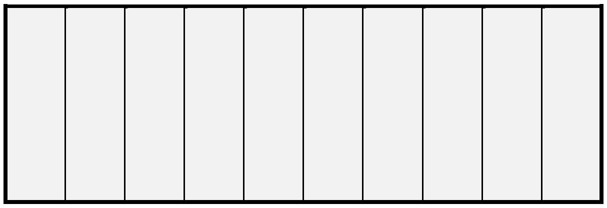
3 parts = $\frac{3}{4}$ (fraction) = 0.75 (decimal) = 75% (percent)



The bar on the left is divided into five parts.

1 part = $\frac{1}{5}$ (fraction) = 0.2 (decimal) = 20% (percent)

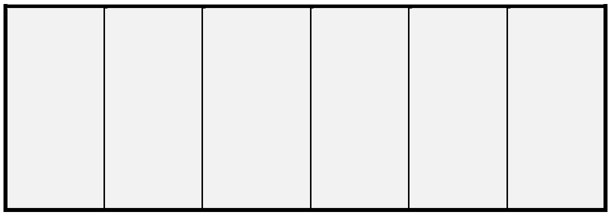
3 parts = $\frac{3}{5}$ (fraction) = 0.6 (decimal) = 60% (percent)



The bar on the left is divided into ten parts.

3 parts = $\frac{3}{10}$ (fraction) = 0.3 (decimal) = 30% (percent)

8 parts = $\frac{4}{5}$ (fraction) = 0.8 (decimal) = 80% (percent)

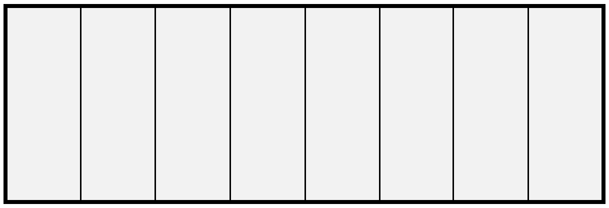


The bar on the left is divided into six parts.

1 part = $\frac{1}{6}$ (fraction) = $0.1\overline{6}$ (decimal) = $16.\overline{6}\%$ (percent)

2 parts = $\frac{1}{3}$ (fraction) = $0.\overline{3}$ (decimal) = $33.\overline{3}\%$ (percent)

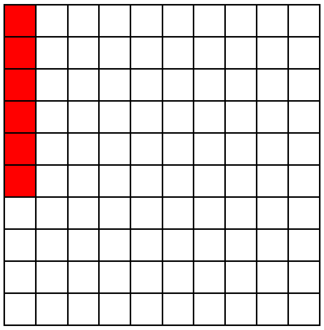
4 parts = $\frac{2}{3}$ (fraction) = $0.\overline{6}$ (decimal) = $66.\overline{6}\%$ (percent)



The bar on the left is divided into eight parts.

2 parts = $\frac{1}{4}$ (fraction) = 0.25 (decimal) = 25% (percent)

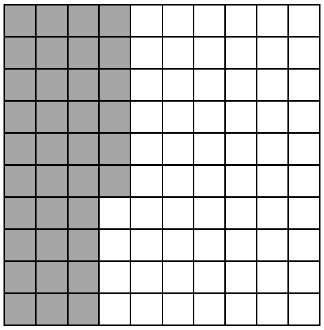
7 parts = $\frac{7}{8}$ (fraction) = 0.875 (decimal) = 87.5% (percent)



1. To the left is a 10 × 10 Grid. Why do you think it is called a 10 × 10 grid?

2. Use the grid to show the fraction $\frac{6}{100}$. Explain why this model is correct.

Discuss why 6/100 = 3/50 using the model.

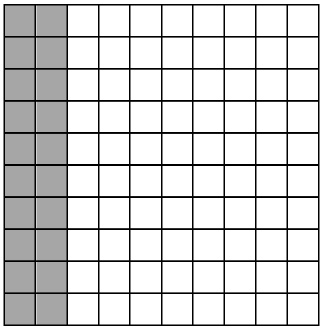


3. What fraction is shown in this 10 × 10 grid? Explain. $\frac{36}{100}$ *or* $\frac{9}{25}$

Shade 36 boxes to show 9/25 (they might shade 9 groups of 4→4 is 1/25 of the whole.)

4. What is the decimal equivalent for this fraction? 0.36

Talk about $\frac{9}{25} = \frac{9}{25} \times \frac{4}{4} = \frac{36}{100}$ = 0.36

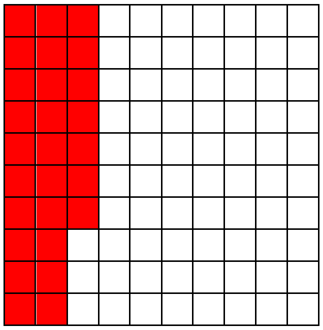


5. What fraction is shown in this 10 × 10 grid? $\frac{20}{100}$ *or* $\frac{1}{5}$

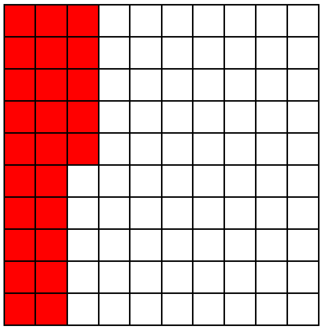
6. What is the decimal equivalent for this fraction? 0.2

7. Shade the given decimal in each grid below:

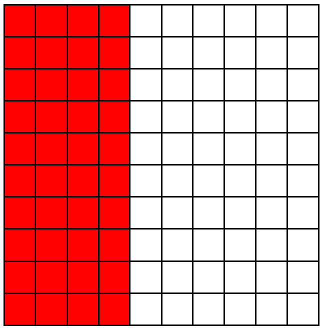
a. 0.27



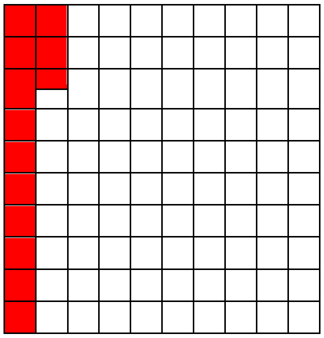
b. 0.35



c. 0.4



d. 0.125



8. Write the fraction equivalent for each decimal, in **simplest form**, here.

a. ___ $\frac{27}{100}$ ___

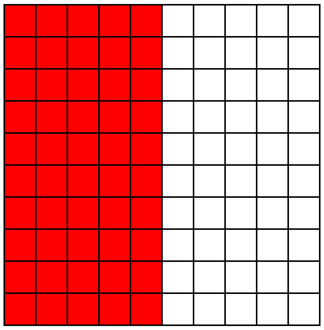
b. ___ $\frac{7}{20}$ ___

c. __ $\frac{2}{5}$ ___

d. ___ $\frac{1}{8}$ _____

9. Shade the fractional part of each grid. Then write the fraction as a decimal and a percent.
(Shading may vary.)

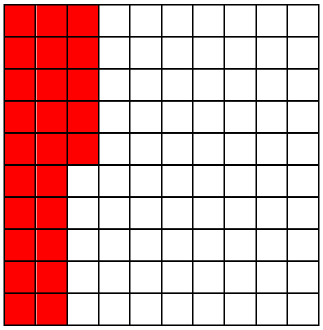
a. $\frac{1}{2}$



decimal: 0.5

percent: 50%

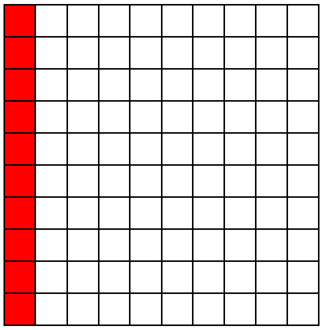
b. $\frac{1}{4}$



decimal: 0.25

percent: 25%

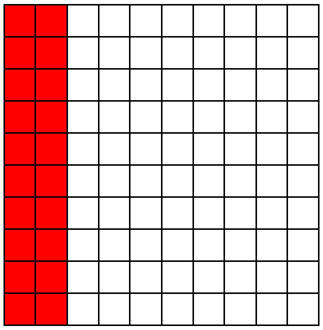
c. $\frac{1}{10}$



decimal: 0.1

percent: 10%

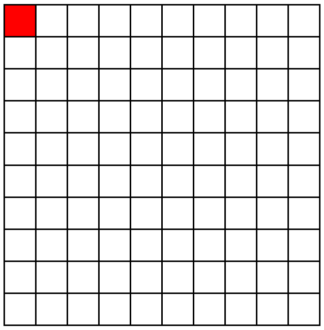
d. $\frac{1}{5}$



decimal: 0.2

percent: 20%

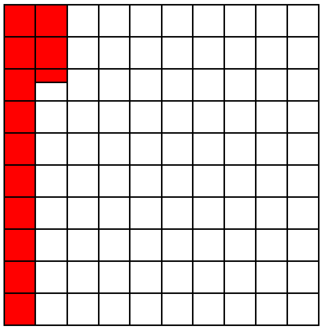
e. $\frac{1}{100}$



decimal: 0.01

percent: 1%

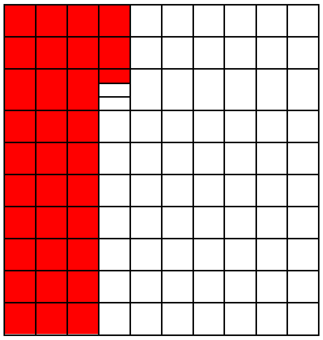
f. $\frac{1}{8}$



decimal: 0.125

percent: 12.5%

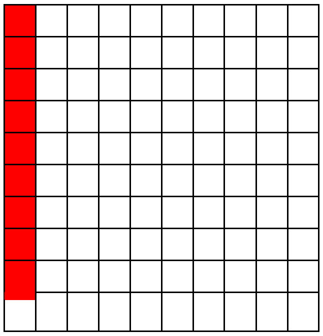
g. $\frac{1}{3}$



decimal: $0.\overline{3}$

percent: $33.\overline{3}\%$

h. $\frac{1}{9}$



decimal: $0.\overline{1}$

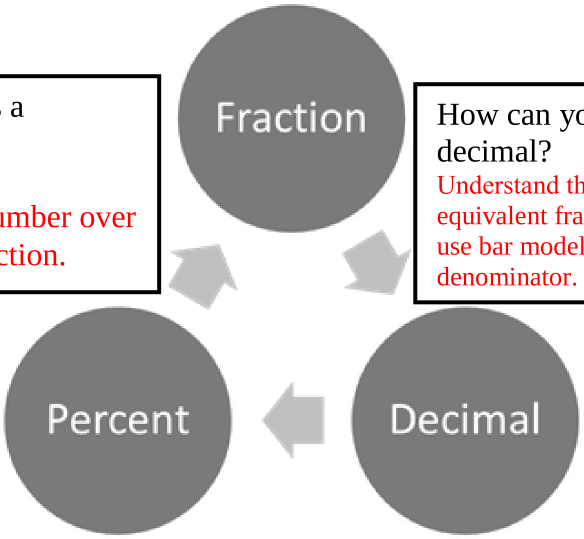
percent: $11.\overline{1}\%$

10. Use long division to show how you can convert each fraction to a decimal and then a percent. Then use equivalent fractions to do the same.

a. $\frac{1}{2}$	b. $\frac{1}{4}$	c. $\frac{1}{10}$	d. $\frac{1}{5}$	e. $\frac{1}{100}$	f. $\frac{1}{8}$	g. $\frac{1}{3}$	h. $\frac{1}{9}$
$\begin{array}{r} 0.5 \\ 2\overline{)1.0} \end{array}$	0.25	0.10	0.20	0.01	0.125	0.33…	0.111…
50%	25%	10%	20%	1%	12.5%	$33.\overline{3}\%$	$11.\overline{1}\%$

1.2a Homework: Converting Between Fractions, Decimals, and Percents

1. Fill in the boxes below to show how you can convert between fractions, decimals, and percentages.



How can you write a percent as a fraction?

Write the percent as a whole number over 100, then simplify the fraction.

How can you write a fraction as a decimal?
Understand that a decimal is “out of 100;” find an equivalent fraction with a denominator of 100 or use bar model. OR Divide the numerator by the denominator.

How can you write a decimal as a percent?
Understand that decimal is out of 100; move the decimal point to the right two places and put in the percentage symbol.

Show how to use the steps you described above to complete the following problems.

2. Write 45% as a fraction.

$\frac{45}{100}\ or \frac{9}{20}$

3. Write $\frac{3}{5}$ as a decimal.

4. Write $\frac{1}{9}$ as a decimal.

$0.\overline{1}$

5. Write 0.45 as a percent.

6. Write 1 as a percent.

100%

Fill in each blank with the equivalent fraction, decimal or percent. Use bar notation for repeated decimals.

	Fraction	Decimal	Percent
7.	$\frac{6}{10}$	0.6	
8.	$\frac{4}{25}$		16%
9.		0.42	42%
10.	$\frac{4}{5}$	0.8	
11.	$\frac{8}{25}$		32%
12.	$\frac{9}{20}$	0.45	
13.		0.21	21%
14.		0.06	6%
15.	$\frac{7}{100}$		7%
16.	$\frac{1}{8}$	0.125	
17.		0.99	
18.	$\frac{4}{5}$		80%
19.	$\frac{1}{4}$		25%
20.		0.2	20%
21.	$\frac{6}{15}$	0.4	
22.	$\frac{3}{2}$	1.5	
23.	$\frac{5}{2}$		250 %
24.	$\frac{3}{1}$	3.0	
25.	$\frac{8}{11}$	$0.\overline{72}$	
26.	$\frac{2}{3}$		$66.\overline{6}\%$

<u>Show your work over here!</u>

Spiral Review

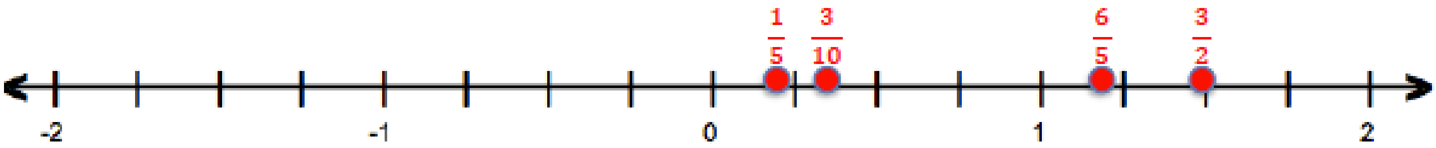
1. Order the following fractions from *least to greatest*.

$$\frac{1}{2}, \quad \frac{4}{5}, \quad \frac{1}{3}, \quad \frac{2}{5}$$

$$\frac{1}{3}, \quad \frac{2}{5}, \quad \frac{1}{2}, \quad \frac{4}{5}$$

2. Write three-fourth as a fraction, decimal, and percent. ¾ 0.75. 75%

3. Place the fractions on the number line below. $\frac{6}{5}, \frac{3}{10}, \frac{1}{5}, \frac{3}{2}$



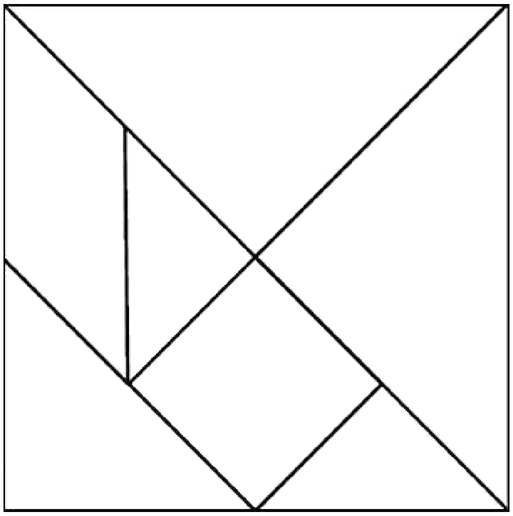
4. Without using a calculator, determine which fraction is bigger in the pair. Justify your answer.

[✓ $\frac{3}{7}$] or $\frac{3}{8}$

For each we have 3 parts of a whole that was cut into either 7 or 8. 1/7 is larger than 1/8, so 3 groups of 1/7 is more than three groups of 1/8.

1.2b Class Activity: Using Tangrams to Understand Fractions and Decimals

Below is a square created out of tangrams.



1. With a partner, take two different colored tangram sets and recreate the square shown above.

2. Using your different colored shapes, compare the small, medium, and large triangles to each other. How do they compare in size? The medium triangle is 2 times as big as the small triangle. The large triangle is 2 times as big as the medium triangle. Thus the large triangle is 4 times as big as the small triangle.

3. Now look at the small square and parallelogram. What do you observe?
They are the same portion of the whole—they have the same area. You may want to return to this idea in Chapters 5 and 8.

4. Complete each equation. Find as many solutions as possible. The first one is started for you.

a. one small square =

one small square = 2 small triangles	one small square = $\frac{1}{2}$ of the large triangle
One small square = 1 medium triangle	One small square = 1 parallelogram

b. one large triangle=

One large triangle = 4 small triangles	One large triangle = 2 small squares
One large triangle = 2 medium triangles	One large triangle = 2 parallelograms

c. one parallelogram =

One parallelogram = 2 small triangles	One parallelogram = $\frac{1}{2}$ of the large triangle
One parallelogram = 1 medium triangle	One parallelogram = 1 small square

d. one medium triangle =

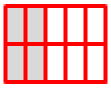
One medium triangle = 2 small triangles	One medium triangle = $\frac{1}{2}$ of the large triangle
One medium triangle = 1 small square	One medium triangle = 1 parallelogram

5. Find the value of each shape relative to the entire square. Remember the square represents 1 whole. Record your findings in the table below.

Name of Shape	Fraction	Decimal	Percent
Large triangle	1/4	0.25	25%
Medium triangle	1/8	0.125	12.5%
Small triangle	1/16	0.0625	6.25%
Small Square	1/8	0.125	12.5%
Parallelogram	1/8	0.125	12.5%

Reduce/simplify each fraction. Draw a model to show the equivalence between the original fraction and the reduced one:

6. $\frac{4}{10} =$ $\frac{2}{5}$



7. $\frac{12}{18} =$ $\frac{2}{3}$

8. $\frac{4}{12} =$ $\frac{1}{3}$

Find an equivalent fraction for each. Draw a model to show the equivalence between the original fraction and the new one.

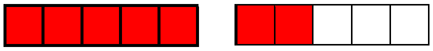
9. $\frac{1}{3} = \frac{?}{9}$? = 3

10. $\frac{3}{7} = \frac{18}{?}$? = 42

11. $\frac{4}{5} = \frac{?}{25}$? = 20

Change each to a mixed number. Draw a bar model to show the equivalence between the original fraction and the new one.

12. $\frac{7}{5} =$ $1\frac{2}{5}$

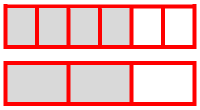


13. $\frac{25}{3} =$ $8\frac{1}{3}$

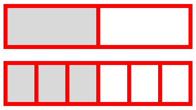
14. $\frac{18}{4} =$ $4\frac{1}{2}$

1.2b Homework: Equivalent Fractions

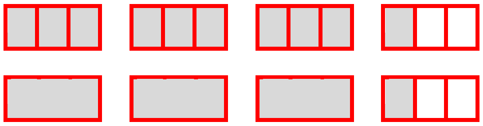
7. Simplify each fraction. Draw a model for questions “a” and “b” to show equivalent fractions.

a. $\frac{4}{6} = \frac{2}{3}$ 	b. $\frac{3}{9} =$	c. $\frac{10}{18} = \frac{5}{9}$
d. $\frac{14}{21} =$	e. $\frac{9}{21} = \frac{3}{7}$	f. $\frac{7}{35} =$

2. Write an equivalent fraction. Draw a bar model for questions “a” and “b” to show the equivalent fractions.

a. $\frac{1}{2} = \frac{}{6}$ $\frac{1}{2} = \frac{3}{6}$ 	b. $\frac{2}{5} = \frac{}{15}$	c. $\frac{2}{3} = \frac{}{15}$ $\frac{2}{3} = \frac{10}{15}$
d. $\frac{4}{7} = \frac{}{14}$	e. $\frac{5}{8} = \frac{}{24}$ $\frac{5}{8} = \frac{15}{24}$	f. $\frac{3}{4} = \frac{}{24}$

3. Change each fraction to a mixed number. Draw a bar model for questions “a” and “d”.

a. $\frac{10}{3} = 3\frac{1}{3}$ 	b. $\frac{29}{4} =$	c. $\frac{25}{9} = 2\frac{7}{9}$
d. $\frac{25}{12} =$	e. $\frac{20}{7} = 2\frac{6}{7}$	f. $\frac{19}{5} =$

Spiral Review

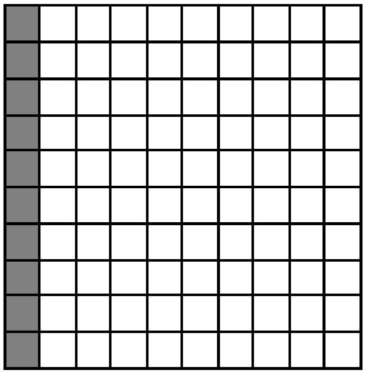
1. Write $\frac{2}{3}$ as a decimal and percent.

$\frac{2}{3}$	$0.\overline{66}$	$66.\overline{6}\%$ or 66 2/3%

2. Write this model in fraction form then simplify.

$\frac{10}{100}$,

$\frac{1}{10}$



Use long division to show how you can convert this fraction to a decimal and then a percent

a) $\frac{1}{5}$

$$\begin{array}{r|l} & .2 \\ \hline 5 & 1.0 \\ & 1\ 0 \\ \hline & 0 \end{array}$$

.2 , 20%

b) $\frac{3}{8}$

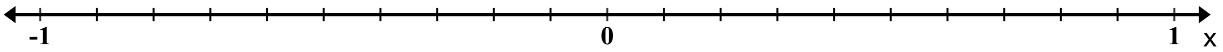
0.375 37.5 %

1.2c Class Activity: Rational Number Ordering and Estimation

1. Plot each fraction on the number line below.

a. $\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, \frac{2}{5}$

b. $-\frac{1}{2}, -\frac{1}{3}, -\frac{3}{4}, -\frac{2}{5}$



c. Compare the fractions using <, >, or =.

$\frac{1}{2}$ __>__ $\frac{1}{3}$ $\qquad$ $-\frac{1}{2}$ ____ $-\frac{1}{3}$ $\qquad$ $\frac{2}{5}$ ____ $\frac{3}{4}$ $\qquad$ $-\frac{2}{5}$ __>__ $-\frac{3}{4}$

d. What differences do you observe when comparing the fractions in part *c*?

Notice that $\frac{1}{3} < \frac{1}{2}$ but that $-\frac{1}{3} > -\frac{1}{2}$.

e. How does the number line help you determine which number is larger?
Numbers to the right are always larger than numbers to the left.

2. Classify these fractions as close to 0, close to $\frac{1}{2}$, or close to 1.

$\frac{1}{2}, \frac{1}{9}, \frac{5}{8}, \frac{5}{6}, \frac{7}{8}, \frac{1}{5}, \frac{5}{9}, \frac{3}{8}, \frac{1}{7}$

Close to 0	**Close to $\frac{1}{2}$**	**Close to 1**
$\frac{1}{9}, \frac{1}{5}, \frac{1}{7}$	$\frac{1}{2}, \frac{5}{8}, \frac{5}{9}, \frac{3}{8}$	$\frac{5}{6}, \frac{7}{8}$

3. Order the fractions from least to greatest.

Fractions: ______, ______, ______, ______, ______, ______, ______, ______, ______

$\frac{1}{9}, \frac{1}{7}, \frac{1}{5}, \frac{3}{8}, \frac{1}{2}, \frac{5}{9}, \frac{5}{8}, \frac{5}{6}, \frac{7}{8}$

4. Classify these fractions as close to 0, close to $-\frac{1}{2}$, close to -1.

$-\frac{1}{2}, -\frac{4}{28}, -\frac{3}{5}, -\frac{2}{7}, -\frac{5}{14}, -\frac{2}{11}, -\frac{4}{5}, -\frac{7}{9}$

If students struggle with this, have them plot each as positive numbers and then reflect over 0.

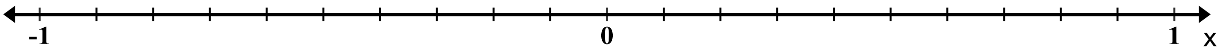
Close to 0	**Close to $-\frac{1}{2}$**	**Close to -1**
$-\frac{4}{28}, -\frac{2}{11}$	$-\frac{1}{2}, -\frac{2}{7}, -\frac{2}{7}, -\frac{5}{14}$	$-\frac{4}{5}, -\frac{7}{9}$

5. Order the above numbers from least to greatest.

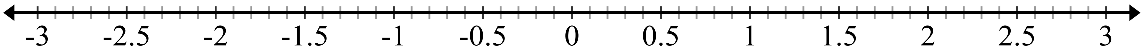
Fractions: _______, _______, _______, _______, _______, _______, _______, _______

$-\frac{4}{5}, -\frac{7}{9}, -\frac{3}{5}, -\frac{1}{2}, -\frac{5}{14}, -\frac{2}{7}, -\frac{2}{11}, -\frac{4}{28}$

Now approximate their location on the number line below:



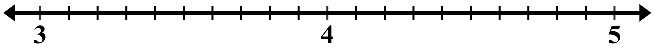
6. Approximate where each value is located on the number line below. State a "common" fraction and decimal you can use to help you find the approximate location for each. The first one is done for you.



a. $0.32 \approx 0.333 \approx \frac{1}{3}$	b. $0.67 \approx 0.666 \approx \frac{2}{3}$	c. $0.76 \approx .75 \approx \frac{3}{4}$	d. $0.98 \approx \frac{1}{1}$
e. $-0.24 \approx -0.25 \approx -\frac{1}{4}$	f. $2.32 \approx 2.333 \approx 2\frac{1}{3}$	g. $-3.38 \approx -3.4 \approx -3\frac{2}{5}$	h. $1.76 \approx 1.75 \approx 1\frac{3}{4}$

Plot each fraction on the number line. Fill in the blank using < , > , or = .

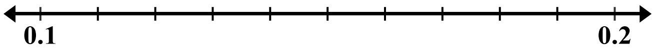
7. $4\frac{3}{10}$ ___>__ $4\frac{2}{7}$



8. $-4\frac{3}{10}$ __<___ $-4\frac{2}{7}$

Plot each fraction on the number line. Fill in the blank with < , > , or = . How do you know your answer is correct? Justify your answer.

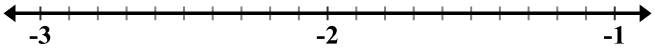
9. $0.1\overline{4} > 0.14$



It may be helpful to ask students where "0" is located on the line for # 9, 10 and 12.

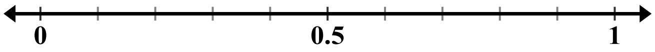
Justification:

10. $-2.15 < -2.13$



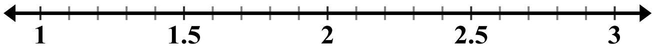
Justification:

11. $0.15 = \frac{3}{20}$



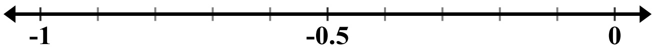
Justification:

12. $2\frac{2}{3} > 2.6$



Justification:

13. $-0.3 > -\frac{1}{3}$



Justification:

14. Is 0.74 to the left or right of $\frac{3}{4}$? Explain.

0.74 is left of ¾ because it's smaller: ¾ = 0.75

15. Is 1.26 to the left or right of $1\frac{1}{4}$? Explain.

1.26 is right of 1 ¼ because it's larger: 1 ¼ = 1.25

16. Give a fraction and decimal approximation of $\frac{71}{102}$.

This is approximately 75/100, so approximately 0.75. The estimate is a bit larger than the original. OR approximately 70/100, so approximately 7/10 = 0.7.

17. Give a fraction and decimal approximation of $\frac{9}{23}$.

Some students may say 40/100 = 2/5 or 0.4 However, 8/24 or 0.33 repeating is far better. Stress with students that you're estimating, but that you want to get as close as possible.

1.2c Homework: Rational Number Ordering and Estimation

ESTIMATE **each of the following. Justify your answer with either words or a model. Indicate if your estimate is larger or smaller than the exact answer.**

	Fraction	**≈ Decimal**	**≈ Percent**	**Justification**
1.	$\frac{11}{21}$	0.50	50%	11 is a little more than half of 21; it's about 10/20
2.	$\frac{16}{27}$		$66.\overline{6}\%$	16 is a little less than 2/3 of 27; it's about 18/27
3.	$\frac{89}{99}$			
4.	$\frac{13}{17}$	0.75		
5.	$\frac{5}{23}$			This is close to $\frac{5}{25}$ *or* $\frac{1}{5}$. Students might also say 5/20 or 1/4.

Fill in the blank with <, >, or =. Justify each with a picture or number-line or explanation.

6. $1\frac{4}{25}$ 1.2 Justification:

7. $-1\frac{4}{25}$ > -1.2 Justification:

8. -2.34 $-2\frac{4}{11}$ Justification:

9. $\frac{5}{11}$ $=$ $0.\overline{45}$ Justification:

10. $4\frac{5}{6}$ $4\frac{10}{13}$ Justification:

11. $-3\frac{19}{20} < -3.94$ Justification:

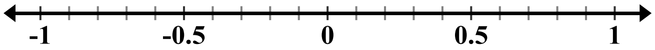
12. Order the following set of numbers from least to greatest: $1\frac{3}{4}$, 1.73, $1\frac{3}{5}$, 1.78

13. Order the following set of numbers from least to greatest: $-0.26, -\frac{3}{10}, -0.35, -\frac{6}{25}$

$-0.35, -\frac{3}{10}, -0.26, -\frac{6}{25}$

14. Plot each rational number on the number line. Write them in order from least to greatest:

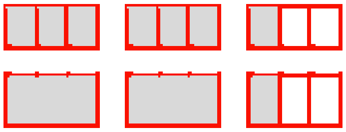
$\frac{3}{8}$, - 0.38, $\frac{3}{7}$, - 0.43



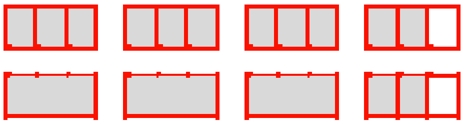
Spiral Review

Change each fraction to a mixed number. Draw a bar model for each.

1. $\frac{7}{3}$ $2\frac{1}{3}$



2. $\frac{11}{3}$ $3\frac{2}{3}$



3. Compare these two fractions: $\frac{6}{12}$ $\boxed{<}$ $\frac{8}{14}$

1.2d Class Activity: Probability, Fractions, and Percentage

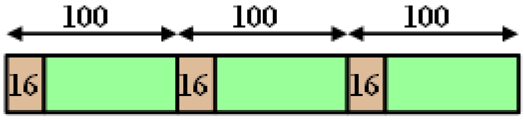
It is important in this unit to realize that a 25% decrease is equal to 75% of the whole. It is a good idea to always have students identify the part, whole and percent in the problems at the end.

1. A bag contains 100 marbles. The table below shows how many red, blue, green and yellow marbles are in the bag. Use that data to complete the table below. The first row is completed for you.

Color	**Number of Marbles**
Red	16
Blue	24
Green	45
Yellow	15

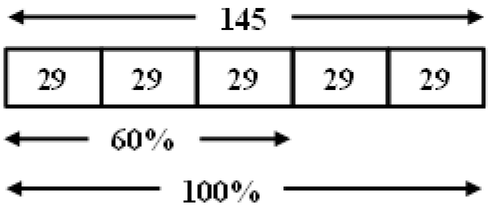
Color of Marble	**Probability of drawing the colored marble**	**Ratio of colored marble to all marbles**	**Percentage of colored marble within all marbles**
1. Red	$\frac{16}{100}=\frac{4}{25}$	$\frac{16}{100}=\frac{4}{25}$	16%
2. Blue	$\frac{24}{100}=\frac{6}{25}$	$\frac{24}{100}=\frac{6}{25}$	24%
3. Green	$\frac{45}{100}=\frac{9}{20}$	$\frac{45}{100}=\frac{9}{20}$	45%
4. Yellow	$\frac{15}{100}=\frac{3}{20}$	$\frac{15}{100}=\frac{3}{20}$	15%
5. Orange	$\frac{0}{100}=0$	$\frac{0}{100}=0$	0%
6. Red OR blue	$\frac{16+24}{100}=\frac{40}{100}=\frac{2}{5}$	$\frac{16+24}{100}=\frac{40}{100}=\frac{2}{5}$	40%
7. Red, blue, green OR yellow	$\frac{16+24+45+15}{100}=\frac{100}{100}=1$		100%

2. Suppose you want to create a bag with 300 red, blue, green and yellow marbles that has the same probability for drawing each color as #1 above. Find how many of each color you will need.

Color	**Probability→Amount**	**Percent→Amount**	**Model**
Red	$\frac{4}{25}=\frac{4}{25}{}^{\prime}\frac{12}{12}=\frac{48}{300}$ or $\frac{4}{25}=\frac{16}{100}=\frac{3{}^{\prime}\,16}{3{}^{\prime}\,100}=\frac{48}{300}$	$16\% = \frac{16}{100}$ $\frac{16\cdot 3}{100\cdot 3}=\frac{48}{300}$ 48 red marbles	
Blue	Make sense of problems and model with mathematics	$24\% = \frac{24}{100}$ $\frac{24\times 3}{100\times 3}=\frac{72}{300}$ 72 blue marbles	
Green	$\frac{45\times 3}{100\times 3}=\frac{135}{300}$	$45\% = \frac{45}{100}$ 135 green marbles	
Yellow	$\frac{15\times 3}{100\times 3}=\frac{45}{300}$	$15\% = \frac{15}{100}$ 45 yellow marbles	

Modeling to find the part, whole or percent

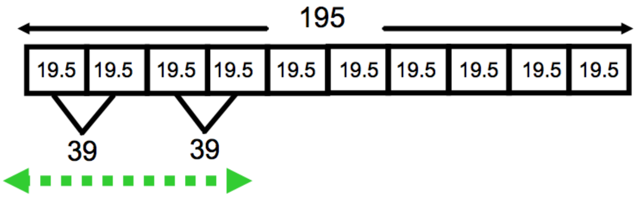
Example 1: Sally wants to compute 60% of 145. She knows that 60% $=\frac{60}{100}=\frac{6}{10}=\frac{3}{5}$. This means that 145 should be divided into either100, 10 or 5 parts. Dividing 145 into 10 parts would mean 1.45 in each part, dividing into 10 parts would mean 14.5 in each part; dividing into 5 parts would mean 29 in each part. She decided to use 5:



Sixty percent of 145 is the same as 3/5 of 145; thus 60% of 145 is 29 + 29 + 29 or 87.

Example 2: What percent of 195 is 78?

We know that 10% of 195 is 19.5. So one way to think about this is to divide 195 into ten parts:

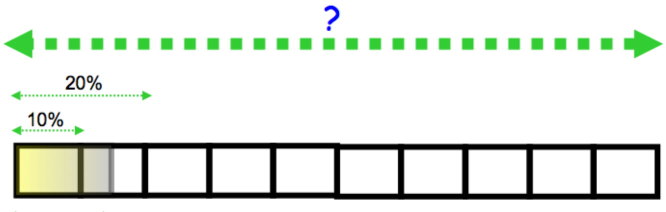


Four out of the ten boxes make 78, thus 40% of 197 is 78

Above we can see that two 10% portions make 39; thus four 10% portions make 78. So, 78 is 40% of 195.

Example 3: 15% of what is 45?

We know that percents are relationships of 100 and that if we can find 10%, we can compute anything fairly easily. If 45 is 15% of the whole, then 15 is 5% of the whole (15% divided into 3 is 5%, and 45 divided into 3 is 15.) That means that 5% of the whole is 15, so 10% is 30:



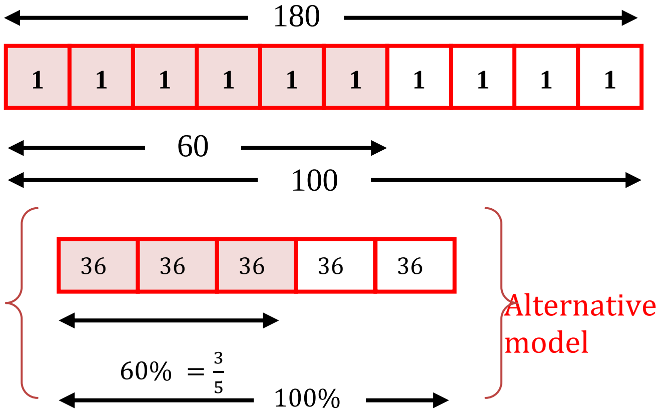
45 is 15% of the whole

If each 10% portion is 30, than the whole is 30 × 10 or 300.

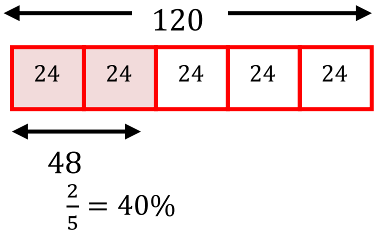
Use a model like the one shown previously to answer the following questions.

The key to drawing these models is to convert the percent to a fraction. Help students see the connection between fractional and percent portions.

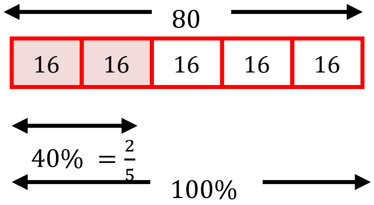
3. Find 60% of 180. 108



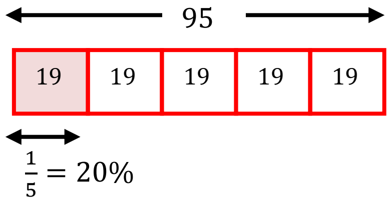
7. What percent of 120 is 48? 40%



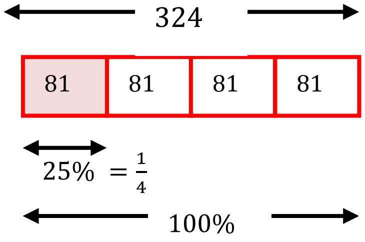
4. Find 40% of 80. 32



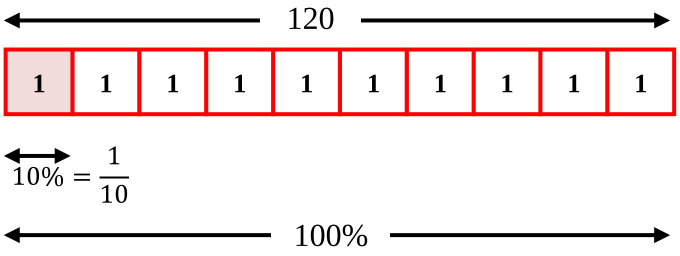
8. What percent of 95 is 19? 20%



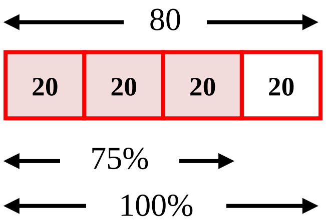
5. Find 25% of 324. 81



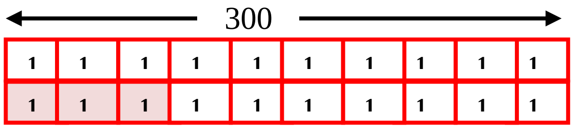
6. 9. 12 is 10 percent of what number? 120



7. What percent of 80 is 60? 75%

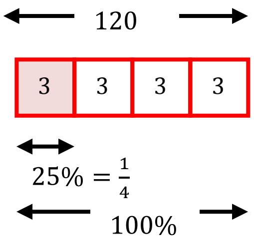


10. 45 is 15 percent of what number? 300



$15\% = \frac{3}{20}$, 45 into 3 parts is 15 per part

11. 30 is 25 percent of what number? 120



1.2d Homework: Probability, Fractions, and Percentage

Express each fraction as a percent.

1. $\frac{1}{5} =$ 20%

2. $\frac{9}{10} =$ 90%

3. $\frac{3}{20} =$

4. $\frac{2}{5} =$

Express each percent as a fraction in simplest forms

5. 60% =

6. 80% =

7. 75% $\frac{3}{4}$

8. 10% =

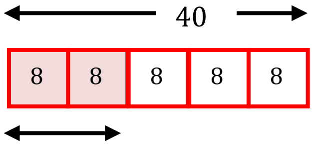
9. 5% = $\frac{1}{20}$

10. 25%

Solve each using a model and mental math. Show your work:

11. Find 80% of 150.

12. Find 40% of 40. 16



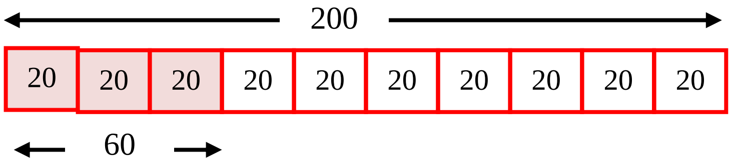
$40\% = \frac{2}{5}$

13. What percent of 30 is 15?

14. What percent of 240 is 60?

15. 40 is 8% of what number? 500

16. 60 is 30% of what number? 200



$30\% = \frac{3}{10}$, 60 into 3 parts is 20 per part

Use a model to ESTIMATE each. Indicate if your estimate is slightly higher or lower than the exact answer.

Answers may vary.

17. Estimate 74% of 24.

74% is close to 75% =3/4 and 3/4 of 24 = 18; high estimate

18. Makayla got a score of 77% on her English final. If there were 48 questions on the test, approximately how many questions did she get right?

19. Milo can run 10 miles in 60 minutes. He needs to reduce his time by 18%, approximately how many minutes does he have to take off his time?

12 minutes, high estimate

20. The cycle store has a bike regularly priced at $660, Tom negotiated a 32 % discount. What fraction can you use to estimate the money he saved? Approximately how much did he pay for the bike?

Plot each fraction on the number line. Fill in the blank with <,>, or +. How do you know your answer is correct. Justify your answer.

Spiral Review

1. Circle the number that's greater: 0.17 0.107 Justification:

0.17 is greater

2. Circle the number that's greater: 2.65 2.63 Justification:

2.65 is greater

3. **I 0.77 to the left or right of $\frac{3}{4}$ on a number line? Explain.**

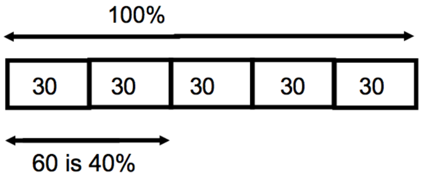
0.77 is to the right of $\frac{3}{4}$ because it's bigger: $\frac{3}{4}$ = 0.75

4. Write 32% as a decimal and fraction. 0.32; 8/25

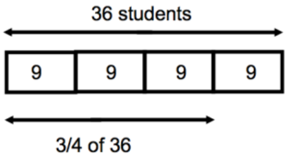
1.2e Class Activity: Applications with Models, Multi-Step Problems

Percent and fraction questions: Use a model to find the solutions.

1. 60 is 40% of what number? 150



2. There are 36 students in a math class. $\frac{3}{4}$ of the students take an art class after their math class, the rest take a social studies class. How many students take art after math? 27 students

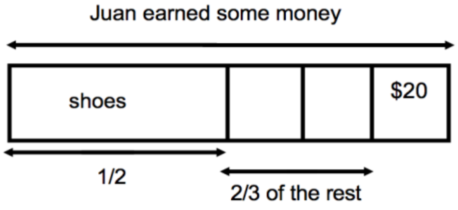


3. You get 80% correct on a history quiz with 150 questions. How many questions did you get correct?

   120 questions

4. Juan earned money for creating a webpage for a local business. He used $\frac{1}{2}$ of the money he earned for new shoes and $\frac{2}{3}$ of the rest for music. He has $20 left. How much money did he earn for his work? $120

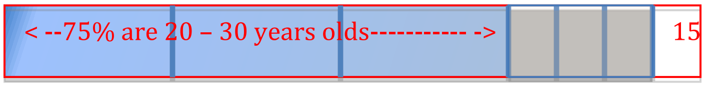
   $20 • 3 • 2 = $120



Emphasize that "2/3 of the rest" is not the same as 2/3 of the whole.

5. Lydia volunteers with an organization that helps older citizens take care of their yards. 75% of the volunteers in the organization are 20-30 years old. Of the remaining portion, 75% are over 30 and 25% are under 20. If there are 15 people under 20, how many people are in the organization?

   240 people



6. There are 360 7th grade students at Eisenhower Middle School. One-fourth of the students went to Clermont Elementary. Of the rest, half went to Central Elementary and the others came from a variety of other elementary schools. How many students came from Central Elementary?

   135 students from Central Elementary

7. A snowboard at a local shop normally costs $450. Over Labor Day weekend, the snowboard is on sale for 50% off. Customers who make their purchase before 8:00 AM earn an additional 10% off of the sale price. If Mia buys the snowboard before 8:00 AM, how much will she pay?

   $202.50

8. A local business is reviewing their expenditures. They found that they spend $\frac{1}{3}$ of their income on payroll, another $\frac{1}{2}$ goes back into the business to purchase inventory and pay for the facility, $\frac{1}{3}$ of the remainder goes to paying off their original small business loan. If they have $100,000 left to reinvest in their company, what are their total expenditures?

   $900,000

9. At a Monument Valley High School, three fourths of the 7th grade students went to Salt Lake City on a field trip. Half of the rest of the students went to Monticello for a different field trip. If there were 12 students that did not go on either trip, how many 7th grade students are there in all?

   96 students in 7th grade

10. Camilla earned $160 over the summer. If she put 80% of her earnings into her savings account and spent 75% of the rest on a gift for her mother, how much money did she have left over?
    $8

11. Mila rode in a bike tour across Utah. On one particular day, 40% of her ride was uphill. Of the rest of her ride, 1/3 was downhill and 2/3 was flat. If the flat portion of her ride was 36 miles, how far did she ride that day?

    90 miles

12. Marco’s football team was 20 yards from the goal when they got possession of the football. At the end of one play, they got half way to the goal. After the second play, they made half that distance closer to the goal. After the third play, they got half the remaining distance. How far were they from the goal line before the fourth play? 20(1/2)(1/2)(1/2) = 20/8 = 5/2 = 2.5 yards from the goal.

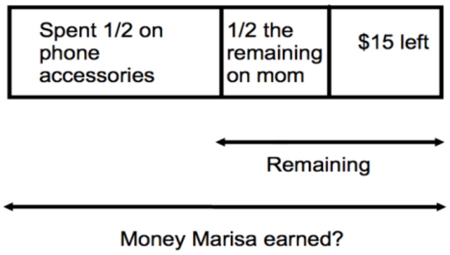
1.2e Homework: Rational Numbers with Models, Multi-Step Problems

Use a model to fill in the table below:

	Fraction	Decimal	Percent	MODEL
1.	$\frac{3}{5}$	0.6	60%	
2.	$\frac{4}{25}$			
3.	$\frac{3}{20}$	0.15	15%	
4.	$\frac{2}{3}$			
5.	$\frac{9}{20}$	0.45	45%	

Use a model to solve each problem.

6. Marisa earned money for helping a local business with organizing their inventory. She spent half of her earnings on accessories for her phone, and half of the remaining money on a gift for her mother. If she has $15 left, how much did she make organizing the inventory? $60



$15 was remaining. It is ½ of the remaining, so was $30. $30 is ½ of the total, so the total is $60.

7. Pedro has $80 left from the money he earned during the summer helping at his father's business. He had spent $\frac{1}{3}$ of his earnings on climbing equipment, $\frac{1}{3}$ on camping gear and $\frac{1}{2}$ of the remainder on entertainment during the summer. How much did he earn during the summer?

8. Jose estimates that 50% of his income goes toward living expenses (rent, utilities and food). Of the rest, 50% goes to paying for his car and 25% to other expenses. If Jose has $300 left at the end of the month, how much does he earn?

9. Marco earned $360 helping his grandfather at his business. He spent $\frac{1}{4}$ of his earnings on a gift for his mother and put $\frac{2}{3}$ of the rest into a savings account. How much does he have left over for fun?

$90

10. Julia found a great pair of boots for $240, but that was more than she wanted to spend. A few months later they were on sale for 40% off. She searched online and found a coupon for 25% off the sale price. How much will she pay for the boots? $108

11. Jose earned money over the summer working at his family's store. He put three-fourths of the money he earned in his savings account and spent half of the rest. If he as $120 left over, how much did her earn?

12. Jessi bought 18 gallons of paint to paint her house and garage. If she used 75% of the paint on the house and half of the rest on the garage, how much paint did she have left over?
2.25 gallons

Spiral Review

Express each fraction as a percent.

1. $\frac{1}{5} =$ **20%**

3. $\frac{2}{5} =$

Express each percent as a fraction in simplest form.

3. 40% $\frac{2}{5}$

4. 25% $\frac{1}{4}$

1.2f Self-Assessment: Section 1.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Express probability using appropriate fractions, decimals, and percents. [1]	I sometimes can express probabilities as either a fraction, decimal, or percent, but not all three.	Most of the time I can express probabilities as two of the following: fraction, decimal, or percent.	I can express probabilities as a fraction, decimal, and percent.	I can express probabilities as a fraction, decimal, and percent. I can also justify the most appropriate form(s) of a number for a given context.
2. Express and convert between rational numbers in different forms. [2]	I struggle to convert between rational numbers in different forms and/or I'm not sure what a rational number is.	I can convert between rational numbers in two of the different forms: fraction, decimal, or percent.	I can convert between rational numbers in all three different forms (fraction, decimal, and percent).	I can convert between rational numbers in all three different forms. I can also explain when each may be more appropriate.
3. Draw models to show equivalence among fractions and rational numbers.	I struggle to draw models of rational numbers.	I can draw models of some, but not all forms of rational numbers: fractions, decimals, and percents.	I can draw models of all forms of rational numbers. I can use the models to show equivalence.	I can draw models of all forms of rational numbers. I can explain how the models show equivalence in different forms.
4. Compare rational numbers in different forms. [3]	I can order most sets of positive rational numbers like in #3 Set A on the following page. I have trouble with other forms	I can order most sets of positive and negative rational numbers if they are in the same form like in #3 Set A or B on the following page.	I can order sets of positive and negative rational numbers in two forms like in #3 Set A, B or C on the following page.	I can compare rational numbers in different forms such as #3 Sets A, B, C and D. I can also justify my comparison.
5. Find the percent of a quantity using a model. [4]	I can draw a model of a percent. I struggle to use that model to find percent of a quantity.	I can use a model to correctly find the percent of a quantity such as Problem 4b on the following page.	I can use a model to find an unknown quantity given a percent and can find the percent of a quantity such as Problem 4a and 4b.	I can use a model to find an unknown quantity given a percent and the percent of a quantity such as Problem 4a and 4b. I can explain how the model is related to the "mathematical" process for finding percents values.
6. Solve problems with rational numbers using models. [5]	I can draw a model of a rational number. I struggle to use that model to solve problems.	I can use a model to solve single-step problems with rational numbers such as Problem 5a or 5b on the following page.	I can use a model to solve one-step and multi-step problems with rational numbers such as Problem 5a – d on the following page.	I can use a model to solve one-step and multi-step problems with rational numbers such as Problem 5a – d. I can also explain how the model is related to the computational process.

Sample Problems for Section 1.2

1. Rufina has a bag of marbles. She has 30 striped marbles, 4 cat's eye marbles and 10 solid color marbles. Rufina draws one marble out of her bag. Express each of the following probabilities as a fraction, decimal and percent.
   a. P(striped)____

   b. P(solid)____

2. Complete the following chart of equivalent rational numbers.

Fraction	Decimal	Percent
$\frac{13}{25}$		
$\frac{84}{300}$		
	0.74	
	0.1325	
		81%
		1.5%

3. Order the numbers in each box from least to greatest.

Set A

$\frac{1}{2}, \frac{3}{4}, \frac{4}{9}$	5.15, 5.002, 4.7, 5.1	$\frac{2}{3}, \frac{3}{10}, \frac{1}{4}, \frac{4}{9}$

Set B

$-\frac{1}{2}, \frac{2}{7}, \frac{1}{2}, -\frac{8}{9}$	$-7.34, -7.002, -7.7, -7.2$	$\frac{3}{7}, -\frac{10}{2}, \frac{4}{3}, -\frac{9}{8}$

Set C

$\frac{2}{7}, 0.24, \frac{1}{2}, 0.8$	$2.54, \frac{5}{2}, 2.7, \frac{15}{7}$	$0.1, \frac{10}{99}, \frac{4}{36}, 0.\overline{66}$

Set D

$-2.15, \frac{17}{7}, 2.7, -2.105$	$-\frac{4}{5}, -\frac{5}{8}, 0.4, \frac{3}{8}$	$1.4, -\frac{10}{8}, -\frac{4}{3}, 1.08$

4. Find each.

   a. 40% of what number is 50?

   b. What is 85% of 420?

5.

a. Shauna wants to buy a sweater at the store. The sweater is normally $28, but today it is on sale for 15% off. How much is the sweater?	b. In Mr. Garcia's 7th Grade Math class, $\frac{4}{5}$ of the students brought a pencil to class. If 6 people did not have pencils, how many students are in Mr. Garcia's class?
c. Uzumati has $80. He spends $\frac{1}{5}$ on groceries. Then he spends 75% of what's left on two new Blu-ray's. How much money does he have left?	d. Ms. Wells surveyed her 205 students. She found that 80% of the students were wearing blue jeans. One student was wearing shorts. Of the remaining students, $\frac{3}{8}$ were wearing skirts. How many students were wearing skirts?

Section 1.3: Solve Percent Problems Including Discounts, Interest, Taxes, Tips, and Percent Increase or Decrease.

Section Overview:

In this section, students continue to solve contextual problems with fractions, decimals and percent but begin to transition from relying solely on models to writing numeric expressions. In future chapters, students will extend their understanding by writing equations and proportional equations using variables.

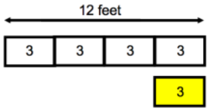
Concepts and Skills to be Mastered (from standards)

1. Find a percent or fractional portion of a whole with or without a model.
2. Solve percent problems involving percent increase and decrease (including discounts, interest, taxes, tips, etc.).
3. Develop numeric expressions from percent and fraction models.

1.3a Class Activity: Model Percent and Fraction Problems

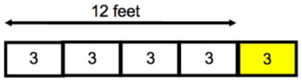
Use a model to solve each of the following multi-step problems. Then write a number sentence that reflects your model and answer.

1. Larry has a piece of rope that's 12 feet long.
   a. He cuts off 25% of the rope off. How long is the rope now? 9 feet long



12 – (.25)12 = 12 – 3 = 9 or 12 – (.25)12 = (.75)12 = 9

   b. Joe has a rope that is 25% longer than Larry's 12 foot long rope. How long is Joe's rope? 15 feet long



12 + (.25) 12 = 12 + 3 =15 or 12 + (.25) 12 = (1.25) 12 =15

2. Lydia invested $150. If Lydia earned 10% on her investment. How much money would she now have?

$165

How much money would Lydia have if instead she lost 10% on her original investment?

$135

3. A refrigerator costs $1200 wholesale. If the mark-up on the refrigerator is 20%, what is the new price?

$1440

4. Rico's resting heart rate is 50 beats per minute. His target exercise rate is 350% of his resting rate. What is his target rate? 175 beats per minute

50(3) + 50(1/2) = 150 + 25 = 175

25	25	25	25	25	25	25

← 50 beats →

5. A pair of boots was originally priced at $200. The store put them on sale for 25% off. A month later, the boots were reduced an additional 50% off the previous sale price. What is the price now?

$75

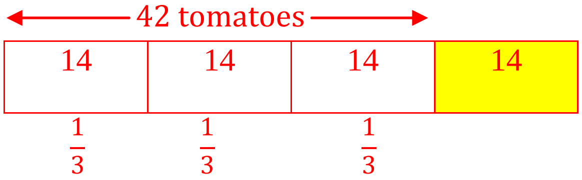
6. Marie went out for dinner with her friend. The dinner cost $24. Tax is 5% and Marcie wants to leave a 15% tip (she computed tip on the cost of dinner and tax.) How much will Marcie pay all together for dinner? 24 + 0.05(24) = 25.20; 25.20 + 0.15(25.20) = 28.98; mention that you'd likely leave $29.00

$28.98

1.3a Homework: Model Percent and Fraction Problems

Use a model to answer each of the questions below. Then write a number sentence that reflects your model and answer.

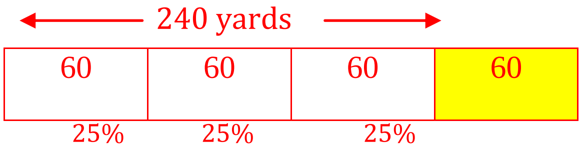
1. Last year Cory harvested 42 tomatoes from his backyard garden. This year, his harvest increased by 1/3. How many tomatoes did he harvest this year?



$42 + 14 = 56$ tomatoes

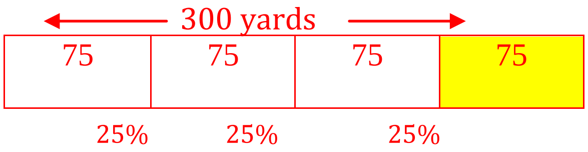
2. Jerry is taking care of a vacant lot in his neighborhood. There are approximately 64 thistle weeds in the lot. He decided to try a homemade weed killer his grandmother suggested. Five weeks later, the thistles have decreased by 75%. Approximately how many thistles are in the vacant lot now?

3. Maria is learning to play golf. She has been working particularly hard on driving. Before lessons, her drives average 240 yards. After her first lesson, her drives increase 25%. After her second lesson, they increase another 25%. How far are her average drives after two lessons?



Lesson #1

$240 + 60 = 300$ yards



Lesson #2

$240 + 75 = 375$ yards

4. Two stores have the same skateboard on sale. The original price of the skateboard is $200. At store AAA, it's on sale for 30% off with a rewards coupon that allows the purchaser to take an *additional* 20% off the sale price at the time of purchase. At store BBB, the skateboard is on sale for 50% off. Will the price for the skateboard be the same at both stores? If not, which store has the better deal?

5. Two schools start with 1000 students. The first school's enrollment increase 20% in 2012 and then decreases 20% in 2013; the second school's enrollment stays constant in both 2012 and 2013. Which school has the most students now?

First school: 2012 enrollment 1200 students, 2013 enrollment 960

Second school: 2012 enrollment 1000 students, 2013 enrollment 1000

The second school has more students in 2013

Review questions. Write all probabilities as fractions, decimals and percentages.

6. Suppose you were to roll a fair 6-sided number cube once, then flip a coin. List all the possible outcomes.

1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T

7. What is the probability of getting a 2 and heads?

8. What is the probability that you would roll an even number and flip heads? $\frac{3}{12} = \frac{1}{4} = 0.25 = 25\%$

9. What is the probability that you would roll an even number <u>or</u> flip heads?

Spiral Review

1. What is 85% of 420?

2. Fill in the equivalent decimal and percent for this fraction:

Fraction	Decimal	Percent
$\frac{9}{15}$	.6	60%

3. Jana wants to buy a shirt at the store. The shirt is normally $24, but today it's on sale for 15% off.
   a. How much will she save?

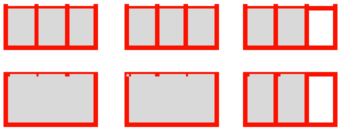
15% = $3.60

   b. How much will Jana pay for the shirt?

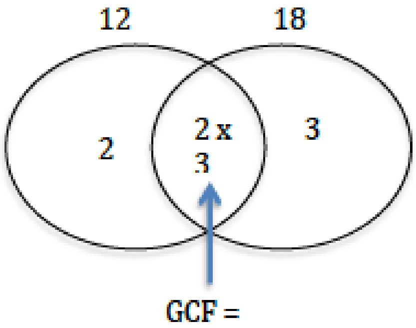
$24.00 - $3.60 = $20.40

4. Change this fraction to a mixed number. Draw a bar model.

$\frac{8}{3}$ $2\frac{2}{3}$

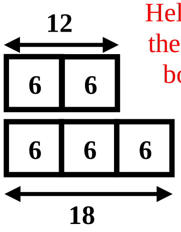
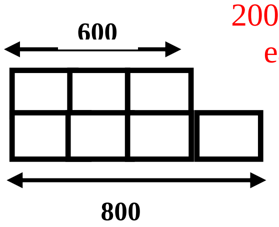
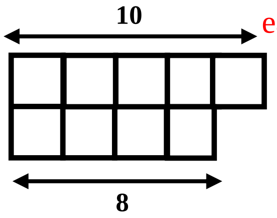


5. What is the greatest common factor of 12 and 18? 6



1.3b Class Activity: Percent and Fraction Problems Transition to Numeric Expressions

Part A: For each problem below, use a model to answer the question.

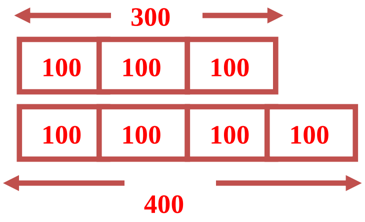
Context	Model	Fraction Change	Percent Change	Fractional Portion of the original	Percent of the original
1a. Li planted a 12 foot tall tree in her yard. 10 years later it is 18 feet tall.	 Help students see that the number inside the boxes is the greatest common factor of the two numbers.	$\frac{1}{2}$ factor increase of the original.	Grew by 50% of the original.	$\frac{3}{2}$ of the original height	150% of the original height
2a. Pedro had $600 in his savings account. Five years later he has $800.	 200 goes in each box	1/3 factor increase of the original.	It grew by $33.\overline{3}\%$	$\frac{4}{3}$ of the original amount	$133.\overline{3}\%$of the original amount
3a. It used to take Naya 10 minutes to walk to school. Now it takes her 8 minutes.	 2 goes in each box	1/5 factor decrease of the original.	It decreased by 20%	It is $\frac{4}{5}$ the original time	It is 80% the original time
4a. Mario's old car got 20 miles to the gallon. His new car gets 24 miles to the gallon.		$\frac{1}{5}$ factor increase of the original.	It gets 20% more mpg	It is $\frac{6}{5}$ the original mileage	It is 120% the original mileage
5a. The Castro's old dishwasher used 12 gallons of water per load. Their new dishwasher uses 8 gallons per load.		$\frac{1}{3}$ factor decrease of the original.	It uses $33.\overline{3}\%$ less water	It uses $\frac{2}{3}$ the original water	It uses $66.\overline{6}\%$ the original water
6a. A pair of running shoes was originally $75, they are on sale for $60.		$\frac{1}{5}$ factor decrease of the original.	It is 20% off the original price	It is $\frac{4}{5}$ the original price	It is 80% the original price
7a. The pair of running shoes was originally $75, they are on sale for $50.		$\frac{1}{3}$ factor decrease of the original	It is $33.\overline{3}\%$ off the original price	It is $\frac{2}{3}$ the original price	It is $66.\overline{6}\%$ of the original price

Part B: These are the same questions as above in Part A. Use the model you created for each problem to write a numeric expression to answer the question.

Context	Fraction Change Expression	Percent Change Expression
1b. Li planted a 12 foot tall tree in her yard. 10 years later it is 18 feet tall.	$12(1)+12\left(\frac{1}{2}\right)$ $=12\left(\frac{3}{2}\right)$ $=18$	12(1) + 12(.5) = 12(1.5) = 18
2b. Pedro had $600 in his savings account. Five years later he has $800.	$600(1)+600\left(\frac{1}{3}\right)$ $=600\left(\frac{4}{3}\right)$ $=800$	$600(1)+600(.\overline{33})$ $=600(1.\overline{33})$ $=800$
3b. It used to take Naya 10 minutes to walk to school. Now it takes her 8 minutes.	$10(1)-10\left(\frac{1}{5}\right)$ $=10\left(\frac{4}{5}\right)$ $=8$	$10(1)-10(0.2)$ $=10(.8)$ $=8$
4b. Mario's old car got 20 miles to the gallon. His new car gets 24 miles to the gallon.	$20(1)+20\left(\frac{1}{5}\right)$ $=20\left(\frac{6}{5}\right)$ $=24$	$20(1)+20(.2)$ $=20(1.2)$ $=24$
5b. The Castro's old dishwasher used 12 gallons of water per load. Their new dishwasher uses 8 gallons per load.	$12(1)-12\left(\frac{1}{3}\right)$ $=12\left(\frac{2}{3}\right)$ $=8$	$12(1)-12(.\overline{33})$ $=12(.\overline{66})$ $=8$
6b. A pair of running shoes was originally $75, they are on sale for $60.	$75(1)-75\left(\frac{1}{5}\right)$ $=75\left(\frac{4}{5}\right)$ $=60$	$75(1)-75(.2)$ $=75(.8)$ $=60$
7b. The pair of running shoes was originally $75, they are on sale for $50.	$75(1)-75\left(\frac{1}{3}\right)$ $=75\left(\frac{2}{3}\right)$ $=50$	$75(1)-75(.\overline{33})$ $=75(.\overline{66})$ $=50$

1.3b Homework: Percent and Fraction Problems Transition to Numeric Expressions

For each problem below, use a model to answer the question.

Context	Model	Fraction Change	Percent Change	Fraction	Percent
1. At the beginning of the year there were 32 students in Ms. Herrera's class. There are now 36 students in her class.					
2. At Lincoln Middle School 300 students participated in Science Fair in 2011. In 2012 400 students participated.		They increased by $\frac{1}{3}$	They increased by 33.$\overline{3}$%	They have $\frac{4}{3}$ their 2011 students	They have 133.$\overline{3}$% of their 2011 students
3. Celesta used to spend 90 minutes a day on the phone. She now spends 60 minutes a day.		Use decreased by $\frac{1}{3}$	Use decreased by 33.$\overline{3}$%	Celesta uses $\frac{2}{3}$ as many minutes	Celesta uses 66.$\overline{6}$% % as many minutes
4. During the school year, Jose worked 15 hours a week. During the summer he works 30 hours a week.					
5. A local business used to spend $800 a month on employee rewards. They now spend $1000.		$\frac{1}{4}$ factor increase of the original.	They spend 25% more on rewards	They spend $\frac{5}{4}$ their original amount	They spend 125% their original amount

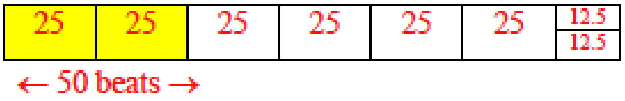
For each context, write a fraction change and percent change expression.

Context	Fraction Change Expression	Percent Change Expression
6. Tio's mom used to drink 4 diet sodas a day. Now she drinks 1 a day.		
7. In a small town in Utah, 40,000 homes used to have land-line phones. Now 35,000 homes have land-line phones.	$40{,}000(1) - 40{,}000\left(\frac{1}{8}\right)$ $= 40{,}000\left(\frac{7}{8}\right)$ $= 35{,}000$	$40{,}000(1) - 40{,}000(.125)$ $= 40{,}000(.875)$ $= 35{,}000$
8. A small business's profits in 2011 were \$120,000. In 2012 they were \$150,000		
9. Nina and her friend went to dinner. Their bill came to \$40. Nina paid \$60 to cover the bill, tax and tip.	$40(1) + 40\left(\frac{1}{2}\right)$ $= 40\left(\frac{3}{2}\right)$ $= 60$	$40(1) + 40(.5)$ $= 40(1.5)$ $= 60$
10. A local business used to spend \$800 a month on employee rewards. They now spend \$1000.		

Spiral Review

1. Alyssa's resting heart rate is 50 beats per minute. Her target exercise rate is 325% of her resting rate. What is her target rate? 162.5 beats per minute

$50(3) + 50(1/4) = 150 + 12.50 = 162.5$



2. Samantha has a bag of marbles. She has 21 striped marbles, 5 cat's eye marbles and 10 solid color marbles. Samantha draws one marble out of her bag. Express each of the following probabilities as a fraction, decimal and percent.

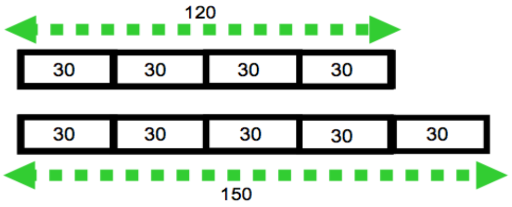
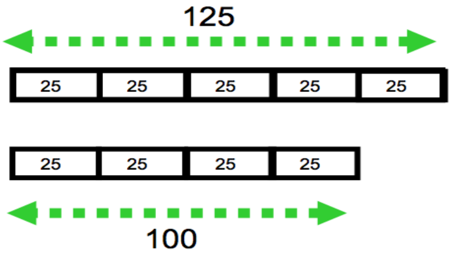
a. P(striped)____ $\frac{21}{35}$ ***or*** $\frac{3}{5}$, **.6, 60%**________________________________

b. P(solid)____ $\frac{10}{35}$ ***or*** $\frac{2}{7}$, **.29, 29%**________________________________

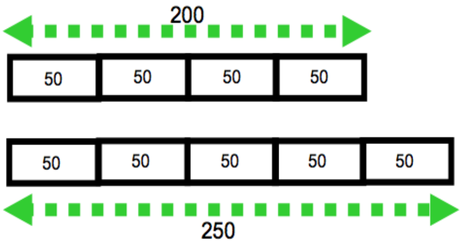
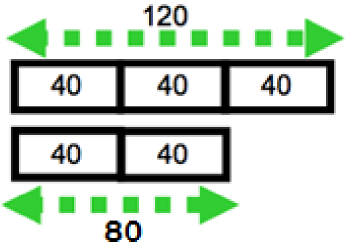
3. Put in order from least to greatest: $2.54, \frac{5}{2}, 2.7, \frac{15}{7}$

4. Write $\frac{23}{7}$ as a mixed number. Draw a bar model as needed. $3\frac{2}{7}$

1.3c Class Activity: Create a Context for Each Model or Numeric Representation

Context	Model or numeric representation
1.	
2.	300(1) – 300(1/5) = 240
3.	48(1) + 48(.25) = 48(1.25) = 60
4.	
5.	60(1) + 60(1/3) = 60(4/3) = 80
6.	80(1) – 80(.25) = 80(.75) = 60

1.3c Homework: Create a Context for Each Model or Numeric Representation

Context	Model or numeric representation
1.	
2.	500(1) – 500(1/5) = 400
3.	60(1) + 60(.75) = 60(1.75) = 105
4.	
5.	80(1) + 80(.40) = 80(1.40) = 112
6.	120(1) – 120(1/4) = 120(3/4) = 90

Spiral Review

1. The downtown bakery normally sells one dozen muffins for $12. Today, the muffins are 25% off.

   a. What is the discount for one dozen muffins?

   $3.00

   b. How much will two dozen muffins cost with the discount?

   $18.00

2. Find 25% of $500.00.

   $125.00

3. What is the greatest common factor of 30 and 75? 15

4. Matt earned $800 helping his grandfather at his business. If he spent 1/5 of his earnings on a gift for his mom, how much money did he have left over? $640.

5. Write 3 as a fraction and percent:

$\frac{3}{1}$	3.0	300%
Fraction	Decimal	Percent

1.3d Self-Assessment: Section 1.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Find a percent or fractional portion of a whole with or without a model. [1a, 1bi, 1ci]	I can sometimes find the percent or fractional portion of a whole.	I can find the percent or fractional portion of a whole with a calculator accurately.	I can find the percent or fractional portion of a whole with a calculator and with mental math strategies accurately.	I can find the percent or fractional portion of a whole with a calculator or with mental math strategies accurately. I can explain how a model and my strategy are related to the question and answer.
2. Solve percent problems involving percent increase and decrease (including discounts, interest, taxes, tips, etc.). [1b, 1c, 2]	I sometimes have a hard time getting started, but once I know what to do, I can do the problem.	I know how to find the percent of the whole, but I sometimes mess up on the next step.	I can solve percent problems involving percent increase and decrease.	I can solve percent problems involving percent increase and decrease accurately with either a model or numeric expression. I can also explain how the model and expression are related to the answer.
3. Develop numeric expressions from percent and fraction models. [3]	I don't know how to write a numeric expression from percent and fraction models.	I can match a numeric expression to the correct percent or fraction model.	I can write a numeric expression from percent and fraction models.	I can write a numeric expression from percent and fraction models. I can also draw percent and fraction models from a numeric expression.

Sample Problems for Section 1.3

1. Solve each of the following problems with or without a model.

   a. Ana has 75 necklaces. She sells $\frac{1}{3}$ of the necklaces. How many necklaces did she sell?

   b. Bubba invested \$760. If he earned 20% on his investment,

      i. How much interest did he earn?

      ii. How much money does he have now?

   c. The bakery normally sells one dozen cupcakes for \$24. Today, the cupcakes are 25% off.

      i. What is the discount for one dozen cupcakes?

      ii. If 5% tax is added to the total after the discount, how much will two dozen cupcakes cost?

2. Solve the following problems with or without a model.

   a. Daniel left a \$9 tip for the waiter at a restaurant. If the tip was 15% of the bill, how much was the bill?

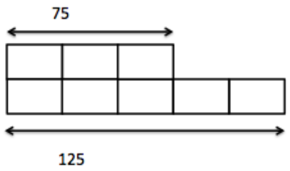
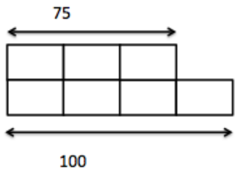
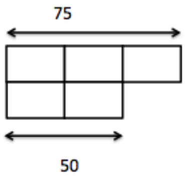
   b. Ellie borrowed money from a company that charges 20% interest for loans. If she repaid them a total of \$420, how much was her original loan?

   c. Flo sold 14 cars last year. This year, she sold 35 cars. What was the percent change?

   d. Gabrielle had \$90 in her savings account last month. Now she has \$63 in her account. What was the percent change?

3.

a. Match the context with the correct model, and numeric expression.

Context	1. Hans had \$75 in his bank account. Two years later, he had \$100.	2. Isi could do 75 sit-ups last year. Now she can only do 50 sit-ups.	3. Jenna's family used 75 gallons of water per day last month. This month, her family used 125 gallons of water per day.
Model	A. 	B. 	C. 
Numeric Expression	X. $75(1) + 75\left(\frac{1}{3}\right)$	Y. $75(1) - 75\left(\frac{1}{3}\right)$	Z. $75(1) + 75\left(\frac{2}{3}\right)$

Context 1, Model ________, Numeric Expression____________

Context 2, Model________, Numeric Expression____________

Context 3, Model________, Numeric Expression_____________

b. For Context 1: What percent of Hans' original amount of money does he have 2 years later?

c. For Context 2: What is the percent change in the number of sit-ups Isi can do?

d. For Context 3: What is the percent change in the amount of water Jana's family now uses compared to last month?

4. Write a context to match the following numeric expressions:

	$24(1) + 24\left(\frac{1}{4}\right)$
	$700(1) - 700(0.3)$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