mathematics
vision project

Transforming Mathematics Education

SECONDARY
MATH ONE

An Integrated Approach

MODULE 2 HONORS

Linear & Exponential Functions

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The Mathematics Vision Project

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2.1 Connecting the Dots: Piggies and Pools

A Develop Understanding Task

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1. My little sister, Savannah, is three years old. She has a piggy bank that she wants to fill. She started with five pennies and each day when I come home from school, she is excited when I give her three pennies that are left over from my lunch money. Use a table, a graph, and an equation to create a mathematical model for the number of pennies in the piggy bank on day n.

2. Our family has a small pool for relaxing in the summer that holds 1500 gallons of water. I decided to fill the pool for the summer. When I had 5 gallons of water in the pool, I decided that I didn't want to stand outside and watch the pool fill, so I had to figure out how long it would take so that I could leave, but come back to turn off the water at the right time. I checked the flow on the hose and found that it was filling the pool at a rate of 2 gallons every minute. Use a table, a graph, and an equation to create a mathematical model for the number of gallons of water in the pool at t minutes.

3. I'm more sophisticated than my little sister so I save my money in a bank account that pays me 3% interest on the money in the account at the end of each month. (If I take my money out before the end of the month, I don't earn any interest for the month.) I started the account with \$50 that I got for my birthday. Use a table, a graph, and an equation to create a mathematical model of the amount of money I will have in the account after m months.

4. At the end of the summer, I decide to drain the 1500 gallon swimming pool. I noticed that it drains faster when there is more water in the pool. That was interesting to me, so I decided to measure the rate at which it drains. I found that 3% was draining out of the pool every minute. Use a table, a graph, and an equation to create a mathematical model of the gallons of water in the pool at t minutes.

5. Compare problems 1 and 3. What similarities do you see? What differences do you notice?

6. Compare problems 1 and 2. What similarities do you see? What differences do you notice?

7. Compare problems 3 and 4. What similarities do you see? What differences do you notice?

2.1

READY, SET, GO! Name Period Date

READY

Topic: Recognizing arithmetic and geometric sequences

Predict the next 2 terms in the sequence. State whether the sequence is arithmetic, geometric, or neither. Justify your answer.

1. 4 , -20 , 100 , -500 , . . .

2. 3 , 5 , 8 , 12 , . . .

3. 64 , 48 , 36 , 27 , . . .

4. 1.5 , 0.75 , 0 , -0.75 , . . .

5. 40 , 10 , $\frac{5}{2}$, $\frac{5}{8}$, …

6. 1, 11, 111, 1111, . . .

7. -3.6, -5.4, -8.1, -12.15, . . .

8. -64, -47, -30, -13, . . .

9. Create a predictable sequence of at least 4 numbers that is NOT arithmetic or geometric.

SET

Topic: Discrete and continuous relationships

Identify whether the following statements represent a *discrete* or a *continuous* relationship.

10. The hair on your head grows ½ inch per month.

11. For every ton of paper that is recycled, 17 trees are saved.

12. Approximately 3.24 billion gallons of water flow over Niagara Falls daily.

13. The average person laughs 15 times per day.

14. The city of Buenos Aires adds 6,000 tons of trash to its landfills every day.

15. During the Great Depression, stock market prices fell 75%.

GO

Topic: Solving one-step equations

Either find or use the unit rate for each of the questions below.

16. Apples are on sale at the market 4 pounds for \$2.00. What is the price (in cents) for one pound?

17. Three apples weigh about a pound. About how much would one apple cost?
(Round to the nearest cent.)

18. One dozen eggs cost \$1.98. How much does 1 egg cost? (Round to the nearest cent.)

19. One dozen eggs cost \$1.98. If the charge at the register for only eggs, without tax, was \$11.88, how many dozen were purchased?

20. Best Buy Shoes had a back to school special. The total bill for four pairs of shoes came to \$69.24 (before tax.) What was the average price for each pair of shoes?

21. If you only purchased 1 pair of shoes at Best Buy Shoes instead of the four described in problem 20, how much would you have paid, based on the average price?

Solve for x. Show your work.

22. $6x = 72$

23. $4x = 200$

24. $3x = 50$

25. $12x = 25.80$

26. $\frac{1}{2}x = 17.31$

27. $4x = 69.24$

28. $12x = 198$

29. $1.98x = 11.88$

30. $\frac{1}{4}x = 2$

31. Some of the problems 22 – 30 could represent the work you did to answer questions 16 – 21. Write the number of the equation next to the story it represents.

2.2 Shh! Please Be Discreet (Discrete)!

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A Solidify Understanding Task

1. The Library of Congress in Washington D.C. is considered the largest library in the world. They often receive boxes of books to be added to their collection. Since books can be quite heavy, they aren't shipped in big boxes. If, on average, each box contains about 8 books, how many books are received by the library in 6 boxes, 10 boxes, or n boxes?
   a. Use a table, a graph, and an equation to model this situation.

   b. Identify the domain of the function.

2. Many of the books at the Library of Congress are electronic. If about 13 e-books can be downloaded onto the computer each hour, how many e-books can be added to the library in 3 hours, 5 hours, or n hours (assuming that the computer memory is not limited)?
   a. Use a table, a graph, and an equation to model this situation.

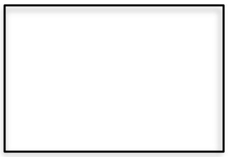
   b. Identify the domain of the function.

3. The librarians work to keep the library orderly and put books back into their proper places after they have been used. If a librarian can sort and shelve 3 books in a minute, how many books does that librarian take care of in 3 hours, 5 hours, or *n* hours? Use a table, a graph, and an equation to model this situation.

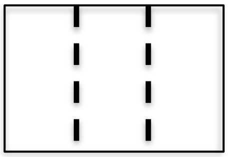
4. Would it make sense in any of these situations for there to be a time when 32.5 books had been shipped, downloaded into the computer or placed on the shelf?

5. Which of these situations (in problems 1-3) represent a discrete function and which represent a continuous function? Justify your answer.

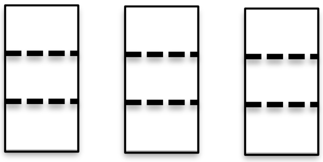
6. A giant piece of paper is cut into three equal pieces and then each of those is cut into three equal pieces and so forth. How many papers will there be after a round of 10 cuts? 20 cuts? *n* cuts?



Zero Cuts



One Cut



Two Cuts

a. Use a table, a graph, and an equation to model this situation.

b. Identify the domain of the function.

c. Would it make sense to look for the number of pieces of paper at 5.2 cuts? Why?

d. Would it make sense to look for the number of cuts it takes to make 53.6 papers? Why?

7. Medicine taken by a patient breaks down in the patient's blood stream and dissipates out of the patient's system. Suppose a dose of 60 milligrams of anti-parasite medicine is given to a dog and the medicine breaks down such that 20% of the medicine becomes ineffective every hour. How much of the 60 milligram dose is still active in the dog's bloodstream after 3 hours, after 4.25 hours, after *n* hours?

a. Use a table, a graph, and an equation to model this situation.

b. Identify the domain of the function.

c. Would it make sense to look for an amount of active medicine at 3.8 hours? Why?

d. Would it make sense to look for when there is 35 milligrams of medicine? Why?

8. Which of the functions modeled in #6 and #7 are discrete and which are continuous? Why?

9. What needs to be considered when looking at a situation or context and deciding if it fits best with a discrete or continuous model?

10. Describe the differences in each representation (table, graph, and equation) for discrete and continuous functions.

11. Which of the functions modeled above are linear? Which are exponential? Why?

READY, SET, GO! Name Period Date

READY

Topic: Comparing rates of change in linear situations.

State which situation has the greatest rate of change

1. The amount of stretch in a short bungee cord stretches 6 inches when stretched by a 3 pound weight. A slinky stretches 3 feet when stretched by a 1 pound weight.

2. A sunflower that grows 2 inches every day or an amaryllis that grows 18 inches in one week.

3. Pumping 25 gallons of gas into a truck in 3 minutes or filling a bathtub with 40 gallons of water in 5 minutes.

4. Riding a bike 10 miles in 1 hour or jogging 3 miles in 24 minutes.

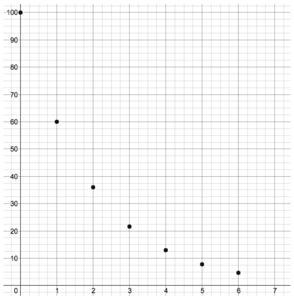
SET

Topic: Discrete and continuous relationships

Identify whether the following items best fit with a *__discrete__* or a *__continuous__* model. Then determine whether it is a *__linear__* *(arithmetic)* or *__exponential__* *(geometric)* relationship that is being described.

5. The freeway construction crew pours 300 ft of concrete in a day.
6. For every hour that passes, the amount of area infected by the bacteria doubles.
7. To meet the demands placed on them the brick layers have started laying 5% more bricks each day.
8. The average person takes 10,000 steps in a day.
9. The city of Buenos Aires has been adding 8% to its population every year.
10. At the headwaters of the Mississippi River the water flows at a surface rate of 1.2 miles per hour.
11. a. $f(n) = f(n-1) + 3; f(1) = 5$

b.



c. $g(x) = 2^x(7)$

GO

Topic: Solving one-step equations

Solve the following equations. Remember that what you do to one side of the equation must also be done to the other side. *(Show your work, even if you can do these in your head.)*

Example: Solve for x . $1x + 7 = 23$ $Add\ -7\ to\ both\ sides\ of\ the\ equation.$

$$\begin{array}{l} 1x + 7 = 23 \\ \quad\ -7 = -7 \\ \hline 1x + 0 = 16 \\ Therefore\ \ 1x = 16 \end{array}$$

Example: Solve for x. $9x = 63$ $Multiply\ both\ sides\ of\ the\ equation\ by\ \frac{1}{9}.$

$$\begin{gathered} 9x = 63 \\ \left(\frac{1}{9}\right)9x = \left(\frac{1}{9}\right)63 \\ \left(\frac{9}{9}\right)x = \frac{63}{9} \\ 1x = 7 \end{gathered}$$

Note that multiplying by $\frac{1}{9}$ gives the same result as dividing everything by 9.

11. $1x + 16 = 36$

12. $1x - 13 = 10$

13. $1x - 8 = -3$

14. $8x = 56$

15. $-11x = 88$

16. $425x = 850$

17. $\frac{1}{6}x = 10$

18. $-\frac{4}{7}x = -1$

19. $\frac{3}{4}x = -9$

2.3 Linear, Exponential or Neither?

A Practice Understanding Task

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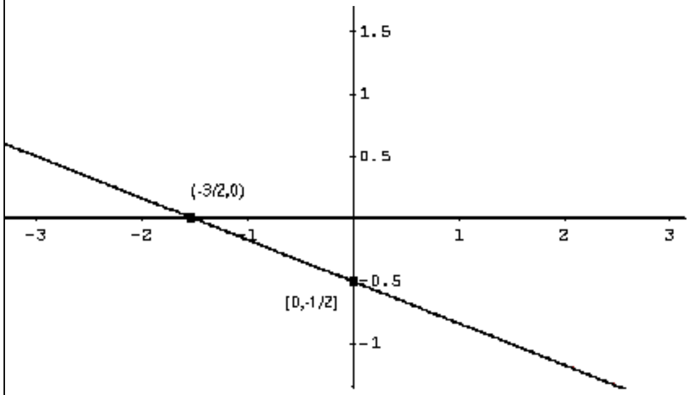
For each representation of a function, decide if the function is linear, exponential, or neither.
Give at least 2 reasons for your answer.

1.



Linear Exponential Neither

Why?

2.

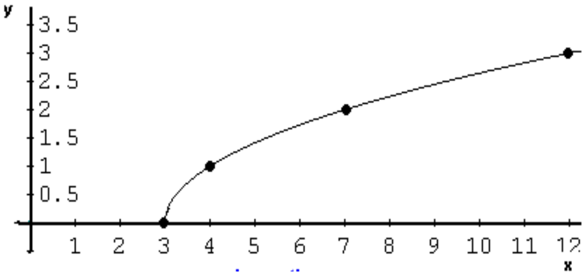
Tennis Tournament

Rounds	1	2	3	4	5
Number of Players left	64	32	16	8	4

There are 4 players remaining after 5 rounds

Linear Exponential Neither

Why?

3. $y = 4x$	Linear Exponential Neither Why?
4. This function is decreasing at a constant rate .	Linear Exponential Neither Why?
5. 	Linear Exponential Neither Why?

<table>
<tr>
<td>6.
A person's height as a function of a person's age (from age 0 to 100)</td>
<td>Linear Exponential Neither

Why?</td>
</tr>
<tr>
<td>7.
$-3x = 4y + 7$</td>
<td>Linear Exponential Neither

Why?</td>
</tr>
<tr>
<td>8.
<table>
<tr><th><i>x</i></th><th><i>y</i></th></tr>
<tr><td>-2</td><td>23</td></tr>
<tr><td>0</td><td>5</td></tr>
<tr><td>2</td><td>-13</td></tr>
<tr><td>4</td><td>-31</td></tr>
<tr><td>6</td><td>-49</td></tr>
</table>
</td>
<td>Linear Exponential Neither

Why?</td>
</tr>
</table>

9.

Height in Inches	Shoe Size
62	6
74	13
70	9
67	11
53	4
58	7

Linear Exponential Neither

Why?

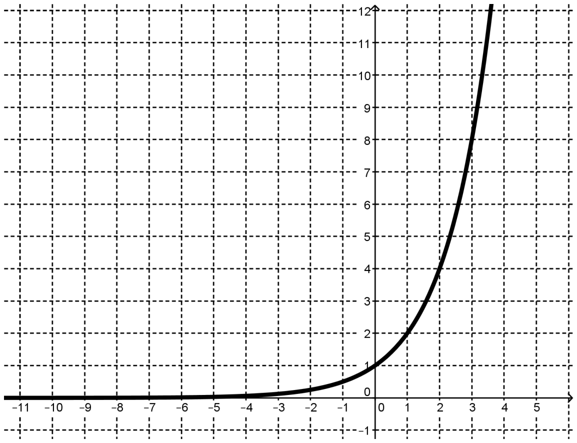
10.

The number of cell phone users in Centerville as a function of years, if the number of users is increasing by 75% each year.

Linear Exponential Neither

Why?

11.



Linear Exponential Neither

Why?

12. The time it takes you to get to work as a function the speed at which you drive	Linear Exponential Neither Why?
13. $y = 7x^2$	Linear Exponential Neither Why?
14. Each point on the graph is exactly 1/3 of the previous point.	Linear Exponential Neither Why?

15. $f(1) = 7, f(2) = 7, f(n) = f(n-1) + f(n-2)$	Linear Exponential Neither Why?
16. $f(1) = 1, f(n) = \frac{2}{3} f(n-1)$	Linear Exponential Neither Why?

READY, SET, GO!

Name Period Date

READY

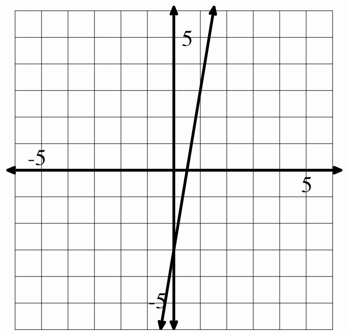
Topic: Comparing rates of change in both linear and exponential situations.

Identify whether situation "a" or situation "b" has a greater rate of change.

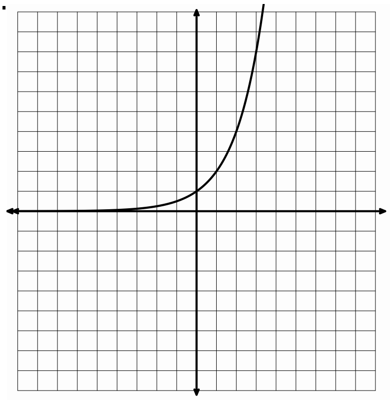
1. a.

x	y
-10	-48
-9	-43
-8	-38
-7	-33

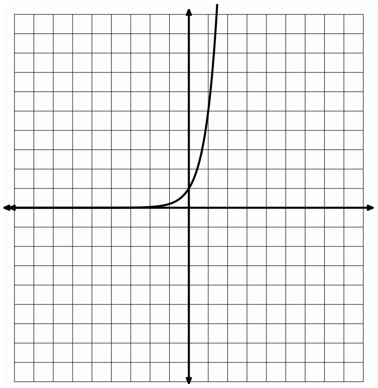
b.



2. a.



b.



3. a. Lee has $25 withheld each week from his salary to pay for his subway pass.

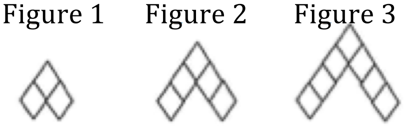
b. Jose owes his brother $50. He has promised to pay half of what he owes each week until the debt is paid.

4. a.

x	6	10	14	18
y	13	15	17	19

b. The number of rhombi in each shape.

Figure 1 Figure 2 Figure 3



5. a. $y = 2(5)^x$

b. In the children's book, *The Magic Pot,* every time you put one object into the pot, two of the same object come out. Imagine that you have 5 magic pots.

SET

Topic: Recognizing linear and exponential functions.

Based on each of the given representations of a function determine if it is linear, exponential or neither.

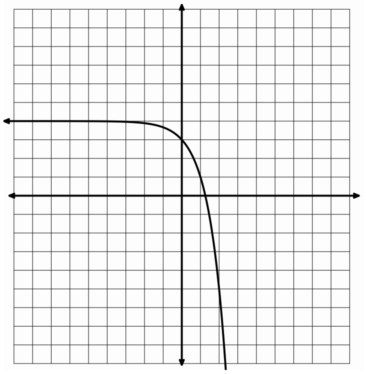
6. The population of a town is decreasing at a rate of 1.5% per year.

7. Joan earns a salary of $30,000 per year plus a 4.25% commission on sales.

8. 3x +4y = -3

9. The number of gifts received each day of "The 12 Days of Christmas" as a function of the day. ("On the 4th day of Christmas my true love gave to me, 4 calling birds, 3 French hens, 2 turtledoves, and a partridge in a pear tree.")

10.



11.

Side of a square	Area of a square
1 inch	1 in^2
2 inches	4 in^2
3 inches	9 in^2
4 inches	16 in^2

GO

Topic: Geometric means

For each geometric sequence below, find the missing terms in the sequence.

12.

x	1	2	3	4	5
y	2				162

13.

x	1	2	3	4	5
y	1/9			-3	

14.

x	1	2	3	4	5
y	10				0.625

15.

x	1	2	3	4	5
y	g				gz^4

16.

x	1	2	3	4	5
y	-3				-243

Find the rate of change (slope) in each of the exercises below.

17.

x	g(x)
-5	11
-3	4
-2	0.5
0	-6

18.

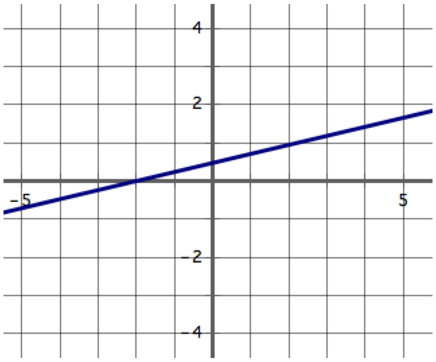
t	h(t)
3	13
8	23
18	43
23	53

19.

n	f(n)
-7	20
-5	24
-1	32
2	38

20. (2, 5) (8, 29)

21.



22. (-3, 7) (8, 29)

2.4 Getting Down to Business

A Solidify Understanding Task

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Calcu-rama had a net income of 5 million dollars in 2010, while a small competing company, Computafest, had a net income of 2 million dollars. The management of Calcu-rama develops a business plan for future growth that projects an increase in net income of 0.5 million per year, while the management of Computafest develops a plan aimed at increasing its net income by 15% each year.

a. Create standard mathematical models (table, graph and equations) for the projected net income over time for both companies. (Attend to precision and be sure that each model is accurate and labeled properly so that it represents the situation.)

b. Compare the two companies. How are the representations for the net income of the two companies similar? How do they differ? What relationships are highlighted in each representation?

c. If both companies were able to meet their net income growth goals, which company would you choose to invest in? Why?

d. When, if ever, would your projections suggest that the two companies have the same net income? How did you find this? Will they ever have the same net income again?

e. Since we are creating the models for these companies we can choose to have a discrete model or a continuous model. What are the advantages or disadvantages for each type of model?

READY, SET, GO! Name Period Date

READY

Topic: Comparing arithmetic and geometric sequences.

The first and fifth terms of a sequence are given. Fill in the missing numbers if it is an arithmetic sequence. Then fill in the numbers if it is a geometric sequence.

+80 +80 +80 +80

Example:

Arithmetic	4	84	164	244	324
Geometric	4	12	36	108	324

x 3 x 3 x 3 x 3

1.

Arithmetic	3				48
Geometric	3				48

2.

Arithmetic	-6250				-10
Geometric	-6250				-10

3.

Arithmetic	-12				-0.75
Geometric	-12				-0.75

SET

Topic: Distinguishing specifics between sequences and linear or exponential functions.

Answer the questions below with respect to the relationship between sequences and the larger families of functions.

4. If a relationship is modeled with a continuous function which of the domain choices is a possibility?
A. $\{x \mid x \in R, x \geq 0\}$ B. $\{x \mid x \in W\}$ C. $\{x \mid x \in Z, x \geq 0\}$ D. $\{x \mid x \in N\}$

5. Which one of the options below is the mathematical way to represent the Natural Numbers?
A. $\{x \mid x \in R, x \geq 0\}$ B. $\{x \mid x \in Q, x \geq 0\}$ C. $\{x \mid x \in Z, x \geq 0\}$ D. $\{x \mid x \in N\}$

6. Only one of the choices below would be used for a continuous exponential model, which one is it?
   A. $f(x) = f(x-1) \cdot 4, f(1) = 3$ B. $g(x) = 4^x(5)$
   C. $h(t) = 3t - 5$ D. $k(n) = k(n-1) - 5, k(1) = 32$

7. Only one of the choices below would be used for a continuous linear model, which one is it?
   A. $f(x) = f(x-1) \cdot 4, f(1) = 3$ B. $g(x) = 4^x(5)$
   C. $h(t) = 3t - 5$ D. $k(n) = k(n-1) - 5, k(1) = 32$

8. What domain choice would be most appropriate for an arithmetic or geometric sequence?
   A. $\{x \mid x \in R, x \geq 0\}$ B. $\{x \mid x \in Q, x \geq 0\}$ C. $\{x \mid x \in Z, x \geq 0\}$ D. $\{x \mid x \in N\}$

9. What attributes will arithmetic or geometric sequences always have?
   (There could be more than one correct choice. Circle all that apply.)

   A. Continuous B. Discrete C. Domain: $\{x \mid x \in N\}$ D. Domain: $\{x \mid x \in R\}$

   E. Negative x-values F. Something constant G. Recursive Rule

10. What type of sequence fits with linear mathematical models?

    What is the difference between this sequence type and the overarching umbrella of linear relationships? (Use words like discrete, continuous, domain and so forth in your response.)

11. What type of sequence fits with exponential mathematical models?

    What is the difference between this sequence type and the overarching umbrella of exponential relationships? (Use words like discrete, continuous, domain and so forth in your response.)

GO

Topic: Writing explicit equations for linear and exponential models.

Write the explicit equations for the tables and graphs below. This is something you really need to know. Persevere and do all you can to figure them out. Remember the tools we have used. (#21 is bonus give it a try.)

12.

x	$f(x)$
2	-4
3	-11
4	-18
5	-25

13.

x	$f(x)$
-1	2/5
0	2
1	10
2	50

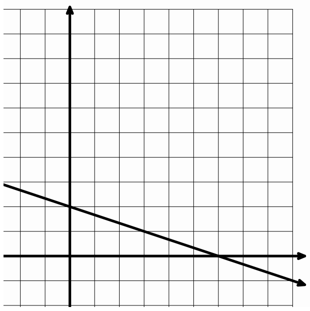
14.

x	$f(x)$
2	-24
3	-48
4	-96
5	-192

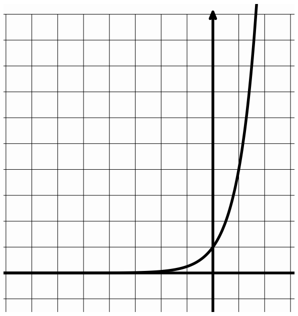
15.

x	$f(x)$
-4	81
-3	27
-2	9
-1	3

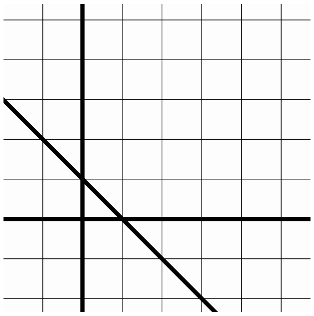
16



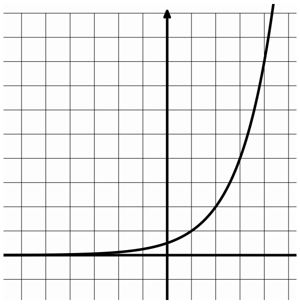
17.



18



19.



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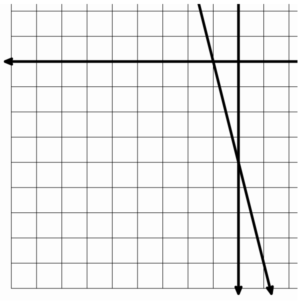

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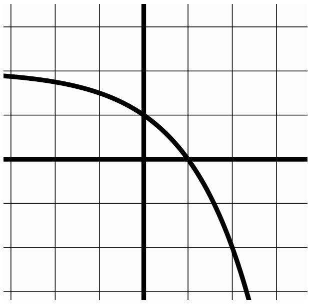

2.5 Making My *Point*

A Solidify Understanding Task

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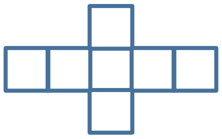
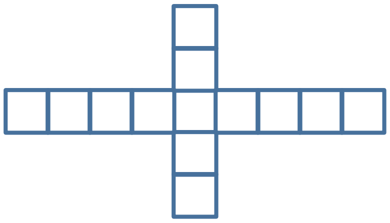
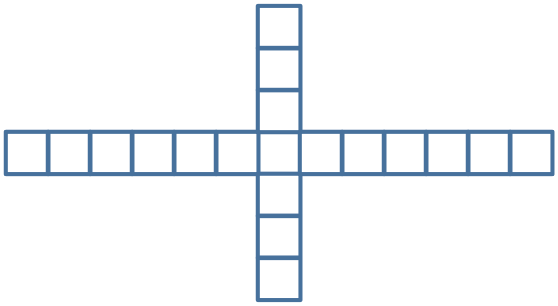
Zac and Sione were working on predicting the number of quilt blocks in this pattern:







When they compared their results, they had an interesting discussion:

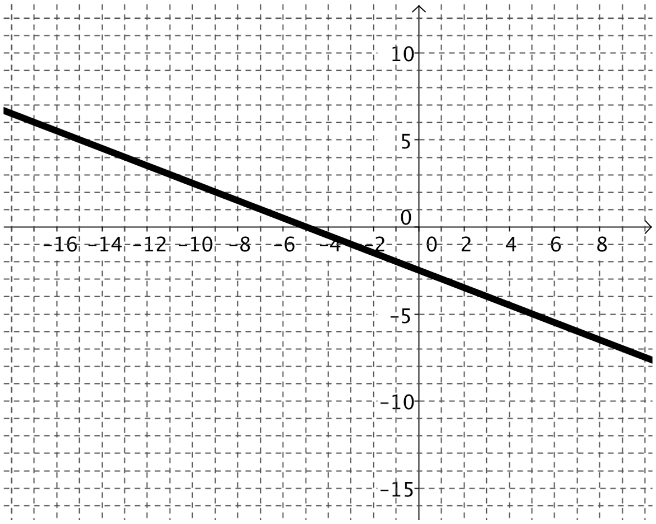
Zac: I got $y = 6n + 1$ because I noticed that 6 blocks were added each time so the pattern must have started with 1 block at $n = 0$.

Sione: I got $y = 6(n - 1) + 7$ because I noticed that at $n = 1$ there were 7 blocks and at $n = 2$ there were 13, so I used my table to see that I could get the number of blocks by taking one less than the n, multiplying by 6 (because there are 6 new blocks in each figure) and then adding 7 because that's how many blocks in the first figure. Here's my table:

1	2	3	n
7	13	19	$6(n-1)+7$

1. What do you think about the strategies that Zac and Sione used? Are either of them correct? Why or why not? Use as many representations as you can to support your answer.

The next problem Zac and Sione worked on was to write the equation of the line shown on the graph below.



When they were finished, here is the conversation they had about how they got their equations:

Sione: It was hard for me to tell where the graph crossed the y axis, so I found two points that I could read easily, (-9, 2) and (-15, 5). I figured out that the slope was $-\frac{1}{2}$ and made a table and checked it against the graph. Here's my table:

x	-15	-13	-11	-9	n
$f(x)$	5	4	3	2	$-\frac{1}{2}(n+9)+2$

I was surprised to notice that the pattern was to start with the *n*, add 9, multiply by the slope and then add 2.

I got the equation: $f(x) = -\frac{1}{2}(x+9)+2$.

Zac: Hey—I think I did something similar, but I used the points, (7,-6) and (9,-7).

I ended up with the equation: $f(x) = -\frac{1}{2}(x-9)-7$. One of us must be wrong because yours says that you add 9 to the *n* and mine says that you subtract 9. How can we both be right?

2. What do you say? Can they both be right? Show some mathematical work to support your thinking.

Zac: My equation made me wonder if there was something special about the point (9, -7) since it seemed to appear in my equation $f(x) = -\frac{1}{2}(x-9)-7$ when I looked at the number pattern. Now I'm noticing something interesting—the same thing seems to happen with your equation, $f(x) = -\frac{1}{2}(x+9)+2$ and the point (-9, 2)

3. Describe the pattern that Zac is noticing.

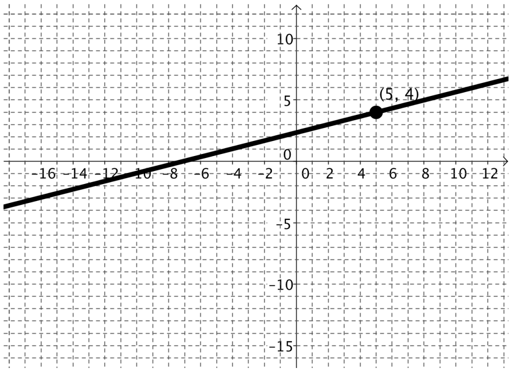
4. Find another point on the line given above and write the equation that would come from Zac's pattern.

5. What would the pattern look like with the point (a, b) if you knew that the slope of the line was m?

6. Zac challenges you to use the pattern he noticed to write the equation of line that has a slope of 3 and contains the point (2,-1). What's your answer?

Show a way to check to see if your equation is correct.

7. Sione challenges you to use the pattern to write the equation of the line graphed below, using the point (5, 4).



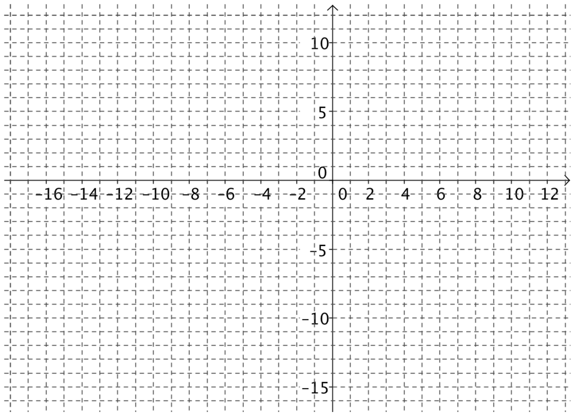
Show a way to check to see if your equation is correct.

8. **Zac:** "I'll bet you can't use the pattern to write the equation of the line through the points (1,-3) and (3,-5). Try it!"

Show a way to check to see if your equation is correct.

9. **Sione:** I wonder if we could use this pattern to graph lines, thinking of the starting point and using the slope. Try it with the equation: $f(x) = -2(x+1) - 3$.
Starting point: Slope:

Graph:



10. Zac wonders, "What is it about lines that makes this work?" How would you answer Zac?

11. Could you use this pattern to write the equation of any linear function? Why or why not?

READY, SET, GO! Name Period Date

READY

Topic: Writing equations of lines.

Write the equation of a line in slope-intercept form: y = mx + b, using the given information.

1. m = -7, b = 4
2. m = 3/8, b = -3
3. m = 16, b = -1/5

Write the equation of the line in point-slope form: $y = m(x - x_1) + y_1$, using the given information.

4. m = 9, (0. -7)
5. m = 2/3, (-6, 1)
6. m = -5, (4, 11)

7. (2,-5) (-3, 10)
8. (0, -9) (3, 0)
9. (-4, 8) (3, 1)

SET

Topic: Graphing linear and exponential functions

Make a graph of the function based on the following information. Add your axes. Choose an appropriate scale and label your graph. Then write the equation of the function.

10. The beginning value is 5 and its value is 3 units smaller at each stage.

Equation:

11. The beginning value is 16 and its value is ¼ smaller at each stage.

Equation:

12. The beginning value is 1 and its value is 10 times as big at each stage.

Equation:

13. The beginning value is -8 and its value is 2 units larger at each stage.

Equation:

GO

Topic: Equivalent equations

Prove that the two equations are equivalent by simplifying the equation on the right side of the equal sign. The justification in the example is to help you understand the steps for simplifying. You do NOT need to justify your steps.

Example:	Justification
$2x - 4 = 8 + x - 5x + 6(x - 2)$	Add $x - 5x$ and distribute the 6 over $(x - 2)$
$= 8 - 4x + 6x - 12$	Combine like terms.
$= -4 + 2x$	
$2x - 4 = 2x - 4$	Commutative property of addition

14. $x - 5 = 5x - 7 + 2(3x + 1) - 10x$

15. $6 - 13x = 24 - 10(2x + 8) + 62 + 7x$

16. $14x + 2 = 2x - 3(-4x - 5) - 13$

17. $x + 3 = 6(x + 3) - 5(x + 3)$

18. $4 = 7(2x + 1) - 5x - 3(3x + 1)$

19. $x = 12 + 8x - 3(x + 4) - 4x$

20. Write an expression that equals $(x - 13)$. It must have at least two sets of parentheses and one minus sign. Verify that it is equal to $(x - 13)$.

2.6 Form Follows Function

A Practice Understanding Task

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In our work so far, we have worked with linear and exponential equations in many forms. Some of the forms of equations and their names are:

Linear Functions

Equation	**Name**
$y = \frac{1}{2}x + 1$	Slope Intercept Form $y = mx + b$, where m is the slope and b is the y-intercept
$y = \frac{1}{2}(x - 4) + 3$	Point Slope Form $y = m(x - x_1) + y_1$, where m is the slope and (x_1, y_1) the coordinates of a point on the line
$f(0) = 1, f(n) = f(n-1) + \frac{1}{2}$	Recursion Formula $f(n) = f(n-1) + \text{D}$, Given an initial value *f(a)* D = constant difference in consecutive terms (used only for discrete functions)

Exponential Functions

Equation	**Name**
$y = 10(3)^x$	Explicit Form $y = a(b)^x$
$f(0) = 10, f(n+1) = 3f(n)$	Recursion Formula $f(n+1) = Rf(n)$ Given an initial value *f(a)* *R* = constant ratio between consecutive terms (used only for discrete functions)

Knowing a number of different forms for writing and graphing equations is like having a mathematical toolbox. You can select the tool you need for the job, or in this case, the form of the equation that makes the job easier. Any master builder will tell you that the more tools you have the better. In this task, we'll work with our mathematical tools to be sure that we know how to use them all efficiently. As you model the situations in the following problems, think about the important information in the problem and the conclusions that can be drawn from it. Is the function linear or exponential? Does the problem give you the slope, a point, a ratio, a y-intercept? Is the function discrete or continuous? This information helps you to identify the best tools and get to work!

1. In his job selling vacuums, Joe makes $500 each month plus $20 for each vacuum he sells. Write the equation that describes Joe's monthly income *I* as a function of the *n*, the number of vacuums sold.

   Name the form of the equation you wrote and why you chose to use that form.

   This function is: linear exponential neither (choose one)

   This function is: continuous discrete neither (choose one)

2. Write the equation of the line with a slope of -1 through the point (-2, 5)

   Name the form of the equation you wrote and why you chose to use that form.

   This function is: linear exponential neither (choose one)

   This function is: continuous discrete neither (choose one)

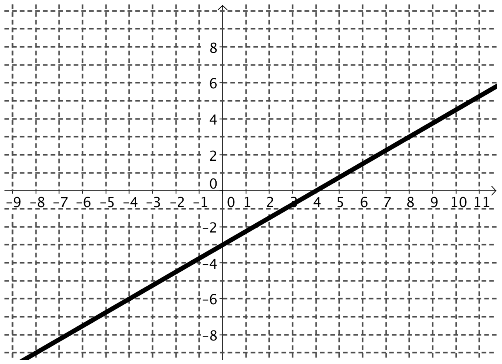
3. Write the equation of the geometric sequence with a constant ratio of 5 and a first term of -3.

Name the form of the equation you wrote and why you chose to use that form.

This function is: linear exponential neither (choose one)

This function is: continuous discrete neither (choose one)

3. Write the equation of the function graphed below:



Name the form of the equation you wrote and why you chose to use that form.

This function is: linear exponential neither (choose one)

This function is: continuous discrete neither (choose one)

4. The population of the resort town of Java Hot Springs in 2003 was estimated to be 35,000 people with an annual rate of increase of about 2.4%. Write the equation that models the number of people in Java Hot Springs, with t = the number of years from 2003?

Name the form of the equation you wrote and why you chose to use that form.

This function is: linear exponential neither (choose one)

This function is: continuous discrete neither (choose one)

5. Yessica's science fair project involved growing some seeds to see what fertilizer made the seeds grow fastest. One idea she had was to use an energy drink to fertilize the plant. (She thought that if they make people perky, they might have the same effect on plants.) This is the data that shows the growth of the seed each week of the project.

Week	1	2	3	4	5
Height (cm)	1.7	2.9	4.1	5.3	6.5

Write the equation that models the growth of the plant over time.

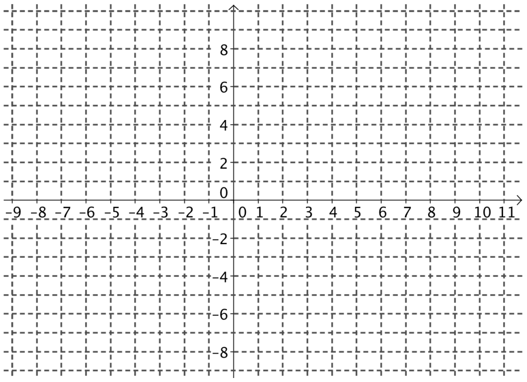
Name the form of the equation you wrote and why you chose to use that form.

This function is: linear exponential neither (choose one)

This function is: continuous discrete neither (choose one)

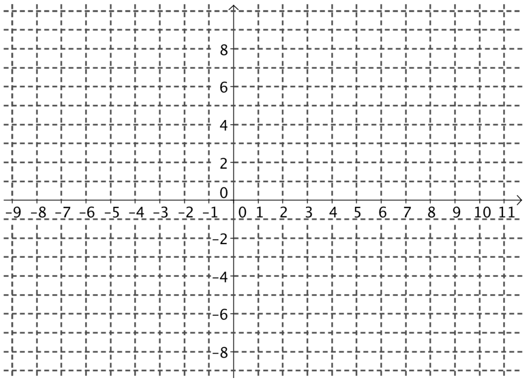
An equation gives us information that we can use to graph the function. Pick out the important information given in each of the following equations and use the information to graph the function.

6. $y = \frac{1}{2}x - 5$



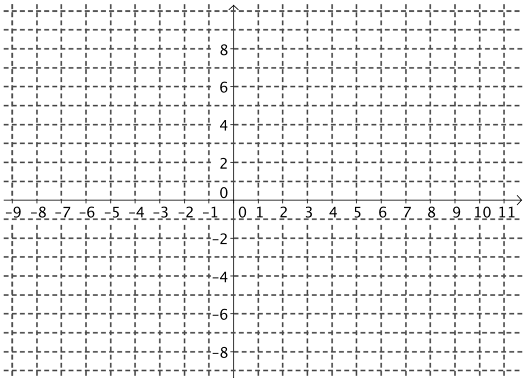
What do you know from the equation that helps you to graph the function?

7. $y = 2^n$



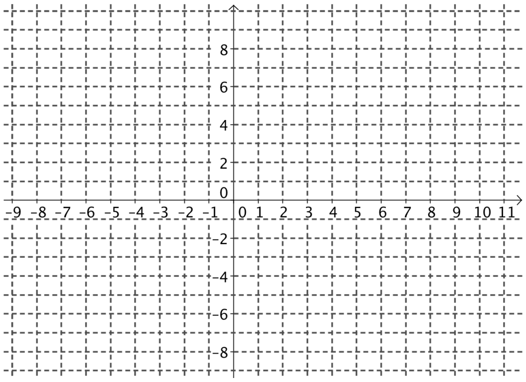
What do you know from the equation that helps you to graph the function?

8. $y = -2(x + 6) + 8$



What do you know from the equation that helps you to graph the function?

9. $f(1) = -5, f(n) = f(n-1) + 1$



What do you know from the equation that helps you to graph the function?

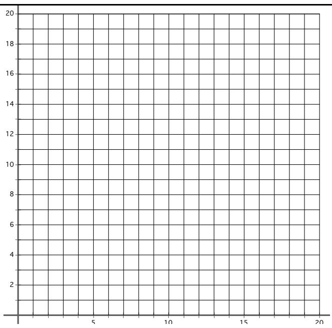
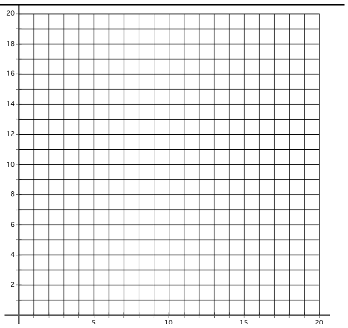
2.6

READY, SET, GO! Name Period Date

READY

Topic: Comparing linear and exponential models.

Comparing different characteristics of each type of function by filling in the cells of each table as completely as possible.

	$y = 4 + 3x$	$y = 4(3^x)$
1. Type of growth		
2. What kind of sequence corresponds to each model?		
3. Make a table of values	x y	x y
4. Find the rate of change		
5. Graph each equation. Compare the graphs. What is the same? What is different?		
6. Find the y-intercept for each function.		

7. Find the y-intercepts for the following equations

a) $y = 3x$

b) $y = 3^x$

8. Explain how you can find the y-intercept of a linear equation and how that is different from finding the y-intercept of a geometric equation.

SET

Topic: Efficiency with different forms of linear and exponential functions.

For each exercise or problem below use the given information to determine which of the forms would be the most efficient to use for what is needed. (See task 2.6, Linear: slope-intercept, point-slope form, recursive, Exponential: explicit and recursive forms)

9. Jasmine has been working to save money and wants to have an equation to model the amount of money in her bank account. She has been depositing $175 a month consistently, she doesn't remember how much money she deposited initially, however on her last statement she saw that her account has been open for 10 months and currently has $2475 in it. Create an equation for Jasmine.

Which equation form do you chose? **Write the equation.**

10.

The table below shows the number of rectangles created every time there is a fold made through the center of a paper. Use this table for each question.

Folds	Rectangles
1	2
2	4
3	8
4	16

A. Find the number or rectangles created with 5 folds.

Which equation form do you chose? **Write the equation.**

B. Find the number of rectangles created with 14 folds.

Which equation form do you chose? **Write the equation.**

11. Using a new app that I just downloaded I want to cut back on my calorie intake so that I can lose weight. I currently weigh 90 kilograms, my plan is to lose 1.2 kilograms a week until I reach my goal. How can I make an equation to model my weight loss for the next several weeks.

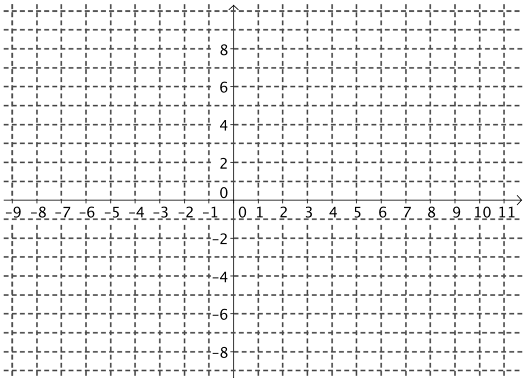
Which equation form do you chose? **Write the equation.**

12. Since Scott started doing his work out plan Janet has been inspired to set her self a goal to do more exercise and walk a little more each day. She has decided to walk 10 meters more every day. On the day 20 she walked 800 meters. How many meters will she walk on day 21? On day 60?

Which equation form do you chose? **Write the equation.**

For each equation provided state what information you see in the equation that will help you graph it, then graph it. Also, use the equation to fill in any four coordinates on the table.

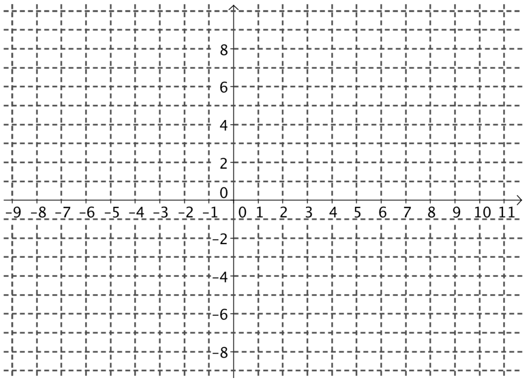
13. $y = \left(\frac{1}{2}\right)^n 8$



What do you know from the equation that helps you to graph the function?

14. $y = 5(x - 2) - 6$

What do you know from the equation that helps you to graph the function?



GO

Topic: Solving one-step equations with justification.

Recall the two properties that help us solve equations.

The Additive property of equality states:
You can add any number to both sides of an equation and the equation will still be true.

The Multiplicative **property of equality states:**
You can multiply any number to both sides of an equation and the equation will still be true.

Solve each equation. Justify your answer by identifying the property(s) you used to get it.

Example 1:

$x - 13 = 7$	Justification
$+13 \quad +13$	additive property of equality
$x + 0 = 20$	addition
$x = 20$	additive identity (You added 0 and got x.)

Example 2:

$5x = 35$	Justification
$\frac{5}{5}x = \frac{35}{5}$	multiplicative property of equality (multiplied by $\frac{1}{5}$)
$1x = 7$	multiplicative identity (A number multiplied by its reciprocal = 1)

15. $3x = 15$ Justification

16. $x - 10 = 2$ Justification

17. $-16 = x + 11$ Justification

18. $6 + x = 10$ Justification

19. $6x = 18$ Justification

20. $-3x = 2$ Justification

2.7H I Can See—Can't You?

A Solidify Understanding Task

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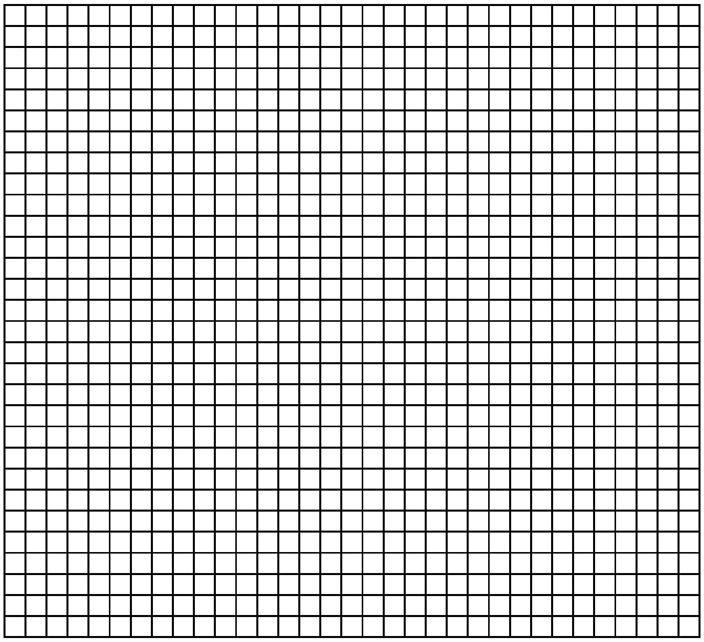
Kwan's parents bought a home for $50,000 in 1997 just as real estate values in the area started to rise quickly. Each year, their house was worth more until they sold the home in 2007 for $309,587.

1. Model the growth of the home's value from 1997 to 2007 with both a linear and an exponential equation. Graph the two models below.

Linear model:

Exponential model:



2. What was the change in the home's value from 1997 to 2007?

The average rate of change is defined as the change in y (or *f(x)*) divided by the change in x.

3. What was the average rate of change of the linear function from 1997 to 2007?

4. What is the average rate of change of the exponential function in the interval from 1997 to 2007?

5. How do the average rates of change from 1997 to 2007 compare for the two functions? Explain.

6. What was the average rate of change of the linear function from 1997 to 2002?

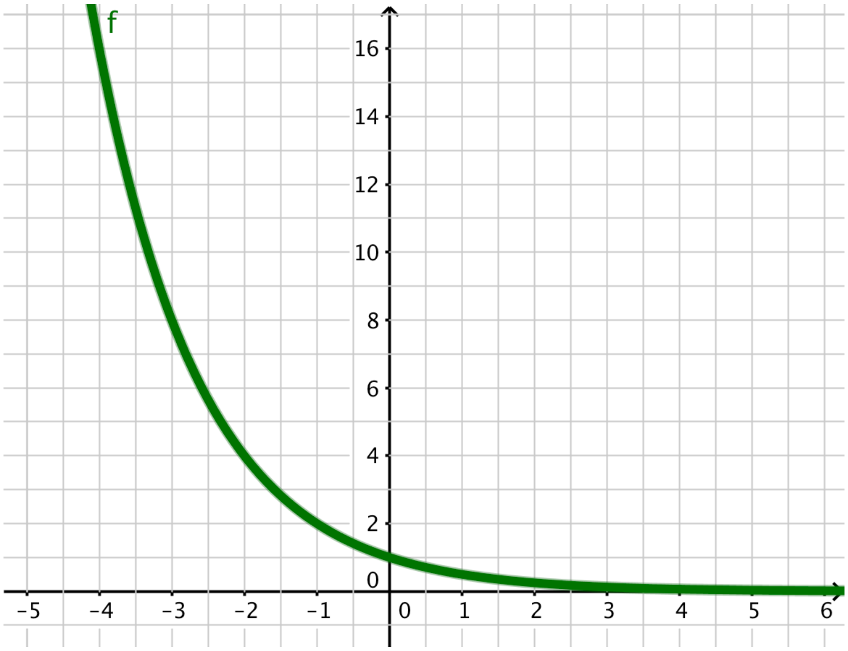
7. What is the average rate of change of the exponential function in the interval from 1997 to 2002?

8. How do the average rates of change from 1997 to 2002 compare for the two functions? Explain.

9. How can you use the equation of the exponential function to find the average rate of change over a given interval?

   How does this process compare to finding the slope of the line through the endpoints of the interval?

Consider the following graph:



10. What is the equation of the graph shown?

11. What is the average rate of change of this function on the interval from x = -3 to x = 1?

12. What is the average rate of change of this function in the interval from x = -3 to x = 0?

13. What is the average rate of change of this function in the interval from x = -3 to x = -1?

14. What is the average rate of change of this function in the interval from x = -3 to x = -1.5?

15. Draw the line through the point at the beginning and end of each of the intervals in 11, 12, 13 and 14. What is the slope of each of these lines?

16. Which of these average rates of change best represents the change at the point (-2, 4)? Explain your answer.

17. How does the average rate of change compare to the change factor for an exponential function? What is described by each of these quantities?

2.7H

READY, SET, GO! Name Period Date

READY

Topic: Finding an appropriate viewing window.

When viewing the secant line of an exponential function on a calculator, you want a window that shows the two points on the curve that are being connected. Since exponential functions get very large or small in just a few steps, you may want to change the scale as well as the dimensions of the window. Don't be afraid to experiment until you are satisfied with what you see.

The graphs below depict an exponential function and a secant line. The equations are given. Identify the dimensions of the viewing window. Include the scale for both the x and y values. Check your answer by matching your calculator screen to the one displayed here.

1. $Y_1 = 4(0.2)^x$ *and* $Y_2 = -1.92x + 4$

WINDOW

a. X min = ____________________

b. X max = ____________________

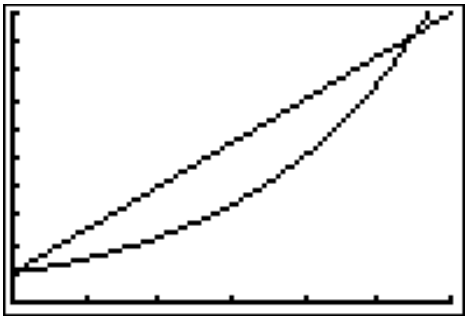
c. X scl = ____________________

d. Y min = ____________________

e. Y max = ____________________

f. Y scl = ____________________

2. $Y_1 = 1.5^X$ and $Y_2 = 1.5x + 1$



WINDOW

a. X min = ____________________

b. X max = ____________________

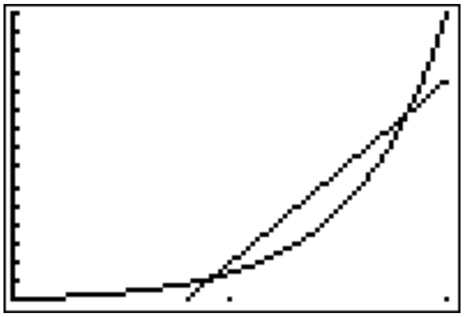
c. X scl = ____________________

d. Y min = ____________________

e. Y max = ____________________

f. Y scl = ____________________

3. $Y_1 = 150\,(10)^X$ and $Y_2 = 9500x - 7500$



WINDOW

a. X min = ______________________

b. X max = ______________________

c. X scl = ______________________

d. Y min = ______________________

e. Y max = ______________________

f. Y scl = ______________________

SET

Topic: Using slope to compare change in linear and exponential models.

The tables below show the values for a linear model and an exponential model. Use the slope formula between *each set of 2 points* to calculate the rate of change.

Example: Find the slope between the points (30 , 1) and (630 , 2) then between (630 , 2) and (1230 , 3). Do the same between each pair of points in the table for the exponential model.

Linear Model

x	y
1	30
2	630
3	1230
4	1830
5	2430

Exponential Model

x	y
1	30
2	90
3	270
4	810
5	2430

5. Compare the change between each pair of points in the linear model to the change between each pair of points in the exponential model. Describe your observations and conclusions.

6. Find the average of the 4 rates of change of the exponential model. How does the average of the rates of change of the exponential model compare to the rates of change of the linear model?

7. Without using a graphing calculator, make a rough sketch on the same set of axes of what you think the linear model and the exponential model would look like.

8. How did your observations in #5 influence your sketch?

9. Explain how a table of 5 consecutive values can begin and end with the same y-values and be so different in the middle 3 values. How does this idea connect to the meaning of a secant line?

GO

Topic: Finding slope between points.

Use your calculator and the slope formula to find the slope of the line that passes through the two points.

10. A (-10, 17) , B (10, 97)

11. P (57, 5287) , Q (170, 4948)

12. R (6.055, 23.1825) , S (5.275, 12.0675)

13. G (0.0012, 0.125) , H (2.5012, 6.375)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