mathematics vision project

Transforming Mathematics Education

SECONDARY MATH ONE

An Integrated Approach

Standard Teacher Notes

MODULE 7

Congruence, Construction & Proof

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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CONGRUENCE, CONSTRUCTION AND PROOF

7.1 Under Construction

A Develop Understanding Task

Anciently, one of the only tools builders and surveyors had for laying out a plot of land or the foundation of a building was a piece of rope.

There are two geometric figures you can create with a piece of rope: you can pull it tight to create a line segment, or you can fix one end, and—while extending the rope to its full length—trace out a circle with the other end. Geometric constructions have traditionally mimicked these two processes using an unmarked straightedge to create a line segment and a compass to trace out a circle (or sometimes a portion of a circle called an arc). Using only these two tools you can construct all kinds of geometric shapes.

Suppose you want to construct a rhombus using only a compass and straightedge. You might begin by drawing a line segment to define the length of a side, and drawing another ray from one of the endpoints of the line segment to define an angle, as in the following sketch.

Now the hard work begins. We can't just keep drawing line segments, because we have to be sure that all four sides of the rhombus are the same length. We have to stop drawing and start constructing.

Constructing a rhombus

Knowing what you know about circles and line segments, how might you locate point C on the ray in the diagram above so the distance from B to C is the same as the distance from B to A?

1. Describe how you will locate point C and how you know $\overline{BC} \cong \overline{BA}$, then construct point C on the diagram above.

Now that we have three of the four vertices of the rhombus, we need to locate point D, the fourth vertex.

2. Describe how you will locate point D and how you know $\overline{CD} \cong \overline{DA} \cong \overline{AB}$, then construct point D on the diagram above.

Constructing a Square (A rhombus with right angles)

The only difference between constructing a rhombus and constructing a square is that a square contains right angles. Therefore, we need a way to construct perpendicular lines using only a compass and straightedge.

We will begin by inventing a way to construct a perpendicular bisector of a line segment.

3. Given $\overline{RS}$ below, fold and crease the paper so that point R is reflected onto point S. Based on the definition of reflection, what do you know about this "crease line"?

You have "constructed" a perpendicular bisector of $\overline{RS}$ by using a paper-folding strategy. Is there a way to construct this line using a compass and straightedge?

4. Experiment with the compass to see if you can develop a strategy to locate points on the "crease line". When you have located at least two points on the "crease line" use the straightedge to finish your construction of the perpendicular bisector. Describe your strategy for locating points on the perpendicular bisector of $\overline{RS}$.

Now that you have created a line perpendicular to $\overline{RS}$ we will use the right angle formed to construct a square.

5. Label the midpoint of $\overline{RS}$ on the diagram above as point M. Using segment $\overline{RM}$ as one side of the square, and the right angle formed by segment $\overline{RM}$ and the perpendicular line drawn through point M as the beginning of a square. Finish constructing this square on the diagram above. (Hint: Remember that a square is also a rhombus, and you have already constructed a rhombus in the first part of this task.)

7.1 Under Construction – Teacher Notes

A Develop Understanding Task

Purpose: Compass and straightedge constructions can be justified based on properties of quadrilaterals and the definitions of the rigid-motion transformations. While we have used dynamic geometric software and paper folding in previous tasks, in this learning cycle we restrict the work to compass and straightedge in order to generate ideas about defending how these constructions result in the desired objects—a goal which is sometimes obscured by other tools. In this task students invent strategies for constructing a rhombus and a square. Embedded in this work are smaller constructions, such as copying a segment (using radii of congruent circles) or creating the perpendicular bisector of a segment. These constructions also contain the potential for building more sophisticated constructions, such as bisecting an angle by constructing a diagonal of a rhombus.

Core Standards Focus:

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

See also Mathematics I note for G.CO.12, G.CO.13: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Standards for Mathematical Practice of Focus in the Task:

SMP 1 – Make sense of problems and persevere in solving them

SMP 4 – Use appropriate tools strategically

SMP 7 – Look for and make use of structure

Additional Resources for Teachers:

The hands-on activity with compass and straightedge can be replicated using dynamic geometry software. The mathematical decisions students need to make—such as where to construct congruent circles and what information they provide—are similar decisions regardless of the construction tool being used. Some constructions, however, are more difficult using technology since many dynamic geometry software programs do not have a tool that replicates a rigid compass (one that can be picked up and moved to another location while retaining the measure of the radius). The circle construction tool in many dynamic geometry programs does not retain a "memory" of the radius of a circle drawn in another location. This "collapsible compass" tool can be more cumbersome to complete the construction work intended by this task. If you chose to use dynamic geometry software, try out each of the constructions in advance to consider the implications of such a decision.

The Teaching Cycle:

Launch (Whole Class):

Read the introductory paragraphs of the task with students to suggest why we are going to restrict ourselves to using two tools: a compass and a straightedge. Remind students of the definition of a circle that we encountered in *Leap Year*: the set of all points in a plane that are equidistant from a fixed point called the center of the circle. Point out that students may need to think about this definition as they work on this task. Ask students what they would need to know about two different circles in order to know that the circles are congruent?

Read through the remainder of the first page of the task up to the words “we have to stop drawing and start constructing.” Tell students to follow the prompts as they work on the two tasks of constructing a rhombus and a square.

Explore (Small Group):
Allow students sufficient time to explore both constructions.

The key to constructing the rhombus is creating a set of congruent circles, centered at each of the vertices of the evolving rhombus. Since radii of congruent circles are congruent line segments, we can use these congruent segments to define the sides of the rhombus.

The key to locating points on the perpendicular bisector is to construct congruent circles on each endpoint of the segment, with the radius larger than half of the segment. The points where these circles intersect, along with the endpoints of the segment, form pairs of congruent triangles. Students might use their intuition to assume that the triangles are congruent, they might rely on transformational ideas to try to justify why these triangles are congruent, or they may recall experimenting with conditions that create congruent triangles in Math 7. It is helpful to ask students to explain why they think this construction works, even though their explanations will be somewhat intuitive until they have established the ASA, SAS and SSS triangle congruence criteria in subsequent tasks in this module. It is important to surface the need for justification, and some initial ideas about how to justify this construction.

Since finishing the square is the same as the rhombus construction once the right angle has been constructed, you may move to the whole class discussion of the task once all students have explored a strategy for constructing the perpendicular bisector of the given segment.

Discuss (Whole Class):
Select students to present each of the constructions. Highlight the emerging ideas underlying each construction, as described above, as they appear in students’ work during the presentations. For

example, ask questions like, "How do we know these two segments are congruent?" or "What convinces us that these two segments are perpendicular?"

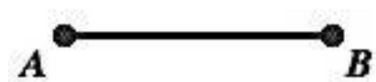
Aligned Ready, Set, Go: Congruence, Construction and Proof 7.1

READY, SET, GO! Name Period Date

READY

Topic: Tools for construction and geometric work.

1. Using your compass draw several concentric circles that have point A as a center and then draw those same sized concentric circles that have B as a center. What do you notice about where all the circles with center A intersect all the corresponding circles with center B?

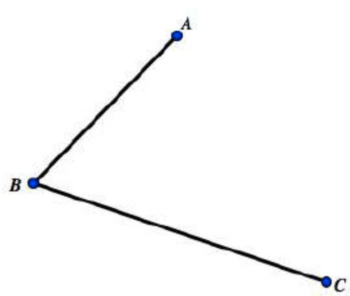


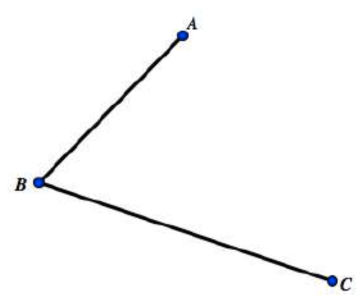
2. In the problem above you have demonstrated one way to find the midpoint of a line segment. Explain another way that a line segment can be bisected without the use of circles.

SET

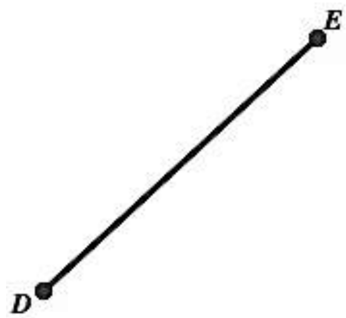
Topic: Constructions with compass and straight edge.

3. Bisect the angle below do it with compass and straight edge as well as with paper folding.

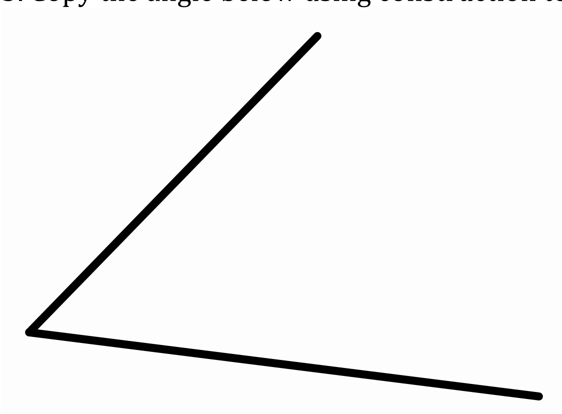




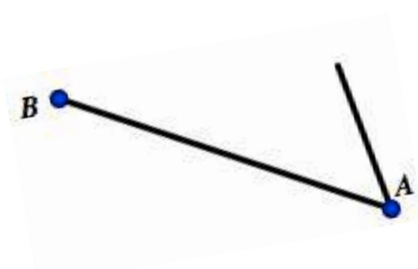
4. Copy the segment below using construction tools of compass and straight edge, label the image ***D'E'***.



5. Copy the angle below using construction tool of compass and straight edge.

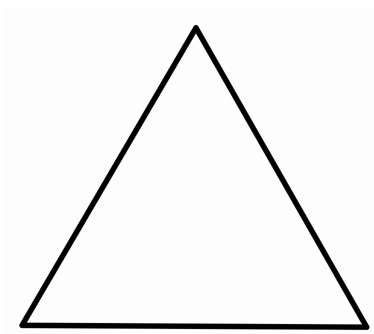


6. Construct a rhombus on the segment AB that is given below and that has point A as a vertex. Be sure to check that your final figure is a rhombus.



7. Construct a square on the segment CD that is given below. Be sure to check that your final figure is a square.

8. Given the equilateral triangle below, find the center of rotation of the triangle using compass and straight edge.



GO

Topic: Solving systems of equations

Solve each system of equations. Utilize substitution, elimination, graphing or matrices.

9. $\begin{cases} x = 11 + y \\ 2x + y = 19 \end{cases}$

10. $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$

11. $\begin{cases} x + 2y = 11 \\ x - 4y = 2 \end{cases}$

12. $\begin{cases} y = -x + 1 \\ y = 2x + 1 \end{cases}$

13. $\begin{cases} y = -2x + 7 \\ -3x + y = -8 \end{cases}$

14. $\begin{cases} 4x - y = 7 \\ -6x + 2y = 8 \end{cases}$

7. 2 More Things Under Construction

A Develop Understanding Task

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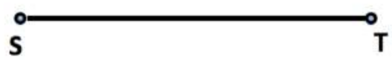
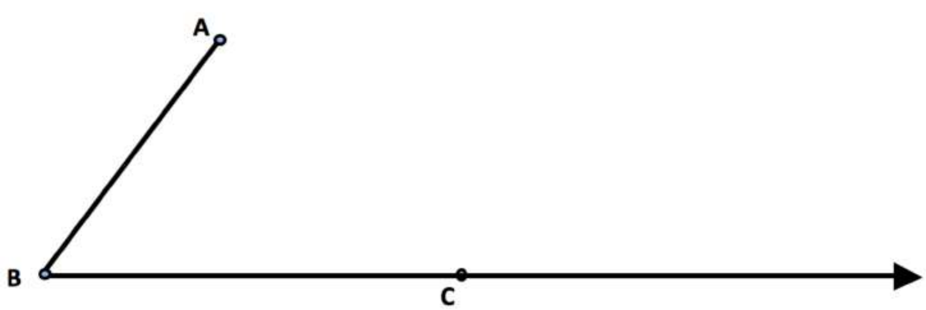
Like a rhombus, an equilateral triangle has three congruent sides. Show and describe how you might locate the third vertex point on an equilateral triangle, given $\overline{ST}$ below as one side of the equilateral triangle.



Constructing a Parallelogram

To construct a parallelogram we will need to be able to construct a line parallel to a given line through a given point. For example, suppose we want to construct a line parallel to segment $\overline{AB}$ through point C on the diagram below. Since we have observed that parallel lines have the same slope, the line through point C will be parallel to $\overline{AB}$ only if the angle formed by the line and $\overrightarrow{BC}$ is congruent to $\angle ABC$. Can you describe and illustrate a strategy that will construct an angle with vertex at point C and a side parallel to $\overline{AB}$?



Constructing a Hexagon Inscribed in a Circle

Because regular polygons have rotational symmetry, they can be *inscribed* in a circle. The *circumscribed* circle has its center at the center of rotation and passes through all of the vertices of the regular polygon.

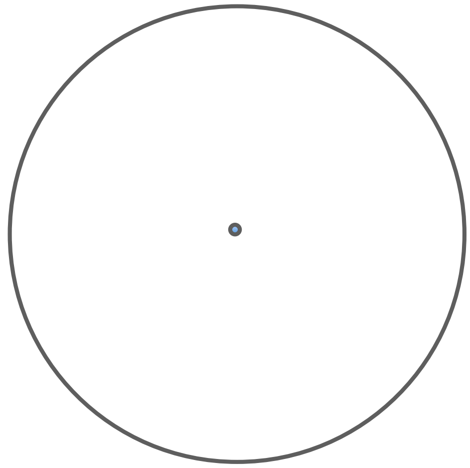
We might begin constructing a hexagon by noticing that a hexagon can be decomposed into six congruent equilateral triangles, formed by three of its lines of symmetry.

1. Sketch a diagram of such a decomposition.

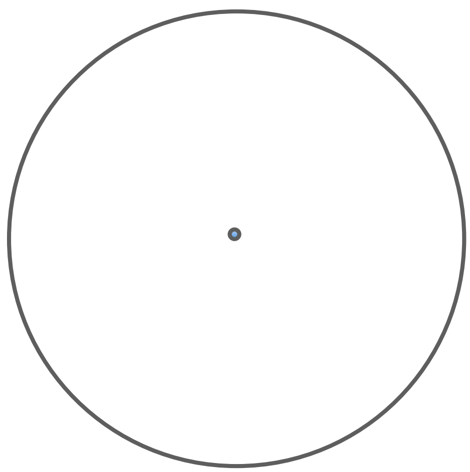
2. Based on your sketch, where is the center of the circle that would circumscribe the hexagon?

3. The six vertices of the hexagon lie on the circle in which the regular hexagon is inscribed. The six sides of the hexagon are *chords* of the circle. How are the lengths of these chords related to the lengths of the radii from the center of the circle to the vertices of the hexagon? That is, how do you know that the six triangles formed by drawing the three lines of symmetry are equilateral triangles? (Hint: Considering angles of rotation, can you convince yourself that these six triangles are equiangular, and therefore equilateral?)

4. Based on this analysis of the regular hexagon and its circumscribed circle, illustrate and describe a process for constructing a hexagon inscribed in the circle given below.



5. Modify your work with the hexagon to construct an equilateral triangle inscribed in the circle given below.



6. Describe how you might construct a square inscribed in a circle.

7. 2 More Things Under Construction – Teacher Notes

A Develop Understanding Task

Purpose: This task continues the construction work of the previous task, allowing students to "invent" some additional construction strategies and to continue to explore the idea that these constructions are based on properties of quadrilaterals and the definitions of the rigid-motion transformations. In this task students invent strategies for constructing an equilateral triangle, a parallelogram and a hexagon inscribed in a circle. Embedded in this work are smaller constructions, such as copying an angle (by constructing congruent triangles) and creating a line parallel to another line through a given point. These constructions also contain the potential for building more sophisticated constructions, such as inscribing a square or equilateral triangle in a circle.

Core Standards Focus:
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

See also Mathematics I note for G.CO.12, G.CO.13: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Standards for Mathematical Practice of Focus in the Task:

SMP 7 – Look for and make use of structure, and

SMP 4 – Use appropriate tools strategically

Additional Resources for Teachers:

The hands-on activity with compass and straightedge can be replicated using dynamic geometry software. Some constructions, however, are more difficult using technology since many dynamic geometry software programs do not have a tool that replicates a rigid compass (one that can be picked up and moved to another location while retaining the measure of the radius). The circle construction tool in many dynamic geometry programs does not retain a "memory" of the radius of a circle drawn in another location. This "collapsible compass" tool can be more cumbersome to complete the construction work intended by this task. If you chose to use dynamic geometry software, try out each of the constructions in advance to consider the implications of such a decision.

The Teaching Cycle:

Launch (Whole Class):

Give students a few minutes individually to construct the equilateral triangle. If students need prompting to do this, remind them that an equilateral triangle, like a rhombus or a square, contains congruent sides. How can they use the ideas of the rhombus construction in this work? Once students are back into "construction mode" you can set them to work on the other constructions in this task.

Explore (Small Group):

Allow students sufficient time to explore each construction.

The key to constructing the parallelogram is to create parallel lines by copying an angle (so the slant or slope will be the same). In this task, students might copy an angle by constructing congruent triangles in which the angle we desire to copy is embedded.

The key to inscribing a hexagon inside a circle is to notice that a hexagon can be decomposed into six equilateral triangles by drawing the lines of symmetry of the hexagon that pass through opposite vertices. If the hexagon is inscribed in a circle, these lines of symmetry form diameters of the circle. These lines of symmetry intersect the circle at six equally spaced intervals, and the line segments between these points of intersection are the same length as the radii of the circle. Therefore, we can use a compass setting equal to the radius of the circle to mark off these six points where the vertices of the hexagon can be located.

Discuss (Whole Class):

Select students to present each of the constructions. Highlight the emerging ideas underlying each construction, as described above, as they appear in students' work during the presentations. For example, ask questions like, "How do we know these two segments are congruent?" or "What convinces us that these two lines are parallel?"

Aligned Ready, Set, Go: Congruence, Construction and Proof 7.2

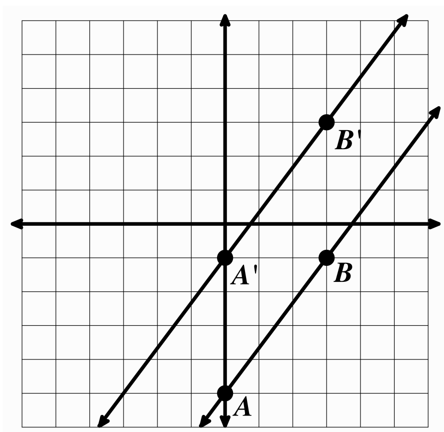
READY, SET, GO! Name Period Date

READY

Topic: Transformation of lines, connecting geometry and algebra.

For each set of lines use the points on the line to determine which line is the image and which is the pre-image, write image by the image line and pre image by the original line. Then define the transformation that was used to create the image. Finally find the equation for each line.

1.

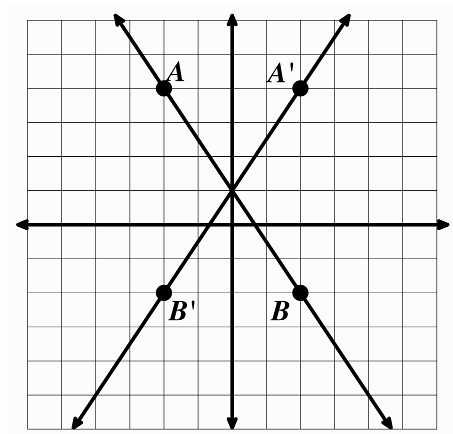


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

2.

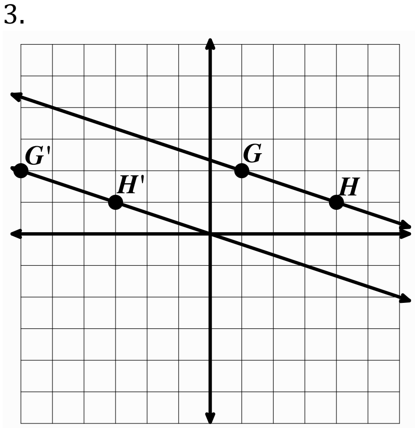


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

3.

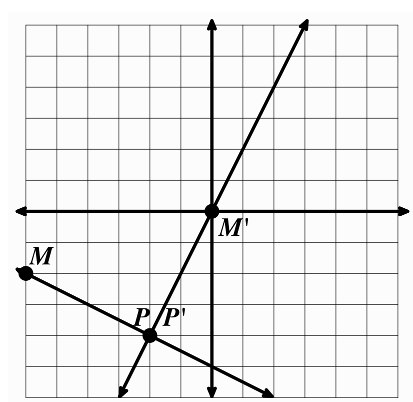


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

4.



a. Description of Transformation:

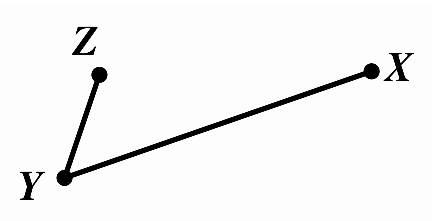
b. Equation for pre-image:

c. Equation for image:

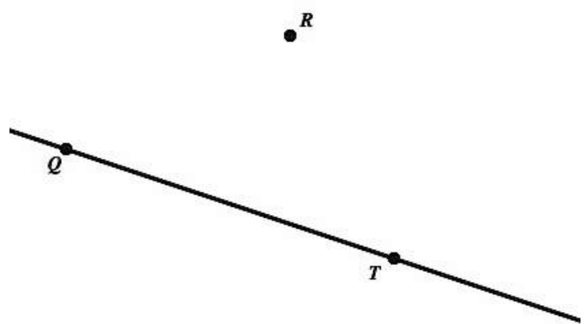
SET

Topic: Geometric constructions with compass and straight edge.

5. Construct a parallelogram given sides $\overline{XY}$ and $\overline{YZ}$ and $\angle XYZ$.



6. Construct a line parallel to $\overline{QT}$ and through point ***R.***



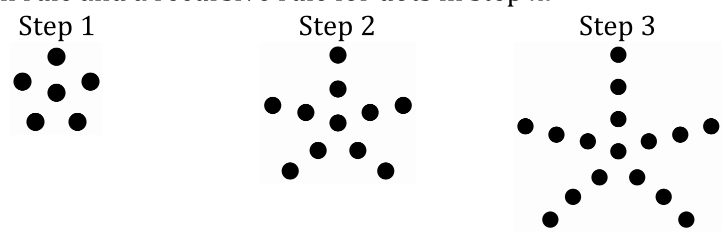
7. Given segment $\overline{AB}$ show all points ***C*** such that Δ ABC is an isosceles triangle.

8. Given segment $\overline{AB}$ show all points C such that Δ ABC is a right triangle.

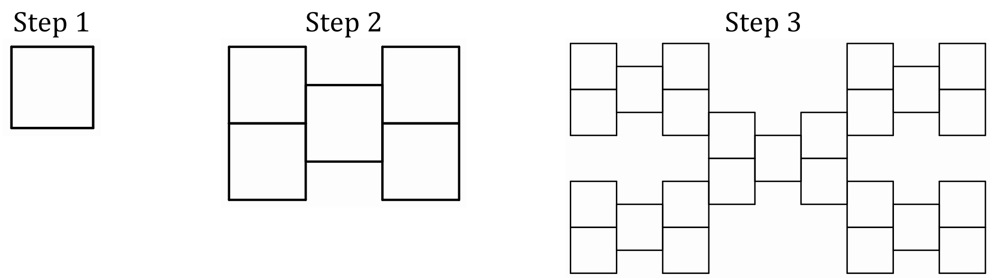
GO

Topic: Creating explicit and recursive rules for visual patterns

9. Find an explicit function rule and a recursive rule for dots in step *n*.



10. Find an explicit function rule and a recursive rule for squares in step *n.*



Find an explicit function rule and a recursive rule for the values in each table.

11.

Step	Value
1	1
2	11
3	21
4	31

12.

n	f(n)
2	16
3	8
4	4
5	2

13.

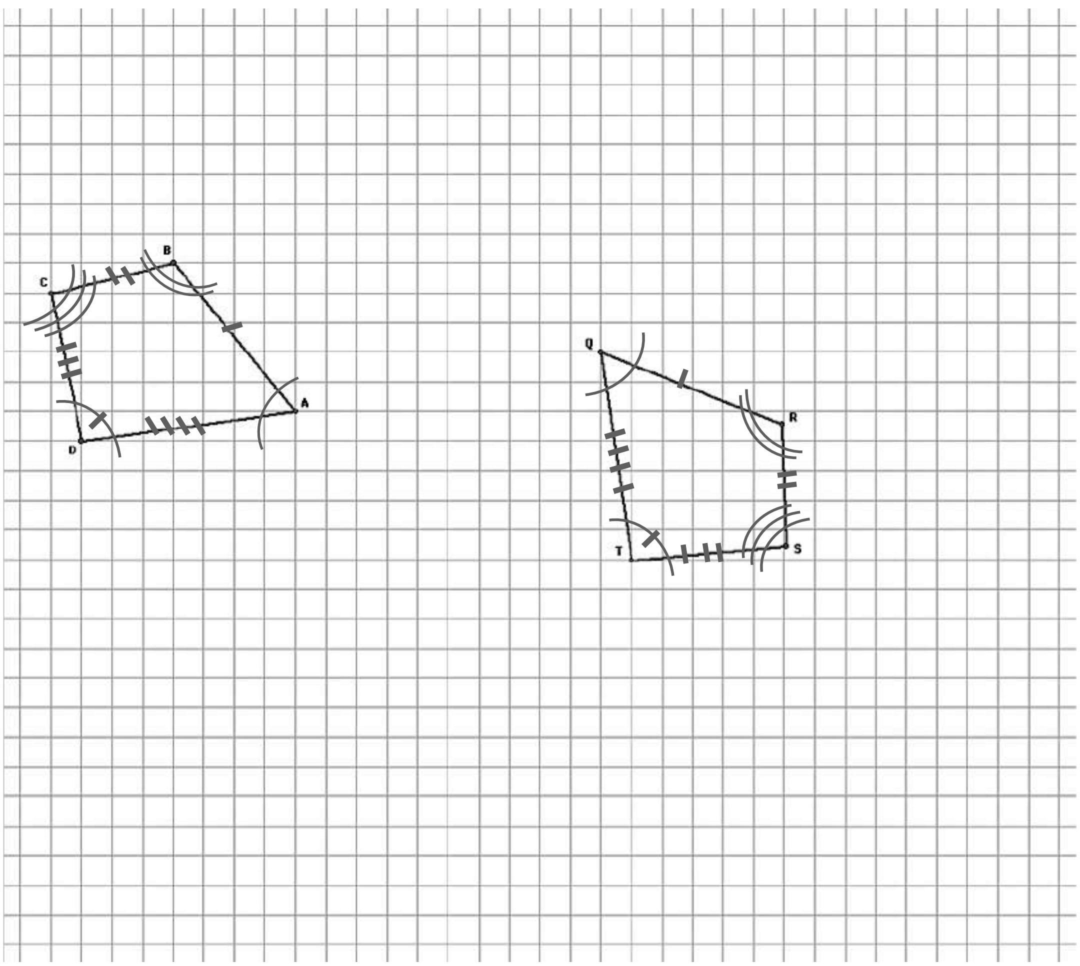
n	f(n)
1	-5
2	25
3	-125
4	625

7. 3 Can You Get There From Here?

A Develop Understanding Task

The two quadrilaterals shown below, quadrilateral *ABCD* and quadrilateral *QRST* are congruent, with corresponding congruent parts marked in the diagrams.

Describe a sequence of rigid-motion transformations that will carry quadrilateral *ABCD* onto quadrilateral *QRST*. Be very specific in describing the sequence and types of transformations you will use so that someone else could perform the same series of transformations.



7. 3 Can You Get There From Here? – Teacher Notes

A Develop Understanding Task

Purpose: In Math 8 students came to understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. In this task students are given two congruent figures and asked to describe a sequence of transformations that exhibits the congruence between them. While exploring potential sequences of transformations, students will notice how corresponding parts of congruent figures have to be carried onto one another, and they may look for ways that this can be accomplished in a consistent sequence of steps.

Core Standards Focus:

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Related Standards: G.CO.6

Standards for Mathematical Practice:

SMP 1 – Make sense of problems and persevere in solving them

SMP 7 – Look for and make use of structure

The Teaching Cycle:

Launch (Whole Class):

Give students the handout and discuss the markings used to indicate the corresponding parts of the quadrilaterals that are congruent. For example, ask what angle in quadrilateral *QRST* is congruent to angle *B* in quadrilateral *ABCD* or what segment in quadrilateral *QRST* is congruent to $\overline{BC}$. Point out the convention for naming congruent figures so that corresponding angles and sides can be determined from the names of the figures without having to have a marked diagram. You may want to spend some time practicing this convention by naming two triangles and asking students to mark congruent angles and sides based on the naming convention.

Provide transparencies or tracing paper for students to experiment with as they look for a sequence of transformations that will carry parts of one quadrilateral onto the corresponding congruent parts of the other. Challenge them to try to make this work strategic versus guess and check. What features of the figures themselves might support this work so it can be replicated in other situations?

Explore (Small Group):
Allow students time to experiment with potential sequences of transformations that will carry quadrilateral *ABCD* onto quadrilateral *QRST*. Watch for the strategies they use to get congruent parts to match. As students work, encourage them to look for ways that this can be accomplished in a small number of possible steps. Challenge them to develop a consistent and efficient strategy for determining a sequence of transformations that will always work to carry a given figure onto another congruent figure.

Once students are convinced they have a list of transformations to carry quadrilateral *ABCD* onto quadrilateral *QRST,* challenge them to find a sequence of transformations that will carry quadrilateral *QRST* onto quadrilateral *ABCD.* Encourage students to look for how these two sequences of transformations are similar. Students who finish faster than other students should be challenged to make their list of transformations work for other such congruent figures, helping them to recognize that a translation, a rotation and a reflection may all be necessary to complete the sequence.

Discuss (Whole Class):
Select and sequence student presentations so that each strategy involves fewer transformations, and is therefore more consistent (e.g., less haphazard) and efficient. Work towards a strategy that translates a vertex on quadrilateral *ABCD* to the corresponding vertex on quadrilateral *QRST*, then rotates quadrilateral *ABCD* about that vertex until a pair of corresponding congruent sides match

up, and then reflects quadrilateral *ABCD* over the sides that has been superimposed so that quadrilateral *ABCD* coincides with quadrilateral *QRST*. This sequence uses an easy to identify point of rotation—a vertex of one of the figures—and an easy to identify line of reflection—a side of one of the figures. This sequence of transformations may not be the shortest, but it is consistent and easy to apply when tools such as reflection devices or tracing paper are not available.

Have students check out a "translation-rotation-reflection" sequence to see if it can be applied to carrying quadrilateral *QRST* onto quadrilateral *ABCD.*

Aligned Ready, Set, Go: Congruence, Construction and Proof 7.3

READY, SET, GO! Name Period Date

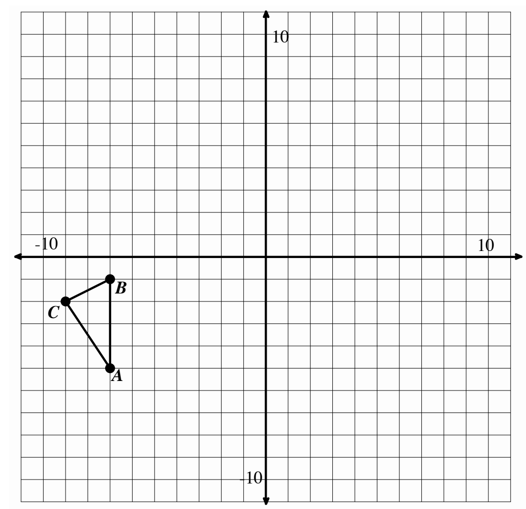
READY

Topic: Multiple transformations

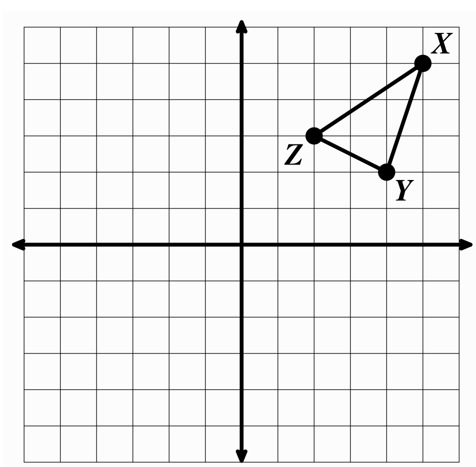
The given figures are to be used as pre-images. Perform the stated transformations to obtain an image. Label the corresponding parts of the image in accordance with the pre-image.

1. Reflect triangle ***ABC*** over the line $y = x$ and label the image ***A'B'C'***.

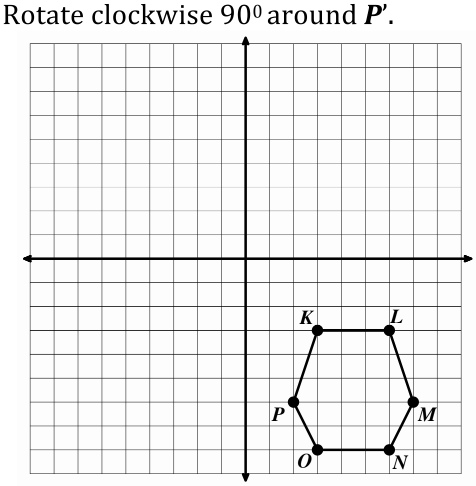
   Rotate triangle ***A'B'C'*** 180^0 counter clockwise around the origin and label the image ***A''B''C''***.



2. Reflect over the line $y = -x$.

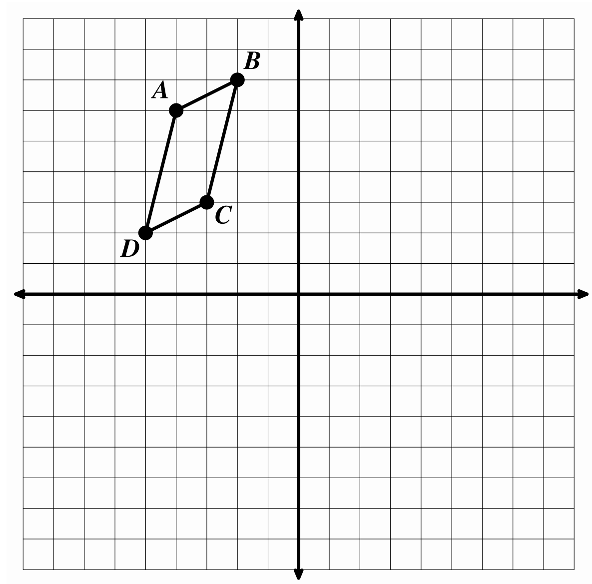


3. Reflect over y-axis and then Rotate clockwise 90^0 around ***P'***.



4. Reflect quadrilateral ABCD over the line $y = 2 + x$ and label the image A'B'C'D'.

Rotate quadrilateral A'B'C'D' counter-clockwise 90^0 around (-2, -3) as the center of rotation label the image A''B''C''D''.

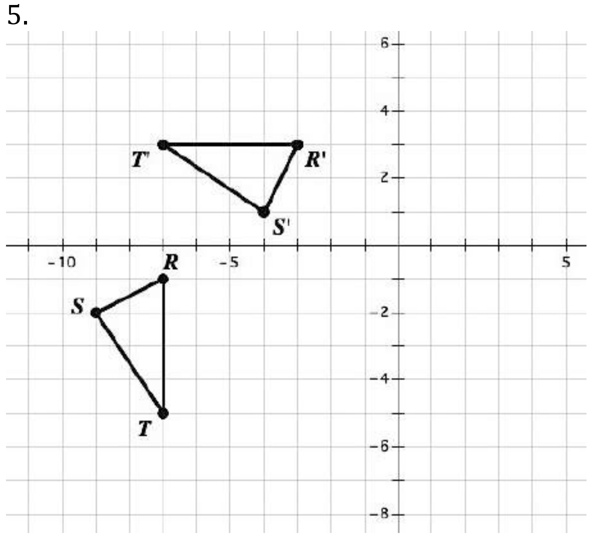


SET

Topic: Find the sequence of transformations.

Find a sequence of transformations that will carry triangle ***RST*** onto triangle ***R'S'T'***.
Clearly describe the sequence of transformations below each grid.

5.



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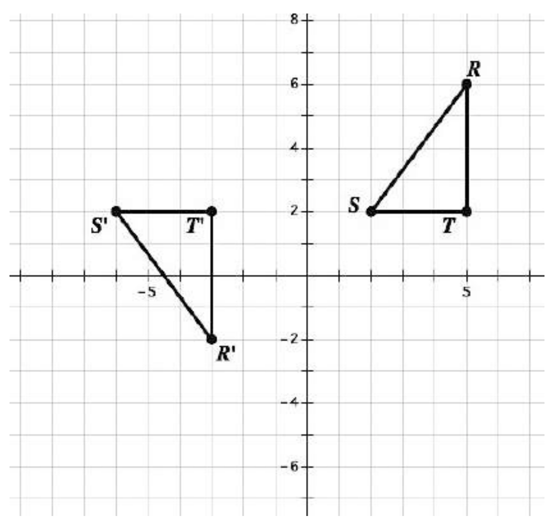
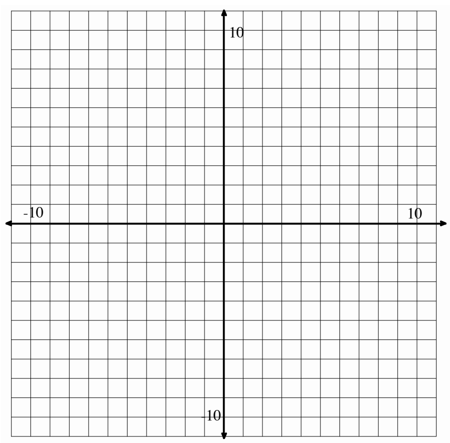


GO

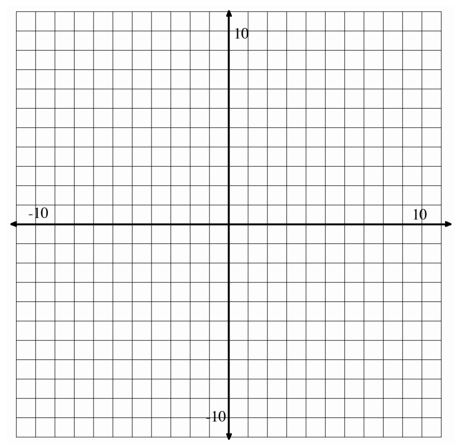
Topic: Graphing systems of functions and making comparisons.

Graph each pair of functions and make an observation about how the functions compare to one another.

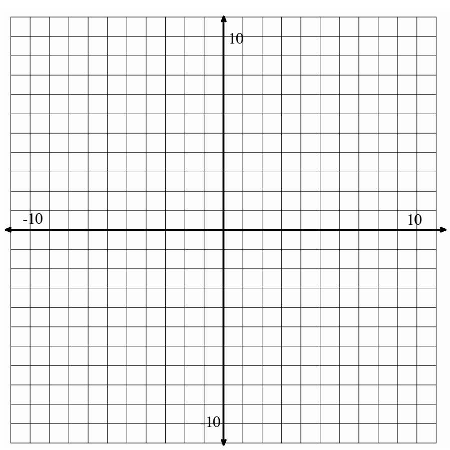
7. $y = \frac{1}{3}x - 1$
$y = -3x - 1$



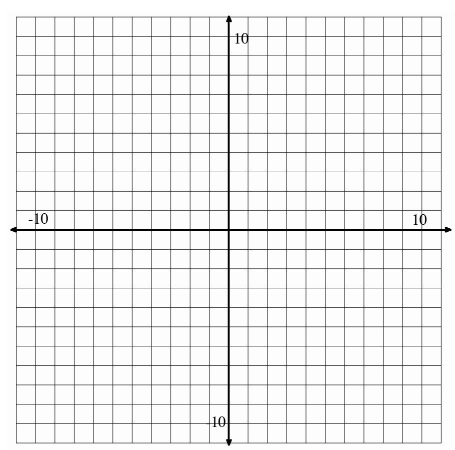
8. $y = -\frac{2}{3}x + 5$
$y = \frac{3}{2}x + 5$



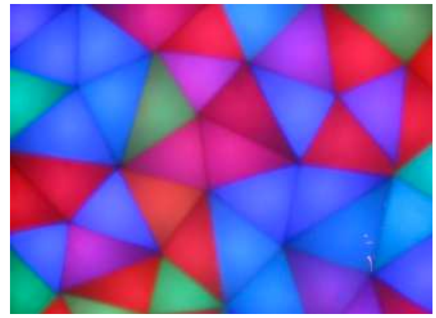
9. $y = \frac{1}{4}x + 2$
$y = -\frac{1}{4} + 2$



10. $y = 2^x$
$y = -2^x$



7. 4 Congruent Triangles

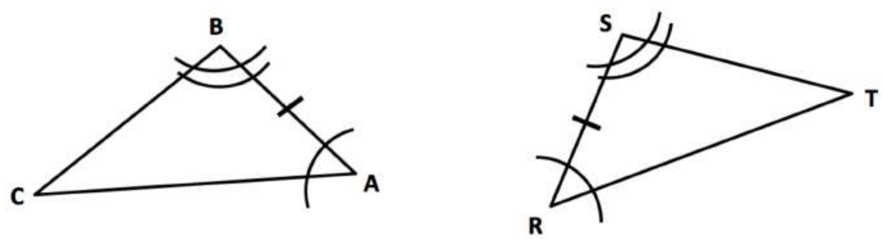


A Solidify Understanding Task

We know that two triangles are congruent if all pairs of corresponding sides are congruent and all pairs of corresponding angles are congruent. We may wonder if knowing less information about the triangles would still guarantee they are congruent.

For example, we may wonder if knowing that two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle—a set of criteria we will refer to as ASA—is enough to know that the two triangles are congruent. And, if we think this is enough information, how might we justify that this would be so.

Here is a diagram illustrating ASA criteria for triangles:



1. Based on the diagram, which angles are congruent? Which sides?

2. To convince ourselves that these two triangles are congruent, what else would we need to know?

3. Use tracing paper to find a sequence of transformations that will show whether or not these two triangles are congruent.

4. List your sequence of transformations here:

Your sequence of transformations is enough to show that these two triangles are congruent, but how can we guarantee that *all* pairs of triangles that share ASA criteria are congruent?

Perhaps your sequence of transformations looked like this:

- **translate** point A until it coincides with point R
- **rotate** $\overline{AB}$ about point R until it coincides with $\overline{RS}$
- **reflect** ΔABC across $\overleftrightarrow{RS}$

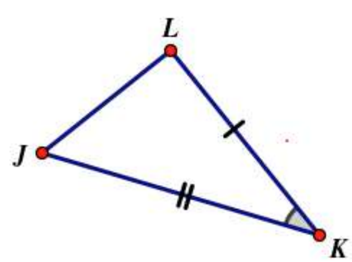
We can use the word "coincides" when we want to say that two points or line segments occupy the same position on the plane. When making arguments using transformations we will use the word a lot.

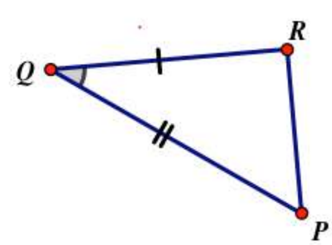
Now the question is, how do we know that point C has to land on point T after the reflection, making all of the sides and angles coincide?

5. *Answer this question as best you can to justify why ASA criteria guarantees two triangles are congruent. To answer this question, it may be helpful to think about how you know point C can't land anywhere else in the plane except on top of T.*

Using tracing paper, experiment with these additional pairs of triangles. Try to determine if you can find a sequence of transformations that will show if the triangles are congruent. Be careful, there may be some that aren't. If the triangles appear to be congruent based on your experimentation, write an argument to explain how you know that this type of criteria will always work. That is, what guarantees that the unmarked sides or angles must also coincide?

6. Given criteria: __________ Are the triangles congruent? _________

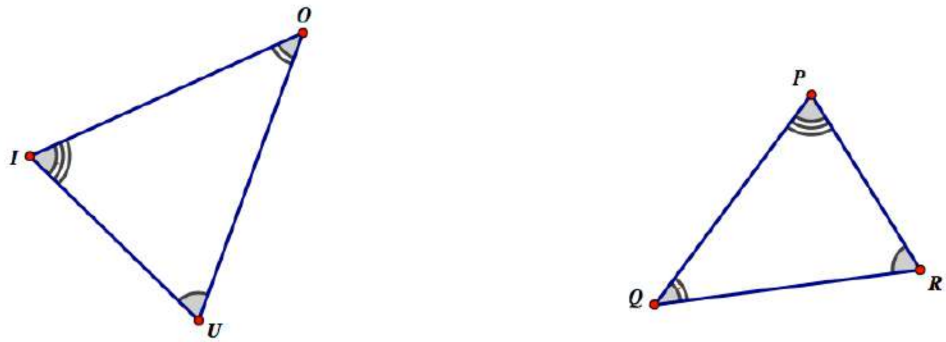




List your transformations in the order performed:

If the triangles are congruent, justify why this will always be true based on this criteria:

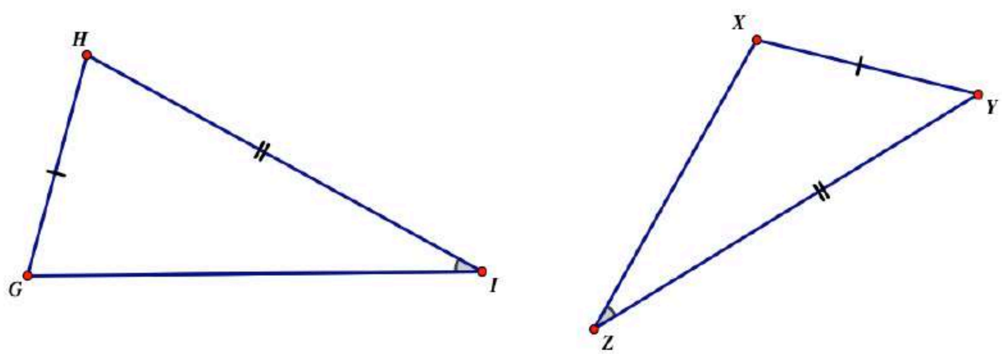
7. Given information: _________ Are the triangles congruent? ________



List your transformations in the order performed:

If the triangles are congruent, justify why this will always be true based on this criteria:

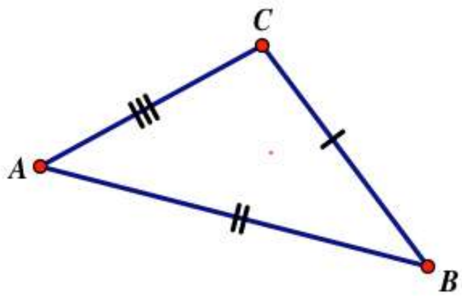
8. Given information: _________ Are the triangles congruent? ________

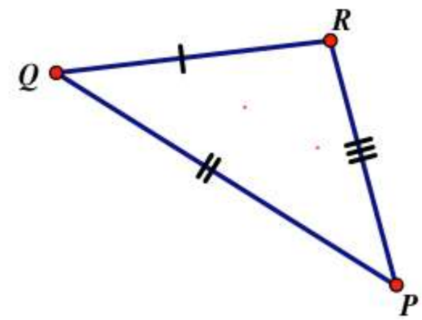


List your transformations in the order performed:

If the triangles are congruent, justify why this will always be true based on this criteria:

9. Given information: _________ Are the triangles congruent? ________





List your transformations in the order performed:

If the triangles are congruent, justify why this will always be true based on this criteria:

10. Based on these experiments and your justifications, what criteria or conditions seem to guarantee that two triangles will be congruent? List as many cases as you can. Make sure you include ASA from the triangles we worked with first.

11. Your friend wants to add AAS to your list, even though you haven't experimented with this particular case. What do you think? Should AAS be added or not? What convinces you that you are correct?

12. Your friend also wants to add HL (hypotenuse-leg) to your list, even though you haven't experimented with right triangles at all, and you know that SSA doesn't work in general from problem 8. What do you think? Should HL for right triangles be added or not? What convinces you that you are correct?

7. 4 Congruent Triangles – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to establish ASA, SAS and SSS as sufficient criteria for claiming that two triangles are congruent and to show how the rigid-motion transformations, along with the given congruence criteria about the two triangles, allows us to prove that the two triangles are congruent. As students work on such proofs they often overlook or reveal misconceptions about how to use the given congruence criteria in their work. Consequently, they might create a sequence of transformations that they claim carries one triangle onto the other, similar to the work they did in the previous task *Can You Get There From Here*, but in doing so they often *assume* the triangles *are* congruent, rather than proving them *to be* congruent. Therefore, the purpose of this task is less about students creating their own arguments, and more about considering the details of how such arguments can be made. Students begin by analyzing a couple of different arguments about ASA criteria for congruent triangles—one that harbors some misconceptions and one that is more explicit about the details. Then they explore other criteria for congruent triangles, such as SSS and SAS, and begin to formulate their own arguments about how they might justify such criteria using transformations.

Note: As this module continues to unfold, students will be asked to continue formulating their own arguments or justifications that something is true, and this task provides a model for doing so. Recognize that students are developing their understandings about justification and proof, and that this work does not get fully formalized until Mathematics II. Based on the students in your class, you will need to decide how much of this work should be based on experimentation, perhaps using technology, and how much should be based on justifying the results of their experimentation.

Core Standards Focus:

G.CO.6 Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

See also Mathematics I note for G.CO.6, G.CO.7, and G.CO.8: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Related Standards: G.CO.5

Standards for Mathematical Practice:

SMP 3 – Construct viable arguments and critique the reasoning of others

The Teaching Cycle:

Launch (Whole Class):

Clarify for students how this task is different from the previous one by saying something similar to the following: "In the previous task you found that if two figures are congruent you can always find a sequence of rigid-motion transformations that will carry one figure onto the other. In this task we will reverse this big idea: *If we can convince ourselves that a sequence of rigid-motion transformations has carried one figure onto another, then the two figures are congruent*. So, unlike in the previous task where we started with congruent figures, in this task we start with figures that we don't know are congruent and try to convince ourselves that they are."

You will need to introduce students to the use of notation for naming corresponding congruent parts of two triangles, such as SAS ("side-angle-side", meaning two sides and the included angle between the two sides of one triangle are congruent to corresponding sides and an included angle of another triangle), ASA ("angle-side-angle", meaning two angles and the included side between the two angles of one triangle are congruent to corresponding angles and an included side of another triangle), and SSS ("side-side-side", meaning the three sides of one triangle are congruent to the corresponding three sides of another triangle).

Work through questions 1-5 as a whole class. Give students some time to work individually on questions 3 and 4, and then discuss their ideas for question 5. It is particularly important to assist students in thinking about how to make an argument that supports their claims for question 5. See the description of "Proof by Contradiction" in the Instructional Supports section below for ideas on how to move this conversation towards modeling justification and proof, without giving away too much of the mathematics.

Explore (Small Group):

Since this lesson is about learning how to construct an argument based on transformations, it will be best to use a think-pair-share structure during much of the explore phase of the lesson. Here is the basic structure:

Have students individually work on a particular pair of triangles, share their observations and justifications with a partner, then discuss their observations and justifications as a whole class.
The point of this discussion is to move away from an argument based on intuition ("it looks like the reflected triangle will coincide with the other triangle) and towards arguments made on the basis of preserved angles and distance. During this part of the discussion students should focus on critiquing each other's arguments by noting concerns and shortcomings in each other's arguments.
Point out that often in geometry we need to follow another person's work step by step.
Prompt students to draw appropriate auxiliary lines or circles to help make their arguments.

Discuss (Whole Class):

Share students' tentative arguments for why SAS (question 6) and SSS (question 9) congruence works. As students share their arguments, other students should take on the role of a skeptic, looking for places where the argument doesn't make sense or isn't convincing. This is hard work, and the nature of proof will be revisited in Secondary Mathematics II. You can feel that the lesson is successful if students are starting to see the need for greater precision in making arguments, and not basing arguments on "it looks like it works." Students should feel assured at the end of the lesson that ASA, SAS and SSS are sufficient criteria for determining that two triangles are congruent—that we don't need to know all pairs of consecutive angles and sides are congruent in order to know the triangles are congruent.

End the discussion by discussing questions 11 and 12. These problems point out that we can often conclude that other things are true from the initial given conditions, such as the third pair of angles being congruent in ASA situations since the sum of the interior angles of a triangle is 180°, or the third pair of sides being congruent by the Pythagorean theorem when given two pairs of congruent sides in a right triangle. These two problems illustrate the nature of proof: using statements we know are true to justify other claims. Don't skip this work, since these two problems are simple proofs that should be accessible to most students.

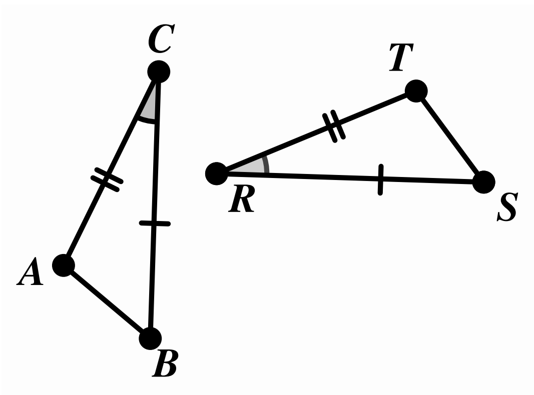
Aligned Ready, Set, Go: Congruence, Construction and Proof 7.4

READY, SET, GO! Name Period Date

READY

Topic: Corresponding parts of figures and transformations.

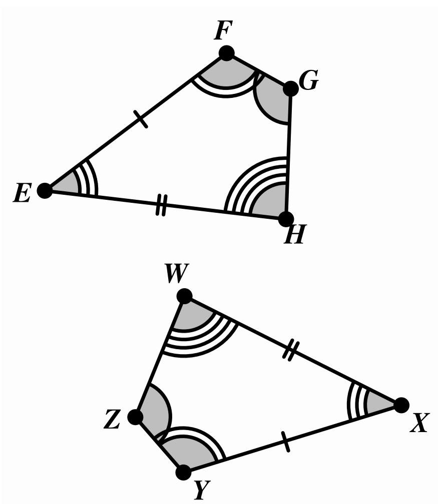
Given the figures in each sketch with congruent angles and sides marked, first list the parts of the figures that correspond (For example, in #1, $\angle C \cong \angle R$) Then determine if a reflection occurred as part of the sequence of transformations that was used to create the image.



Congruencies

$\angle C \cong \angle R$

Reflected? Yes or No



Congruencies

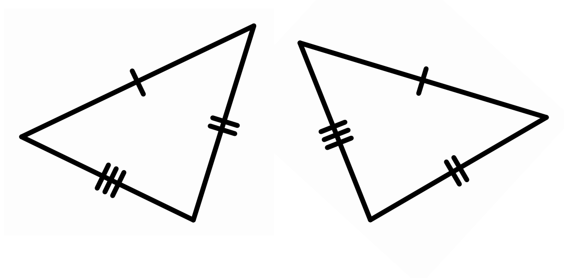
Reflected? Yes or No

SET

Topic: Triangle Congruence

Explain whether or not the triangles are congruent, similar, or neither based on the markings that indicate congruence.

3.



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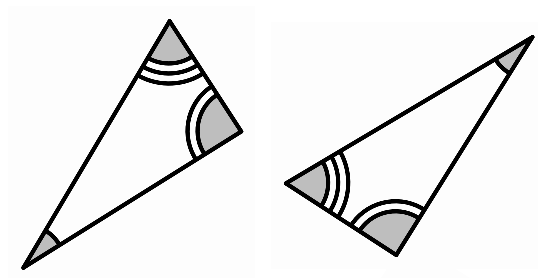

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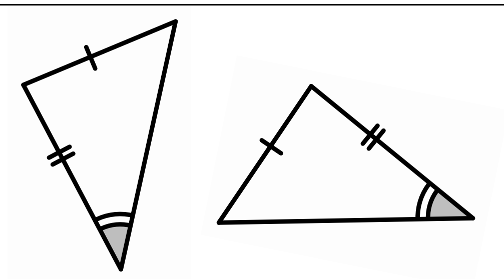

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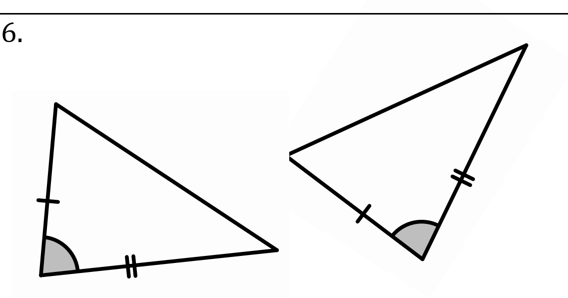

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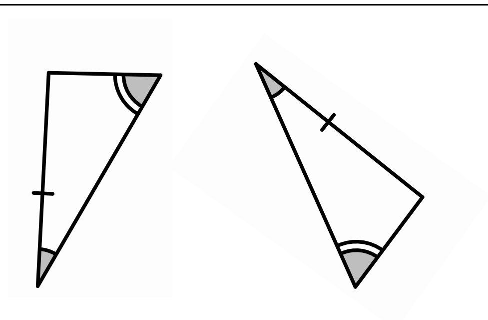

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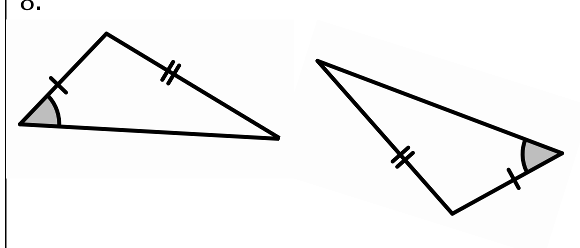


Use the given congruence statement to draw and label two triangles that have the proper corresponding parts congruent to one another.

9. $\Delta ABC \cong \Delta PQR$

10. $\Delta XYZ \cong \Delta KLM$

GO

Topic: Solving equations and finding recursive rules for sequences.

Solve each equation for *t*.

11. $\frac{3t-4}{5} = 5$

12. $10 - t = 4t + 12 - 3t$

13. $P = 5t - d$

14. $xy - t = 13t + w$

Use the given sequence of number to write a recursive rule for the *n*th value of the sequence.

15. 5, 15, 45, …

16. $\frac{1}{2}$, 0, -$\frac{1}{2}$, -1, …

17. 3, -6, 12, -24, …

18. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, …

7.5 Congruent Triangles to the Rescue

A Practice Understanding Task

Part 1

Zac and Sione are exploring isosceles triangles—triangles in which two sides are congruent:

> *Zac*: I think every isosceles triangle has a line of symmetry that passes through the vertex point of the angle made up by the two congruent sides, and the midpoint of the third side.
>
> *Sione*: That's a pretty big claim—to say you know something about *every* isosceles triangle. Maybe you just haven't thought about the ones for which it isn't true.
>
> *Zac*: But I've folded lots of isosceles triangles in half, and it always seems to work.
>
> *Sione*: *Lots* of isosceles triangles are not *all* isosceles triangles, so I'm still not sure.

1. What do you think about Zac's claim? Do you think every isosceles triangle has a line of symmetry? If so, what convinces you this is true? If not, what concerns do you have about his statement?

2. What else would Zac need to know about the crease line through in order to know that it is a line of symmetry? (Hint: Think about the definition of a line of reflection.)

3. Sione thinks Zac's "crease line" (the line formed by folding the isosceles triangle in half) creates two congruent triangles inside the isosceles triangle. Which criteria—ASA, SAS or SSS—could he use to support this claim? Describe the sides and/or angles you think are congruent, and explain how you know they are congruent.

4. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "base angles" of an isosceles triangle (the two angles that are not formed by the two congruent sides)?

5. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "crease line"? (You might be able to make a couple of claims about this line—one claim comes from focusing on the line where it meets the third, non-congruent side of the triangle; a second claim comes from focusing on where the line intersects the vertex angle formed by the two congruent sides.)

Part 2

Like Zac, you have done some experimenting with lines of symmetry, as well as rotational symmetry. In the tasks *Symmetries of Quadrilaterals* and *Quadrilaterals—Beyond Definition* you made some observations about sides, angles, and diagonals of various types of quadrilaterals based on your experiments and knowledge about transformations. Many of these observations can be further justified based on looking for congruent triangles and their corresponding parts, just as Zac and Sione did in their work with isosceles triangles.

Pick one of the following quadrilaterals to explore:

- A **rectangle** is a quadrilateral that contains four right angles.
- A **rhombus** is a quadrilateral in which all sides are congruent.
- A **square** is both a rectangle and a rhombus, that is, it contains four right angles and all sides are congruent

1. Draw an example of your selected quadrilateral, with its diagonals. Label the vertices of the quadrilateral A, B, C, and D, and label the point of intersection of the two diagonals as point N.

2. Based on (1) your drawing, (2) the given definition of your quadrilateral, and (3) information about sides and angles that you can gather based on lines of reflection and rotational symmetry, list as many pairs of congruent triangles as you can find.

3. For each pair of congruent triangles you list, state the criteria you used—ASA, SAS or SSS—to determine that the two triangles are congruent, and explain how you know that the angles and/or sides required by the criteria are congruent (see the following chart).

Congruent Triangles	Criteria Used (ASA, SAS, SSS)	How I know the sides and/or angles required by the criteria are congruent
If I say $\Delta RST \cong \Delta XYZ$	based on SSS	then I need to explain: • how I know that $\overline{RS} \cong \overline{XY}$, and • how I know that $\overline{ST} \cong \overline{YZ}$, and • how I know that $\overline{TR} \cong \overline{ZX}$ so I can use SSS criteria to say $\Delta RST \cong \Delta XYZ$

4. Now that you have identified some congruent triangles in your diagram, can you use the congruent triangles to justify something else about the quadrilateral, such as:

   - the diagonals bisect each other
   - the diagonals are congruent
   - the diagonals are perpendicular to each other
   - the diagonals bisect the angles of the quadrilateral

   Pick one of the bulleted statements you think is true about your quadrilateral and try to write an argument that would convince Zac and Sione that the statement is true.

7. 5 Congruent Triangles to the Rescue – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to provide students with practice in identifying the criteria they might use—ASA, SAS or SSS—to determine if two triangles embedded in another geometric figure are congruent, and then to use those congruent triangles to make other observations about the geometric figures based on the concept that corresponding parts of congruent triangles are congruent. A secondary purpose of this task is to allow students to continue to examine what it means to make an argument based on the definitions of transformations, as well as based on properties of congruent triangles. The focus should be on using congruent triangles and transformations to identify other things that can be said about a geometric figure, rather than on the specific properties of triangles or quadrilaterals that are being observed. These observations will be more formally proved in Secondary II. The observations in this task also provide support for the geometric constructions that are explored in the next task.

Core Standards Focus:

G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

See also Mathematics I note for G.CO.6, G.CO.7, G.CO.8: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Related Standards: G.CO.10

Standards for Mathematical Practice of Focus in the Task:

SMP 3 – Construct viable arguments and critique the reasoning of others

SMP 7 – Look for and make use of structure

Additional Resources for Teachers:

A copy of the images used in this task can be found at the end of this set of teacher notes. These images can be printed for use with students who may be accessing the task on a computer or tablet.

The Teaching Cycle:

Launch (Whole Class):

Make sure that students know the definition of an isosceles triangle and give them several isosceles triangles to fold—essentially recreating Zac's paper-folding experiment as described in part 1 of the task (see attached handout of isosceles triangles). Ask students if they see any congruent triangles inside of the folded isosceles triangle, and what criteria for congruent triangles—ASA, SAS or SSS—they could use to convince themselves that these interior triangles are congruent. Work through the additional questions in part 1 with the class, giving students time to think about each question individually or with a partner.

Help students see the difference between verifying Zac's claim ("every isosceles triangle has a line of symmetry that passes through the vertex point of the angle made up of the two congruent sides, and the midpoint of the third side") through experimentation—paper folding—and a justification based on transformations and congruent triangle criteria. It appears from folding one side of the isosceles triangle onto the other that two congruent triangles are formed. This can be justified using the SSS triangle congruence criterion: the line through the vertex and the midpoint of the opposite side is common to both interior triangles (S1); the midpoint of the opposite side forms two corresponding congruent segments in the interior triangles (S2); and by definition of an isosceles triangle the other pair of sides in the interior triangles are congruent (S3). Since the interior triangles are congruent by SSS, we can also conclude that the three corresponding angles are congruent. This leads to such additional properties as: the base angles of the isosceles triangle are congruent; the vertex angle is bisected by the line through the vertex and midpoint of the opposite side; and the line through the vertex and midpoint of the opposite side is perpendicular to the base since the angles formed are congruent and together form a straight angle. Collectively, these statements justify Zac's claim that every isosceles triangle has a line of symmetry.

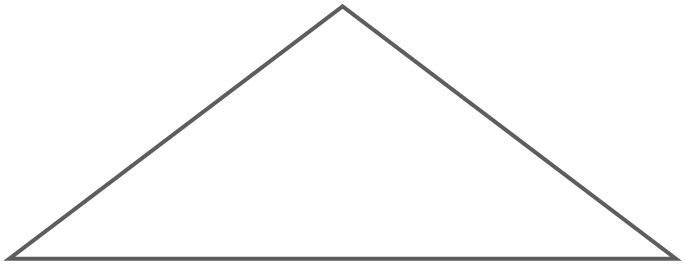
Explore (Small Group):

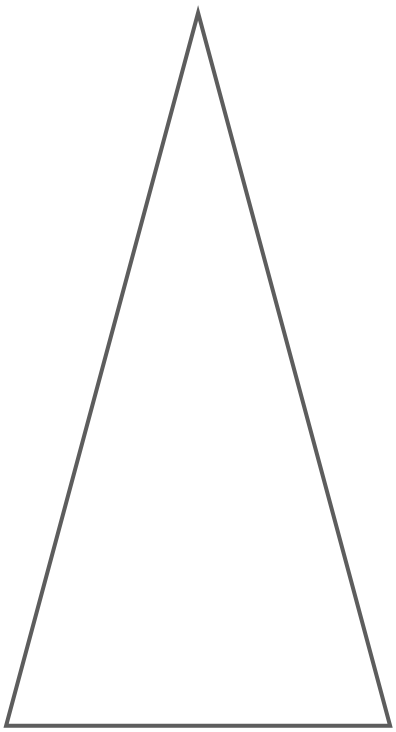
The guided discussion of part 1 of this task will prepare students to work more independently on part 2. You may want to assign different groups to a particular quadrilateral, so all of the quadrilaterals get explored. Center the exploration time on part 2, questions 2 and 3—looking for congruent triangles, and listing the criteria that was used to claim that the triangles are congruent. Fast finishers can work on part 2, question 4—justifying other properties of quadrilaterals based on corresponding parts of congruent triangles.

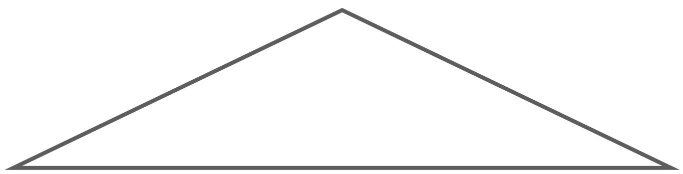
Discuss (Whole Class):

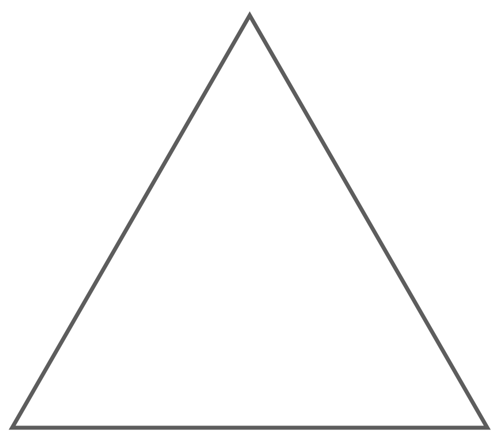
The focus of the discussion should be on part 2, question 2—identifying congruent triangles formed in different types of quadrilaterals by drawing in the diagonals. As students claim two triangles are congruent, ask them to explain the triangle congruence criteria—ASA, SAS or SSS—they used to justify their claim. As time allows, discuss some of the other claims that can be made about the quadrilaterals based on corresponding parts of congruent triangles.

Aligned Ready, Set, Go: Congruence, Construction and Proof 7.5









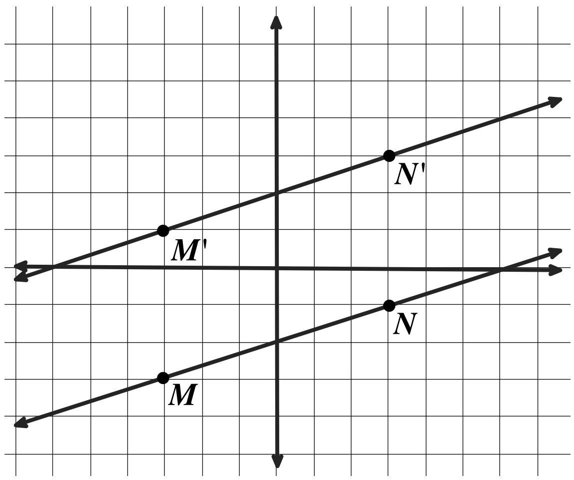
READY, SET, GO! Name Period Date

READY

Topic: Transformations of lines, connecting geometry and algebra.

For each set of lines use the points on the line to determine which line is the image and which is the pre-image, write image by the image line and pre image by the original line. Then define the transformation that was used to create the image. Finally find the equation for each line.

1.

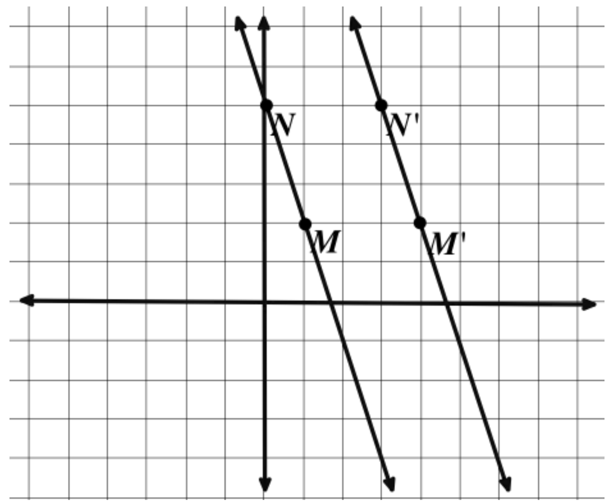


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

2.

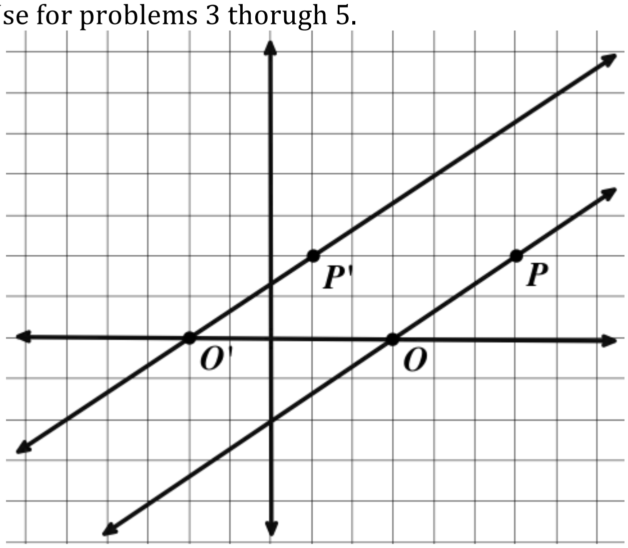


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

Use for problems 3 thorugh 5.



3. a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

4. Write an equation for a line with the same slope that goes through the origin.

5. Write the equation of a line perpendicular to these and though the point **O'.**

After working with these equations and seeing the transformations on the coordinate graph it is good timing to consider similar work with tables.

6. Match the table of values below with the proper function rule.

I

x	*f(x)*
-1	16
0	14
1	12
2	10

II

x	*f(x)*
-1	14
0	12
1	10
2	8

III

x	*f(x)*
-1	12
0	10
1	8
2	6

IV

x	*f(x)*
-1	10
0	8
1	6
2	4

V

x	*f(x)*
-1	8
0	6
1	4
2	2

A. $f(x) = -2(x-1)+8$

B. $f(x) = -2(x-1)+12$

C. $f(x) = -2(x-2)+8$

D. $f(x) = -2(x+1)+8$

E. $f(x) = -2(x+1)+10$

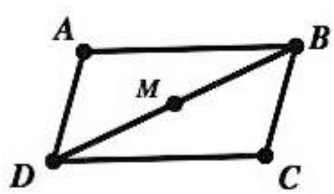
SET

Topic: Use Triangle Congruence Criteria to justify conjectures.

In each problem below there are some true statements listed. From these statements a conjecture (a guess) about what might be true has been made. Using the given statements and conjecture statement create an argument that justifies the conjecture.

7. True statements:

Point M is the midpoint of $\overline{DB}$
$\angle ABD \cong \angle BDC$
$\overline{AB} \cong \overline{DC}$

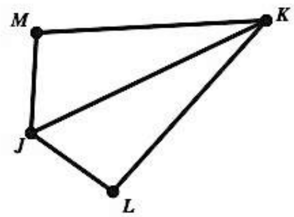


Conjecture: $\angle A \cong \angle C$

a. Is the conjecture correct?

b. Argument to prove you are right:

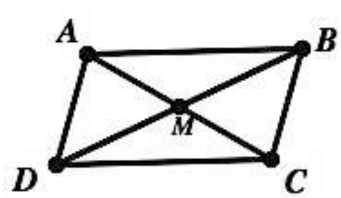
8. True statements
$\angle KJL \cong \angle KJM$
$\overline{JL} \cong \overline{JM}$



Conjecture: $\overline{JK}$ bisects $\angle MKL$
a. Is the conjecture correct?

b. Argument to prove you are right:

9. True statements
ΔADM is a 180°
rotation of ΔCMB



Conjecture: $\Delta ABM \cong \Delta CDM$
a. Is the conjecture correct?

b. Argument to prove you are right:

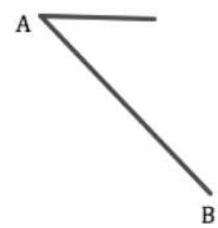
GO

Topic: Constructions with compass and straight edge.

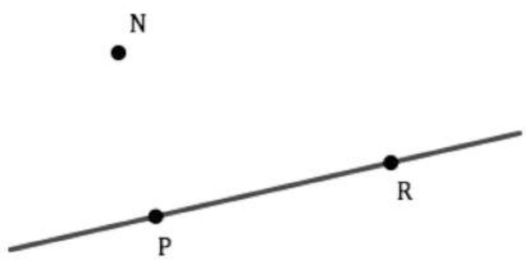
10. Why do we use a geometric compass when doing constructions in geometry?

Perform the indicated constructions using a compass and straight edge.

11. Construct a rhombus, use segment AB as one side and angle A as one of the angles.

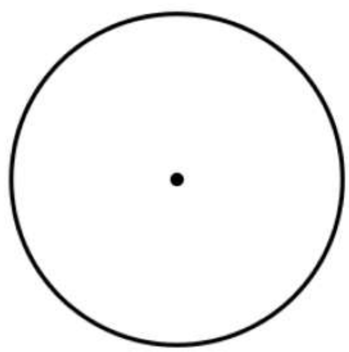


12. Construct a line parallel to line PR and through the point N.

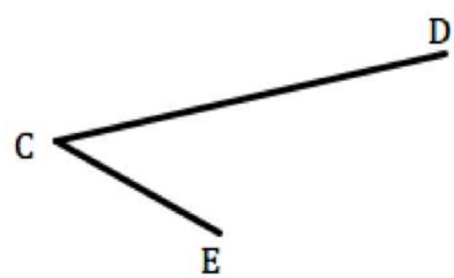


13. Construct an equilateral triangle with segment RS as one side.

14. Construct a regular hexagon inscribed in the circle provided.

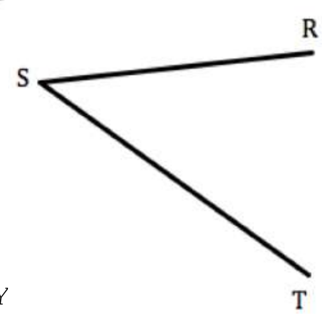


15. Construct a parallelogram using CD as one side and CE as the other side.



16. Bisect the line segment LM.

17. Bisect the angel RST.



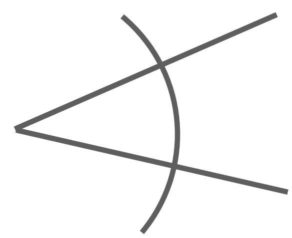
7. 6 Justifying Constructions

A Solidify Understanding Task

Compass and straightedge constructions can be justified using such tools as:

- the definitions and properties of the rigid-motion transformations
- identifying corresponding parts of congruent triangles
- using observations about sides, angles and diagonals of special types of quadrilaterals

Study the steps of the following procedure for *constructing an angle bisector*, and complete the illustration based on the descriptions of the steps.

Steps	**Illustration**
Using a compass, draw an arc (portion of a circle) that intersects each ray of the angle to be bisected, with the center of the arc located at the vertex of the angle.	
Without changing the span of the compass, draw two arcs in the interior of the angle, with the center of the arcs located at the two points where the first arc intersected the rays of the angle.	
With the straightedge, draw a ray from the vertex of the angle through the point where the last two arcs intersect.	

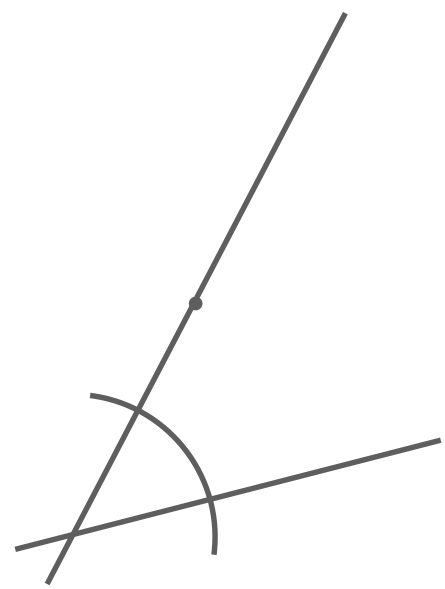
Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Study the steps of the following procedure for *constructing a line perpendicular to a given line through a given point*, and complete the illustration based on the descriptions of the steps.

Steps	**Illustration**
Using a compass, draw an arc (portion of a circle) that intersects the given line at two points, with the center of the arc located at the given point.	
Without changing the span of the compass, locate a second point on the other side of the given line, by drawing two arcs on the same side of the line, with the center of the arcs located at the two points where the first arc intersected the line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Study the steps of the following procedure for constructing *a line parallel to a given line through a given point*, and complete the illustration based on the descriptions of the steps.

Steps	**Illustration**
Using a straightedge, draw a line through the given point to form an arbitrary angle with the given line.	
Using a compass, draw an arc (portion of a circle) that intersects both rays of the angle formed, with the center of the arc located at the point where the drawn line intersects the given line.	
Without changing the span of the compass, draw a second arc on the same side of the drawn line, centered at the given point. The second arc should be as long or longer than the first arc, and should intersect the drawn line.	
Set the span of the compass to match the distance between the two points where the first arc crosses the two lines. Without changing the span of the compass, draw a third arc that intersects the second arc, centered at the point where the second arc intersects the drawn line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

7. 6 Justifying Constructions – Teacher Notes

A Develop Understanding Task

Purpose: In previous tasks students have invented strategies for constructing various polygons: rhombuses, squares, parallelograms, equilateral triangles, and regular hexagons. Embedded within these constructions are the ideas of the standard constructions: copying a segment, copying an angle, bisecting a segment, bisecting an angle, constructing parallel and perpendicular lines through given points. In this task students examine the details of the standard constructions and justify them based on properties of quadrilaterals, corresponding parts of congruent triangles, and the definitions of the rigid-motion transformations.

Core Standards Focus:

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

See also Mathematics I note for G.CO.12, G.CO.13: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Standards for Mathematical Practice of Focus in the Task:

SMP 3 – Construct viable arguments and critique the reasoning of others

SMP 7 – Look for and make use of structure

The Teaching Cycle:

Launch (Whole Class):

Point out that in each chart students have been given an incomplete illustration of the construction steps that are outlined in the first column of the chart. They are to follow the steps to complete the illustration, then write a justification as to why the construction works.

Explore (Small Group):

Monitor students as they complete the illustrations of each construction. Listen for the ways students justify why the construction works. You may need to help students "see" the underlying congruent triangles or quadrilaterals on which the constructions are based. For example, the construction of the angle bisector is based on constructing a rhombus and its diagonal. Since we have conjectured that the diagonal of a rhombus bisects the angles through which it is drawn, we can justify this construction on that basis. Alternatively, we could look for congruent triangles formed by this line, or use this line as a line of reflection, in order to justify this construction in other ways. Select students to present who have developed thoughtful arguments justifying each construction.

Discuss (Whole Class):

Have selected students demonstrate how the steps for the constructions lead to the desired objects, and then justify the construction based on congruent triangles, properties of quadrilaterals, or definitions of transformations.

Aligned Ready, Set, Go: Congruence, Construction and Proof 7.6

READY, SET, GO! Name Period Date

READY

Topic: Rotational symmetry in regular polygons and with transformations.

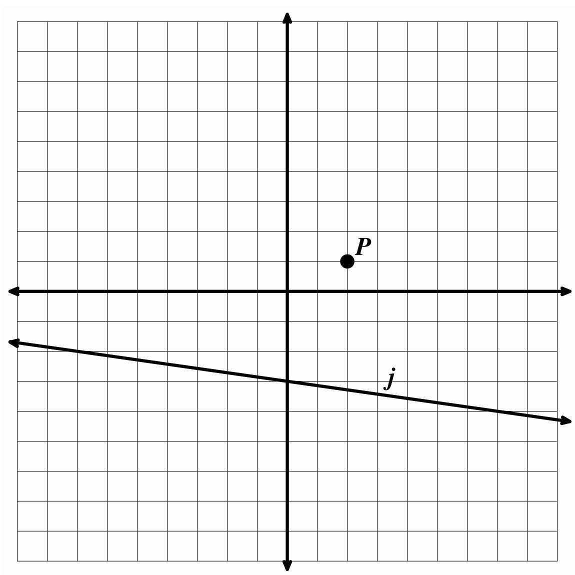
1. What angles of rotational symmetry are there for a regular pentagon?

2. What angles of rotational symmetry are there for a regular hexagon?

3. If a regular polygon has an angle of rotational symmetry that is 40^0, how many sides does the polygon have?

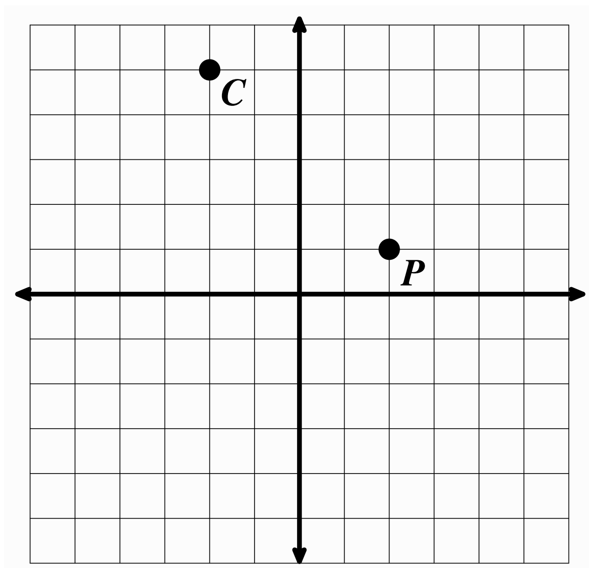
On each given coordinate grid below perform the indicated transformation.

4.



Reflect point ***P*** over line ***j***.

5.



Rotate point ***P*** 90^0 clockwise around point ***C***.

SET

Topic: Use Triangle Congruence Criteria to justify conjectures.

6. Construct an isosceles triangle that incorporates $\overline{CD}$ as one of the sides. Construct the circumscribed circle around the triangle.

7. Construct a regular hexagon that incorporates $\overline{CD}$ as one of the sides. Construct the circumscribed circle around the hexagon.

8. Construct a square that incorporates $\overline{CD}$ as one of the sides. Construct the circumscribed circle around the square.

GO

Topic: Finding Distance and Slope.

For each pair of given coordinate points find distance between them and find the slope of the line that passes through them. Show all your work.

9. (-2, 8) , (3, -4)
   a. Slope: b. Distance:

10. (-7, -3) , (1, 5))
   a. Slope: b. Distance:

11. (3, 7) , (-5, 9)
   a. Slope: b. Distance:

12. (1, -5) (-7, 1)
   a. Slope: b. Distance:

13. (-10, 31) (20, 11)
   a. Slope: b. Distance:

14. (16, -45) (-34, 75)
   a. Slope: b. Distance: