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Transforming Mathematics Education

SECONDARY
MATH TWO

An Integrated Approach

MODULE 1
Quadratic Functions

MODULE 1 - TABLE OF CONTENTS
QUADRATIC FUNCTIONS

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1.6 How Does it Grow – A Practice Understanding Task

Incorporating quadratics with the understandings of linear and exponential functions (F.LE.1, F.LE.2, F.LE.3)

READY, SET, GO Homework: Quadratic Functions 1.6

1.1 Something to Talk About

A *Develop Understanding Task*

Cell phones often indicate the strength of the phone's signal with a series of bars. The logo below shows how this might look for various levels of service.
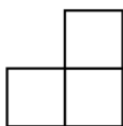

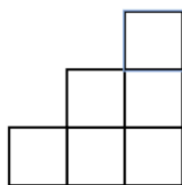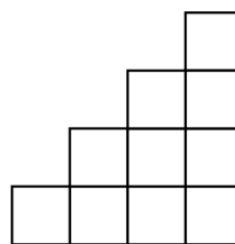Figure 1   Figure 2   Figure 3   Figure 4

1. Assuming the pattern continues, draw the next figure in the sequence.

2. How many blocks will be in the figure 10?

3. Examine the sequence of figures and find a rule or formula for the number of tiles in any figure number.

1.1 Something to Talk About – Teacher Notes
A Develop Understanding Task

Purpose: The purpose of this task is to surface ideas and representations for quadratic functions. The task is designed to elicit tables, graphs, and equations, both recursive and explicit to describe a growing pattern. The classroom discussion will focus on the growth shown in the various representations, developing the idea that quadratic functions show linear rates of change.

Core Standards Focus:

F.BF.1   Write a function that describes a relationship between two quantities. *

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

A.SSE.1   Interpret expressions that represent a quantity in terms of its context.*

A.CED.2   Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

*Focus on situations that exhibit a quadratic or exponential relationship.

The Teaching Cycle:

Launch (Whole Class):

Begin the task by drawing students' attention to the pattern and asking students to draw the next figure in the sequence (question #1). Have a student show figure 5 and talk about what they noticed about the pattern to draw figure 5. Ask students to work on the rest of the task.

Explore (Small Group):

As students are working in their groups, pay attention to ways that they are using the pattern. Watch for students that are using tables to keep track of the growth of the pattern and find the number of blocks in figure 10. Listen for students that are using reasoning about the pattern to predict figure 10. Some

students may be writing various equations, based upon the way they see the pattern growing. If they are noticing the part that is added onto the pattern each time, they may write a recursive equation. Since this is an important representation for the discussion, encourage this thinking, even if the notation isn't entirely correct. If students are paying attention to the relationship between the figure number and the number of blocks in the figure, they may try to write an explicit equation for the pattern. You may expect students to think about the "empty space" or the squares that could be used to complete a rectangle. They may notice that if the figure is copied and rotated, it can be fit upon itself to form a rectangle like so:
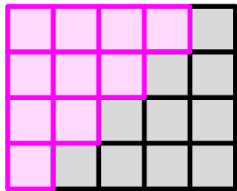
Figure 4

This reasoning may help them to find the explicit formula for the pattern. As students work, encourage as many representations as possible.

Discuss (Whole Class):

When students have finished #2 and had a chance to work on #3 (some students may not have found explicit equations), begin the discussion with the number of squares in figure 10 (problem #2). Begin by having a group that used a table to present their work.

Figure No.	Number of squares	First Difference
1	1	
		2
2	3	
		3
3	6	
		4
4	10	
		5
5	15	
6	21	
7	27	
8	36	
9	45	
10	55	
n		

Ask students what patterns they see in the table. Ask how those same patterns show up in the figures. One important pattern to notice is the first difference, or the change in outputs on each line of the table.

Ask a student that has created a graph to show their representation. An example is shown below. Have a brief discussion of why the points in this graph should not be connected based upon this context.
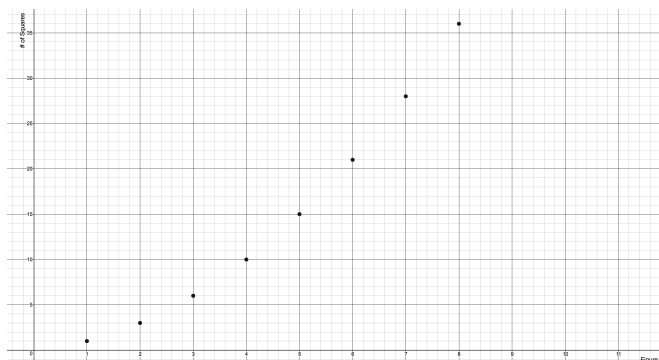
Ask students what they know about the relation based upon the table and graph. Some conclusions that should be drawn:

- The relation is a function.

- The function is discrete.

- The domain and range of the function are both the set of natural numbers.

- The function is not linear because it does not have a constant rate of change.

- The function is not exponential because it does not have a constant ratio between terms.

- The function is always increasing.

- The minimum value of the function is (1,1).

Students may refer to the relationship as a sequence, comparing it to arithmetic and geometric sequences. This idea is appropriate but can be extended to include the broader categories of linear and exponential functions.

Turn the discussion to finding an equation to represent the pattern. Ask students to share their recursive equations. You may wish to start with a student that has used a recursive idea but has not used function notation. The idea is that the function starts at 1, and the nth term can be found by taking the previous term and adding n. Ask students to show how this can be seen in the table, moving from one output row to the next. In function notation, this is represented by:

$$f(1) = 1, f(n) = f(n-1) + n$$

Ask a student how they can use the equation to describe the figures as the pattern goes from figure 4 to figure 5.

It is much harder to use the table to find the explicit equation than the recursive equation. Most students that are successful in writing an explicit formula for the pattern use the visual pattern, thinking about the rectangle made by copying and rotating the figure. Even if they have not written an equation, have a student that considered this strategy to show how they used the drawing for figure 4, as below:

"Using figure 4, I notice that the rectangle formed has width of 5 and a height of 4. When I tried it with other rectangles I saw that the width was always one more than the height."
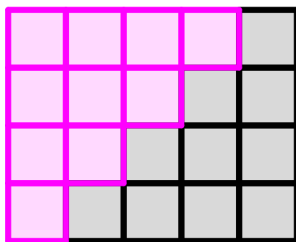
Help students to see that the height of a rectangle formed in this fashion is *n* and the width is *n*+1. The area of the rectangle is $n(n + 1)$, but since the actual figure is only half of the rectangle, the function that describes the number of blocks in each figure is: $f(n) = \frac{1}{2}n(n + 1)$.

Conclude by going back to the table and the first difference. Ask students what type of function appears to be formed by the first difference. This will bring up the second difference, the rate of change of the first difference. In this case the second difference is always 1, which means that the first difference is linear. Tell students that functions with a linear rate of change are called quadratic functions.

In addition, tell students that because the domain is restricted in this case, we are only seeing half the graph. Display the graph of $f(x) = \frac{1}{2}x(x + 1)$, demonstrating the typical quadratic graph. Ask students what they notice about the graph. Ask students why the equation would give a graph that decreases, then increases, and is symmetrical. Students will work extensively with the graphs of quadratic equations in module 2, but it is important that they notice many of the features of the graph as they are developing an understanding of quadratic relationships in module 1.

Aligned Ready, Set, Go: *Quadratic Functions 1.1*

Name   Period   Date

READY

Topic: Distributive Property

Simplify the following expressions

1. $3(2x + 7)$

2. $-12(5x - 4)$

3. $5a(-3a + 13)$

4. $9x(6x - 2)$

5. $\frac{2x}{3}(12x + 18)$

6. $\frac{4a}{5}(10a - 25b)$

7. $\frac{-4x}{11}(121x + 22)$

SET

Topic: Recognizing Linear Exponential and Quadratic Functions

In each set of 3 functions, one will be linear and one will be exponential. One of the three will be a new category of function. List the characteristics in each table that helped you to identify the linear and the exponential functions. What are some characteristics of the new function? Find an explicit and recursive equation for each.

8. Linear, exponential, or a new kind of function?

a.

x	$f(x)$
6	64
7	128
8	256
9	512
10	1024

b.

x	$f(x)$
6	36
7	49
8	64
9	81
10	100

c.

x	$f(x)$
6	11
7	13
8	15
9	17
10	19

Type and characteristics?

Type and characteristics?

Type and characteristics?

Explicit equation:

Explicit equation:

Explicit equation:

Recursive equation:

Recursive equation:

Recursive equation:

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9. Linear, exponential, or a new kind of function?

d.

x	$f(x)$
-2	-17
-1	-12
0	-7
1	-2
2	3

Type and characteristics?

Explicit equation:

Recursive equation:

e.

x	$f(x)$
-2	1/25
-1	1/5
0	1
1	5
2	25

Type and characteristics?

Explicit equation:

Recursive equation:

f.

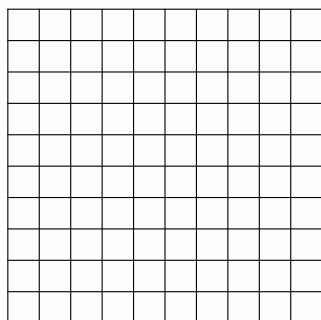| x	$f(x)$
-2	9
-1	6
0	5
1	6
2	9

Type and characteristics?
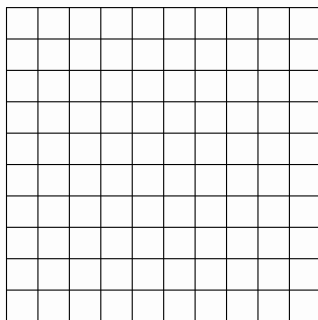
Explicit equation:

Recursive equation:

10. Graph the functions from the tables in #8 and #9. Add any additional characteristics you notice from the graph. Place your axes so that you can show all 5 points. Identify your scale. Write your explicit equation above the graph.
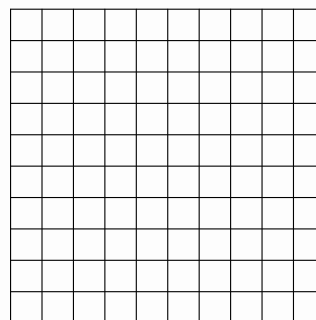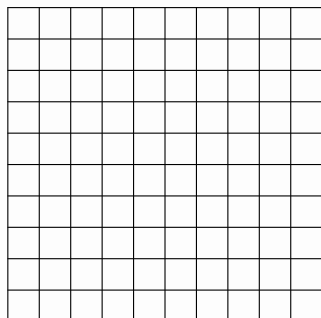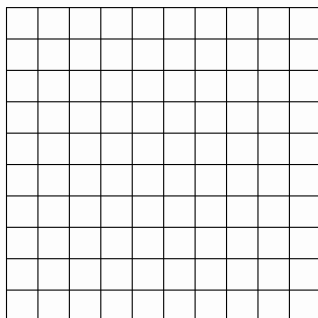
a. Equation:

b. Equation:

c. Equation:

d. Equation:

e. Equation:

f. Equation:

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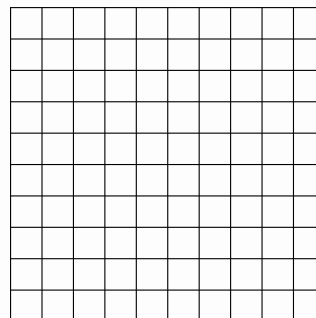
GO

Topic: Rates of Change

Identify the rate of change in each of the representations below.

11.
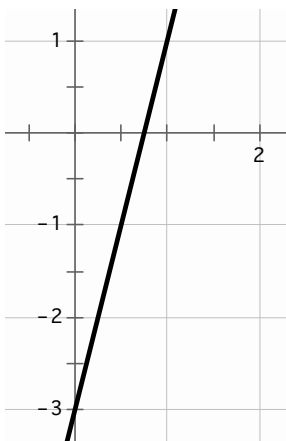
12.
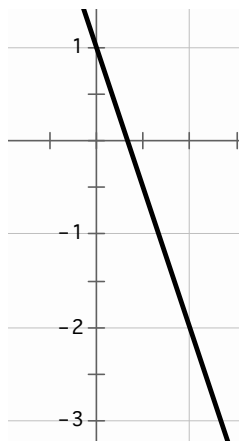
13.

x	f(x)
25	65
26	68
27	71
28	74

14.

$$f(0) = 7; f(n + 1) = f(n) + 5$$

15.
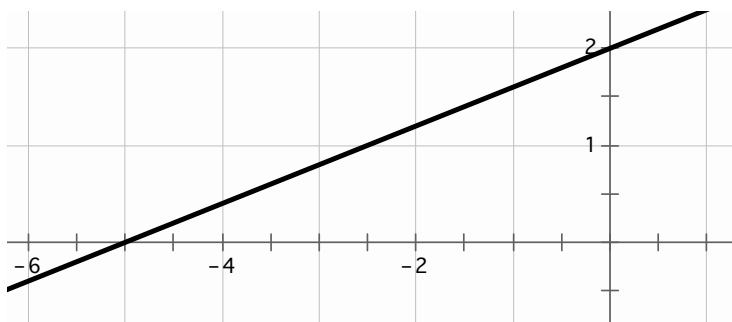
16.

Slope of $\overleftrightarrow{AB}$

A(-3, 12) B(-11, -16)

17. George is loading freight into an elevator. He notices that the weight limit for the elevator is 1000 lbs. He knows that he weighs 210 lbs. He has loaded 15 boxes into the elevator. Each box weighs 50 lbs. Identify the rate of change for this situation.

18.

Independent variable	4	5	6	7	8
Dependent variable	5	5.5	6	6.5	7

19.

$$f(-4) = 24 \ and \ f(6) = -36$$

1.2 I Rule!

A Solidify Understanding Task

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Marco has started a new blog about sports at Imagination High School (mascot: the fighting unicorns) that he has decided to call "I Site". He created a logo for the web site that looks like this:

He is working on creating the logo in various sizes to be placed on different pages on the website. Marco developed the following designs:

Size 1   Size 2   Size 3   Size 4
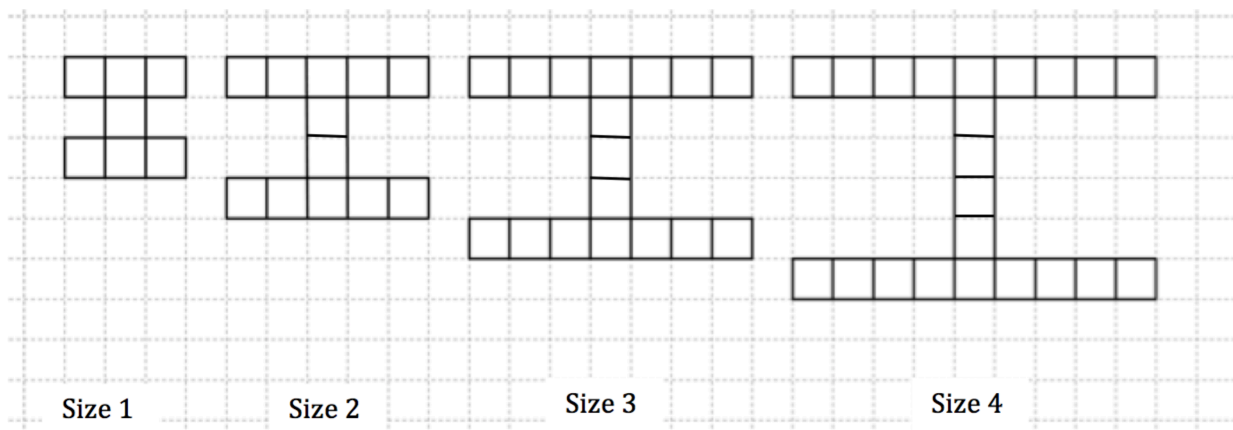
1.   How many squares will be needed to create the size 100 logo?

2.   Develop a mathematical model for the number of squares in the logo for size *n*.

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Marco decides to experiment with making his logo "blockier" so that it looks stronger. Here's what he came up with:

Size 1

Size 2

Size 3
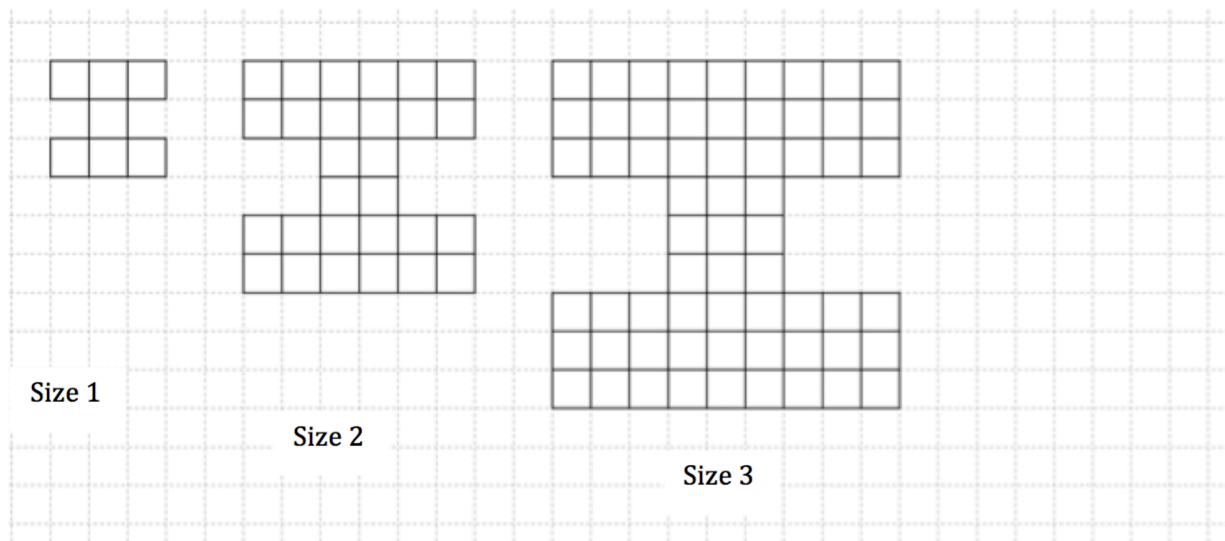
3.   Assuming that Marco continues with the pattern as it has begun, draw the next figure, size 4, and find the number of blocks in the figure.
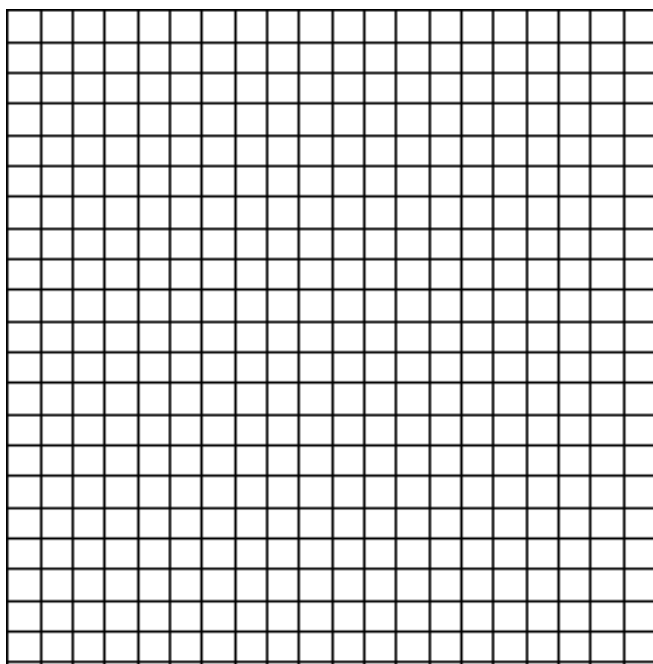
4. Develop a mathematical model for the number of blocks in a logo of size *n*.

5. Compare the models that you developed for the first set of logos to the second set of logos. In what ways are they similar? In what ways are they different?

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7

1.2 I Rule! – Teacher Notes

A *Solidify Understanding Task*

Purpose: The purpose of this task is to solidify students' understanding of quadratic functions and their representations, by providing both an example and a non-example of a quadratic function. The task provides an opportunity for students to compare the growth of linear functions to the growth of quadratic functions. Equations, both recursive and explicit, graphs, and tables are used to describe the relationship between the number of blocks and the figure number in this task.

Core Standards Focus:

A-SE.1 Interpret expressions that represent a quantity in terms of its context.*

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-BF.1 Write a function that describes a relationship between two quantities. *

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

*Focus on situations that exhibit a quadratic or exponential relationship.

Construct and compare linear, quadratic, and exponential models and solve problems.

Compare linear and exponential growth studied in Secondary Mathematics I to quadratic growth.

The Teaching Cycle:

Launch (Whole Class):

Begin by having students read the task and understanding the context. Ask them to compare the two sets of logos in the first and second parts of the task and share what they notice about the mathematical/geometric attributes of the figures. Have students get started on questions 1 and 2.

Explore (Small Group):

Part 1: Monitor students as they work, looking for their use of tables, graphs, and equations. If students are stuck, ask how they see the figures changing. It may be useful to provide colored pencils to help them keep track of what changes and what stays the same in each figure. Encourage students to use as many representations as possible for their mathematical model.
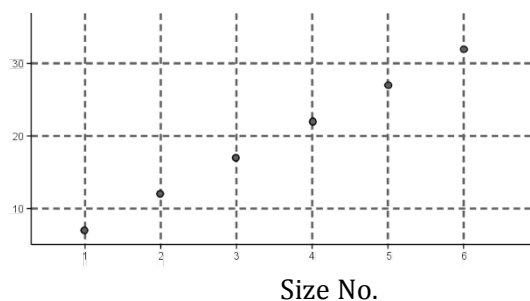
Discuss (Whole Class):

Part 1:

Start the discussion with students presenting a table and a graph. Ask students what type of function this is and how they know. Using the first difference, highlight the ideas that the rate of change is always 5.

Size No.	Number of Squares
1	7
2	12
3	17
4	22
5	27
n	

Number of Squares

Size No.

Ask students to show how they see the change of 5 in each of figures. Then, ask a student to present a recursive equation and connect it to the figures. The recursive formula is:

$$f(1) = 7, f(n) = f(n-1) + 5$$

Ask a student to show how they used the diagram to find an explicit formula. A possible explanation is, "I noticed that for n =2 there were 5 groups of 2 blocks (shown in red) and then 2 more blocks left over. When I tried it on the other sizes it worked the same way so I decided the equation is: $f(n) = 5n + 2$."

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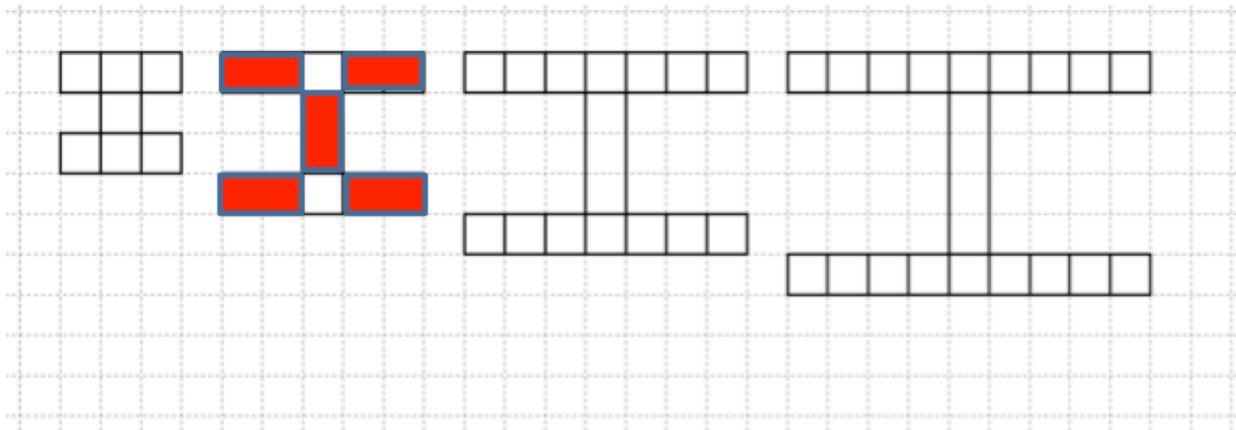
At this point, ask students to work on the second part of the task with the "blockier" logos.

Explore (Small Group):

As you monitor student work, ensure that students are able to correctly draw the next figure. Listen for how they are using the diagram to think about how the figures are changing. Encourage students to use all the representations including getting both an explicit and recursive equation if time permits.

Discuss (Whole Group):

Part 2. Begin the discussion much like Part 1. Have students show a table and graph, making connections to the figures. Direct students to look at the first differences in the table and how they see the growing differences on the graph. Much as in the discussion in "Something to Talk About", help students to see that the first differences are linear, making this a quadratic function.

Size No.	Number of Squares	First Difference	Second Difference
1	7		
		21	
2	28		14
		35	
3	63		14
		49	
4	112		14
		63	
5	175		14
		77	
6	252		
n			

Number of squares

Size number
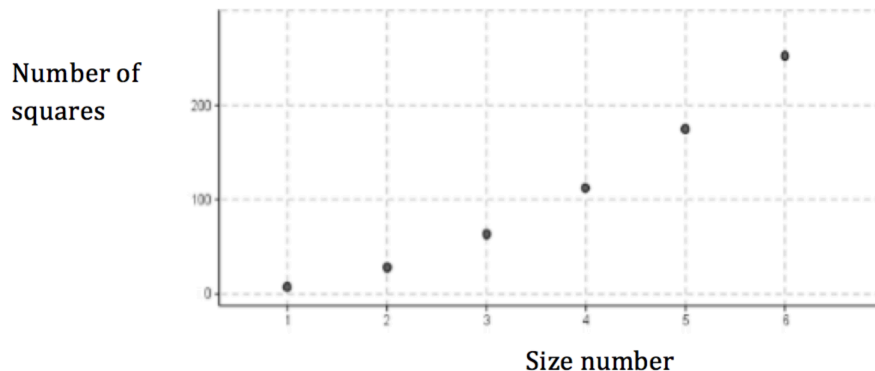
Ask students to present an explicit equation and to show how they used the diagram to get the equation. One possible way to think about it is shown below, with a student saying, "I noticed that each time there are seven large squares. For n = 2, the squares are made up of 4 smaller squares, for n = 3, the squares are made up of 9 smaller squares, and so on, giving the equation: $f(n) = 7n^2$.

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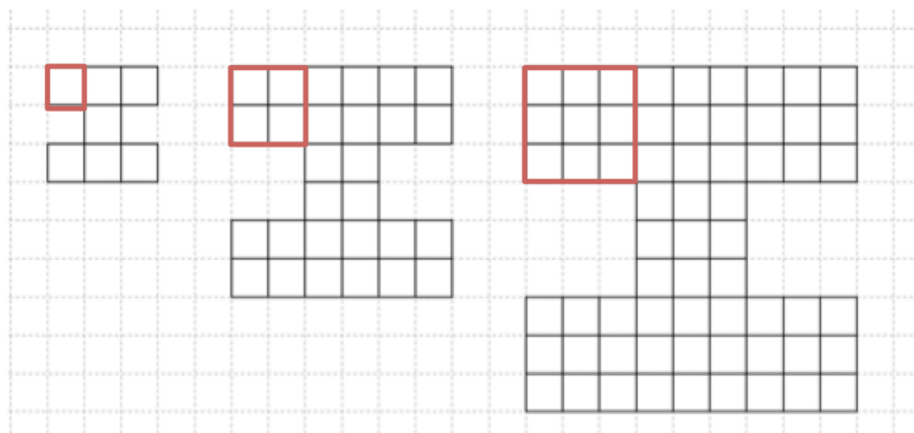
Ask students to share their thinking about a recursive equation. They have used the diagram a similar way, noticing that each time the big squares are wrapped with an "L" shape that adds $2n - 1$ squares each time as shown below:
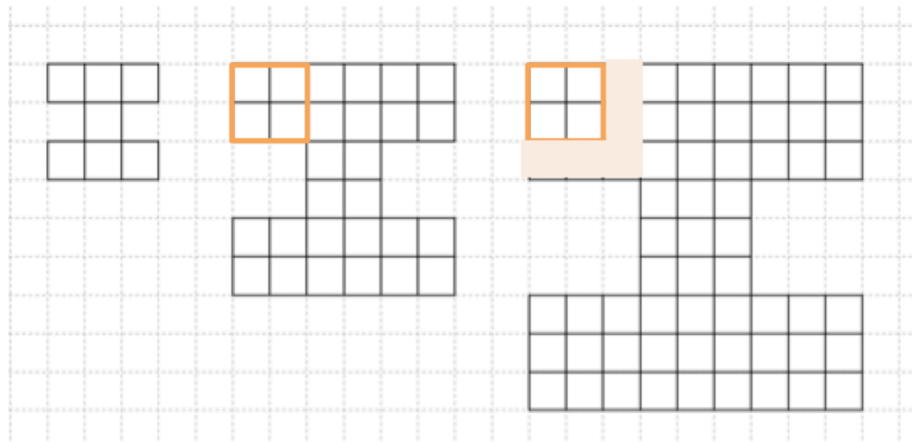
This thinking yields the equation:

$$f(1) = 7, f(n) = f(n-1) + 7(2n - 1)$$

Ask students how they see the rate of change in the table showing up in the recursive equation. They may notice that the part added on to the previous term is the change, which for a quadratic function will be a linear expression.

End the discussion with a comparison of linear and quadratic functions. Lead the class in a discussion to complete a table such as:

	Linear	**Quadratic**
Rate of change	Constant	Linear
Graph	Line	Parabola
Equation	Highest powered term is x	Highest powered term is x^2 term
Table	First difference is constant	First difference is linear, second different is constant.

Aligned Ready, Set, Go: *Quadratic Functions 1.2*

READY

Topic: Distributive Property

Simplify. First use the distributive property and then combine the like terms.

Example:

$$3x(4x + 1) + 2(4x + 1) \rightarrow (12x^2 + 3x) + (8x + 2) \rightarrow 12x^2 + \underset{\text{like terms}}{[3x + 8x]} + 2 \rightarrow \underbrace{12x^2 + 11x + 2}_{\text{Simplified form}}$$

1. $2x(5x + 3) + 7(5x + 3)$

2. $8x(x + 1) + 2(x + 1)$

3. $6x(x - 10) - 1(x - 10)$

4. $1x(3x + 4) + 5(3x + 4)$

5. $3x(8x + 3) - 4(8x + 3)$

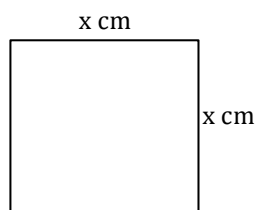6. $5x(2x + 6) + 2(2x + 6)$

7. $7x(-5x + 2) - 13(-5x + 2)$

8. $-4x(12x + 3) + 3(12x + 3)$

SET

Topic: Comparing Area and perimeter

Calculate the area and perimeter of each figure below. The area may be written as a product. Include the correct unit on your answer. (Your answers will contain a variable.)
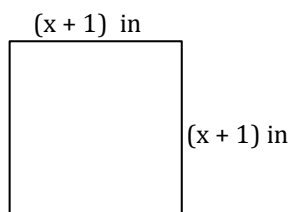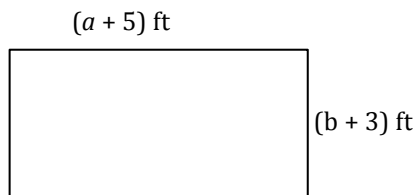
9.

x cm

x cm

a. Perimeter: _____

b. Area: _____

10.

(x + 1) in

(x + 1) in

a. Perimeter: _____

b. Area: _____

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11.

(a + 5) ft

(b + 3) ft

a. Perimeter: _____

b. Area: _____

12.

b mi

a mi
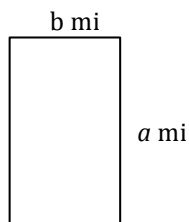
a. Perimeter: _____

b. Area: _____

13.

(x + 3) m

(x – 2) m
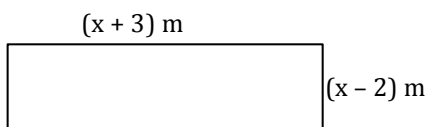
a. Perimeter: _____

b. Area: _____

14.

(x + 4) in

(x + 1) in
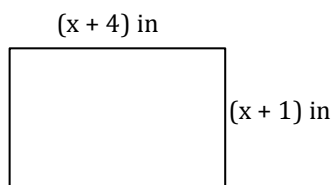
a. Perimeter: _____

b. Area: _____

15.   Compare the perimeter to the area in each of problems (9-14).

 In what way are the numbers and units in the perimeters and areas different?

GO
Topic: Greatest Common Factor

Find the GCF for the given terms.

16. $15abc^2$ and $25a^3bc$

17. $12x^5y$ and $32x^6y$

18. $17pqr$ and $51pqr^3$

19. $7x^2$ and $21x$

20. $6x^2$, $18x$, and -12

21. $4x^2$ and $9x$

22. $11x^2y^2$, $33x^2y$, and $3xy^2$

23. $16a^2b$, $24ab$, and $16b$

24. $49s^2t^2$ and $36s^2t^2$

1.3 Scott's Macho March

A Solidify Understanding Task

CC BY Hector Alejandro
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After looking in the mirror and feeling flabby, Scott decided that he really needed to get in shape. He joined a gym and added push-ups to his daily exercise routine. He started keeping track of the number of push-ups he completed each day in the bar graph below, with day one showing he completed three push-ups. After four days, Scott was certain he could continue this pattern of increasing the number of push-ups for at least a few months.
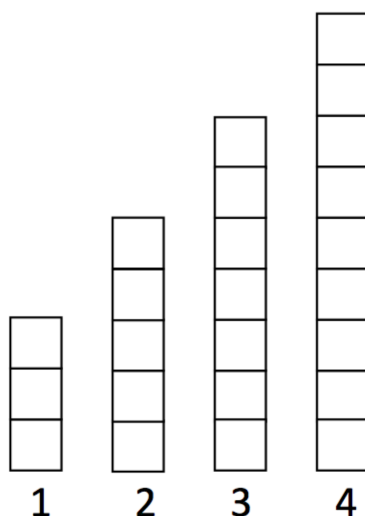
1. Model the number of push-ups Scott will complete on any given day. Include both explicit and recursive equations.

Scott's gym is sponsoring a "Macho March" promotion. The goal of "Macho March" is to raise money for charity by doing push-ups. Scott has decided to participate and has sponsors that will donate money to the charity if he can do a total of at least 500 push-ups, and they will donate an additional $10 for every 100 push-ups he can do beyond that. So now Scott is going to track the total number of push-ups done up to any given day of the month.

2.   Estimate the total number of push-ups that Scott will do in a month if he continues to increase the number of push-ups he does each day in the pattern shown above.

3.   Draw the diagram that shows the total number of pushups that Scot has done in the month at the end of each day.

4.   How many push-ups will Scott have done after a week?

5. Model the total number of push-ups that Scott has completed on any given day during "Macho March". Include both recursive and explicit equations.

6. Will Scott meet his goal and earn the donation for the charity? Will he get a bonus? If so, how much? Explain.

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12

1.3 Scott's Macho March – Teacher Notes
A Solidify Understanding Task

Purpose: The purpose of this task is to solidify student understanding of quadratic functions by giving another opportunity to create a quadratic model for a context. This task introduces the idea that quadratic functions are models for the sum of a linear function, which obviously creates a linear rate of change. Again, students have the opportunity to use algebraic, numeric, and graphical representations to model a story context with a visual model.

Standards Focus:

F-BF.1: Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Construct and compare linear, quadratic and exponential models and solve problems.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function *h(n)* gives the number of person-hours it takes to assemble *n* engines in a factory, then the positive integers would be an appropriate domain for the function.

The Teaching Cycle:

Launch (Whole Class):

Begin the task by clarifying the context to be sure that students understand that in the first part of the task they are looking at the number of pushups that Scott does each day and in the second part

of the task, they are looking at the sum of the number of pushups that Scott has completed on a particular day. Question 1 may be familiar for students that did the "Scott's Workout" task in the previous year.

Explore (Small Group): Monitor student thinking as they work. Since students should be very familiar with representations for linear functions, don't allow any group to spend too much time on question #1. Encourage students to use tables, graphs, and recursive and explicit equations as they work on the task. Listen to students and identify different groups to present and explain their work on one representation each. If students are having difficulty writing the equation, ask them to be sure that they have the other representations first. Like the previous tasks, it will be helpful to use the visual model to develop the explicit equation for the quadratic.

Discuss (Whole Class): When the various groups are prepared to present, start the discussion with a table for problem #1. Be sure that the columns of the table are labeled. After students have presented their table, ask students to identify the difference between consecutive terms and the type of function (linear). Have students present both an explicit and recursive equation and to connect their equations to the geometric representation and the table. Ask students to identify the domain and range of the function.

n Days	$f(n)$ Push-ups
1	3
2	5
3	7
4	9
5	11
...	...
n	$3 + 2(n-1)$

> 2
> 2
> 2

Recursive equation: $f(1) = 3, f(n) = f(n-1) + 2$

Explicit equation: $f(n) = 3 + 2(n-1)$

Briefly discuss questions 2 and 3, focusing on strategies students used to come up with a reasonable estimate in #2. Turn the discussion to question #4. Extend the table presented earlier to include the sum of the pushups.

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n Days	$f(n)$ Push-ups each day	$M(n)$ Total number of pushups in the month	First Difference in M(n)	Second Difference in M(n)
1	3	3		
2	5	8		
3	7	15		
4	9	24		
5	11	35		
...	...			
n	$3 + 2(n-1)$			

Begin to complete the first difference table and ask students what they notice. They should see that the first difference is the same as the number of pushups each day. Ask students why this turns out to be the case. At this point, identify that the function is quadratic, based on the linear rate of change.

Ask students to present a graph.

Total number of pushups

Number of days
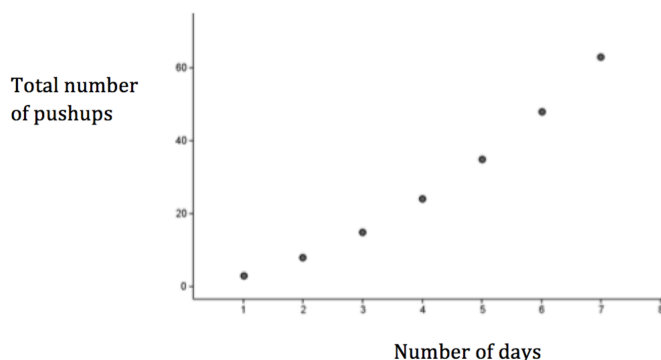
Depending on the scale selected, their graphs may look nearly linear. Connect the graph to the type of growth shown in the table to explain why the graph isn't really linear.

Turn the discussion to the recursive equation. Ask the class to be prepared to connect their equations with the visual model in the problem. Students that have noticed that each figure is composed of the previous figure plus that day's pushups will write an equation similar to:

$$M(1) = 3, M(n) = M(n-1) + (2n+1)$$

Compare the equation for the linear and quadratic function. Ask students to identify similarities and differences. The important thing to notice is that the change in the function is the expression added to the end of the recursive formula. If the change is a single number, as in the linear function, that shows a constant rate of change. If the change is a linear expression, then the function is quadratic.

Have students present how they thought about the visual representation to write an explicit formula. Students may use a strategy similar to "Something to Talk About" by copying and reflecting the figure on top of itself to form a rectangle. In this case, they will get an equation that is: $M(n) = \frac{n[(2n+1)+3)]}{2}$, with the numerator of the fraction representing the area of the rectangle and then dividing by 2 to account for the fact that the actual figure is only half of the area of the rectangle. Other students may have used other strategies to find an explicit equation, but they should all simplify to be $M(n) = n(n + 2)$. Ask students for the domain and range.

Conclude the discussion with a summary of what the class knows about quadratic functions thus far. The discussion should include features such as:

- The first difference is linear and the second difference is constant.
- They are models for the sum of the terms in a linear function.
- They model situations with change in 2 dimensions (like area)
- The equations are the product of 2 linear factors.
- The equations have a squared term.
- The graphs curve. (At this point, students have not experienced a full parabola graph because of the limited domains.)

Aligned Ready, Set, Go: *Quadratic Functions 1.3*

READY

Topic: Multiplying two binomials

In the previous RSG, you were asked to use the distributive property on two different terms in the same problem. Example: $Multiply\ and\ simplify\ 3x(4x + 1) + 2(4x + 1)$.

You may have noticed that the binomial $(4x + 1)$ occurred twice in the problem.

Here is a simpler way to write the same problem: $(3x + 2)(4x + 1)$.

You will use the distributive property twice. First multiply $3x(4x + 1)$; then multiply $+2(4x + 1)$. Add the like terms. Write the x² term first, the x-term second, and the constant term last.

$$3x(4x + 1) + 2(4x + 1) \rightarrow (12x^2 + 3x) + (8x + 2) \rightarrow 12x^2 + [3x + 8x] + 2 \rightarrow \underbrace{12x^2 + 11x + 2}$$
$$\text{like terms}$$
$$\text{Simplified form}$$

Multiply the two binomials. (Your answer should have 3 terms and be in this form $ax^2 + bx + c$.)

1. $(x + 5)(x - 7)$ 2. $(x + 8)(x + 3)$ 3. $(x - 9)(x - 4)$

4. $(x + 1)(x - 4)$ 5. $(3x - 5)(x - 1)$ 6. $(5x - 7)(3x + 1)$

7. $(4x - 2)(8x + 10)$ 8. $(x + 6)(-2x + 5)$ 9. $(8x - 3)(2x - 1)$

SET

Topic: Distinguishing between linear and quadratic patterns

Use first and second differences to identify the pattern in the tables as *linear, quadratic*, or *neithe*r. Write the recursive equation for the patterns that are linear or quadratic.

10.

x	y
-3	-23
-2	-17
-1	-11
0	-5
1	1
2	7
3	13

a. Pattern:
b. Recursive equation:

11.

x	y
-3	4
-2	0
-1	-2
0	-2
1	0
2	4
3	10

a. Pattern:
b. Recursive equation:

12.

x	y
-3	-15
-2	-10
-1	-5
0	0
1	5
2	10
3	15

a. Pattern:
b. Recursive equation:

13.

x	y
-3	24
-2	22
-1	20
0	18
1	16
2	14
3	12

  a. Pattern:

  b. Recursive equation:

14.

x	y
-3	48
-2	22
-1	6
0	0
1	4
2	18
3	42

  a. Pattern:

  b. Recursive equation:

15.

x	y
-3	4
-2	1
-1	0
0	1
1	4
2	9
3	16

  a. Pattern:

  b. Recursive equation:

16.

Figure 1   Figure 2   Figure 3   Figure 4   Figure 5
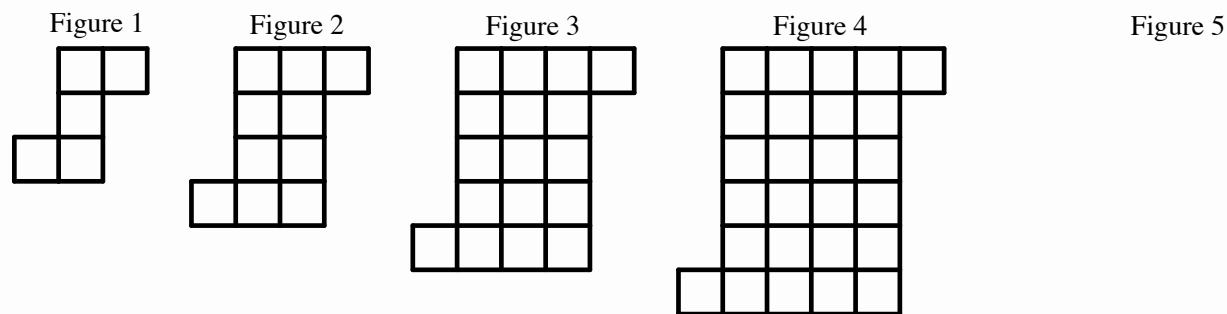
  a. Draw figure 5.

  b. Predict the number of squares in figure 30. Show what you did to get your prediction.

GO

Topic: Interpreting recursive equations to write a sequence

Write the first five terms of the sequence.

17. $f(0) = -5; f(n) = f(n-1) + 8$

18. $f(0) = 24; f(n) = f(n-1) - 5$

19. $f(0) = 25; f(n) = 3f(n-1)$

20. $f(0) = 6; f(n) = 2f(n-1)$

1.4 Rabbit Run

A Solidify Understanding Task

Misha has a new rabbit that she named "Wascal". She wants to build Wascal a pen so that the rabbit has space to move around safely. Misha has purchased a 72 foot roll of fencing to build a rectangular pen.

1. If Misha uses the whole roll of fencing, what are some of the possible dimensions of the pen?

2. If Misha wants a pen with the largest possible area, what dimensions should she use for the sides? Justify your answer.

3. Write a model for the area of the rectangular pen in terms of the length of one side. Include both an equation and a graph.

4. What kind of function is this? How do you know?

5. How does this function compare to the second type of block **I** logos in *I Rule*?

1.4 Rabbit Run – Teacher Notes
A *Solidify Understanding Task*

Purpose: The purpose of this task is to solidify and extend student thinking about quadratic functions to include those with a maximum point. Students will use the graph of the function to discuss the domain and range of a continuous quadratic function in addition to identifying the maximum value and finding the intervals on which the function is increasing and decreasing.

Core Standards Focus:

F.BF.1 Write a function that describes a relationship between two quantities. *

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms factors, and coefficients.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

*Focus on situations that exhibit a quadratic or exponential relationship.

Related Standards: F.IF.4

Note: Graphing technology (either calculators or computer software such as Geogebra or Desmos) would be useful for this task.

The Teaching Cycle:

Launch (Whole Class):

Begin the task by familiarizing students with the context of building a rectangular rabbit pen that can be surrounded by 72 feet of fencing. Ask students to think of some possible dimensions for the pen. Be sure that they are thinking about perimeters, not areas, so that they know that a rectangle with dimensions of 8 x 9 won't work. After they have successfully found one or two possible dimensions for the pen, ask if all the different rectangles will have the same area. Compare the areas of a couple of the rectangles that they have found and tell students that the one of their jobs in

this task will be to find the rectangle with the most area because that will give the rabbit the most room to move around.

Explore (Small Group):

As students are working, circulate to see that they are trying various values for the dimensions of the rectangle that have a perimeter of 72. It may help them to be systematic and organized in thinking about the possibilities for the dimensions so they can use the patterns they observe to find the relationship between the length and width and write the equation for the area.

Discuss (Whole Class):

Start the discussion with a table such as:

Length x	Width	Area $A(x)$
1	35	35
2	34	68
3	33	99
4	32	128
5	31	155
6	30	180
x	$36 - x$	$x(36 - x)$

At this point, it may be useful to display the rest of the table using technology so that students can see a possible maximum value in the table. Students should notice that once they reach a length of 18, they begin to get the same rectangles, but with the length and width switched (different orientation).

Discuss the domain and range of the $A(x)$. This is the first continuous context, which students may not immediately recognize. They should be able to write the domain in interval notation or set builder notation. Ask if this is a quadratic function and how they know. Students should be justifying their answer using the first difference in their tables. This may raise some controversy because the first difference is positive, then negative. Resolve this controversy by separating the table into two sections (at the line of symmetry) and checking for a linear first difference and constant second difference in each section.

Ask a student to present an equation for the function: $A(x) = x(36 - x)$. Ask students how this equation is connected to the table and how it fits what they know about quadratic functions. Ask what is different about this equation compared to other quadratics that they have seen. Since it is important for them to recognize that the x^2 term is negative, you may ask them to distribute the x to see another form of the equation.

Turn the discussion to the graph (using technology). Ask students about the features of the graph that they notice. They should come up with a list like:

- The function is continuous.
- The function increases in the interval (0,18) and decreases in (18,36).
- The function has a maximum at (18, 324).
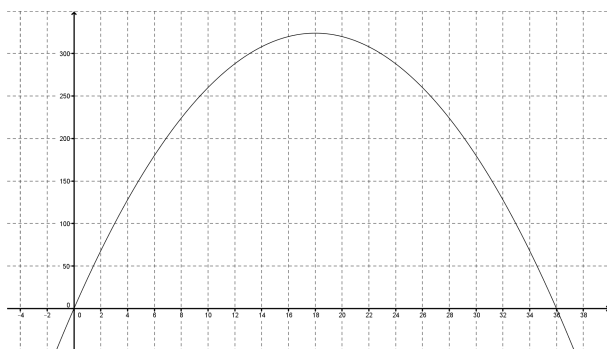
Ask how this graph compares with the graphs of other quadratic functions that they have seen. Since this is the first full parabola they have seen, you will need to tell them that because of the limited domains that were used in previous contexts, they have only seen part of the parabolas that are representative of quadratic functions. This one opens down and has a maximum, graphs of quadratic functions can also open upward and have a minimum.

Aligned Ready, Set, Go: *Quadratic Functions 1.4*

READY

Topic: Applying slope formula

Calculate the slope of the line between the given points. Use your answer to indicate which line is the steepest.

1. A (-3, 7)   B (-5, 17)

2. H (12, -37)   K (4, -3)

3. P (-11, -24)   Q (21, 40)

4. R (55, -75)   W(-15, -40)

SET

Topic: Investigating perimeters and areas

Adam and his brother are responsible for feeding their horses. In the spring and summer the horses graze in an unfenced pasture. The brothers have erected a portable fence to corral the horses in a grazing area. Each day the horses eat all of the grass inside the fence. Then the boys move it to a new area where the grass is long and green. The porta-fence consists of 16 separate pieces of fencing each 10 feet long. The brothers have always arranged the fence in a long rectangle with one length of fence on each end and 7 pieces on each side making the grazing area 700 sq. ft. Adam has learned in his math class that a rectangle can have the same perimeter but different areas. He is beginning to wonder if he can make his daily job easier by rearranging the fence so that the horses have a bigger grazing area. He begins by making a table of values. He lists all of the possible areas of a rectangle with a perimeter of 160 ft., while keeping in mind that he is restricted by the lengths of his fencing units. He realizes that a rectangle that is oriented horizontally in the pasture will cover a different section of grass than one that is oriented vertically. So he is considering the two rectangles as different in his table. Use this information to answer questions 5 – 9 on the next page.

Horizontal   Vertical

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5. Fill in Adam's table with all of the arrangements for the fence. (The first one is done for you.)

	Length in fencing" units	idth in "fencing" units	Length in ft.	Width in ft.	Perimeter (ft)	Area (ft)²
	1 unit	7 units	10 ft	70 ft	160 ft	700 ft²
a.	2 units				160 ft	
b.	3 units				160 ft	
c.	4 units				160 ft	
d.	5 units				160 ft	
e.	6 units				160 ft	
f.	7 units				160 ft	

6. Discuss Adam's findings. Explain how you would rearrange the sections of the porta-fence so that Adam will be able to do less work.

7. Make a graph of Adam's investigation. Let length be the independent variable and area be the dependent variable. Label the scale.

8. What is the shape of your graph?

9. Explain what makes this function be a quadratic.

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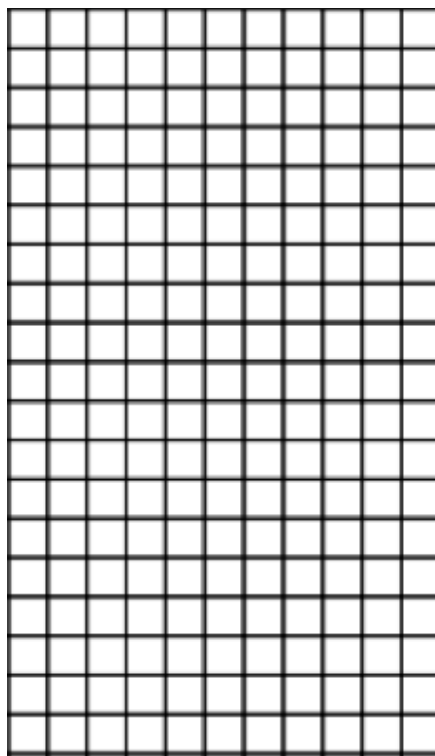
GO

Topic: Comparing linear and exponential rates of change

Indicate which function is changing faster.

10.
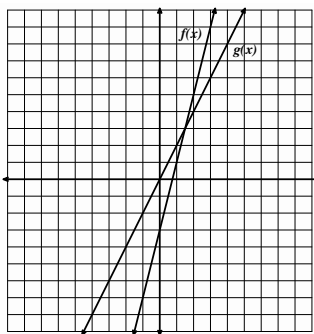
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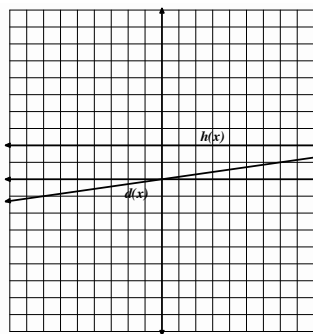
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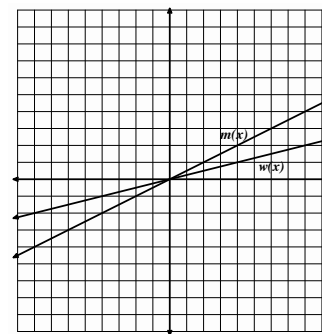
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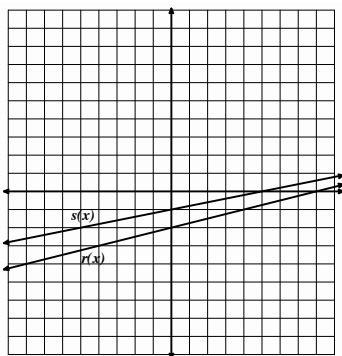
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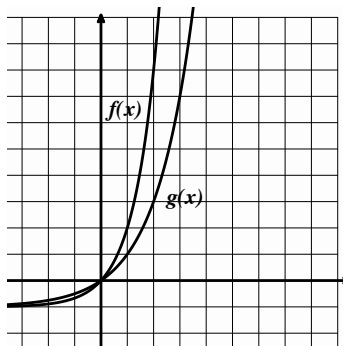
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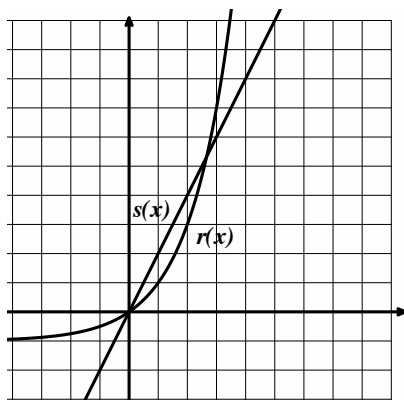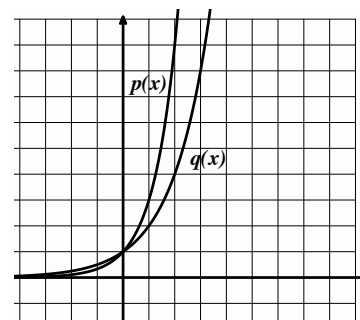
16 a. Examine the graph at the left from 0 to 1.

Which graph do you think is growing faster?

b. Now look at the graph from 2 to 3.

Which graph is growing faster in this interval?

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19

1.5 The Tortoise and The Hare

A Solidify Understanding Task

In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race. The distance from the starting line of the hare is given by the function:

$$d = t^2 \ (d \text{ in meters and } t \text{ in seconds})$$

Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

$$d = 2^t \ (d \text{ in meters and } t \text{ in seconds})$$

1. At what time does the hare catch up to the tortoise?

2. If the racecourse is very long, who wins: the tortoise or the hare? Why?

3. At what time(s) are they tied?

4. If the racecourse were 15 meters long who wins, the tortoise or the hare? Why?

5. Use the properties $d = 2^t$ and $d = t^2$ to explain the **speeds** of the tortoise and the hare in the following time intervals:

Interval	Tortoise $d = 2^t$	Hare $d = t^2$
$[0, 2)$		
$[2, 4)$		
$[4, \infty)$		

1.5 The Tortoise and The Hare – Teacher Notes
A Solidify Understanding Task

Special Note to Teachers: Graphing technology is necessary for this task.

Purpose: The purpose of this task is to compare quadratic and exponential functions by examining tables and graphs for each. They will consider rates of change for each function type in various intervals and ultimately, see that an increasing exponential function will exceed a quadratic function.

Core Standards Focus:

F-BF.1 Write a function that describes a relationship between two quantities. *

      a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.*

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

*Focus on situations that exhibit a quadratic or exponential relationship.

F-LE.3 Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

The Teaching Cycle:

Launch (Whole Class):

Begin the task by simply making sure that students understand the problem situation and the functions that have been given to model the distance traveled by the tortoise and the hare. To avoid confusion, it may be useful to clarify that the hare gave the tortoise a one meter head start, which in this case means that at t = 0, the tortoise was at 1 meter. The equation $d = 2^t$ accounts for the head start because at t = 0, the equation yields 1.

The other possible source of confusion is found in the table in #5. It is asking students to describe the speed (the prompt doesn't indicate average or instantaneous in hopes that students may describe both), not simply the distance traveled in each interval.

Explore (Small Group):

Observe students as they work. The task doesn't prescribe a particular method for thinking about the behavior of each of the two functions, so tables, graphs, and equations should all be expected. It is good mathematical thinking to write an equation by setting the two functions equal to each other, but students may get stuck laboring over an algebraic solution. Encourage them to use their thinking about equality with a different representation. You may need to prompt students to change the windows on their to graphs to display particular pieces of the graphs that are interesting.

Discuss (Whole Class):

Begin the discussion of questions #1 and 3 with a student that wrote an equation that sets the two functions equal. Have the class explain why this strategy makes sense, but tell the class that this is a case where the algebraic solution to the equation is not accessible. Then have a student that used a graph to find the intersections to explain. Have them share how the graphing strategy relates to the equation with the two functions equal that was written. Project a graph of the two functions on the same axes for discussion, possibly beginning with one like this, which is probably where most students started:
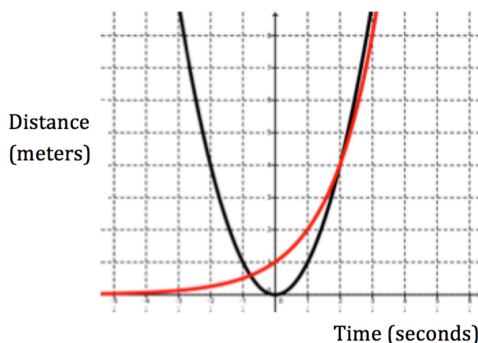
Briefly discuss the domain of the two functions in this context begins at t = 0. On the graph show, the tortoise is in red. Ask students what they notice about the graph, at what time the hare caught

up with the tortoise and who looks like they will win in this view. Ask students which character was the fastest (had the greatest speed or rate of change) in the interval. Since they were both 4 meters from the starting line after 2 seconds, the hare must have been going faster. Students should notice the steeper slope of the hare's graph in this interval. Ask a student that used a table to show how to see the greater speed in the table.

Ask a student to present the two graphs in the interval from t = 2 to t = 4.
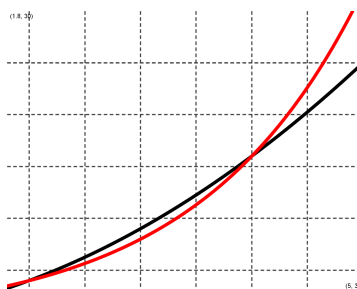
Ask students what they can say about the speed of the hare vs the speed of the tortoise in this interval. Ask them to calculate the average speed of both characters and compare that to what they described earlier. Tell them the difference between instantaneous speed at time t and the average speed in an interval. Finding the instantaneous speed can be done in Calculus (spoiler alert), but it can be estimated by finding the average speed in a small interval near the point of interest. We often use the first difference in a table as an approximation of the instantaneous rate of change, although it is really the average rate of change in the interval from one row of the table to the next. Now ask, which character wins the race and why. Have a student show a graph similar to the one below: (Window $3 < t < 10, 8 < d < 1024$)
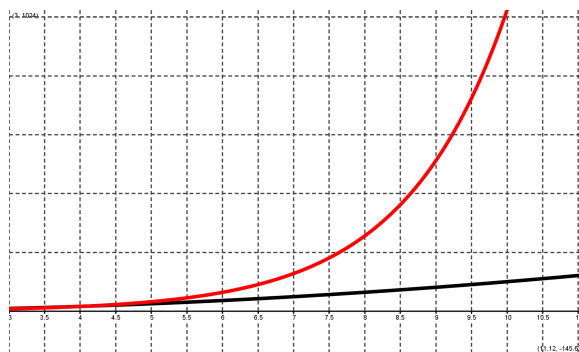
Ask students to discuss the rates of change in this interval. Why does the graph of the quadratic seem so flat compared to the exponential function? Make the point that an increasing exponential function will eventually exceed any quadratic function because the quadratic is increasing linearly, but the exponential is increasing exponentially. Confirm the idea by sharing tables for each of the two functions, looking at the first difference for each.

Exit Ticket for Students: *In the case of the tortoise and the hare, the exponential function exceeded the quadratic function for large values of t. I wonder if this would be the case if the quadratic function were doubled. Compare $y = 2x^2$ and $y = 2^x$ and decide which function grows faster for large values of x. Explain why your answer is correct.*

Aligned Ready, Set, Go: *Quadratic Functions 1.6*

READY, SET, GO!	Name	Period	Date

READY

Topic: Recognizing Functions

Identify which of the following representations are functions. If the representation is NOT a function state how you would fix it so it was.

1. D = {(4,-1) (3, -6) (2, -1) (1, 2) (0, 4) (2, 5)}	2. The number of calories you have burned since midnight at any time during the day.
3.	4. 	x	-12	-8	-6	-4	 |---|---|---|---|---| 	f (x)	25	25	25	25	
5.	6.

For item 4:
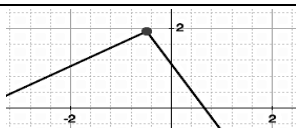
x	-12	-8	-6	-4
f (x)	25	25	25	25
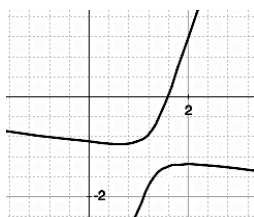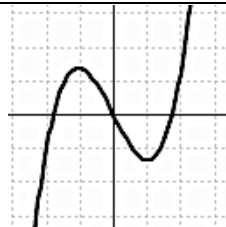
SET

Topic: Comparing rates of change in linear, quadratic, and exponential functions

The graph at the right shows a time vs. distance graph of two cars traveling in the same direction along the freeway.

Figure 1

7. Which car has the cruise control on? How do you know?

8. Which car is accelerating? How do you know?

9. Identify the interval in *figure 1* where car A seems to be going faster than car B.

10. For what interval in *figure 1* does car B seem to be going faster than car A?

11. What in the graph indicates the speed of the cars?

Figure 2

12. A third car *C* is now shown in the graph (*see figure 2*). All 3 cars have the same destination. If the destination is a distance of 12 units from the origin, which car do you predict will arrive first? Justify your answer.

22
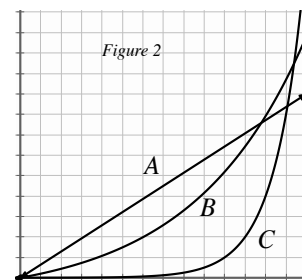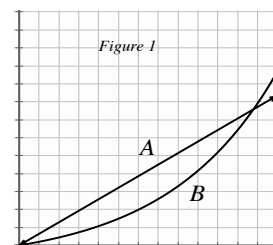
GO

Topic: Identifying domain and range from a graph

State the domain and range of each graph. Use interval notation where appropriate.

13a. Domain _____
   b. Range _____
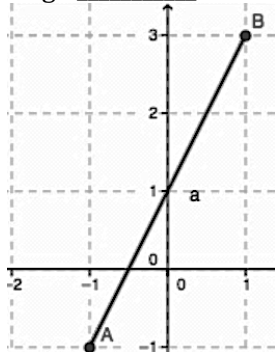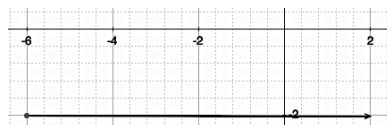
14a. Domain _____
   b. Range _____
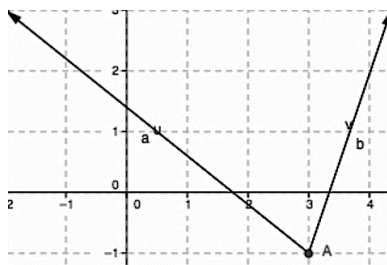
15a. Domain _____
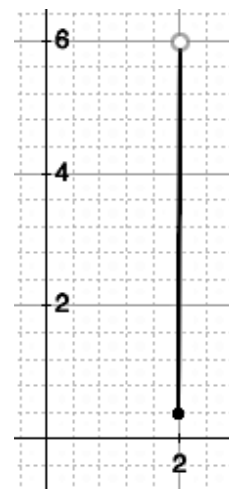   b. Range _____
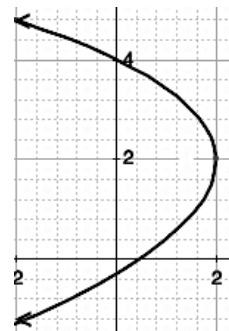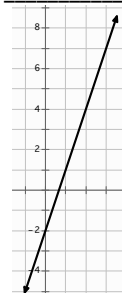
16a. Domain _____
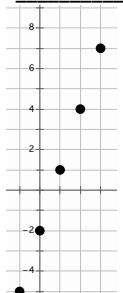   b. Range _____
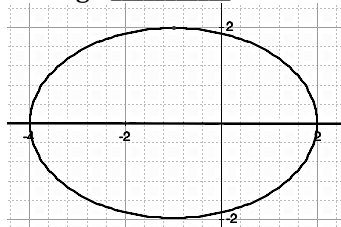
17a. Domain _____
   b. Range _____

18a. Domain _____
   b. Range _____

19a. Domain _____
   b. Range _____

20a. Domain _____
   b. Range _____

21. Are the domains of #19 and #20 the same? Explain.

1.6 How Does It Grow?

A Practice Understanding Task

For each relation given:

a.   Identify whether or not the relation is a function;

b.   Determine if the function is linear, exponential, quadratic or neither;

c.   Describe the type of growth

d.   Create one more representation for the relation.

1.   A plumber charges a base fee of $55 for a service call plus $35 per hour for each hour worked during the service call. The relationship between the total price of the service call and the number of hours worked.

2.
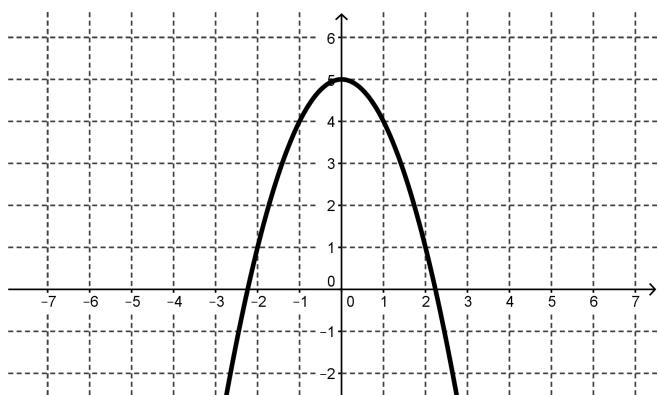
3.
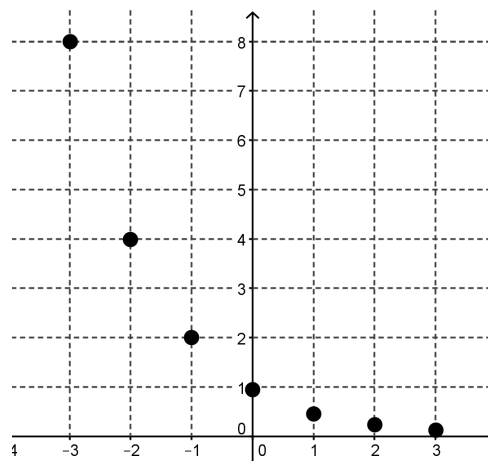
4. $y = \frac{1}{3}(x-2)^2 + 4$

5.
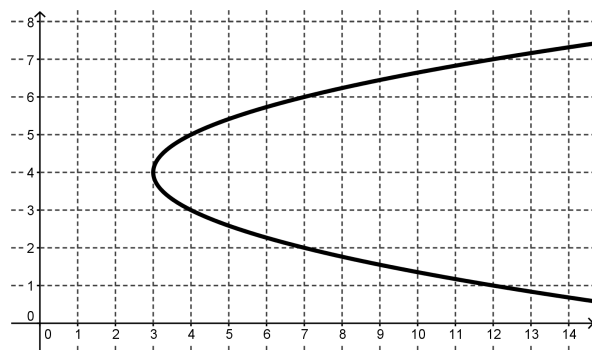
6. $y = \frac{1}{3}(x-2) + 4$

7.   The relationship between the speed of a car and the distance it takes to stop when traveling at that speed.

Speed (mph)	Stopping Distance (ft)
10	12.5
20	50
30	112.5
40	200
50	312.5
60	450
70	612.5

8.   The relationship between the number of dots in the figure and the time, *t.*

t = 1   t = 2   t = 3   t = 4
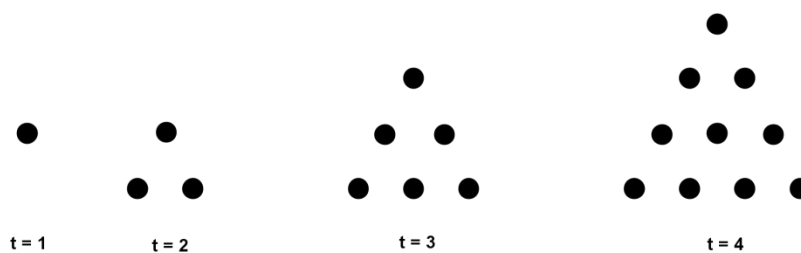
9.   The rate at which caffeine is eliminated from the bloodstream of an adult is about 15% per hour. The relationship between the amount of caffeine in the bloodstream and the number of hours from the time the adult drinks the caffeinated beverage if the initial amount of caffeine in the bloodstream is 500 mg.
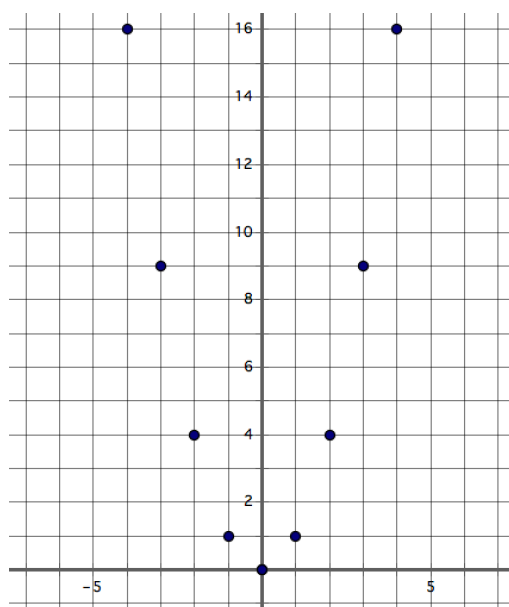
10.

.
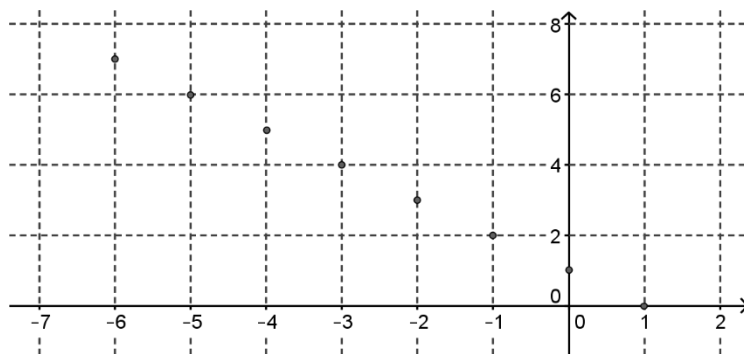
11. $y = (4x + 3)(x - 6)$

12.   Mary Contrary wants to build a rectangular flower garden surrounded by a walkway 4 meters wide. The flower garden will be 6 meters longer than it is wide.

a. The relationship between the width of the garden and the perimeter of the walkway.

b. The relationship between the width of the garden and area of the walkway.

13. $y = \left(\frac{1}{3}\right)^{x-2} + 4$

14.

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28

1.6 How Does It Grow? – Teacher Notes
A Practice Understanding Task

Purpose: The purpose of this task is to refine student understanding of quadratic functions by distinguishing between relationships that are quadratic, linear, exponential or neither. Examples include relationships given with tables, graphs, equations, visuals, and story context. Students are asked to draw upon their understanding of representations to determine the type of change shown and to create a second representation for the relationships given.

Core Standards Focus:

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

   *Focus on situations that exhibit a quadratic or exponential relationship.

F-LE Construct and compare linear, quadratic, and exponential models and solve problems.

Special Note to Teachers: Graphing technology is recommended for this task.

The Teaching Cycle:

Launch (Whole Class):

 To activate student background knowledge, ask students to take the first 3 minutes to write down what they know about linear, quadratic, and exponential functions. Ask students to share a few of their ideas with the class without recording their answers. Go over the instructions for the task to ensure that students know what they are to do.

Explore (Small Group):

Monitor students as they work, looking for problems that are generating discussion, student misconceptions, or important ideas for the discussion. Watch for discussions of whether or not all the various function types can be continuous or discrete. Listen for how they deal with problem #5, which is a parabola, but not a function. (Such cases will be explored in a later module.) Since most of the quadratics that students have worked with so far have domains that begin at 0, students may

not immediately that graphs like #14 are quadratic. (Graphing quadratics is a major topic in module 2). Many of the equations in this task are in unfamiliar forms, which will push students to build tables or graphs to identify the type of change. Graphing technology should be available for student use in graphing functions. The focus for this work should be on rates of change and representations.

Discuss (Whole Class):

Begin the discussion by asking which relationships they classified as quadratic functions. (Answer: 2, 4, 7, 8, 11,12b, and 14). For each of these, have a student show a second representation and talk about how they identified the type of change in their two representations. When this is complete, ask a student to identify one exponential function and share the second representation and their justification in deciding the function is exponential. (Problems 3, 9, and 13 are exponential.) Repeat the process with the linear functions. Close the class by asking students to write for 3 more minutes to add to their list of things they know about linear, quadratic, and exponential functions.

**Aligned Ready, Set, Go:** *Quadratic Functions 1.6*

READY

Topic: Transforming lines

1. Graph the following linear equations on the grid. The equation y = x has been graphed for you. For
   each new equation explain what the number 3 does to the graph of y = x. Pay attention to the
   y-intercept, the x-intercept, and the slope. Identify what changes in the graph and what stays the
   same.

   a.   y = x + 3

   b.   y = x – 3
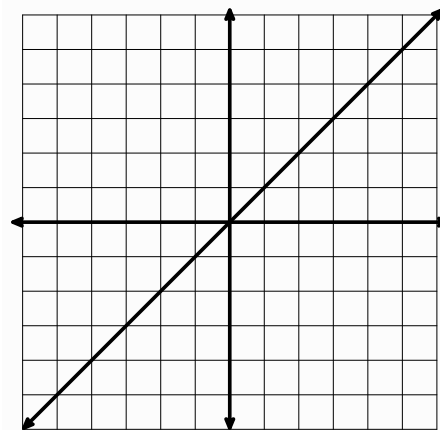
   c.   y = 3x
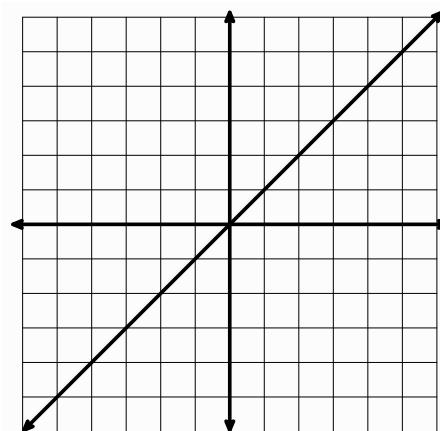
2. The graph of y = x is given. (*See figure 2.*) For each equation predict what you think the number -2
   will do to the graph. Then graph the equation.

   a.   y = x + (-2)
        Prediction:

   b.   y = x – (-2)
        Prediction:

   c.   y = -2x
        Prediction:

SET

Topic: Distinguish between linear, exponential and quadratic functions

For each relation given:

a.   **Identify whether or not the relation is a function. (If it's not a function, skip b – d.)**
b.   **Determine if the function is Linear, Exponential, Quadratic or Neither.**
c.   **Describe the type of growth.**
d.   **Express the relation in the indicated form.**

3. I had 81 freckles on my nose before I began using vanishing cream. After the first week I had 27, the next week 9, then 3 . . .

a. Function?
b. Linear, Exponential, Quadratic or Neither
c. How does it grow?
d. Make a graph. Label your axes and the scale Show all 4 points.
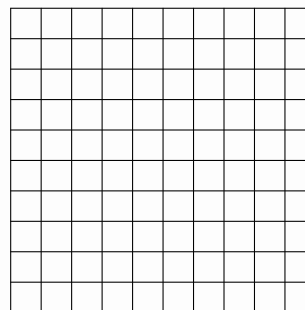
4.

x	y
0	81
1	$80\frac{2}{3}$
2	$80\frac{1}{3}$
3	80
4	$79\frac{2}{3}$

a. Function?
b. Linear, Exponential, Quadratic or Neither
c. How does it grow?
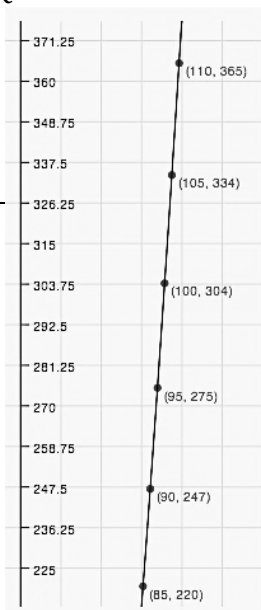d. Write the explicit equation.

5.
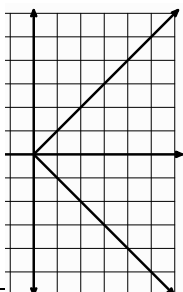
a. Function?
b. Linear, Exponential, Quadratic or Neither
c. How does it grow?
d. Create a table
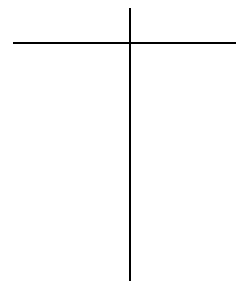
6. Speed in mph of a baseball vs. distance in ft.

a. Function?
b. Linear, Exponential, Quadratic or Neither
c. How does it grow?
d. Predict the distance the baseball flies,
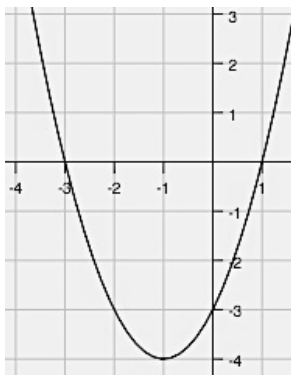   if it leaves the bat at a speed of 115 mph.

371.25
360
348.75
337.5
326.25
315
303.75
292.5
281.25
270
258.75
247.5
236.25
225

(110, 365)
(105, 334)
(100, 304)
(95, 275)
(90, 247)
(85, 220)

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GO

Topic: Matching function representations

Match the function on the left with the equivalent function on the right.
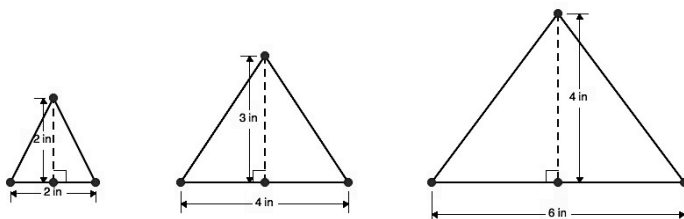
_____ 7. $f(x) = -2x + 5$

a. $f(x) = 5(2)^x$

_____8.

b.

_____9. I put $7000 in a savings account that pays 3% interest compounded annually. I plan to leave it in the bank for 20 years. The amount I will have then.

c. $f(1) = 2;\ f(n+1) = f(n) + 2n + 2$

_____10. The area of the triangles below.
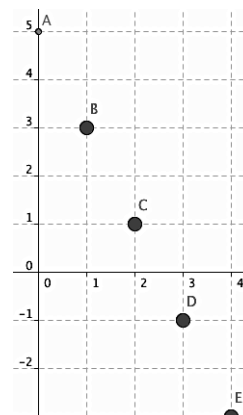
d.

_____11. $f(0) = 5;\ f(n) = 2 * f(n-1)$

e. $y + x = 0$

_____12. $f(0) = 5;\ f(n) = f(n-1) - 2$

f. $y = (x - 1)(x + 3)$

_____13.

x	-7.75	-¼	½	11.6
$f(x)$	7.75	¼	-½	-11.6
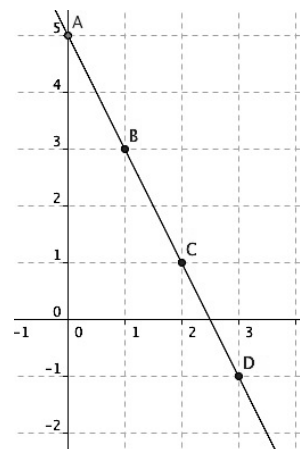
g. $A = 7000(1.03)^{20}$

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