mathematics vision project

Transforming Mathematics Education

SECONDARY MATH TWO

An Integrated Approach

Standard Teacher Notes

MODULE 2

Structures of Expressions

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The Mathematics Vision Project

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2.1 Transformers: Shifty y's

A Develop Understanding Task

Optima Prime is designing a robot quilt for her new grandson. She plans for the robot to have a square face. The amount of fabric that she needs for the face will depend on the area of the face, so Optima decides to model the area of the robot's face mathematically. She knows that the area A of a square with side length x units (which can be inches or centimeters) is modeled by the function, $A(x) = x^2$ square units.

1. What is the domain of the function $A(x)$ in this context?

2. Match each statement about the area to the function that models it:

Matching Equation (A,B, C, or D)	Statement	Function Equation	
	The length of each side is increased by 5 units.	A	$A(x) = 5x^2$
	The length of each side is multiplied by 5 units.	B	$A = (x+5)^2$
	The area of a square is increased by 5 square units.	C	$A = (5x)^2$
	The area of a square is multiplied by 5.	D	$A = x^2 + 5$

Optima started thinking about the graph of $y = x^2$ (in the domain of all real numbers) and wondering about how changes to the equation of the function like adding 5 or multiplying by 5 affect the graph. She decided to make predictions about the effects and then check them out.

3. Predict how the graphs of each of the following equations will be the same or different from the graph of $y = x^2$.

	Similarities to the graph of $y = x^2$	**Differences from the graph of $y = x^2$**
$y = 5x^2$		
$y = (x + 5)^2$		
$y = (5x)^2$		
$y = x^2 + 5$		

4. Optima decided to test her ideas using technology. She thinks that it is always a good idea to start simple, so she decides to go with $y = x^2 + 5$. She graphs it along with $y = x^2$ in the same window. Test it yourself and describe what you find.

5. Knowing that things make a lot more sense with more representations, Optima tries a few more examples like $y = x^2 + 2$ and $y = x^2 - 3$, looking at both a table and a graph for each. What conclusion would you draw about the effect of adding or subtracting a number to $y = x^2$? Carefully record the tables and graphs of these examples in your notebook and explain why your conclusion would be true for any value of k, given, $y = x^2 + k$.

6. After her amazing success with addition in the last problem, Optima decided to look at what happens with addition and subtraction inside the parentheses, or as she says it, "adding to the x before it gets squared". Using your technology, decide the effect of h in the equations: $y = (x + h)^2$ and $y = (x - h)^2$. (Choose some specific numbers for h.) Record a few examples (both tables and graphs) in your notebook and explain why this effect on the graph occurs.

7. Optima thought that #6 was very tricky and hoped that multiplication was going to be more straightforward. She decides to start simple and multiply by -1, so she begins with $y = -x^2$. Predict what the effect is on the graph and then test it. Why does it have this effect?

8. Optima is encouraged because that one was easy. She decides to end her investigation for the day by determining the effect of a multiplier, a, in the equation: $y = ax^2$. Using both positive and negative numbers, fractions and integers, create at least 4 tables and matching graphs to determine the effect of a multiplier.

2.1 Transformers: Shifty y's – Teacher Notes

A Develop Understanding Task

Special Note to Teachers: Graphing technology is required for this task.

Purpose: The purpose of this task is to develop understanding of the effect on the graph of a quadratic function of replacing $f(x)$ by $f(x)\ + k, kf(x), f(kx)$ and $f(x+k)$. The task begins with a brief story context to anchor student thinking about the effect of changing parameters on the graph. Students use technology to investigate the graphs, create tables and generalize about the transformations of quadratic functions.

Core Standards Focus:

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, kf(x), f(kx)$ and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

The Teaching Cycle:

Launch (Whole Class):

Begin the task by asking students how to calculate the area of a square with a side of length 2. Use this short example to introduce the context given at the beginning of the task, and that the area of a square is a quadratic function of the length of a side of the square. Ask students to use the context to think about the answers to question 1 and 2, encouraging the use of drawings to show how the change in parameters actually changes the given square. Give them a short time to work on their own, and then discuss each of the answers, sharing reasoning about whether the change is to the length of the side or directly to area. Help students notice that if 5 is added or multiplied by the length of a side, then the units of the 5 are linear units (like inches or feet). If 5 is multiplied or added to the area, the units of the 5 are square units (like square inches or square feet). It is also useful to notice that if the 5 is "applied" to the length of the side, it is inside the argument of the function. If the 5 is "applied" to the area it is outside the square function.

Matching Equation (A,B, C, or D)	**Statement**	**Function Equation**	
B	The length of any given side is increased by 5 units.	A	$A = 5x^2$
C	The length of any given side is multiplied by 5 units.	B	$A = (x+5)^2$
D	The area of a square is increased by 5 square units.	C	$A = (5x)^2$
A	The area of a square is multiplied by 5.	D	$A = x^2 + 5$

Ask students to make predictions about the changes in the graphs. Although the domain of the area context is $(0,\infty)$, they should think about the entire domain of the function for the remainder of the task. If students have not used the technology that they will be employing for the remainder of the task before, it is important to be sure that they can obtain graphs and find appropriate viewing windows before they proceed.

Explore (Small Group):
Monitor students as they work to ensure that they are able to get appropriate graphs and draw conclusions from the graphs. This is a chance to experiment mathematically, so students should have fun with testing their predictions and explaining their results. Watch for students that are relating the numeric results in the tables to the graphs to help explain why the graphs are transformed as they are. Listen particularly for explanations of why the horizontal shifts are the opposite sign of the parameter so that the reasoning can be used in the discussion.

Discuss (Whole Class):

When students have completed their investigation, go through each question 4-8. Ask students to relate their tables to the graphs and explain why their conclusions make sense in each case. Some important points to highlight for each question are:

Questions 4 and 5: Use the context to bring out the idea that the area (or the result of squaring) is the y-value or height of the graph. It should make sense that adding a number, k, to each area (or y-value) will shift the graph up k units. Ask students what happens if k is negative.

Question 6: Although students will probably be able to see that for $k > 0$, $x + k$ shifts the function to the left and x – k shifts the function to the right, this is generally much harder to explain. Help students to draw upon the tables to compare values and articulate something like, "if you add 5 to the length of the side, then the area that you will get at $x = 1$ is the area that would have been at $x = 6$. That's why adding 5 shifts the graph 5 units to the left." Another easy explanation is that if 5 units are added to x, then everything happens 5 units sooner. So, what would happen at 0 is now happening at -5, what was happening at 1 is now happening 5 units sooner, at -4, and so on. It is also helpful to solidify the common use of the notation $f(x) = (x - k)^2$ to represent both $x + k$ and $x - k$, with the shift depending on the sign of k.

Question 7: This should be an easy one to explain based on either the table or the context. If you change the sign of every output (or multiply by -1), it will create a maximum where the minimum was previously, and change the intervals that were increasing to decreasing and vice-versa.

Question 8: Students will probably notice that multiplying by a whole number makes the parabola "skinnier" and multiplying by a fraction makes the parabola wider. This may seem a little counter-intuitive until they connect to the tables and think that a multiplier of 3 multiplies each output, making the curve decrease and increase three times faster. It will be very useful for later tasks to establish that the graph of the parent function, $y = x^2$ starts at the vertex (0, 0) and then counts: over 1, up 1, over 2, up 4, over 3, up 9 and repeats on the other side of the line of symmetry, x = 0. A multiplier multiplies the outputs, so the graph of $y = 2x^2$ will start at the vertex (0, 0) and count: over 1, up 2, over 2, up 8, over 3 up 18 and so on. Beginning the practice of counting three points on either side of the line of symmetry will help students build fluency in quickly graphing parabolas for later work.

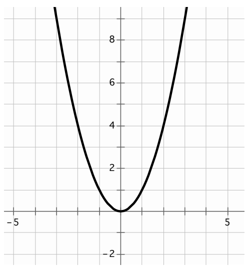
Aligned Ready, Set, Go: ***Quadratic Functions 2.1***

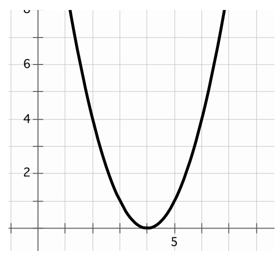
READY, SET, GO! Name Period Date

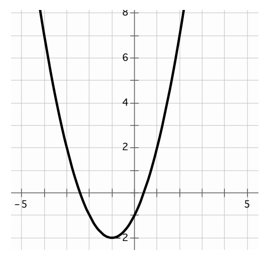
READY

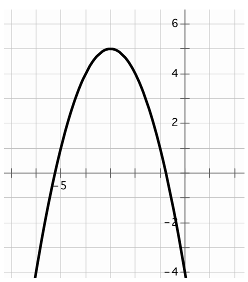
Topic: Finding key features in the graph of a quadratic equation

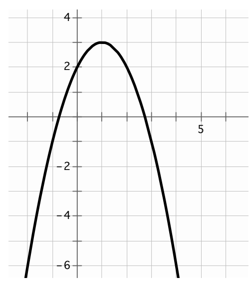
Make a point on the vertex and draw a dotted line for the axis of symmetry. Label the coordinates of the vertex and state whether it's a maximum or a minimum. Write the equation for the axis of symmetry.

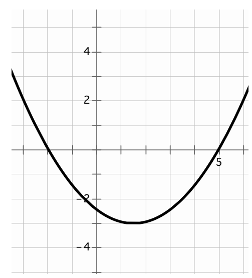
1. 

2. 

3. 

4. 

5. 

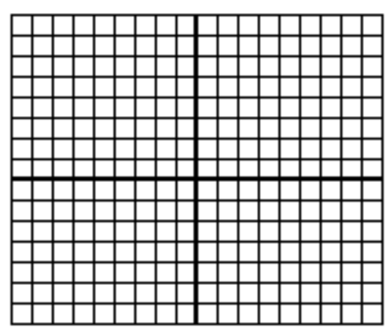
6. 

7. What connection exists between the coordinates of the vertex and the equation of the axis of symmetry?

8. Look back at #6. Try to find a way to find the exact value of the coordinates of the vertex. Test your method with each vertex in 1 - 5. Explain your conjecture.

9. How many x-intercepts can a parabola have?

10. Sketch a parabola that has no x-intercepts, then explain what has to happen for a parabola to have no x-intercepts.



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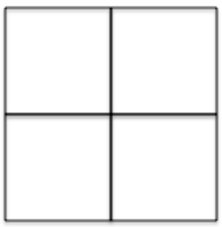
SET

Topic: Transformations on quadratics

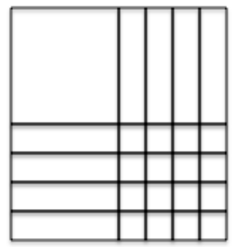
Matching: Choose the area model that is the best match for the equation.

___11. $x^2 + 4$ ___12. $(x + 4)^2$ ___13. $(4x)^2$ ___14. $4x^2$

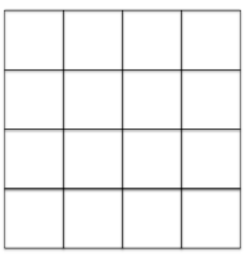
a.



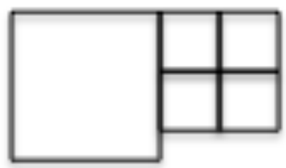
b.



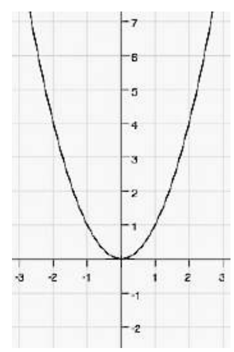
c.



d.



A table of values and the graph for $f(x) = x^2$ is given. Compare the values in the table for g(x) to those for f(x). Identify what stays the same and what changes. a) Use this information to write the vertex form of the equation of g(x). b) Graph g(x). c) Describe how the graph changed from the graph of f(x). Use words such as right, left, up, and down. d) Answer the question.

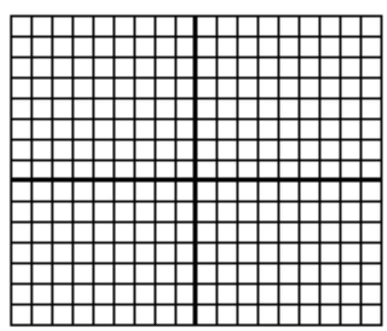


x	-3	-2	-1	0	1	2	3
f(x) = x²	9	4	1	0	1	4	9

15 a) g(x) =

x	-3	-2	-1	0	1	2	3
g(x)	2	-3	-6	-7	-6	-3	2

b)



c) In what way did the graph move?

d) What part of the equation indicates this move?

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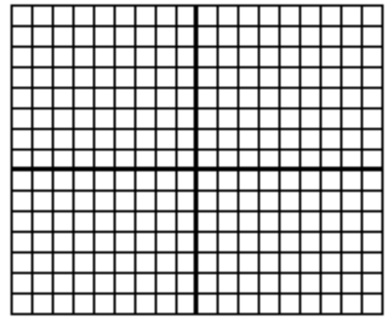
16 a) $g(x) =$

b)



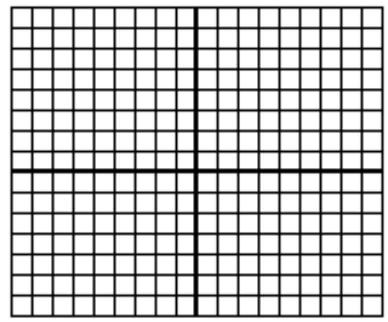
x	-3	-2	-1	0	1	2	3
$g(x)$	11	6	3	2	3	6	11

c) In what way did the graph move?

d) What part of the equation indicates this move?

17 a) $g(x) =$

b)



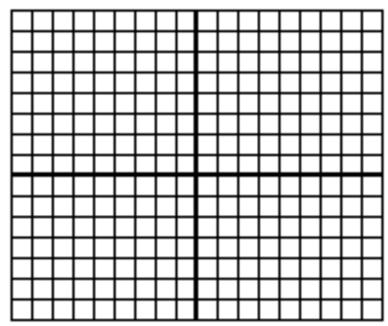
x	-4	-3	-2	-1	0	1	2
$g(x)$	9	4	1	0	1	4	9

c) In what way did the graph move?

d) What part of the equation indicates this move?

18 a) $g(x) =$

b)



x	0	1	2	3	4	5	6
$g(x)$	9	4	1	0	1	4	9

c) In what way did the graph move?

d) What part of the equation indicates this move?

GO

Topic: Finding Square Roots

Simplify the following expressions

19. $\sqrt{49a^2b^6}$

20. $\sqrt{(x+13)^2}$

21. $\sqrt{(x-16)^2}$

22. $\sqrt{(36x+25)^2}$

23. $\sqrt{(11x-7)^2}$

24. $\sqrt{9m^2(2p^3-q)^2}$

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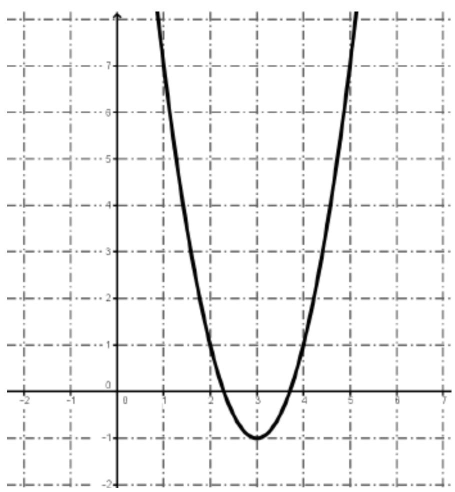
2.2 Transformers: More Than Meets the y's

A Solidify Understanding Task

Write the equation for each problem below. Use a second representation to check your equation.

1. The area of a square with side length x, where the side length is decreased by 3, the area is multiplied by 2 and then 4 square units are added to the area.

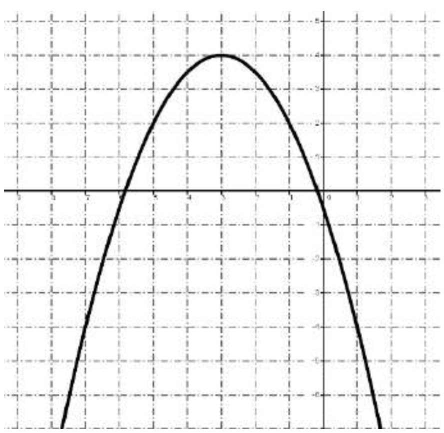
2.



3.

x	***f(x)***
-4	7
-3	2
-2	-1
-1	-2
0	-1
1	2
2	7
3	14
4	23

4.



Graph each equation without using technology. Be sure to have the exact vertex and at least two correct points on either side of the line of symmetry.

5. $f(x) = -x^2 + 3$

6. $g(x) = (x + 2)^2 - 5$

7. $h(x) = 3(x - 1)^2 + 2$

8. Given: $f(x) = a(x - h)^2 + k$
   a. What point is the vertex of the parabola?

   b. What is the equation of the line of symmetry?

   c. How can you tell if the parabola opens up or down?

   d. How do you identify the dilation?

9. Does it matter in which order the transformations are done? Explain why or why not.

2.2 Transformers: More Than Meets the y's – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to extend student understanding of the transformation of quadratic functions to include combinations of vertical stretches, reflections over the *x*-axis, and vertical and horizontal shifts. Students will write equations given story contexts, graphs, and tables. They will use their knowledge of transformations to graph equations and then they will apply their understanding to a general formula for the graph of a quadratic function in vertex form.

Core Standards Focus:

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

F-BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) +k*, *kf(x), f(kx)* and *f(x+k)* for specific values of *k* (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

The Teaching Cycle:

Launch (Whole Class):

Begin class by reminding students of the work they did in "Transformers: Shifty y's". Give the following equations and ask how each equation is a transformation of the parent function, $f(x) = x^2$:

a) $f(x) = x^2 - 3$
b) $f(x) = 3x^2$
c) $f(x) = \frac{1}{3}x^2$
d) $f(x) = (x - 3)^2$

Ask what equation they could write that would reflect the graph over the x-axis and what equation they could write that would shift the graph to the left 3.

Tell students that in the work today they will be combining these transformations and using them to write equations and find the graph of quadratic functions. For questions 1 – 4, they should write an equation and then use the equation to create another representation to check their work. For instance, on #2, they are given a graph. They should write an equation, use it to create a table of values and then check to see that the table and the graph match up.

Explore (Small Group):

Monitor students as they work to be sure that they are entering the problem successfully. Because students are probably not yet solid on the graph of $f(x) = x^2$, they may need to start by creating a table and graph of $f(x)$ and using it to compare to the function given in the problem. This may help them to identify the transformations and write the equation. If they are stuck on problems 5-7, they may need to start with a table. Listen for students that have productive comments to share for the discussion of questions 8 and 9.

Discuss (Whole Class):

Start the discussion with students presenting their work for problems 2 and 3. In each case, have a student identify what transformations have been made to the graph of $f(x) = x^2$ and then explain how they appear in the equation.

Move the discussion to the questions 5, 6, and 7. Have the students that present their work for each problem start by identifying the transformations from the equations and then build the graph. Have at least one of the students show a table along with the graph to demonstrate how the transformations appear in the table, as well as the graph. After each graph is presented, ask the class to identify:

a. The maximum or minimum point of the graph (the vertex):
b. Intervals on which the function is increasing or decreasing.
c. The domain and range of the function.

d. The equation of the line of symmetry.

This will lead students to generalize their experiences with questions 5-7 to answer question #8. Discuss the use of the formula and check for understanding with a couple of examples like: $f(x) = -2(x+3)^2 + 7$.

Turn the discussion to question #9. Use student comments from the exploration to explain that if a function has multiple transformations, they are applied starting from the inside and working outward, in the following order:

1. Horizontal translation
2. Reflection, stretching, shrinking
3. Vertical Translation.

Close the discussion with a quick practice of accurately graphing quadratic functions from the equation. Model quickly identifying the location of the vertex, drawing the line of symmetry, deciding if it opens up or down and then counting the points: over 1, up $1\times a$, over 2, up $4\times a$, over 3, up $9\times a$, etc. Establish a routine of beginning the class period with using this method to quickly and accurately graph a few quadratic functions each day for the next few days to build fluency.

Aligned Ready, Set, Go: *Quadratic Functions 2.2*

READY, SET, GO! Name Period Date

READY

Topic: Standard form of quadratic equations

The standard form of a quadratic equation is defined as $\mathbf{y = ax^2 + bx + c, (a \neq 0)}$.

Identify a, b, and c in the following equations.

Example: Given $\mathbf{4x^2 + 7x - 6}$**, a = 4, b = 7, and c = -6**

1. $y = 5x^2 + 3x + 6$

a = __________
b = __________
c = __________

2. $y = x^2 - 7x + 3$

a = __________
b = __________
c = __________

3. $y = -2x^2 + 3x$

a = __________
b = __________
c = __________

4. $y = 6x^2 - 5$

a = __________
b = __________
c = __________

5. $y = -3x^2 + 4x$

a = __________
b = __________
c = __________

6. $y = 8x^2 - 5x - 2$

a = __________
b = __________
c = __________

Multiply and write each product in the form $\mathbf{y = ax^2 + bx + c}$**. Then identify a, b, and c.**

7. $y = x(x - 4)$

a = __________
b = __________
c = __________

8. $y = (x - 1)(2x - 1)$

a = __________
b = __________
c = __________

9. $y = (3x - 2)(3x + 2)$

a = __________
b = __________
c = __________

10. $y = (x + 6)(x + 6)$

a = __________
b = __________
c = __________

11. $y = (x - 3)^2$

a = __________
b = __________
c = __________

12. $y = -(x + 5)^2$

a = __________
b = __________
c = __________

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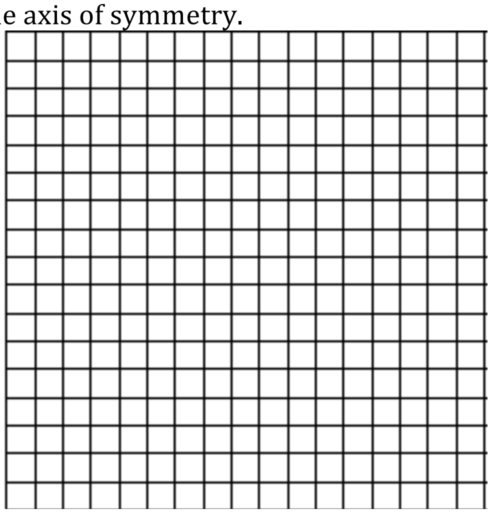
SET

Topic: Graphing a standard $y=x^2$ parabola

13. Graph the equation $y = x^2$.
Include at least 3 accurate points on each side of the axis of symmetry.



a. State the vertex of the parabola.

b. Complete the table of values for $y = x^2$.

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	

Topic: Writing the equation of a transformed parabola in vertex form.

Find a value for ω such that the graph will have the specified number of x-intercepts.

14. $\mathbf{y = x^2 + \omega}$
2 (x-intercepts)

15. $\mathbf{y = x^2 + \omega}$
1 (x-intercept)

16. $\mathbf{y = x^2 + \omega}$
no (x-intercepts)

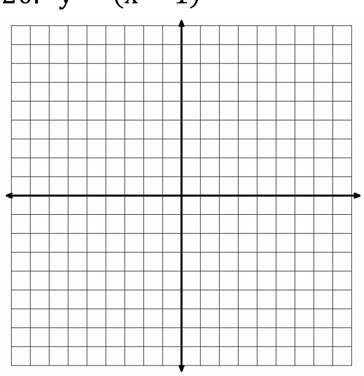
17. $y = -x^2 + \omega$
2 (x-intercepts)

18. $y = -x^2 + \omega$
1 (x-intercept)

19. $y = -x^2 + \omega$
no (x-intercepts)

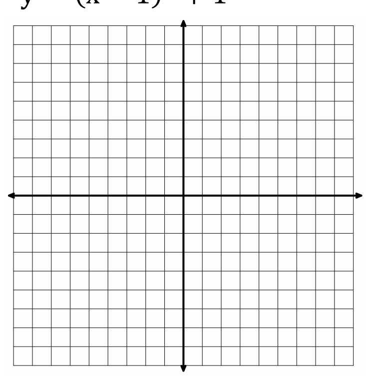
Graph the following equations. State the vertex.
(Be accurate with your key points and shape!)

20. $y = (x - 1)^2$



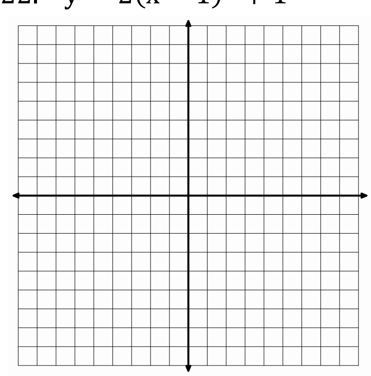
Vertex? ___________________

21. $y = (x - 1)^2 + 1$



Vertex? ___________________

22. $y = 2(x - 1)^2 + 1$



Vertex? ___________________

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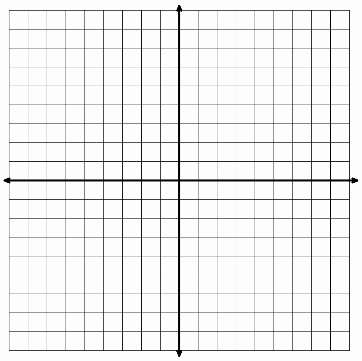
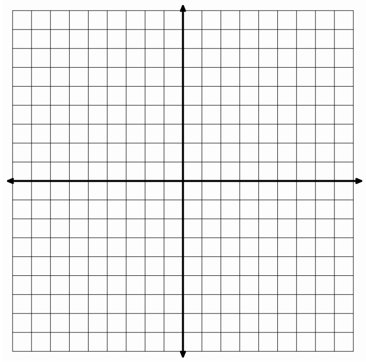
23. $y = (x+3)^2$



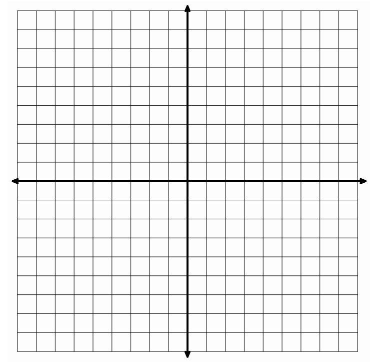
Vertex? ___________________

24. $y = -(x+3)^2 - 4$



Vertex? ___________________

25. $y = -0.5(x+1)^2 + 4$



Vertex? ___________________

GO

Topic: Features of Parabolas

Use the table to identify the vertex, the equation for the axis of symmetry (AoS), and state the number of x-intercept(s) the parabola will have, if any. State whether the vertex will be a *minimum* or a *maximum*.

26.

x	*y*
-4	10
-3	3
-2	-2
-1	-5
0	-6
1	-5
2	-2

a. Vertex: ________

b. AoS: ___________

c. x-int(s): _______

d. MIN or MAX

27.

x	*y*
-2	49
-1	28
0	13
1	4
2	1
3	4
4	13

a. Vertex: ________

b. AoS: ___________

c. x-int(s): _______

d. MIN or MAX

28.

x	*y*
-7	-9
-6	3
-5	7
-4	3
-3	-9
-2	-29
-1	-57

a. Vertex: ________

b. AoS: ___________

c. x-int(s): _______

d. MIN or MAX

29.

x	*y*
-8	-9
-7	-8
-6	-9
-5	-12
-4	-17
-3	-24
-2	-33

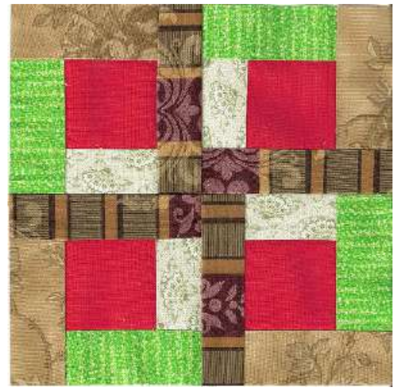
a. Vertex: ________

b. AoS: ___________

c. x-int(s): _______

d. MIN or MAX

2.3 Building the Perfect Square



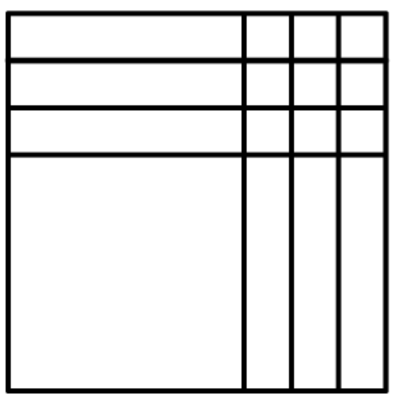
A Develop Understanding Task

Quadratic Quilts

Optima has a quilt shop where she sells many colorful quilt blocks for people who want to make their own quilts. She has quilt designs that are made so that they can be sized to fit any bed. She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. If Optima adds 3 inches to the side of the square, what is the area of the square?

When Optima draws a pattern for the square in problem #1, it looks like this:



2. Use both the diagram and the equation, $A(x) = (x + 3)^2$ to explain why the area of the quilt block square, $A(x)$, is also equal to $x^2 + 6x + 9$.

The customer service representatives at Optima's shop work with customer orders and write up the orders based on the area of the fabric needed for the order. As you can see from problem #2 there are two ways that customers can call in and describe the area of the quilt block. One way describes the length of the sides of the block and the other way describes the areas of each of the four sections of the block.

For each of the following quilt blocks, draw the diagram of the block and write two equivalent equations for the area of the block.

3. Block with side length: $x + 2$.

4. Block with side length: $x + 1$.

5. What patterns do you notice when you relate the diagrams to the two expressions for the area?

6. Optima likes to have her little dog, Clementine, around the shop. One day the dog got a little hungry and started to chew up the orders. When Optima found the orders, one of them was so chewed up that there were only partial expressions for the area remaining. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.

   a. $x^2 + 4x$

   b. $x^2 + 6x$

c. $x^2 + 8x$

d. $x^2 + 12x$

7. If $x^2 + bx + c$ is a perfect square, what is the relationship between b and c? How do you use b to find c, like in problem 6?

Will this strategy work if b is negative? Why or why not?

Will the strategy work if b is an odd number? What happens to c if b is odd?

2.3 Building the Perfect Square – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to develop students' understanding of the procedure of completing the square using area models. In the task, students will use diagrams of area models to make sense of the terms in a perfect square trinomial and discover the relationship between coefficients of a quadratic expression. They will use this understanding to complete the square to find equivalent forms of quadratic expressions. In this task, the quadratic expressions will be limited to those in which the coefficient of the x^2 term is 1. This task is the beginning of a learning cycle that ends in students using the completing the square procedure to find the vertex form of a quadratic function and graph the associated parabola.

Core Standards Focus:

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

The Teaching Cycle:

Launch (Whole Class):

Begin the task by reading through the initial context. Ask students what the area would be if the base square was 1, 2, and 3. Ask them to work on question 1 and check that the class understands that the area function would be $A(x) = (x + 3)^2$. Let students spend a few minutes working individually to answer question #2 and then ask students to explain their thinking. Reinforce the idea that $x + 3$ represents the length of a side, and so the units would be inches. The expression, $(x + 3)^2$, represents an area, and so the units would be square inches. Ask students to test a few numbers for x to verify the squaring relationship with numbers. Use both the equation and a diagram to show that the area model is a way to see the distributive property, both for variables and for numbers.

Explore (Small Group):

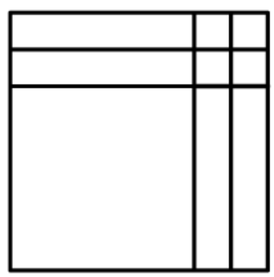
Let students work on drawing the diagrams and completing the task. Ensure that students are connecting the diagrams to the algebraic expressions. Listen for their reasoning and encourage them to make generalizations that can be used in the later part of the task.

Discuss (Whole Class):

Begin with problem #6 and ask a student to present the diagram that he/she made and the equations that he/she wrote. Ask for students to explain how they set up the diagram and especially how they figured out how many small squares to fill in. Label both the sides of the diagram and the different portions of the area. Relate the diagram to the equations.

Diagram and equation(s) for part a):

$$A(x) = (x+2)^2 = x^2 + 4x + 4$$



Depending on how students are responding, you may wish to continue to drawing the diagrams and looking for patterns in the expressions. When students have articulated the idea that the number of blocks in the small square is obtained by taking half of the coefficient of the x term and squaring it, go to question #8. Ask for their generalizations in words, and then be sure that the ideas are recorded algebraically. For instance, if a student says, "To get c, you take half of b and then square it," be sure to model writing:

$$\left(\frac{b}{2}\right)^2$$

Be sure that students are solid on the relationship between b and c before moving on. Tell students that they will be applying these ideas to their work in upcoming tasks.

Aligned Ready, Set, Go: *Quadratic Functions 2.3*

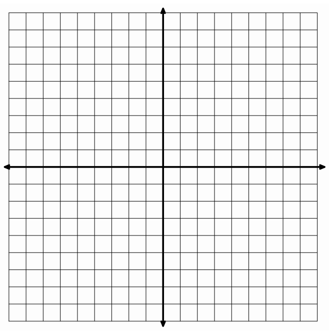
READY, SET, GO! Name Period Date

READY

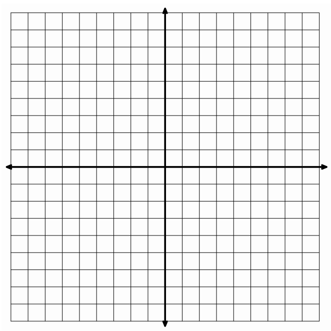
Topic: Graphing lines using the intercepts

Find the x-intercept and the y-intercept. Then graph the equation.

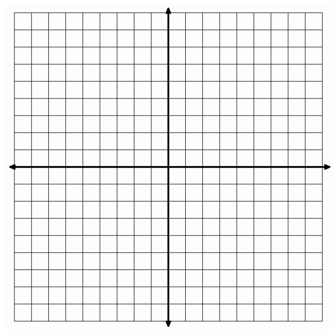
1. $3x + 2y = 12$
   a. x-intercept:
   b. y-intercept:



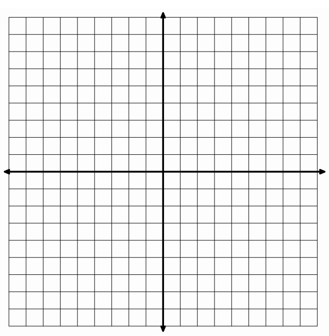
2. $8x - 12y = -24$
   a. x-intercept:
   b. y-intercept:



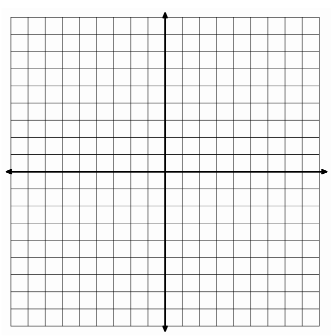
3. $3x - 7y = 21$
   a. x-intercept:
   b. y-intercept:



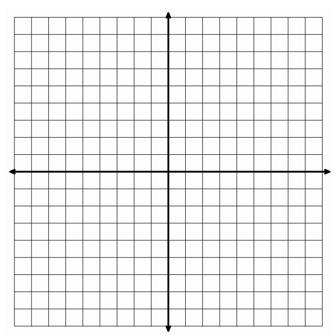
4. $5x - 10y = 20$
   a. x-intercept:
   b. y-intercept:



5. $2y = 6x - 18$
   a. x-intercept:
   b. y-intercept:



6. $y = -6x + 6$
   a. x-intercept:
   b. y-intercept:



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SET

Topic: Completing the square by paying attention to the parts

Multiply. Show each step. Circle the pair of like terms before you simplify to a trinomial.

7. $(x+5)(x+5)$ 8. $(3x+7)(3x+7)$ 9. $(9x+1)^2$ 10. $(4x+11)^2$

11. Write a rule for finding the coefficient "B" of the x-term (the middle term) when multiplying and simplifying $(\mathrm{ax}+\mathrm{q})^2$.

In problems 12 – 17, (a) Fill in the number that completes the square.
(b) Then write the trinomial as the product of two factors.

12. a) $\mathrm{x}^2+8\mathrm{x}+$ ____
 b)

13. a) $\mathrm{x}^2+10\mathrm{x}+$ ____
 b)

14. a) $\mathrm{x}^2+16\mathrm{x}+$ ____
 b)

15. a) $\mathrm{x}^2+6\mathrm{x}+$ ____
 b)

16. a) $\mathrm{x}^2+22\mathrm{x}+$ ____
 b)

17. a) $\mathrm{x}^2+18\mathrm{x}+$ ____
 b)

In problems 18 – 26, (a) Find the value of "B," that will make a perfect square trinomial.
(b) Then write the trinomial as a product of two factors.

18. $\mathrm{x}^2+\mathrm{Bx}+16$
 a)
 b)

19. $\mathrm{x}^2+\mathrm{Bx}+121$
 a)
 b)

20. $\mathrm{x}^2+\mathrm{Bx}+625$
 a)
 b)

21. $\mathrm{x}^2+\mathrm{Bx}+225$
 a)
 b)

22. $\mathrm{x}^2+\mathrm{Bx}+49$
 a)
 b)

23. $\mathrm{x}^2+\mathrm{Bx}+169$
 a)
 b)

24. $\mathrm{x}^2+\mathrm{Bx}+\frac{25}{4}$
 a)
 b)

25. $\mathrm{x}^2+\mathrm{Bx}+\frac{9}{4}$
 a)
 b)

26. $\mathrm{x}^2+\mathrm{Bx}+\frac{49}{4}$
 a)
 b)

GO

Topic: Features of horizontal and vertical lines

Find the intercepts of the graph of each equation. State whether it's an x- or y- intercept.

27. y = -4.5 28. x = 9.5 29. x = -8.2 30. y = 112

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2.4 A Square Deal

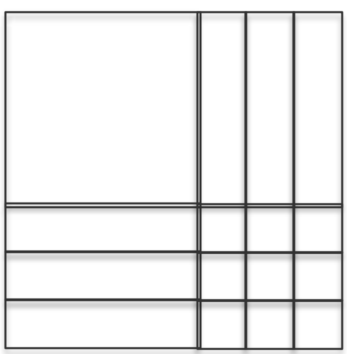
A Solidify Understanding Task

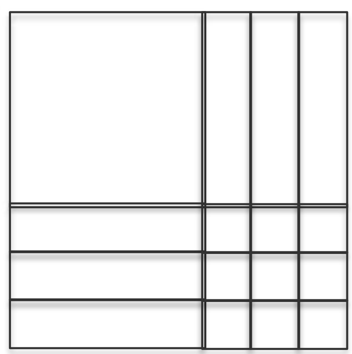
Quadratic Quilts, Revisited

Remember Optima's quilt shop? She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. Sometimes a customer orders more than one quilt block of a given size. For instance, when a customer orders 4 blocks of the basic size, the customer service representatives write up an order for $A(x) = 4x^2$. Model this area expression with a diagram.

2. One of the customer service representatives finds an envelope that contains the blocks pictured below. Write the order that shows two equivalent equations for the area of the blocks.





3. What equations for the area could customer service write if they received an order for 2 blocks that are squares and have both dimensions increased by 1 inch in comparison to the basic block? Write the area equations in two equivalent forms. Verify your algebra using a diagram.

4. If customer service receives an order for 3 blocks that are each squares with both dimensions increased by 2 inches in comparison to the basic block? Again, show 2 different equations for the area and verify your work with a model.

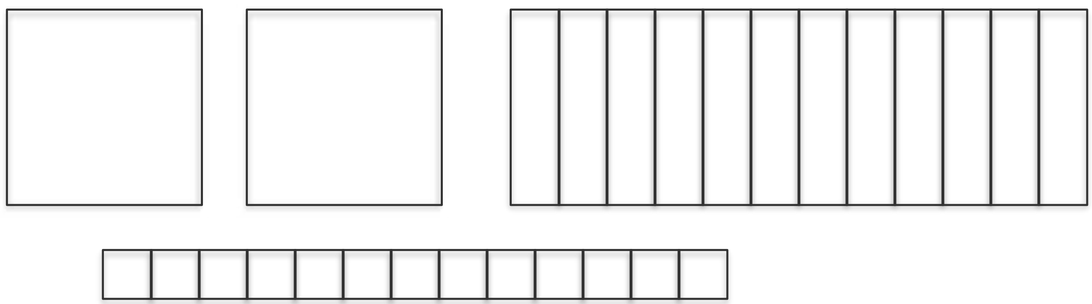
5. Clementine is at it again! When is that dog going to learn not to chew up the orders? (She also chews Optima's shoes, but that's a story for another day.) Here are some of the orders that have been chewed up so they are missing the last term. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.

$2x^2 + 8x$

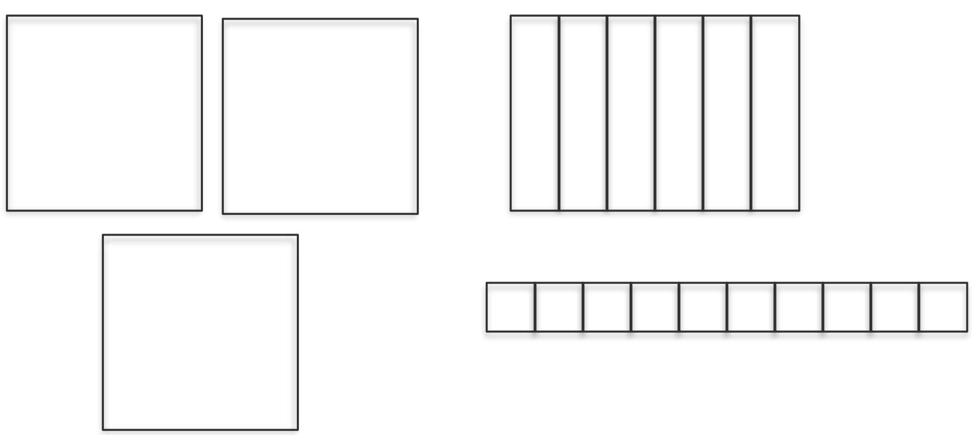
$3x^2 + 24x$

Sometimes the quilt shop gets an order that turns out not to be more or less than a perfect square. Customer service always tries to fill orders with perfect squares, or at least, they start there and then adjust as needed.

6. Now here's a real mess! Customer service received an order for an area $A(x) = 2x^2 + 12x + 13$. Help them to figure out an equivalent expression for the area using the diagram.



7. Optima really needs to get organized. Here's another scrambled diagram. Write two equivalent equations for the area of this diagram:



8. Optima realized that not everyone is in need of perfect squares and not all orders are coming in as expressions that are perfect squares. Determine whether or not each expression below is a perfect square. Explain why the expression is or is not a perfect square. If it is not a perfect square, find the perfect square that seems "closest" to the given expression and show how the perfect square can be adjusted to be the given expression.

A. $A(x) = x^2 + 6x + 13$

B. $A(x) = x^2 - 8x + 16$

C. $A(x) = x^2 - 10x - 3$

D. $A(x) = 2x^2 + 8x + 14$

E. $A(x) = 3x^2 - 30x + 75$

F. $A(x) = 2x^2 - 22x + 11$

9. Now let's generalize. Given an expression in the form $ax^2 + bx + c$ $(a \neq 0)$, describe a step-by-step process for completing the square.

2.4 A Square Deal – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to extend students' understanding of the procedure of completing the square using area models by introducing situations in which the coefficient of the x^2 term is not 1. In the task, students will use area models to represent expressions in the form $ax^2 + bx + c$ and to generalize and apply the process of completing the square to several examples.

Core Standards Focus:

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

The Teaching Cycle:

Launch:

Begin the task by asking students to do questions #1 and #2 individually. Discuss their strategies with the whole class so that all students know how to use the model to represent the algebraic expression when more the one square is involved. Ask students to notice similarities with the work that they have done in "Building a Perfect Square".

Explore:

Monitor students as they are working to see that they are connecting their diagrams to the algebraic expressions. Listen for students that are describing steps in the algebraic procedure of completing the square as they think about the diagrams. As students progress to #6 and #7 make sure that they understand that the blocks given may not be perfect squares and they need to find expressions that show the perfect squares and whatever pieces are additional or missing.

Discuss:

Begin the discussion with students explaining their thinking about #6. Have students show how they re-arranged the blocks in the diagram and how the diagram connects with the algebraic expression. Be sure that students describe the following ideas:

- The coefficient of the x^2 term describes the number of blocks
- The coefficient of the x term has to be divided evenly between the blocks and then split with half going on either side of the x^2 block.
- The small squares are "set aside" until they are used to fill in the square and then the extras or missing squares are counted.

Ask several students to share their work on some of the problems in #8a-f. Help students to generalize the work so that the algebraic procedure is clarified. Discuss #9. By the end of the discussion the class should have arrived at a verbal procedure for completing the square

Aligned Ready, Set, Go: ***Quadratic Functions 2.4***

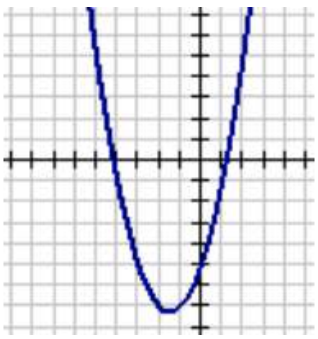
READY, SET, GO! Name Period Date

READY

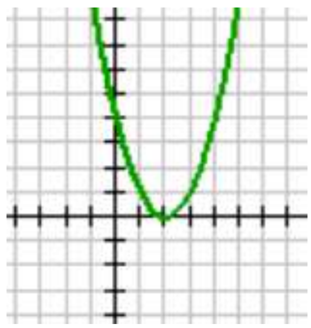
Topic: Find y-intercepts in parabolas

State the y-intercept for each of the graphs. Then match the graph with its equation.

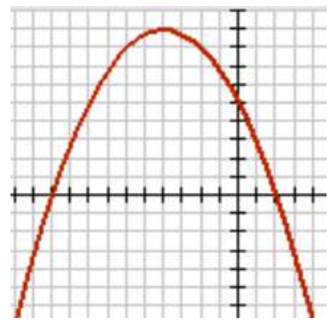
1.



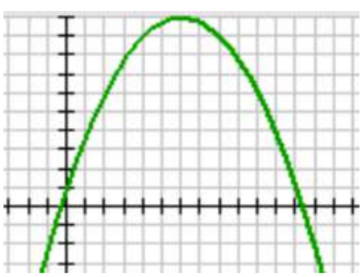
2.



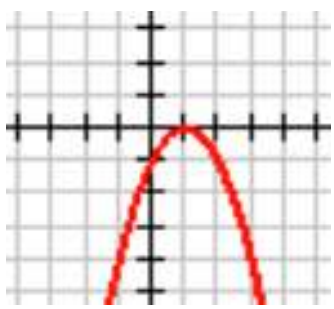
3.



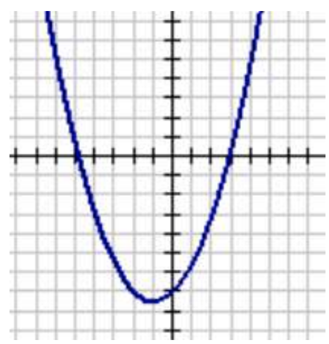
4.



5.



6.



a. $f(x) = -x^2 + 2x - 1$

b. $f(x) = -.25x^2 - 2x + 5$

c. $f(x) = x^2 + 3x - 5$

d. $f(x) = .5x^2 + x - 7$

e. $f(x) = -.25x^2 + 3x + 1$

f. $f(x) = x^2 - 4x + 4$

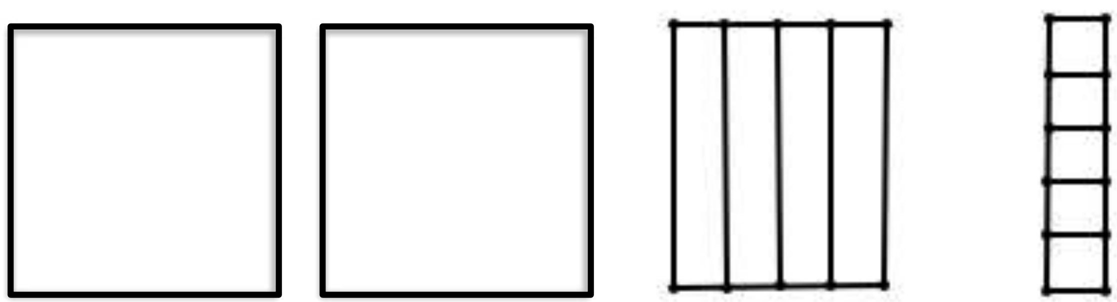
SET

Topic: Completing the square when a >1.

Fill in the missing constant so that each expression represents 5 perfect squares. Then state the dimensions of the squares in each problem.

7. $5x^2 + 30x +$ ______ 8. $5x^2 - 50x +$ ______ 9. $5x^2 + 10x +$ ______

10. Given the scrambled diagram below, write two equivalent equations for the area.



Determine if each expression below is a perfect square or not. If it is not a perfect square, find the perfect square that seems "closest" to the given expression and show how the perfect square can be adjusted to be the given expression.

11. $A(x) = x^2 + 10x + 14$ 12. $A(x) = 2x^2 + 16x + 6$ 13. $A(x) = 3x^2 + 18x - 12$

GO

Topic: Evaluating functions.

Find the indicated function value when $f(x) = 4x^2 - 3x - 25$ *and* $g(x) = -2x^2 + x - 5$.

14. $f(1)$ 15. $f(5)$ 16. $g(10)$ 17. $g(-5)$ 18. $f(0) + g(0)$

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2.5 Be There or Be Square

A Practice Understanding Task

Quilts and Quadratic Graphs

Optima's niece, Jenny works in the shop, taking orders and drawing quilt diagrams. When the shop isn't too busy, Jenny pulls out her math homework and works on it. One day, she is working on graphing parabolas and notices that the equations she is working with looks a lot like an order for a quilt block. For instance, Jenny is supposed to graph the equation: $y = (x-3)^2 + 4$. She thinks, "That's funny. This would be an order where the length of the standard square is reduced by 3 and then we add a little piece of fabric that has as area of 4. We don't usually get orders like that, but it still makes sense. I better get back to thinking about parabolas. Hmmm…"

1. Fully describe the parabola that Jenny has been assigned to graph.

2. Jenny returns to her homework, which is about graphing quadratic functions. Much to her dismay, she finds that she has been given: $y = x^2 - 6x + 9$. "Oh dear", thinks Jenny. "I can't tell where the vertex is or identify any of the transformations of the parabola in this form. Now what am I supposed to do?"

   "Wait a minute—is this the area of a perfect square?" Use your work from *Building the Perfect Square* to answer Jenny's question and justify your answer.

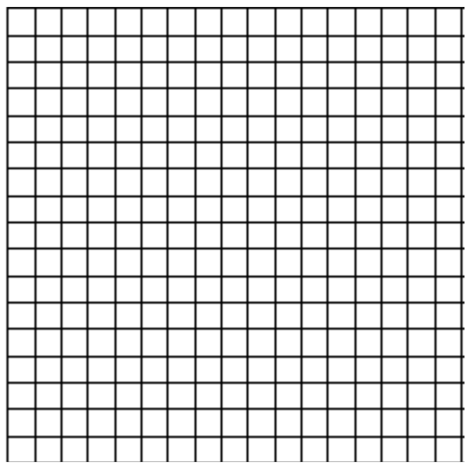
3. Jenny says, "I think I've figured out how to change the form of my quadratic equation so that I can graph the parabola. I'll check to see if I can make my equation a perfect square." Jenny's equation is: $y = x^2 - 6x + 9$.

See if you can change the form of the equation, find the vertex, and graph the parabola.

a. $y = x^2 - 6x + 9$ New form of the equation: ___________________________________

b. Vertex of the parabola: ______________

c. Graph (with at least 3 accurate points on each side of the line of symmetry):

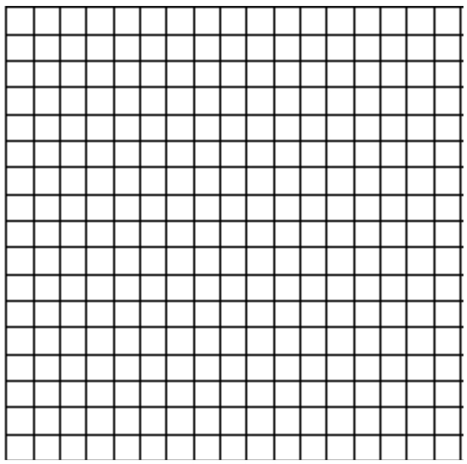


4. The next quadratic to graph on Jenny's homework is $y = x^2 + 4x + 2$. Does this expression fit the pattern for a perfect square? Why or why not?

a. Use an area model to figure out how to complete the square so that the equation can be written in vertex form, $y = a(x - h)^2 + k$.

b. Is the equation you have written equivalent to the original equation? If not, what adjustments need to be made? Why?

c. Identify the vertex and graph the parabola with three accurate points on both sides of the line of symmetry.

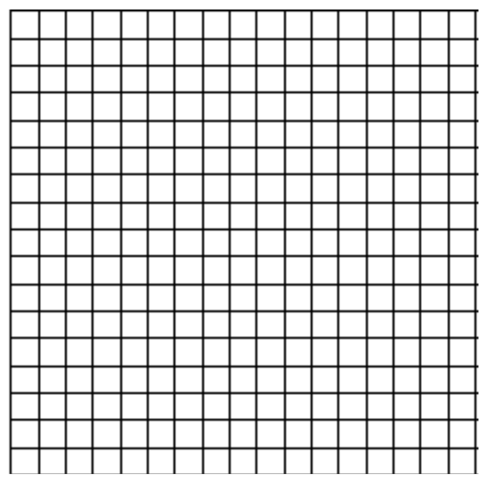


5. Jenny hoped that she wasn't going to need to figure out how to complete the square on an equation where b is an odd number. Of course, that was the next problem. Help Jenny to find the vertex of the parabola for this quadratic function:

$$g(x) = x^2 + 7x + 10$$

6. Don't worry if you had to think hard about #5. Jenny has to do a couple more:
   a. $g(x) = x^2 - 5x + 3$
   b. $g(x) = x^2 - x - 5$

7. It just gets better! Help Jenny find the vertex and graph the parabola for the quadratic function: $h(x) = 2x^2 - 12x + 17$



8. This one is just too cute—you've got to try it! Find the vertex and describe the parabola that is the graph of: $f(x) = \frac{1}{2}x^2 + 2x - 3$

2.5 Be There or Be Square – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is for students to connect the completing the square procedure learned in the previous two tasks to graphing parabolas. The task asks students to complete the square to change the equation of a quadratic function in standard form into vertex form. Students will need to extend their understanding of completing the square in an expression to an equation to maintain the equality, requiring that they add and subtract an equivalent term to one side of the equation (effectively adding zero) or that they add the same thing to both sides of the equation. After getting the equation into vertex form, students graph the equation of the parabola.

Core Standards Focus:
F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

The Teaching Cycle:

Launch:

Begin the task by asking students to work individually on questions #1 and #2. These questions are designed to activate students' prior knowledge of perfect square trinomials and to make the connection between graphing parabolas in vertex form. Students should be able to quickly name the vertex and describe the parabola for #1 based on the work they did in tasks 2.1 and 2.2.

On question #2, it is important that the pattern for the relationship among terms in a perfect square is described so that students can draw upon it to work the rest of the task.

Explore:

Monitor students as they work to see that they are drawing upon their previous work to complete the square and change the form of the equation. Watch for when students get to #4 that they are checking to see that they are maintaining the equality by either adding the same thing to both sides of the equation or adding and subtracting an equivalent expression to one side of the equation.

Identify students that have used these strategies to "maintain the balance of the equation" so that the strategies can be discussed and compared.

Discuss:

Focus the discussion on problem 4. Ask a student to demonstrate how they drew the diagram and knew that they needed 2 more small squares to complete the square. Of course, adding 2 squares changes the value of the expression, so adjustments need to be made. This is a very important point to talk through with the class, getting several explanations from students. They will probably have different strategies for accounting for this change (described in the "Explore" section) and it is worth discussing and comparing them. Identify the properties of equality that are the basis for each of the strategies. Quickly graph the parabola and go on to problem 5.

Ask a student to present his/her work on problem 5. The important part to talk through is how they managed to divide 7 by 2 and then square it. Some students may have used decimals, but fractions should be encouraged so that the expressions remain exact values. (Decimals could be exact values too, but students tend to round numbers off.)

Proceed with student demonstrations of the remaining problem with discussions of how to work through the algebraic details.

Move the discussion with #7. Ask a student to present that has used a diagram, then ask another student to demonstrate how they worked algebraically. Ask students to articulate connections between the diagram and the algebraic work.

Aligned Ready, Set, Go: ***Quadratic Functions 2.5***

READY, SET, GO! Name Period Date

READY

Topic: Recognizing Quadratic Equations

Identify whether or not each equation represents a quadratic function. Explain how you knew it was a quadratic.

1. $x^2 + 13x - 4 = 0$

Quadratic or no?

Justification:

2. $3x^2 + x = 3x^2 - 2$

Quadratic or no?

Justification:

3. $x(4x - 5) = 0$

Quadratic or no?

Justification:

4. $(2x - 7) + 6x = 10$

Quadratic or no?

Justification:

5. $2^x + 6 = 0$

Quadratic or no?

Justification:

6. $32 = 4x^2$

Quadratic or no?

Justification:

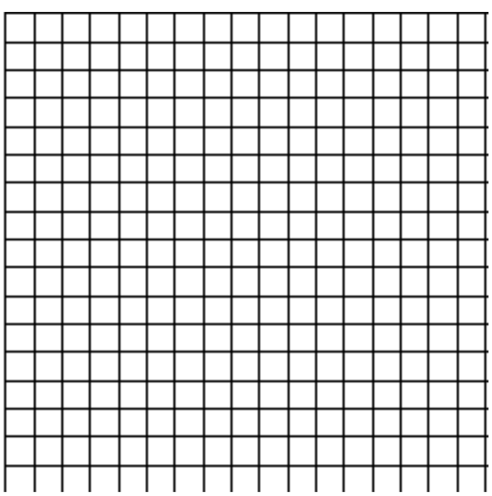
SET

Topic: Changing from standard form of a quadratic to vertex form.

Change the form of each equation to vertex form: $y = a(x - h)^2 + k$. State the vertex and graph the parabola. Show at least 3 accurate points on each side of the line of symmetry.

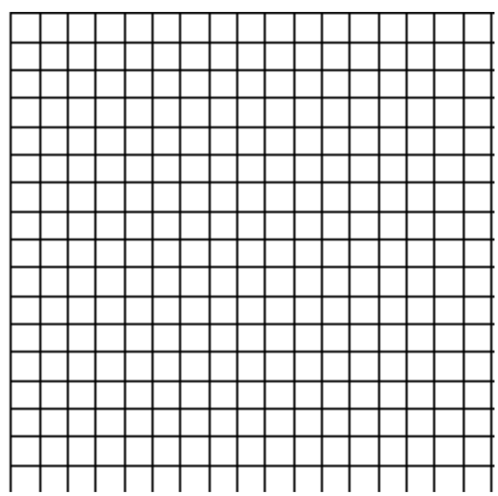
7. $y = x^2 - 4x + 1$

vertex:



8. $y = x^2 + 2x + 5$

vertex:



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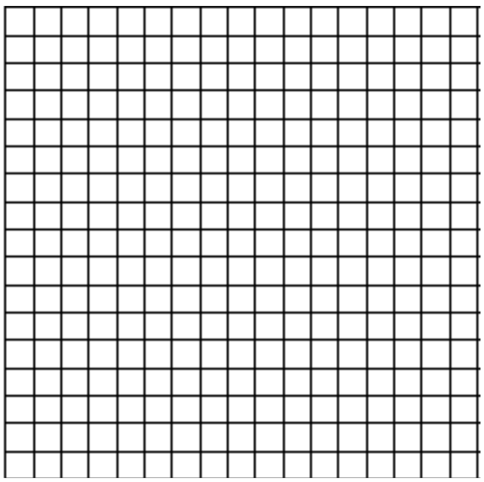
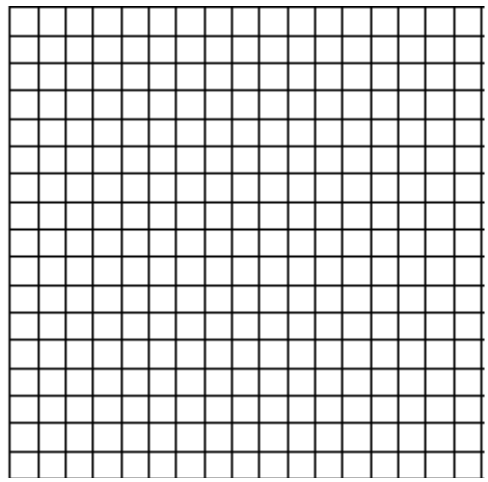
9. $y = x^2 + 3x + \frac{13}{4}$

vertex:



10. $y = \frac{1}{2}x^2 - x + 5$

vertex:



11. One of the parabolas in problems 9 – 10 should look "wider" than the others. Identify the parabola. Explain why this parabola looks different.

Fill in the blank by completing the square. Leave the number that completes the square as an improper fraction. Then write the trinomial in factored form.

12. $x^2 - 11x + \_\_\_$

13. $x^2 + 7x + \_\_\_$

14. $x^2 + 15x + \_\_\_$

15. $x^2 + \frac{2}{3}x + \_\_\_$

16. $x^2 - \frac{1}{5}x + \_\_\_$

17. $x^2 - \frac{3}{4}x + \_\_\_$

GO

Topic: Writing recursive equations for quadratic functions.

Identify whether the table represents a linear or quadratic function. If the function is linear, <u>write both the explicit and recursive equations.</u> If the function is quadratic, write only the recursive equation.

18.

x	$f(x)$
1	0
2	3
3	6
4	9
5	12

Type of function:

Equation(s):

19.

x	$f(x)$
1	7
2	10
3	16
4	25
5	37

Type of function:

Equation(s):

20.

x	$f(x)$
1	8
2	10
3	12
4	14
5	16

Type of function:

Equation(s):

21.

x	$f(x)$
1	28
2	40
3	54
4	70
5	88

Type of function:

Equation(s):

2.6 Factor Fixin'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A Develop Understanding Task

At first, *Optima's Quilts* only made square blocks for quilters and Optima spent her time making perfect squares. Customer service representatives were trained to ask for the length of the side of the block, x, that was being ordered, and they would let the customer know the area of the block to be quilted using the formula $A(x) = x^2$.

Optima found that many customers that came into the store were making designs that required a combination of squares and rectangles. So, *Optima's Quilts* has decided to produce several new lines of rectangular quilt blocks. Each new line is described in terms of how the rectangular block has been modified from the original square block. For example, one line of quilt blocks consists of starting with a square block and extending one side length by 5 inches and the other side length by 2 inches to form a new rectangular block. The design department knows that the area of this new block can be represented by the expression: $A(x) = (x + 5)(x + 2)$, but they do not feel that this expression gives the customer a real sense of how much bigger this new block is (e.g., how much more area it has) when compared to the original square blocks.

1. Can you find a different expression to represent the area of this new rectangular block? You will need to convince your customers that your formula is correct using a diagram.

Here are some additional new lines of blocks that *Optima's Quilts* has introduced. Find two different algebraic expressions to represent each rectangle, and illustrate with a diagram why your representations are correct.

2. The original square block was extended 3 inches on one side and 4 inches on the other.

3. The original square block was extended 4 inches on only one side.

4. The original square block was extended 5 inches on each side.

5. The original square block was extended 2 inches on one side and 6 inches on the other.

Customers start ordering custom-made block designs by requesting how much additional area they want beyond the original area of x^2. Once an order is taken for a certain type of block, customer service needs to have specific instructions on how to make the new design for the manufacturing team. The instructions need to explain how to extend the sides of a square block to create the new line of rectangular blocks.

The customer service department has placed the following orders on your desk. For each, describe how to make the new blocks by extending the sides of a square block with an initial side length of x. Your instructions should include diagrams, written descriptions and algebraic descriptions of the area of the rectangles in using expressions representing the lengths of the sides.

6. $x^2 + 5x + 3x + 15$

7. $x^2 + 4x + 6x + 24$

8. $x^2 + 9x + 2x + 18$

9. $x^2 + 5x + x + 5$

Some of the orders are written in an even more simplified algebraic code. Figure out what these entries mean by finding the sides of the rectangles that have this area. Use the sides of the rectangle to write equivalent expressions for the area.

10. $x^2 + 11x + 10$

11. $x^2 + 7x + 10$

12. $x^2 + 9x + 8$

13. $x^2 + 6x + 8$

14. $x^2 + 8x + 12$

15. $x^2 + 7x + 12$

16. $x^2 + 13x + 12$

17. What relationships or patterns do you notice when you find the sides of the rectangles for a given area of this type?

18. A customer called and asked for a rectangle with area given by: $x^2 + 7x + 9$. The customer service representative said that the shop couldn't make that rectangle. Do you agree or disagree? How can you tell if a rectangle can be constructed from a given area?

2.6 Factor Fixin' – Teacher Notes

A Develop Understanding Task

Note to teacher: Graph paper and colored pencils will be useful for this task.

Purpose: The purpose of this task is for students to understand equivalent expressions obtained from factoring trinomials by using rectangular area models. Students will write expressions in both factored form and standard form, using area diagrams and the distributive property to show that the expressions are equivalent. In the task, students use area model diagrams to identify the sides of the rectangle, and thus, the factors. The expressions to be factored in this task are in the form $x^2 + bx + c$ and restricted to only positive numbers so that students will notice the number relationships between b and c. In the next task, students will work with expressions that contain both positive and negative numbers.

Core Standards Focus:

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.,

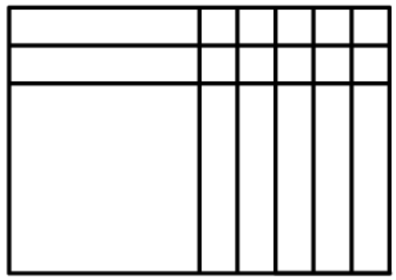
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

The Teaching Cycle:

Launch (Whole Class):

This task is based on a context that students are familiar with from "Building the Perfect Square". Since they have used area model diagrams to model perfect squares, it should not be difficult to extend their thinking to rectangles. Start the task by having them work on problem 1. Ask a student to share the diagram, which should be something like this:

$(x+2)(x+5) = x^2 + 7x + 10$



Ask the class how to label each of the sides and parts of the area. Ask students to connect the diagram with the equation and to justify the expression for the area. Be sure that students recognize the distributive property in both representations. Ask students if they notice any number patterns in the diagrams or the equation. At this point, it's not important to introduce the patterns unless students happen to see them. The purpose of the question is to get them to think about patterns as they work.

Explore (Small Group):

This task is meant to be a puzzler, so let students work through it a bit without leading too much. Problems 6-9 are designed to help students use the diagrams to write equivalent expressions. Encourage them to use the diagrams to write and justify their expressions. Problems 10-16 are designed to get students to recognize the number patterns. Listen for students that are noticing useful patterns in the numbers that will help the class with factoring.

Discuss (Whole Class):

Start the discussion with #10 and #11. In each case, have the presenting student talk about how they constructed the rectangle and found the sides. After both problems have been demonstrated ask students what they notice about the relationship between the last number in the trinomial and the coefficient of the middle term. Students may not yet be articulating the relationship, so the next two problems (#12 and #13) can be used to verify or refine what they have noticed. In each case, be sure that they label their diagrams and write the factored form of the expression for area. Work problems 14, 15, and 16, writing the factored form of the expression next to the trinomial. Again, ask students to look for patterns and to describe how to choose the factors of 12 that go into the factored form such as noticing that the factors of the last term must add up to the middle term. Help students to formalize the language and write it on the board so they can remember it.

When the discussion gets to #18, let students explain how they know that the expression doesn't factor. A good way to tell if a trinomial factors, in general, is to multiply the lead coefficient by the last term. If there are factors of that product that add to give the middle coefficient, the trinomial can be factored.

Aligned Ready, Set, Go: ***Quadratic Functions 2.6***

READY, SET, GO! Name Period Date

READY

Topic: Creating Binomial Quadratics
Multiply. (Use the distributive property, write in standard form.)

1. x(4x – 7) 2. 5x(3x + 8) 3. 3x(3x – 2)

4. The answers to problems 1, 2, & 3 are quadratics that can be represented in standard form $ax^2 + bx + c$. Which coefficient, **a**, **b**, or **c** equals 0 for all of the exercises above?

Factor the following. (Write the expressions as the product of two linear factors.)

5. $x^2 + 4x$ 6. $7x^2 - 21x$ 7. $12x^2 + 60x$ 8. $8x^2 + 20x$

Multiply
9. $(x + 9)(x - 9)$ 10. $(x + 2)(x - 2)$ 11. $(6x + 5)(6x - 5)$ 12. $(7x + 1)(7x - 1)$

13. The answers to problems 9, 10, 11, &12 are quadratics that can be represented in standard form $ax^2 + bx + c$. Which coefficient, **a**, **b**, or **c** equals 0 for all of the exercises above?

SET

Topic: Factoring Trinomials
Factor the following quadratic expressions into two binomials.
14. $x^2 + 14x + 45$ 15. $x^2 + 18x + 45$ 16. $x^2 + 46x + 45$

17. $x^2 + 11x + 24$ 18. $x^2 + 10x + 24$ 19. $x^2 + 14x + 24$

20. $x^2 + 12x + 36$ 21. $x^2 + 13x + 36$ 22. $x^2 + 20x + 36$

23. $x^2 - 15x - 100$ 24. $x^2 + 20x + 100$ 25. $x^2 + 29x + 100$

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26. Look back at each "row" of factored expressions in problems 14 to 25 above. Explain how it is possible that the coefficient ***(b)*** of the middle term can be different numbers in each problem when the "outside" coefficients ***(a)*** and (c) are the same. (Recall the standard form of a quadratic is $\boldsymbol{ax^2 + bx + c}$.)

GO

Topic: Taking the square root of perfect squares.

Only some of the expressions inside the radical sign are perfect squares. Identify which ones are perfect squares and take the square root. Leave the ones that are not perfect squares under the radical sign. Do not attempt to simplify them. (Hint: Check your answers by squaring them. You should be able to get what you started with, if you are right.)

27. $\sqrt{(17xyz)^2}$

28. $\sqrt{(3x-7)^2}$

29. $\sqrt{121a^2b^6}$

30. $\sqrt{x^2+8x+16}$

31. $\sqrt{x^2+14x+49}$

32. $\sqrt{x^2+14x-49}$

33. $\sqrt{x^2+10x+100}$

34. $\sqrt{x^2+20x+100}$

35. $\sqrt{x^2-20x+100}$

2.7 The x Factor

A Solidify Understanding Task

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Now that *Optima's Quilts* is accepting orders for rectangular blocks, their business in growing by leaps and bounds. Many customer want rectangular blocks that are bigger than the standard square block on one side. Sometimes they want one side of the block to be the standard length, x, with the other side of the block 2 inches bigger.

1. Draw and label this block. Write two different expressions for the area of the block.

Sometimes they want blocks with one side that is the standard length, x, and one side that is 2 inches less than the standard size.

2. Draw and label this block. Write two different expressions for the area of the block. Use your diagram and verify algebraically that the two expressions are equivalent.

There are many other size blocks requested, with the side lengths all based on the standard length, x. Draw and label each of the following blocks. Use your diagrams to write two equivalent expressions for the area. Verify algebraically that the expressions are equal.

3. One side is 1" less than the standard size and the other side is 2" more than the standard size.

4. One side is 2" less than the standard size and the other side is 3" more than the standard size.

5. One side is 2" more than the standard size and the other side is 3" less than the standard size.

6. One side is 3" more than the standard size and the other side is 4" less than the standard size.

7. One side is 4" more than the standard size and the other side is 3" less than the standard size.

8. An expression that has 3 terms in the form: $ax^2 + bx + c$ is called a trinomial. Look back at the trinomials you wrote in questions 3-7. How can you tell if the middle term (bx) is going to be positive or negative?

9. One customer had an unusual request. She wanted a block that is extended 2 inches on one side and decreased by 2 inches on the other. One of the employees thinks that this rectangle will have the same area as the original square since one side was decreased by the same amount as the other side was increased. What do you think? Use a diagram to find two expressions for the area of this block.

10. The result of the unusual request made the employee curious. Is there a pattern or a way to predict the two expressions for area when one side is increased and the other side is decreased by the same number? Try modeling these two problems, look at your answer to #8, and see if you can find a pattern in the result.

a. $(x+1)(x-1)$

b. $(x+3)(x-3)$

11. What pattern did you notice? What is the result of $(x+a)(x-a)$?

12. Some customers want both sides of the block reduced. Draw the diagram for the following blocks and find a trinomial expression for the area of each block. Use algebra to verify the trinomial expression that you found from the diagram.

a. $(x-2)(x-3)$

b. $(x-1)(x-4)$

13. Look back over all the equivalent expressions that you have written so far, and explain how to tell if the third term in the trinomial expression ax^2+bx+c will be positive or negative.

14. Optima's quilt shop has received a number of orders that are given as rectangular areas using a trinomial expression. Find the equivalent expression that shows the lengths of the two sides of the rectangles.

a. $x^2 + 9x + 18$

b. $x^2 + 3\text{x} - 18$

c. $x^2 - 3x - 18$

d. $x^2 - 9x + 18$

e. $x^2 - 5x + 4$

f. $x^2 - 3x - 4$

g. $x^2 + 2x - 15$

15. Write an explanation of how to factor a trinomial in the form: $x^2 + bx + c$.

2.7 The x Factor – Teacher Notes

A Solidify Understanding Task

Note to teacher: Graph paper and colored pencils will be useful for this task.

Purpose: The purpose of this task is for students to understand equivalent expressions obtained from factoring trinomials. In the task, students use area model diagrams to identify the sides of the rectangle, and thus, the factors. In the previous task, *Factor Fixin'*, students factored trinomials in which all of the terms are positive. This task builds on that work to include factoring expressions that have both positive and negative terms. The problems are carefully selected to help students see number patterns that they can use to become fluent with factoring. Students write expressions in both factored form and standard form.

Core Standards Focus:

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

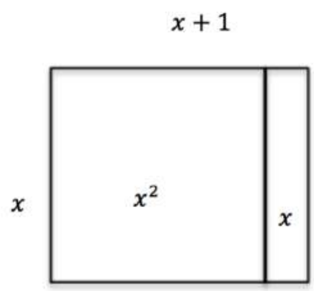
A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

The Teaching Cycle:

Launch (Whole Class):

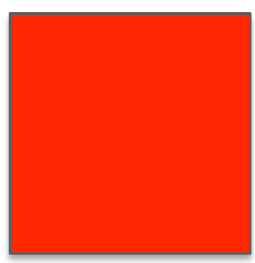
This task is based on a context that students are familiar with from *Factor Fixin'*. The area models used here become more difficult because students have to think about the effect on the area of both adding to a side and subtracting from a side. This leads to some complications that must be carefully worked through. Start the task by having them work on problem 1. Ask a student to share the diagram, which should be something like this:



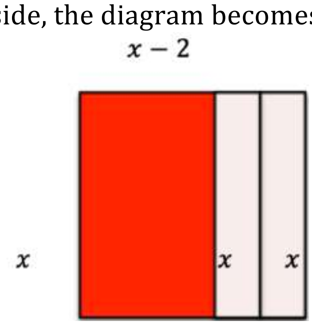
Ask the class how to label each of the sides and parts of the area. Ask students to connect the diagram with the equation and to justify the expression for the area. Be sure that students recognize the distributive property in both representations. This should not be difficult, based upon previous work.

Ask students to do problem #2 and then proceed in sharing in a similar fashion. The diagram will show the length of the standard block, x, and then that 2 inches have been removed from one side. The steps for creating the diagram follow:

Begin with the standard square:



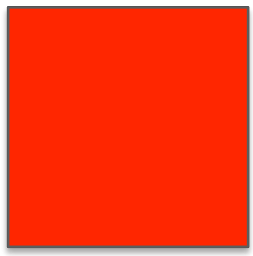
When two inches are removed from one side, the diagram becomes like the one below, with the red area representing the order:



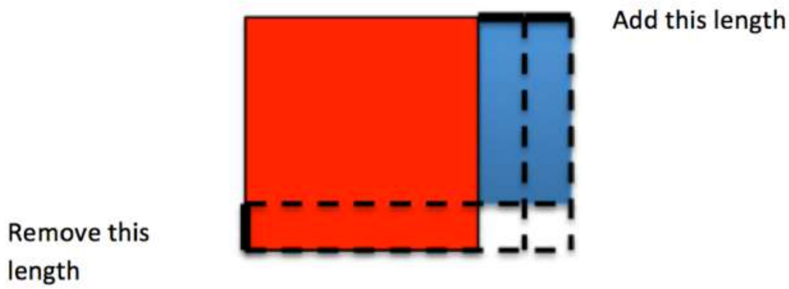
The expressions for the area of the order are: $x(x-2) = x^2 - 2x$. Be sure that students can connect the algebraic expressions to the diagram, especially that they can explain how the diagram shows $x^2 - 2x$.

Ask students to do #3 and discuss it in a similar way to #1 and #2. The diagram is built as follows:

Begin with the standard square:



Add 2 inches on one side and remove an inch on the other side. It helps to mark it on the sides first and then fill in the resulting areas, like so:



The resulting rectangular area is given by: $(x-1)(x+2) = x^2 + 2x - x - 2 = x^2 + x - 2$

Be sure that students can use the diagram to explain the areas and they see how the distributive property shows that the algebraic expressions are equivalent.

At this point, students can proceed through the task. You may wish to have them work to do the lesson in three parts with part 1 ending with a discussion of problem 8, the second part with problems 8 and 12, and the third part with problem #14.

Explore (Small Group):

Encourage them to use the diagrams to write and justify their expressions. Listen for students that are noticing useful patterns in the numbers that will help the class with factoring. If students are having difficulty with the diagrams, you may want to offer the idea of coloring the positive spaces one color and the negative spaces another. Students will probably need some help in thinking about the case when length is removed from both sides. Identify students that have worked through this idea and drawn diagrams for #11a to share in the discussion.

Discuss (Whole Class):

Start the discussion with #6 and #7. In each case, ask a student to share their diagram and relate the diagram to each of the algebraic expressions. Use the diagram to show why the expressions are equivalent, making sure that students are able to explain why the middle term in the trinomial is either positive or negative. Ask the class to come up with a guide for looking at two factors and knowing if the middle term of the trinomial is positive or negative. Also ask what we know about the factors if the middle term is positive or negative. Students should recognize that a negative middle term doesn't necessarily mean that the signs of the factors will be either positive or negative.

Ask a student to share their conjecture about #12. Have a discussion of the conjecture with students either supporting or disputing the conjecture. End this segment of the discussion with an agreed-upon statement like: The third term will be positive if both factors are either positive or negative. The third term will be negative if the two factors have different signs.

The last part of the discussion should focus on #14 and the number patterns in factoring. At the end of the discussion students should be clear that in a trinomial of the form $x^2 + bx + c$, the factors of the last term must add up to the middle term. To this end, the discussion should compare the results of #14a, b, c, and d to highlight the number patterns. For the most part, the discussion can focus on the algebraic expressions, although the diagrams may still be useful here for explaining why the patterns occur. When the number patterns have been articulated, help students to formalize the language and write it on the board so they can remember it.

Use the remaining examples in #14 to illustrate the patterns found throughout the task including how to determine the signs of the factors and how to select and place the appropriate numbers into the factors.

Aligned Ready, Set, Go: *Quadratic Functions 2.7*

READY, SET, GO! Name Period Date

READY

Topic: Exploring the density of the number line.
Find <u>three</u> numbers that are between the two given numbers.

1. $5\frac{3}{4}$ *and* $6\frac{1}{3}$
2. $-2\frac{1}{4}$ *and* $-1\frac{1}{2}$
3. $\frac{1}{4}$ *and* $\frac{5}{8}$
4. $\sqrt{3}$ *and* $\sqrt{5}$
5. 4 *and* $\sqrt{23}$
6. $-9\frac{3}{4}$ *and* -8.5
7. $\sqrt{\frac{1}{4}}$ *and* $\sqrt{\frac{4}{9}}$
8. $\sqrt{13}$ *and* $\sqrt{14}$

SET

Topic: Factoring Quadratics
The area of a rectangle is given in the form of a trinomial expression. Find the equivalent expression that shows the lengths of the two sides of the rectangle.

9. $x^2 + 9x + 8$
10. $x^2 - 6x + 8$
11. $x^2 - 2x - 8$
12. $x^2 + 7x - 8$
13. $x^2 - 11x + 24$
14. $x^2 - 14x + 24$
15. $x^2 - 25x + 24$
16. $x^2 - 10x + 24$
17. $x^2 - 2x - 24$
18. $x^2 - 5x - 24$
19. $x^2 + 5x - 24$
20. $x^2 - 10x + 25$
21. $x^2 - 25$
22. $x^2 - 2x - 15$
23. $x^2 + 10x - 75$
24. $x^2 - 20x + 51$
25. $x^2 + 14x - 32$
26. $x^2 - 1$
27. $x^2 - 2x + 1$
28. $x^2 + 12x - 45$

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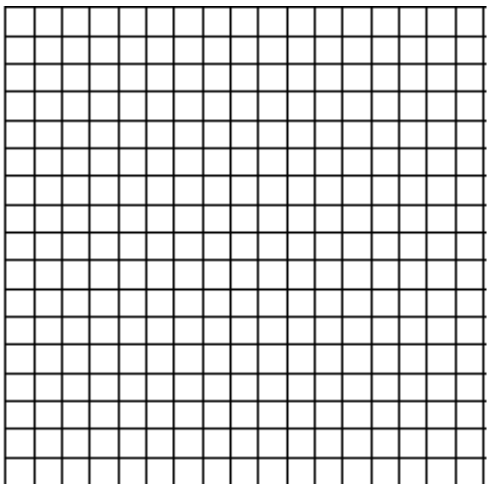
GO

Topic: Graphing Parabolas

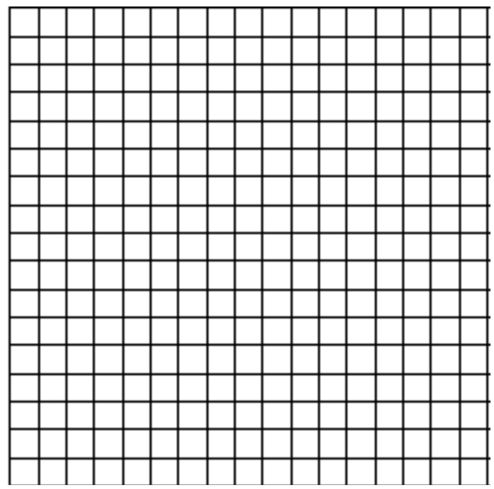
Graph each parabola. Include the vertex and at least 3 accurate points on each side of the axis of symmetry. Then describe the transformation in words.

29. $f(x) = x^2$



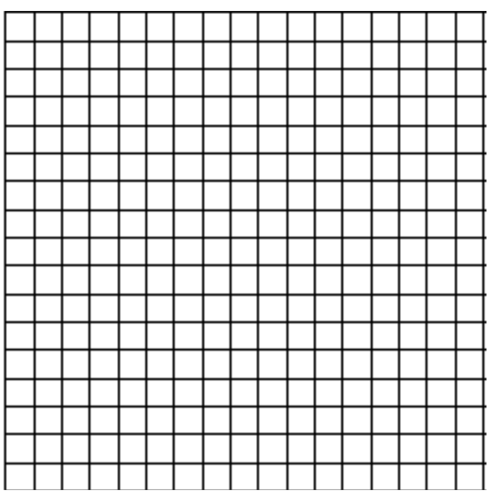
Description:

30. $g(x) = x^2 - 3$



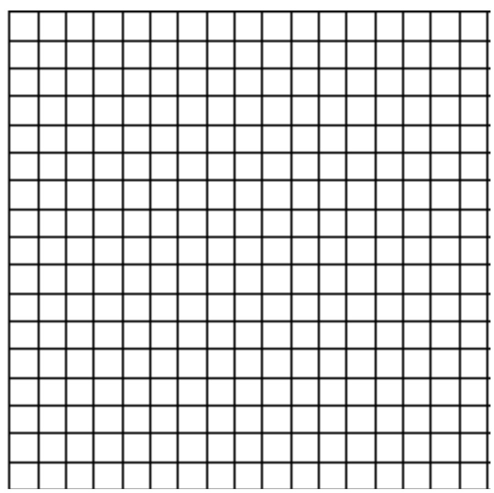
Description:

31. $h(x) = (x-2)^2$



Description:

32. $b(x) = -(x+1)^2 + 4$



Description:

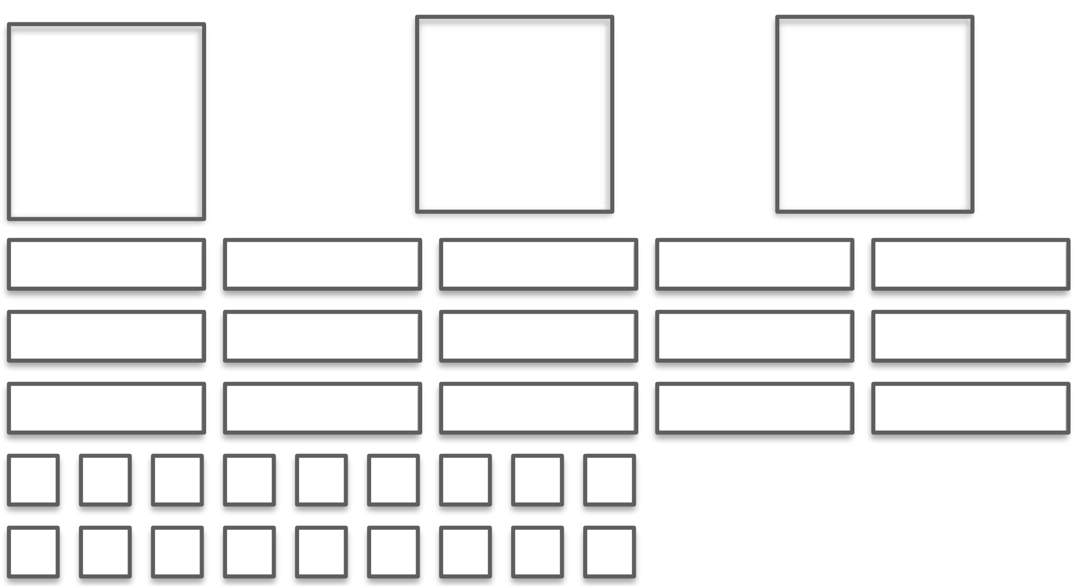
2.8H The Wow Factor

A Solidify Understanding Task

Optima's Quilts sometimes gets orders for blocks that are multiples of a given block. For instance, Optima got an order for a block that was exactly twice as big as the rectangular block that has a side that is 1" longer than the basic size, x, and one side that is 3" longer than the basic size.

1. Draw and label this block. Write two equivalent expressions for the area of the block.

2. Oh dear! This order was scrambled and the pieces are shown here. Put the pieces together to make a rectangular block and write two equivalent expressions for the area of the block.



3. What do you notice when you compare the two equivalent expressions in problems #1 and #2?

4. Optima has a lot of new orders. Use diagrams to help you find equivalent expressions for each of the following:

a. $5x^2 + 10x$

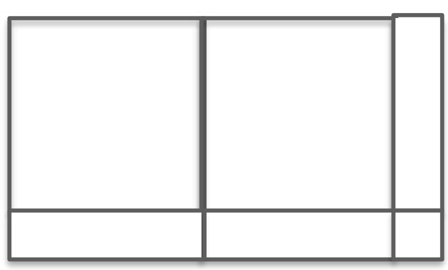
b. $3x^2 + 21x + 36$

c. $2x^2 + 2x - 4$

d. $2x^2 - 10x + 12$

e. $3x^2 - 27$

Because she is a great business manager, Optima offers her customers lots of options. One option is to have rectangles that have side lengths that are more than one x. For instance, Optima made this cool block:



5. Write two equivalent expressions for this block. Use the distributive property to verify that your answer is correct.

6. Here we have some partial orders. We have one of the expressions for the area of the block and we know the length of one of the sides. Use a diagram to find the length of the other side and write a second expression for the area of the block. Verify your two expressions for the area of the block are equivalent using algebra.

a. Area: $2x^2 + 7x + 3$ Side: $(x + 3)$

Equivalent expression for area:

b. Area: $5x^2 + 8x + 3$ Side: $(x + 1)$

Equivalent expression for area:

c. Area: $2x^2 + 7x + 3$ Side: $(2x + 1)$

Equivalent expression for area:

7. What are some patterns you see in the two equivalent expressions for area that might help you to factor?

8. Business is booming! More and more orders are coming in! Use diagrams or number patterns (or both) to write each of the following orders in factored form:

a. $3x^2 + 16x + 5$

b. $2x^2 - 13x + 15$

c. $3x^2 + x - 10$

d. $2x^2 + 9x - 5$

9. In *The x Factor*, you wrote some rules for deciding about the signs inside the factors. Do those rules still work in factoring these types of expressions? Explain your answer.

10. Explain how Optima can tell if the block is a multiple of another block or if one side has a multiple of x in the side length.

11. There's one more twist on the kind of blocks that Optima makes. These are the trickiest of all because they have more than one x in the length of both sides of the rectangle! Here's an example:

Write two equivalent expressions for this block. Use the distributive property to verify that your answer is correct.

12. All right, let's try the tricky ones. They may take a little messing around to get the factored expression to match the given expression. Make sure you check your answers to be sure that you've got them right. Factor each of the following:

a. $6x^2 + 7x + 2$

b. $10x^2 + 17x + 3$

c. $4x^2 - 8x + 3$

d. $4x^2 + 4x - 3$

e. $9x^2 - 9x - 10$

12. Write a "recipe" for how to factor trinomials in the form, $ax^2 + bx + c$.

2.8H The Wow Factor – Teacher Notes

A Solidify Understanding Task

Note to teacher: Graph paper and colored pencils will be useful for this task.

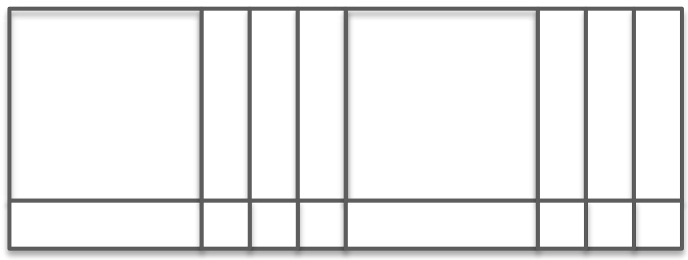
Purpose: This task provides opportunity to extend the work of factoring and working with the area model for quadratics to those of the form $ax^2 + bx + c$, with an a-value other than one. Students will work to see how the area model connects with quadratics of this form and how both factored form and standard form connect with the area model. The task begins with expressions that have a common factor between terms, and continues to other expressions with $a \neq 0$. The distributive property will be used to verify the work and move to efficiency as combinations of the factors of a are considered with the combinations of the factors of c.

Core Standards Focus:
F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
a. Factor a quadratic expression to reveal the zeros of the function it defines.

The Teaching Cycle:

Launch (Whole Class):
This task is based on a context that students are familiar with from *Factor Fixin'*. The area models become increasingly difficult. Launch the task with question #1, which asks students to model a quilt block that is exactly twice as big as the rectangular block that has a side that is 1" longer than

the basic size, x, and one side that is 3" longer than the basic size. Their diagrams could look like this, with two identical block combined together:



After students have drawn the diagrams, support them in writing the expressions that model the block:

$$2(x+1)(x+3) = 2(x^2+4x+3) = 2x^2+8x+6$$

Ask students what they notice about these expressions. Be sure to solicit the idea that in the expression $2x^2+8x+6$, each of the terms is divisible by 2 because the original block was doubled. It is important to point out that in the equivalent expressions, the 2 can be factored out, leaving the expression $2(x^2+4x+3)$, which can be further factored to $2(x+1)(x+3)$. Tell students that an expression is not considered fully factored unless all the common factors have been divided out. Ask student to work on problems 2-4.

Explore Part 1(Small Group):

As students are working, be sure that they are using the diagrams to write expressions that are actually equivalent. It is sometimes tempting to "divide out" the common factor without accounting for it in the final expression. If this is observed, ask the student to relate his/her final expression to the diagram and to account for all the pieces of the original diagram in the final algebraic expression.

Discuss Part 1 (Whole Class):

Focus the discussion on problem 4, beginning with question a. Ask a student to share his/her diagram and the equivalent expression that he/she wrote. In this problem, the block $x(x+2)$ is repeated 5 times. Ask students how they knew how many times to repeat the block and then how they knew the dimensions of the block. Flesh out the idea that the common factor can be seen in all the original terms and factored out to form an equivalent expression, which may then be factored further. Repeat this process with questions c and e, highlighting the equivalent expressions and how to algebraically check to see that the expressions are truly equivalent. Question e is a special product, the difference of two squares. This was introduced in a previous task, but students will probably require support in recognizing the pattern.

After this discussion, re-launch the task by working with students on question 5. Support them in noticing that in the expression $2x^2 + 3x + 1$, the beginning term has a coefficient of 2, but 2 is not a common factor among the terms. This creates a new factoring pattern that will be explored in problems 6-10. Tell students that in problem 6 they are given one side of the block so that it will be easier to find the other side. Relate to them that the purpose of this is so that they begin to notice patterns that they can use to answer questions 7-10.

Explore Part 2 (Small Group):
As students are working in this section, support them in noticing useful patterns that can be discussed. Encourage them to use the diagrams if they are stuck. Some students may be thinking about the problems entirely algebraically. If they are being successful, help them to articulate how they are able to find the equivalent expressions, so that the idea can be shared with the class.

Discuss Part 2 (Whole Class):
Ask a student to share their diagram and expressions for #8, question *a*. Ask the class what they have noticed about the numbers in the factors and how they relate to the given trinomial. Record what they have noticed about the number patterns. Follow this by having a student share his/her work on question c. Ask if the number patterns that were noticed in question *a* are still working for this question. Then ask about the signs of the factored expression, compared to the trinomial. Do the sign rules that they found in *The x-Factor* still hold in factoring these problems? Ask for reasons why they would be the same.

Launch the remainder of the task by working with students on problem 11. Ask them to compare the equivalent expressions that they wrote and what number patterns they see. Have students complete the task.

Explore Part 3 (Small Group):

Students may be getting tired of drawing the diagrams and beginning to rely more on the algebraic expressions. This is a great step forward, but may not yet be accessible for all students. Continue to encourage students that need the diagrams to use them, and then support them to notice the number and sign patterns that result.

Discuss Part 3 (Whole Class):

Discuss as many of the problems in #11 as time allows. Students will be using a guess and check strategy, but will need support in making it efficient. Emphasize the number and sign patterns that have been used previously. End the discussion with question 12. As a process for finding the factors is developed, be sure that it includes determining whether or not there is a common factor among terms.

Aligned Ready, Set, Go: *Quadratic Functions 2.8H*

2.8H

READY, SET, GO! Name Period Date

READY

Topic: Comparing arithmetic and geometric sequences

The first and fifth terms of each sequence are given. Fill in the missing numbers.

Example:

		+80	+80	+80	+80
Arithmetic	4	84	164	244	324
Geometric	4	12	36	108	324
		× 3	× 3	× 3	× 3

1.

Arithmetic	3				1875
Geometric	3				1875

2.

Arithmetic	-1458				-18
Geometric	-1458				-18

3.

Arithmetic	1024				4
Geometric	1024				4

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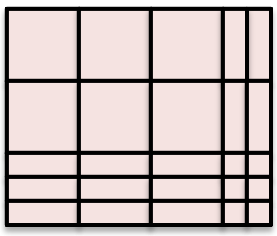
SET

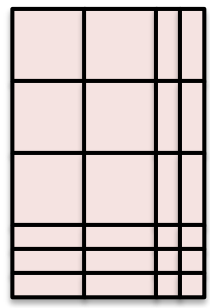
Topic: Writing an area model as a quadratic expression

Write two equivalent expressions for the area of each block. Let x be the side length of each of the large squares.

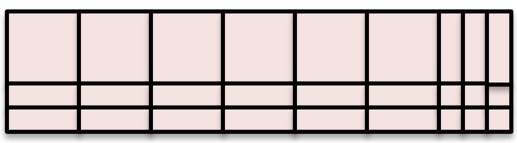
4.



5.



6.



7. Problems 4, 5, and 6 all contain the same number of squares measuring x^2 and 1^2.

   A. What is different about them?

   B. How does this difference affect the quadratic expression that represents them?

   C. Describe how the arrangement of the squares and rectangles affects the factored form.

Topic: Factoring quadratic expressions when $\boldsymbol{a > 1}$

Factor the following quadratic expressions.

8. $4x^2 + 7x - 2$ 9. $2x^2 - 7x - 15$ 10. $6x^2 + 7x - 3$ 11. $4x^2 - x - 3$

12. $4x^2 + 19x - 5$

13. $3x^2 - 10x + 8$

14. $6x^2 + x - 2$

15. $3x^2 - 14x - 24$

16. $2x^2 + 9x + 10$

17. $5x^2 + 31x + 6$

18. $5x^2 + 7x - 6$

19. $4x^2 + 8x - 5$

20. $3x^2 - 75$

21. $3x^2 + 7x + 2$

22. $4x^2 + 8x - 5$

23. $2x^2 + x - 6$

GO

Topic: Finding the equation of the line of symmetry of a parabola

Given the x-intercepts of a parabola, write the equation of the line of symmetry.

24. x-intercepts: (-3, 0) and (3, 0)

25. x-intercepts: (-4, 0) and (16, 0)

26. x-intercepts: (-2, 0) and (5, 0)

27. x-intercepts: (-14, 0) and (-3, 0)

28. x-intercepts: (17, 0) and (33, 0)

29. x-intercepts: $(-0.75, 0)$and $(2.25, 0)$

2.9 Lining Up Quadratics

A Practice Understanding Task

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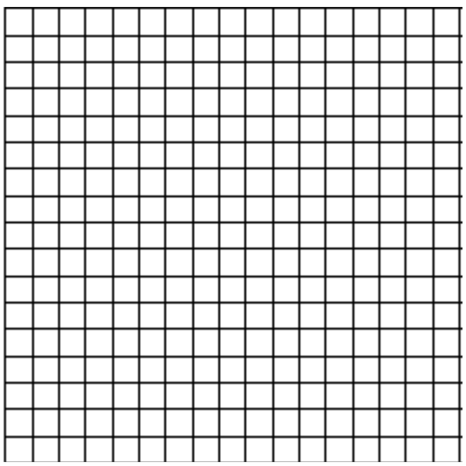
Graph each function and find the vertex, the y-intercept and the x-intercepts. Be sure to properly write the intercepts as points.

1. $y = (x - 1)(x + 3)$



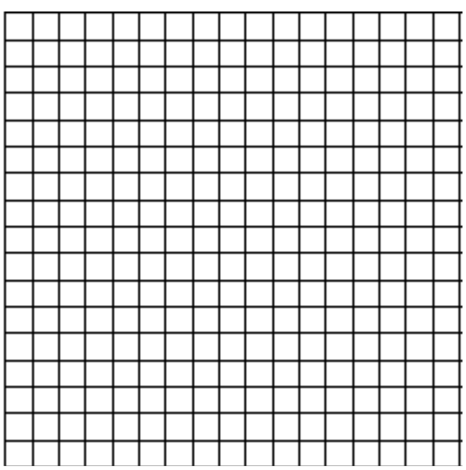
Line of Symmetry ________

Vertex _________

x -intercepts ________ ________

y-intercept _______

2. $f(x) = 2(x - 2)(x - 6)$



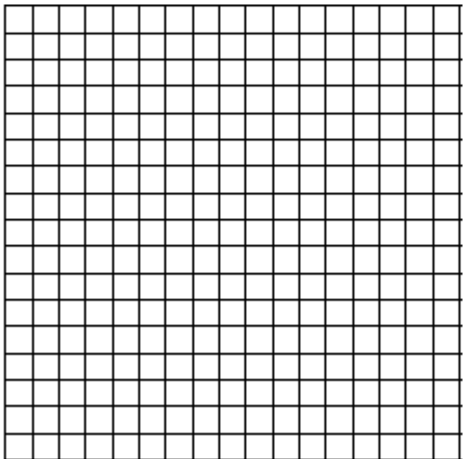
Line of Symmetry ________

Vertex _________

x -intercepts ________ ________

y-intercept _______

3. $g(x) = -x(x+4)$



Line of Symmetry __________

Vertex _________

x -intercepts ________ ________

y-intercept _______

4. Based on these examples, how can you use a quadratic function in factored form to:

a. Find the line of symmetry of the parabola?

b. Find the vertex of the parabola?

c. Find the x -intercepts of the parabola?

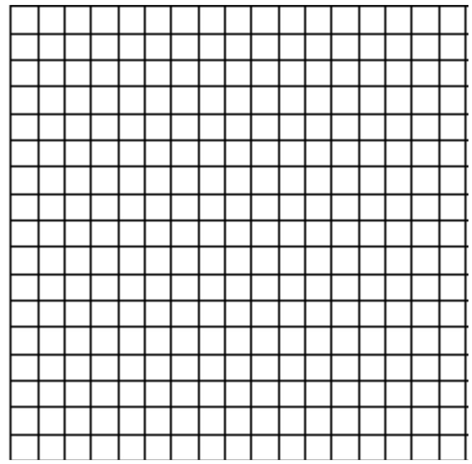
d. Find the y-intercept of the parabola?

e. Find the vertical stretch?

5. Choose any two **linear** functions and write them in the form: $f(x) = m(x - c)$, where m is the slope of the line. Graph the two functions.

Linear function 1:

Linear function 2:

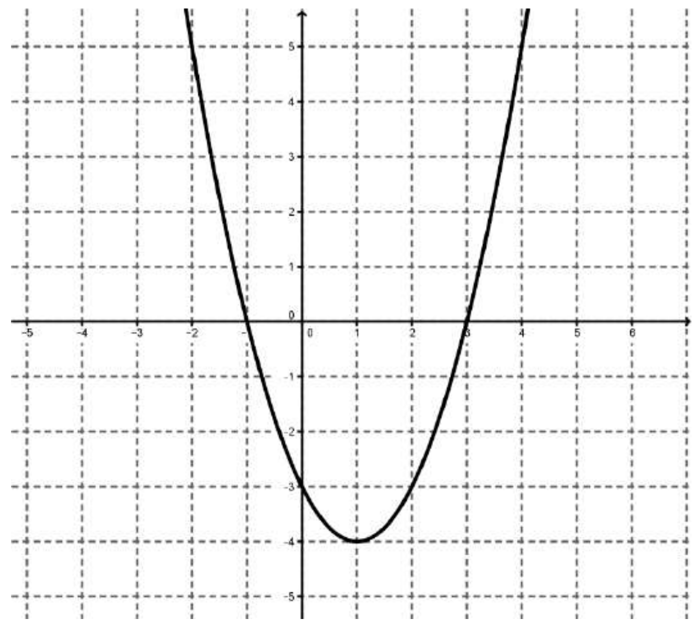


6. On the same graph as #5, graph the function *P(x)* that is the product of the two linear functions that you have chosen. What shape is created?

7. Describe the relationship between x-intercepts of the linear functions and the x-intercepts of the function *P(x).* Why does this relationship exist?

8. Describe the relationship between y-intercepts of the linear functions and the y-intercepts of the function $P(x)$. Why does this relationship exist?

9. Given the parabola to the right, sketch two lines that could represent its linear factors.



10. Write an equation for each of these two lines.

11. How did you use the x and y intercepts of the parabola to select the two lines?

12. Are these the only two lines that could represent the linear factors of the parabola? If so, explain why. If not, describe the other possible lines.

13. Use your two lines to write the equation of the parabola. Is this the only possible equation of the parabola?

2.9 Lining Up Quadratics – Teacher Notes

A Solidify Understanding Task

Special Note to Teachers: Graphing technology is useful for this task.

Purpose: The purpose of this task is two-fold. The first purpose is for students to explore and generalize how the features of the equation can be used to graph the quadratic function. The second purpose is for students to deepen their understanding of quadratic functions as the product of two linear factors. In the task, students are asked to graph parabolas from equations in factored form. They are given several cases to provide an opportunity to notice how the x-intercepts, y-intercept, and vertical stretch are readily visible in the equation. This also sets them up to notice the relationship between the x -intercepts and the y-intercept. The task extends this thinking by asking students to start with any two linear functions, multiply them together and find the function that is created, which is quadratic. They graph both the initial lines and the parabola to find the relationship between x-intercepts and y-intercept and to highlight the idea that quadratic functions are the product of two linear factors.

Core Standards Focus:

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-BF.1 Write a function that describes a relationship between two quantities.

b. Combine standard functions types using arithmetic operations.

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

The Teaching Cycle:

Launch (Whole Class):
Begin the task by telling students that questions 1-4 are an exploration that has been set up for them to notice as many things as they can about using factored form of the equation of a quadratic function. It is recommended that graphing technology be provided for students to use in this part of the task. Tables of values could also be used to generate graphs of the functions, although this will be much slower. Ask students to work on just questions 1-4 and then call them back for a discussion.

Explore (Small Group):
While students are working, listen for students who can describe the process of finding the line of symmetry half way between the x -intercepts and then using the x-value to find the y-value of the vertex. This will be tricky for many students and may require a few scaffolding questions for support. Encourage students to not only draw conclusions based on the examples that are given, but to consider if their conclusions will be true for any quadratic function.

Discuss (Whole Class):
Begin the discussion with question #2. Use technology to project the graph for the class, and ask a student to explain how he/she found the x-intercepts. Ask another student to explain how he/she used the x-intercepts to find the line of symmetry, and then another student to explain how to use the line of symmetry to find the vertex. Move the discussion to question #4 and formalize the thinking of the class using algebraic notation. For each question, ask if the process will work in all cases and to justify their generalizations. Demonstrate how to do a quick graph of a parabola with the following method using problem #2 as an example:

$f(x) = 2(x-2)(x-6)$

Step 1: Identify the x-intercepts, (2, 0) and (6, 0) and mark them with points on the graph.

Step 2: Identify the line of symmetry by determining the midpoint between the two intercepts and drawing the line. In this case, the line of symmetry is $x = 4$. Graph the line of symmetry.

Step 3: Find the vertex by substituting $x = 4$ into the function and solving for y. This works because we know the vertex lies on the line of symmetry.

Step 4: Identify any reflection or vertical stretch on the function. In this case, we have a vertical stretch by a factor of 2. Start at the vertex and use the quick graph counting method to get the points of the parabola. Along the way, the count should get to the two x-intercepts or else a mistake has been made.

Re-launch (Whole class):

Start the second part of the task with question #5. Lead students through this question, prompting them to write two linear functions of their choice and to graph them. Explain that for question #6, students are to multiply the two functions together and graph the result. They don't need to go through the algebra of multiplying the two functions together and combining terms. You may need to show a quick example. Once everyone has this part completed, they should be ready to work together on the rest of the task.

Explore (Small Group):

Because the language in this part of the task is somewhat abstract, be prepared to help students understand what the question is asking for without giving away the answers to the questions. The most important thing to be shared in the discussion is student reasoning about why they are getting the results that they are. Since they have all chosen different linear functions, encourage students to share their results with each other and to talk about why the conclusions are the same.

Discuss (Whole Class):

After students have been given time to work through the task on their own, lead the class through the questions, sharing results and pressing for good explanations of the results.

Aligned Ready, Set, Go: *Structures of Expressions 2.9*

2.9

READY, SET, GO! Name Period Date

READY

Topic: Multiplying Binomials Using a Two-Way Table

Multiply the following binomials using the given two-way table to assist you.

Example: $(2x + 3)(5x - 7)$

$(5x - 7)$

$(2x + 3)$

$10x^2$	$-14x$
$+15x$	-21

$= 10x^2 + x - 21$

1. $(3x - 4)(7x - 5)$

2. $(9x + 2)(x + 6)$

3. $(4x - 3)(3x + 11)$

4. $(7x + 3)(7x - 3)$

5. $(3x - 10)(3x + 10)$

6. $(11x + 5)(11x - 5)$

7. $(4x + 5)^2$

8. $(x + 9)^2$

9. $(10x - 7)^2$

10. The "like-term" boxes in #'s 7, 8, and 9 reveal a special pattern. Describe the relationship between the middle coefficient ***(b)*** and the coefficients ***(a)*** and ***(c).***

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SET

Topic: Factored Form of a Quadratic Function

Given the *factored form* of a quadratic function, identify the vertex, intercepts, and vertical stretch of the parabola.

11. $y = 4(x-2)(x+6)$
    a. Vertex:______________
    b. x-inter(s) ___________
    c. y-inter _______________
    d. Stretch ______________

12. $y = -3(x+2)(x-6)$
    a. Vertex:______________
    b. x-inter(s) ___________
    c. y-inter: ______________
    d. Stretch ______________

13. $y = (x+5)(x+7)$
    a. Vertex:______________
    b. x-inter(s) __________
    c. y-inter ______________
    d. Stretch ______________

14. $y = \frac{1}{2}(x-7)(x-7)$
    a. Vertex:______________
    b. x-inter(s) ___________
    c. y-inter _______________
    d. Stretch ______________

15. $y = -\frac{1}{2}(x-8)(x+4)$
    a. Vertex:______________
    b. x-inter(s) ___________
    c. y-inter: ______________
    d. Stretch ______________

16. $y = \frac{3}{5}(x-25)(x-9)$
    a. Vertex:______________
    b. x-inter(s) __________
    c. y-inter ______________
    d. Stretch ______________

17. $y = \frac{3}{4}(x-3)(x+3)$
    a. Vertex:______________
    b. x-inter(s) ___________
    c. y-inter _______________
    d. Stretch ______________

18. $y = -(x-5)(x+5)$
    a. Vertex:______________
    b. x-inter(s) ___________
    c. y-inter: ______________
    d. Stretch ______________

19. $y = \frac{2}{3}(x+10)(x+10)$
    a. Vertex:______________
    b. x-inter(s) __________
    c. y-inter ______________
    d. Stretch ______________

Need help? Visit **www.rsgsupport.org**

GO

Topic: Vertex Form of a Quadratic Equation

Given the *vertex form* of a quadratic function, identify the vertex, intercepts, and vertical stretch of the parabola.

20. $y = (x+2)^2 - 4$
    a. Vertex:____________
    b. x-inter(s) __________
    c. y-inter ____________
    d. Stretch ____________

21. $y = -3(x+6)^2 + 3$
    a. Vertex:____________
    b. x-inter(s) __________
    c. y-inter: ____________
    d. Stretch ____________

22. $y = 2(x-1)^2 - 8$
    a. Vertex:____________
    b. x-inter(s) __________
    c. y-inter ____________
    d. Stretch ____________

23. $y = 4(x+2)^2 - 64$
    a. Vertex:____________
    b. x-inter(s) __________
    c. y-inter ____________
    d. Stretch ____________

24. $y = -3(x-2)^2 + 48$
    a. Vertex:____________
    b. x-inter(s) __________
    c. y-inter: ____________
    d. Stretch ____________

25. $y = (x+6)^2 - 1$
    a. Vertex:____________
    b. x-inter(s) __________
    c. y-inter ____________
    d. Stretch ____________

26. Did you notice that the parabolas in problems 11, 12, & 13 are the same as the ones in problems 23, 24, & 25 respectively? If you didn't, go back and compare the answers in problems 11, 12, & 13 and problems 23, 24, & 25.

Prove that a. $4(x-2)(x+6) = 4(x+2)^2 - 64$

b. $-3(x+2)(x-6) = -3(x-2)^2 + 48$

c. $(x+5)(x+7) = (x+6)^2 - 1$

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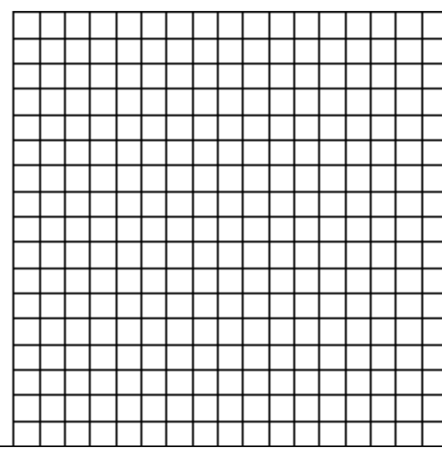
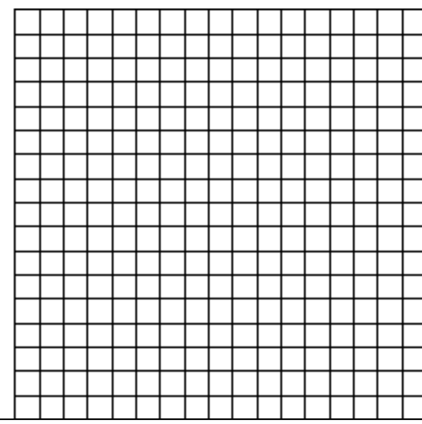
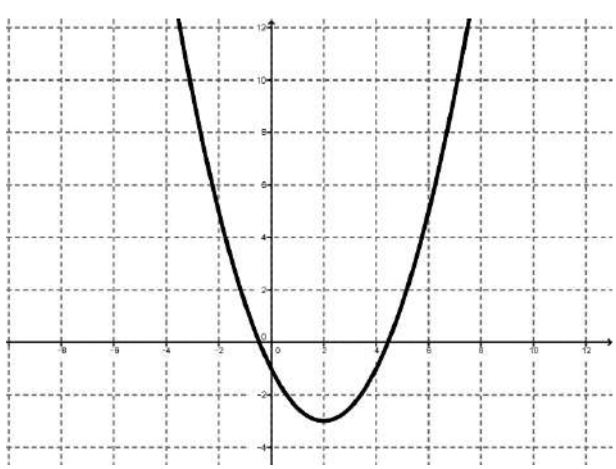
2.10 I've Got a Fill-in

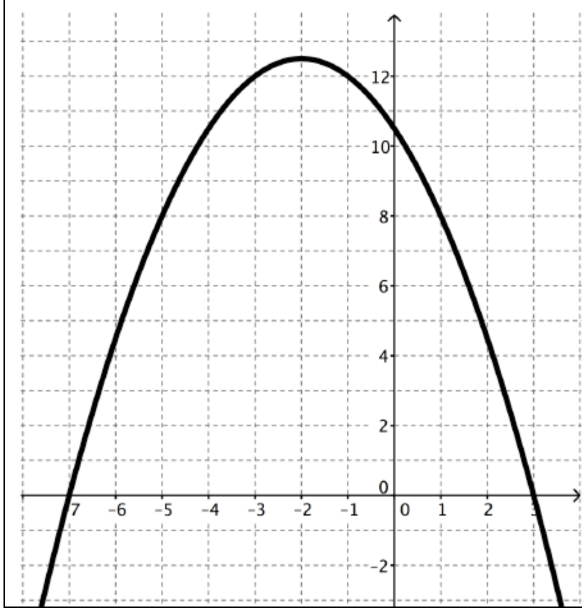
A Practice Understanding Task

For each problem below, you are given a piece of information that tells you a lot. Use what you know about that information to fill in the rest.

1. You get this:	Fill in this:
$y = x^2 - x - 12$	Factored form of the equation:
	Graph of the equation: 

2.	You get this:	Fill in this:
	$y = x^2 - 6x + 3$	Vertex form of the equation:
		Graph of the equation: 

3.	You get this:	Fill in this:
		Vertex form of the equation:
		Standard form of the equation:

4.	You get this:	Fill in this:
		Factored form of the equation:
		Standard form of the equation:

5.	You get this:	Fill in this:
	$y = -x^2 - 6x + 16$	Either form of the equation other than standard form.
		Vertex of the parabola
		x-intercepts and y-intercept

6. You get this:	Fill in this:
$y = 2x^2 + 12x + 13$	Either form of the equation other than standard form.
	Vertex of the parabola
	x-intercepts and y-intercept

7. You get this:	Fill in this:
$y = -2x^2 + 14x + 60$	Either form of the equation other than standard form.
	Vertex of the parabola
	x-intercepts and y-intercept

2.10 I've Got a Fill-In – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to build fluency in writing equivalent expressions for quadratic equations using factoring, completing the square, and the distributive property. Students will use the equations that they have constructed to analyze and graph quadratic functions.

Core Standards Focus:

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-BF.1 Write a function that describes a relationship between two quantities.

b. Combine standard function types using arithmetic operations.

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

The Teaching Cycle:

Launch (Whole Class):

Students are familiar with the ideas in the task, so tell them that this task is to help them to be more flexible and fluent in using completing the square and factoring to graph quadratic functions. Be sure they understand the instructions and that in every case, they should be prepared to justify and explain their work. Before students get started it might be useful to note that the two graphs are both scaled by two's.

Explore (Small Group):

As students are working, listen for problems that are difficult or controversial for the discussion. In particular, watch for student approaches to problems 5-7. Listen for how they are deciding to either factor or complete the square, and then how they are working with the expressions with $a \neq 1$. Also note if students use area model diagrams or tables to support their thinking, as these are strategies that can be shared with the class.

Discuss (Whole Class):

Begin the discussion with problem #3. Ask a student to present their work, explaining how they identified the vertex and the vertical stretch of the parabola and used it to write the equation in vertex form. Ask another student to show how they used the vertex form of the equation to get the standard form. Ask the class how they could verify that the two equations are equivalent and then test them to see if they are correct.

Move the discussion to #5 and ask the class what they know about the parabola just by looking at the equation. They should be able to predict that it opens downward, that it doesn't have a vertical stretch and that the y-intercept is 16. If some students wrote the equation in vertex form and others used factored form, ask both to present. Ask the class to verify that the two forms are equivalent. Use technology to project the graph of the function and discuss how the features appear in the equations. Discuss the merits of each form and what information can be easily used in each form.

Continue the discussion with problems 6 and 7, proceeding just like problem 5. Note that problem #6 does not have x-intercepts and can't be factored. Students will be able to find the vertex and know that the graph will not cross the x-axis.

Aligned Ready, Set, Go: ***Structures of Expressions 2.10***

READY, SET, GO! Name Period Date

READY

A golf-pro practices his swing by driving golf balls off the edge of a cliff into a lake. The height of the ball above the lake (measured in meters) as a function of time (measured in seconds and represented by the variable t) from the instant of impact with the golf club is

$$58.8 + 19.6t - 4.9t^2.$$

The expressions below are equivalent:

$-4.9t^2 + 19.6t + 58.8$	standard form
$-4.9(t-6)(t+2)$	factored form
$-4.9(t-2)^2 + 78.4$	vertex form

1. Which expression is the most useful for finding how many seconds it takes for the ball to hit the water? Why?

2. Which expression is the most useful for finding the maximum height of the ball? Justify your answer.

3. If you wanted to know the height of the ball at exactly 3.5 seconds, which expression would help the most to find the answer? Why?

4. If you wanted to know the height of the cliff above the lake, which expression would you use? Why?

SET

Topic: Finding multiple representations of a quadratic

One form of a quadratic function is given. Fill-in the missing forms.

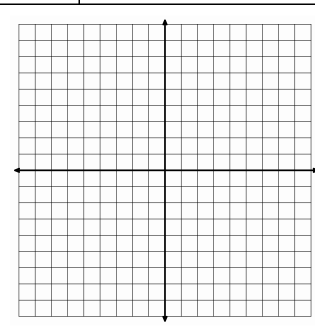
5 a. *Standard Form*	b. *Vertex Form*	c. *Factored Form* $y = (x+5)(x-3)$

d. *Table* (Include the vertex and at least 2 points on each side of the vertex.)

x	y

Show the first differences and the second differences.

e. *Graph*



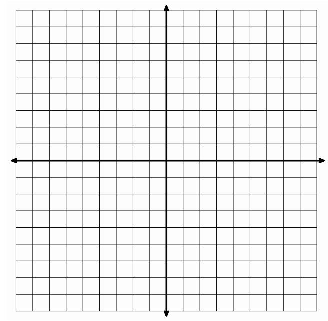
6 a. *Standard Form*	b. *Vertex Form* $y = -3(x-1)^2 + 3$	c. *Factored Form*

d. *Table* (Include the vertex and at least 2 points on each side of the vertex.)

x	y

Show the first differences and the second differences.

e. *Graph*



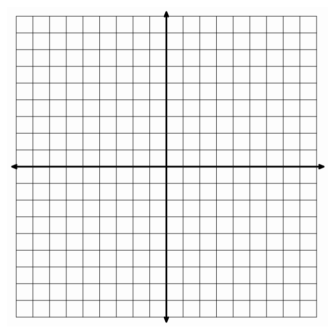
7 a. *Standard Form* $y = -x^2 + 10x - 25$	b. *Vertex Form*	c. *Factored Form*

d. *Table* (Include the vertex and at least 2 points on each side of the vertex.)

x	y

Show the first differences and the second differences.

e. *Graph*



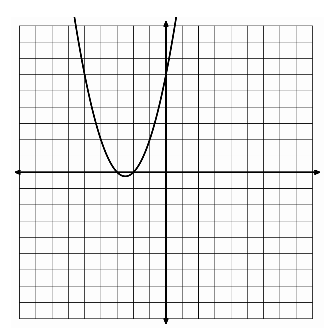
8 a. *Standard Form*	b. *Vertex Form*	c. *Factored Form*

d. *Table* (Include the vertex and at least 2 points on each side of the vertex.)

x	y

Show the first differences and the second differences.

e. *Graph*



9 a. *Standard Form*

b. *Vertex Form*

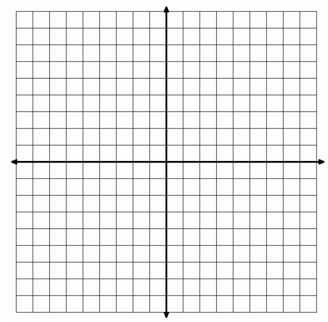
c. *Factored Form*
Skip this for now

d. *Table*

x	y
0	12
1	2
2	-4
3	-6
4	-4
5	2
6	12

Show the first differences and the second differences.

e. *Graph*



GO

Topic: Factoring Quadratics

Verify each factorization by multiplying.

10. $x^2 + 12x - 64 = (x + 16)(x - 4)$

11. $x^2 - 64 = (x + 8)(x - 8)$

12. $x^2 + 20x + 64 = (x + 16)(x + 4)$

13. $x^2 - 16x + 64 = (x - 8)(x - 8)$

Factor the following quadratic expressions, if possible. (Some will not factor.)

14. $x^2 - 5x + 6$

15. $x^2 - 7x + 6$

16. $x^2 - 5x - 36$

17. $m^2 + 16m + 63$

18. $s^2 - 3s - 1$

19. $x^2 + 7x + 2$

20. $x^2 + 14x + 49$

21. $x^2 - 9$

22. $c^2 + 11c + 3$

23. Which quadratic expression above could represent the area of a square? Explain.

24. Would any of the expressions above NOT be the side-lengths for a rectangle? Explain.