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Transforming Mathematics Education

SECONDARY MATH TWO

An Integrated Approach

Standard Teacher Notes

MODULE 4

More Functions, More Features

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The Mathematics Vision Project

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MORE FUNCTIONS, MORE FEATURES

4.7 More Features, More Functions – A Practice Understanding Task

Using prior knowledge to identify features of a function as well as to create functions when given features (F.IF.4)

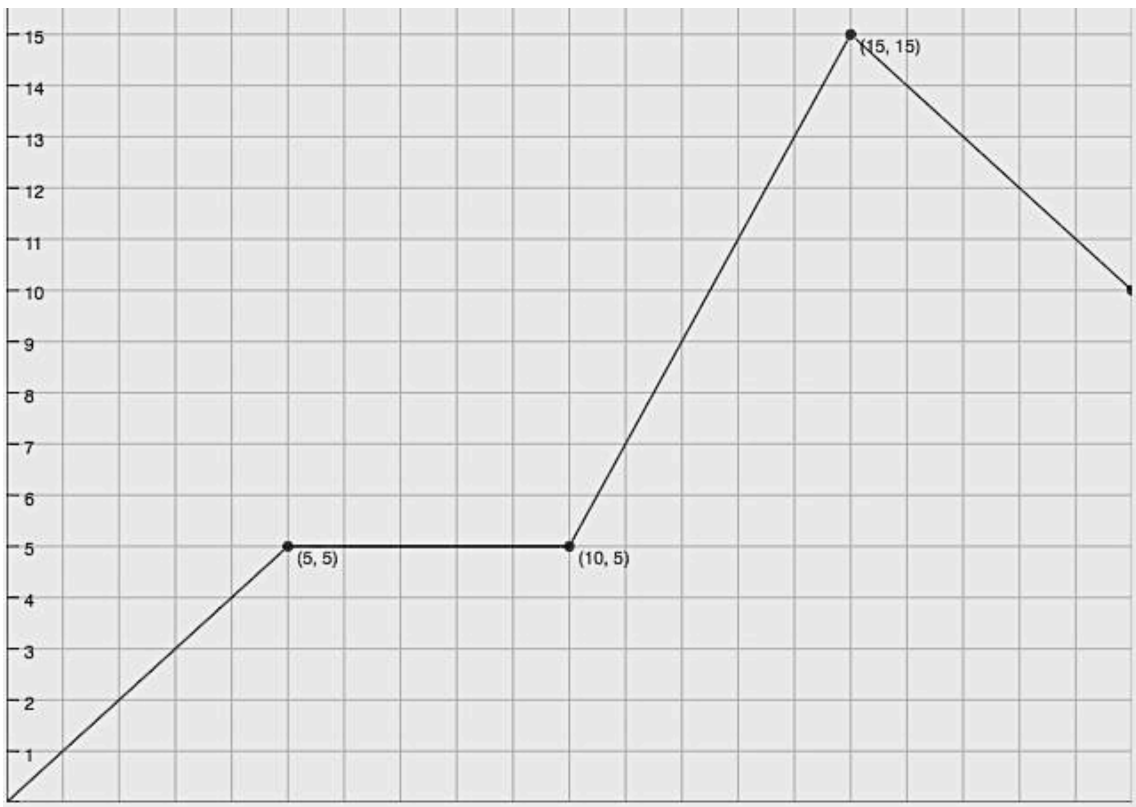
READY, SET, GO Homework: More Functions, More Features 4.7

4.1 Some of This, Some of That

A Develop Understanding Task

Part I: Connect context and graphical representations

1. Create a story that matches the graph below. Label axes and be as specific as possible in describing what is happening to connect your story to the graph.



2. If you were to write a function to match each part of your story (or section of the graph), how many would you write? Explain.

3. Identify and wrtie the function and corresponding domain for each section of the graph.

$$f(x) = \begin{cases} \textit{function here} & , & \textit{domain here} \\ & , & \\ & , & \\ & , & \\ & , & \end{cases}$$

4. Make connections between the graph, functions, and context (story you created).

The function you created above is called a **piecewise function.** In mathematics, a piecewise-defined function is a function defined by more than one sub-function (or *piece* of a function), with each section only existing in a certain interval of the functions domain.

Part II: Connecting function notation to a piecewise defined function

5. Find $f(12)$. Use the story you created to explain this meaning.
6. Which sub-function would you use to algebraically find the value of $f(12)$?
7. Find the following:
   a. $f(7) =$
   b. $f(x) = 3$
   c. $f(x) = 13$
   d. $f(15) =$

4.1 Some of This, Some of That – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is for students to learn about piecewise functions using their background knowledge of domain, linear functions and function notation. Students will develop an understanding of piecewise-defined functions by

- Using their knowledge of domain to talk about the four 'pieces' of the graph that are made up of different linear functions.
- Creating a story for the graph, recognizing how and where the story changes based on each section of the graph.
- Interpreting values for different sections of the graph and identifying which equation from the piecewise function to use based on input/output values.

Core Standards Focus:

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain of the function.*

F.IF.7b Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Related Standards: F.IF.2, F.IF.4, F.IF.6, F.IF.8

The Teaching Cycle:

Launch (Whole Class):
Prior to beginning this task, you may wish to have students begin with a warm-up that has them write a linear equation from a graph or have them graph a linear function that is given in point-slope form. This will help access their background knowledge for the task without 'giving it away'.

Launch the task by having students look at the given graph and let them know that they are to create a story that matches the graph. Read the first problem, emphasizing that their story needs to be as specific as possible to connect the story to the graph, and that they need to complete the graph by labeling the axes. Have them work quietly and individually on their story, then have them continue to work through the rest of Part I with a partner.

Explore (Small Group): *Part I*

After students create a story connected to the graph, have them work in pairs to complete the rest of Part I. As you monitor, listen for how students adjust their story depending on which section of the graph they are writing about. If students are not being specific enough, ask questions such as "When does this happen in your story?" (to push them to think about domain) or "Can you be more specific about what is happening here?" (domain or rate of change). Look for students who have made connections between the graph, their story, and the equations for the piecewise-defined function. Select two or three students to share their story, having them connect their story to the graph and to the function they created, emphasizing how all three representations change depending on the interval of the domain. Look for students who came up with equations in different ways (perhaps slope-intercept vs point-slope) as well as students with different stories. Informally assess which students are paying attention to the domain restrictions for functions vs those who are not.

Discuss (Whole Class): *Part I*

The goal of the whole group discussion is to have ALL students be able to do the following:

- connect the story they create to the mathematics (graph, domain, rate of change);
- write linear equations to match the graph, including the domain for each section of the graph; and
- recognize that the graph is a function and that the restriction on the domain for each equation is important.

An example of how to orchestrate the discussion is as follows:

Have copies of the graph available to use during the whole group discussion (for example, already having the graph on chart paper). Select two or three students to share their story, having them connect their story to the graph and to the function they created, emphasizing how all three representations change depending on the interval of the domain. When student one is sharing, make sure all three representations are visible to the whole group and have the student explain how all three representations (story, equation, and graph) connect to each section of the graph. The same should happen when student two shares. When selecting students, you may wish to select one student who emphasizes the story/graph connection while another student emphasizes how they came up with their equations (including slope and point on the graph). If they do not restrict the domain on their own, ask the class about this, distinguishing between the domain of the function and the domain of the situation. ("What is the domain of this function?" "What is the domain of this function in this situation?"). After clarifying the function of one section of the graph, ask another group to share. Choose a group (if possible) that used point-slope to write their function.

Explore (Small Group): ***Part II***

Have students continue to work with their partner to complete Part II of this task. Listen for students to recognize which equation to use in the piecewise-defined function as they answer questions 5, 6, and 7.

Discuss (Whole Class): ***Part II***

The goal of this section is for students to reason abstractly and quantitatively by recognizing where each value is located within the equation and connect it to what is happening in the story at this time. For the whole group discussion, select students to explain their answer to the whole group.

Aligned Ready, Set, Go: More Features, More Functions RSG 4.1

READY, SET, GO! Name Period Date

READY

Topic: Reading function values in a piece-wise defined graph.

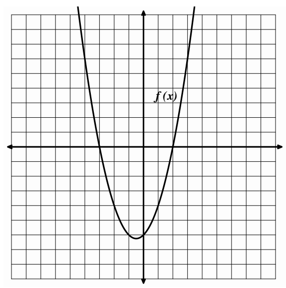
Use the graph to find the indicated function value.

1a. $f(-3) =$

b. $f(-2) =$

c. $f(0) =$

d. $f(2) =$

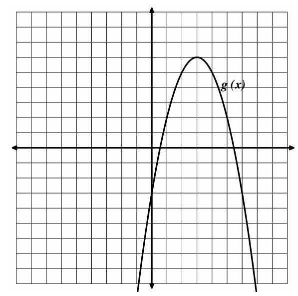


2a. $g(0) =$

b. $g(2) =$

c. $g(3) =$

d. $g(5) =$

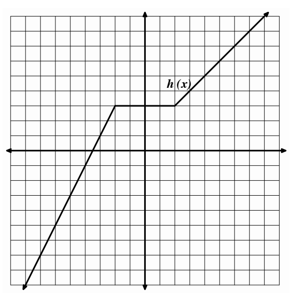


3a. $h(-4) =$

b. $h(0) =$

c. $h(2) =$

d. $h(4) =$

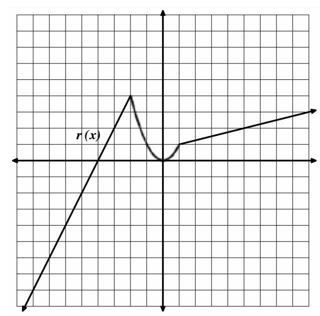


4a. $r(-3) =$

b. $r(-1) =$

c. $r(0) =$

d. $r(5) =$



5. Isaac lives 3 miles away from his school. School ended at 3 pm and Isaac began his walk home with his friend Tate who lives 1 mile away from the school, in the direction of Isaac's house. Isaac stayed at Tate's house for a while and then started home. On the way he stopped at the library. Then he hurried home. The graph at the right is a **piece-wise defined function** that shows Isaac's distance from home during the time it took him to arrive home.

a. How much time passed between school ending and Isaac's arrival home?

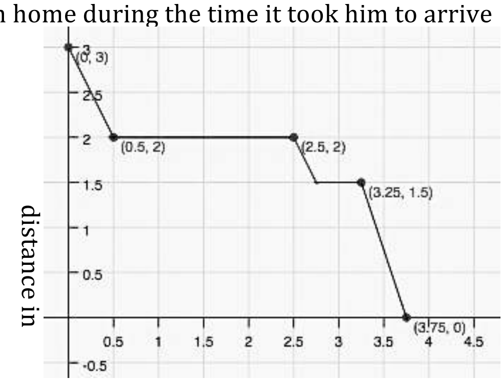
b. How long did Isaac stay at Tate's house?

c. How far is the library from Isaac's house?

d. Where was Isaac, 3 hours after school ended?

e. Use function notation to write a mathematical expression that says the same thing as question d.

f. When was Isaac walking the fastest? How fast was he walking?



time in

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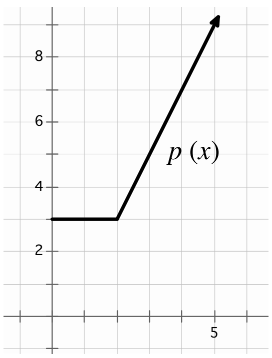
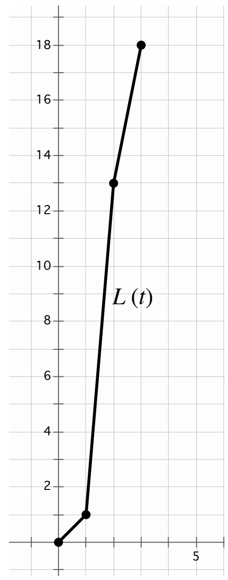
SET

Topic: Writing piece-wise defined functions

6. A parking garage charges $3 for the first two hours that a car is parked in the garage. After that, the hourly fee is $2 per hour. Write a piece-wise function *p(x)* for the cost of parking a car in the garage for *x* hours. (The graph of *p(x)* is shown.)





7. Lexie completed an 18 mile triathlon. She swam 1 mile in 1 hour, bicycled 12 miles in 1 hour, and then ran 5 miles in 1 hour. The graph of Lexie's distance versus time is shown. Write a piecewise function *L(t)* for the graph.

GO

Topic: Using the point-slope formula to write the equations of lines.

Write the equation of the line (in point-slope form) that contains the given slope and point.

8. *p:* (1, 2); m = 3

9. *p:* (1, -2); m = -1

10. p: (5, -1); m = 2

Write the equation of the line (in point-slope form) that contains the given points.

11. *K* (0, 0); *L* (-4, 5)

12. *X* (-1, 7); *Y* (3, -1)

13. *T* (-1, -9); *V* (5, 18)

4.2 Bike Lovers

A Solidify Understanding Task

Michelle and Rashid love going on long bike rides.

Every Saturday, they have a particular route they bike together that takes four hours. Below is a piecewise function that estimates the distance they travel for each hour of their bike ride.

$$f(x) = \begin{cases} 16x, & 0 \le x \le 1 \\ 10(x-1)+16, & 1 < x \le 2 \\ 14(x-2)+26, & 2 < x \le 3 \\ 12(x-3)+40, & 3 < x \le 4 \end{cases}$$

1. What part of the bike ride are they going the fastest? Slowest?

2. What is the domain of this function?

3. Find f(2). Explain what this means in terms of the context.

4. How far have they traveled at 3 hours? Write the answer using function notation.

5. What is the total distance they travel on this bike ride?

6. Sketch a graph of the bike ride as a function of distance traveled over time.

Rashid also has a route he likes to do on his own and has the following continuous piecewise function to represent the average distance he travels in minutes:

$$g(x) = \begin{cases} \frac{1}{4}(x) & 0 \le x \le 20 \\ \frac{1}{5}(x-20)+5 & 20 < x \le 50 \\ \frac{2}{7}(x-50)+11 & 50 < x \le 92 \\ \frac{1}{8}(x-a)+b & 92 < x \le 100 \end{cases}$$

7. What is the domain for this function? What does the domain tell us?

8. What is the average rate of change during the interval [20, 50]?

9. Over which time interval is the greatest average rate of change?

10. Find the value of each, then complete each sentence frame:
    a. g(30) = _________. This means…
    b. g(64)= ___________. This means…
    c. g(10) = __________. When finding output values for given input values in a piecewise function, you must …

11. Complete the last equation by finding values for a and b.

12. Sketch a graph of the bike ride as a function of distance traveled as a function of time.

Use the following continuous piecewise defined function, where x represents time in minutes and h(x) represents distance traveled in km, to answer the following questions.

$$h(x) = \begin{cases} \frac{1}{4}x^2 & 0 \le x \le 10 \\ \frac{1}{2}(x-10)+c & 10 < x \le 20 \\ 2(x-20)+30 & 20 < x \le 30 \end{cases}$$

13. Find the value of c.

14. Sketch the graph (label axes).

15. What is the domain of h(x)?

16. What is the range of h(x)?

17. Which five minute interval of time has the greatest average rate of change?
    a. [0, 5] b. [5, 10] c. [10, 15] d. [25, 30]

    What is the average rate of change over this interval?

18. Find h(8).

19. Find h(15).

4.2 Bike Lovers – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is for students to solidify their understanding of piecewise functions using their background knowledge of domain and linear functions. Students will solidify their understanding of piecewise functions by

- answering questions relating to a piecewise function
- using their knowledge of domain to graph each section of the piecewise function
- graphing complete piecewise-defined functions from equations
- interpreting the context of a piecewise function

Core Standards Focus:
F.IF.2 Use function notation, and evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain of the function.

F.IF.7b Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Related Standards: F.IF.4, F.IF.6, F.IF.8

The Teaching Cycle:

Launch (Whole Class):

Begin this task by showing the graph and the correlating piecewise function from the 4.1 task *Some of This, Some of That* and ask students to explain features of a piecewise function. Highlighted features include:

- each piece has a specific function that is graphed depending on the domain given
- the functions are written in ascending order based on the domain

Once all students understand this, have them work through the task with a partner or in a small group. Provide the context for questions 1 – 6 by reading it aloud and then letting students know that they need to be able to provide explanations for each of their answers.

Explore (Small Group):
As you monitor, look for common student misconceptions to discuss during the whole group discussion. For example, some students may not realize that to find f(2), they should use the equation that includes 2 in its domain. Asking struggling students if they should enter 2 in for all four equations may be a good way to have them think about this idea without telling them that they should only plug this into the equation that has the corresponding domain. Also, as you monitor, listen for student explanations about their graphs. If your class as a whole is comfortable with the concepts involved with piecewise functions and is answering questions with little confusion, let them continue to complete the task before going to the whole group discussion. If you have more than 15-20% of your class struggling, then stop after a few pairs have graphed question six and have a whole group discussion prior to having students finish the task.

Discuss (Whole Group) Part I: Q1 – Q6
During this part of the whole group discussion, select a student to explain their graph for question six either by having them re-create this on chart paper or by having them use a document camera. It is important that they show how each part of the graph is connected to the specific equation in the given domain. Then go over questions 1-5, focusing on questions 2 and 3. The purpose of this discussion is to highlight how the time determines which equation is to be used to determine the distance traveled.

(If your class was struggling and stopped to have this discussion, then have them complete the task before you move on to the remaining whole group discussion.)

Discuss (Whole Group) Part II:Q7 – Q19
The focus for the second situation (questions 7-12) is to solidify the connections between domain, function, and graph. Select a student to graph Rashid's bike ride (and answer questions 7 and 12),

then choose another student to explain their answers to questions 8 and 9, another student to answer questions 10 and 11. Select another student to show their solution to questions 13 and 14, giving another example of finding values in a continuous piecewise function and also to show the graph.

Questions 13-19 are similar to the first 12 questions. If time runs out, these questions may be used as a warm-up for the next day, or for additional support for students who are not quite comfortable with analyzing piecewise-defined functions but need/want extra practice.

Aligned Ready, Set, Go: More Functions, More Features RSG 4.2

READY, SET, GO! Name Period Date

READY

Topic: Solving absolute value equations.

Solve for x. (You will have two answers.)

1. $|x| = 7$

2. $|x - 6| = 3$

3. $|w + 4| = 11$

4. $-9|m| = -63$

5. $|3d| = 15$

6. $|3x - 5| = 11$

7. $-|m + 3| = -13$

8. $|-4m| = 64$

9. $2|x + 1| - 7 = -3$

10. $5|c + 3| - 1 = 9$

11. $-2|2p - 3| - 1 = -11$

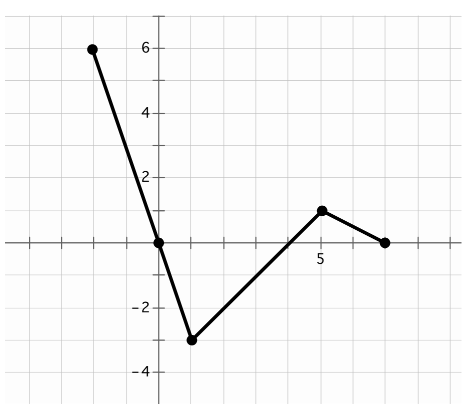
12. Explain why the equation $|m| = -3$ has no solution.

SET

Topic: Reading the domain and range from a graph

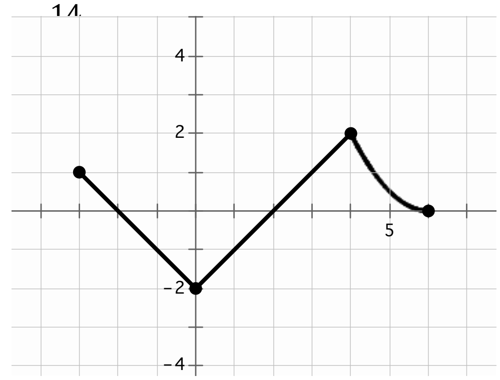
State the domain and range of the piece-wise functions in the graph. Use interval notation.

13.



a. Domain: b. Range:

14.

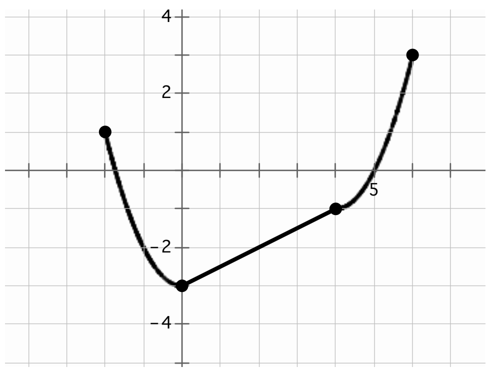


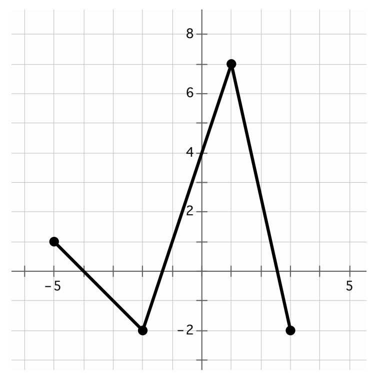
a. Domain: b. Range:

For each of the graphs below write the interval that defines each piece of the graph. Then write the domain of the entire piece-wise function.

Example: (Look at the graph in #14. Moving left to right. Piece-wise functions use set notation.)
Interval 1 $-3 \le x < 0$
Interval 2 $0 \le x < 4$
Interval 3 $4 \le x \le 6$
Domain: $[-3,6]$ (We can use interval notation on the domain, if it's continuous.)

Pay attention to your inequality symbols! You do not want the pieces of your graph to overlap. Do you know why?



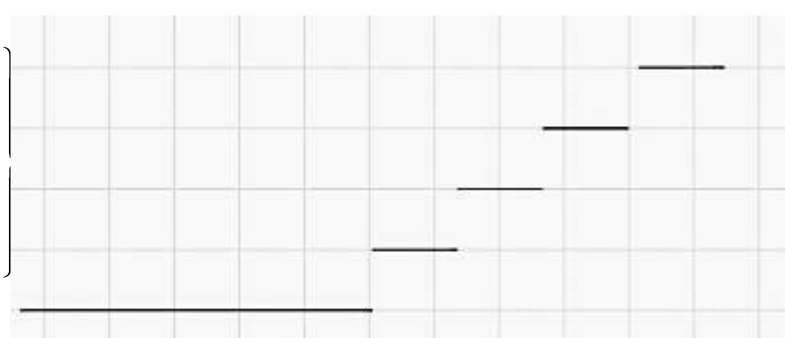


15. a. Interval 1 ______________
b. Interval 2 ______________
c. Interval 3 ______________
d. Domain: ______________

16. a. Interval 1 ______________
b. Interval 2 ______________
c. Interval 3 ______________
d. Domain: ______________

17. So far you've only seen continuous piece-wise defined functions, but piece-wise functions can also be non-continuous. In fact, you've had some real life experience with one kind of non-continuous piece-wise function. The graph below represents how some teachers calculate grades. Finish filling in the piece-wise equation. Then label the graph with the corresponding values.

$$f(x)=\begin{cases} A, & \_\_\_\_\_ \quad x \quad \_\_\_\_\_ \\ B, & \_\_\_\_\_ \quad x \quad \_\_\_\_\_ \\ C, & \_\_\_\_\_ \quad x \quad \_\_\_\_\_ \\ D, & \_\_\_\_\_ \quad x \quad \_\_\_\_\_ \\ F, & \_\_\_\_\_ \quad x \quad \_\_\_\_\_ \end{cases}$$



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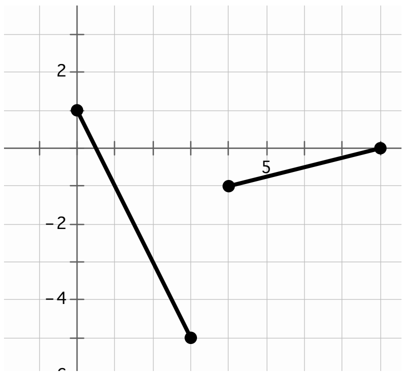
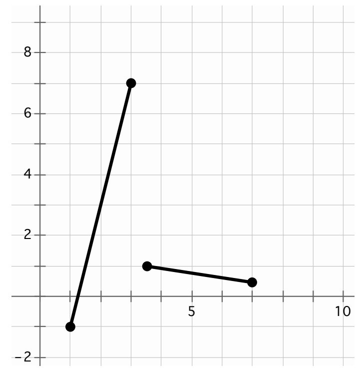
Write the piece-wise equations for the given graphs.

18.



19.



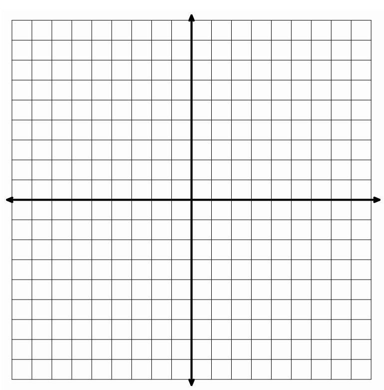
GO

Topic: Transformations on quadratic equations

Beginning with the parent function $f(x) = x^2$, write the equation of the new function $g(x)$ that is a transformation of $f(x)$ as described. Then graph it.

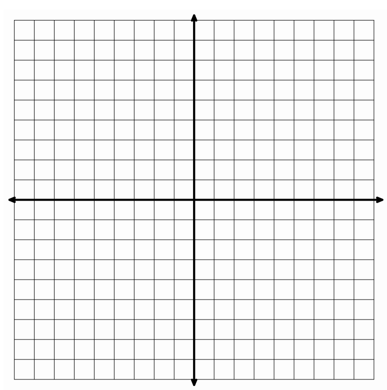
20. Shift $f(x)$ left 3 units, stretch vertically by 2, reflect $f(x)$ vertically, and shift down 5 units.

$g(x) =$ ________________



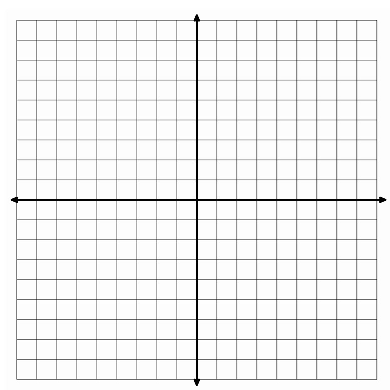
21. Shift $f(x)$ right 1, stretch vertically by 3, and shift up 4 units.

$g(x) =$ ________________



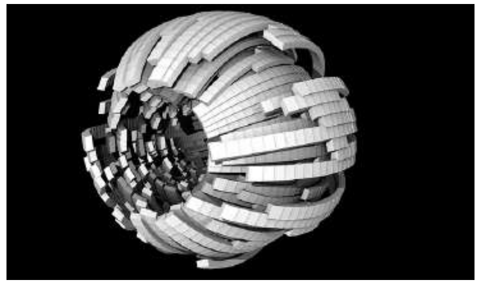
22. Shift $f(x)$ up 3 units, left 6, reflect vertically, and stretch by ½

$g(x) =$ ________________



4.3 More Features, More Functions

A Solidify Understanding Task



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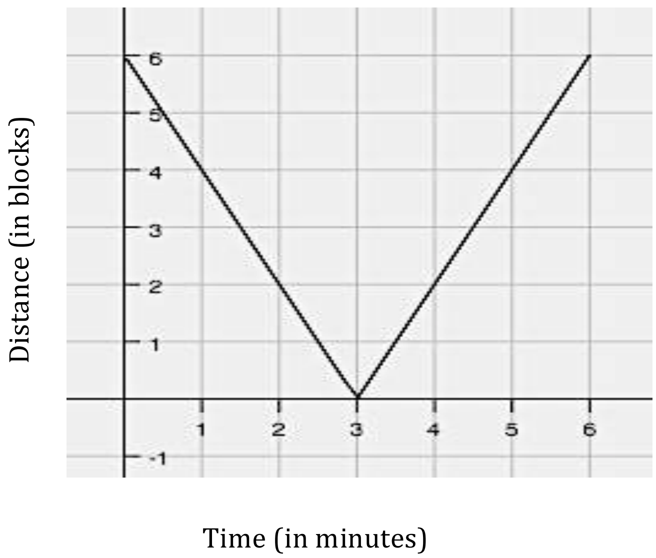
Part I

Michelle likes riding her bike to and from her favorite lake on Wednesdays. She created the following graph to represent the distance she is away from the lake while biking.



1. Interpret the graph by writing three observations about Michelle's bike ride.

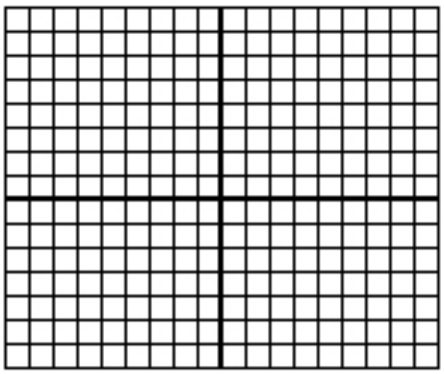
2. Write a piecewise function for this situation, with each linear function being in point-slope form using the point (3,0). What do you notice?

3. This particular piecewise function is called a linear absolute value function. What are the traits you are noticing about linear absolute value functions?

Part II

In this part of the task, you will solidify your understanding of piecewise and use your knowledge of transformations to make sense of absolute value functions. Follow the directions and answer the questions below.

4. Graph the linear function $f(x) = x$.



5. On the same set of axes, graph $g(x) = |f(x)|$.

6. Explain what happens graphically from $f(x)\ to\ g(x)$.

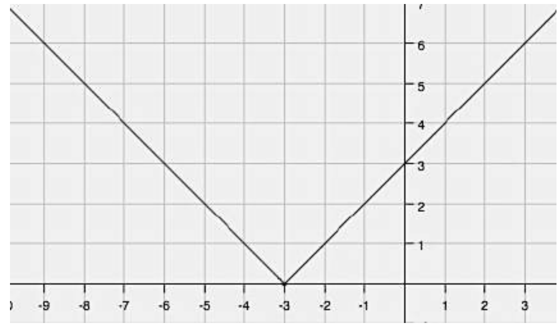
7. Write the piecewise function for $g(x)$. Explain your process for creating this piecewise function and how it connects to your answer in question 3.

8. Complete the table of values from [-4, 4] for *f(x)* and *g(x).* Explain how this connects to your answer in questions 3 and 4.

x	*f(x)*	*g(x)*
-4		
-3		
-2		
-1		
0		
1		
2		
3		
4		

Part III

9. The graph below is another example of an absolute value function. The equation of this function can be written two ways:



as an absolute value function: $f(x) = |x+3|$

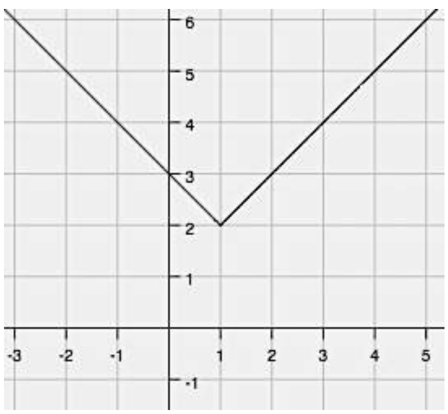
or as a piecewise:

$$f(x) = \begin{cases} -(x+3), & x < -3 \\ (x+3), & x \geq -3 \end{cases}$$

How do these two equations relate to each other?

Below are graphs and equations of more linear absolute value functions. Write the piecewise function for each. See if you can create a strategy for writing these equations.

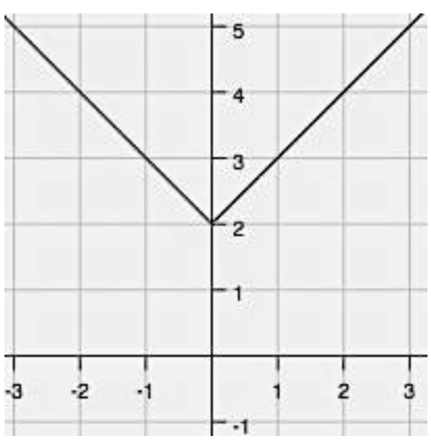
10.



Abolute value: $f(x) = |x-1| + 2$

Piecewise: $f(x) =$

11.

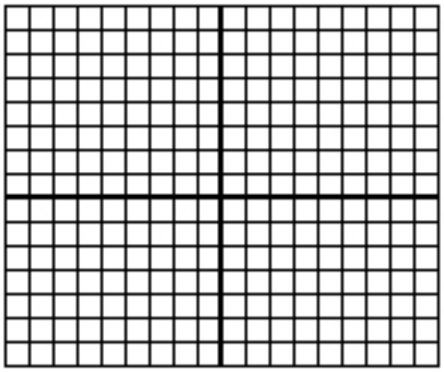


Abolute value: $f(x) = |x| + 2$

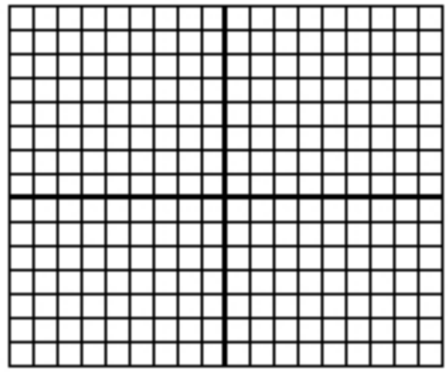
Piecewise: $f(x) =$

Graph the following linear absolute value piecewise functions.

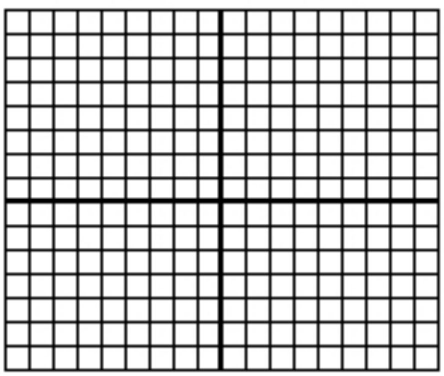
12. $f(x) = |x - 4| = \begin{cases} -(x-4), & x < 4 \\ (x-4), & x \geq 4 \end{cases}$



13. $f(x) = |x| + 1 = \begin{cases} -(x) + 1, & x < 0 \\ (x) + 1, & x \geq 0 \end{cases}$



14.



Piecewise: $f(x) = \begin{cases} -3(x+2) + 1, & x < -2 \\ 3(x+2) + 1, & x \geq -2 \end{cases}$

Absolute Value: $f(x) =$

15. Explain your method for doing the following:

a) Writing piecewise linear absolute value functions from a graph.

b) Writing piecewise linear absolute value functions from an absolute value function.

c) Graphing absolute value functions (from either a piecewise or an absolute value equation).

4.3 More Features, More Functions – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is for students to solidify their understanding of piecewise functions using linear absolute value. Students will also learn how to graph, write, and create linear absolute value functions by looking at structure and making sense of piecewise defined functions.

Core Standards Focus:

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain of the function.*

F.IF.7b Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Related Standards: F.IF.4, F.IF.6

The Teaching Cycle:

Launch (Whole Class):

Read the context of the bike ride, then have students answer the questions from part I individually (this could also serve as a warm-up to begin class). After a few minutes, ask students to popcorn out observations about this situation. Focus on answers such as "Michelle rode her bike at the same rate both directions" and "It took three minutes to get to lake and three minutes to get back". Next, show the function in point-slope form $f(x) = \begin{cases} -2(x-3)+0, & x < 3 \\ 2(x-3)+0, & x \geq 3 \end{cases}$ as a review of writing piecewise. Prior to the explore phase, make sure students understand that linear absolute value functions have opposite slopes for each section of the graph and that the function values have symmetry about the vertex (which also happens to be where the piecewise function changes from one equation to the other).

Explore (Small Group):

As you monitor, press for student understanding of where the graphs of *f(x)* and *g(x)* are the same and where they are different. Have students explain how they see the relationship between the graphs, equations, and tables before they move on to Part III.

As students work in small groups for Part III, the goal is for students to make connections between the graph and the two different ways to write linear absolute value functions (as an absolute value and as a piecewise-defined function). Select different groups of students to share their strategies to move from one representation to another (graph to absolute value equation, graph to piecewise, piecewise to graph, piecewise to absolute value equation, absolute value equation to piecewise).

For groups who struggle, have them explain what they understand about question 6 and then ask probing questions to connect them to their prior knowledge. Question 6 provides opportunities to make connections to piecewise, transformations, and linear functions.

Have them try questions 7 and 8 while you move on to another group. Answers to 7 and 8 (piecewise using point-slope form):

$f(x) = \begin{cases} -(x-1)+2, & x < 1 \\ (x-1)+2, & x \geq 1 \end{cases}$ and $f(x) = \begin{cases} -(x)+2, & x < 0 \\ (x)+2, & x \geq 0 \end{cases}$

Discuss (Whole Class):

The goal of the whole group discussion is to have ALL students able to do the following:

- Explain the vertical reflection of linear functions when graphing linear absolute value;
- Use transformations to graph absolute value functions;
- Write piecewise functions from a graph and from an absolute value equation; and
- Understand how the domain is used to go from one section of a graph to another in a piecewise function.

You may wish to start the whole group discussion by having a student explain their understanding of the relationship between *f(x)* and *g(x)* in part II. Be sure to include all representations (graph, equation, and table).

Sequence the rest of the whole group discussion so that different students share their strategies for how they go from one representation to another (graphing and writing equations for absolute value piecewise functions). Connect prior understandings (transformations, domain, linear functions, piecewise, etc). Highlight that the function inside the absolute value has a domain change anywhere that the interior function would change from positive to negative.

Aligned Ready, Set, Go: More Functions, More Features RSG 4.3

READY, SET, GO! Name Period Date

READY

Topic: Finding the x-intercept(s) for a quadratic function

Find the x-intercepts of the following quadratic functions.

1. $y = x^2 + 3x - 10$

2. $y = x^2 + 8x + 7$

3. $y = 6x^2 + 7x - 20$

4. $y = (x-2)^2 - 9$

5. $y = -(x+3)^2 + 9$

6. $y = \frac{1}{2}(x-1)^2 - 2$

SET

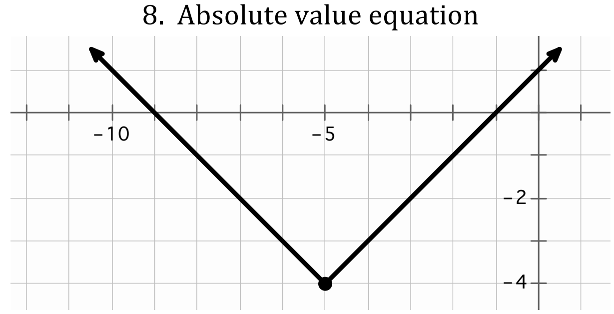
Topic: Absolute value equations

Use the given information to write the indicated form of the function.

7. Piecewise equation

x	$f(x)$
-1	9
0	6
1	3
2	0
3	3
4	6

8. Absolute value equation

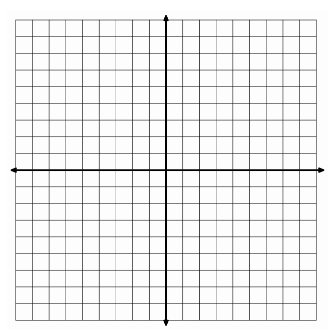


9. Make a table of values. Be sure to include the vertex in the table.

$h(x) = 5|x-6| - 8$

x	$h(x)$

10. Graph $f(x) = \begin{cases} -\frac{2}{3}(x-6)+4, & x < 6 \\ \frac{2}{3}(x-6)+4, & x \geq 6 \end{cases}$



GO

Topic: Interpreting absolute value

Evaluate each expression for the given value of the variable.

11. $-s$; $s = 4$

12. $-t$; $t = -7$

13. $-x$; $x = 0$

14. $-w$; $w = -11$

15. $|v|$; $v = -25$

16. $-(a)$; $a = -25$

17. $-(-n)$; $n = -2$

18. $|-(-p)|$; $p = -6$

19. $|-(-q)|$; $q = 8$

20. $-|-(-r)|$; $r = -9$

4.4 Reflections of a Bike Lover

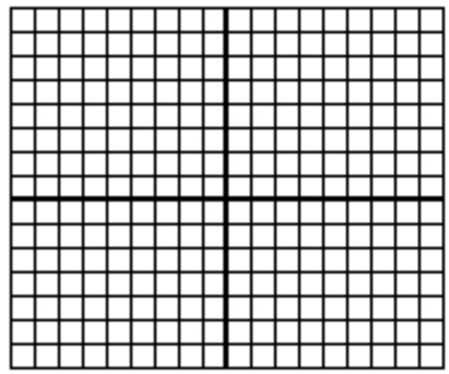
A Practice Understanding Task

CC BY Netherlands Tourism
https://goo.gl/images/2Za8Zc

1. Graph the function $f(x) = x^2 - 4$

2. Graph $g(x) = |f(x)|$ on the same set of axes as *f(x)*.

3. Explain what happens graphically.

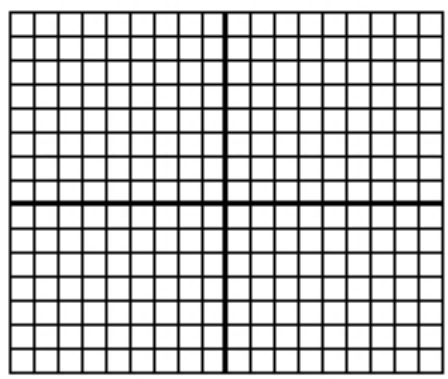


4. Write the piecewise function for $g(x)$.

5. Graph the function $f(x) = (x + 1)^2 - 9$

6. Graph $g(x) = |f(x)|$.

7. Explain what happens graphically.



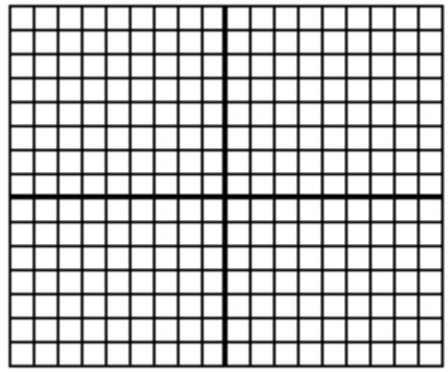
8. Write the piecewise function for $g(x)$.

9. What do you have to think about when writing any absolute value piecewise function?

Graph the following absolute value functions and write the corresponding piecewise functions for each.

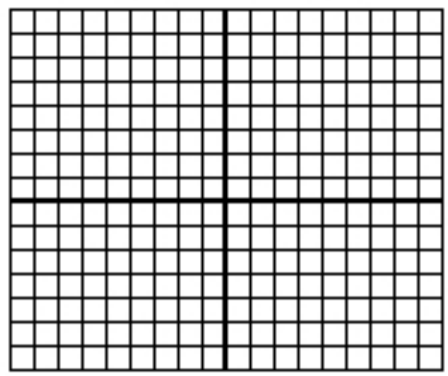
10. $g(x) = |x^2 - 4| + 1$

    Piecewise:



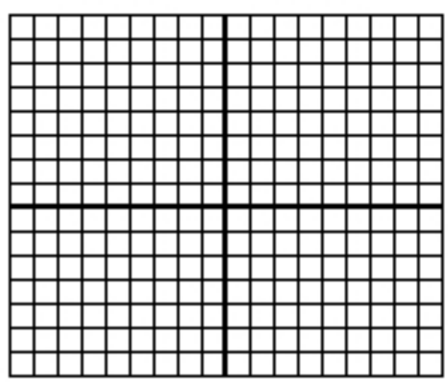
11. $g(x) = |(x+2)^2 - 4| + 3$

    Piecewise:



12. $g(x) = |2^x - 4|$

    Piecewise:



4.4 Reflections of a Bike Lover – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is for students to practice graphing absolute value functions and determine a process for writing them as piecewise-defined functions. To become more precise in their language, students will extend their knowledge of linear absolute value to other functions. Understanding where the piecewise function changes (based on sign change of function within the absolute value) is key. Students will also practice graphing with transformations.

Core Standards Focus:

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain of the function.*

F.IF.7b Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Related Standards: F.IF.2, F.IF.4, F.IF.6, F.BF.3

The Teaching Cycle:

Launch (Whole Class):

Begin this task by having students share with a partner what they know about writing absolute value functions as piecewise-defined functions. You may even want to have a function and graph posted for this short "turn and talk". Listen in and then choose a student or two to share (get out one or more of these attributes):

- You go from one equation to the other where the graph reflects over the x-axis
- The domain for each section of the piecewise is determined by where the interior function changes sign
- When the function inside the absolute value produces negative answers, you write the 'opposite equation' in for that part of the domain of the piecewise function

- The reflection about the x-axis occurs graphically when the output values of the linear function within the absolute value is negative

This does not need to all come out at this time as it is also part of the explanation students are to complete when answering questions within the task. Once students seem to understand the idea, have them work on the task.

Explore (Small Group):

In small groups, have students answer the questions on the task and talk about their process for writing piecewise functions. As you monitor, ask students to show you the connections between the absolute value function, the graph, and the piecewise-defined function they write. Also listen for how students explain the connections between output values for $f(x)$ compared to $g(x)$. As students complete the task, choose a group to chart their answer to question ten and ask them to be the first group to share during the whole group discussion. Also ask them to include an explanation for graphing absolute value equations.

Discuss (Whole Class):

As you begin the whole group discussion, select and sequence students to share their understanding of absolute value functions so that the difficulty increases from one problem to the next (it is recommended to share out approximately three of the problems, starting with the first problem and including the last problem). When the first group shares, have them post their chart on the process they use for graphing and writing the piecewise functions of absolute value equations as they explain their solution to the first question. Ask the class if they have any questions if this is not already established as a norm. You may also ask the class if they agree with the graph and/or process.

As you progress through having the next two groups share their graphs and piecewise functions, ask the class if there is anything they would adjust or add to the chart paper relating to the process. By the end of the task, students should be comfortable with taking any absolute value function and writing it as a piecewise-defined function. Students should also be comfortable with the transformations for creating these graphs.

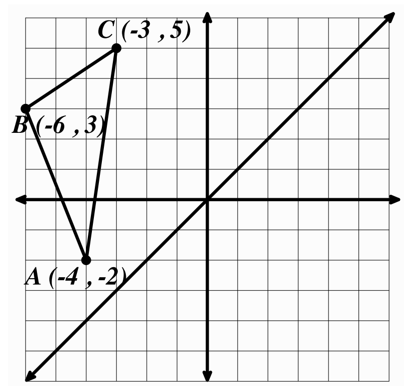
Aligned Ready, Set, Go: More Functions, More Features RSG 4.4

READY, SET, GO! Name Period Date

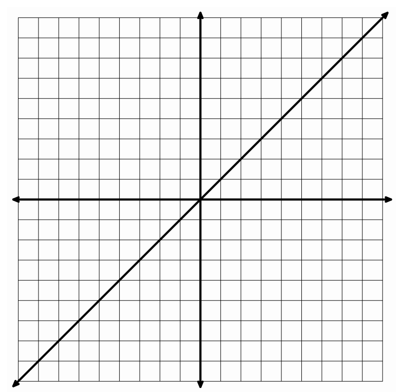
READY

Topic: Reflecting Images

1. Reflect ΔABC across the line y = x. Label the new image as $\Delta A'B'C'$. Label the coordinates of *points* $A'B'C'$. Connect segments AA', BB', and CC'. Describe how these segments are related to each other and to the line y = x .



2. On the graph provided to the right, draw a 5-sided figure in the 4th quadrant. Label the vertices of the pre-image. Include the coordinates of the vertices. Reflect the pre-image across the line y = x. Label the image, including the coordinates of the vertices.



3. A table of values for a four-sided figure is given in the first two columns. Reflect the image across the line y = x, and write the coordinates of the reflected image in the space provided.

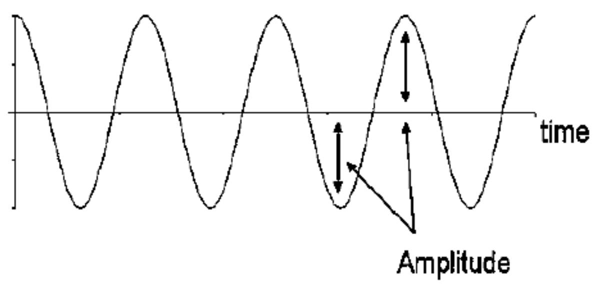
A	$(-6,2)$	A'	
B	$(-4,5)$	B'	
C	$(-2,3)$	C'	
D	$(-3,-1)$	D'	

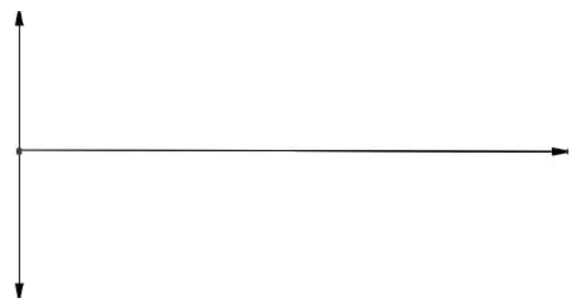
SET

Topic: Absolute value and non-linear functions

4. *Figure 1* is the graph of a sound wave. The height (or depth) of the graph indicates the magnitude and direction *f(x)* reaches from the norm or the undisturbed value. In this case that would be the x-axis. When we are only concerned with the distance from the x-axis, we refer to this distance as the **amplitude.** Since distance alone is always positive, **amplitude** can be described as the absolute value of *f(x).* Use the graph of a sound wave to sketch a graph of the absolute value of the amplitude or $\boldsymbol{y = |f(x)|}$.

figure 1



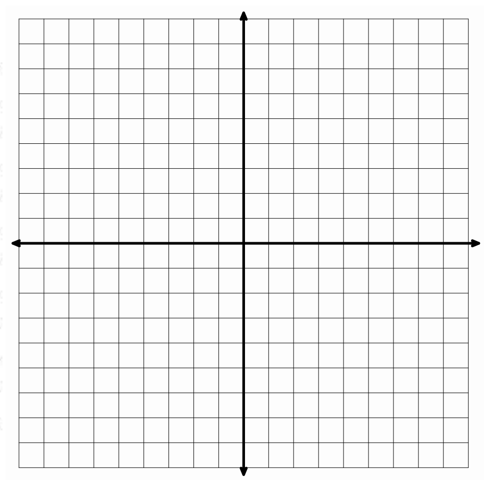


5. *Figure 2* is a table of values for $g(x) = (x+3)^2 - 9$ What values in the table would need to change if the function were redefined as $h(x) = |g(x)|$?

6. Graph $h(x) = |g(x)|$.

figure 2

x	$g(x)$
-8	16
-7	7
-6	0
-4	-5
-3	-8
-2	-9
-1	-8
0	0
1	7
2	16



7. Write the piece-wise equation for $h(x) = |g(x)|$, as defined in question 6. Let the domain be all real numbers in the interval [-8, 2].

GO

Topic: Simplifying Radical Expressions

Simplify. Write the answers in simplest radical form. Some answers may consist of numbers with no radical sign.

8. $\left(-7-2\sqrt{5}\right)+\left(6+8\sqrt{5}\right)$

9. $\left(-10-\sqrt{13}\right)-\left(-11+5\sqrt{13}\right)$

10. $\left(4-\sqrt{50}\right)+\left(7+3\sqrt{18}\right)-\left(12-2\sqrt{72}\right)$

11. $\sqrt{98}+\sqrt{8}$

12. $\left(-2-7\sqrt{5}\right)+\left(2\sqrt{125}\right)-3\sqrt{625}$

13. $\left(3r^2-8\sqrt{3b^2}\right)-\left(2r^2-3\sqrt{27b^2}\right)$

14.
Write an equivalent form using exponents $\sqrt{x}+\sqrt{x^3}+\sqrt{x^5}+\sqrt{x^7}+\sqrt{x^9}+\sqrt{x^{11}}+\sqrt{x^{13}}+\sqrt{x^{15}}$.
Assume that $x \geq 0$.

4.5 What is Your Pace?

A Develop Understanding Task

Chandler and Isaac both like to ride bikes for exercise. They were discussing whether or not they have a similar pace so that they could plan a time to bike together. Chandler said she bikes about 12 miles per hour (or 12 miles in 60 minutes). Isaac looked confused and said he does not know how many miles he bikes in an hour because he calculates his pace (or rate) differently.

1. Use multiple representations (table, graph, equation, diagram) to model Chandler's information. Be sure to label each representation.

2. Explain connections between the representations created.

3. How many miles will Chandler travel at 30 minutes?

Isaac says he calculates his pace differently. He explains that he bikes a five minute mile, meaning that for every five minutes he bikes, he travels one mile.

4. Use multiple representations (table, graph, equation, diagram) to model Isaac's information. Be sure to label each representation.

5. How is this different than how Chandler describes her rate? Who goes at a faster rate?

6. How many miles will Isaac travel at 30 minutes?

7. Using the equations, tables, and graphs for Isaac and Chandler, make a list of observations about how these situations relate to each other.

8. The two situations in this task are considered **inverse functions** of each other. If Chandlers function is written as *f(x)* then we can say the inverse of this function is written as $f^{-1}(x)$. What do you think are characteristics of inverse functions?

4.5 What is Your Pace? – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is for students to surface ideas about inverse functions. The context for this task allows students to think about times when the input/output values can be switched, depending on how the situation is described. Students use their background knowledge of creating tables, equations, and graphs of linear functions while also focusing on the following:

- determine independent and dependent variables;
- recognize that a function and its inverse switch their x and y values;
- recognize that a function and its inverse seem to 'reflect' over the line $f(x) = x$;
- understand that the domain and range switch places (also part of the domain and range or independent/dependent variables switch places); and
- identify the relationship of a function and its inverse and how there are times when you want to solve for the input variable and make it the output variable, hence, creating the inverse.

Core Standards Focus:
F.BF.4: Find inverse functions.
a. Solve an equation of the form *f(x) = c* for a simple function *f* that has an inverse and write an expression for the inverse.
c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

Related Standards: F.IF.1, F.IF.2, F.BF.1

The Teaching Cycle:
Launch (Whole Class):
This task is about how fast two friends ride their bikes. You may wish to share some Tour De France statistics with your students (http://www.bikeraceinfo.com/tdf/tdfstats.html) or just provide information about how the pace for pro cyclists is quite a bit more than amateurs. As you read the

scenario, have students work independently to create the different representations for Chandler's pace, then work with a partner to complete the task.

Explore (Small Group):
Students should move smoothly through Chandler's situation and complete the various representations. Since the task focuses on inverse functions, as you monitor, you may notice some students not labeling at this time. If some students are labeling, do not correct them as this will be part of what they work through in small groups as they go through the task. If you notice a few students strugglingwith creating different representations for Chandler's situation, provide appropriate scaffolding to move them forward. Creating different representations for this linear function is review and should move fairly quickly.

After a few minutes, as students complete their representations for Chandler's situation, have them work in pairs to complete the task. As you monitor, listen for students to realize the input/output values have switched places in the context, and also that Isaac and Chandler actually bike at the same pace. Some students may think that Isaac is going faster because his pace is "5" and Chandler's is "1/5". Have students communicate with each other to discuss who is going 'faster' and why (this little productive struggle will emphasize the need for appropriate labels and show why context is important). Look for students who make connections and see the relationships between the graphs, equations, and the tables that will be valuable in the whole group discussion (which will focus on characteristics of a function and its inverse).

Discuss (Whole Class):
The whole group discussion will focus on the last two questions of the task. In comparing a function and its inverse, you will want the representations of both bike riders visible for all students as observations are shared. The whole group discussion should first start by having different students who made observations share. Answers should include the following (and should include visual models for each while the observation is being discussed):

- input/output values switch places (evidence for this observation may occur in the graph or via a table of values)
- as a result of the x and y coordinates switching places, the two graphs are symmetric about

- the line *y=x* (evidence: graph)
- inverse functions can be created by 'switching' the independent and dependent variables. (evidence: table of values, graph, equation)
- inverse functions can be created by switching the independent and dependent variables and then isolating the new dependent variable or this can be said as the inverse function is created by isolating the independent variable of the original function, then switching the independent/dependent variable around.

Throughout sharing these observations, press students to show examples (going between context and algebra). When showing the graph, for example, highlight points that mean the same but the ordered pair is switched: (60, 12) for Chandler is the same as (12, 60) for Isaac. In both situations, the distance and time are the same, but how you say them is switched.

Chandler: At 60 minutes, Chandler bikes 12 miles.

Isaac: At 12 miles, Isaac has biked for 60 minutes.

In this summary, be sure to have students understand what is happening with each representation:

- Table of values: *(x,y)* → *(y,x)* or in this case, (*t, d*)→ (*d, t*)
- Graph: the two equations are reflections of *y=x* (be careful not to place two functions on same set of axes, as the axes represent different values for each of the functions)
- Equation: to find the inverse of a function, you are switching the independent and dependent variables and creating a new inverse function.
- Equations for this task:
  - Chandler: $\mathrm{d(t)} = \frac{12}{60}\,\mathrm{t}$; Isaac as the inverse would be $\mathrm{d}^{-1}(\mathrm{t})$.
  - Isaac: $\mathrm{t(d)} = \frac{60}{12}\,\mathrm{d}$; Chandler as the inverse would be $\mathrm{t}^{-1}(\mathrm{d})$.

Aligned Ready, Set, Go: More Features, More Functions RSG 4.5

READY, SET, GO! Name Period Date

READY

Topic: Square Roots

The area of a square is given. Find the length of the side.

1. $16\ in^2$

2. $(x-11)^2\ ft^2$

3. $(25a^2+60a+36)\ cm^2$

4. If the length of the side of a square is $(\mathbf{x-24})$ **cm**, what do we know about the value of x?

Complete the table of values for $f(\mathrm{x})=\sqrt{\mathrm{x}}$. Write answers in simplest radical form.

5.

x	$f(x)$
1	
4	
9	
16	
25	
36	
49	
64	
81	
100	

6.

x	$f(x)$
25	
50	
75	
100	
125	
150	
175	
200	
225	
250	

7.

x	$f(x)$
x^2-2x+1	
x^2-4x+4	
x^2-6x+9	
$x^2-8x+16$	
$x^2-10x+25$	
$x^2-12x+36$	
$x^2-14x+49$	
$x^2-16x+64$	
$x^2-18x+81$	
$x^2-20x+100$	

SET

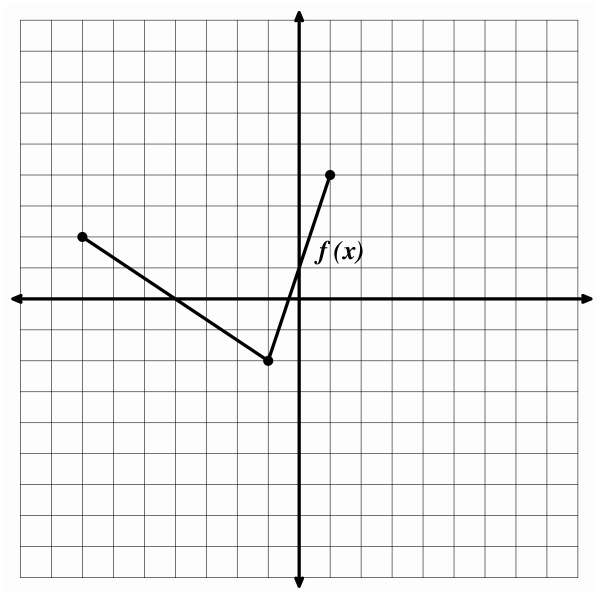
Topic: Inverse functions

8. **Given**: $\boldsymbol{f(x)} = \{(\mathbf{-13}, \mathbf{5})(\mathbf{-9}, \mathbf{-9})(\mathbf{-5}, \mathbf{-2})(\mathbf{-1}, \mathbf{-5})(\mathbf{0}, \mathbf{-4})(\mathbf{4}, \mathbf{6})(\mathbf{9}, \mathbf{10})(\mathbf{14}, \mathbf{32})\}$

   Find $\boldsymbol{f^{-1}(x)} = \{(\quad , \quad)(\quad , \quad)(\quad , \quad)(\quad , \quad)(\quad , \quad)(\quad , \quad)(\quad , \quad)(\quad , \quad)\}$

9. The function $\boldsymbol{f}(\mathbf{x})$is shown on the graph. Graph $\boldsymbol{f}^{-\mathbf{1}}(\mathbf{x})$ on the same set of axes.

10. Is the graph of $f^{-1}(x)$ also a function?
    Justify your answer.



11. I am going on a long trip to Barcelona, Spain. I am only taking one suitcase and it is packed very full. I plan to arrive completely exhausted at my hotel in the middle of the night. The only thing I will want to take out of my suitcase is a pair of pajamas. So when I packed my suitcase at home, did I want to put my pajamas in first, somewhere in the middle, or last? Explain.

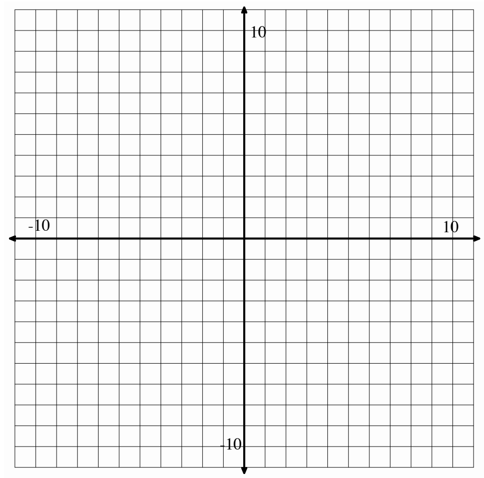
12. Write the inverse function for the table of values.

Input x	-10	-6	-2	2	6
Output $g(x)$	-2	-1	0	1	2

Input x					
Output $g^{-1}(x)$					

13. Use the points in problem 12. Graph $g(x)$ in black and $g^{-1}(x)$ in a different color on the coordinate grid at the right. Graph the line of reflection for the corresponding points.

14. Is $g^{-1}(x)$ also a function? Justify your answer.



GO

Topic: Multiplying square roots

Multiply. Write your answers in simplest radical form.

15. $\sqrt{3}(4+5\sqrt{3})$

16. $6\sqrt{11}(2-\sqrt{11})$

17. $(1-7\sqrt{2})(1-\sqrt{2})$

18. $(3+2\sqrt{13})(3-2\sqrt{13})$

19. $(4+3\sqrt{5})(4-3\sqrt{5})$

20. $(1-3\sqrt{6})(5-2\sqrt{6})$

4. 6 Bernie's Bikes

A Solidify Understanding Task

Bernie owns *Bernie's Bike Shop* and is advertising his company by taking his logo and placing it around town on different-sized signs. After creating a few signs, he noticed a relationship between the amount of ink he needs for his logo and the size of the sign.

1. The table below represents some of the signs Bernie has created and the relationship between the amount of ink needed versus the size of the sign. Complete the information below to help Bernie see this relationship (don't forget to label your graph).

Length of sign (in feet)	Ink needed (in ounces)
3	9
4	16
2	4
15	225
x	

Function: **Domain:** **Range:**

Graph:

2. Using question 1, complete the information below for the ***inverse*** of this function (don't forget to label your graph).

Function: **Domain:** **Range:**

Graph:

3. Explain in words what the inverse function represents.

Bernie likes the look of his signs when the vertical height is 5 inches more than twice the horizontal length.

4. Complete the table and write a function for the vertical height of signs with a given horizontal length. Use Bernie's verbal rule to help you decide what numbers go in the table.

Length of sign (in feet)	Vertical height (in feet)
3	
4	
2	
15	
x	

Verbal rule: ***The vertical height is 5 inches more than twice the horizontal length.***

Function: $\boldsymbol{f(x)} =$

Domain:

Range:

5. Create a table for the inverse function, $f^{-1}(x)$. Write a verbal rule that will help Bernie understand what your inverse function does. Label the table to clearly indicate what the input variable x represents for your inverse function.

Verbal rule:

Function: $\boldsymbol{f^{-1}(x)} =$

Domain:

Range:

6. Bernie's rule contained two operations, multiplying by 2 and adding 5, performed in that order. List the operations in the correct order that appear in your inverse rule:

7. In general, describe how you can find the inverse rule for a function by listing the order of operations in the original function.

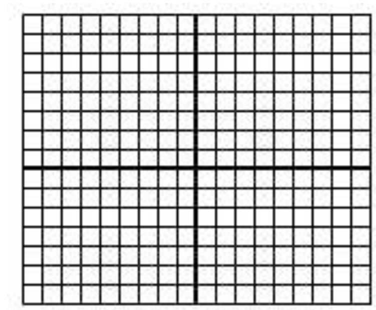
Part II

Determine the inverse rule for each function, then sketch the graphs and state the domain and range for both the original function and it's inverse.

8. $f(x) = x^2 - 1$; $f^{-1}(x) =$

Domain: Domain:

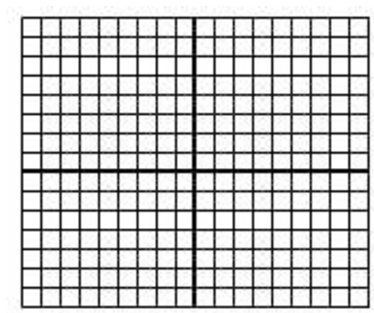
Range: Range:



9. $g(x) = 3x + 2$; $g^{-1}(x) =$

Domain: Domain:

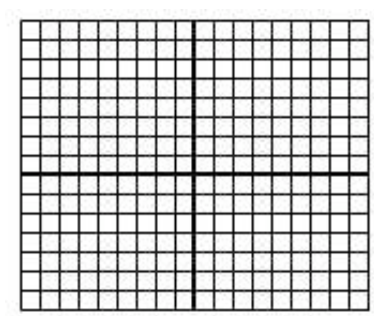
Range: Range:



10. $f(x) = (x + 3)^2$; $f^{-1}(x) =$

Domain: Domain:

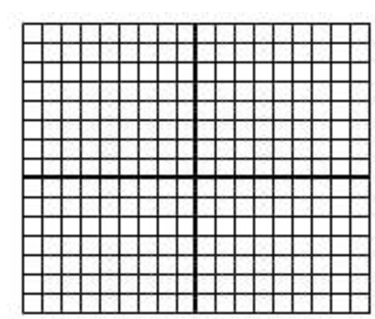
Range: Range:



11. $f(x) = x^3$; $f^{-1}(x) =$

Domain: Domain:

Range: Range:



4. 6 Bernie's Bikes – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is for students to solidify their understanding about inverse functions. Students will understand how to find the inverse of a function and know when to restrict the domain so that they can produce an invertible function from a non-invertible function. Students will also become familiar with square root functions as a result of this task and will connect square root functions to their domain, range, and graphs.

Core Standards Focus:

F.BF.4: Find inverse functions.

a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

Related Standards: F.IF.1, F.IF.2, F.BF.1

The Teaching Cycle:

Launch (Whole Class):

Start this task by asking the whole group "What do you know about a function and its inverse?" After the whole group highlights their current understanding of features of a function and its inverse, have them complete Part I of the task.

Explore (Small Group) Part I:

Have students spend a few minutes working on this task individually before having them work in pairs so that they can have time to make sense of the task themselves. As you monitor, look for how students are graphing the function $f(x) = x^2$ and identifying the domain and range. The choices for this may vary at this time and will actually contribute to the whole group discussion so don't try to have everyone with the exact same answer right now. At some point, have students work in pairs to continue working on the task, focusing more on their understanding of a function and its inverse and less on how they are determining the domain of the situation. Questions 4 – 7 are intended to help students articulate how the operations in the original function are related to the operations in the inverse function, and to develop a strategy for finding the inverse function rule. Listen for students who are noticing that the inverse function consists of the inverse operations of the operations in the original function, and that these inverse operations are performed in the reverse order.

Discuss (Whole Class) Part I:

As most students complete Part I of the task, bring the class together for the first whole group discussion. One focus of this discussion is to see how the function $f(x) = x^2$ and its inverse, $f^{-1}(x) = \sqrt{x}$ show up in the table, graph, function and situation. We will also examine how and why the domain restriction would occur if the original domain had been all real numbers. Since the domain of the original function will determine how students will handle the inverse function, sequence this discussion so that students share based on how you want to build understanding of inverse functions.

An example of how to do this is to start with having a group whose equation and representations are correct, however, their original domain is the set of all real numbers. Begin by having this group show their answers to question 1, then explain to the class how they answered question 2. First focus on what is right about their understanding (the function itself, knowing that you switch the input/output values, etc). Before moving to the next group to share, make sure everyone sees how the domain and range answers also trade places and that the class talks about how the inverse is not actually a function. Tell the class you want to talk about this more, but come back to it after you have a second group share. For the next group, select someone whose domain goes from 0 to ∞, or a number that they feel would represent the largest possible side length of a sign. Have this group share what is the same as the first group, but then highlight the differences (the graph, table, and domain/range).

At this time, ask the whole group if they have questions and check for understanding that ALL students understand how to find the inverse of a function and that, in this particular situation, the original domain was actually restricted due to the situation, which allowed for the inverse to also be a function. It is important to note at this time that students have now learned how to graph the square root function. To summarize Bernie's Bike situation, go back to the example where the original domain was all real numbers. Highlight that there are times when a function's inverse will not also be a function. Ask the class, "If you wanted to find the inverse of $f(x) = x^2$ and there was not a context that naturally limits the domain, what restrictions could be made with the original function such that the inverse would also be a function?" Make sure students understand this idea and that they also know how to find an inverse function from an equation, a table of values, and from a graph. At this time, have students work in small groups to complete the task.

Next discuss questions 4 – 7 to solidify a strategy for finding the inverse of a function by paying attention to the operations in the original function. This discussion is facilitated by considering the wording in Bernie's rule, which identifies two operations: doubling and adding 5. Have students present their work who can explain how both of these operations need to be undone, in order to find the horizontal length for a given vertical height. Help students notice that the order we undo each operation matters by asking, "Should we take half of the vertical height and subtract 5 from the result, or should we subtract 5 from the vertical height and take half of the result?" Because students have already created a table of values for the inverse function, it should be clear that the second process produces the correct result. That is, the inverse operations need to be performed in the reverse order.

The following list includes the key ideas that should come out of this discussion:

- table of values: (x,y)→ (y,x)

- graph: the two equations are reflections over the line $y = x$ (when working with a context, be careful not to place two functions on same set of axes, as the axes represent different quantities for each of the functions)
- equation: to find the inverse of a function, you use inverse operations to undo the operations of the original function, but in the reverse order to create the new inverse function; this is essentially the same work as solving for the independent variable of the original function to make it the dependent variable of the inverse function.
- The domain and range of the original function switch places for the inverse function
- If we chose to use x to describe the independent variable in both the original function and the inverse function, it is important to note that the quantity represented by the variable x has changed meaning.

Explore (Small Group) Part II:
As students complete part II of the task, look for student understanding of finding the inverse of a given function. For this part of the task, students can graph both functions on the same set of axes to highlight the 'reflection over $y = x$'. Help students notice again the relationships between a function and its inverse.

Discuss (Whole Class) Part II:
For this whole group discussion, have different students who have completed the problems correctly chart their answers to questions 8 - 11. Go over each question with the whole group, asking them what they notice in each situation regarding the function and its inverse. Point out the cases where the domain of the original function has to be restricted so that the inverse relationship is also a function.
For this discussion, also highlight the square root function and its features. Questions 8 and 10 offer opportunities to make connections between transformations of functions and the equation, the domain and range, and the corresponding graph.

At the end of this task, students should have solidifief their understanding of inverse functions, and they will should also identify the features of the square root function, and how to graph and find the domain and range of square root functions.

Aligned Ready, Set, Go: More Features, More Functions 4.6

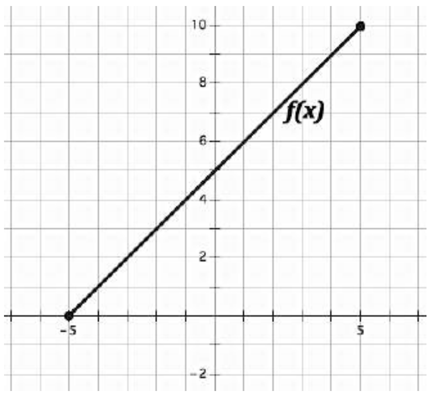
READY, SET, GO! Name Period Date

READY

Topic: Identifying Features of Functions

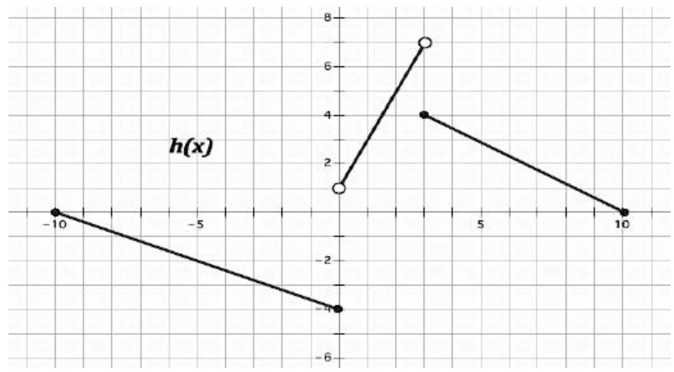
Given each representation of a function, determine the domain and range. Then indicate whether the function is discrete, continuous, or discontinuous and increasing, decreasing, or constant.

1.



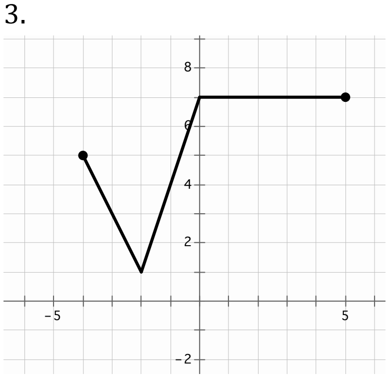
Description of Function:

2.



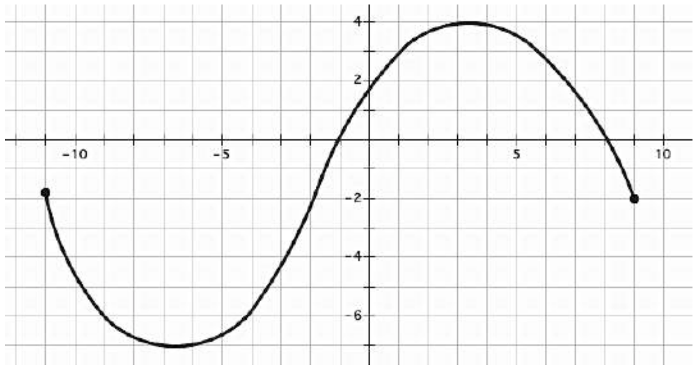
Description of Function:

3.



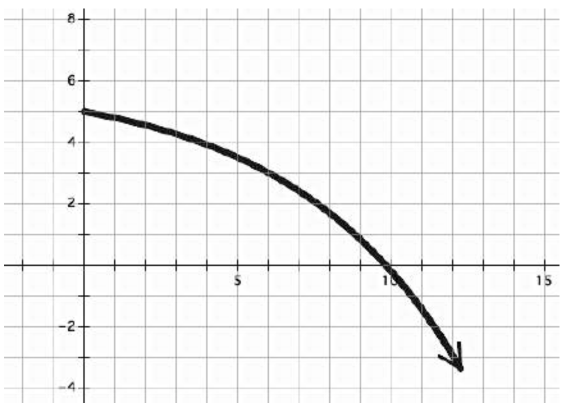
Description of Function:

4.



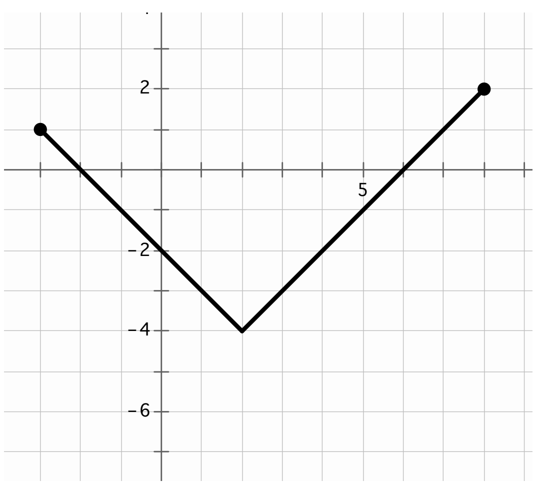
Description of Function:

5.



Description of Function:

6.



Description of Function:

SET

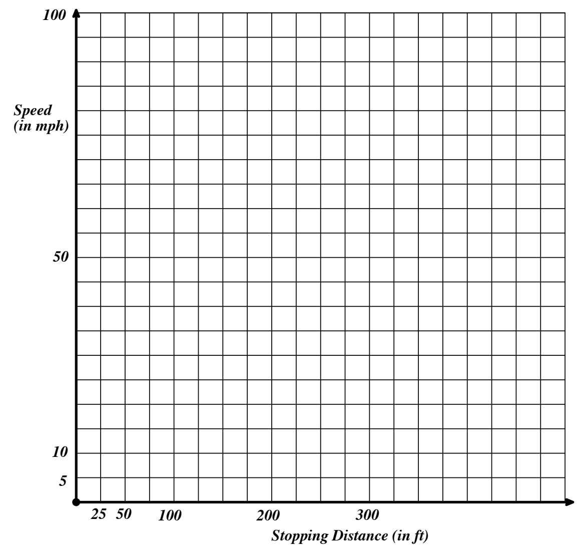
Topic: Square root functions

The speed limit for driving in a school zone is 20mph. That seems so slow if you're riding in a car. But have you ever wondered how quickly you could come to a complete stop going that speed (even if you had super quick reflexes)? It would take you over 13 feet! The **speed of a vehicle *s*** and the **stopping distance *d*** are related by the function $s(d) = \sqrt{30d}$.

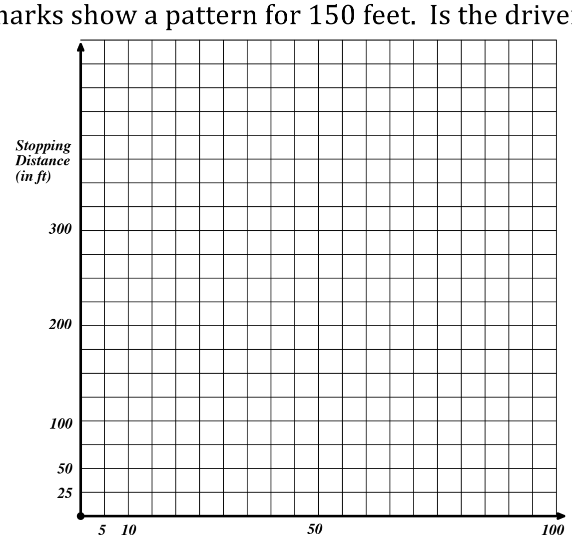
Fill in the table of values for $s(d)$. (Round to nearest whole number.) Then graph $s(d)$ and answer the questions.

7.

$d\ ft$	$s(d)\ mph$
25	
50	
100	
200	
300	



8. If you were a police officer investigating the site of an accident, you would be able to measure the length of the skid marks on the road and then approximate the speed of the driver. The driver swears he was sure he was going under 60 mph. The tire marks show a pattern for 150 feet. Is the driver's sense of his speed accurate? Justify your answer.



9. Use your answers in problem 8 to make a graph of stopping distance as a function of speed.

10. How are the two graphs related?

Need help? Visit **www.rsgsupport.org**

GO

Topic: Solving literal equations for a given variable

Solve each equation for the indicated variable.

11. $C = 2\pi r$; Solve for r.

12. $A = \pi r^2$; Solve for r.

13. $V = \pi r^2 h$; Solve for h.

14. $V = \pi r^2 h$; Solve for r.

15. $V = e^3$; Solve for e.

16. $A = \frac{b_1+b_2}{2}h$; Solve for h

4.7 More Features, More Functions

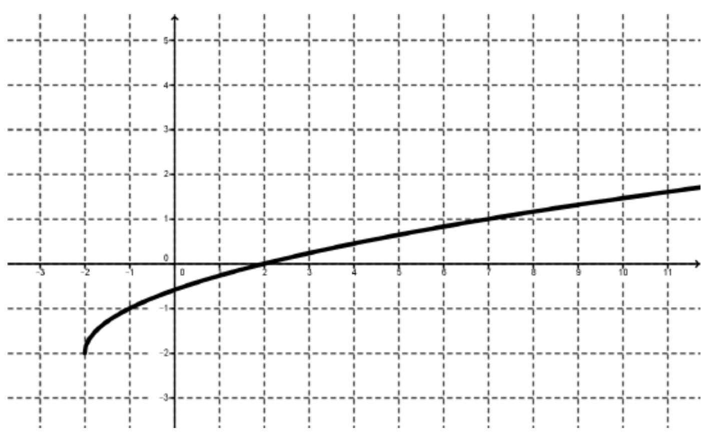
A Practice Understanding Task

Part I: Features of Functions

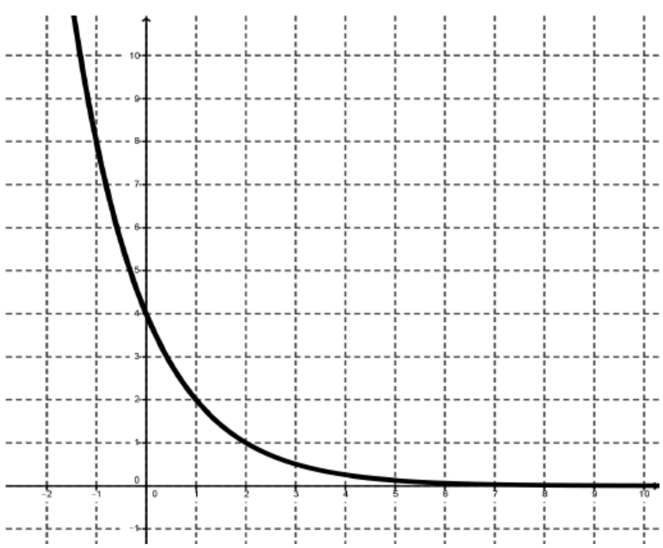
Find the following for each function (all graphs have a scale value of one on both the x-axis and y-axis)

a. Equation of the function
b. Domain and range
c. Intercepts
d. Location and value of maxima/minima
e. Intervals where function is increasing or decreasing
f. Sketch the inverse of the function (on a new set of axes or overlay on the given graph)

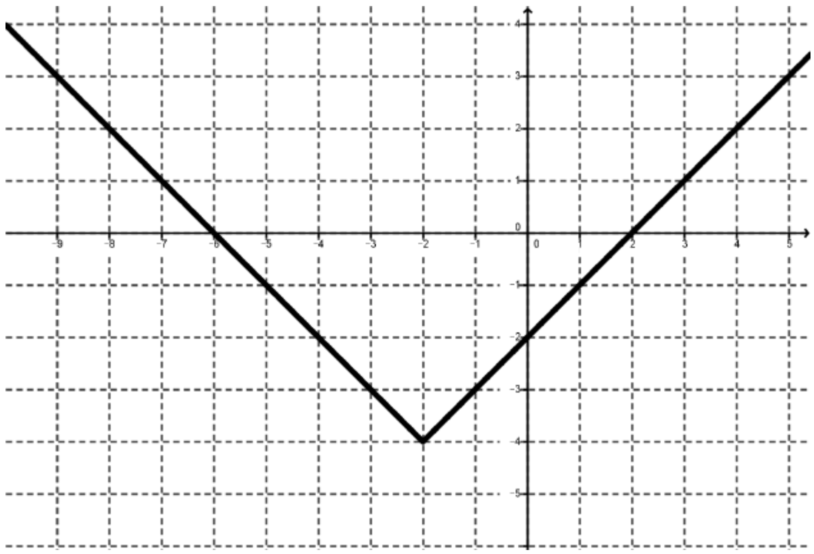
1.



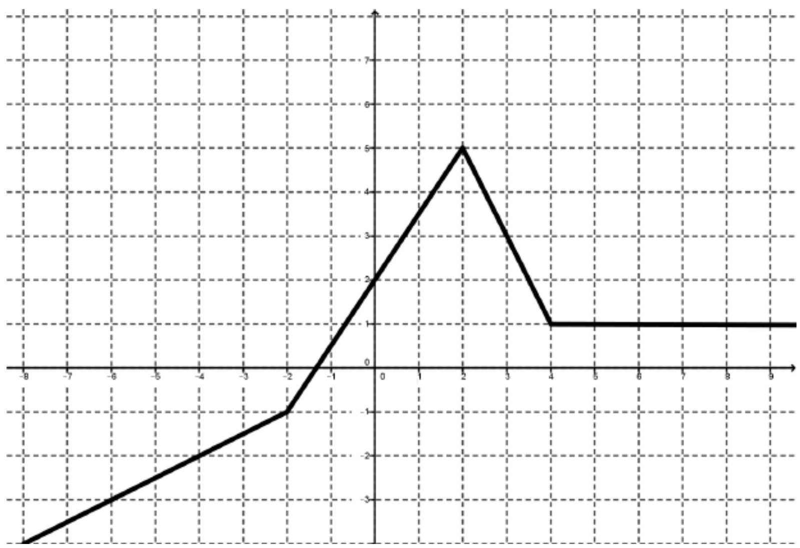
2.



3.



4.



Part II: Creating Functions

Directions: Write **two** different functions that meet the given requirements.

5. A function that is always increasing

6. A function that is symmetrical about the y-axis

7. A function with a minimum of -2 at $x = 5$

8. A function that is decreasing from $(-\infty, -3)$ then increasing from $(-3, \infty)$

9. A function with zero real roots

10. A function that has a domain from $[3, \infty)$ and a range from $[0, \infty)$

11. A function with a constant rate of change

12. A function whose second difference is a constant rate of change

13. A function whose inverse is also a function

4.7 More Features, More Functions – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is for students to show their understanding of features of functions. This is a culminating task for modules 1 through 4 and asks students to describe features of specific functions, then to create two different functions when given specific features.

Core Standards Focus:

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetry, end behavior; and periodicity.*

Related Standards:

F.IF.1, F.IF.3, F.IF.5, F.IF.7, F.BF.1, F.BF.2, F.BF.3, F.LQE.1

The Teaching Cycle:

Launch (Whole Class):

To launch, go over the directions of the two parts to this task and explain to students that this task is for them to practice what they know about functions, using all of the functions they know. You may wish to have students spend a bit of time working on this task individually before working with a partner or in a small group.

Explore (Individual, then Small Group):

As you monitor, redirect students who have an incorrect answer by asking them questions. Look for common errors among students so that you can discuss these more thoroughly during the whole group discussion. At some point, when students are working on Part II, select students to share who have different functions but meet the requirements. Push students to think about the different kinds of function types and to be as creative as possible.

Discuss (Whole Class):

Choose questions from Part I to go over in the whole group discussion based on which problems seem to need the most attention. Going over only those problems that need addressing in Part I will allow more time for sharing of answers from Part II.

The discussion of Part II can best be determined based on observations you make during the explore phase. You may wish to go over 4-6 different problems and probe as many different kind of functions as possible. For example, question 6 could include linear, exponential, and square root functions, while question 7 could include absolute value and quadratic functions (and even a constant function). The goal of this part of the task is to have students fluently be able to discuss the types of features certain functions have. Another idea to pull out in this discussion would be to ask students to compare and contrast problems. “Are there equations that would work for questions 6 and 10 and other equations that would work for 6 but not 10 (and vice versa)?” or be more general “Are there any two problems that could have the same function? Explain. Would this work for any function in both of these problems?” An example of this could be that students could say they have the same exponential function for questions 6 and 10, however, that there are also linear functions that work in question 6 that could not be in question 10.

Aligned Ready, Set, Go: More Features, More Functions RSG 4.7

READY, SET, GO! Name Period Date

READY

Topic: Geometric Symbols

Make a sketch that matches the geometric symbols. Label your sketch appropriately.

1. ΔRST

2. $\overrightarrow{AB}$

3. $\angle XYZ$

4. $\overleftrightarrow{GH}$

5. $\overline{JK} \perp \overline{PQ}$

6. Point S bisects $\overline{MN}$.

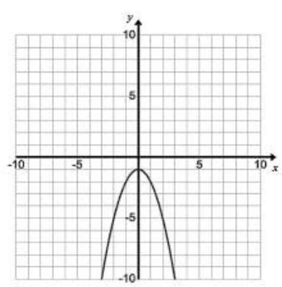
7. $\overrightarrow{AB}$ bisects $\angle XYZ$

SET

Topic: Features or Functions

Find the following key features for each function:

8.



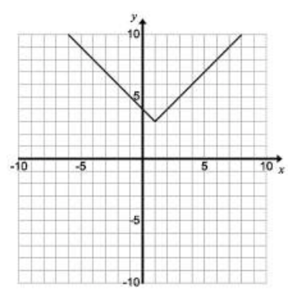
a. Domain and range

b. Intercepts

c. Location and value of maxima/minima

d. Intervals where function is increasing or decreasing

9.



a. Domain and range

b. Intercepts

c. Location and value of maxima/minima

d. Intervals where function is increasing or decreasing

10.

$$f(x) = \begin{cases} -(x+3), & x < -3 \\ (x+3), & x \geq -3 \end{cases}$$

a. Domain and range

b. Intercepts

c. Location and value of maxima/minima

d. Intervals where function is increasing or decreasing

Write a function that meet the given requirements.

11. A function that is always decreasing

12. A function that is symmetrical about the line x=3

13. A function with a minimum of 5 at $x = 1$

14. A function that is increasing from $(-\infty, 2)$ then decreasing from $[2, \infty)$

15. A function with one real root

16. A function that has a domain from $[-2, \infty)$

17. A function with a range from $[0, \infty)$

18. A function with a common factor of 2

19. A function that is also a geometric sequence

20. A function with x-intercepts at (-1, 0) and (1,0)

GO

Topic: Inverse Function

Find the inverse of each function. If the inverse is not a function, restrict the domain.

21. $f(x) = x^2; f^{-1}(x) =$

22. $g(x) = 2x + 4; g^{-1}(x) =$

23. $f(x) = (x + 1)^2; f^{-1}(x) =$

24. $h(x) = \frac{1}{3}x + 6; h^{-1}(x) =$

25. $\boldsymbol{f(x) = \{(-3,5)(-2,-9)(-1,-2)(0,-5)(1,-4)(2,6)(3,10)(4,8)\}};$

$\boldsymbol{f^{-1}(x) =}$ $\{(\ ,\ )(\ ,\ )(\ ,\ )(\ ,\ )(\ ,\ )(\ ,\ )(\ ,\ )(\ ,\ )\}$

Write the piecewise-defined function for the following absolute value functions

26. $h(x) = |x + 3|$

27. $f(x) = |x^2 - 4| + 1$

28. $g(x) = 5|x + 3|$

29. $f(x) = |x^2 - 16|$