mathematics vision project

Transforming Mathematics Education

SECONDARY MATH THREE

An Integrated Approach

Standard Teacher Notes

MODULE 4

Rational Expressions & Functions

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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MODULE 4 - TABLE OF CONTENTS

RATIONAL EXPRESSIONS AND FUNCTIONS

4.1 Winner, Winner – *A Develop Understanding Task*
Introducing rational functions and asymptotic behavior **(F.IF.7d A.CED.2, F.IF.5)**
Ready, Set, Go Homework: *Rational Functions 4.1*

4.2 Shift and Stretch – *A Solidify Understanding Task*
Applying transformations to the graph of $f(x) = \frac{1}{x}$. **(F.BF.3, F.IF.7d, A.CED.2)**
Ready, Set, Go Homework: *Rational Functions 4.2*

4.3 Rational Thinking – *A Solidify Understanding Task*
Discovering the relationship between the degree of the numerator and denominator and the horizontal asymptotes. **(F.IF.7d A.CED.2, F.IF.5)**
Ready, Set, Go Homework: *Rational Functions 4.3*

4.4 Are You Rational? – *A Solidify Understanding Task*
Reducing rational functions and identifying improper rational functions and writing them in an equivalent form. **(A.APR.6, A.APR.7, A.SSE.3)**
Ready, Set, Go Homework: *Rational Functions 4.4*

4.5 Just Act Rational – *A Solidify Understanding Task*
Adding, subtracting, multiplying, and dividing rational expressions. **(A.APR.7, A.SSE.3)**
Ready, Set, Go Homework: *Rational Functions 4.5*

4.6 Sign on the Dotted Line – *A Practice Understanding Task*
Developing a strategy for determining the behavior near the asymptotes and graphing rational functions. **(F.IF.4, F.IF.7d)**
Ready, Set, Go Homework: *Rational Functions 4.6*

4.7 We All Scream – *A Practice Understanding Task*
Modeling with rational functions, and solving equations that contain rational expressions. **(A.REI.A.2, A.SSE.3)**
Ready, Set, Go Homework: *Rational Functions 4.7*

4.1 Winner, Winner

A Develop Understanding Task

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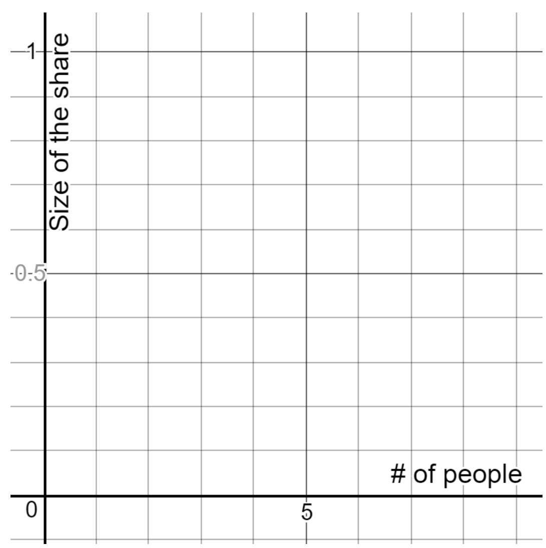
One of the most interesting functions in mathematics is $f(x) = \frac{1}{x}$ because it brings up some mathematical mind benders. In this task, we will use story context and representations like tables and graphs to understand this important function.

Let's being by thinking about the interval $[1, \infty)$.

1. Imagine that you won the lottery and were given one big pot of money. Of course, you would want to share the money with friends and family. If you split the money evenly between yourself and one friend, what would be each person's share of the prize money?

2. If three people shared the prize money, what would be each person's share?

3. Model the situation with a table, equation, and graph.



4. Just in case you didn't think about the really big numbers in your model, how much of the pot would each person get if 1000 people get a share? If 100,000 people get a share? If 100,000,000 people get a share?

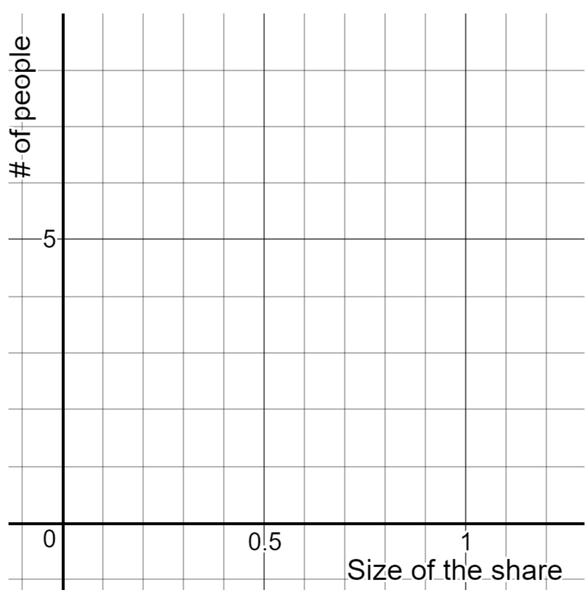
5. Use mathematical notation to describe the behavior of this function as $x \to \infty$.

Next, let's look at the interval (0, 1] and consider a new way to think about splitting the prize money.

6. Imagine that you want each person's share to be $^1/_2$ of the prize. How many people could share the prize?

7. If you want each person's share to be $^1/_3$ of the prize, how many people could share the prize?

8. Model this situation with a table, graph and equation.



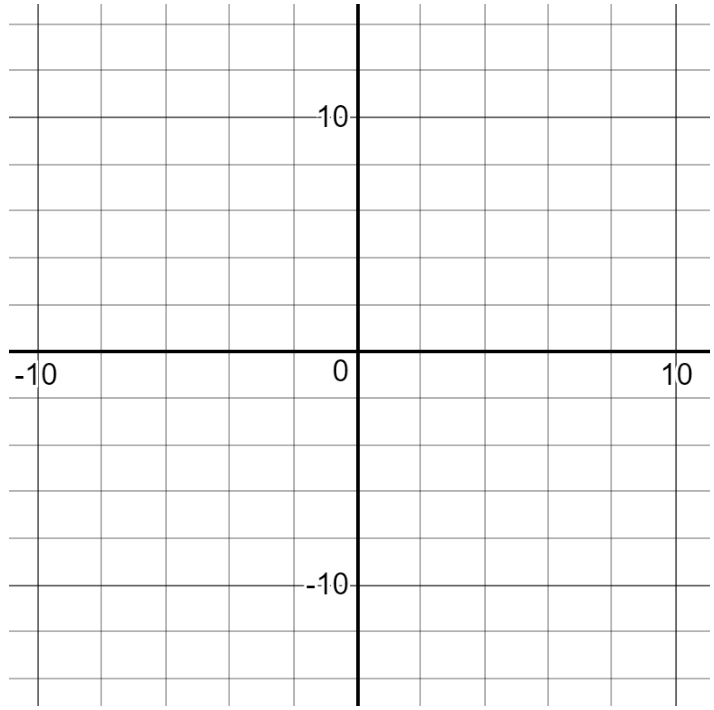
9. What do you notice when you compare the two models that you have written?

Now, let's put it all together to graph the entire function, $f(x) = \frac{1}{x}$.

10. Create a table for $f(x) = \frac{1}{x}$ that includes negative input values.

11. How do the values of $f(x)$ in the interval from $(-\infty, 0)$ compare to the values of $f(x)$ from $(0, \infty)$? Use this comparison to predict the graph of $f(x) = \frac{1}{x}$.

12. Graph $f(x) = \frac{1}{x}$.



13. Describe the features of $f(x) = \frac{1}{x}$, including domain, range, intervals of increase or decrease, x- and y- intercepts, end behavior, and any maximum(s) or minimum(s).

4.1 Winner, Winner – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to introduce students to rational functions and to explore the behavior of the function, $f(x) = \frac{1}{x}$, that produces the vertical and horizontal asymptotes. Students will use a story context to think numerically about the values of $f(x) = \frac{1}{x}$ in quadrant I and extend that thinking to graph the function over the entire domain. Students will consider all of the features of the function including, domain, range, intervals of increase and decrease, and intercepts.

Core Standards Focus:

F.IF.7d Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

d. Graph rational functions, identifying zeros when suitable factorizations are available, and showing end behavior.

A.CED.2 (Create equations that describe numbers or relationships) Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

Related: F.IF.6, A.CED.3

Standards for Mathematical Practice:

SMP 2 – Reason abstractly and quantitatively

Vocabulary: Horizontal asymptote

The Teaching Cycle:

Launch (Whole Group):

Begin by telling students that today's work will focus on the function $f(x) = \frac{1}{x}$. To activate their background a little, ask students what they notice about the function, just by looking at the equation. They may recognize things like:

- The function is undefined at 0.
- The function values are going to be a lot of fractions.
- The function is the reciprocal of $y = x$.

At this point, there is no need to pursue any of their observations, just keep them in mind during the class discussion to see how they come into play as students work with the function.

Explain the context of the task, that they have won the lottery and have a big prize to share. The amount of money is not important, they should think about the prize as one big "pot of money" and how they can divide it up evenly. Ask students about question 1. Be sure that students understand that the pot would be shared equally between two people so that each person gets half of the prize. Ask students to do question 2 and discuss it briefly before asking them to work on the rest of the task.

Explore (Individual and then Small Groups):

As students are working, listen for students that are discussing ideas about the behavior of the function as $x \to \infty$. They are likely to argue about whether or not the value of the function is ever zero, since practically speaking, the share of the prize would become so small that each person is getting nearly nothing. Students who have weak understanding of fractions may not immediately understand that as the denominator of the fraction increases, the value of the fraction decreases. The story context can be used to support them in thinking about this idea.

There may be some discussion of whether or not to model this situation as a discrete function in the interval $[1, \infty)$ because the number of people can't be a fraction. Since this is not the focus of the task, you may wish to ask students to make a continuous model, with the understanding that any answer using the model will require interpretation to be sure that it is reasonable in the context.

As students move to consider the interval $(0,1)$, they may need more support for thinking about dividing by a fraction. The story context is provided for this purpose to help them to see a situation where dividing by a fraction makes sense. This understanding is important for students to consider the behavior of the function near the vertical asymptote. Students should also notice that they are working with the x and y variables switched from the previous section, and yet, the function is still $f(x) = \frac{1}{x}$.

Discuss (Whole Group):

Begin the discussion with question #3. Ask a student to present his/her table and to describe how the table models the context. Then, ask the class how the table is connected to the equation, $f(x) = \frac{1}{x}$. Ask a second student to present the graph of the function in the interval $[1, \infty)$. Focus on why the function approaches zero for large values of x and introduce the term "horizontal asymptote". Be careful not to describe a horizontal asymptote as a line that the function approaches but never crosses. There are several functions that occur later in the module where the graph actually crosses the horizontal asymptote. A horizontal asymptote describes the end behavior of a function, either as $x \to \infty$ or $x \to -\infty$.

Turn the discussion to question 8. Again, ask a previously-selected student to share a table and to connect the values in the table to the story context. Ask the class what happens to the number of people that the prize can be shared with if the portion of the prize that each person gets is increasingly smaller. Help students to understand how this behavior creates a vertical asymptote at $x = 0$. Share a graph of the function in the interval $(0, 1)$. Ask students about the relationship between the two different tables that they created in questions 3 and 8. They should notice that the variables have been switched, and thus, the functions are inverses. Ask how this could be the case if the equation is $f(x) = \frac{1}{x}$ in both situations. Emphasize that the two different ways of thinking about the story context are presented here so that they can see the similarities in the vertical and horizontal asymptotes.

Ask a student to present his/her work on question 10 and then ask another student to present a graph of the entire function. Ask the class what the relationship is between the graph in Quadrant I and the graph in Quadrant III. They should recognize that it is a rotation of 180 degrees around the origin, so $f(x) = \frac{1}{x}$ is an odd function. Finally, discuss the features of $f(x) = \frac{1}{x}$. Be sure that students can use proper notation to describe the domain and range, as well as recognizing that the function is decreasing through the entire domain.

Aligned Ready, Set, Go: ***Rational Expressions and Functions 4.1***

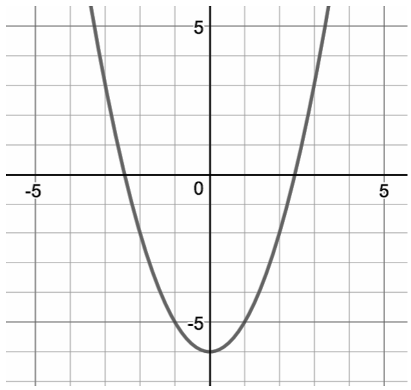
READY, SET, GO! Name Period Date

READY

Topic: Recalling transformations on quadratic functions

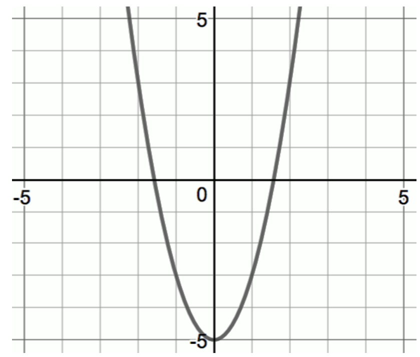
Describe the transformation of each function. Then write the equation in vertex form.

1. Description:



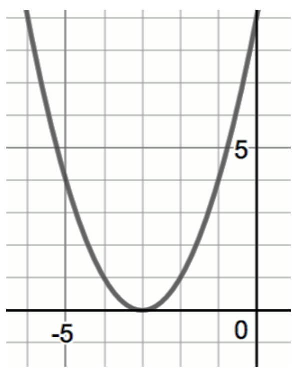
Equation:

2. Description:



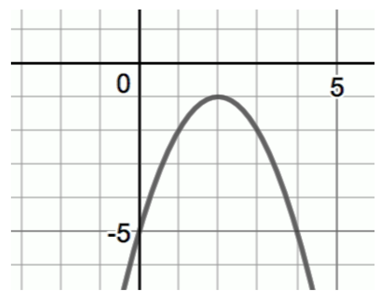
Equation:

3. Description:



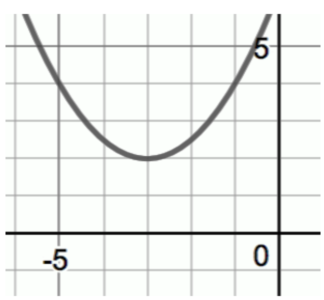
Equation:

4. Description:



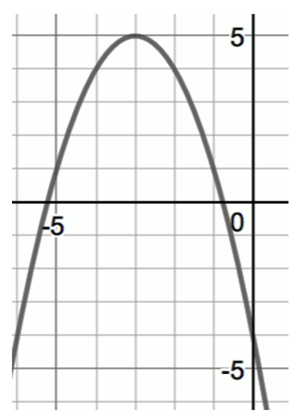
Equation:

5. Description:



Equation:

6. Description:



Equation:

SET

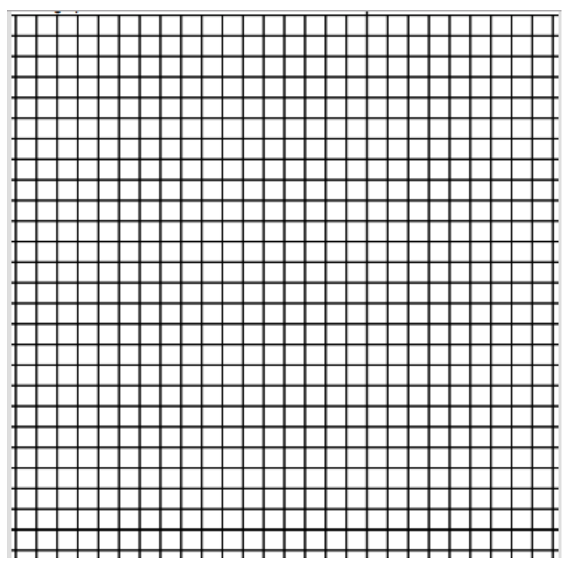
Topic: Exploring a rational function

Chile is celebrating her Quinceañera. Hannah knows the perfect gift to buy Chile, but it costs $360. Hannah can't afford to pay for this on her own so thinks about asking some friends to join in and share the cost.

7. How much would each person spend if there were two people dividing the cost of the gift? How much would each person spend if there were five people dividing the cost? Ten people? One hundred?

8. The function that models this situation is $f(x) = \frac{360}{x}$. Define the meaning of the numerator and the denominator within the context of the story.

9. Create a table and a graph to show how the amount each person would contribute to the gift would change, depending on the number of people contributing.



10. Hannah created a fundraising site on the internet. Within 5 days, enough people had registered so that each friend, including Hannah, only needed to donate $0.50.

    a. How many people had registered in 5 days?

    b. By the day of the event, enough people had registered that each friend, including Hannah, only donated 10¢. How many friends had registered?

GO

Topic: Reviewing the horizontal asymptote in an exponential function

All exponential functions have a horizontal asymptote. All of the graphs below show exponential functions. Match the function rule with the correct graph. Then write the equation of the horizontal asymptote.

11. $f(x) = 2^x$

Equation of horizontal asymptote:

12. $g(x) = 2^x - 3$

Equation of horizontal asymptote:

13. $h(x) = 2^{x-3}$

Equation of horizontal asymptote:

14. $m(x) = -(2^x) - 3$

Equation of horizontal asymptote:

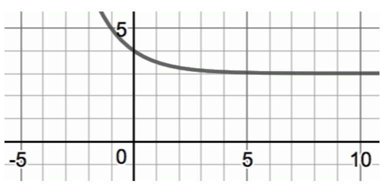
15. $q(x) = 2^{(-x)} + 3$

Equation of horizontal asymptote:

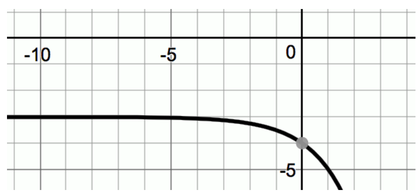
16. $r(x) = -2^{(-x)}$

Equation of horizontal asymptote:

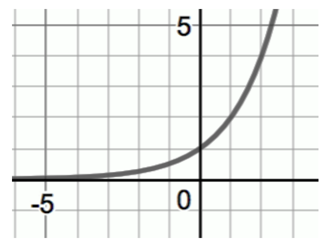
a.



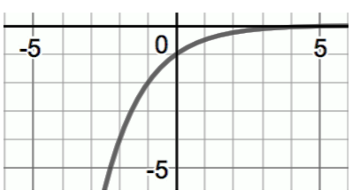
b.



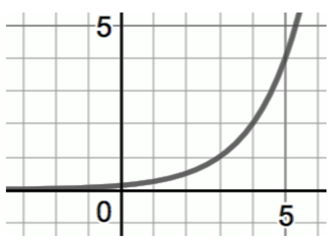
c.



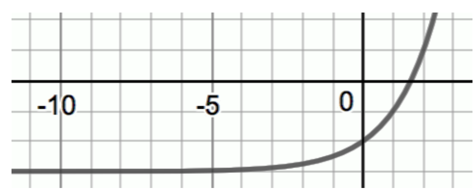
d.



e.



f.



17. Use $f(x) = ab^{(x-h)} + k$ to explain which values affect the position of the horizontal asymptote in an exponential function. Be precise.

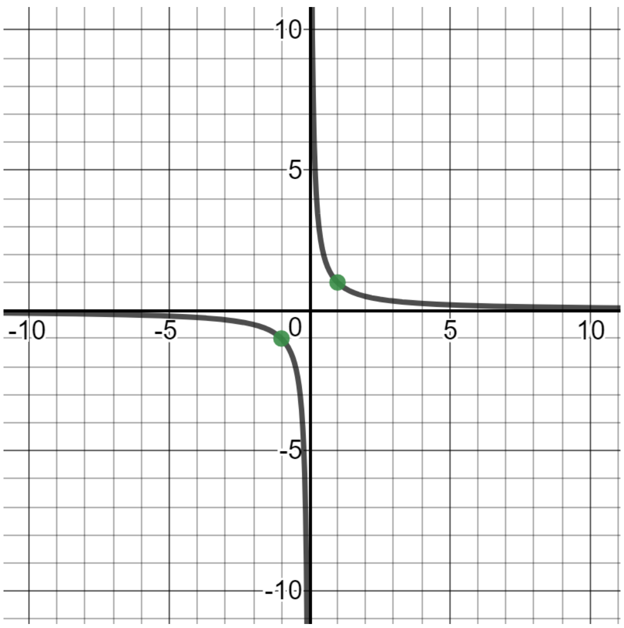
18. Why does an exponential function have a horizontal asymptote?

4.2 Shift and Stretch

A Solidify Understanding Task

In *4.1 Winner, Winner* you were introduced to the function $y = \frac{1}{x}$. Before exploring the family of related functions, let's clarify some of the features of $y = \frac{1}{x}$ that can help with graphing.

Here's a graph of $y = \frac{1}{x}$.



1. Use the graph to identify each of the following:

Horizontal Asymptote: ____________________

Vertical Asymptote: ____________________

Anchor Points:

(1, _____) and (-1, _____)

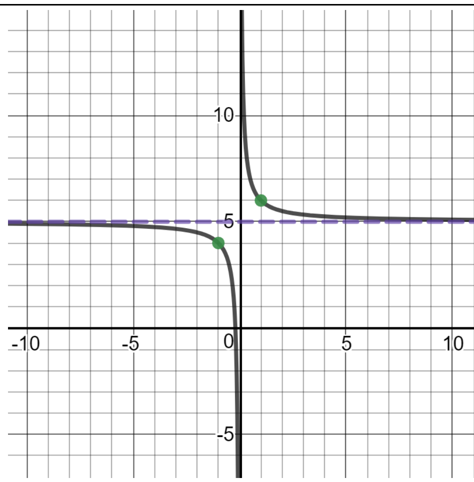
$\left(\frac{1}{2}, \_\_\_\_\_\right)$ and $\left(-\frac{1}{2}, \_\_\_\_\_\right)$

(2, _____) and (-2, _____)

Now you're ready to use this information to figure out how the graph of $y = \frac{1}{x}$ can be transformed. As you answer the questions that follow, look for patterns that you can generalize to describe the transformations of $y = \frac{1}{x}$.

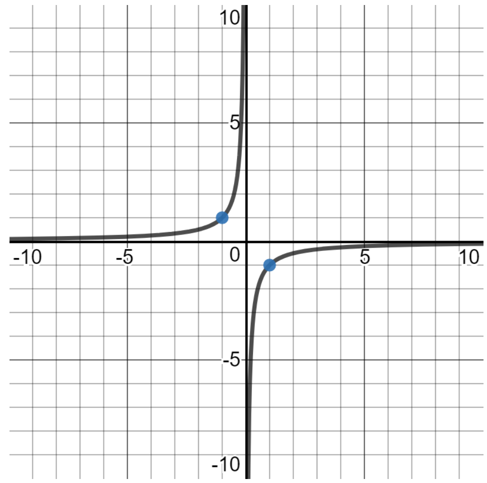
In each of the following problems, you are given either a graph or a description of a function that is a transformation of $y = \frac{1}{x}$. Use your amazing math skills to find an equation for each.

2.



Equation:

3.

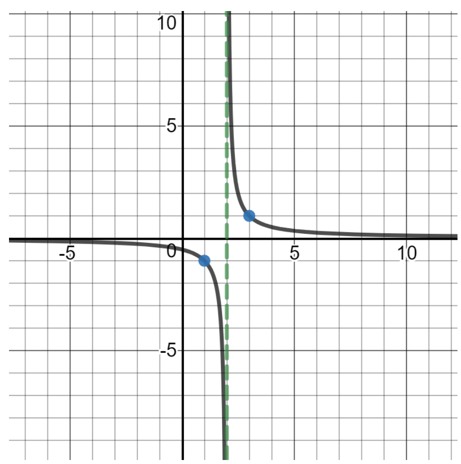


Equation:

4. The function has a vertical asymptote at $x = -3$ and a horizontal asymptote at $y = 0$. It contains the points (-2, 1) and (-4, -1). The y-intercept is $(0, \frac{1}{3})$.

Equation:

5.

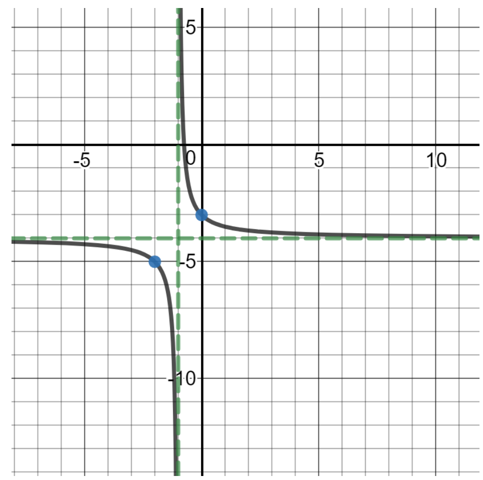


Equation:

6. The function has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$. It contains the points (1,2), (-1, -2), (2, 1), (-2,1), $\left({}^{1}/_{2}, 4\right)$ and $\left({}^{-1}/_{2}, -4\right)$.

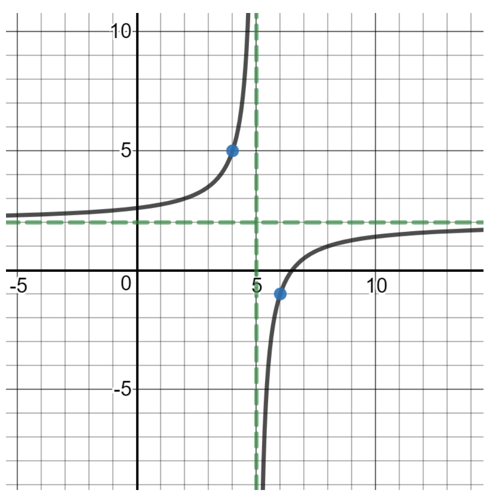
Equation:

7.



Equation:

8.



Equation:

9. The function has a vertical asymptote at $x = -6$ and a horizontal asymptote at $y = -3$. It crosses the x-axis at $-6\frac{1}{3}$. It contains the points (-5, -4) and (-7, -2).

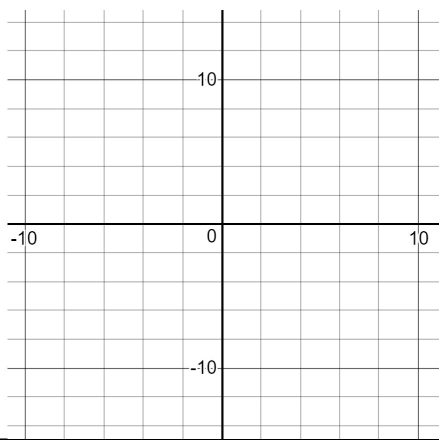
Equation:

10. Match each equation to the phrase that describes the transformation from $y = \frac{1}{x}$.

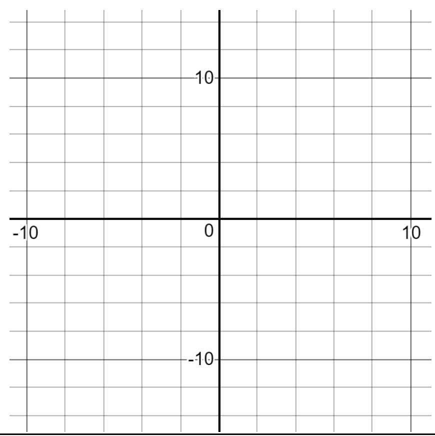
$y = \frac{1}{x+b}$ ________	A) Reflection over the x-axis.
$y = b + \frac{1}{x}$ ________	B) Vertical shift of b, making the horizontal asymptote $y = b$.
$y = \frac{b}{x}$ ________	C) Horizontal shift left b, making the vertical asymptote $x = -b$.
$y = \frac{-1}{x}$ ________	D) Vertical stretch by a factor of b
$y = \frac{1}{x-b}$ ________	E) Horizontal shift right b, making the vertical asymptote $x = b$.

11. Graph each of the following equations without using technology.

$$y = -2 + \frac{3}{x - 4}$$



$$y = 1 - \frac{1}{x + 3}$$



12. Describe the features of the function:

$$y = k + \frac{b}{x - h}$$

Vertical Asymptote:

Horizontal Asymptote:

Vertical Stretch Factor:

Anchor Points:

Domain:

Range:

4.2 Shift and Stretch – Teacher Notes

A Solidify Understanding Task

Purpose:
The purpose of this task is to extend students' understanding of rational functions by applying transformations to the graph of $f(x) = \frac{1}{x}$. The task helps students to focus on the vertical and horizontal asymptotes and how they can be used to easily graph simple rational functions. This task prepares students for more complicated rational functions in upcoming tasks.

Core Standards Focus:
F.IF.7d Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
d. Graph rational functions, identifying zeros when suitable factorizations are available, and showing end behavior.

F.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

A.CED.2 (Create equations that describe numbers or relationships) Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Standards for Mathematical Practice:

SMP 5 – Use appropriate tools strategically

SMP 8 – Look for and express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Group):

Start the task by reminding students of their previous work in *4.1 Winner, Winner.* Discuss question #1 together. Clarify for students that the term "anchor points" in this context simply means a few useful points for sketching the graph. In this case, the anchor points help to get the shape of the curve right. Tell students that in the upcoming problems, they will be given a graph or a description of a function and they will use their knowledge of $f(x) = \frac{1}{x}$ to write an equation for the function. Once they have written the equation, they should use graphing technology to check their work and make adjustments to their equations, if necessary. Ask students to pay careful attention to how changes in the equation correspond to changes in the graph so that they will be ready to generalize by the end of the task.

Explore (Individual, followed by small group):

Monitor students as they work to be sure that they are identifying the changes in the graphs and matching them to an equation. The most difficult problem for students is usually the horizontal shift because students don't see that the argument of the function is in the denominator. Support students in thinking about inputs and outputs, specifically that whenever a number is added to the x, it is affecting the input of the function. When the change occurs after the function operation has been applied, in this case, taking the reciprocal of the input, then the change affects the output of the function. Changes that affect the outputs, or y-values, are vertical shifts or stretches. Changes to the inputs are horizontal shifts and stretches. In this task, horizontal stretches are not considered. These ideas are not new to students, but this type of function can press students because the form seems different.

Discuss :

If students have struggled with applying the transformations during the exploration period, begin the discussion with problem #2 and have students present their work on each of the problems, 2-8. If most of the class has been able to see the transformations during the exploration period, then begin with question #5 and then discuss question #8. In either case, be sure to ask the class why the changes in the graph result from the changes in the equation.

After discussing the examples, help students to generalize by working problem #10 and being sure that each student knows the vertical and horizontal shifts, along with the vertical stretches. Then, ask a previously selected student to demonstrate how he/she graphed either of the problems on #11.

Aligned Ready, Set, Go: ***Rational Expressions and Functions 4.2***

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Connecting the zeroes of a polynomial with the domain of a rational function

Find the zeroes of each polynomial.

1. $p(x) = (x+4)(x-2)(x-7)$

2. $p(x) = (2x-6)(8x-1)(x-5)$

3. $p(x) = (9x+3)(x^2-9)$

4. $p(x) = x^2 + 25$

Find the domain of each of the rational functions.

5. $q(x) = \frac{1}{(x+4)(x-2)(x-7)}$

6. $q(x) = \frac{1}{(2x-6)(8x-1)(x-5)}$

7. $q(x) = \frac{1}{(9x+3)(x^2-9)}$

8. $q(x) = \frac{1}{x^2+25}$

SET

Topic: Practicing transformations on rational functions

Identify the vertical asymptote, horizontal asymptote, domain, and range of each function. Then sketch the graph on the grids provided. (Grids on next page.)

9. $f(x) = \frac{4}{x}$

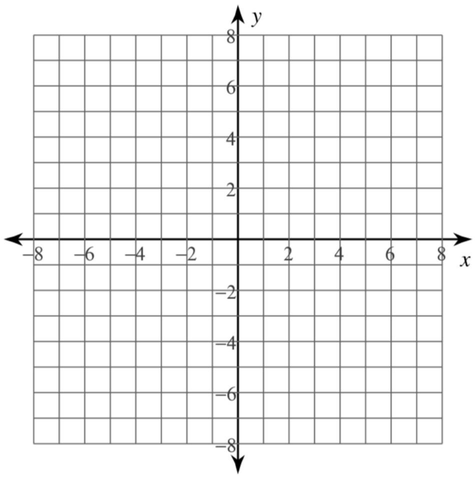
V.A. H. A.

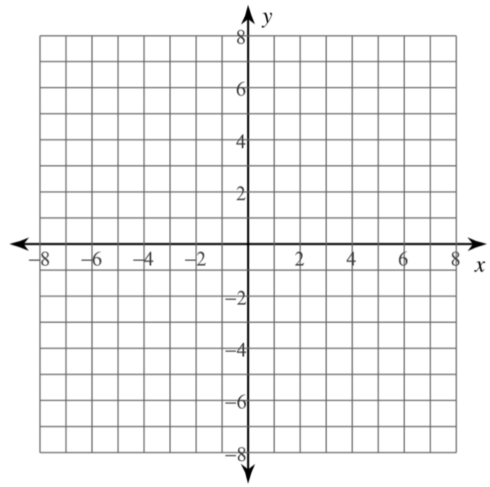
Domain: Range:

10. $f(x) = \frac{3}{x} + 2$

V.A. H. A.

Domain: Range:





11. $f(x) = -\frac{5}{x-3}$

V.A.

H. A.

Domain:

Range:

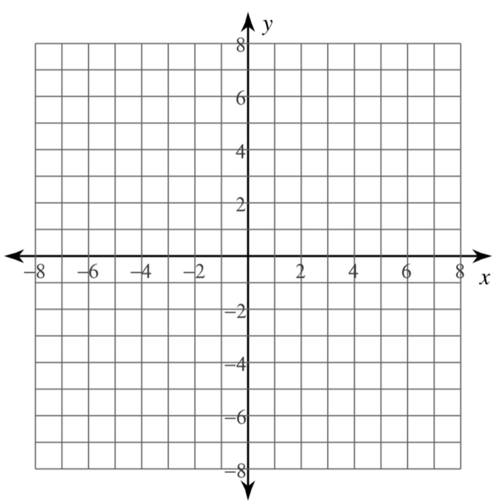
12. $f(x) = \frac{1}{(x+5)} - 4$

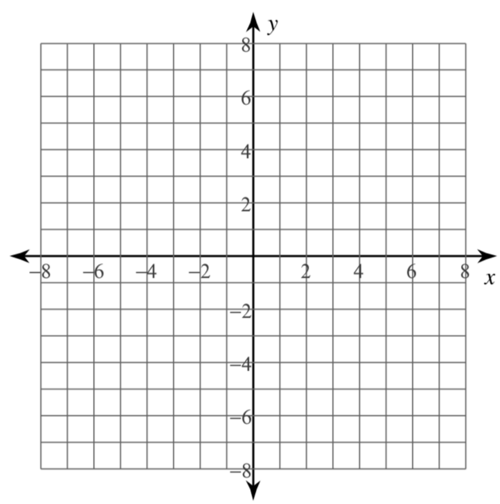
V.A.

H. A.

Domain:

Range:





13. Write a function of the form $f(x) = \frac{a}{x-h} + k$ with a vertical asymptote at $x = -15$ and a horizontal asymptote at $y = -6$.

GO

Topic: Finding the roots and factors of a polynomial

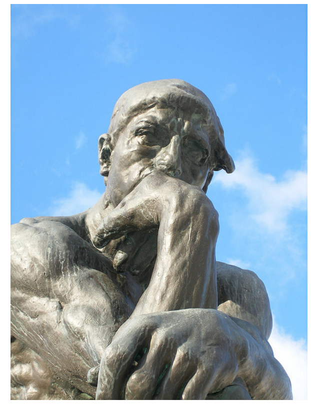
Use the given root to find the remaining roots. Then write the function in factored form.

14. $f(x) = x^3 - x^2 - 17x - 15$	$x = -1$	15. $f(x) = x^3 - 3x^2 - 61x + 63$	$x = 1$
16. $f(x) = 6x^3 - 18x^2 - 60x$	$x = 0$	17. $f(x) = x^3 - 14x^2 + 57x - 72$	$x = 8$

18. A relationship exists between the roots of a function and the constant term of the function. Look back at the roots and the constant term in each problem. Make a statement about anything you notice.

4.3 Rational Thinking

A Solidify Understanding Task



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The broad category of functions that contains the function $f(x) = \frac{1}{x}$ is called rational functions. A rational number is a ratio of integers. A rational function is a ratio of polynomials. Since polynomials come in many forms, constant, linear, quadratic, cubic, etc., we can expect rational functions to come in many forms too. Some examples are:

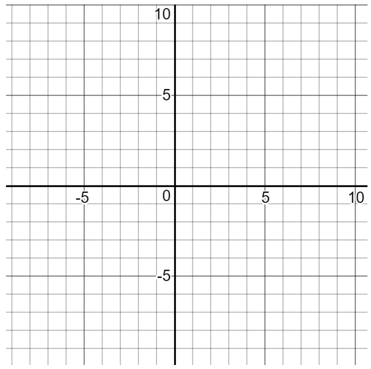
$f(x) = \frac{1}{x}$	$f(x) = \frac{x+1}{x-3}$	$f(x) = \frac{x-4}{x^3+2x^2-4x+1}$	$f(x) = \frac{x^2-4}{(x-3)(x+1)}$
Degree of the numerator = 0 Degree of the denominator = 1	Degree of the numerator = 1 Degree of the denominator = 1	Degree of the numerator = 1 Degree of the denominator = 3	Degree of the numerator = 2 Degree of the denominator = 2

In today's task, you are going to look for patterns in the forms so that you can complete the following chart:

	How to find the vertical asymptote:	How to find the horizontal asymptote:	How to find the intercepts:
Degree of the numerator < Degree of the denominator			
Degree of the numerator = Degree of the denominator			

You are given several different rational functions. Start by identifying the degree of the numerator and denominator and using technology to graph the function. As you are working, look for patterns that will help you complete the table. You need to find a quick way to identify the horizontal and vertical asymptotes when you see the equation of a rational function, as well as noticing other patterns that will help you analyze and graph the function quickly. The last two graphs are there so you can experiment with your own rational functions and test your theories.

1. $y = \frac{x+3}{x-2}$



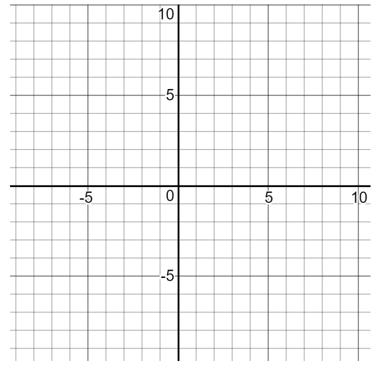
Deg. of Num. ____ Deg. of Denom. ____

Horizontal Asymptote _________

Vertical Asymptote _________

Intercepts ____________________________

2. $y = \frac{x}{(x+2)(x-3)}$



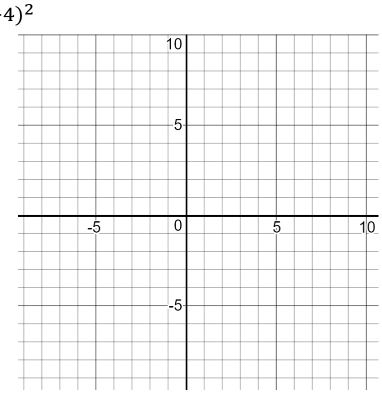
Deg. of Num. ____ Deg. of Denom. ____

Horizontal Asymptote _________

Vertical Asymptote _________

Intercepts ____________________________

3. $y = \frac{1}{(x+4)^2}$



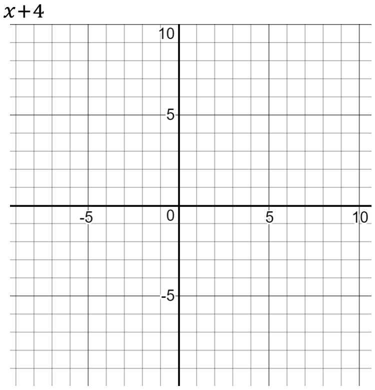
Deg. of Num. ____ Deg. of Denom. ____

Horizontal Asymptote _________

Vertical Asymptote _________

Intercepts ____________________________

4. $y = \frac{3x-1}{x+4}$



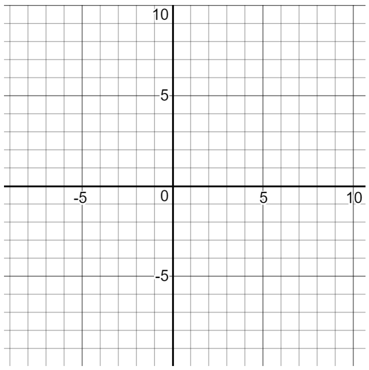
Deg. of Num. ____ Deg. of Denom. ____

Horizontal Asymptote _________

Vertical Asymptote _________

Intercepts ____________________________

5. $y = \frac{x^2-3x}{(x-2)(x+1)(x+3)}$



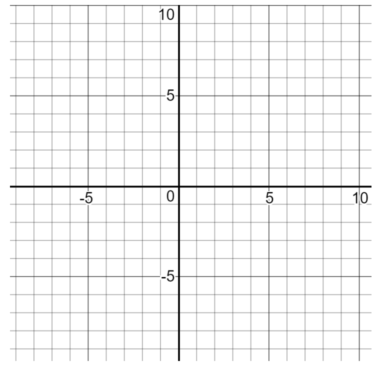
Deg. of Num. ____ Deg. of Denom. ____

Horizontal Asymptote _________

Vertical Asymptote _________

Intercepts _____________________________

6. $y = \frac{2x^2+3}{(2x+5)(2x-5)}$



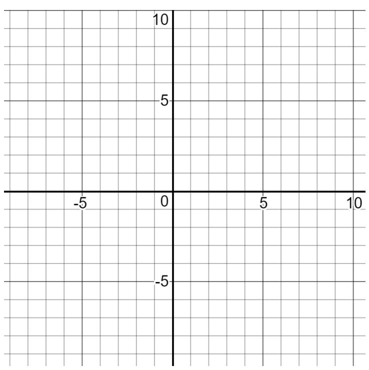
Deg. of Num. ____ Deg. of Denom. ____

Horizontal Asymptote _________

Vertical Asymptote _________

Intercepts _____________________________

7. Your Own Rational Function:



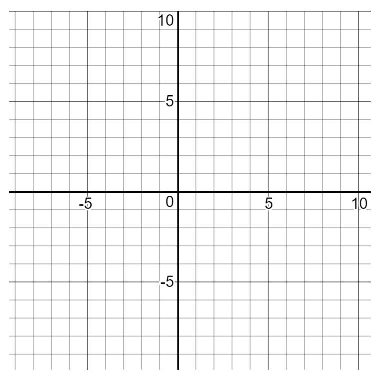
Deg. of Num. ____ Deg. of Denom. ____

Horizontal Asymptote _________

Vertical Asymptote _________

Intercepts _____________________________

8. Your Own Rational Function:



Deg. of Num. ____ Deg. of Denom. ____

Horizontal Asymptote _________

Vertical Asymptote _________

Intercepts _____________________________

9. Now that you have tried some examples, it's time to draw some conclusions and complete the two rows of the table on the first page. Be prepared to discuss your conclusions and why you think that they are correct.

4.3 Rational Thinking – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to introduce the term "rational function" and to use several examples of rational functions to discover the relationship between the degree of the numerator and denominator and the horizontal asymptotes. Students will also find vertical asymptotes and intercepts for rational functions and use this information to complete a graphic organizer. Graphing technology is used as an important tool for exploring rational functions in this task.

Core Standards Focus:
F.IF.7d Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
d. Graph rational functions, identifying zeros when suitable factorizations are available, and showing end behavior.

A.CED.2 (Create equations that describe numbers or relationships) Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
Related: F.IF.6, A.CED.3

Standards for Mathematical Practice:
SMP 3 – Construct viable arguments and critique the reasoning of others
SMP 7 – Look for and make use of structure
Vocabulary: Rational function

The Teaching Cycle:

Launch (Whole Group):

Launch the task by defining the term, "rational function" and discussing the examples given at the beginning of the task. Make sure that students are able to identify the degree of the polynomials in the numerator and denominator. Show students the chart that they will be completing when they get through the task. Tell them that they will use graphing technology to figure out how to find the horizontal and vertical asymptotes and the intercepts for rational functions. In this task they will work with rational functions where the degree of the numerator is less than or equal to the denominator. Remind them that they already know a lot about one rational function: $f(x) = \frac{1}{x}$.

Explore (Individual, Followed by Small Group):

Support students as they work in properly identifying the degree of the numerator and denominator so that it is easier for them to draw conclusions. As they are graphing the rational functions, you may wish to encourage them to graph the asymptotes along with the function, so that it is easier to see that they are correct. Many students will use the technology to find the intercepts, so you may need to prompt them to think about how they could find the intercepts without technology. It will help if students are required to write the intercepts as points, so that they are reminded that a y-intercept occurs when $x = 0$ and an x-intercept occurs when $y = 0$.

The most challenging relationship for students to discover is that when the degree of the numerator is the same as the degree of the denominator, the horizontal asymptote is the ratio of the lead terms. Based on the examples, students may begin to think that the horizontal asymptote in this case is always $y = 1$. You may need to challenge their thinking by pressing them to look closely at problem #6, where the asymptote is $y = 1/2$. If students are not coming up with a conjecture for the horizontal asymptote in the case where the degree of the numerator is the same as the degree of the denominator, encourage them to choose functions of this type for the last two problems.

Discuss (Whole Group):

Begin the discussion with the graphic organizer table, asking students where they found the vertical asymptotes. Ask students to make arguments about why the vertical asymptotes occur where the denominator is zero. Be sure to make the point that the function is undefined at the vertical asymptotes because division by zero is undefined.

Next, ask the class to describe how to find the horizontal asymptote when the degree of the numerator is less than the degree of the denominator. Ask previously-selected students to explain how they drew this conclusion based on problems #2, #3, and #5. In addition, remind students of their experience with $y = 1/x$ confirms the conclusion that the horizontal asymptotes will be at $y = 0$. Ask students why this occurs. Press the class to think about their previous work with the end behavior of polynomials. They learned that as values of x become very large, the higher degree polynomials grow much faster than a lower degree polynomial. If the higher degree polynomial is in the denominator of a fraction, then that fraction will eventually approach zero.

Next, ask the class to share ideas for how to find the horizontal asymptote when the degree of the numerator and denominator are the same. Use problems #1, #4, and #6 to confirm or deny the conjectures that the class has made. Students may need help in recognizing and describing the idea that the horizontal asymptote will be the ratio of the lead terms. Again, ask students why this may occur and refer back to their work with the end behavior of polynomials. Press for the idea that for large values of x the difference in the rate of change between the two polynomials is caused by the lead coefficients. Use technology to show a numeric example of how this occurs, using problem #4 as an example.

Finally discuss how to get the intercepts. Most students should be able to say that the x-intercepts are found by substituting $y = 0$ into the equation. Select a student to share that has extended that idea and noticed that this turns out to be the same as looking for the roots of the numerator. Finding the y-intercept is no different than any other function, but quick methods for doing should be highlighted. Close the lesson by asking students to be sure that their graphic organizer is now completely correct and giving time to make modifications, if needed.

Aligned Ready, Set, Go: *Rational Expressions and Functions 4.3*

READY, SET, GO! Name Period Date

READY

Topic: Doing arithmetic with rational numbers

Perform the indicated operation. Be thoughtful about each step you perform in the procedure. Show your work.

1. $\frac{5}{17} + \frac{8}{17}$
2. $\frac{3}{10} + \frac{16}{25}$
3. $\frac{4}{5} + \frac{7}{11}$
4. $\left(\frac{2}{3}\right) \cdot \left(\frac{4}{5}\right)$
5. $\left(\frac{2}{3}\right) \cdot \left(\frac{9}{16}\right)$
6. $\left(\frac{10}{33}\right) \cdot \left(\frac{11}{15}\right)$
7. Explain the procedure for adding two fractions.
   a. When the denominators are the same:

   b. When the denominators are different:

8. Explain the procedure for multiplying two fractions.

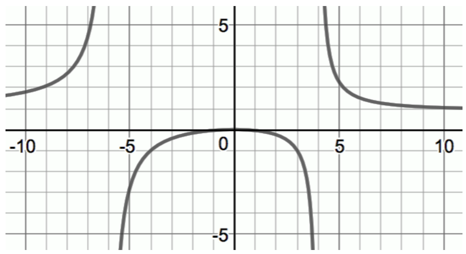
9. When multiplying two fractions, is it better to reduce before you multiply or after you multiply? Explain your reasoning.

SET

Topic: Identifying key features of a rational function

Fill in the specified features of each rational function. Sketch the asymptotes on the graph and mark the location of the intercepts.

10. $y = \frac{x^2}{(x+6)(x-4)}$



Degree of num. ________ Degree of denom.________

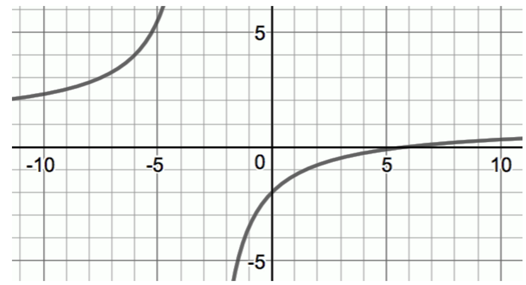
Equation of horizontal asymptote:

Equation of vertical asymptote(s):

y- intercept: (write as a point)

x- intercept(s): (write as points)

11. $y = \frac{(x-6)}{x+3}$



Degree of num. ________ Degree of denom.________

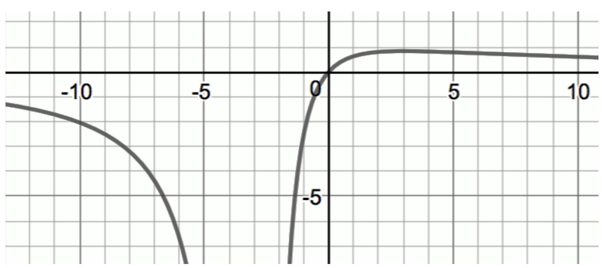
Equation of horizontal asymptote:

Equation of vertical asymptote(s):

y- intercept: (write as a point)

x- intercept(s): (write as points)

12. $y = \frac{10x}{(x+3)^2}$



Degree of num. ________ Degree of denom.________

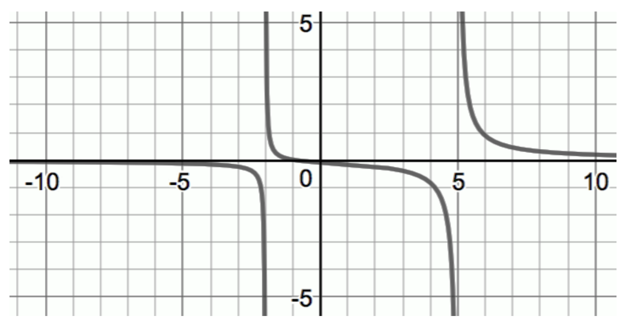
Equation of horizontal asymptote:

Equation of vertical asymptote(s):

y- intercept: (write as a point)

x- intercept(s): (write as points)

13. $y = \frac{(x+1)}{(x+2)(x-5)}$



Degree of num. ________ Degree of denom.________

Equation of horizontal asymptote:

Equation of vertical asymptote(s):

y- intercept: (write as a point)

x- intercept(s): (write as points)

GO

Topic: Reducing fractions

Reduce the following fractions to lowest form. Then explain the mathematics that makes it possible to rewrite the fraction in its new form. (Improper fractions should not be written as mixed numbers.) **If a fraction can't be reduced, explain why.**

14. $\frac{12}{15}$ Explanation:

15. $\frac{26}{11}$ Explanation:

17. $\frac{51}{17}$ Explanation:

18. $\frac{6}{13}$ Explanation:

19. $\frac{114}{27}$ Explanation:

20. $\frac{-14{,}529}{14{,}529}$ Explanation:

4.4 Are You Rational?

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A Solidify Understanding Task

Back in Module 3 when we were working with polynomials, it was useful to draw connections between polynomials and integers. In this task, we will use connections between rational numbers and rational functions to help us to think about operations on rational functions.

1. In your own words, define **rational number.**

Circle the numbers below that are rational and refine your definition, if needed.

$3 \qquad -5 \qquad \frac{2}{3} \qquad \frac{20}{3} \qquad 14 \qquad 2.7 \qquad \sqrt{5} \qquad 2^3 \qquad 3^{-3} \qquad log_2 9 \qquad \frac{7}{0}$

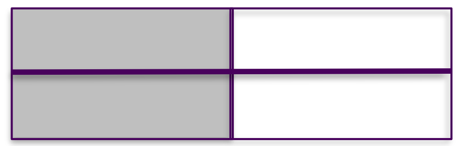
2. The formal definition of a **rational function** is as follows:

A function $\boldsymbol{f(x)}$ is called a rational function if and only if it can be written in the form $\boldsymbol{f(x) = \frac{P(x)}{Q(x)}}$ where $\boldsymbol{P \text{ and } Q}$ are polynomials in $\boldsymbol{x}$ and $\boldsymbol{Q}$ is not the zero polynomial.

Interpret this definition in your own words and then write three examples of rational functions.

3. How are rational numbers and rational functions similar? Different?

Now we are going to use what we know about rational numbers to perform operations on rational expressions. The first thing we often need to do is to simplify or "reduce" a rational number or expression. The numbers and expressions are not really being reduced because the value isn't actually changing. For instance, 2/4 can be simplified to ½, but as the diagram shows, these are just two different ways of expressing the same amount.



Let's try using what we know about simplifying rational numbers to simplify rational expressions. Fill in any missing parts in the fractions below.

Given:	$\frac{24}{30}$	4. $\frac{x^2-x-6}{x^2-4}$	5. $\frac{x^2+8x+15}{x^2+9x+18}$
Look for common factors:	$\frac{2\cdot 2\cdot 2\cdot 3}{2\cdot 3\cdot 5}$	$\frac{\qquad}{(x+2)(x-2)}$	
Divide numerator and denominator by the same factor(s):	$\frac{\cancel{2}\cdot 2\cdot 2\cdot \cancel{3}}{\cancel{2}\cdot \cancel{3}\cdot 5}$		
Write the simplified form:	$\frac{4}{5}$	$\frac{x-3}{x-2}$	$\frac{x+5}{x+6}$

6. Why does dividing the numerator and denominator by the same factor keep the value of the expression the same?

7. If you were given the expression $\frac{x}{x^2-1}$, would it be acceptable to reduce it like this:

$$\frac{\cancel{x}}{\cancel{x^2}-1}=\frac{1}{x-1}$$

Explain your answer.

In *4.3 Rational Thinking,* we learned to predict vertical and horizontal asymptotes, and to find intercepts for graphing rational functions.

8. Given $f(x) = \frac{x^2-x-6}{x^2-4}$, predict the vertical and horizontal asymptotes and find the intercepts.

9. Use technology to view the graph. Were your predictions correct? What occurs on the graph at $x = -2$?

Rational numbers can be written as either proper fractions or improper fractions.

10. Describe the difference between proper fractions and improper fractions and write two examples of each.

A rational expression is similar, except that instead of comparing the numeric value of the numerator and denominator, the comparison is based on the *degree* of each polynomial. Therefore, *a rational expression is proper if the degree of the numerator is less than the degree of the denominator, and improper otherwise.* In other words, improper rational expressions can be written as $\frac{a(x)}{b(x)}$, where $a(x) and\ b(x)$ are polynomials and the degree of $a(x)$ is greater than or equal to the degree of $b(x)$.

11. Label each rational expression as proper or improper.

$\frac{(x+1)}{(x-2)(x+2)}$ $\qquad$ $\frac{x^3-3x^2+5x-1}{x^2-4x+4}$ $\qquad$ $\frac{(x+3)(x+2)}{x^4-4}$ $\qquad$ $\frac{x+3}{x+5}$ $\qquad$ $\frac{x^3-5x+2}{x-10}$

As we may remember, improper fractions can be rewritten in an equivalent form we call a mixed number. If the numerator is greater than the denominator then we divide the numerator by the denominator and write the remainder as a proper fraction. In math terms we would say:

If $a > b$, then the fraction $\frac{a}{b}$ can be rewriten as $\frac{a}{b} = q + \frac{r}{b}$, where q represents the quotient and r represents the remainder.

12. Rewrite each improper fraction as an equivalent mixed number.

a) $\frac{37}{5} =$

b) $\frac{150}{12} =$

Rational expressions work the very same way. If the expression is improper, the numerator can be divided by the denominator and the remainder is written as a fraction. In mathematical terms, we would say:

$\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}$ where $q(x)$ represents the quotient and $r(x)$ represents the remainder.

Try it yourself! Label each rational expression as proper or improper. If it is improper, then divide the numerator by the denominator and write it in an equivalent form.

13. $\frac{x^2+5x+7}{x+2}$

14. $\frac{-5x+10}{x^3+6x^2+3x-1}$

15. $\frac{x^2+2x+5}{x+3}$

16. $\frac{3x+8}{x-1}$

In *4.3 Rational Thinking,* when we looked at the graphs of rational functions, we did not consider the case when the numerator of the fraction is greater than the denominator. So, let's take a closer look at the rational function from #13.

16. Let $f(x) = \frac{x^2+5x+7}{x+2}$. Where do you expect the vertical asymptote and the intercepts to be?

17. Use technology to graph the function. Relate the graph of the function to the equivalent expression that you wrote. What do you notice?

18. Let's try the same thing with #15. Let $f(x) = \frac{x^2+2x+5}{x+3}$. Find the vertical asymptote, the intercepts, and then relate the graph to the equivalent expression for $f(x)$.

19. Using the two examples above, write a process for predicting the graphs of rational functions when the degree of the numerator is greater than the degree of the denominator.

4.4 Are You Rational? – Teacher Notes

A Solidify Understanding Task

Purpose:
In this task, rational functions are formally defined and connected to rational numbers. Students will use these connections to reduce rational functions and to identify rational functions that are improper and write them in an equivalent form. Students will graph rational functions that can be reduced, seeing that the graph is equivalent to the reduced function except that there is a hole in the graph, produced by the factor that is reduced. They will also graph improper rational functions and identify the slant asymptote.

Core Standards Focus:
F.IF.7d Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
d. Graph rational functions, identifying zeros when suitable factorizations are available, and showing end behavior.

A.APR.6 Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form such that $\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}$ where $a(x), b(x), q(x)\ and\ r(x)$ are polynomials with degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system.

A.APR.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Standards for Mathematical Practice:

SMP 2 – Reason abstractly and quantitatively

SMP 8 – Look for and express regularity in repeated reasoning

Vocabulary: Slant asymptote

The Teaching Cycle:

Launch--Part 1 (Whole Group):

Begin the task by asking students question #1, "What is a rational number?" Ask them to look at the numbers given in question #1 and to decide which are rational. Discuss responses and define rational numbers as "any number that can be expressed as the quotient or ratio, p/q of two integers, a numerator p and a non-zero denominator q." Then, discuss the formal definition of rational functions that is given in question #2 and connect it to the information definition given in task 4.3, that rational functions are a ratio of polynomials. Share a few of the examples that students have written and tell students that rational functions behave much like rational numbers. Tell students that in this task, they will be working with rational functions that can be reduced or are improper. You may wish to go over the process for reducing rational numbers before asking students to start working with rational functions. Then, tell students that they should rely on their experiences with rational numbers to help them think about rational functions. Ask students to work problems up to #9 before having a class discussion and re-launching the remainder of the task.

Explore (Individually, Followed by Small Group):

Monitor students as they work on questions #4 and #5 to see that they are making sense of the necessary steps in working with rational expressions. The scaffolding is given in the task, but students will need to be able to factor the functions and reduce them. The answers are given in the task so that students can focus on the process that will get them the answer. Make sure that they are working on the process and correcting errors if their work is not leading them to the right answers. Help students to focus on the idea that reducing is just finding an equivalent form by dividing the same factor out of the numerator and denominator, which is just dividing by one.

Select students to share that have articulated this idea in some way, so that other students can hear several versions of the idea.

As students progress, support them in using technology for question #9 to explore the area of the graph near -2 where there is a hole. As the curve is traced on some graphing technology, like Desmos, the value at -2 is shown to be undefined. On other technology, like some graphing calculators, there is no value shown at -2. Either way, students will need support for interpreting the graph during the discussion. Listen for their ideas that will be useful to share.

Discuss—Part 1 (Whole Group):

Begin the discussion by asking a student share his/her work on question #4. Focus on how the numerator was factored and the common factors are reduced. Similarly, ask a student to share question #5. Ask previously-selected students to share ideas for question #6. Emphasize that common factors must be divided from the numerator and denominator so the value of the fraction is unchanged. Then, ask students about question #7. Since this problem is based upon a common misconception, there may be some controversy. Let students make arguments about whether or not it is correct. After hearing arguments, bring the class to consensus that it is not correct because x is not a factor in the denominator, so "reducing" it will not result in an equivalent expression.

Discuss questions #8 and #9. Ask students for their predictions and then project a graph of the function. Ask students why there is not an asymptote at $x = -2$. Help them to see that the original function is not defined at -2, so there is no value there, and all the other values are the same as the function that remains after the factor $(x + 2)$ is reduced.

Launch--Part 2 (Whole Group):

Re-launch the second part of the task by discussing questions #10 and #11 together. Discuss question #12, emphasizing the process for writing a mixed numeral from an improper fraction. Tell students that this is the same process for improper rational functions and then ask them to work on the remaining questions in the task.

Explore (Individually, Followed by Small Group):

Students may need a little nudge to decide to use long division of polynomials. Remind them that fractions are just another way to write division, so the hint of what to do is right in the expression that they are given.

As students are working on question 17, watch for a student that notices that the function is approaching the line $y = x + 3$. This may take a little prompting with questions like, "What is the end behavior of the function? What line does it appear to be approaching? How is that line related to the equivalent function that you found?"

Discuss—Part 2 (Whole Group):

Begin the discussion with question #13. Ask a student to share his/her work in writing an equivalent expression. Then, project a graph of the function and ask the previously selected student to describe the relationship between the equivalent expression and the graph, specifically how to identify the end behavior of the function. Tell students that this type of end behavior is called a slant (or oblique) asymptote. Repeat the same process with question #15. Discuss question #19, bringing the class to consensus on how to graph a rational function where the degree of the numerator is greater than the degree of the denominator that includes:

- Dividing the numerator by the denominator to find an equivalent expression.
- Finding the vertical asymptote by finding the roots of the denominator.
- Using the equivalent expression to identify the slant asymptote.
- Find the intercepts using the same process as other rational functions.

If time allows, ask a student to share his/her work in writing an equivalent expression for question #16. Then ask the class what they would expect of the graph of the original function. They should notice that the degree of the numerator is the same as the degree of the denominator, so they can predict the vertical and horizontal asymptotes. Connect their predicted asymptotes to the equivalent form. They should be able to see that this function turns out to be a transformation of $y = 1/x$.

Aligned Ready, Set, Go: ***Rational Expressions and Functions 4.4***

READY, SET, GO! Name Period Date

READY

Topic: Connecting features of polynomials and rational functions

Find the roots and domain for each function.

1. $f(x) = (x+5)(x-2)(x-7)$

2. $g(x) = x^2 + 7x + 6$

3. $k(x) = \frac{1}{(x+5)(x-2)(x-7)}$

4. $h(x) = \frac{1}{(x^2+7x+6)}$

5. Make a conjecture that compares the domain of a polynomial with the domain of the reciprocal of the polynomial. *(Note that the reciprocal of a polynomial is a rational function.)*

6. Do the roots of the polynomial tell you anything about the graph of the reciprocal of the polynomial? Explain.

7. Find the y-intercept for #1 and #2. What is the y-intercept for #3 and #4?

SET

Topic: Distinguishing between proper and improper rational functions.

Determine if each of the following is a proper or an improper rational function.

8. $$f(x) = \frac{x^3 + 3x^2 + 7}{7x^2 - 2x + 1}$$

9. $$f(x) = x^3 - 5x^2 - 4$$

10. $$f(x) = \frac{3x^2 - 2x + 7}{x^5 - 5}$$

11. $f(x) = \frac{x^3+4x^2+2x}{10x+7}$

12. $f(x) = \frac{5x^2-4x+4}{7x^5-2x+3}$

13. Which of the above functions have the following end behavior?

$$as\ x \to \infty, f(x) \to 0\ and\ as\ x \to -\infty, f(x) \to 0$$

14. Complete the statement:
ALL *proper* rational functions have end behavior that ______________________________

Determine if each rational expression is proper or improper. If improper, use long division to rewrite the rational expressions such that $\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}$ where $q(x)$ represents the quotient and $r(x)$ represents the remainder.

15. $\frac{2x^3-7x^2+6}{x-1}$

16. $\frac{(x+1)}{(x-2)(x+2)}$

17. $\frac{x^3-3x^2+5x-1}{x^2-4x+4}$

18. $\frac{x^3-5x+2}{x-10}$

GO

Topic: Finding the domain of rational functions that can be reduced

State the domain of the following rational functions.

19. $y = \frac{(x-2)}{(x-2)(x+5)}$

20. $y = \frac{(x+6)}{(x-4)(x+6)}$

21. $y = \frac{(x-7)(x+10)}{(x+10)(x-3)(x-7)}$

a) Each of the previous functions has only one vertical asymptote. Write the equation of the vertical asymptote for #19, #20, and #21 below.

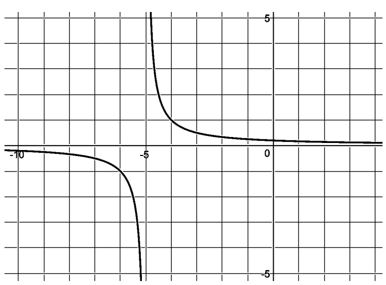
19a) V.A.

20a) V.A.

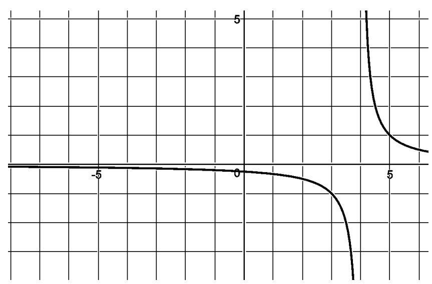
21a) V.A.

b) The graphs of #19, #20, and #21 are below. For each graph, sketch in the vertical asymptote. Put an open circle on the graph anywhere it is undefined.

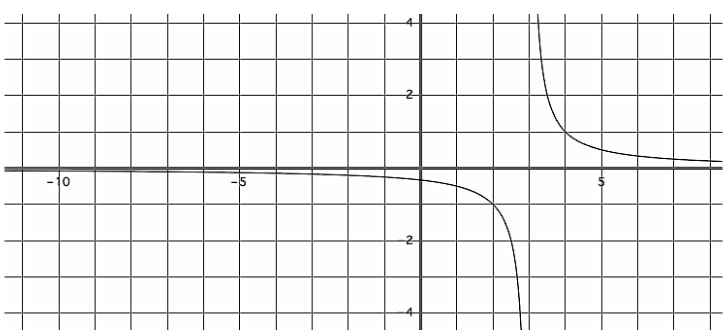
19b)



20b)



21b)



4.5 Just Act Rational

A Solidify Understanding Task

In *4.4 Are You Rational?*, you saw how connecting rational numbers can help us to think about rational functions. In this task, we'll extend that work to consider operations on rational expressions.

Let's begin with multiplication. In each of the tables below there are missing descriptions and missing parts of expressions. Your job is to follow the process for multiplying rational numbers and use it to complete the descriptions of the process and to work the analogous problems with rational expressions.

1.

Description of the Procedure:	**Example Using Numbers**	**Rational Expression A**	**Rational Expression B**
Given:	$\frac{3}{4}\cdot\frac{5}{6}$	$\frac{x(x-3)}{(x+1)}\cdot\frac{5}{x^2}$	$\frac{(x+1)(x-2)}{(x+2)}\cdot\frac{(x+5)}{(x-2)(x+2)}$
	$\frac{3\cdot 5}{4\cdot 6}$		
	$\frac{\cancel{3}\cdot 5}{2\cdot 2\cdot 2\cdot\cancel{3}}$		
Write the simplified form:	$\frac{5}{8}$	$\frac{5(x-3)}{x(x+1)}$ Or $\frac{5x-15}{x(x+1)}$	$\frac{(x+1)(x+5)}{(x+2)^2}$ Or $\frac{x^2+6x+5}{(x+2)^2}$

2. In multiplication, does it matter in which step the simplifying is done? Why?

Now let's try the same process with division.

3. Complete the table below by filling in the missing descriptions or steps for dividing the rational expressions.

Description of the Procedure:	Example Using Numbers	Rational Expression A	Rational Expression B
Given:	$\frac{3}{4} \div \frac{5}{6}$	$\frac{(x-2)}{(x+2)} \div \frac{(x+5)}{x(x+2)}$	$\frac{(x+1)(x-6)}{(x+2)} \div \frac{(x+1)}{(x-3)(x+2)}$
	$\frac{3}{4} \cdot \frac{6}{5}$		
Multiply and simplify:	$\frac{3 \cdot 6}{4 \cdot 5} = \frac{3 \cdot \cancel{2} \cdot 3}{2 \cdot \cancel{2} \cdot 5}$		
	$\frac{9}{10}$	$\frac{x(x-2)}{(x+5)}$ Or $\frac{x^2-2x}{(x+5)}$	$(x-3)(x-6)$

4. How could you check your answer after performing an operation on a pair of rational expressions?

Are you ready for addition? Try it!

5. Complete the table below by filling in the missing descriptions or steps for adding the rational expressions.

Description of the Procedure:	**Example Using Numbers**	**Rational Expression A**
Given	$\frac{2}{3}+\frac{1}{7}$	$\frac{3}{(x+7)}+\frac{4}{(x-4)}$
Determine the factors needed for a common denominator	$\frac{2}{3}\left(\frac{7}{7}\right)+\frac{1}{7}\left(\frac{3}{3}\right)$	$\frac{3}{(x+7)}\left(\frac{x-4}{x-4}\right)+\frac{4}{(x-4)}\left(\frac{x+7}{x+7}\right)$
	$\frac{14}{21}+\frac{3}{21}$	
	$\frac{14+3}{21}$	
Simplify	$\frac{17}{21}$	$\frac{7x+16}{(x+7)(x-4)}$

6. After writing both terms with a common denominator, are they equivalent to the original terms? Explain.

7. Complete the table below by filling in the missing descriptions and steps for adding the rational expressions.

Description of the Procedure:	**Rational Expression B**
Given	$\frac{2x-3}{(x+3)}+\frac{x+5}{(x-2)}$
Determine the factors needed for a common denominator	
Simplify	$\frac{3x^2+x+21}{(x-2)(x+3)}$

8. Is it possible to get an answer to an addition problem that still needs to be reduced? If so, how can you tell if your answer needs to be further simplified?

9. At long last, we have subtraction. Complete the table below by filling in the missing descriptions and steps for adding the rational expressions.

Description of the Procedure:	**Example Using Numbers**	**Rational Expression A**
Given:	$\frac{7}{8} - \frac{3}{5}$	$\frac{3x+1}{(x+5)} - \frac{x-4}{(x-2)}$
	$\frac{7}{8}\left(\frac{5}{5}\right) - \frac{3}{5}\left(\frac{8}{8}\right)$	
	$\frac{35}{40} - \frac{24}{40}$	
	$\frac{35-24}{40}$	
Simplify	$\frac{11}{40}$	$\frac{2x^2-6x+18}{(x+5)(x-2)}$ Or $\frac{2(x^2-3x+9)}{(x+5)(x-2)}$

10. What strategies will you use to be sure that you don't make sign errors when subtracting?

4.5 Just Act Rational – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is for students to learn to add, subtract, multiply and divide rational expressions. The task is designed so that students, again, connect operations with rational numbers to operations with rational expressions.

Core Standards Focus:

A.APR.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Standards for Mathematical Practice:

SMP 1 – Make sense of problems and persevere in solving them

SMP 8 – Look for express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Group):

Begin the task by reminding students that rational numbers and rational expressions are analogous and that they operate the same way. Tell students that in today's task they are going to use what they know about rational numbers to perform operations on rational expressions. They shouldn't worry if they're not too great with fractions. There is a lot of support in the task to help them. Begin by demonstrating how to use the support given. Think-aloud for students about how to get from the first step to the second in the multiplication problem. It could sound something like this:

> "I see that I'm given $\frac{3}{4} \cdot \frac{5}{6}$. In the next step, I have $\frac{3 \cdot 5}{4 \cdot 6}$, so I see that the numerators have been multiplied and the denominators have been multiplied to get a single fraction. I will write that in the box provided. Now, I can do the same thing with the rational expression,

$\frac{x(x-3)}{(x+1)} \cdot \frac{5}{x^2}$. I will multiply the numerators together and the denominator together and make a single fraction: $\frac{5x(x-3)}{x^2(x+1)}$."

Stop there and tell students that they will be using the same strategy throughout the task. They should decide what was done with the numbers, label it, and then do the same thing with the rational expressions. Tell students that the answers to the problems are provided at the end so that they can make sure that their process is correct. If they get to the end of the problem and their work doesn't lead to the answer given, they need to go back and find their mistake.

Explore (Individual, Followed by Small Group) :

Monitor students as they work, helping them to see and label each step of the operations. Expect many mistakes as students work with the rational expressions. Watch for students that notice their mistakes and find ways to correct them so that they can describe their process for the class. Some of the common errors to watch for so that they can be addressed during the discussion:

- Improperly reducing terms, not factors
- Not using parentheses and making mistakes in distributing (usually when changing to common denominator).
- Sign errors in subtraction.

Discuss (Whole Group) :

Begin the discussion with a student sharing his/her work on Rational Expression B in multiplication. Ask the student to explain how the answer was reduced, and then ask the class if it makes a difference in which step the fraction is reduced. Make sure the discussion includes the idea that reducing is dividing by 1, so it can happen in either step since multiplication and division are at the same level in order of operations. Discuss why it is correct to divide $x - 2$ from the numerator and denominator in this case to simplify the fraction.

Next, ask a student to explain how his/her work on problem 3, Rational Expression B in division. Ask the student to explain the process for division, and then ask the class why the denominator of the fraction is gone. Again, emphasize that the denominator is 1 (not 0) because the result of dividing the common factors is one.

Follow with a student that presents his/her work on problem 7, Rational Expression B in addition. Ideally, find a student that did not properly distribute in the first attempt and then found a way to correct the problem. Help students to see when to put parentheses around the numerator to remind them that they must distribute each term when they multiply.

End the discussion with problem 9, Rational Expression A in subtraction. Again, it would be ideal if a student shares that initially made mistakes and describes how they corrected them. The two potential errors with subtraction are failing to distribute properly when getting a common denominator and making sign errors when subtracting. Ask students for strategies to help them avoid these errors and record the errors.

Close the discussion by going back to each of the organizer tables and recording the steps for the operation. Tell students that the organizers are to help them remember the procedure for each operation.

Aligned Ready, Set, Go: ***Rational Expressions and Functions 4.5***

READY, SET, GO! Name Period Date

READY

Topic: Recalling trigonometric functions

Use the given triangle to write the values of $sinA, cosA, and\ tanA$ and $sinB, cosB, and\ tanB$.

1.

sin A =

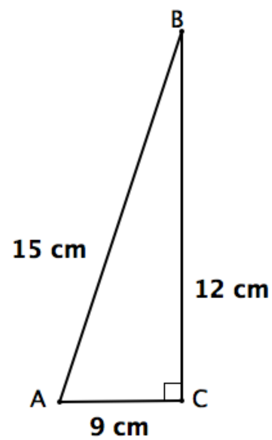
cos A =

tan A =

sin B =

cos B =

tan B =



2.

sin A =

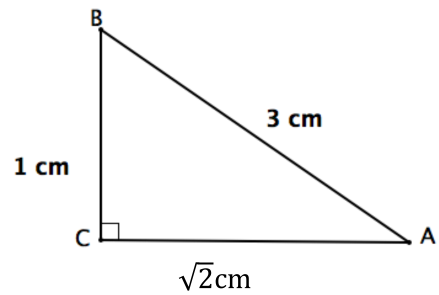
cos A =

tan A =

sin B =

cos B =

tan B =



3.

sin A =

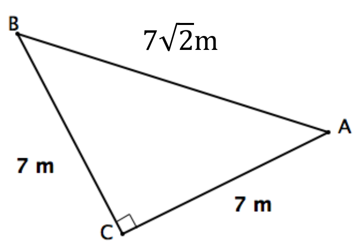
cos A =

tan A =

sin B =

cos B =

tan B =



4.

sin A =

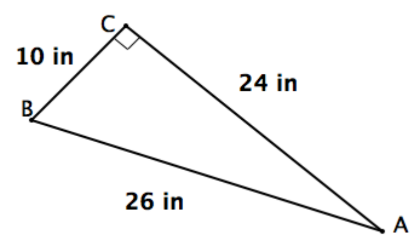
cos A =

tan A =

sin B =

cos B =

tan B =



SET

Topic: Adding, subtracting, multiplying, and dividing rational functions

5. Angela simplified the following rational expressions. Only one of the three problems is correct. Determine which one she answered correctly. Then identify Angela's errors in the two that are incorrect and correct them.

a. $\frac{5x}{(x-3)}+\frac{2}{(x-1)}$

$$\frac{5x(x-1)}{(x-3)(x-1)}+\frac{2(x-3)}{(x-3)(x-1)}$$

$$\frac{5x^2-x+2x-3}{(x-3)(x-1)}$$

$$\frac{5x^2+x-3}{(x-3)(x-1)}$$

b. $\frac{x}{(x+3)}-\frac{4(x+3)}{(x-1)}$

$$\frac{x}{1}-\frac{4}{(x-1)}$$

$$\frac{x(x-1)}{(x-1)}-\frac{4}{(x-1)}$$

$$\frac{x^2-x-4}{(x-1)}$$

c. $\frac{(x+1)(x-2)}{(x+2)}\times\frac{(x+5)}{(x-2)(x+2)}$

$$\frac{(x+1)(x-2)(x+5)}{(x+2)(x-2)(x+2)}$$

$$\frac{(x+1)(x+5)}{(x+2)(x+2)}$$

$$\frac{x^2+6x+5}{x^2+4x+4}$$

Simplify each expression. Reduce when possible.

6. $\frac{2x+6}{(x+1)}-\frac{4}{(x+1)}$

7. $\frac{2x}{x+2}+\frac{x-1}{x-5}$

8. $\frac{x^2+6x+8}{x^2-5x+4}\cdot\frac{x^2+3x-4}{x^2+4x+4}$

9. $\frac{4x+8}{5x-20}\div\frac{x^2-3x-10}{x^2-4x}$

10. $\frac{2x}{(x^2-4)}+\frac{4}{(x+2)}$

11. $\frac{x-10}{x-4}-\frac{x+2}{4-x}$

Topic: Comparing rational numbers and rational expressions

12. Rational numbers and rational expressions are comparable because they have similar features. Complete the table below by writing the comparable situation for each statement written.

Rational Numbers	Rational Expressions
Whole numbers are rational numbers with a denominator of one.	a)
b)	Rational expressions are undefined when the denominator is equal to zero.
When you add, subtract, multiply or divide two rational numbers, the result is also a rational number.	c)
Rational fractions are classified as proper fractions when the numeric value of the numerator is smaller than the denominator.	d)

GO

Topic: Finding values of x that affect the domain of a rational expression

Identify the values of *x* for which the expression is undefined, if any.

13. $\frac{10}{x-4}$

14. $\frac{22}{x}$

15. $\frac{x-7}{x+15}$

16. $\frac{2x}{5}$

4.6 Sign on the Dotted Line

A Practice Understanding Task

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Josue and Francia are working on graphing all kinds of rational functions when they have this little dialogue:

Josue: It's easy to figure out where the asymptotes and intercepts are on a rational function.

Francia: Yes, and it's almost like the asymptotes split the graph into sections. All you need to know is what the graph is doing in each section.

Josue: It seems almost easier than that. It's like all you need to know is whether it's going up or down either side of the vertical asymptote and then use logic to figure it out from there.

Francia: Don't overlook the intercepts. They give some pretty important clues.

Josue: Yeah, yeah. I wonder if we can figure out an easy way to determine the behavior near the asymptotes.

Francia: Seems easy enough to just plug in numbers and see what the outputs are, but maybe you don't even need exact values. Hmmm. We need to think about this.

Josue and Francia are definitely on to something. Everyone wants to find a way to be able to predict and sketch graphs easily. In this task, you're going to work on just that. **Start by finding asymptotes and intercepts, then figure out a strategy that you can use every time to quickly sketch the graph. After using your strategy to graph the function, use technology to check your work and refine your strategy.**

The examples you need to develop your strategy are on the following pages. Some of the functions given need to be combined and/or simplified to make one rational function. If this is the case, write the simplified function in the space next to the graph.

1.

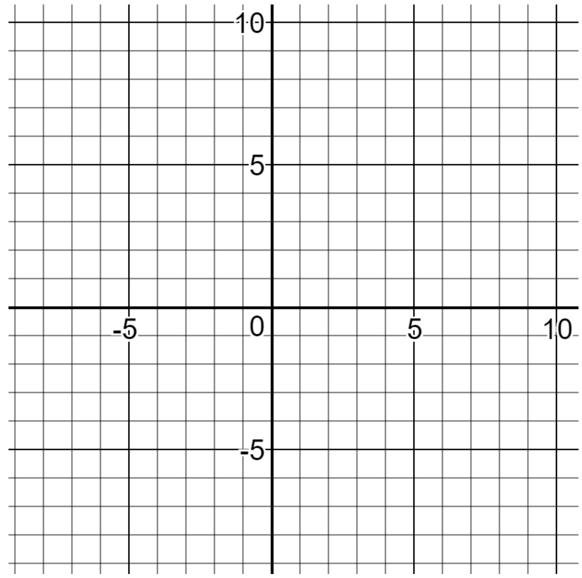
$$y = \frac{(x-5)(x+1)}{(x+2)(x-2)}$$

Vertical Asymptote(s) ______________

Horizontal or Slant Asymptote ___________

Intercepts _________________________

Graph:



2.

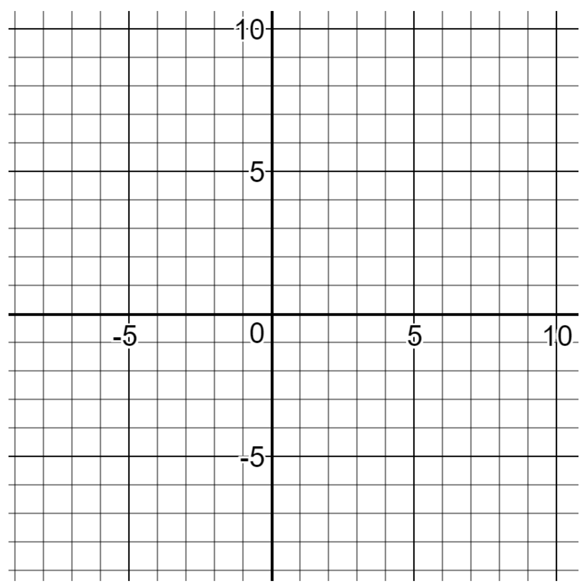
$$y = \frac{(x-3)}{(x+1)} \cdot \frac{x}{(x-4)}$$

Vertical Asymptote(s) ______________

Horizontal or Slant Asymptote ___________

Intercepts _________________________

Graph:



3.

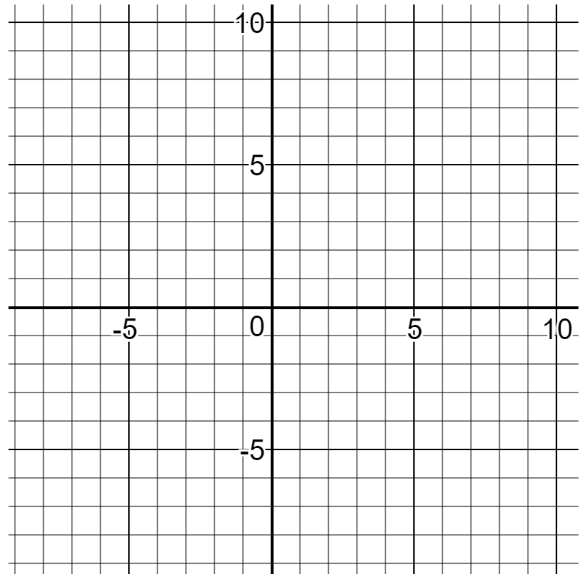
$$y = \frac{x^2 - 6x + 2}{(x - 2)}$$

Vertical Asymptote(s) ____________________

Horizontal or Slant Asymptote ______________

Intercepts ____________________________________

Graph:



4.

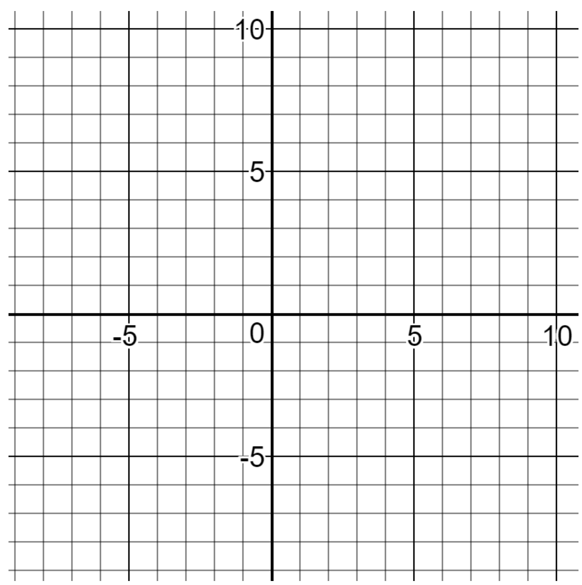
$$y = \frac{4}{(x + 1)} + \frac{(x - 5)}{(x - 3)}$$

Vertical Asymptote(s) ____________________

Horizontal or Slant Asymptote ______________

Intercepts ____________________________________

Graph:



5.

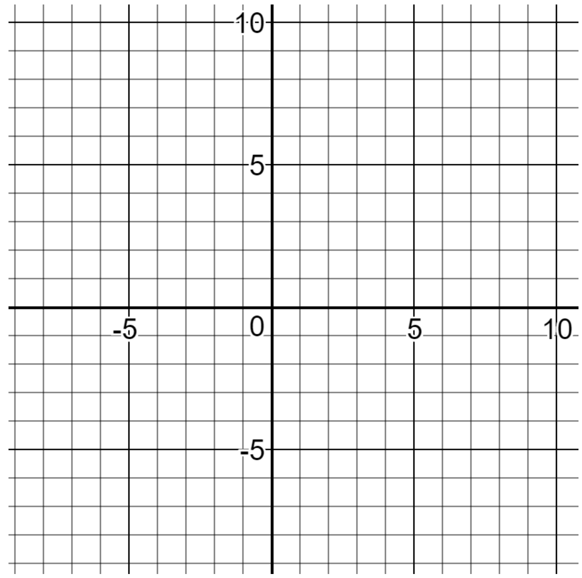
$$y = \frac{3x}{(x^2 + 2x + 1)} \div \frac{x-3}{x+1}$$

Vertical Asymptote(s) ___________________

Horizontal or Slant Asymptote _______________

Intercepts __________________________________

Graph:



6.

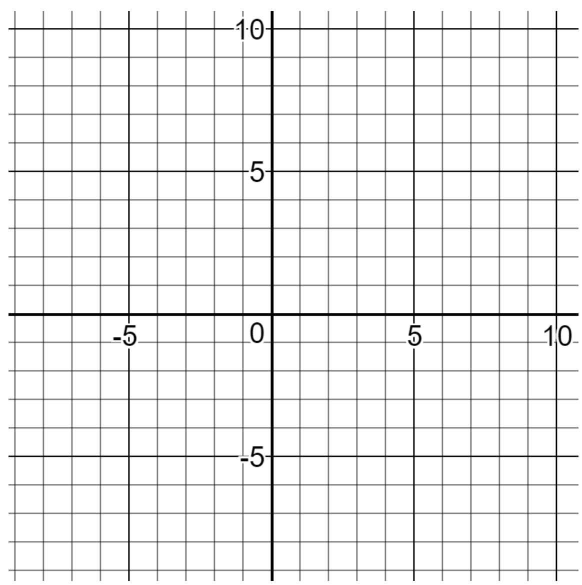
$$y = \frac{x+5}{x+4} - \frac{x+2}{x-1}$$

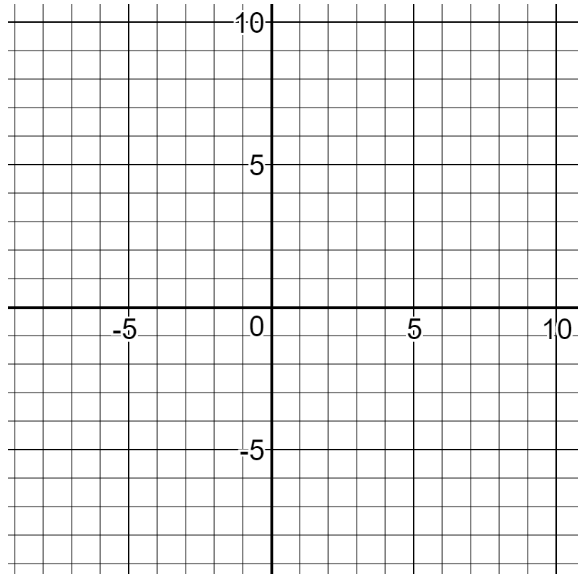
Vertical Asymptote(s) ___________________

Horizontal or Slant Asymptote _______________

Intercepts __________________________________

Graph:



7. $$y = \frac{2x^2 + x - 15}{x^2 + 4x + 3}$$ Vertical Asymptote(s) ________________ Horizontal or Slant Asymptote _____________ Intercepts _____________________________	Graph: 

8. Summarize your strategy for graphing rational functions with a step-by-step process.

4.6 Sign on the Dotted Line – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to have students connect all that they have learned about rational functions so far to sketch the graph of rational functions. Students are asked to develop a strategy for determining the behavior near the asymptotes as part of an overall strategy for graphing rational functions. Some of the functions given in the task require students to combine two rational expressions to make the function more predictable to graph.

Core Standards Focus:

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts, intervals where the function is increasing/decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

F.IF.7d Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
d. Graph rational functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Related: F.IF.5, F.IF.9, A.CED.1

Standards for Mathematical Practice:

SMP 3 – Construct viable arguments and critique the reasoning of others

SMP 7 – Look for and make use of structure

The Teaching Cycle:

Launch (Whole Group):

Start the task by reading the dialogue between Josue and Francia at the beginning of the task. Ask the class what information they can determine about any rational function if they just look at the equation. They should be able to describe how to determine if it will be a transformation of $y = 1/x$ and how to find the asymptotes and intercepts. Ask why it would be helpful to find the behavior near the asymptotes and why Francia might think that she doesn't need exact values. Ideally, a student will say that if one knows the function is positive near a vertical asymptote, then one knows that the graph is approaching infinity on that side of the asymptote.

Describe the expectations for the task, specifically, that students find a quick way to determine the behavior near the asymptotes and use it to develop a strategy for graphing any rational function. Tell students that some of the functions are not given in simplified form, so they may need to combine rational expressions before graphing the function. Stress that they should not use technology to graph a function until they are ready to check their work. In this way, they can try a technique and then modify it if it is not producing a correct graph.

Explore (Individual, Followed by Small Group):
Listen for students who have ideas about how to determine behavior near the asymptote. If a student is stuck, ask them to try a value that is .1 units on either side of a vertical asymptote. This will give a number that is close enough to represent the behavior but is complicated enough that it might lead them to look for a slightly simpler strategy. After they have found a value, ask them how they might think about just the sign of the output value near the asymptote and how they could use that along with the rest of the information about the function.

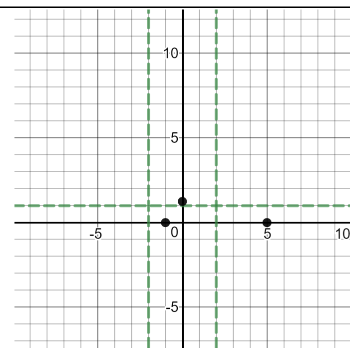
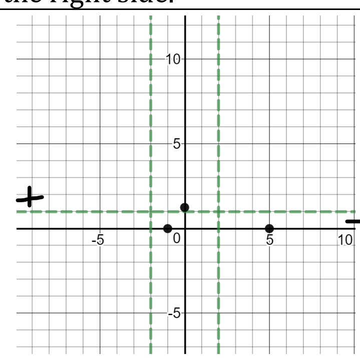
After students have figured out a way to use the signs rather than exact value, they will need to find a way to organize and keep track of their findings. Some people use tables, some people use a sign chart that looks like a number line divided by the vertical asymptotes, some people mark the asymptotes on their graph and then record the appropriate sign near the top or bottom of the asymptotes. Look for students that have found an effective organizational strategy so that they can share during the discussion.

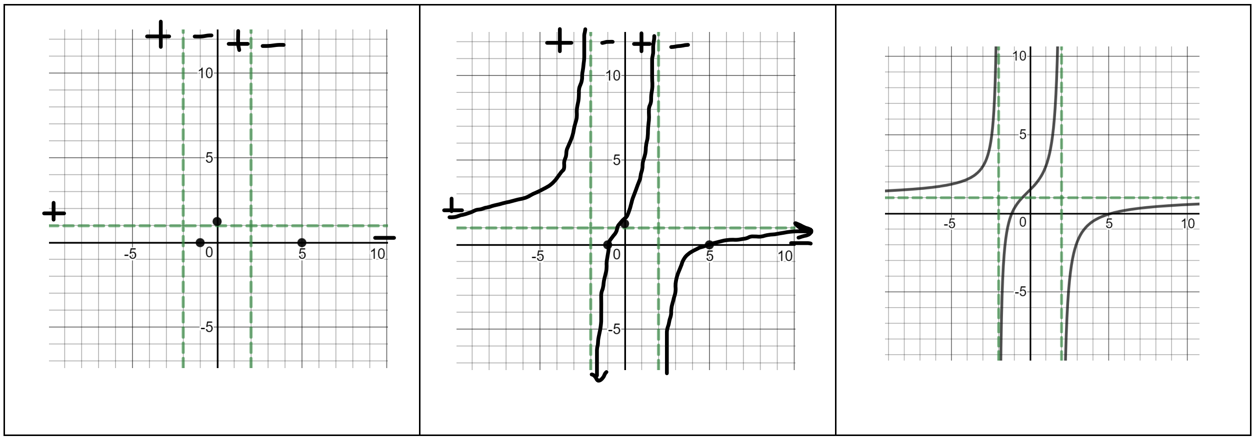
Be on the alert for problems with combining expressions. This task is designed to give students another opportunity to perform operations on rational expressions with the support of a teacher.

Encourage students to check that they have performed the operations correctly by using technology to check their work.

Discuss (Whole Group):

Begin the discussion with asking previously-selected students to share question #1. Since there are several steps in each problem, you may want to have different students share each step. The example below shows one possible way for students to talk about the graph.

Step 1: "I started by finding the vertical asymptotes by setting the denominator equal to zero. I got the horizontal asymptote by seeing that the degree of the numerator and denominator are the same, so I took the ratio of the lead terms. Then, I got the intercepts by substituting $x = 0$ and $y = 0$."	Step 2: "I marked the asymptotes on the graph with dotted lines. Then I plotted the intercepts."	Step 3: "I put in $x = -20$ to see what the left end behavior would be. $y = \frac{(-20-5)(-20+1)}{(-20+2)(-20-2)}$. That makes $\frac{-25(-19)}{-18(-22)}$ which is greater than 1, so I know that the graph is above the asymptote. Then I did a similar process to see that it was below the asymptote on the right side."
Vertical Asymptote(s) $x = 2, x = -2$ Horizontal or Slant Asymptote $y = 1$ Intercepts (5, 0), (-1,0), $(0,\frac{5}{4})$		
Step 4: "I checked x=-2.1. I didn't find exact values, only looked to see if it made the factors positive or negative. $y = \frac{(-)(-)}{(-)(-)} = +$ I did a similar process on the other side of the asymptote and on the asymptote at x=2. Then I marked it on the graph like this:	Step 5: "I used the points that I had and what I knew about the asymptotes to sketch the graph."	Step 6: "I checked it and felt pretty good about myself."



Proceed in a similar fashion with students sharing questions #3, 6, and 7. They all follow a similar process, but each of the problems has a special feature to discuss:

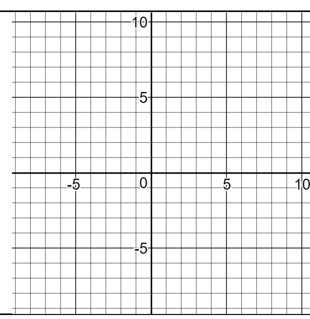
- Question #3 has a slant asymptote, so students must divide the numerator by the denominator to find the end behavior.
- Question #6 requires students to subtract the two rational expressions. The graph crosses the horizontal asymptote.
- Question #7 has a hole in the graph because the rational expression has a common factor in the numerator and denominator.

Finalize the lesson by recording a process for graphing rational functions such as this:

- Simplify the function.
- Find the vertical asymptotes by finding the zeros of the denominator.
- Find the horizontal or slant asymptote.
  - If degree of numerator < degree of denominator, $y = 0$
  - If degree of numerator = degree of denominator, $y =$ ratio of lead terms
  - If degree of numerator > degree of denominator, divide numerator by denominator to get slant asymptote
- Find intercepts by substituting $x = 0$ and $y = 0$.
- Test the behavior near the vertical asymptotes by finding if the values are positive or negative. Keep track of the results using a sign chart of some type.
- Sketch the function.

Aligned Ready, Set, Go: ***Rational Expressions and Functions 4.6***

How to Graph a Rational Function

Steps:	Example:	Remember this:
	$y = \dfrac{x+3}{(x-1)(x+2)}$	
		

READY, SET, GO!	Name	Period	Date

READY

Topic: Identifying extraneous solutions

1. Below is the work done to solve a rational equation. The problem has been worked correctly. Explain why the equation has only **one** solution.

Solve: $\frac{2}{x^2-2x} - \frac{1}{x-2} = 1$	
$\frac{2}{x(x-2)} - \frac{(x)1}{(x)(x-2)} = 1$	*Write using a common denominator.*
$\frac{2-x}{(x)(x-2)} = 1$	*Subtract.*
$(x)(x-2)\frac{2-x}{(x)(x-2)} = 1(x)(x-2)$	*Multiply both sides by the common denominator.*
$2 - x = x^2 - 2x$	*Simplify.*
$x^2 - x - 2 = 0$	*Write a quadratic equation in standard form.*
$(x-2)(x+1) = 0$	*Factor*
$x = 2 \text{ or } x = -1$	*Apply the Zero-Product Property and solve for z.*
	Substitute 2 and -1 into the original equation to see if the numbers are solutions.

Substitute the given numbers into the given equation. Identify which are actual solutions and which, if any, are extraneous.

2. a: -1 *and* $\frac{5}{2}$ $a - \frac{3}{2a+1} = 2$	3. *d: 0 and 3* $\frac{3d}{d^2-d} - \frac{1}{d-1} = 1$	4. m: 1 $\frac{1}{m^2-m} - \frac{1}{m-1} = 0$
Solve 5 and 6. Watch for extraneous solutions.	5. $\frac{1}{x^2-x} - \frac{1}{x-1} = \frac{1}{2}$	6. $2x + \frac{3}{x+2} = 1$

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SET

Topic: Predicting and sketching rational functions

Find the vertical asymptote(s), horizontal or slant asymptote, and intercepts. Then sketch the graph. (Do not use technology to get the graph. The max and mins do not need to be accurate.)

5.

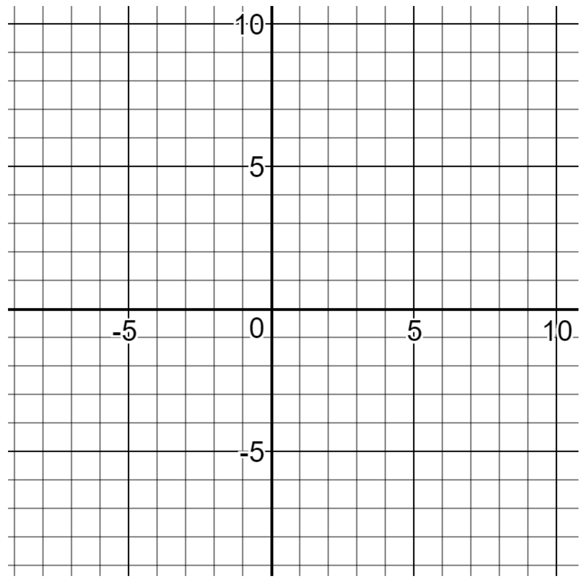
$$y = \frac{(x+4)}{(-2x-6)}$$

Vertical Asymptote(s) ____________________

Horizontal or Slant Asymptote ________________

Intercepts ____________________________________

Graph:



6.

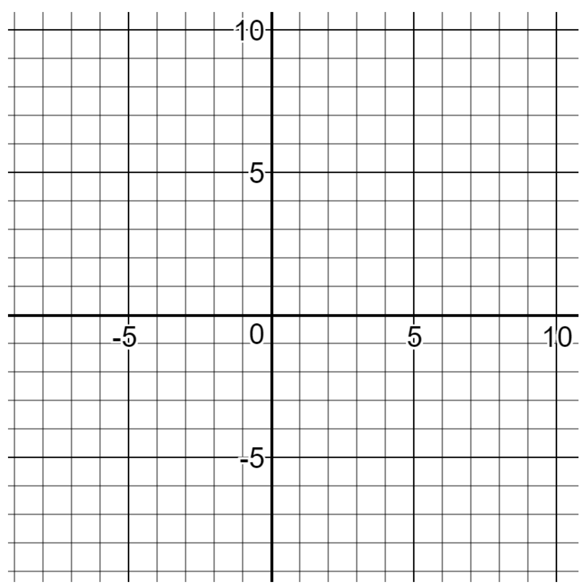
$$y = \frac{3x}{(x-3)} \cdot \frac{(x-4)}{(x+1)}$$

Vertical Asymptote(s) ____________________

Horizontal or Slant Asymptote ________________

Intercepts ____________________________________

Graph:



7.

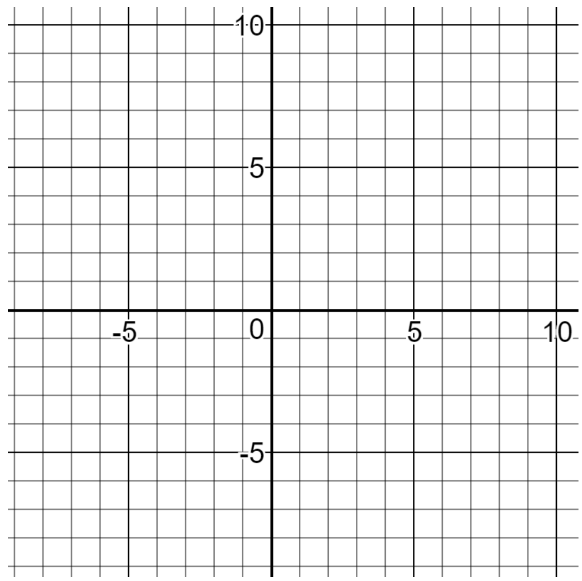
$$y = \frac{(x^2 - 4x)}{(4x - 8)} \div \frac{(x + 2)}{x + 4}$$

Vertical Asymptote(s) ________________

Horizontal or Slant Asymptote ____________

Intercepts ____________________________

Graph:



8.

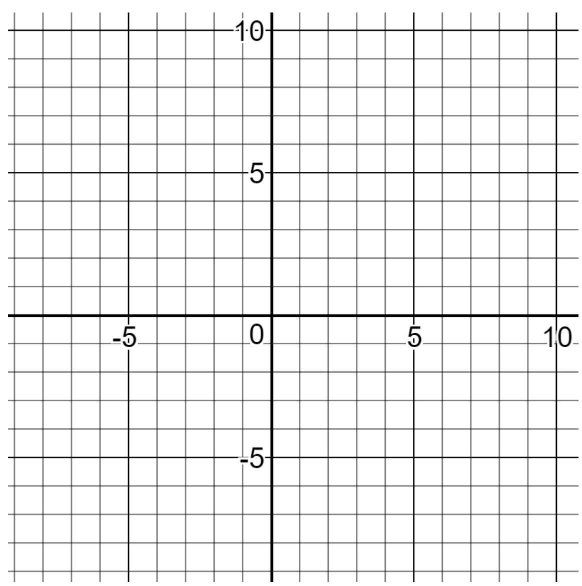
$$y = \frac{(x - 6)}{(x - 3)} + \frac{(x + 3)}{x^2 - 6x + 9}$$

Vertical Asymptote(s) ________________

Horizontal or Slant Asymptote ____________

Intercepts ____________________________

Graph:



GO

Topic: Exploring linear equations

9. What value of k in the equation $kx + 10 = 6y$ would give a line with slope -3?

10. What value of k in the equation $kx - 12 = -15y$ would give a line with slope $\frac{2}{5}$?

11. The standard form of a linear equation is $Ax + By = C$. Rewrite this equation in slope – intercept form. What is the slope? What is the y – intercept?

12. If b is the y – intercept of a linear function whose graph has slope m, then $y = mx + b$ describes the line. Below is an incomplete justification of this statement. Fill in the missing information.

Statements	Reasons
1. $m = \frac{y_2 - y_1}{x_2 - x_1}$	1. slope formula
2. $m = \frac{y-b}{x-0}$	2. By definition, if b is the y – intercept, then $(\quad , b)$ is a point on the line. (x, y) is any other point on the line.
3. $m = \frac{y-b}{x}$	3. ?
4. $m \cdot \quad = y - b$	4. Multiplication Property of Equality (Multiply both sides of the equation by x.)
5. $mx + b = y, or\ y = mx + b$	5. ?

4.7 We All Scream for Ice Cream

A Practice Understanding Task

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The Glacier Bowl is an enormous ice cream treat sold at the neighborhood ice cream parlor. It is so large that any person who can eat it within 30 minutes gets a t-shirt and his picture posted on a wall. Because the Glacier Bowl is so big, it costs $60 and most people split the treat with a group.

Amera and some of her friends are planning to get together to share the bowl of ice cream. They plan to split the cost between them equally.

1. What is an algebraic expression for the amount that each person in the group will pay?

2. At the last minute, one of the friends couldn't go with the group. Write an expression that represents the amount that each person in the group now pays.

3. It turns out that each person in the group had to pay $2 more than they would have if everyone in the original group had shared the ice cream. How many people were in the original group?

4. Explain why your answer(s) makes sense in this situation.

This story and the problem it represents provides an opportunity to model a situation that requires a rational equation. Rational equations can take many forms, but they are solved using principles we have worked with before. Try applying some of the strategies for working with rational expressions that we have used in this module to solve these equations.

5. $\frac{2}{x+4} - \frac{1}{x} = \frac{2}{3x}$

6. $\frac{2x-3}{x+1} = \frac{x+6}{x-2}$

7. $x + \frac{20}{x-4} = \frac{5x}{x-4} - 2$

8. $\frac{x}{x+3} - \frac{4}{x-2} = \frac{-5x^2}{x^2+x-6}$

4.7 We All Scream for Ice Cream – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is two-fold: to model a situation using rational functions, and to solve equations that contain rational expressions. Students will write a rational equation to answer a question about combined rates. They will develop strategies to solve rational equations using their previous work with rational expressions.

Core Standards Focus:

A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Standards for Mathematical Practice:

SMP 1 – Make sense of problems and persevere in solving them

SMP 4 – Model with mathematics

Vocabulary: Extraneous solution

The Teaching Cycle:

Launch (Whole Group):

Begin the task by reading the scenario and ensuring students understand the situation. Ask students to complete questions #1 and #2. Then ask students which expression is greater: $\frac{60}{x}$ or $\frac{60}{x-1}$? Share justifications that relate to the problem situation, such as, we know that each person will pay more if fewer people share the cost. Press students to justify their answer from a strictly numeric perspective also. This will help prepare them to write the equation they will need for the problem.

Explore (Individual, Followed by Small Group) :

Monitor students as they are working to see that they are using the answers to the first two questions to help write an equation. They may need support to think about how to use the additional $2 in the equation. There are several possible approaches. Some students may subtract the two algebraic expressions and say the difference is $2. Some may say that the original amount plus $2 is equal to the amount paid with fewer people in the group. Take note of the possible approaches for writing the equation and for solving the equation so that they can be shared during the discussion.

Once students have an equation, watch for various methods for solving the equation. Most students will think about finding a common denominator and combining the two expressions. Find a student to share that can articulate this strategy for the class. It is likely that this task will produce a number of misconceptions that should be analyzed by the class. If your class is a safe environment for sharing mistakes, then ask a student to share. Otherwise, be prepared to share common misconceptions without identifying the students so that the misconceptions can be addressed.

Discuss (Whole Group):

Begin by asking a student to explain the equation he/she wrote. Most of the possible equations lead to similar steps, and it will be useful for the class to see this. Be sure the student that shares explains how he/she got each rational expression and combined them to form the equation. Next, ask a student to share that has found a common denominator and combined terms before taking the reciprocal of both sides (or using cross-products). Ask students to explain why it is “legal” to take the reciprocal of both sides of the equation or to use cross-products. Be sure to have strong mathematical justification of this step. At this point, ask a student to share who took the reciprocal of each term without combining terms (or any other common misconception that may have occurred). (If you are not comfortable with students sharing errors, then tell the class that this was a strategy you have seen and present it yourself.) Share the entire process and ask the class why the answer doesn’t make sense. Ask why this strategy for solving the equation does not work and is not the same as combining terms and then taking the reciprocal of both sides or using cross products.

If the class has other productive strategies for solving the equation, such as multiplying everything in the equation by the common denominator to get rid of the denominators, you may choose to share them too. Be careful of this strategy because it often leads to the idea that multiplying by a common denominator just makes the denominators go away in any expression. Ask for strong justification of why this works in an equation but is not appropriate when simply adding two terms. Use a numeric example to reinforce the idea.

Next, ask students to share their work on the remaining equations. They are each a little different, so it will be useful to share as many as time allows. Be sure that either #7 or #8 is discussed, so that extraneous solutions can be considered.

Aligned Ready, Set, Go: ***Rational Expressions and Functions 4.7***

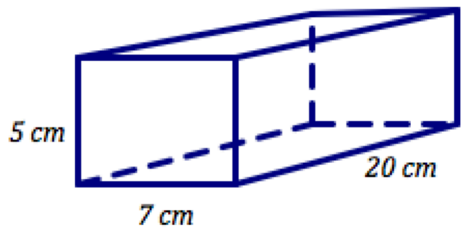
READY, SET, GO! Name Period Date

READY

Topic: Calculating volume, surface area and solving right triangles

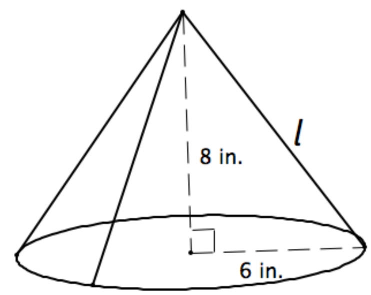
Find the indicated values for the geometric figures below.

1. Volume:
   Surface Area:



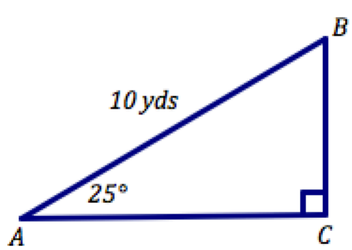
rectangular prism

2. Volume: $V = \frac{1}{3}bh$
   Surface Area: $S_A = \pi r(l + r)$
   where l is the lateral height.

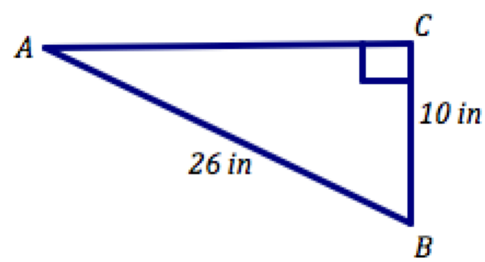


right cone

3. Solve the right triangle.
   $\angle B =$
   $\overline{AC} =$
   $\overline{BC} =$

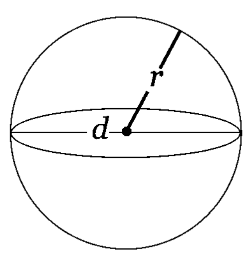


4. Solve the right triangle.
   $\angle B =$
   $\angle A =$
   $\overline{AC} =$



5. Volume: $V = \frac{4}{3}\pi r^3$
   Surface area: $S_A = 4\pi r^2$

$r \approx 3959\ miles$



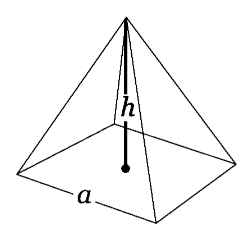
sphere
This is the radius of the earth.

6. Volume: $V = a^2\frac{h}{3}$

   Surface area: $S_A = a^2 + 2a\sqrt{\frac{a^2}{4} + h^2}$

$h = 147\ m$

$a = 230m$



right square pyramid
These are the dimensions of the great pyramid of Giza.

SET

Topic: Solving rational equations

Solve each equation. Identify extraneous solutions.

7. $x + \frac{2}{x} = 3$

8. $\frac{x}{2} - \frac{1}{3x} = \frac{1}{6}$

9. $2x + \frac{3}{x+2} = 1$

10. $\frac{2}{x^2-2x} - \frac{1}{x-2} = 1$

11. $3x - \frac{1}{2x-1} = 4$

12. $\frac{2x}{x^2+3x} - \frac{2}{x+3} = \frac{2}{x}$

Topic: Using work and rate relationships to solve problems

13. Channing takes twice as long as Dakota to complete a school project. It takes them 15 hours to complete the project together. How long would it take each student to complete the project if he works alone?

14. A print shop can print the MVP math book in 24 minutes if both of their print machines are working together to do the job. If a print machine is working alone, the job takes longer. Machine A can print the book 20 minutes faster than machine B. How long does it take each machine to print the book?

15. The problem in #14 generates an extraneous solution, even though neither solution makes a denominator equal zero. What is another reason for having an extraneous solution?

GO

Topic: Simplifying rational expressions

(10 – 11) Reduce to simplest form. (12 – 15) Perform the indicated operations. Reduce each of your answers to its simplest form. (Assume all denominators ≠ 0)

16.

$$\frac{x^2+8x+12}{x^2+3x-18}$$

17.

$$\frac{x^2-3x-40}{x^2-11x+24}$$

18.

$$\frac{x^2+8x+12}{x^2+3x-18}+\frac{x^2-3x-40}{x^2-11x+24}$$

19.

$$\frac{x^2+5x-36}{(x-4)}\cdot\frac{3(x+2)}{x+9}$$

20.

$$\frac{4}{x^2-4}-\frac{1}{x-2}$$

21.

$$\frac{x^2-2x-3}{x+1}\div\frac{x-3}{5}$$