

*Transforming Mathematics Education*

SECONDARY  
MATH THREE

*An Integrated Approach*

MODULE 1

# Functions & Their Inverses

MATHEMATICSVISIONPROJECT.ORG

**The Mathematics Vision Project**

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# MODULE 1 - TABLE OF CONTENTS

## *FUNCTIONS AND THEIR INVERSES*

### **1.1 Brutus Bites Back – A Develop Understanding Task**

Develops the concept of inverse functions in a linear modeling context using tables, graphs, and equations. **(F.BF.1, F.BF.4, F.BF.4a)**

**Ready, Set, Go Homework:** *Functions and Their Inverses 1.1*

### **1.2 Flipping Ferraris – A Solidify Understanding Task**

Extends the concepts of inverse functions in a quadratic modeling context with a focus on domain and range and whether a function is invertible in a given domain. **(F.BF.1, F.BF.4, F.BF.4c, F.BF.4d)**

**Ready, Set, Go Homework:** *Functions and Their Inverses 1.2*

### **1.3 Tracking the Tortoise – A Solidify Understanding Task**

Solidifies the concepts of inverse function in an exponential modeling context and surfaces ideas about logarithms. **(F.BF.1, F.BF.4, F.BF.4c, F.BF.4d)**

**Ready, Set, Go Homework:** *Functions and Their Inverses 1.3*

### **1.4 Pulling a Rabbit Out of a Hat – A Solidify Understanding Task**

Uses function machines to model functions and their inverses. Focus on finding inverse functions and verifying that two functions are inverses. **(F.BF.4, F.BF.4a, F.BF.4b)**

**Ready, Set, Go Homework:** *Functions and Their Inverses 1.4*

### **1.5 Inverse Universe – A Practice Understanding Task**

Uses tables, graphs, equations, and written descriptions of functions to match functions and their inverses together and to verify the inverse relationship between two functions. **(F.BF.4a, F.BF.4b, F.BF.4c, F.BF.4d)**

**Ready, Set, Go Homework:** *Functions and Their Inverses 1.5*



## 1.1 Brutus Bites Back

### *A Develop Understanding Task*

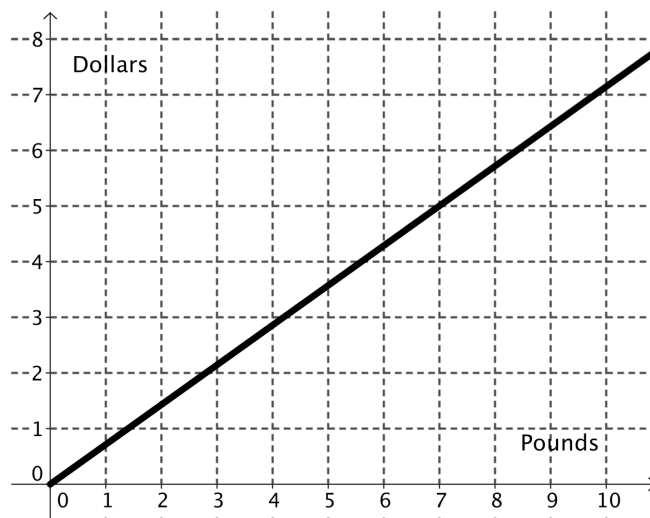
Remember Carlos and Clarita? A couple of years ago, they started earning money by taking care of pets while their owners are away. Due to their amazing mathematical analysis and their loving care of the cats and dogs that they take in, Carlos and Clarita have made their business very successful. To keep the hungry dogs fed, they must regularly buy Brutus Bites, the favorite food of all the dogs.



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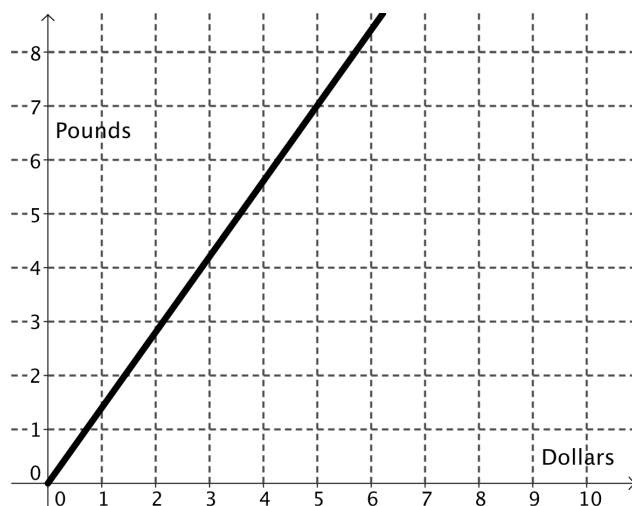
Carlos and Clarita have been searching for a new dog food supplier and have identified two possibilities. The Canine Catering Company, located in their town, sells 7 pounds of food for \$5.

Carlos thought about how much they would pay for a given amount of food and drew this graph:



1. Write the equation of the function that Carlos graphed.

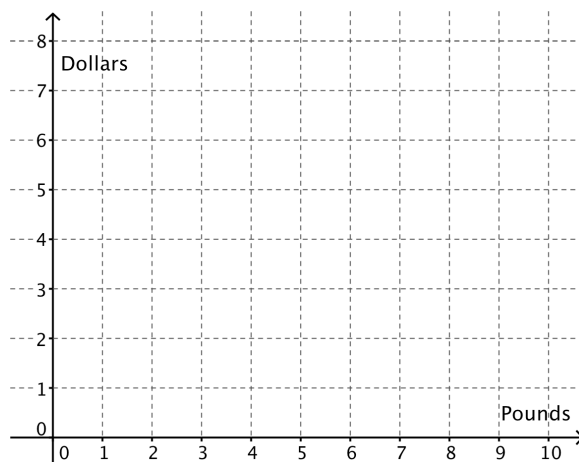
Clarita thought about how much food they could buy for a given amount of money and drew this graph:



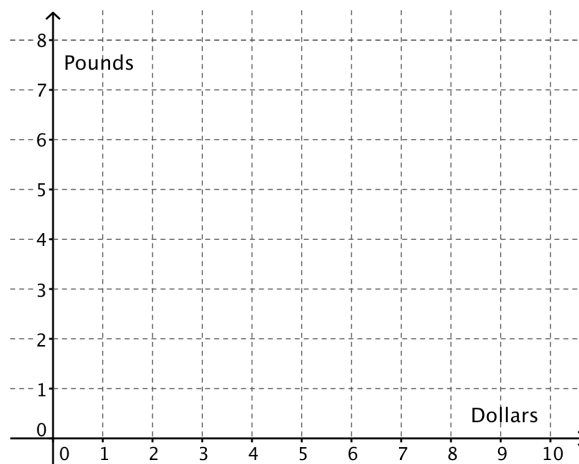
2. Write the equation of the function that Clarita graphed.
3. Write a question that would be most easily answered by Carlos' graph. Write a question that would be most easily answered by Clarita's graph. What is the difference between the two questions?
4. What is the relationship between the two functions? How do you know?
5. Use function notation to write the relationship between the functions.

Looking online, Carlos found a company that will sell 8 pounds of Brutus Bites for \$6 plus a flat \$5 shipping charge for each order. The company advertises that they will sell any amount of food at the same price per pound.

6. Model the relationship between the price and the amount of food using Carlos' approach.



7. Model the relationship between the price and the amount of food using Clarita's approach.



8. What is the relationship between these two functions? How do you know?
9. Use function notation to write the relationship between the functions.
10. Which company should Clarita and Carlos buy their Brutus Bites from? Why?

READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

**READY**

Topic: Inverse operations

Inverse operations “undo” each other. For instance, addition and subtraction are inverse operations. So are multiplication and division. In mathematics, it is often convenient to *undo* several operations in order to solve for a variable.

**Solve for x in the following problems. Then complete the statement by identifying the operation you used to “undo” the equation.**

1.  $24 = 3x$  Undo multiplication by 3 by \_\_\_\_\_
2.  $\frac{x}{5} = -2$  Undo division by 5 by \_\_\_\_\_
3.  $x + 17 = 20$  Undo add 17 by \_\_\_\_\_
4.  $\sqrt{x} = 6$  Undo the square root by \_\_\_\_\_
5.  $\sqrt[3]{(x+1)} = 2$  Undo the cube root by \_\_\_\_\_ then \_\_\_\_\_
6.  $x^4 = 81$  Undo raising x to the 4<sup>th</sup> power by \_\_\_\_\_
7.  $(x - 9)^2 = 49$  Undo squaring by \_\_\_\_\_ then \_\_\_\_\_

**SET**

Topic: Linear functions and their inverses

Carlos and Clarita have a pet sitting business. When they were trying to decide how many each of dogs and cats they could fit into their yard, they made a table based on the following information. Cat pens require 6 ft<sup>2</sup> of space, while dog runs require 24 ft<sup>2</sup>. Carlos and Clarita have up to 360 ft<sup>2</sup> available in the storage shed for pens and runs, while still leaving enough room to move around the cages. They made a table of all of the combinations of cats and dogs they could use to fill the space. They quickly realized that they could fit in 4 cats in the same space as one dog.

cats	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
dogs	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

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8. Use the information in the table to write 5 ordered pairs that have cats as the input value and dogs as the output value.
  
9. Write an explicit equation that shows how many dogs they can accommodate based on how many cats they have. (The number of dogs “ $d$ ” will be a function of the number of cats “ $c$ ” or  $d = f(c)$ .)
  
10. Use the information in the table to write 5 ordered pairs that have dogs as the input value and cats as the output value.
  
11. Write an explicit equation that shows how many cats they can accommodate based on how many dogs they have. (The number of cats “ $c$ ” will be a function of the number of dogs “ $d$ ” or  $c = g(d)$ .)

**Base your answers in #12 and #13 on the table at the top of the page.**

12. Look back at problem 8 and problem 10. Describe how the ordered pairs are different.
  
13.
  - a) Look back at the equation you wrote in problem 9. Describe the domain for  $d = f(c)$ .
  - b) Describe the domain for the equation  $c = g(d)$  that you wrote in problem 11.
  - c) What is the relationship between them?

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**GO**

Topic: Using function notation to evaluate a function.

The functions  $f(x)$ ,  $g(x)$ , and  $h(x)$  are defined below.

$$f(x) = x$$

$$g(x) = 5x - 12$$

$$h(x) = x^2 + 4x - 7$$

**Calculate the indicated function values in the following problems. Simplify your answers.**

14.  $f(10)$

15.  $f(-2)$

16.  $f(a)$

17.  $f(a+b)$

18.  $g(10)$

19.  $g(-2)$

20.  $g(a)$

20.  $g(a+b)$

22.  $h(10)$

23.  $h(-2)$

24.  $h(a)$

25.  $h(a+b)$

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## 1.2 Flipping Ferraris

### *A Solidify Understanding Task*

When people first learn to drive, they are often told that the faster they are driving, the longer it will take to stop. So, when you're driving on the freeway, you should leave more space between your car and the car in front of you than when you are driving slowly through a neighborhood. Have you ever wondered about the relationship between how fast you are driving and how far you travel before you stop, after hitting the brakes?



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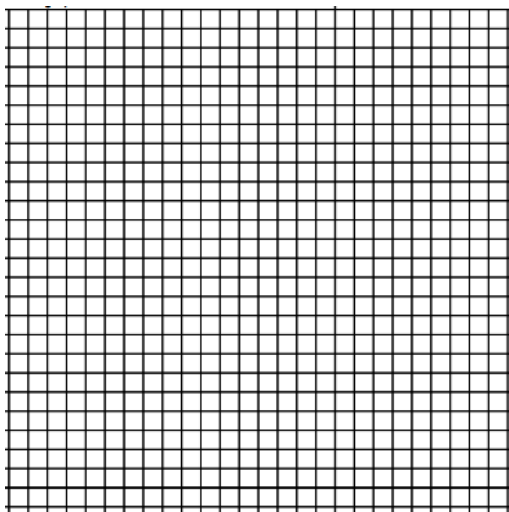
1. Think about it for a minute. What factors do you think might make a difference in how far a car travels after hitting the brakes?

There has actually been quite a bit of experimental work done (mostly by police departments and insurance companies) to be able to mathematically model the relationship between the speed of a car and the braking distance (how far the car goes until it stops after the driver hits the brakes).

2. Imagine your dream car. Maybe it is a Ferrari 550 Maranello, a super-fast Italian car. Experiments have shown that on smooth, dry roads, the relationship between the braking distance ( $d$ ) and speed ( $s$ ) is given by  $d(s) = 0.03s^2$ . Speed is given in miles/hour and the distance is in feet.
  - a) How many feet should you leave between you and the car in front of you if you are driving the Ferrari at 55 mi/hr?
  - b) What distance should you keep between you and the car in front of you if you are driving at 100 mi/hr?
  - c) If an average car is about 16 feet long, about how many car lengths should you have between you and that car in front of you if you are driving 100 mi/hr?

- d) It makes sense to a lot of people that if the car is moving at some speed and then goes twice as fast, the braking distance will be twice as far. Is that true? Explain why or why not.

3. Graph the relationship between braking distance  $d(s)$ , and speed ( $s$ ), below.

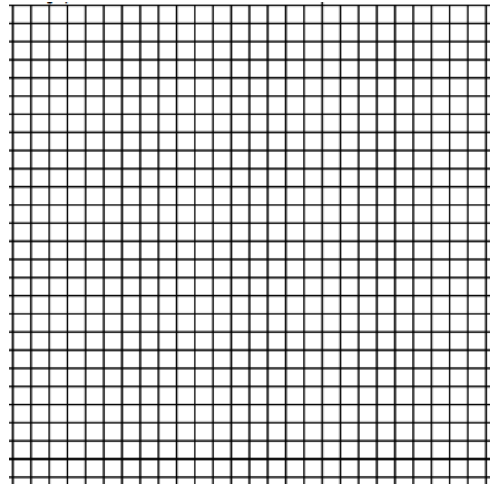


4. According to the Ferrari Company, the maximum speed of the car is about 217 mph. Use this to describe all the mathematical features of the relationship between braking distance and speed for the Ferrari modeled by  $d(s) = 0.03s^2$ .
5. What if the driver of the Ferrari 550 was cruising along and suddenly hit the brakes to stop because she saw a cat in the road? She skidded to a stop, and fortunately, missed the cat. When she got out of the car she measured the skid marks left by the car so that she knew that her braking distance was 31 ft.
- a) How fast was she going when she hit the brakes?
- b) If she didn't see the cat until she was 15 feet away, what is the fastest speed she could be traveling before she hit the brakes if she wants to avoid hitting the cat?

6. Part of the job of police officers is to investigate traffic accidents to determine what caused the accident and which driver was at fault. They measure the braking distance using skid marks and calculate speeds using the mathematical relationships just like we have here, although they often use different formulas to account for various factors such as road conditions. Let's go back to the Ferrari on a smooth, dry road since we know the relationship. Create a table that shows the speed the car was traveling based upon the braking distance.

7. Write an equation of the function  $s(d)$  that gives the speed the car was traveling for a given braking distance.

8. Graph the function  $s(d)$  and describe its features.



9. What do you notice about the graph of  $s(d)$  compared to the graph of  $d(s)$ ? What is the relationship between the functions  $d(s)$  and  $s(d)$ ?

10. Consider the function  $d(s) = 0.03s^2$  over the domain of all real numbers, not just the domain of this problem situation. How does the graph change from the graph of  $d(s)$  in question #3?
11. How does changing the domain of  $d(s)$  change the graph of the inverse of  $d(s)$ ?
12. Is the inverse of  $d(s)$  a function? Justify your answer.

READY, SET, GO!

Name

Period

Date

**READY**

Topic: Solving for a variable

Solve for x.

1.  $17 = 5x + 2$

2.  $2x^2 - 5 = 3x^2 - 12x + 31$

3.  $11 = \sqrt{2x + 1}$

4.  $\sqrt{x^2 + x - 2} = 2$

5.  $-4 = \sqrt[3]{5x + 1}$

6.  $\sqrt[3]{352} = \sqrt[3]{7x^2 + 9}$

7.  $9^x = 243$

8.  $5^x = \frac{1}{125}$

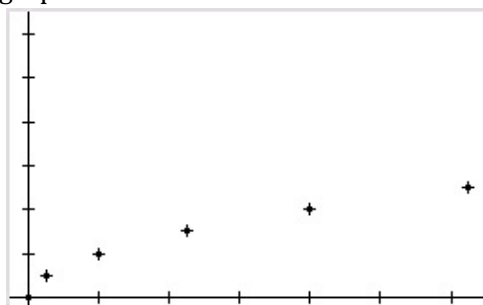
9.  $4^x = \frac{1}{32}$

**SET**

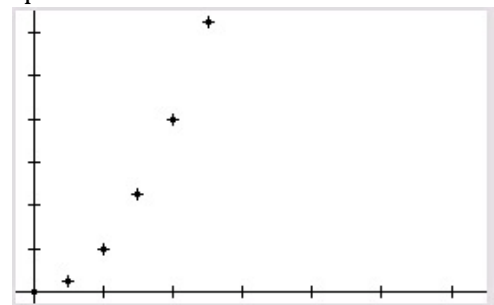
Topic: Exploring inverse functions

10. Students were given a set of data to graph. After they had completed their graphs, each student shared his graph with his shoulder partner. When Ethan and Emma saw each other's graphs, they exclaimed together, "Your graph is wrong!" Neither graph is wrong. Explain what Ethan and Emma have done with their data.

Ethan's graph



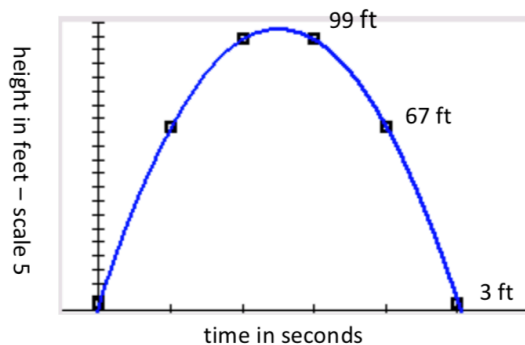
Emma's graph



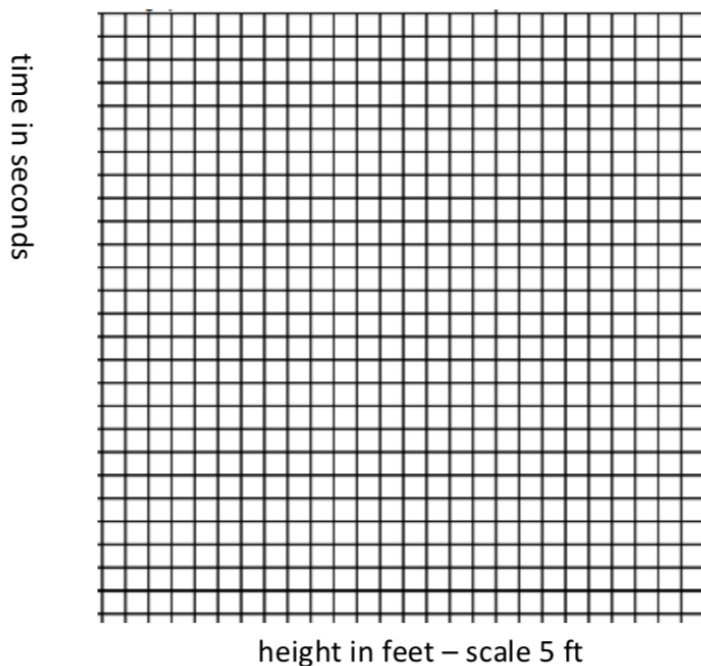
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11. Describe a sequence of transformations that would take Ethan's graph onto Emma's.

12. A baseball is hit upward from a height of 3 feet with an initial velocity of 80 feet per second (about 55 mph). The graph shows the height of the ball at any given second during its flight. Use the graph to answer the questions below.



- Approximate the time that the ball is at its maximum height.
- Approximate the time that the ball hits the ground.
- At what time is the ball 67 feet above the ground?
- Make a new graph that shows the time when the ball is at the given heights.
- Is your new graph a function? Explain.



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**GO**

Topic: Using function notation to evaluate a function

The functions  $f(x)$ ,  $g(x)$ , and  $h(x)$  are defined below.

$$f(x) = 3x$$

$$g(x) = 10x + 4$$

$$h(x) = x^2 - x$$

**Calculate the indicated function values. Simplify your answers.**

13.  $f(7)$

14.  $f(-9)$

15.  $f(s)$

16.  $f(s - t)$

17.  $g(7)$

18.  $g(-9)$

19.  $g(s)$

20.  $g(s - t)$

21.  $h(7)$

22.  $h(-9)$

23.  $h(s)$

24.  $h(s - t)$

Notice that the notation  $f(g(x))$  is indicating that you replace  $x$  in  $f(x)$  with  $g(x)$ .

**Simplify the following.**

25.  $f(g(x))$

26.  $f(h(x))$

27.  $g(f(x))$

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## 1.3 Tracking the Tortoise

### *A Solidify Understanding Task*

You may remember a task from last year about the famous race between the tortoise and the hare. In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race.



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In the task, we modeled the distance from the starting line that both the tortoise and the hare travelled during the race. Today we will consider only the journey of the tortoise in the race.

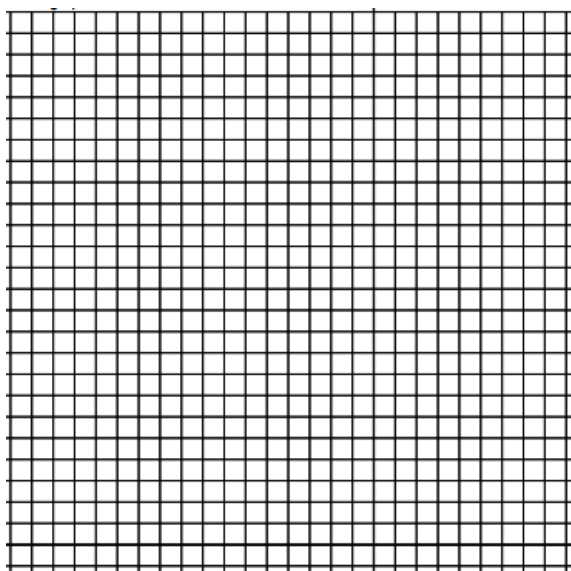
Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

$$d(t) = 2^t \text{ (} d \text{ in meters and } t \text{ in seconds)}$$

The tortoise family decides to watch the race from the sidelines so that they can see their darling tortoise sister, Shellie, prove the value of persistence.

1. How far away from the starting line must the family be, to be located in the right place for Shellie to run by 5 seconds after the beginning of the race? After 10 seconds?
  
  
  
  
  
  
  
  
  
  
2. Describe the graph of  $d(t)$ , Shellie's distance at time  $t$ . What are the important features of  $d(t)$ ?

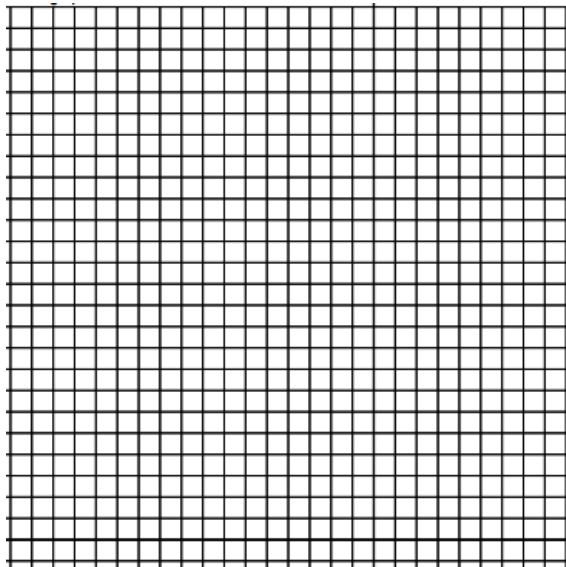
3. If the tortoise family plans to watch the race at 64 meters away from Shellie's starting point, how long will they have to wait to see Shellie run past?
4. How long must they wait to see Shellie run by if they stand 1024 meters away from her starting point?
5. Draw a graph that shows how long the tortoise family will wait to see Shellie run by at a given location from her starting point.



6. How long must the family wait to see Shellie run by if they stand 220 meters away from her starting point?
7. What is the relationship between  $d(t)$  and the graph that you have just drawn? How did you use  $d(t)$  to draw the graph in #5?

8. Consider the function  $f(x) = 2^x$ .  
A) What are the domain and range of  $f(x)$ ? Is  $f(x)$  invertible?

B) Graph  $f(x)$  and  $f^{-1}(x)$  on the grid below.



C) What are the domain and range of  $f^{-1}(x)$ ?

9. If  $f(3) = 8$ , what is  $f^{-1}(8)$ ? How do you know?

10. If  $f\left(\frac{1}{2}\right) = 1.414$ , what is  $f^{-1}(1.414)$ ? How do you know?

11. If  $f(a) = b$  what is  $f^{-1}(b)$ ? Will your answer change if  $f(x)$  is a different function? Explain.

READY, SET, GO!

Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_

**READY**

Topic: Solving exponential equations.

**Solve for the value of  $x$ .**

1.  $5^{x+1} = 5^{2x-3}$

2.  $7^{3x-2} = 7^{-2x+8}$

3.  $4^{3x} = 2^{2x-8}$

4.  $3^{5x-4} = 9^{2x-3}$

5.  $8^{x+1} = 2^{2x+3}$

6.  $3^{x+1} = \frac{1}{81}$

**SET**

Topic: Exploring the inverse of an exponential function

In the fairy tale *Jack and the Beanstalk*, Jack plants a magic bean before he goes to bed. In the morning Jack discovers a giant beanstalk that has grown so large, it disappears into the clouds.

But here is the part of the story you never heard. Written on the bag containing the magic beans was this note.

*Plant a magic bean in rich soil just as the sun is setting. Do not look at the plant site for 10 hours. (This is part of the magic.) After the bean has been in the ground for 1 hour, the growth of the sprout can be modeled by the function  $b(t) = 3^t$ . ( $b$  in feet and  $t$  in hours)*

Jack was a good math student, so although he never looked at his beanstalk during the night, he used the function to calculate how tall it should be as it grew. The table on the right shows the calculations he made every half hour.

Hence, Jack was not surprised when, in the morning, he saw that the top of the beanstalk had disappeared into the clouds.

Time (hours)	Height (feet)
1	3
1.5	5.2
2	9
2.5	15.6
3	27
3.5	46.8
4	81
4.5	140.3
5	243
5.5	420.9
6	729
6.5	1,262.7
7	2,187
7.5	3,788
8	6,561
8.5	11,364
9	19,683
9.5	34,092
10	59,049

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7. Demonstrate how Jack used the model  $b(t) = 3^t$  to calculate how high the beanstalk would be after 6 hours had passed. (You may use the table but write down where you would put the numbers in the function if you didn't have the table.)
8. During that same night, a neighbor was playing with his drone. It was programmed to hover at 243 ft. How many hours had the beanstalk been growing when it was as high as the drone?
9. Did you use the table in the same way to answer #8 as you did to answer #7? Explain.
10. While Jack was making his table, he was wondering how tall the beanstalk would be after the magical 10 hours had passed. He quickly typed the function into his calculator to find out. Write the equation Jack would have typed into his calculator.
11. Commercial jets fly between 30,000 ft. and 36,000 ft. About how many hours of growing could pass before the beanstalk might interfere with commercial aircraft? Explain how you got your answer.
12. Use the table to find  $f(7)$  and  $f^{-1}(11,364)$ .
13. Use the table to find  $f(9)$  and  $f^{-1}(9)$ .
13. Explain why it's possible to answer some of the questions about the height of the beanstalk by just plugging the numbers into the function rule and why sometimes you can only use the table.

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**GO**

Topic: Evaluating functions

The functions  $f(x)$ ,  $g(x)$ , and  $h(x)$  are defined below.

$$f(x) = -2x$$

$$g(x) = 2x + 5$$

$$h(x) = x^2 + 3x - 10$$

**Calculate the indicated function values. Simplify your answers.**

14.  $f(a)$

15.  $f(b^2)$

16.  $f(a + b)$

17.  $f(g(x))$

18.  $g(a)$

19.  $g(b^2)$

20.  $g(a + b)$

21.  $h(f(x))$

22.  $h(a)$

23.  $h(b^2)$

24.  $h(a + b)$

25.  $h(g(x))$

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## 1.4 Pulling a Rabbit Out of the Hat A Solidify Understanding Task

I have a magic trick for you:

- Pick a number, any number.
- Add 6
- Multiply the result by 2
- Subtract 12
- Divide by 2
- The answer is the number you started with!

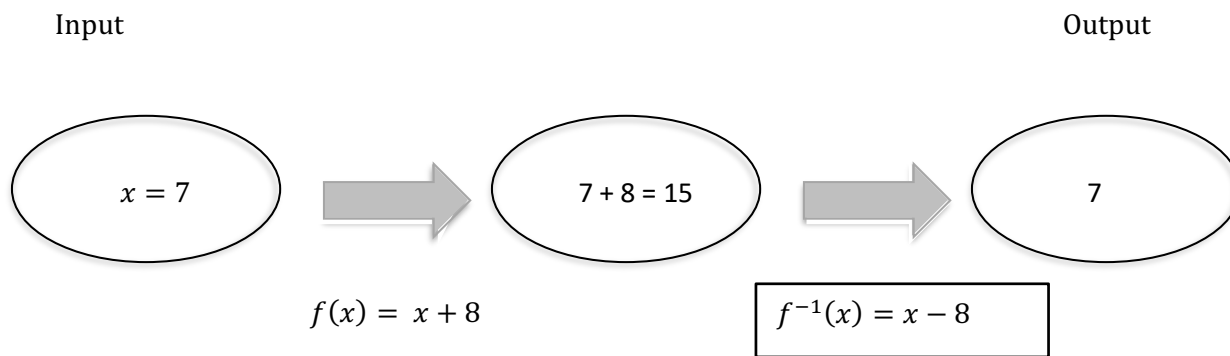


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People are often mystified by such tricks but those of us who have studied inverse operations and inverse functions can easily figure out how they work and even create our own number tricks. Let's get started by figuring out how inverse functions work together.

For each of the following function machines, decide what function can be used to make the output the same as the input number. Describe the operation in words and then write it symbolically.

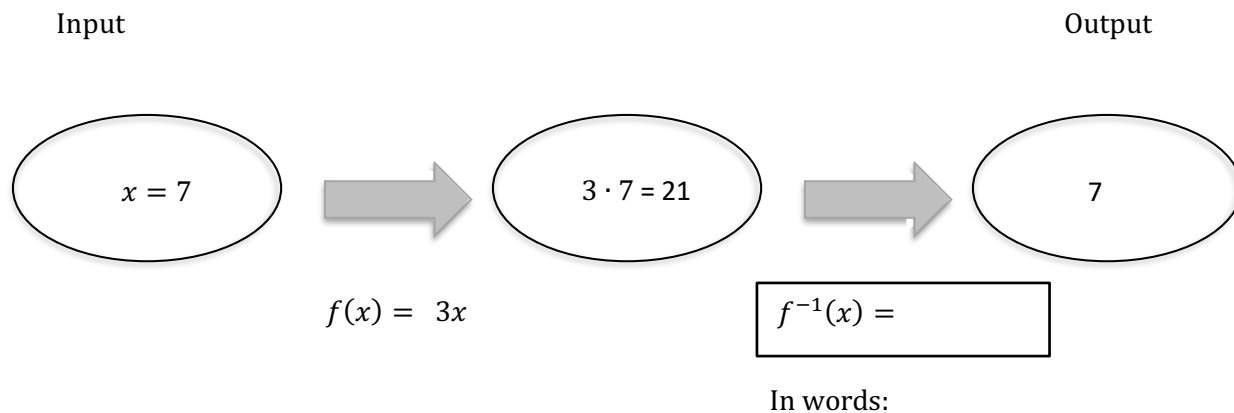
Here's an example:



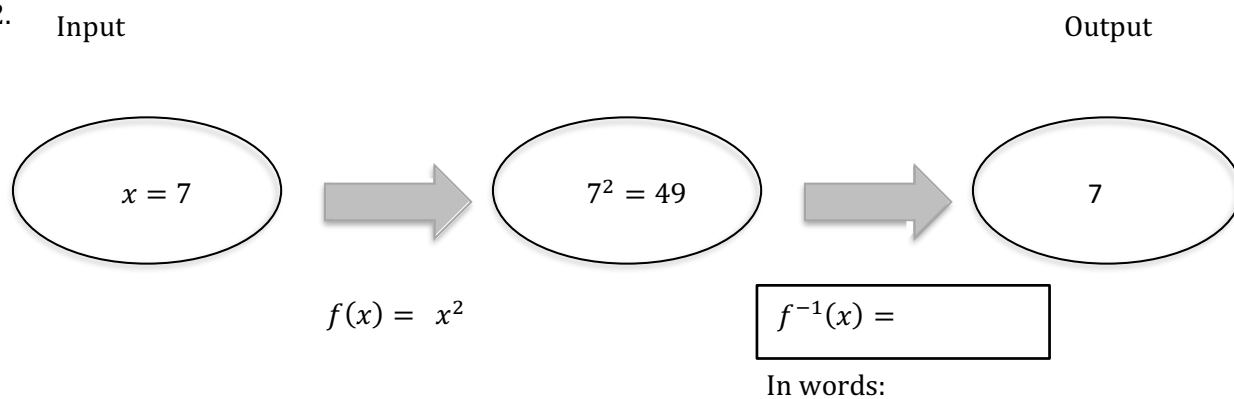
In words: Subtract 8 from the result



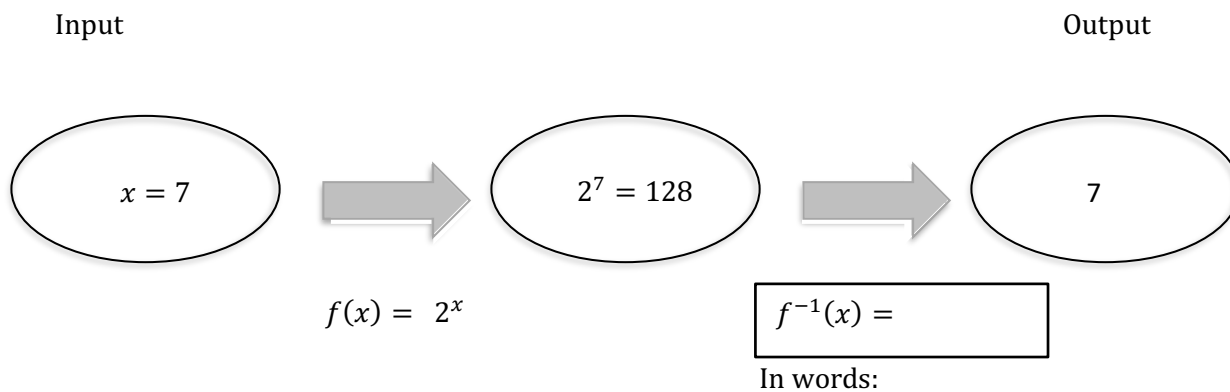
1.



2. Input

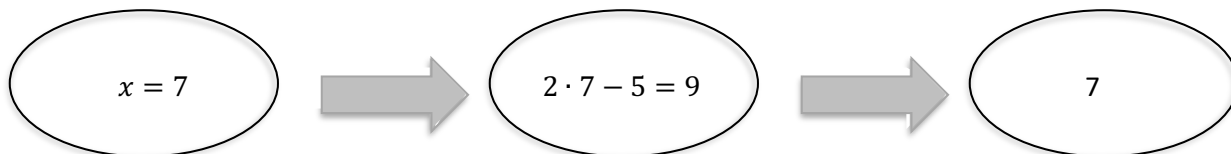


3.



4. Input

Output



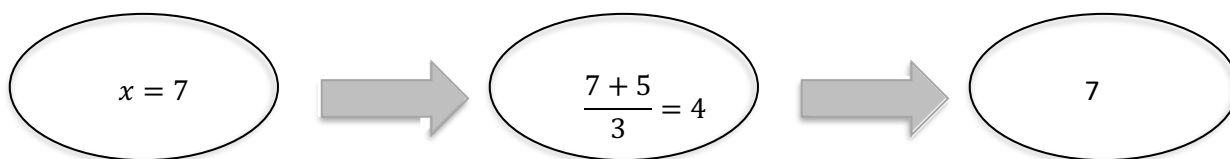
$$f(x) = 2x - 5$$

$$f^{-1}(x) =$$

In words:

5. Input

Output



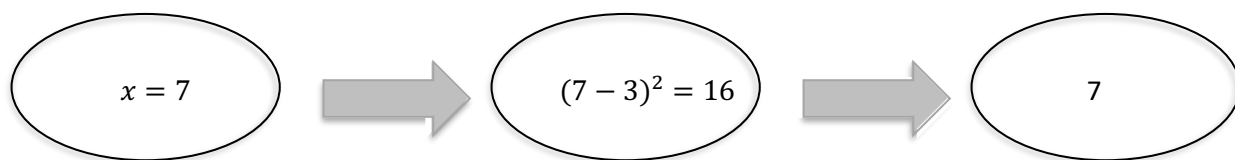
$$f(x) = \frac{x + 5}{3}$$

$$f^{-1}(x) =$$

In words:

6. Input

Output



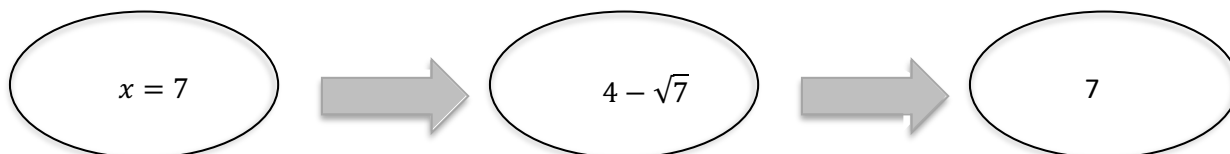
$$f(x) = (x - 3)^2$$

$$f^{-1}(x) =$$

In words:

7. Input

Output



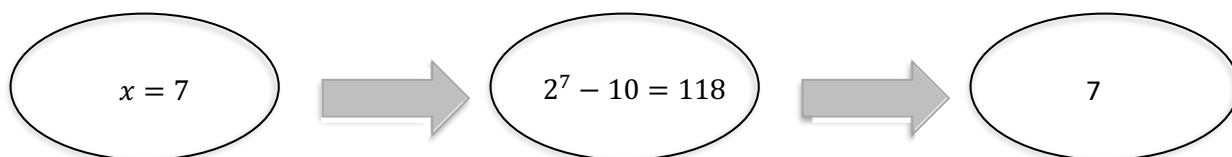
$$f(x) = 4 - \sqrt{x}$$

$$f^{-1}(x) =$$

In words:

8. Input

Output



$$f(x) = 2^x - 10$$

$$f^{-1}(x) =$$

In words:

9. Each of these problems began with  $x = 7$ . What is the difference between the  $x$  used in  $f(x)$  and the  $x$  used in  $f^{-1}(x)$ ?

10. In #6, could any value of  $x$  be used in  $f(x)$  and still give the same output from  $f^{-1}(x)$ ? Explain. What about #7?

11. Based on your work in this task and the other tasks in this module what relationships do you see between functions and their inverses?

READY, SET, GO!

Name

Period

Date

**READY**

Topic: Properties of exponents

**Use the product rule or the quotient rule to simplify. Leave all answers in exponential form with only positive exponents.**

- |                    |                               |                             |                          |
|--------------------|-------------------------------|-----------------------------|--------------------------|
| 1. $3^6 \cdot 3^5$ | 2. $7^2 \cdot 7^6$            | 3. $10^{-4} \cdot 10^7$     | 4. $5^9 \cdot 5^{-6}$    |
| 5. $p^2 p^5$       | 6. $2^6 \cdot 2^{-3} \cdot 2$ | 7. $b^{11} b^{-5}$          | 8. $\frac{7^5}{7^2}$     |
| 9. $\frac{9^8}{9}$ | 10. $\frac{3^5}{3^8}$         | 11. $\frac{7^{-4}}{7^{-8}}$ | 12. $\frac{p^{-3}}{p^5}$ |

**SET**

Topic: Inverse function

13. Given the functions  $f(x) = \sqrt{x} - 1$  and  $g(x) = x^2 + 7$ :
- Calculate  $f(16)$  and  $g(3)$ .
  - Write  $f(16)$  as an ordered pair.
  - Write  $g(3)$  as an ordered pair.
  - What do your ordered pairs for  $f(16)$  and  $g(3)$  imply?
  - Find  $f(25)$ .
  - Based on your answer for  $f(25)$ , predict  $g(4)$ .
  - Find  $g(4)$ . Did your answer match your prediction?
  - Are  $f(x)$  and  $g(x)$  inverse functions? Justify your answer.

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Match the function in the first column with its inverse in the second column.

$f(x)$	$f^{-1}(x)$
16. $f(x) = 3x + 5$	a. $f^{-1}(x) = \log_5 x$
17. $f(x) = x^5$	b. $f^{-1}(x) = \sqrt[3]{x}$
18. $f(x) = \sqrt[5]{x-3}$	c. $f^{-1}(x) = \frac{x-5}{3}$
19. $f(x) = x^3$	d. $f^{-1}(x) = \frac{x}{3} - 5$
20. $f(x) = 5^x$	e. $f^{-1}(x) = \log_3 x$
21. $f(x) = 3(x+5)$	f. $f^{-1}(x) = x^5 + 3$
22. $f(x) = 3^x$	g. $f^{-1}(x) = \sqrt[5]{x}$

**GO**

Topic: Composite functions and inverses

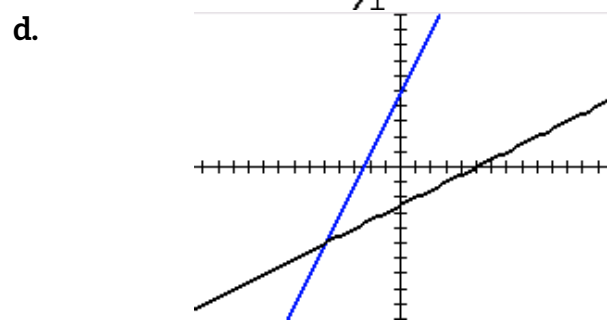
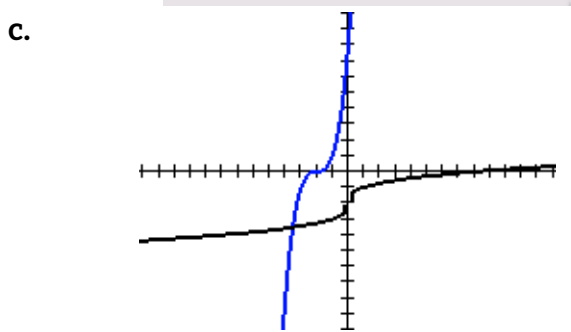
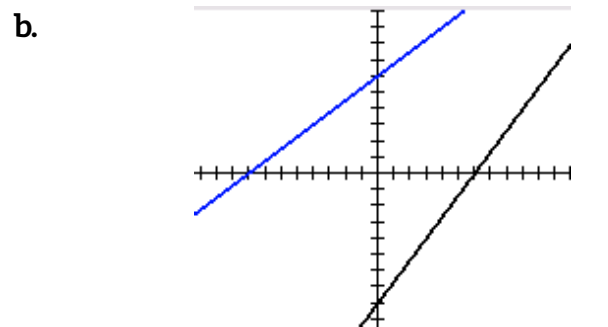
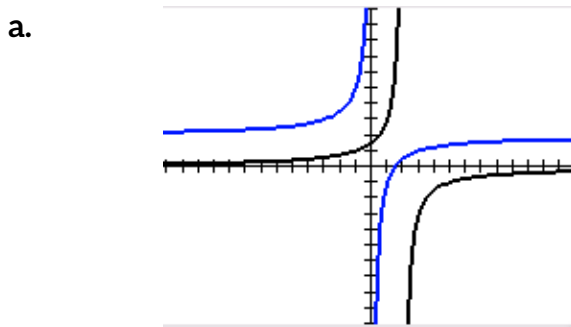
Calculate  $f(g(x))$  and  $g(f(x))$  for each pair of functions.

(Note: the notation  $(f \circ g)(x)$  and  $(g \circ f)(x)$  means the same thing as  $f(g(x))$  and  $g(f(x))$ , respectively.)

23. $f(x) = 2x + 5$	$g(x) = \frac{x-5}{2}$	24. $f(x) = (x+2)^3$	$g(x) = \sqrt[3]{x} - 2$
25. $f(x) = \frac{3}{4}x + 6$	$g(x) = \frac{4(x-6)}{3}$	26. $f(x) = \frac{-3}{x} + 2$	$g(x) = \frac{-3}{x-2}$

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Match the pairs of functions above (23-26) with their graphs. Label  $f(x)$  and  $g(x)$ .



27. Graph the line  $y = x$  on each of the graphs above. What do you notice?
28. Do you think your observations about the graphs in #27 has anything to do with the answers you got when you found  $f(g(x))$  and  $g(f(x))$ ? Explain.
29. Look at graph *b*. Shade the 2 triangles made by the  $y$ -axis,  $x$ -axis, and each line. What is interesting about these two triangles?
30. Shade the 2 triangles in graph *d*. Are they interesting in the same way? Explain.

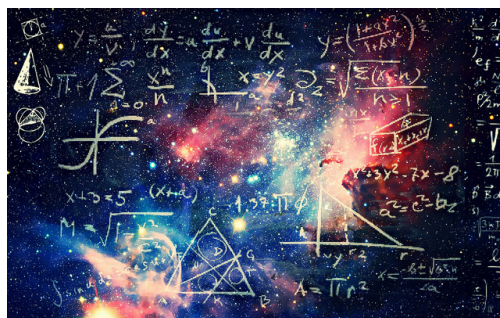
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# 1.5 Inverse Universe

## A Practice Understanding Task

You and your partner have each been given a different set of cards. The instructions are:

1. Select a card and show it to your partner.
2. Work together to find a card in your partner's set of cards that represents the inverse of the function represented on your card.
3. Record the cards you selected and the reason that you know that they are inverses in the space below.
4. Repeat the process until all of the cards are paired up.



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\*For this task only, assume that all tables represent points on a continuous function.

Pair 1: \_\_\_\_\_ Justification of inverse relationship: \_\_\_\_\_

---

Pair 2: \_\_\_\_\_ Justification of inverse relationship: \_\_\_\_\_

---

Pair 3: \_\_\_\_\_ Justification of inverse relationship: \_\_\_\_\_

---

Pair 4: \_\_\_\_\_ Justification of inverse relationship: \_\_\_\_\_

---

Pair 5: \_\_\_\_\_ Justification of inverse relationship: \_\_\_\_\_

---

Pair 6: \_\_\_\_\_ Justification of inverse relationship: \_\_\_\_\_

---

Pair 6: \_\_\_\_\_ Justification of inverse relationship: \_\_\_\_\_

---

Pair 7: \_\_\_\_\_ Justification of inverse relationship: \_\_\_\_\_

---

Pair 8: \_\_\_\_\_ Justification of inverse relationship: \_\_\_\_\_

---

Pair 9: \_\_\_\_\_ Justification of inverse relationship: \_\_\_\_\_

---

Pair 10: \_\_\_\_\_ Justification of inverse relationship: \_\_\_\_\_

---



A1

$$f(x) = \begin{cases} -2x - 2, & -5 < x < 0 \\ -2, & x \geq 0 \end{cases}$$

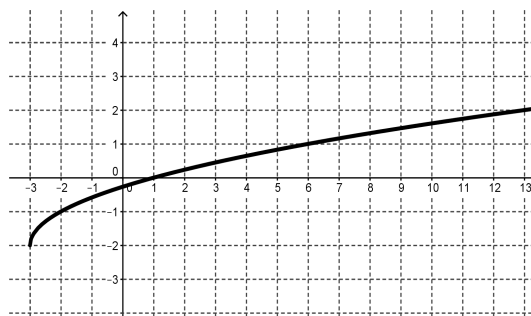
A2

The function increases at a constant rate of  $\frac{a}{b}$  and the y-intercept is  $(0, c)$ .

A3

Each input value,  $x$ , is squared and then 3 is added to the result. The domain of the function is  $[0, \infty)$

A4



A5

$x$	$y$
-2	-3
2	3
0	0
6	5
4	4
$-\frac{4}{3}$	-2

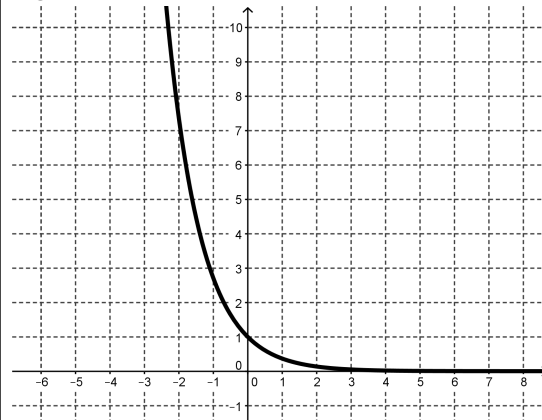
A6

$$y = 3^x$$

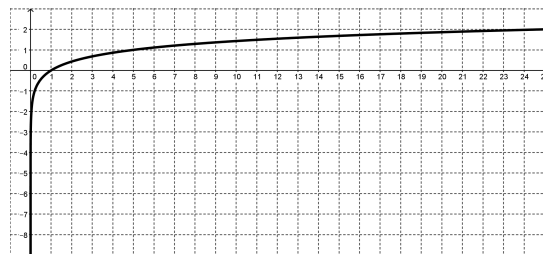
A7

$x$	$y$
-5	-125
-3	-27
-1	-1
1	1
3	27
5	125

A8

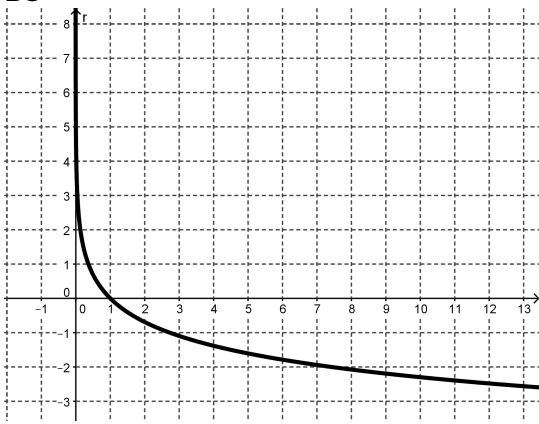


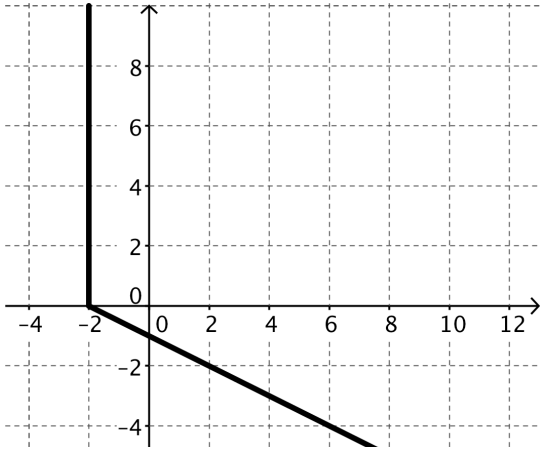
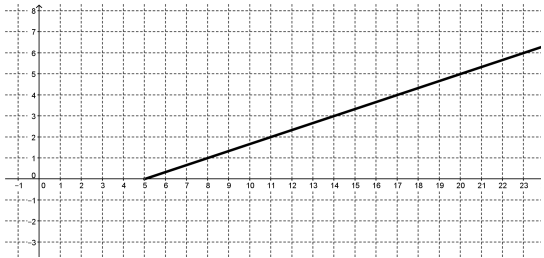
A9



A10

Yasmin started a savings account with \$5. At the end of each week, she added 3. This function models the amount of money in the account for a given week.

<p>B1</p> $y = \log_3 x$	<p>B2</p> $f(x) = \begin{cases} \frac{2}{3}x, & -3 < x < 3 \\ 2x - 4, & x \geq 3 \end{cases}$																
<p>B3</p> <p>The x-intercept is <math>(c, 0)</math> and the slope of the line is <math>\frac{b}{a}</math>.</p>	<p>B4</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="padding: 5px;"><math>x</math></th> <th style="padding: 5px;"><math>y</math></th> </tr> </thead> <tbody> <tr><td style="padding: 5px;">-216</td><td style="padding: 5px;">-6</td></tr> <tr><td style="padding: 5px;">-64</td><td style="padding: 5px;">-4</td></tr> <tr><td style="padding: 5px;">-8</td><td style="padding: 5px;">-2</td></tr> <tr><td style="padding: 5px;">0</td><td style="padding: 5px;">0</td></tr> <tr><td style="padding: 5px;">8</td><td style="padding: 5px;">2</td></tr> <tr><td style="padding: 5px;">64</td><td style="padding: 5px;">4</td></tr> <tr><td style="padding: 5px;">216</td><td style="padding: 5px;">6</td></tr> </tbody> </table>	$x$	$y$	-216	-6	-64	-4	-8	-2	0	0	8	2	64	4	216	6
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-216	-6																
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64	4																
216	6																
<p>B5</p> 	<p>B6</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="padding: 5px;"><math>x</math></th> <th style="padding: 5px;"><math>y</math></th> </tr> </thead> <tbody> <tr><td style="padding: 5px;">3</td><td style="padding: 5px;">0</td></tr> <tr><td style="padding: 5px;">4</td><td style="padding: 5px;">1</td></tr> <tr><td style="padding: 5px;">7</td><td style="padding: 5px;">2</td></tr> <tr><td style="padding: 5px;">12</td><td style="padding: 5px;">3</td></tr> <tr><td style="padding: 5px;">19</td><td style="padding: 5px;">4</td></tr> <tr><td style="padding: 5px;">28</td><td style="padding: 5px;">5</td></tr> <tr><td style="padding: 5px;">39</td><td style="padding: 5px;">6</td></tr> </tbody> </table>	$x$	$y$	3	0	4	1	7	2	12	3	19	4	28	5	39	6
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3	0																
4	1																
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<p><b>B7</b></p> 	<p><b>B8</b></p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="padding: 5px;"><math>x</math></th> <th style="padding: 5px;"><math>y</math></th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">-2</td> <td style="padding: 5px;">-3</td> </tr> <tr> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">-2</td> </tr> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">6</td> </tr> <tr> <td style="padding: 5px;">2</td> <td style="padding: 5px;">13</td> </tr> </tbody> </table>	$x$	$y$	-2	-3	-1	-2	0	1	1	6	2	13
$x$	$y$												
-2	-3												
-1	-2												
0	1												
1	6												
2	13												
<p><b>B9</b></p> 	<p><b>B10</b></p> <p>The function is continuous and grows by an equal factor of 5 over equal intervals. The y-intercept is (0,1).</p>												

READY, SET, GO!

Name

Period

Date

**READY**

Topic: Properties of exponents

Use properties of exponents to simplify the following. Write your answers in exponential form with positive exponents.

1.  $\sqrt[7]{x^2} \cdot \sqrt[7]{x^3}$

2.  $\sqrt[3]{x} \cdot \sqrt[4]{x} \cdot \sqrt[6]{x}$

3.  $\sqrt[6]{a} \cdot \sqrt[3]{a^2} \cdot \sqrt[5]{b^3}$

4.  $\sqrt[5]{32} \cdot \sqrt{9} \cdot \sqrt[3]{27}$

5.  $\sqrt[4]{8} \cdot \sqrt[3]{16} \cdot \sqrt[6]{2}$

6.  $(5^2)^3$

7.  $(7^2)^{-1}$

8.  $(3^{-4})^{-5}$

9.  $\left(\frac{5^{-4}}{5^2}\right)^3$

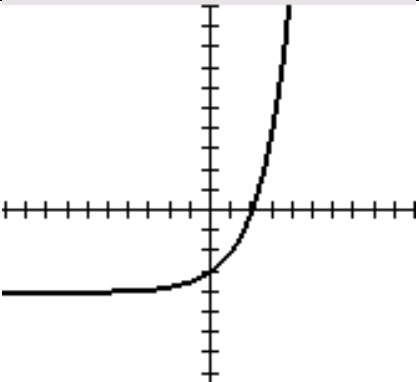
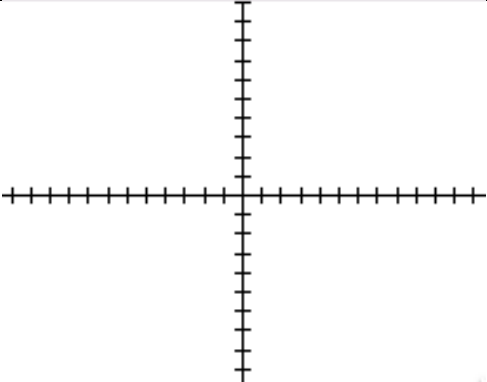


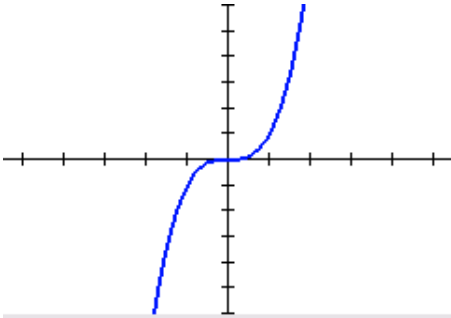
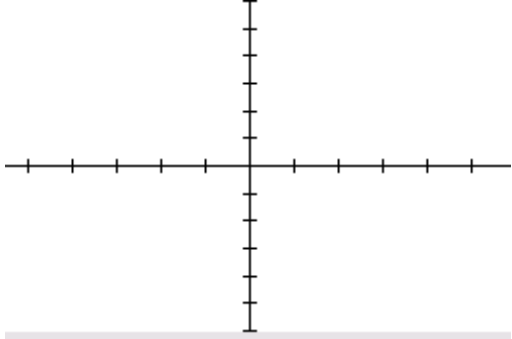
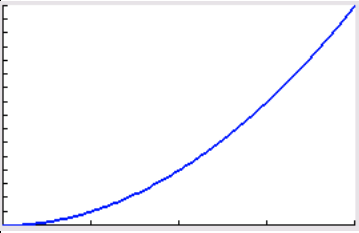
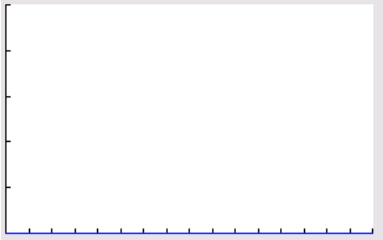
**SET**

Topic: Representations of inverse functions

Write the inverse of the given function in the same format as the given function.

Function $f(x)$		Inverse $f^{-1}(x)$	
10.			
x	$f(x)$		
-8	0		
-4	3		
0	6		
4	9		
8	12		

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11. 																									
12. $f(x) = -2x + 4$																									
13. $f(x) = \log_3 x$																									
14. 																									
15. <table border="1" style="display: inline-table; margin-right: 20px; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">x</th> <th style="padding: 5px;"><math>f(x)</math></th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="padding: 5px;">2</td> <td style="padding: 5px;">4</td> </tr> <tr> <td style="padding: 5px;">3</td> <td style="padding: 5px;">9</td> </tr> <tr> <td style="padding: 5px;">4</td> <td style="padding: 5px;">16</td> </tr> </tbody> </table> 	x	$f(x)$	0	0	1	1	2	4	3	9	4	16	<table border="1" style="display: inline-table; margin-right: 20px; border-collapse: collapse;"> <tbody> <tr><td style="width: 50px; height: 20px;"></td><td style="width: 50px; height: 20px;"></td></tr> <tr><td style="width: 50px; height: 20px;"></td><td style="width: 50px; height: 20px;"></td></tr> <tr><td style="width: 50px; height: 20px;"></td><td style="width: 50px; height: 20px;"></td></tr> <tr><td style="width: 50px; height: 20px;"></td><td style="width: 50px; height: 20px;"></td></tr> <tr><td style="width: 50px; height: 20px;"></td><td style="width: 50px; height: 20px;"></td></tr> <tr><td style="width: 50px; height: 20px;"></td><td style="width: 50px; height: 20px;"></td></tr> </tbody> </table> 												
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**GO**

Topic: Composite functions

**Calculate  $f(g(x))$  and  $g(f(x))$  for each pair of functions.**

(Note: the notation  $(f \circ g)(x)$  and  $(g \circ f)(x)$  mean the same thing, respectively.)

16.  $f(x) = 3x + 7$ ;  $g(x) = -4x - 11$

17.  $f(x) = -4x + 60$ ;  $g(x) = -\frac{1}{4}x + 15$

18.  $f(x) = 10x - 5$ ;  $g(x) = \frac{2}{5}x + 3$

19.  $f(x) = -\frac{2}{3}x + 4$ ;  $g(x) = -\frac{3}{2}x + 6$

20. Look back at your calculations for  $f(g(x))$  and  $g(f(x))$ . Two of the pairs of equations are inverses of each other. Which ones do you think they are?

Why?

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