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Transforming Mathematics Education

# SECONDARY MATH THREE

*An Integrated Approach*

MODULE 2

# Logarithmic Functions

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**The Mathematics Vision Project**

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# 2.1 Log Logic

## *A Develop Understanding Task*

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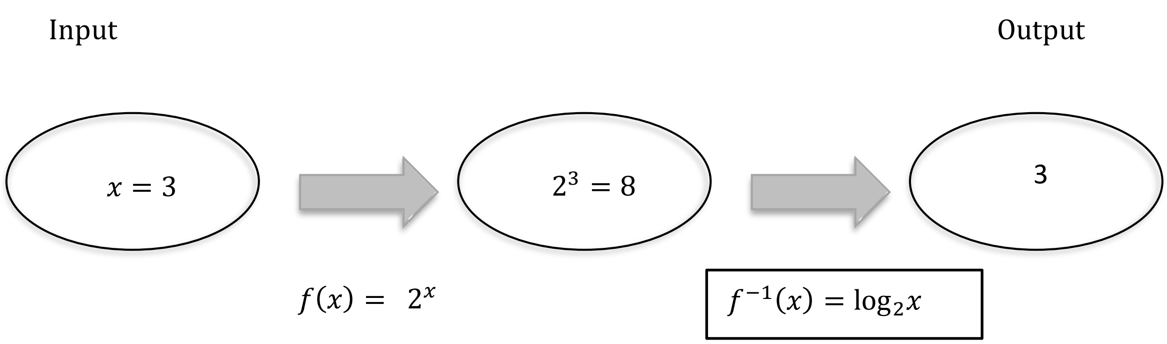
We began thinking about logarithms as inverse functions for exponentials in *Tracking the Tortoise*. Logarithmic functions are interesting and useful on their own. In the next few tasks, we will be working on understanding logarithmic expressions, logarithmic functions, and logarithmic operations on equations.

We showed the inverse relationship between exponential and logarithmic functions using a diagram like the one below:




We could summarize this relationship by saying:

$2^3 = 8$ so, $\log_2 8 = 3$

Logarithms can be defined for any base used for an exponential function. Base 10 is popular. Using base 10, you can write statements like these:

$10^1 = 10$ so, $\log_{10} 10 = 1$

$10^2 = 100$ so, $\log_{10} 100 = 2$

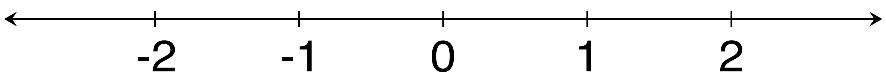
$10^3 = 1000$ so, $\log_{10} 1000 = 3$

The notation may see different, but you can see the inverse pattern where the inputs and outputs switch.

The next few problems will give you an opportunity to practice thinking about this pattern and possibly make a few conjectures about other patterns related to logarithms.

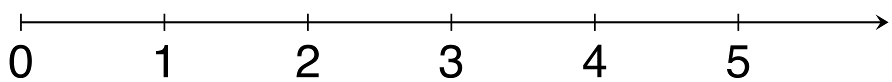
Place the following expressions on the number line. Use the space below the number line to explain how you knew where to place each expression.

1. A. $\log_3 3$ B. $\log_3 9$ C. $\log_3 \frac{1}{3}$ D. $\log_3 1$ E. $\log_3 \frac{1}{9}$



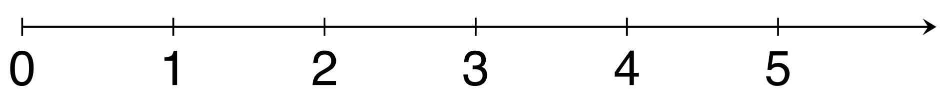

Explain: ______________________________

2. A. $\log_3 81$ B. $\log_{10} 100$ C. $\log_8 8$ D. $\log_5 25$ E. $\log_2 32$



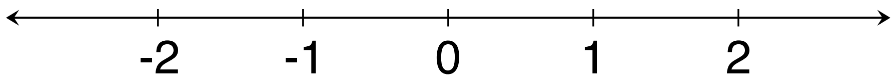

Explain: ______________________________

3. A. $\log_7 7$ B. $\log_9 9$ C. $\log_{11} 1$ D. $\log_{10} 1$



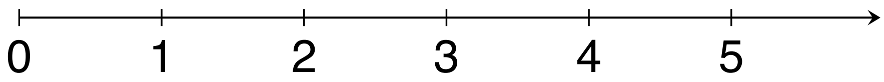

Explain: ______________________________

4. A. $\log_2\left(\frac{1}{4}\right)$ B. $\log_{10}\left(\frac{1}{1000}\right)$ C. $\log_5\left(\frac{1}{125}\right)$ D. $\log_6\left(\frac{1}{6}\right)$



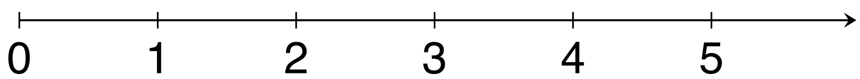

Explain: ______________________________________________

5. A. $\log_4 16$ B. $\log_2 16$ C. $\log_8 16$ D. $\log_{16} 16$



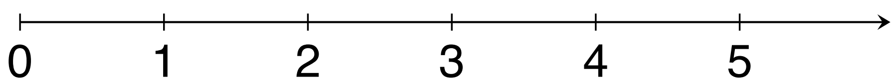

Explain: ______________________________________________

6. A. $\log_2 5$ B. $\log_5 10$ C. $\log_6 1$ D. $\log_5 5$ E. $\log_{10} 5$



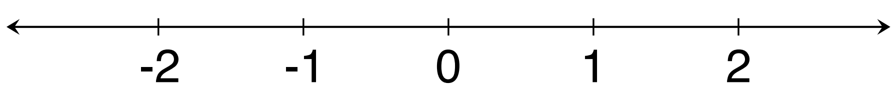

Explain: ______________________________________________

7. A. $\log_{10} 50$ B. $\log_{10} 150$ C. $\log_{10} 1000$ D. $\log_{10} 500$




Explain: ______________________________________________

8. A. $\log_3 3^2$ B. $\log_5 5^{-2}$ C. $\log_6 6^0$ D. $\log_4 4^{-1}$ E. $\log_2 2^3$




Explain: ______________________________________________

Based on your work with logarithmic expressions, determine whether each of these statements is always true, sometimes true, or never true. If the statement is sometimes true, describe the conditions that make it true. Explain your answers.

9. The value of $\log_b x$ is positive.

Explain: ______________________________________________

10. $\log_b x$ is not a valid expression if $x$ is a negative number.

Explain: ______________________________________________

11. $\log_b 1 = 0$ for any base, $b > 0$.

Explain: ______________________________________________

12. $\log_b b = 1$ for any $b > 0$.

Explain: ______________________________________________

13. $\log_2 x < \log_3 x$ for any value of $x$.

Explain: ______________________________________________

14. $\log_b b^n = n$ for any $b > 0$.

Explain: ______________________________________________

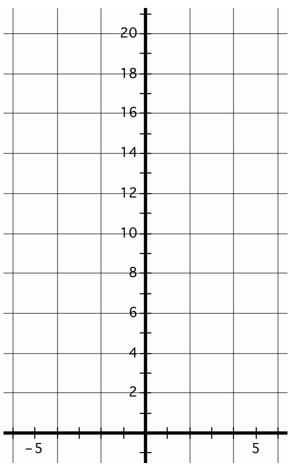
**READY, SET, GO!** Name Period Date

## READY

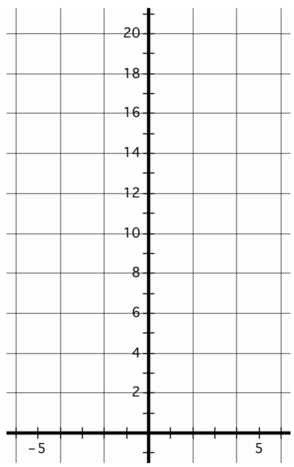
Topic: Graphing exponential equations

**Graph each function over the domain** $\{-4 \leq x \leq 4\}$.

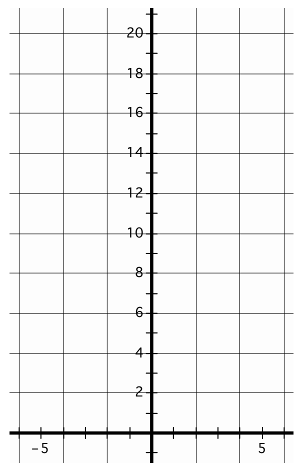
1. $y = 2^x$



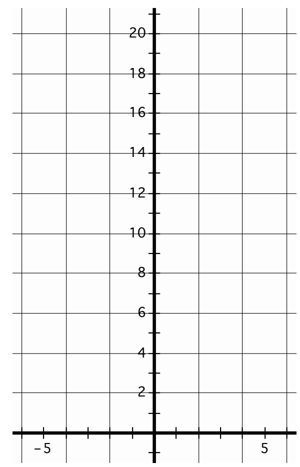

2. $y = 2 \cdot 2^x$




3. $y = \left(\frac{1}{2}\right)^x$




4. $y = 2\left(\frac{1}{2}\right)^x$




5. Compare graph #1 to graph #2. Multiplying by 2 should generate a dilation of the graph, but the graph looks like it has been translated vertically. How do you explain that?

6. Compare graph #3 to graph #4. Is your explanation in #5 still valid for these two graphs? Explain.

## SET

Topic: Writing the logarithmic form of an exponential equation.

**Definition of Logarithm:** *For all positive numbers a, where a ≠ 1, and all positive numbers x,*

$$\boldsymbol{y = log_a x} \textit{ means the same as } \boldsymbol{x = a^y}.$$

*(Note the* ***ba****se of the exponent and the* ***base*** *of the logarithm are both* ***a****.)*

7. Why is it important that the definition of logarithm states that the base of the logarithm does not equal 1?

8. Why is it important that the definition states that the base of the logarithm is positive?

9. Why is it necessary that the definition states that $\boldsymbol{x}$ in the expression $\boldsymbol{log_a x}$ is positive?

**Write the following exponential equations in logarithmic form.**

| **Exponential form** | **Logarithmic form** | **Exponential form** | **Logarithmic form** |
|---|---|---|---|
| 10. $5^4 = 625$ | | 11. $3^2 = 9$ | |
| 12. $\left(\frac{1}{2}\right)^{-3} = 8$ | | 13. $4^{-2} = \frac{1}{16}$ | |
| 14. $10^4 = 10000$ | | 15. $a^y = x$ | |

16. Compare the exponential form of an equation to the logarithmic form of an equation. What part of the exponential equation is the **answer** to the logarithmic equation?

Topic: Considering values of logarithmic functions

**Answer the following questions. If yes, give an example or the answer. If no, explain why not.**

17. Is it possible for a logarithm to equal a negative number?

18. Is it possible for a logarithm to equal zero?

19. Does $log_x 0$ have an answer?

20. Does $log_x 1$ have an answer?

21. Does $log_x x^5$ have an answer?

## GO

Topic: Reviewing properties of Exponents

**Write each expression as an integer or a simple fraction.**

22. $27^0$

23. $11(-6)^0$

24. $-3^{-2}$

25. $4^{-3}$

26. $\frac{9}{2^{-1}}$

27. $\frac{4^3}{8^0}$

28. $3\left(\frac{29^3}{11^5}\right)^0$

29. $\frac{3}{6^{-1}}$

30. $\frac{32^{-1}}{4^{-1}}$

# 2.2 Falling Off a Log

## *A Solidify Understanding Task*

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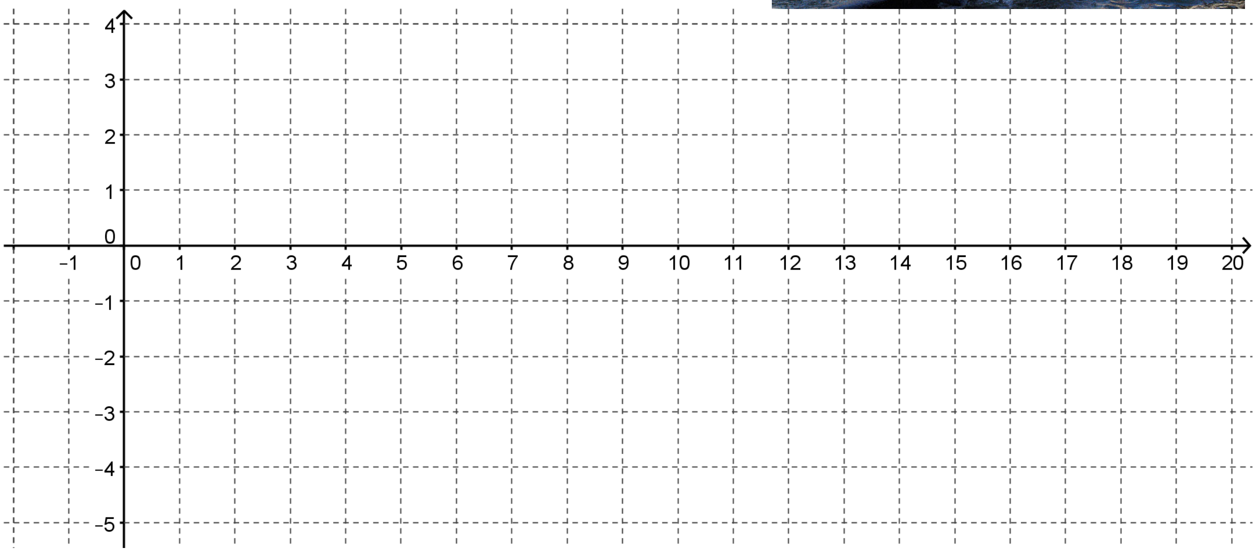
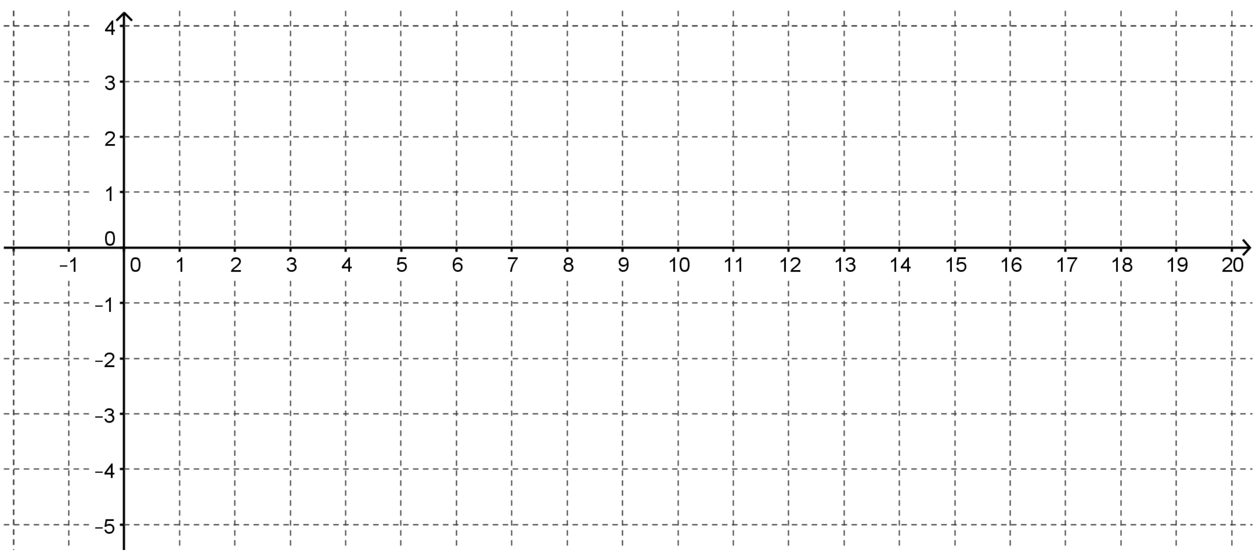
1. Construct a table of values and a graph for each of the following functions. Be sure to select at least two values in the interval $0 < x < 1$.

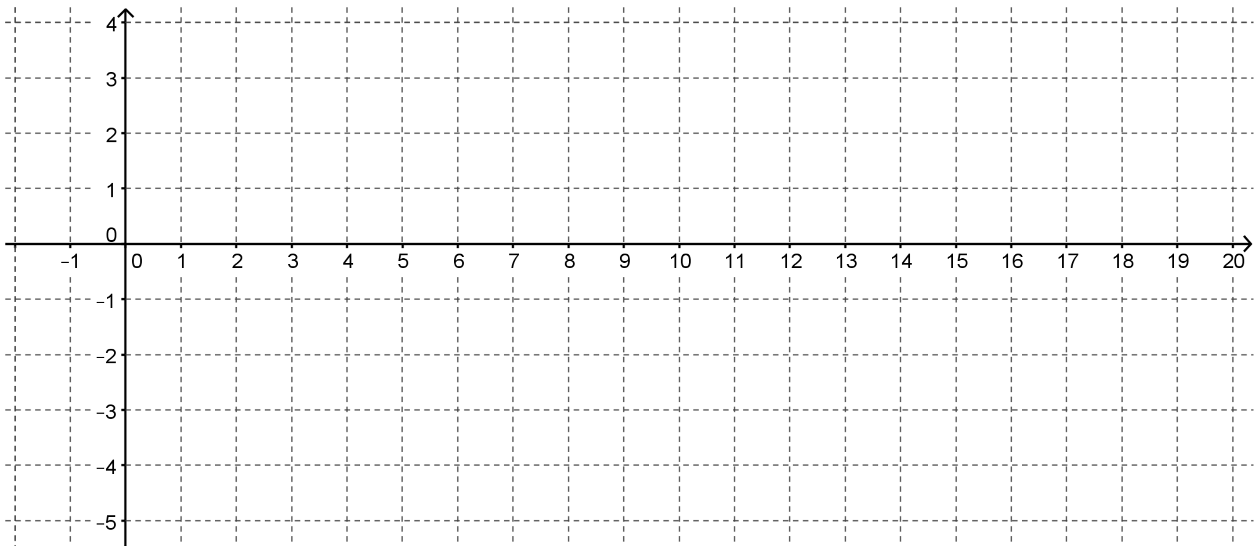
a) $f(x) = \log_2 x$



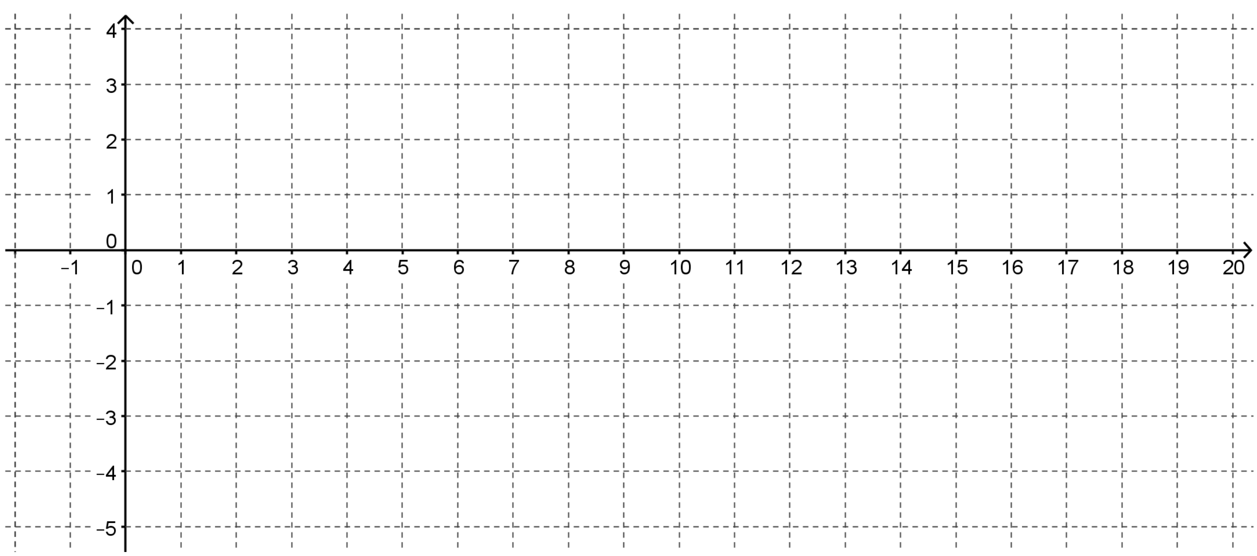
b) $g(x) = \log_3 x$



c) $h(x) = \log_4 x$




d) $k(x) = \log_{10} x$




2. How did you decide what values to use for *x* in your table?

3. How did you use the *x* values to find the y values in the table?

4. What similarities do you see in the graphs?

5. What differences do you observe in the graphs?

6. What is the effect of changing the base on the graph of a logarithmic function?

Let's focus now on $k(x) = \log_{10}x$ so that we can use technology to observe the effects of changing parameters on the function. **Because base 10 is a very commonly used base for exponential and logarithmic functions, it is often abbreviated and written without the base, like this: $\boldsymbol{k(x) = \log x}$.**

7. Use technology to graph $y = \log x$. How does the graph compare to the graph that you constructed?

8. How do you predict that the graph of $y = a + \log x$ will be different from the graph of $y = \log x$?

9. Test your prediction by graphing $y = a + \log x$ for various values of $a$. What is the effect of $a$ on the graph? Make a general argument for why this would be true for all logarithmic functions.

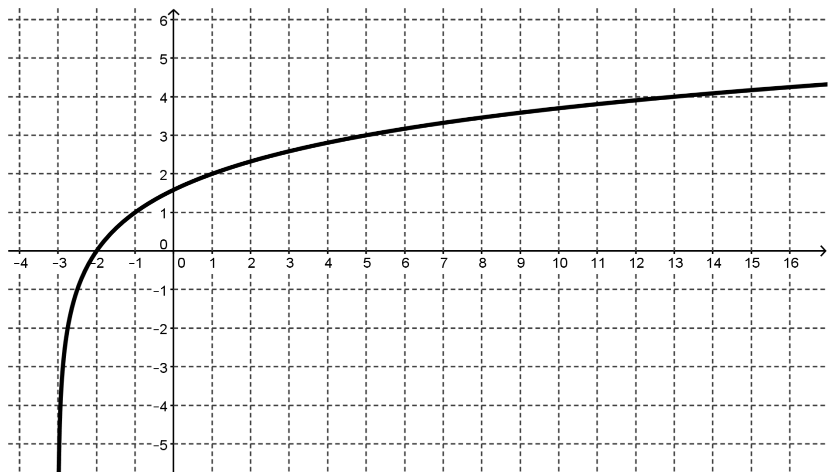
10. How do you predict that the graph of $y = \log(x + b)$ will be different from the graph of $y = \log x$?

11. Test your prediction by graphing $y = \log(x + b)$ for various values of $b$.

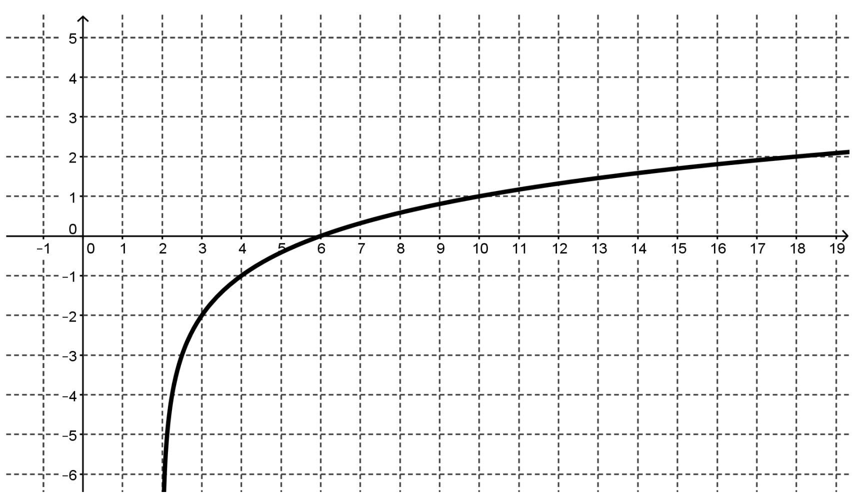
- What is the effect of adding $b$?
- What will be the effect of subtracting $b$ (or adding a negative number)?
- Make a general argument for why this is true for all logarithmic functions.

12. Write an equation for each of the following functions that are transformations of $f(x) = \log_2 x$.

a.

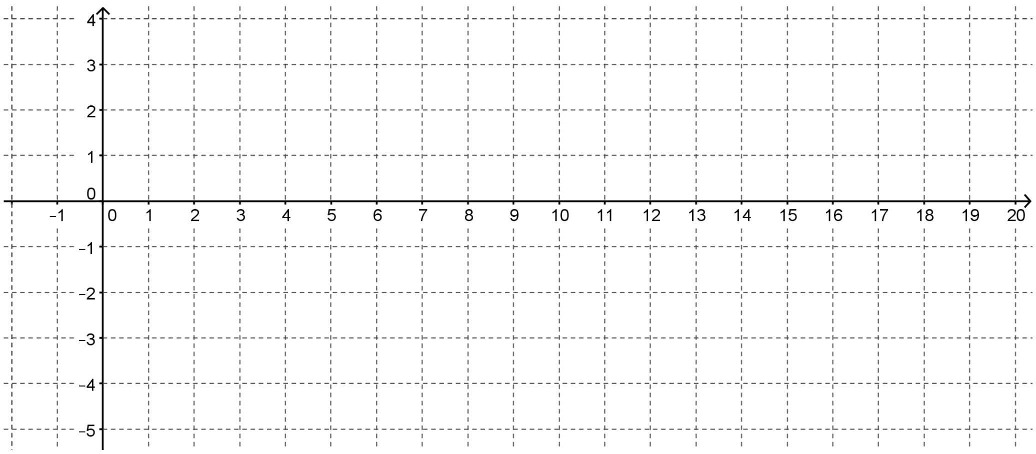


b.

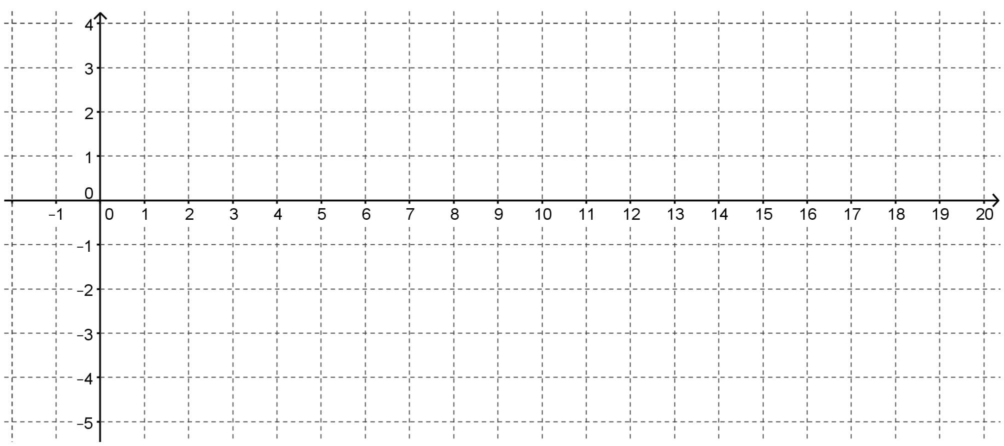


13. Graph and label each of the following functions:

a. $f(x) = 2 + \log_2(x - 1)$




b. $g(x) = -1 + \log_2(x + 2)$




14. Compare the transformation of the graphs of logarithmic functions with the transformation of the graphs of quadratic functions.

READY, SET, GO! Name Period Date

## READY

Topic: Solving simple logarithmic equations

**Find the answer to each logarithmic equation. Then explain how each equation supports the statement,** *"The answer to a logarithmic equation is always the exponent."*

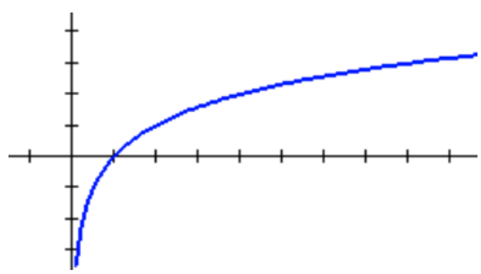
1. $log_5 625 =$
2. $log_3 243 =$
3. $log_5 0.2 =$
4. $log_9 81 =$
5. $log 1{,}000{,}000 =$
6. $log_x x^7 =$

## SET

Topic: Exploring transformations on logarithmic functions

**Answer the questions about each graph.**

7.



a. What is the value of x when $f(x) = 0$?

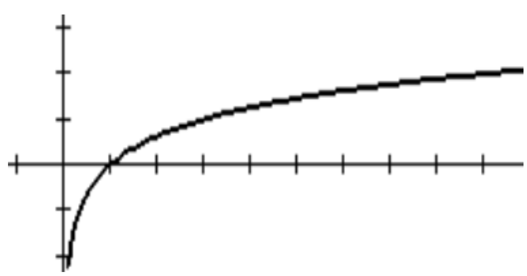
b. What is the value of x when $f(x) = 1$?

c. What is the value of $f(x)$ when $x = 2$?

d. What will be the value of x when $f(x) = 4$?

e. What is the equation of this graph?

8.



a. What is the value of x when $f(x) = 0$?

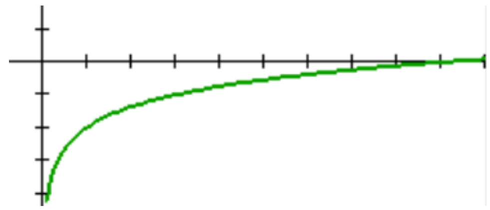
b. What is the value of x when $f(x) = 1$?

c. What is the value of $f(x)$ when $x = 9$?

d. What will be the value of x when $f(x) = 4$?

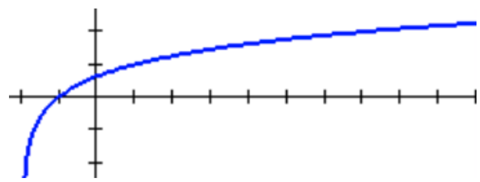
e. What is the equation of this graph?

9. Use the graph and the table of values for the graph to write the equation of the graph.
Explain which numbers in the table helped you the most to write the equation.



| X | Y1 |
|---|---|
| 1 | -2 |
| 2 | -1.369 |
| 3 | -1 |
| 5 | -.535 |
| 7 | -.2288 |
| 9 | 0 |

10. Use the graph and the table of values for the graph to write the equation of the graph.
Explain which numbers in the table helped you the most to write the equation.



| X | Y1 |
|---|---|
| -2 | ERROR |
| -1 | 0 |
| 0 | .63093 |
| 1 | 1 |
| 5 | 1.7712 |
| 6 | 1.8928 |
| 7 | 2 |

## GO

Topic: Using the power to a power rule with exponents

**Simplify each expression. Answers should have only positive exponents.**

11. $(2^3)^4$

12. $(x^3)^2$

13. $(a^3)^{-2}$

14. $(2^3w)^4$

15. $(b^{-7})^3$

16. $(d^{-3})^{-2}$

17. $x^2 \cdot (x^5)^2$

18. $m^{-3} \cdot (m^2)^3$

19. $(x^5)^{-4} \cdot x^{25}$

20. $(5a^3)^2$

21. $(6^{-3})^2$

22. $(2a^3b^2)^2$

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# 2.3 Chopping Logs

## *A Solidify Understanding Task*

Abe and Mary were working on their math homework together when Abe has a brilliant idea!

**Abe:** I was just looking at this *log* function that we graphed in *Falling Off A Log*:

$$y = \log_2(x+b).$$

I started to think that maybe I could just "distribute" the *log* so that I get:

$$y = \log_2 x + \log_2 b.$$

I guess I'm saying that I think these are equivalent expressions, so I could write it this way:

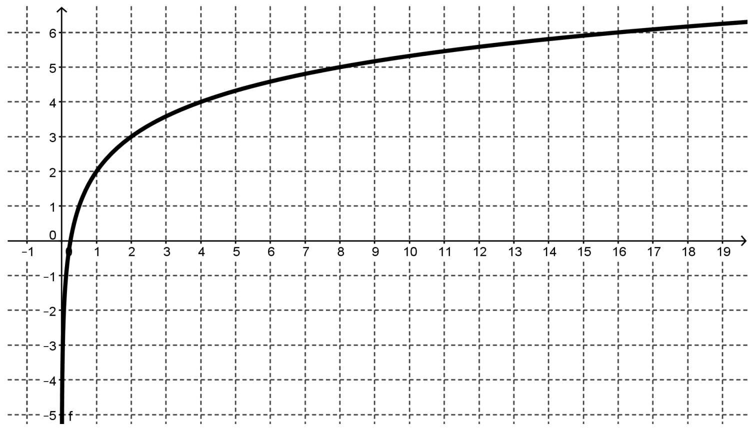
$$\log_2(x+b) = \log_2 x + \log_2 b$$

**Mary:** I don't know about that. *Logs* are tricky and I don't think that you're really doing the same thing here as when you distribute a number.

1. What do you think? How can you verify if Abe's idea works?

2. If Abe's idea works, give some examples that illustrate why it works. If Abe's idea doesn't work, give a counter-example.

**Abe:** I just know that there is something going on with these *logs*. I just graphed $f(x) = \log_2(4x)$. Here it is:




It's weird because I think that this graph is just a translation of $y = \log_2 x$. Is it possible that the equation of this graph could be written more than one way?

3. How would you answer Abe's question? Are there conditions that could allow the same graph to have different equations?

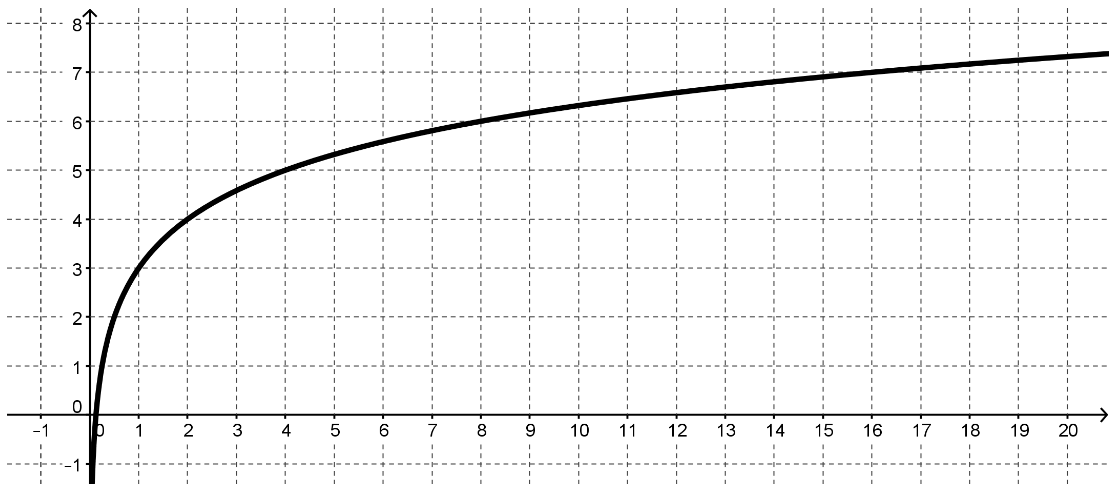
**Mary:** When you say, "a translation of $y = \log_2 x$" do you mean that it is just a vertical or horizontal shift? What could that equation be?

4. Find an equation for $f(x)$ that shows it to be a horizontal or vertical shift of $y = \log_2 x$.

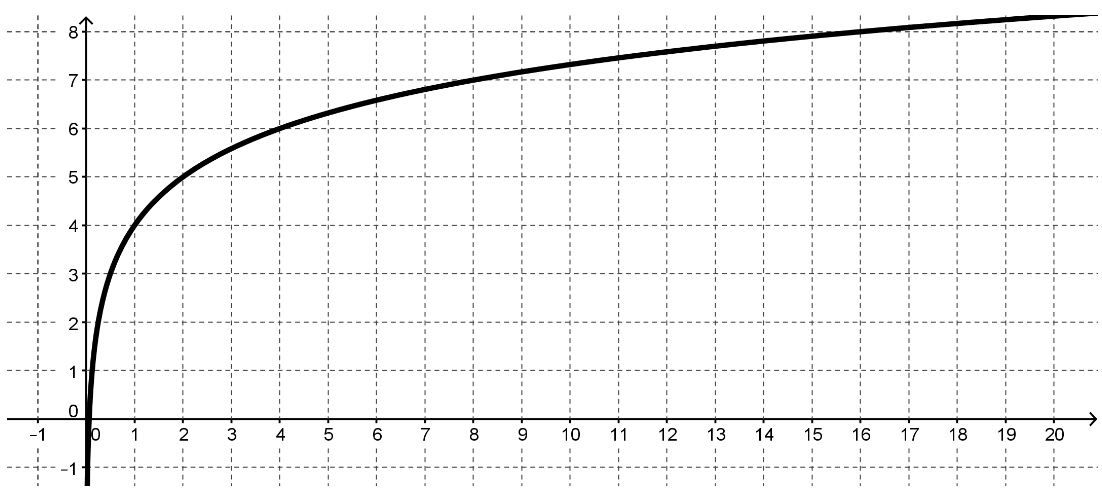
**Mary:** I wonder why the vertical shift turned out to be up 2 when the $x$ was multiplied by 4. I wonder if it has something to do with the power that the base is raised to, since this is a *log* function. Let's try to see what happens with $y = \log_2(8x)$ and $y = \log_2(16x)$.

5. Try to write an equivalent equation for each of these graphs that is a vertical shift of $y = \log_2 x$.

a) $y = \log_2(8x)$ Equivalent equation: ______________________




b. $y = \log_2(16x)$ Equivalent equation: ______________________



**Mary:** Oh my gosh! I think I know what is happening here! Here's what we see from the graphs:

$$\log_2(4x) = 2 + \log_2 x$$

$$\log_2(8x) = 3 + \log_2 x$$

$$\log_2(16x) = 4 + \log_2 x$$

Here's the brilliant part: We know that $\log_2 4 = 2, \log_2 8 = 3,$ and $\log_2 16 = 4$. So:

$$\log_2(4x) = \log_2 4 + \log_2 x$$

$$\log_2(8x) = \log_2 8 + \log_2 x$$

$$\log_2(16x) = \log_2 16 + \log_2 x$$

I think it looks like the "distributive" thing that you were trying to do, but since you can't really distribute a function, it's really just a *log* multiplication rule. I guess my rule would be:

$$\log_2(ab) = \log_2 a + \log_2 b$$

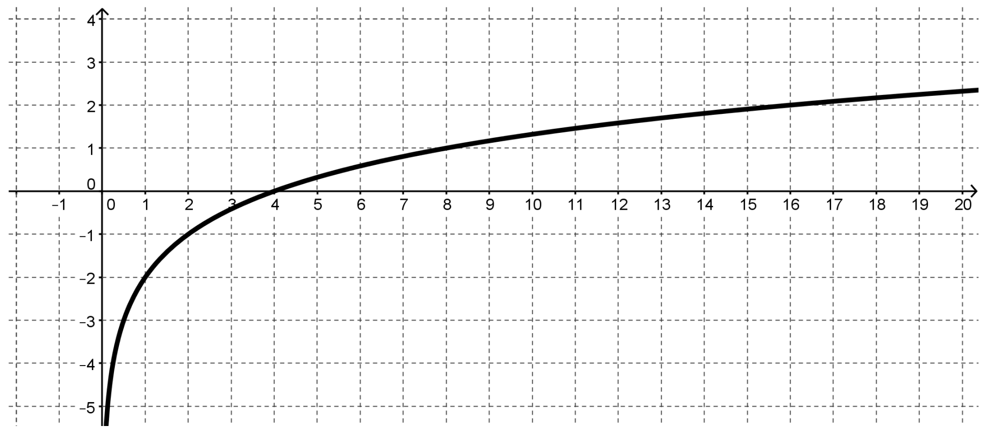
6. How can you express Mary's rule in words?

7. Is this statement true? If it is, give some examples that illustrate why it works. If it is not true provide a counter example.

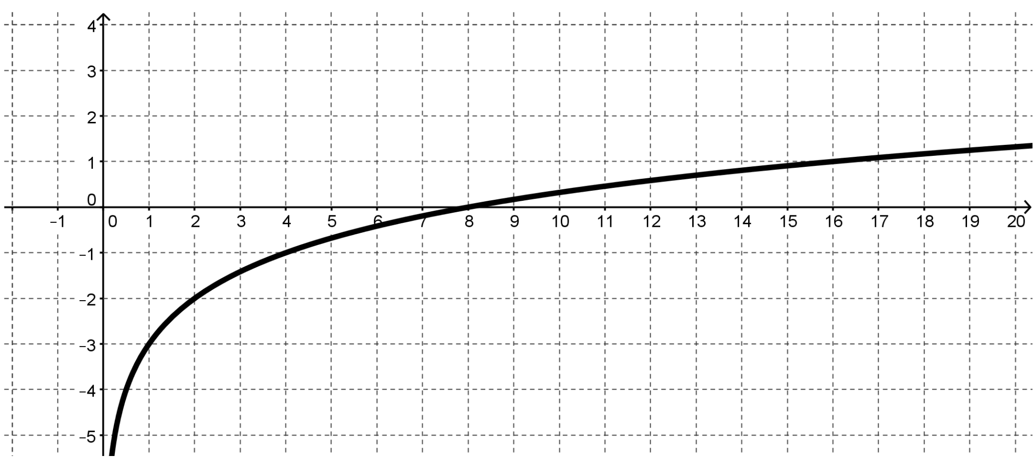
**Mary:** So, I wonder if a similar thing happens if you have division inside the argument of a *log* function. I'm going to try some examples. If my theory works, then all of these graphs will just be vertical shifts of $y = \log_2 x$.

8. Here are Abe's examples and their graphs. Test Abe's theory by trying to write an equivalent equation for each of these graphs that is a vertical shift of $y = \log_2 x$.

a) $y = \log_2\left(\frac{x}{4}\right)$ Equivalent equation: ______________________



b) $y = \log_2\left(\frac{x}{8}\right)$ Equivalent equation: ______________________



9. Use these examples to write a rule for division inside the argument of a *log* that is like the rule that Mary wrote for multiplication inside a *log*.

10. Is this statement true? If it is, give some examples that illustrate why it works. If it is not true provide a counter example.

**Abe:** You're definitely brilliant for thinking of that multiplication rule. But I'm a genius because I've used your multiplication rule to come up with a power rule. Let's say that you start with:

$\log_2(x^3)$

Really that's the same as having: $\log_2(x \cdot x \cdot x)$

So, I could use your multiplying rule and write: $\log_2 x + \log_2 x + \log_2 x$

I notice that there are 3 terms that are all the same. That makes it: $3\log_2 x$

So my rule is: $\log_2(x^3) = 3\log_2 x$

If your rule is true, then I have proven my power rule.

**Mary:** I don't think it's really a power rule unless it works for any power. You only showed how it might work for 3.

**Abe:** Oh, good grief! Ok, I'm going to say that it can be any number *x*, raised to any power, *k*. My power rule is:

$$\log_2(x^k) = k\log_2 x$$

Are you satisfied?

11. Provide an argument about Abe's power rule. Is it true or not?

**Abe:** Before we win the Nobel Prize for mathematics I suppose that we need to think about whether or not these rules work for any base.

12. The three rules, written for any base $b > 1$ are:

    **Log of a Product Rule:** $\log_b(xy) = \log_b x + \log_b y$

    **Log of a Quotient Rule:** $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

    **Log of a Power Rule:** $\log_b(x^k) = k\log_b x$

    Make an argument for why these rules will work in any base $b > 1$ if they work for base 2.

13. How are these rules similar to the rules for exponents? Why might exponents and *logs* have similar rules?

READY, SET, GO! Name Period Date

## READY

Topic: Recalling fractional exponents

**Write the following with an exponent. Simplify when possible.**

1. $\sqrt[5]{x}$
2. $\sqrt[7]{s^2}$
3. $\sqrt[3]{w^8}$
4. $\sqrt[3]{8r^6}$
5. $\sqrt[5]{125m^5}$
6. $\sqrt[3]{(8x)^2}$
7. $\sqrt[3]{9b^8}$
8. $\sqrt{75x^6}$

**Rewrite with a fractional exponent. Then find the answer.**

9. $log_3 \sqrt[5]{3} =$
10. $log_2 \sqrt[3]{4} =$
11. $log_7 \sqrt[5]{343} =$
12. $log_5 \sqrt[5]{3125} =$

## SET

Topic: Using the properties of logarithms to expand logarithmic expressions

**Use the properties of logarithms to expand the expression as a sum or difference, and/or constant multiple of logarithms.** (Assume all variables are positive.)

13. $log_5 7x$
14. $log_5 10a$
15. $log_5 \frac{5}{b}$
16. $log_5 \frac{d}{4}$
17. $log_6 x^3$
18. $log_5 9x^2$
19. $log_2 (7x)^4$
20. $log_3 \sqrt{w}$

21. $log_5 \frac{xyz}{w}$

22. $log_5 \frac{9\sqrt{x}}{y^3}$

23. $log_2 \left(\frac{x^2-4}{x^3}\right)$

24. $log_2 \left(\frac{x^2}{y^5 w^3}\right)$

## GO

Topic: Writing expressions in exponential form and logarithmic form

**Convert to logarithmic form.**

25. $2^9 = 512$

26. $10^{-2} = 0.01$

27. $\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$

**Write in exponential form.**

28. $log_4 2 = \frac{1}{2}$

29. $log_{\frac{1}{3}} 3 = -1$

30. $log_{\frac{2}{5}} \frac{8}{125} = 3$

# 2.4 Log-Arithm-etic

## *A Practice Understanding Task*

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Abe and Mary are feeling good about their *log* rules and bragging about their mathematical prowess to all of their friends when this exchange occurs:

**Stephen:** I guess you think you're pretty smart because you figured out some *log* rules, but I want to know what they're good for.

**Abe:** Well, we've seen a lot of times when equivalent expressions are handy. Sometimes when you write an expression with a variable in it in a different way it means something different.

1. What are some examples from your previous experience where equivalent expressions were useful?

**Mary:** I was thinking about the *Log Logic* task where we were trying to estimate and order certain *log* values. I was wondering if we could use our log rules to take values we know and use them to find values that we don't know.

For instance: Let's say you want to calculate $\log_2 6$. If you know what $\log_2 2$ and $\log_2 3$ are then you can use the product rule and say:

$$\log_2(2 \cdot 3) = \log_2 2 + \log_2 3$$

Stephen: That's great. Everyone knows that $\log_2 2 = 1$, but what is $\log_2 3$?

Abe: Oh, I saw this somewhere. Uh, $\log_2 3 = 1.585$. So Mary's idea really works. You say:

$$\log_2(2 \cdot 3) = \log_2 2 + \log_2 3$$

$$= 1 + 1.585$$

$$= 2.585$$

$$\log_2 6 = 2.585$$

2. Based on what you know about logarithms, explain why 2.585 is a reasonable value for $\log_2 6$.

Stephen: Oh, oh! I've got one. I can figure out $\log_2 5$ like this:

$$\log_2(2+3) = \log_2 2 + \log_2 3$$

$$= 1 + 1.585$$

$$= 2.585$$

$$\log_2 5 = 2.585$$

3. Can Stephen and Mary both be correct? Explain who's right, who's wrong (if anyone) and why.

Now you can try applying the *log* rules yourself. Use the values that are given and the ones that you know by definition, like $\log_2 2 = 1$, to figure out each of the following values. Explain what you did in the space below each question.

$\log_2 3 = 1.585$ $\log_2 5 = 2.322$ $\log_2 7 = 2.807$

The three rules, written for any base $b$ > 1 are:

**Log of a Product Rule:** $\boldsymbol{\log_b(xy) = \log_b x + \log_b y}$

**Log of a Quotient Rule:** $\boldsymbol{\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y}$

**Log of a Power Rule:** $\boldsymbol{\log_b(x^k) = k\log_b x}$

4. $\log_2 9 =$ ______________________________

5. $\log_2 10 =$ ______________________________

6. $\log_2 12 =$ ________________________________

7. $\log_2 \left(\frac{7}{3}\right) =$ ________________________________

8. $\log_2 \left(\frac{30}{7}\right) =$ ________________________________

9. $\log_2 486 =$ ________________________________

10. Given the work that you have just done, what other values would you need to figure out the value of the base 2 log for any number?

Sometimes thinking about equivalent expressions with logarithms can get tricky. Consider each of the following expressions and decide if they are always true for the numbers in the domain of the logarithmic function, sometimes true, or never true. Explain your answers. If you answer "sometimes true", describe the conditions that must be in place to make the statement true.

11. $\log_4 5 - \log_4 x = \log_4\left(\frac{5}{x}\right)$ ____________________________________

12. $\log 3 - \log x - \log x = \log\left(\frac{3}{x^2}\right)$ ____________________________________

13. $\log x - \log 5 = \frac{\log x}{\log 5}$ ____________________________________

14. $5\log x = \log x^5$ ____________________________________

15. $2\log x + \log 5 = \log(x^2 + 5)$ ____________________________________

16. $\frac{1}{2}\log x = \log\sqrt{x}$ ____________________________________

17. $\log(x-5) = \frac{\log x}{\log 5}$ ____________________________________

READY, SET, GO! Name Period Date

## READY

Topic: Solving simple exponential and logarithmic equations

You have solved exponential equations before based on the idea that $\boldsymbol{a^x = a^y, if\ and\ only\ if\ x = y}$.

You can use the same logic on logarithmic equations. $\boldsymbol{log_a x = log_a y, if\ and\ only\ if\ x = y}$

**Rewrite each equation so that you set up a one-to-one correspondence between all of the parts. Then solve for *x*.**

| **Example:** ***Original equation*** | ***Rewritten equation:*** | ***Solution:*** |
|---|---|---|
| a.) $3^x = 81$ | $3^x = 3^4$ | $x = 4$ |
| b.) $log_2 x - log_2 5 = 0$ | $log_2 x = log_2 5$ | $x = 5$ |

1. $3^{x+4} = 243$

2. $\left(\frac{1}{2}\right)^x = 8$

3. $\left(\frac{3}{4}\right)^x = \frac{27}{64}$

4. $log_2 x - log_2 13 = 0$

5. $log_2(2x - 4) - log_2 8 = 0$

6. $log_2(x + 2) - log_2 9x = 0$

7. $\frac{\log 2x}{\log 14} = 1$

8. $\frac{\log(5x-1)}{\log 29} = 1$

9. $\frac{\log 5^{(x-2)}}{\log 625} = 1$

## SET

Topic: Rewriting logs in terms of known logs

**Use the given values and the properties of logarithms to find the indicated logarithm.**

Do not use a calculator to evaluate the logarithms.

**Given:** $\mathbf{\log 16 \approx 1.2}$
$\mathbf{\log 5 \approx 0.7}$
$\mathbf{\log 8 \approx 0.9}$

10. Find $\log \frac{5}{8}$

11. Find $\log 25$

12. Find $\log \frac{1}{2}$

13. Find $\log 80$

14. Find $\log \frac{1}{64}$

**Given** $\boldsymbol{log_3 2 \approx 0.6}$
$\boldsymbol{log_3 5 \approx 1.5}$

15. Find $log_3 16$

16. Find $log_3 108$

17. Find $log_3 \frac{3}{50}$

18. Find $log_3 \frac{8}{15}$

19. Find $log_3 486$

20. Find $log_3 18$

21. Find $log_3 120$

22. Find $log_3 \frac{32}{45}$

## GO

Topic: Using the definition of logarithm to solve for x.

**Use your calculator and the definition of *log x* (recall the base is 10) to find the value of *x*.**
(Round your answers to 4 decimals.)

23. $\log x = -3$

24. $\log x = 1$

25. $\log x = 0$

26. $\log x = \frac{1}{2}$

27. $\log x = 1.75$

28. $\log x = -2.2$

29. $\log x = 3.67$

30. $\log x = \frac{3}{4}$

31. $\log x = 6$

# 2.5 Powerful Tens

*A Practice Understanding Task*


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## Table Puzzles

1. Use the tables to find the missing values of x:

a.

| $x$ | $y = 10^x$ |
|---|---|
| -2 | $\frac{1}{100}$ |
| 1 | 10 |
| | 50 |
| | 100 |
| 3 | 1000 |

b.

| $x$ | $y = 3(10^x)$ |
|---|---|
| | 0.3 |
| 0 | 3 |
| | 94.87 |
| 2 | 300 |
| | 1503.56 |

c. What equations could be written, in terms of *x* only, for each of the rows that are missing the *x* in the two tables above?

d.

| $x$ | $y = \log x$ |
|---|---|
| 0.01 | -2 |
| | -1 |
| 10 | 1 |
| | 1.6 |
| 100 | 2 |

e.

| $x$ | $y = \log(x + 3)$ |
|---|---|
| | -2 |
| -2.9 | -1 |
| | 0.3 |
| 7 | 1 |
| | 3 |

f. What equations could be written, in terms of $x$ only, for each of the rows that are missing the $x$ in the two tables above?

2. What strategy did you use to find the solutions to equations generated by the tables that contained exponential functions?

3. What strategy did you use to find the solutions to equations generated by the tables that contained logarithmic functions?

# Graph Puzzles

4. The graph of $y = 10^{-x}$ is given below. Use the graph to solve the equations for $x$ and label the solutions.

a. $40 = 10^{-x}$

$x =$ ____

Label the solution with an A on the graph.

b. $10^{-x} = 10$

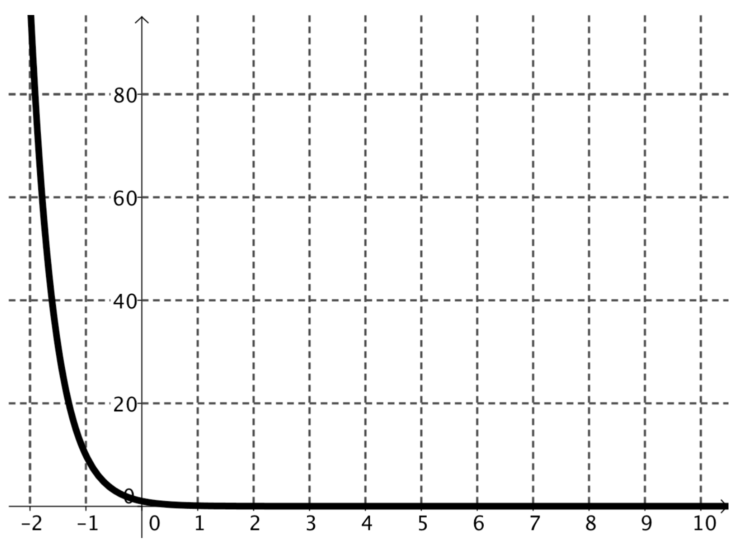
$x =$ ____

Label the solution with a B on the graph.

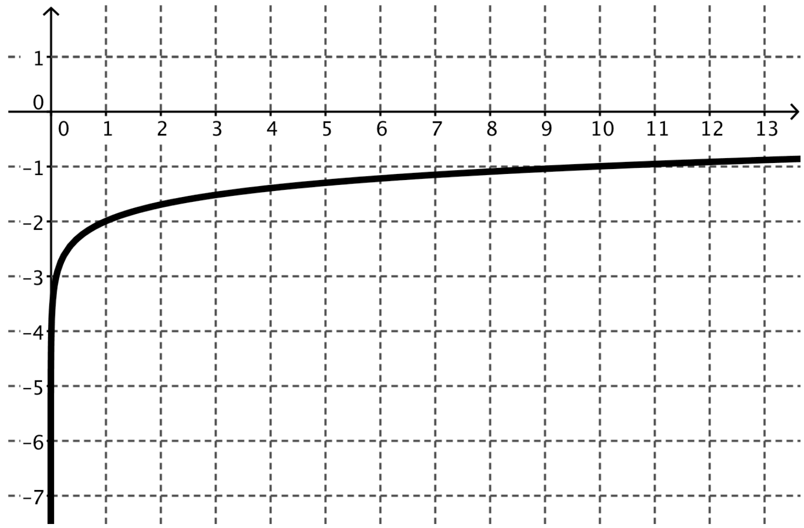
c. $10^{-x} = 0.1$

$x =$ ____

Label the solution with a C on the graph.



5. The graph of y= $-2+\log x$ is given below. Use the graph to solve the equations for $x$ and label the solutions.




a. $-2+\log x=-2$

$x=$ ____

Label the solution with an A on the graph.

b. $-2+\log x=0$

$x=$ ____

Label the solution with a B on the graph.

c. $-4=-2+\log x$

$x=$ ____

Label the solution with a C on the graph.

d. $-1.3=-2+\log x$

$x=$ ____

Label the solution with a D on the graph.

e. $1=-2+\log x$

$x=$ ____

6. Are the solutions that you found in #5 exact or approximate? Why?

# Equation Puzzles:

Solve each equation for *x:*

7. $10^x$=10,000

8. $125=10^x$

9. $10^{x+2}=347$

10. $5(10^{x+2})=0.25$

11. $10^{-x-1}=\frac{1}{36}$

12. $-(10^{x+2})=16$

READY, SET, GO! Name Period Date

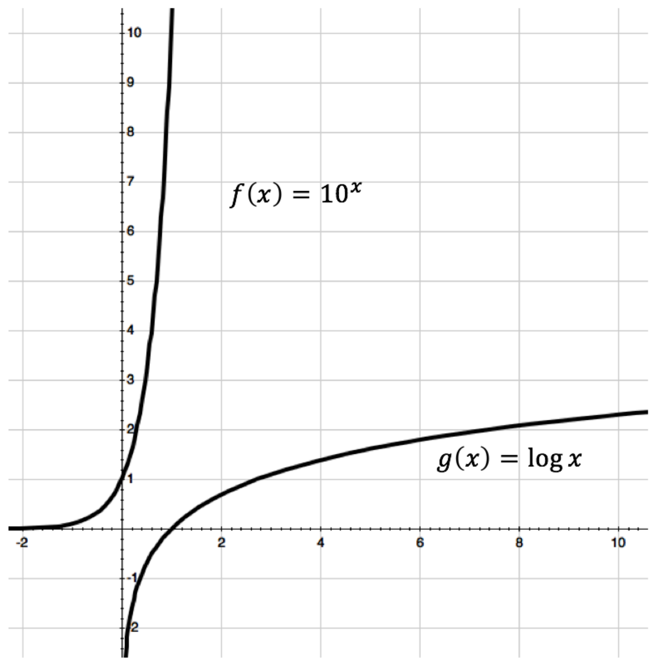
## READY

Topic: Comparing the graphs of the exponential and logarithmic functions

The graphs of $f(x) = 10^x$ *and* $g(x) = \log x$ are shown in the same coordinate plane.

Make a list of the characteristics of each function.

1. $f(x) = 10^x$




2. $g(x) = \log x$

**Each question below refers to the graphs of the functions $f(x) = 10^x$ *and* $g(x) = \log x$. State whether they are true or false. If they are false, correct the statement so that it is true.**

_________ 3. Every graph of the form $g(x) = \log x$ will contain the point (1, 0).

_________ 4. Both graphs have vertical asymptotes.

_________ 5. The graphs of $f(x)$ *and* $g(x)$ have the same rate of change.

_________ 6. The functions are inverses of each other.

_________ 7. The range of $f(x)$ is the domain of $g(x)$.

_________ 8. The graph of $g(x)$will never reach 3.

**Need help?** Visit **www.rsgsupport.org**

## SET

Topic: Solving logarithmic equations *(base 10)* by taking the log of each side

**Evaluate the following logarithms**

9. $\log 10$
10. $\log 10^{-7}$
11. $\log 10^{75}$
12. $\log 10^{x}$
13. $log_3 3^5$
14. $log_8 8^{-3}$
15. $log_{11} 11^{37}$
16. $log_m m^n$

You can use this property of logarithms to help you solve logarithmic equations.

**Note: This property only works when the base of the logarithm matches the base of the exponent.*

**Solve the equations by inserting $\boldsymbol{log_n}$ on both sides of the equation.** (You will need a calculator.)

17. $10^n = 4.305$
18. $10^n = 0.316$
19. $10^n = 14{,}521$
20. $10^n = 483.059$

## GO

Topic: Solving equations involving compound interest

**Formula for compound interest:** If $P$ dollars is deposited in an account paying an annual rate of interest $r$ compounded (paid) $n$ times per year, the account will contain $\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{nt}}$ dollars after $t$ years.

21. How much money will there be in an account at the end of 10 years if \$3000 is deposited at 6% annual interest compounded as follows: (Assume no withdrawals are made.)
    a.) annually
    b.) semiannually
    c.) quarterly
    d.) daily (Use $n$ = 365.)

22. Find the amount of money in an account after 12 years if \$5,000 is deposited at 7.5% annual interest compounded as follows: (Assume no withdrawals are made.)
    a.) annually
    b.) semiannually
    c.) quarterly
    d.) daily (Use $n$ = 365.)