mathematics
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Transforming Mathematics Education

SECONDARY MATH THREE

An Integrated Approach

MODULE 4

Rational Expressions & Functions

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The Mathematics Vision Project

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4.1 Winner, Winner

A Develop Understanding Task

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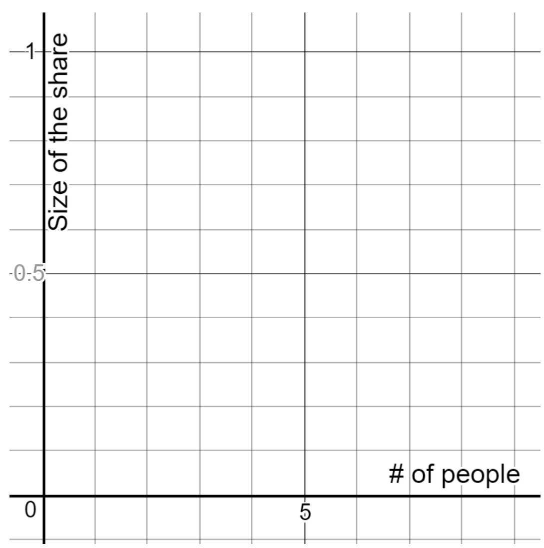
One of the most interesting functions in mathematics is $f(x) = \frac{1}{x}$ because it brings up some mathematical mind benders. In this task, we will use story context and representations like tables and graphs to understand this important function.

Let's being by thinking about the interval $[1, \infty)$.

1. Imagine that you won the lottery and were given one big pot of money. Of course, you would want to share the money with friends and family. If you split the money evenly between yourself and one friend, what would be each person's share of the prize money?

2. If three people shared the prize money, what would be each person's share?

3. Model the situation with a table, equation, and graph.



4. Just in case you didn't think about the really big numbers in your model, how much of the pot would each person get if 1000 people get a share? If 100,000 people get a share? If 100,000,000 people get a share?

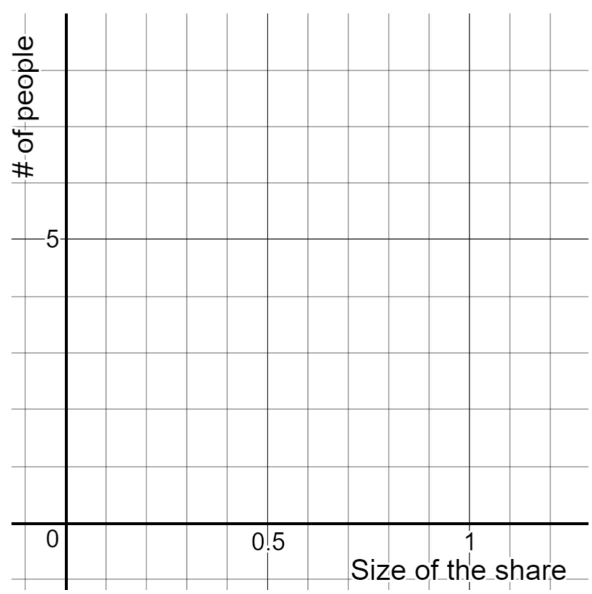
5. Use mathematical notation to describe the behavior of this function as $x \to \infty$.

Next, let's look at the interval (0, 1] and consider a new way to think about splitting the prize money.

6. Imagine that you want each person's share to be $^1/_2$ of the prize. How many people could share the prize?

7. If you want each person's share to be $^1/_3$ of the prize, how many people could share the prize?

8. Model this situation with a table, graph and equation.



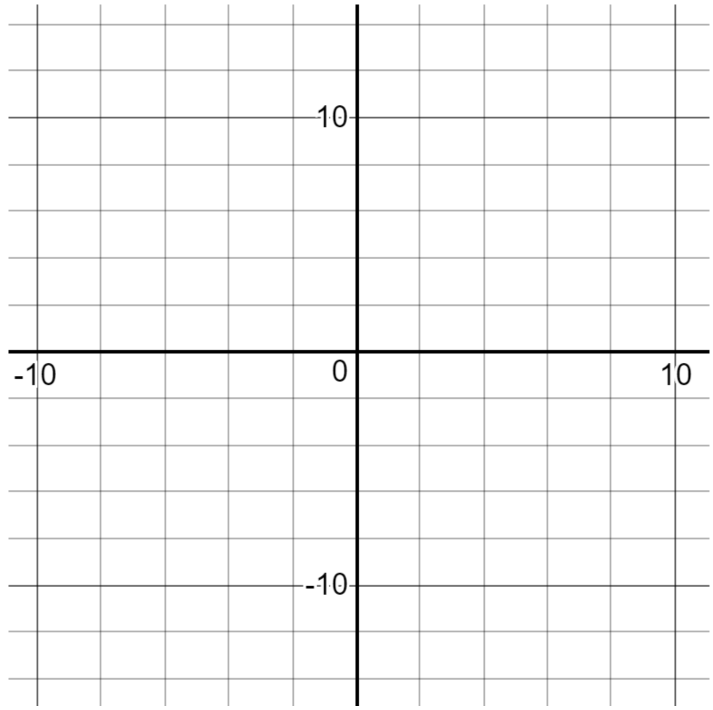
9. What do you notice when you compare the two models that you have written?

Now, let's put it all together to graph the entire function, $f(x) = \frac{1}{x}$.

10. Create a table for $f(x) = \frac{1}{x}$ that includes negative input values.

11. How do the values of $f(x)$ in the interval from $(-\infty, 0)$ compare to the values of $f(x)$ from $(0, \infty)$? Use this comparison to predict the graph of $f(x) = \frac{1}{x}$.

12. Graph $f(x) = \frac{1}{x}$.



13. Describe the features of $f(x) = \frac{1}{x}$, including domain, range, intervals of increase or decrease, x- and y- intercepts, end behavior, and any maximum(s) or minimum(s).

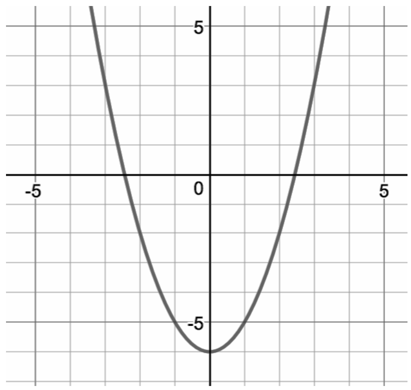
READY, SET, GO! Name Period Date

READY

Topic: Recalling transformations on quadratic functions

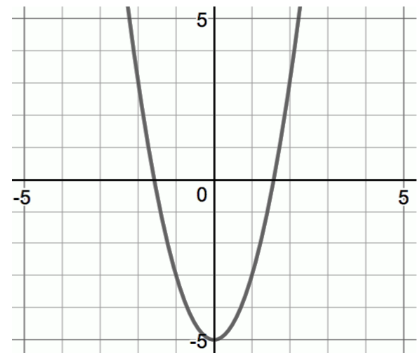
Describe the transformation of each function. Then write the equation in vertex form.

1. Description:



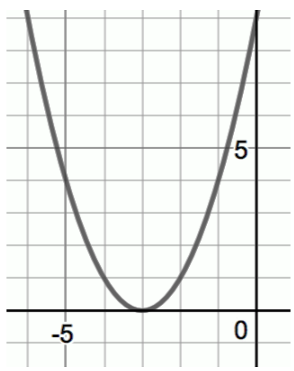
Equation:

2. Description:



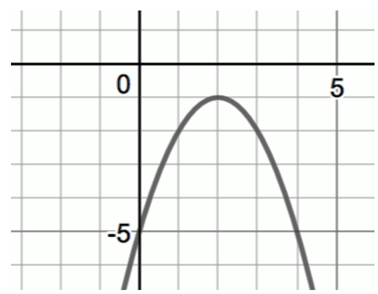
Equation:

3. Description:



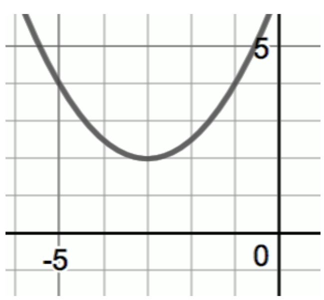
Equation:

4. Description:



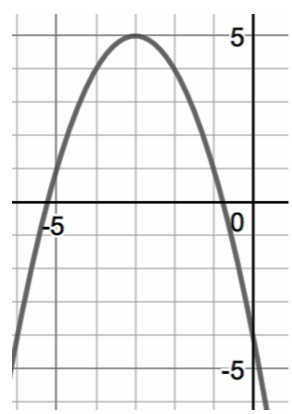
Equation:

5. Description:



Equation:

6. Description:



Equation:

SET

Topic: Exploring a rational function

Chile is celebrating her Quinceañera. Hannah knows the perfect gift to buy Chile, but it costs $360. Hannah can't afford to pay for this on her own so thinks about asking some friends to join in and share the cost.

7. How much would each person spend if there were two people dividing the cost of the gift? How much would each person spend if there were five people dividing the cost? Ten people? One hundred?

8. The function that models this situation is $f(x) = \frac{360}{x}$. Define the meaning of the numerator and the denominator within the context of the story.

9. Create a table and a graph to show how the amount each person would contribute to the gift would change, depending on the number of people contributing.

10. Hannah created a fundraising site on the internet. Within 5 days, enough people had registered so that each friend, including Hannah, only needed to donate $0.50.

    a. How many people had registered in 5 days?

    b. By the day of the event, enough people had registered that each friend, including Hannah, only donated 10¢. How many friends had registered?

GO

Topic: Reviewing the horizontal asymptote in an exponential function

All exponential functions have a horizontal asymptote. All of the graphs below show exponential functions. Match the function rule with the correct graph. Then write the equation of the horizontal asymptote.

11. $f(x) = 2^x$

Equation of horizontal asymptote:

12. $g(x) = 2^x - 3$

Equation of horizontal asymptote:

13. $h(x) = 2^{x-3}$

Equation of horizontal asymptote:

14. $m(x) = -(2^x) - 3$

Equation of horizontal asymptote:

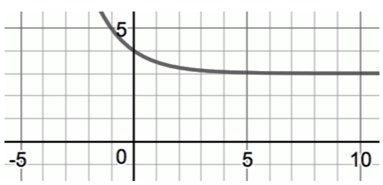
15. $q(x) = 2^{(-x)} + 3$

Equation of horizontal asymptote:

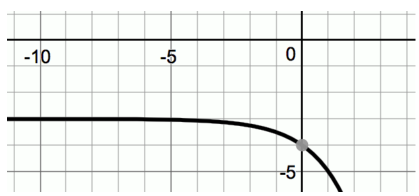
16. $r(x) = -2^{(-x)}$

Equation of horizontal asymptote:

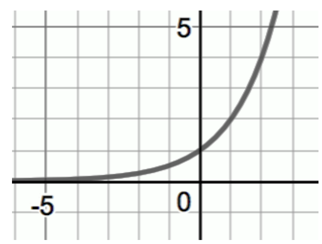
a.



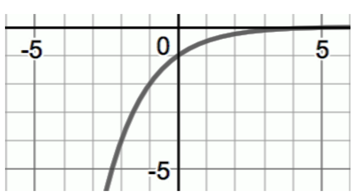
b.



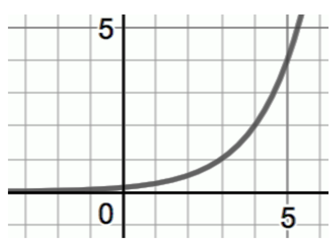
c.



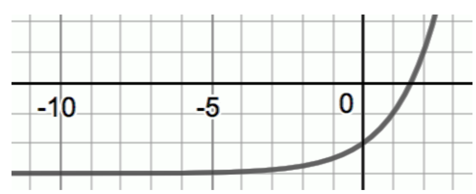
d.



e.



f.



17. Use $f(x) = ab^{(x-h)} + k$ to explain which values affect the position of the horizontal asymptote in an exponential function. Be precise.

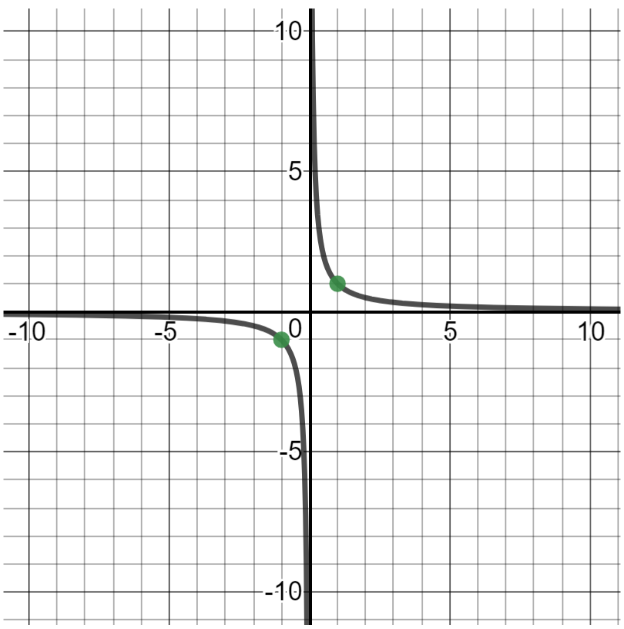
18. Why does an exponential function have a horizontal asymptote?

4.2 Shift and Stretch

A Solidify Understanding Task

In *4.1 Winner, Winner* you were introduced to the function $y = \frac{1}{x}$. Before exploring the family of related functions, let's clarify some of the features of $y = \frac{1}{x}$ that can help with graphing.

Here's a graph of $y = \frac{1}{x}$.



1. Use the graph to identify each of the following:

Horizontal Asymptote: ____________________

Vertical Asymptote: ____________________

Anchor Points:

(1, _____) and (-1, _____)

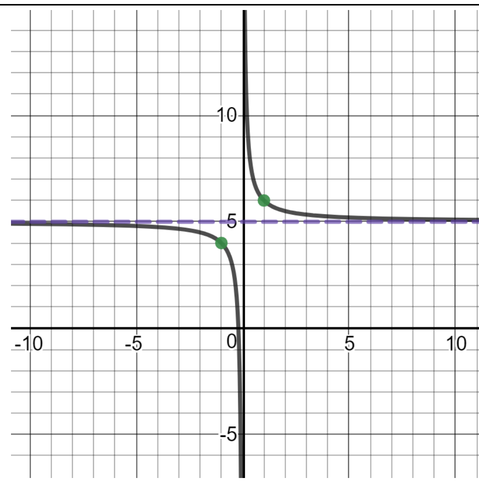
$\left(\frac{1}{2}, \_\_\_\_\_\right)$ and $\left(-\frac{1}{2}, \_\_\_\_\_\right)$

(2, _____) and (-2, _____)

Now you're ready to use this information to figure out how the graph of $y = \frac{1}{x}$ can be transformed. As you answer the questions that follow, look for patterns that you can generalize to describe the transformations of $y = \frac{1}{x}$.

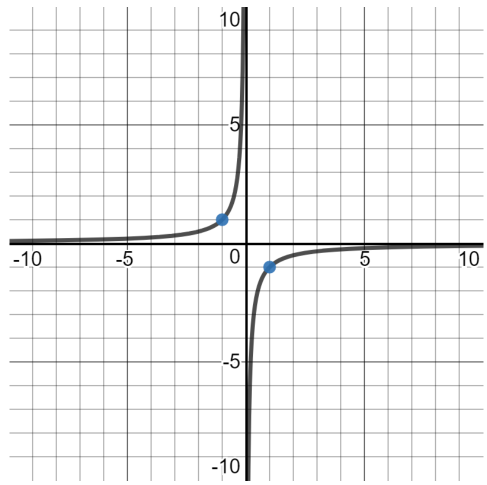
In each of the following problems, you are given either a graph or a description of a function that is a transformation of $y = \frac{1}{x}$. Use your amazing math skills to find an equation for each.

2.



Equation:

3.

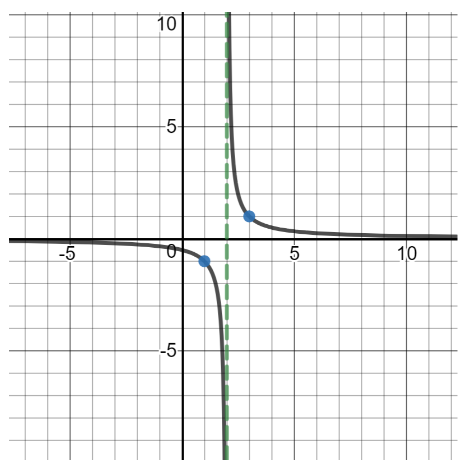


Equation:

4. The function has a vertical asymptote at $x = -3$ and a horizontal asymptote at $y = 0$. It contains the points (-2, 1) and (-4, -1). The y-intercept is $(0, \frac{1}{3})$.

Equation:

5.

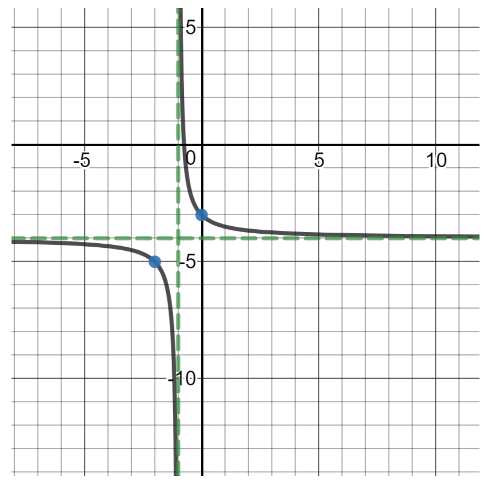


Equation:

6. The function has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$. It contains the points (1,2), (-1, -2), (2, 1), (-2,1), $({}^{1}/_{2}, 4)$ and $({}^{-1}/_{2}, -4)$.

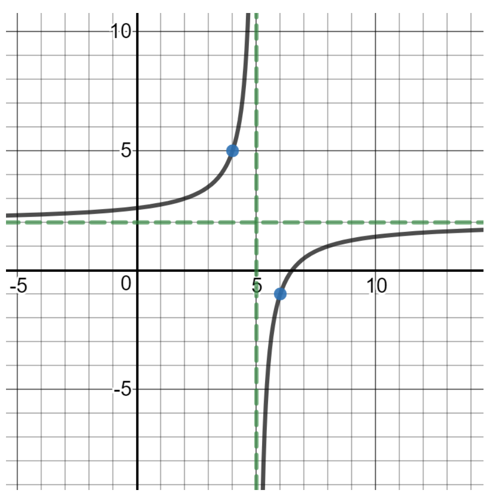
Equation:

7.



Equation:

8.



Equation:

9. The function has a vertical asymptote at $x = -6$ and a horizontal asymptote at $y = -3$. It crosses the x-axis at $-6\frac{1}{3}$. It contains the points (-5, -4) and (-7, -2).

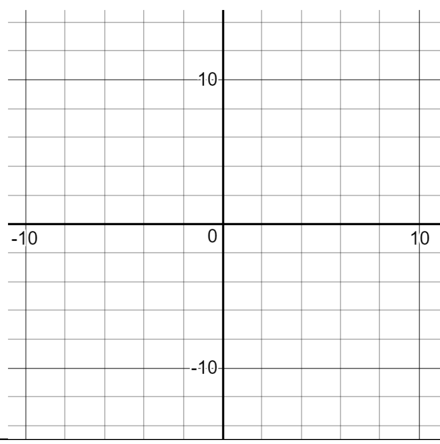
Equation:

10. Match each equation to the phrase that describes the transformation from $y = \frac{1}{x}$.

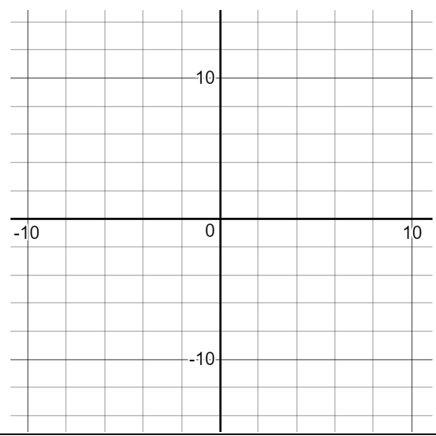
$y = \frac{1}{x+b}$ ________	A) Reflection over the x-axis.
$y = b + \frac{1}{x}$ ________	B) Vertical shift of b, making the horizontal asymptote $y = b$.
$y = \frac{b}{x}$ ________	C) Horizontal shift left b, making the vertical asymptote $x = -b$.
$y = \frac{-1}{x}$ ________	D) Vertical stretch by a factor of b
$y = \frac{1}{x-b}$ ________	E) Horizontal shift right b, making the vertical asymptote $x = b$.

11. Graph each of the following equations without using technology.

$$y = -2 + \frac{3}{x-4}$$



$$y = 1 - \frac{1}{x+3}$$



12. Describe the features of the function:

$$y = k + \frac{b}{x-h}$$

Vertical Asymptote:

Horizontal Asymptote:

Vertical Stretch Factor:

Anchor Points:

Domain:

Range:

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Connecting the zeroes of a polynomial with the domain of a rational function

Find the zeroes of each polynomial.

1. $p(x) = (x+4)(x-2)(x-7)$

2. $p(x) = (2x-6)(8x-1)(x-5)$

3. $p(x) = (9x+3)(x^2-9)$

4. $p(x) = x^2 + 25$

Find the domain of each of the rational functions.

5. $q(x) = \frac{1}{(x+4)(x-2)(x-7)}$

6. $q(x) = \frac{1}{(2x-6)(8x-1)(x-5)}$

7. $q(x) = \frac{1}{(9x+3)(x^2-9)}$

8. $q(x) = \frac{1}{x^2+25}$

SET

Topic: Practicing transformations on rational functions

Identify the vertical asymptote, horizontal asymptote, domain, and range of each function. Then sketch the graph on the grids provided. (Grids on next page.)

9. $f(x) = \frac{4}{x}$

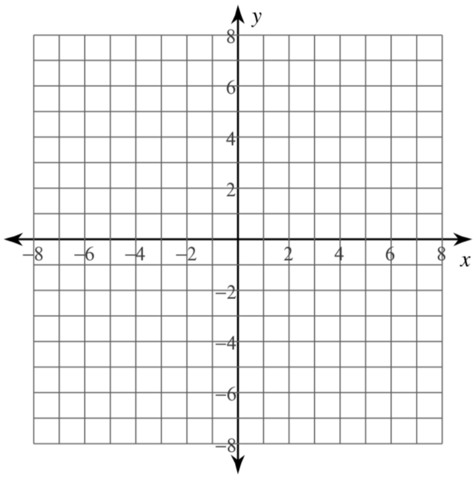
V.A. H. A.

Domain: Range:

10. $f(x) = \frac{3}{x} + 2$

V.A. H. A.

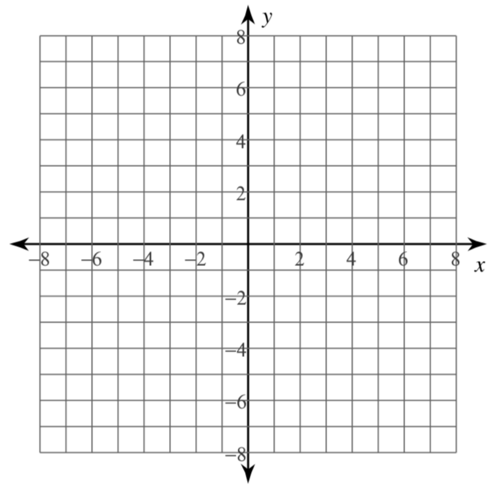
Domain: Range:



11. $f(x) = -\frac{5}{x-3}$

V.A. H. A.

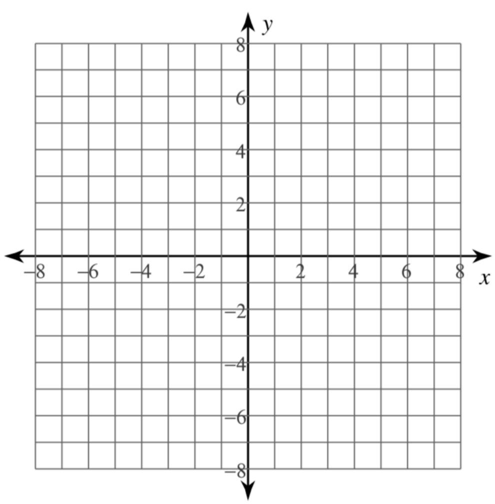
Domain: Range:

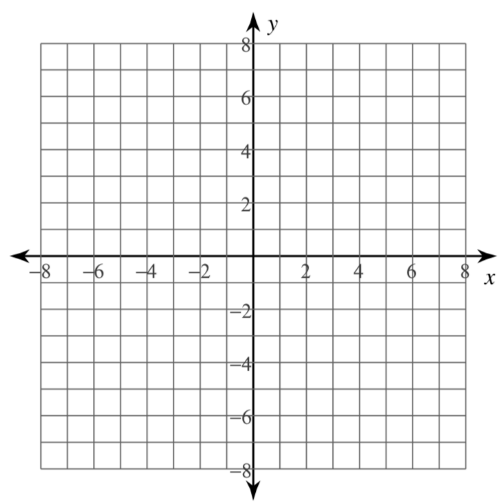


12. $f(x) = \frac{1}{(x+5)} - 4$

V.A. H. A.

Domain: Range:





13. Write a function of the form $f(x) = \frac{a}{x-h} + k$ with a vertical asymptote at $x = -15$ and a horizontal asymptote at $y = -6$.

GO

Topic: Finding the roots and factors of a polynomial

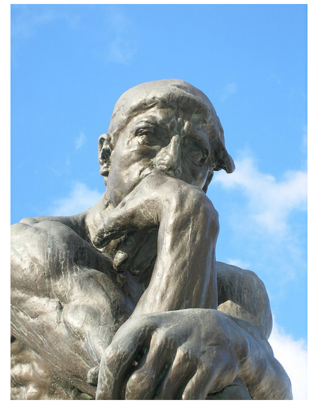
Use the given root to find the remaining roots. Then write the function in factored form.

14. $f(x) = x^3 - x^2 - 17x - 15$	$x = -1$	15. $f(x) = x^3 - 3x^2 - 61x + 63$	$x = 1$
16. $f(x) = 6x^3 - 18x^2 - 60x$	$x = 0$	17. $f(x) = x^3 - 14x^2 + 57x - 72$	$x = 8$

18. A relationship exists between the roots of a function and the constant term of the function. Look back at the roots and the constant term in each problem. Make a statement about anything you notice.

4.3 Rational Thinking

A Solidify Understanding Task



The broad category of functions that contains the function $f(x) = \frac{1}{x}$ is called rational functions. A rational number is a ratio of integers. A rational function is a ratio of polynomials. Since polynomials come in many forms, constant, linear, quadratic, cubic, etc., we can expect rational functions to come in many forms too. Some examples are:

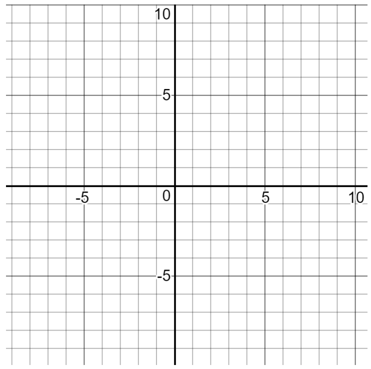
$f(x) = \frac{1}{x}$	$f(x) = \frac{x+1}{x-3}$	$f(x) = \frac{x-4}{x^3+2x^2-4x+1}$	$f(x) = \frac{x^2-4}{(x-3)(x+1)}$
Degree of the numerator = 0 Degree of the denominator = 1	Degree of the numerator = 1 Degree of the denominator = 1	Degree of the numerator = 1 Degree of the denominator = 3	Degree of the numerator = 2 Degree of the denominator = 2

In today's task, you are going to look for patterns in the forms so that you can complete the following chart:

	How to find the vertical asymptote:	How to find the horizontal asymptote:	How to find the intercepts:
Degree of the numerator < Degree of the denominator			
Degree of the numerator = Degree of the denominator			

You are given several different rational functions. Start by identifying the degree of the numerator and denominator and using technology to graph the function. As you are working, look for patterns that will help you complete the table. You need to find a quick way to identify the horizontal and vertical asymptotes when you see the equation of a rational function, as well as noticing other patterns that will help you analyze and graph the function quickly. The last two graphs are there so you can experiment with your own rational functions and test your theories.

1. $y = \frac{x+3}{x-2}$



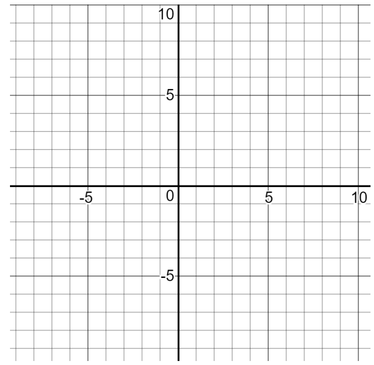
Deg. of Num. _____ Deg. of Denom. _____

Horizontal Asymptote __________

Vertical Asymptote __________

Intercepts ________________________________

2. $y = \frac{x}{(x+2)(x-3)}$



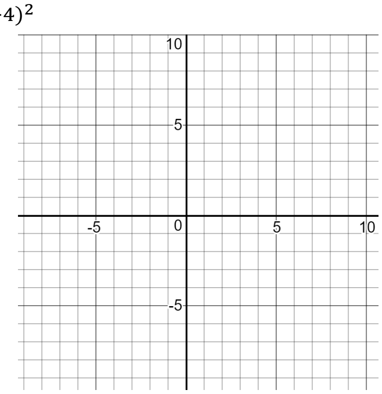
Deg. of Num. _____ Deg. of Denom. _____

Horizontal Asymptote __________

Vertical Asymptote __________

Intercepts ________________________________

3. $y = \frac{1}{(x+4)^2}$



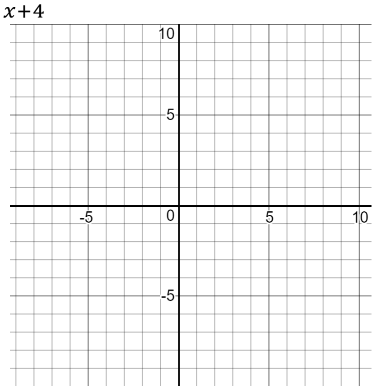
Deg. of Num. _____ Deg. of Denom. _____

Horizontal Asymptote __________

Vertical Asymptote __________

Intercepts ________________________________

4. $y = \frac{3x-1}{x+4}$



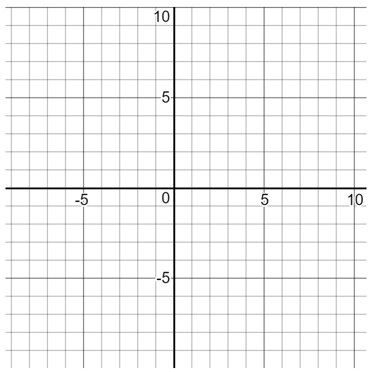
Deg. of Num. _____ Deg. of Denom. _____

Horizontal Asymptote __________

Vertical Asymptote __________

Intercepts ________________________________

5. $y = \frac{x^2-3x}{(x-2)(x+1)(x+3)}$



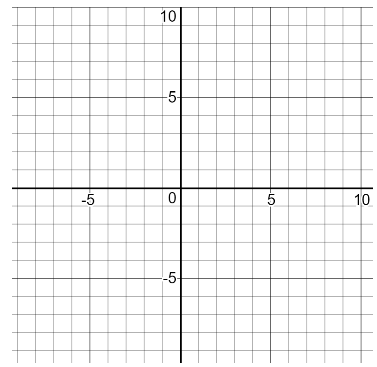
Deg. of Num. ____ Deg. of Denom. ____

Horizontal Asymptote _________

Vertical Asymptote _________

Intercepts ______________________________

6. $y = \frac{2x^2+3}{(2x+5)(2x-5)}$



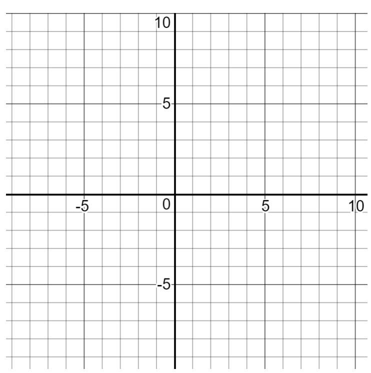
Deg. of Num. ____ Deg. of Denom. ____

Horizontal Asymptote _________

Vertical Asymptote _________

Intercepts ______________________________

7. Your Own Rational Function:



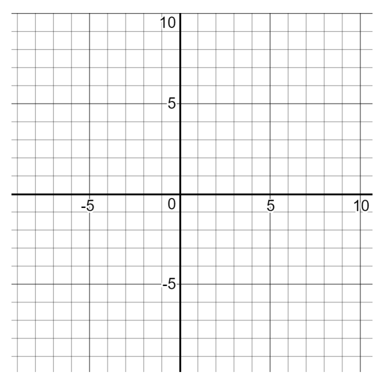
Deg. of Num. ____ Deg. of Denom. ____

Horizontal Asymptote _________

Vertical Asymptote _________

Intercepts ______________________________

8. Your Own Rational Function:



Deg. of Num. ____ Deg. of Denom. ____

Horizontal Asymptote _________

Vertical Asymptote _________

Intercepts ______________________________

9. Now that you have tried some examples, it's time to draw some conclusions and complete the two rows of the table on the first page. Be prepared to discuss your conclusions and why you think that they are correct.

READY, SET, GO! Name Period Date

READY

Topic: Doing arithmetic with rational numbers

Perform the indicated operation. Be thoughtful about each step you perform in the procedure. Show your work.

1. $\frac{5}{17} + \frac{8}{17}$
2. $\frac{3}{10} + \frac{16}{25}$
3. $\frac{4}{5} + \frac{7}{11}$
4. $\left(\frac{2}{3}\right) \cdot \left(\frac{4}{5}\right)$
5. $\left(\frac{2}{3}\right) \cdot \left(\frac{9}{16}\right)$
6. $\left(\frac{10}{33}\right) \cdot \left(\frac{11}{15}\right)$
7. Explain the procedure for adding two fractions.
   a. When the denominators are the same:

   b. When the denominators are different:

8. Explain the procedure for multiplying two fractions.

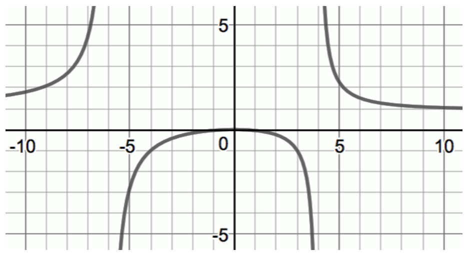
9. When multiplying two fractions, is it better to reduce before you multiply or after you multiply? Explain your reasoning.

SET

Topic: Identifying key features of a rational function

Fill in the specified features of each rational function. Sketch the asymptotes on the graph and mark the location of the intercepts.

10. $y = \frac{x^2}{(x+6)(x-4)}$



Degree of num. ________ Degree of denom.________

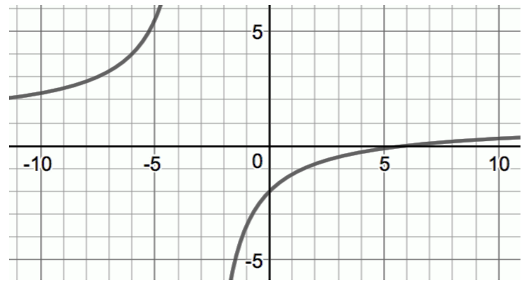
Equation of horizontal asymptote:

Equation of vertical asymptote(s):

y- intercept: (write as a point)

x- intercept(s): (write as points)

11. $y = \frac{(x-6)}{x+3}$



Degree of num. ________ Degree of denom.________

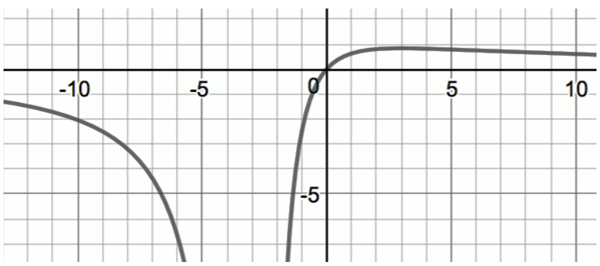
Equation of horizontal asymptote:

Equation of vertical asymptote(s):

y- intercept: (write as a point)

x- intercept(s): (write as points)

12. $y = \frac{10x}{(x+3)^2}$



Degree of num. ________ Degree of denom.________

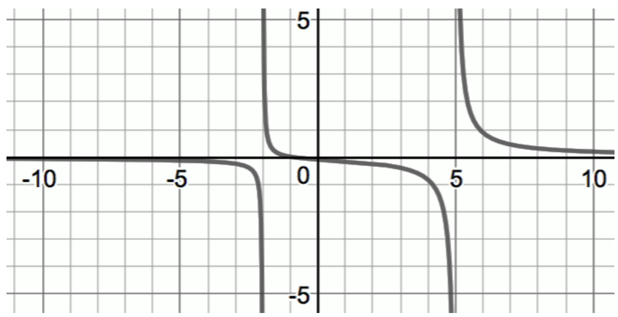
Equation of horizontal asymptote:

Equation of vertical asymptote(s):

y- intercept: (write as a point)

x- intercept(s): (write as points)

13. $y = \frac{(x+1)}{(x+2)(x-5)}$



Degree of num. ________ Degree of denom.________

Equation of horizontal asymptote:

Equation of vertical asymptote(s):

y- intercept: (write as a point)

x- intercept(s): (write as points)

GO

Topic: Reducing fractions

Reduce the following fractions to lowest form. Then explain the mathematics that makes it possible to rewrite the fraction in its new form. (Improper fractions should not be written as mixed numbers.) **If a fraction can't be reduced, explain why.**

14. $\frac{12}{15}$ Explanation:

15. $\frac{26}{11}$ Explanation:

17. $\frac{51}{17}$ Explanation:

18. $\frac{6}{13}$ Explanation:

19. $\frac{114}{27}$ Explanation:

20. $\frac{-14{,}529}{14{,}529}$ Explanation:

4.4 Are You Rational?

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A Solidify Understanding Task

Back in Module 3 when we were working with polynomials, it was useful to draw connections between polynomials and integers. In this task, we will use connections between rational numbers and rational functions to help us to think about operations on rational functions.

1. In your own words, define **rational number.**

Circle the numbers below that are rational and refine your definition, if needed.

$3 \qquad -5 \qquad \frac{2}{3} \qquad \frac{20}{3} \qquad 14 \qquad 2.7 \qquad \sqrt{5} \qquad 2^3 \qquad 3^{-3} \qquad log_2 9 \qquad \frac{7}{0}$

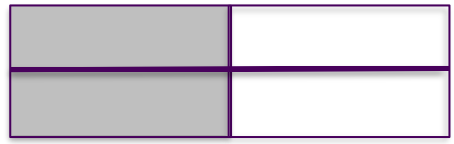
2. The formal definition of a **rational function** is as follows:

A function $f(x)$ is called a rational function if and only if it can be written in the form $f(x) = \frac{P(x)}{Q(x)}$ where P and Q are polynomials in x and Q is not the zero polynomial.

Interpret this definition in your own words and then write three examples of rational functions.

3. How are rational numbers and rational functions similar? Different?

Now we are going to use what we know about rational numbers to perform operations on rational expressions. The first thing we often need to do is to simplify or "reduce" a rational number or expression. The numbers and expressions are not really being reduced because the value isn't actually changing. For instance, 2/4 can be simplified to ½, but as the diagram shows, these are just two different ways of expressing the same amount.



Let's try using what we know about simplifying rational numbers to simplify rational expressions. Fill in any missing parts in the fractions below.

Given:	$\frac{24}{30}$	4. $\frac{x^2-x-6}{x^2-4}$	5. $\frac{x^2+8x+15}{x^2+9x+18}$
Look for common factors:	$\frac{2\cdot 2\cdot 2\cdot 3}{2\cdot 3\cdot 5}$	$\frac{}{(x+2)(x-2)}$	
Divide numerator and denominator by the same factor(s):	$\frac{\cancel{2}\cdot 2\cdot 2\cdot \cancel{3}}{\cancel{2}\cdot \cancel{3}\cdot 5}$		
Write the simplified form:	$\frac{4}{5}$	$\frac{x-3}{x-2}$	$\frac{x+5}{x+6}$

6. Why does dividing the numerator and denominator by the same factor keep the value of the expression the same?

7. If you were given the expression $\frac{x}{x^2-1}$, would it be acceptable to reduce it like this:

$$\frac{\cancel{x}}{\cancel{x^2}-1} = \frac{1}{x-1}$$

Explain your answer.

In *4.3 Rational Thinking,* we learned to predict vertical and horizontal asymptotes, and to find intercepts for graphing rational functions.

8. Given $f(x) = \frac{x^2-x-6}{x^2-4}$, predict the vertical and horizontal asymptotes and find the intercepts.

9. Use technology to view the graph. Were your predictions correct? What occurs on the graph at $x = -2$?

Rational numbers can be written as either proper fractions or improper fractions.

10. Describe the difference between proper fractions and improper fractions and write two examples of each.

A rational expression is similar, except that instead of comparing the numeric value of the numerator and denominator, the comparison is based on the *degree* of each polynomial. Therefore, *a rational expression is proper if the degree of the numerator is less than the degree of the denominator, and improper otherwise.* In other words, improper rational expressions can be written as $\frac{a(x)}{b(x)}$, where $a(x) and\ b(x)$ are polynomials and the degree of $a(x)$ is greater than or equal to the degree of $b(x)$.

11. Label each rational expression as proper or improper.

$\frac{(x+1)}{(x-2)(x+2)}$ $\frac{x^3-3x^2+5x-1}{x^2-4x+4}$ $\frac{(x+3)(x+2)}{x^4-4}$ $\frac{x+3}{x+5}$ $\frac{x^3-5x+2}{x-10}$

As we may remember, improper fractions can be rewritten in an equivalent form we call a mixed number. If the numerator is greater than the denominator then we divide the numerator by the denominator and write the remainder as a proper fraction. In math terms we would say:

If $a > b$, then the fraction $\frac{a}{b}$ can be rewriten as $\frac{a}{b} = q + \frac{r}{b}$, where q represents the quotient and r represents the remainder.

12. Rewrite each improper fraction as an equivalent mixed number.

a) $\frac{37}{5} =$ b) $\frac{150}{12} =$

Rational expressions work the very same way. If the expression is improper, the numerator can be divided by the denominator and the remainder is written as a fraction. In mathematical terms, we would say:

$\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}$ where $q(x)$ represents the quotient and $r(x)$ represents the remainder.

Try it yourself! Label each rational expression as proper or improper. If it is improper, then divide the numerator by the denominator and write it in an equivalent form.

13. $\frac{x^2+5x+7}{x+2}$

14. $\frac{-5x+10}{x^3+6x^2+3x-1}$

15. $\frac{x^2+2x+5}{x+3}$

16. $\frac{3x+8}{x-1}$

In *4.3 Rational Thinking,* when we looked at the graphs of rational functions, we did not consider the case when the numerator of the fraction is greater than the denominator. So, let's take a closer look at the rational function from #13.

16. Let $f(x) = \frac{x^2+5x+7}{x+2}$. Where do you expect the vertical asymptote and the intercepts to be?

17. Use technology to graph the function. Relate the graph of the function to the equivalent expression that you wrote. What do you notice?

18. Let's try the same thing with #15. Let $f(x) = \frac{x^2+2x+5}{x+3}$. Find the vertical asymptote, the intercepts, and then relate the graph to the equivalent expression for $f(x)$.

19. Using the two examples above, write a process for predicting the graphs of rational functions when the degree of the numerator is greater than the degree of the denominator.

READY, SET, GO! Name Period Date

READY

Topic: Connecting features of polynomials and rational functions

Find the roots and domain for each function.

1. $f(x) = (x+5)(x-2)(x-7)$

2. $g(x) = x^2 + 7x + 6$

3. $k(x) = \frac{1}{(x+5)(x-2)(x-7)}$

4. $h(x) = \frac{1}{(x^2+7x+6)}$

5. Make a conjecture that compares the domain of a polynomial with the domain of the reciprocal of the polynomial. *(Note that the reciprocal of a polynomial is a rational function.)*

6. Do the roots of the polynomial tell you anything about the graph of the reciprocal of the polynomial? Explain.

7. Find the y-intercept for #1 and #2. What is the y-intercept for #3 and #4?

SET

Topic: Distinguishing between proper and improper rational functions.

Determine if each of the following is a proper or an improper rational function.

8. $$f(x) = \frac{x^3 + 3x^2 + 7}{7x^2 - 2x + 1}$$

9. $$f(x) = x^3 - 5x^2 - 4$$

10. $$f(x) = \frac{3x^2 - 2x + 7}{x^5 - 5}$$

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11. $f(x) = \frac{x^3+4x^2+2x}{10x+7}$

12. $f(x) = \frac{5x^2-4x+4}{7x^5-2x+3}$

13. Which of the above functions have the following end behavior?

$$as\ x \to \infty, f(x) \to 0\ and\ as\ x \to -\infty, f(x) \to 0$$

14. Complete the statement:
ALL *proper* rational functions have end behavior that ______________________________

Determine if each rational expression is proper or improper. If improper, use long division to rewrite the rational expressions such that $\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}$ where $q(x)$ represents the quotient and $r(x)$ represents the remainder.

15. $\frac{2x^3-7x^2+6}{x-1}$

16. $\frac{(x+1)}{(x-2)(x+2)}$

17. $\frac{x^3-3x^2+5x-1}{x^2-4x+4}$

18. $\frac{x^3-5x+2}{x-10}$

GO

Topic: Finding the domain of rational functions that can be reduced

State the domain of the following rational functions.

19. $y = \frac{(x-2)}{(x-2)(x+5)}$

20. $y = \frac{(x+6)}{(x-4)(x+6)}$

21. $y = \frac{(x-7)(x+10)}{(x+10)(x-3)(x-7)}$

a) Each of the previous functions has only one vertical asymptote. Write the equation of the vertical asymptote for #19, #20, and #21 below.

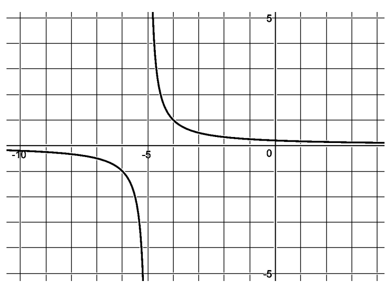
19a) V.A.

20a) V.A.

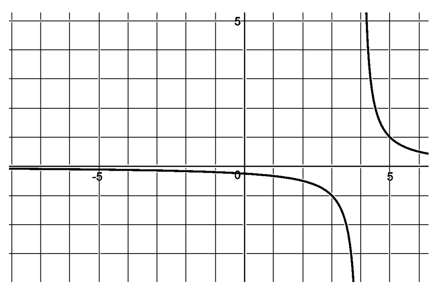
21a) V.A.

b) The graphs of #19, #20, and #21 are below. For each graph, sketch in the vertical asymptote. Put an open circle on the graph anywhere it is undefined.

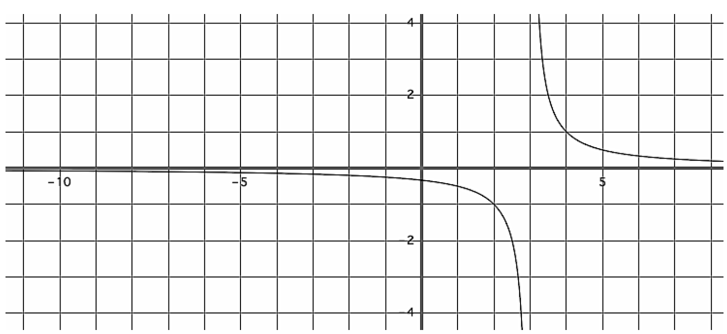
19b)



20b)



21b)



4.5 Just Act Rational

A Solidify Understanding Task

In *4.4 Are You Rational?*, you saw how connecting rational numbers can help us to think about rational functions. In this task, we'll extend that work to consider operations on rational expressions.

Let's begin with multiplication. In each of the tables below there are missing descriptions and missing parts of expressions. Your job is to follow the process for multiplying rational numbers and use it to complete the descriptions of the process and to work the analogous problems with rational expressions.

1.

Description of the Procedure:	**Example Using Numbers**	**Rational Expression A**	**Rational Expression B**
Given:	$\frac{3}{4}\cdot\frac{5}{6}$	$\frac{x(x-3)}{(x+1)}\cdot\frac{5}{x^2}$	$\frac{(x+1)(x-2)}{(x+2)}\cdot\frac{(x+5)}{(x-2)(x+2)}$
	$\frac{3\cdot 5}{4\cdot 6}$		
	$\frac{\cancel{3}\cdot 5}{2\cdot 2\cdot 2\cdot \cancel{3}}$		
Write the simplified form:	$\frac{5}{8}$	$\frac{5(x-3)}{x(x+1)}$ Or $\frac{5x-15}{x(x+1)}$	$\frac{(x+1)(x+5)}{(x+2)^2}$ Or $\frac{x^2+6x+5}{(x+2)^2}$

2. In multiplication, does it matter in which step the simplifying is done? Why?

Now let's try the same process with division.

3. Complete the table below by filling in the missing descriptions or steps for dividing the rational expressions.

Description of the Procedure:	Example Using Numbers	Rational Expression A	Rational Expression B
Given:	$\frac{3}{4} \div \frac{5}{6}$	$\frac{(x-2)}{(x+2)} \div \frac{(x+5)}{x(x+2)}$	$\frac{(x+1)(x-6)}{(x+2)} \div \frac{(x+1)}{(x-3)(x+2)}$
	$\frac{3}{4} \cdot \frac{6}{5}$		
Multiply and simplify:	$\frac{3 \cdot 6}{4 \cdot 5} = \frac{3 \cdot \cancel{2} \cdot 3}{2 \cdot \cancel{2} \cdot 5}$		
	$\frac{9}{10}$	$\frac{x(x-2)}{(x+5)}$ Or $\frac{x^2-2x}{(x+5)}$	$(x-3)(x-6)$

4. How could you check your answer after performing an operation on a pair of rational expressions?

Are you ready for addition? Try it!

5. Complete the table below by filling in the missing descriptions or steps for adding the rational expressions.

Description of the Procedure:	**Example Using Numbers**	**Rational Expression A**
Given	$\frac{2}{3}+\frac{1}{7}$	$\frac{3}{(x+7)}+\frac{4}{(x-4)}$
Determine the factors needed for a common denominator	$\frac{2}{3}\left(\frac{7}{7}\right)+\frac{1}{7}\left(\frac{3}{3}\right)$	$\frac{3}{(x+7)}\left(\frac{x-4}{x-4}\right)+\frac{4}{(x-4)}\left(\frac{x+7}{x+7}\right)$
	$\frac{14}{21}+\frac{3}{21}$	
	$\frac{14+3}{21}$	
Simplify	$\frac{17}{21}$	$\frac{7x+16}{(x+7)(x-4)}$

6. After writing both terms with a common denominator, are they equivalent to the original terms? Explain.

7. Complete the table below by filling in the missing descriptions and steps for adding the rational expressions.

Description of the Procedure:	**Rational Expression B**
Given	$\dfrac{2x-3}{(x+3)}+\dfrac{x+5}{(x-2)}$
Determine the factors needed for a common denominator	
Simplify	$\dfrac{3x^2+x+21}{(x-2)(x+3)}$

8. Is it possible to get an answer to an addition problem that still needs to be reduced? If so, how can you tell if your answer needs to be further simplified?

9. At long last, we have subtraction. Complete the table below by filling in the missing descriptions and steps for adding the rational expressions.

Description of the Procedure:	**Example Using Numbers**	**Rational Expression A**
Given:	$\frac{7}{8} - \frac{3}{5}$	$\frac{3x+1}{(x+5)} - \frac{x-4}{(x-2)}$
	$\frac{7}{8}\left(\frac{5}{5}\right) - \frac{3}{5}\left(\frac{8}{8}\right)$	
	$\frac{35}{40} - \frac{24}{40}$	
	$\frac{35-24}{40}$	
Simplify	$\frac{11}{40}$	$\frac{2x^2-6x+18}{(x+5)(x-2)}$ Or $\frac{2(x^2-3x+9)}{(x+5)(x-2)}$

10. What strategies will you use to be sure that you don't make sign errors when subtracting?

READY, SET, GO! Name Period Date

READY

Topic: Recalling trigonometric functions

Use the given triangle to write the values of $sinA, cosA, and\ tanA$ and $sinB, cosB, and\ tanB$.

1.

sin A =

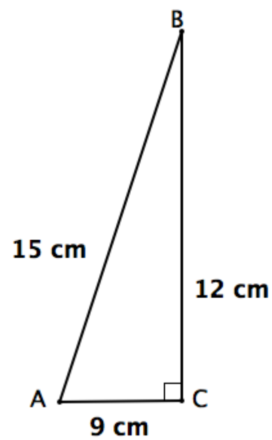
cos A =

tan A =

sin B =

cos B =

tan B =



2.

sin A =

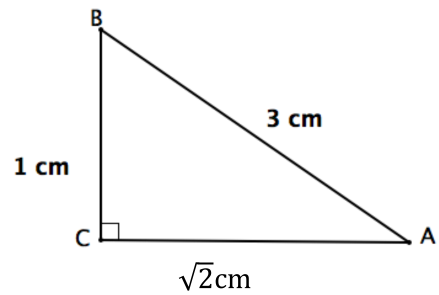
cos A =

tan A =

sin B =

cos B =

tan B =



3.

sin A =

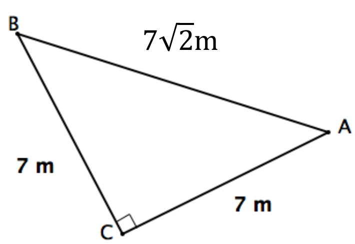
cos A =

tan A =

sin B =

cos B =

tan B =



4.

sin A =

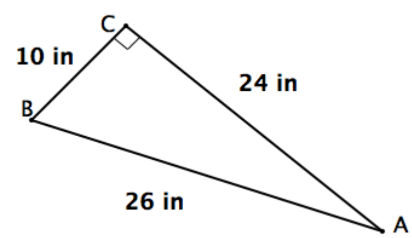
cos A =

tan A =

sin B =

cos B =

tan B =



SET

Topic: Adding, subtracting, multiplying, and dividing rational functions

5. Angela simplified the following rational expressions. Only one of the three problems is correct. Determine which one she answered correctly. Then identify Angela's errors in the two that are incorrect and correct them.

a. $\frac{5x}{(x-3)}+\frac{2}{(x-1)}$

$$\frac{5x(x-1)}{(x-3)(x-1)}+\frac{2(x-3)}{(x-3)(x-1)}$$

$$\frac{5x^2-x+2x-3}{(x-3)(x-1)}$$

$$\frac{5x^2+x-3}{(x-3)(x-1)}$$

b. $\frac{x}{(x+3)}-\frac{4(x+3)}{(x-1)}$

$$\frac{x}{1}-\frac{4}{(x-1)}$$

$$\frac{x(x-1)}{(x-1)}-\frac{4}{(x-1)}$$

$$\frac{x^2-x-4}{(x-1)}$$

c. $\frac{(x+1)(x-2)}{(x+2)}\times\frac{(x+5)}{(x-2)(x+2)}$

$$\frac{(x+1)(x-2)(x+5)}{(x+2)(x-2)(x+2)}$$

$$\frac{(x+1)(x+5)}{(x+2)(x+2)}$$

$$\frac{x^2+6x+5}{x^2+4x+4}$$

Simplify each expression. Reduce when possible.

6. $\frac{2x+6}{(x+1)}-\frac{4}{(x+1)}$

7. $\frac{2x}{x+2}+\frac{x-1}{x-5}$

8. $\frac{x^2+6x+8}{x^2-5x+4}\cdot\frac{x^2+3x-4}{x^2+4x+4}$

9. $\frac{4x+8}{5x-20}\div\frac{x^2-3x-10}{x^2-4x}$

10. $\frac{2x}{(x^2-4)}+\frac{4}{(x+2)}$

11. $\frac{x-10}{x-4}-\frac{x+2}{4-x}$

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Topic: Comparing rational numbers and rational expressions

12. Rational numbers and rational expressions are comparable because they have similar features. Complete the table below by writing the comparable situation for each statement written.

Rational Numbers	**Rational Expressions**
Whole numbers are rational numbers with a denominator of one.	a)
b)	Rational expressions are undefined when the denominator is equal to zero.
When you add, subtract, multiply or divide two rational numbers, the result is also a rational number.	c)
Rational fractions are classified as proper fractions when the numeric value of the numerator is smaller than the denominator.	d)

GO

Topic: Finding values of x that affect the domain of a rational expression

Identify the values of *x* for which the expression is undefined, if any.

13. $\frac{10}{x-4}$

14. $\frac{22}{x}$

15. $\frac{x-7}{x+15}$

16. $\frac{2x}{5}$

4.6 Sign on the Dotted Line

A Practice Understanding Task

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Josue and Francia are working on graphing all kinds of rational functions when they have this little dialogue:

Josue: It's easy to figure out where the asymptotes and intercepts are on a rational function.

Francia: Yes, and it's almost like the asymptotes split the graph into sections. All you need to know is what the graph is doing in each section.

Josue: It seems almost easier than that. It's like all you need to know is whether it's going up or down either side of the vertical asymptote and then use logic to figure it out from there.

Francia: Don't overlook the intercepts. They give some pretty important clues.

Josue: Yeah, yeah. I wonder if we can figure out an easy way to determine the behavior near the asymptotes.

Francia: Seems easy enough to just plug in numbers and see what the outputs are, but maybe you don't even need exact values. Hmmm. We need to think about this.

Josue and Francia are definitely on to something. Everyone wants to find a way to be able to predict and sketch graphs easily. In this task, you're going to work on just that. **Start by finding asymptotes and intercepts, then figure out a strategy that you can use every time to quickly sketch the graph. After using your strategy to graph the function, use technology to check your work and refine your strategy.**

The examples you need to develop your strategy are on the following pages. Some of the functions given need to be combined and/or simplified to make one rational function. If this is the case, write the simplified function in the space next to the graph.

1.

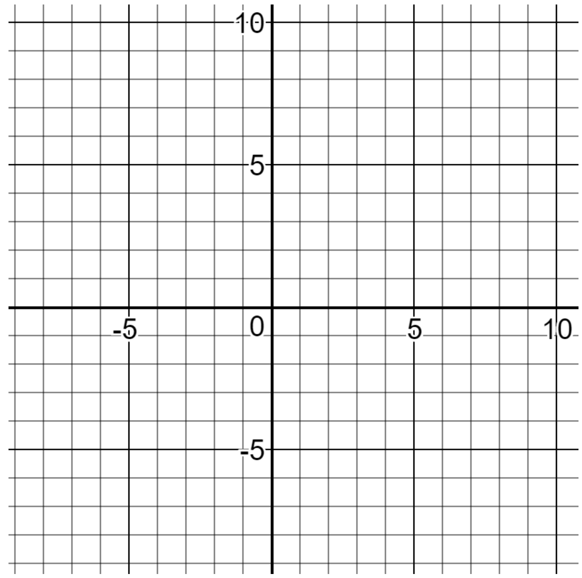
$$y = \frac{(x-5)(x+1)}{(x+2)(x-2)}$$

Vertical Asymptote(s) ________________

Horizontal or Slant Asymptote ____________

Intercepts ____________________________

Graph:



2.

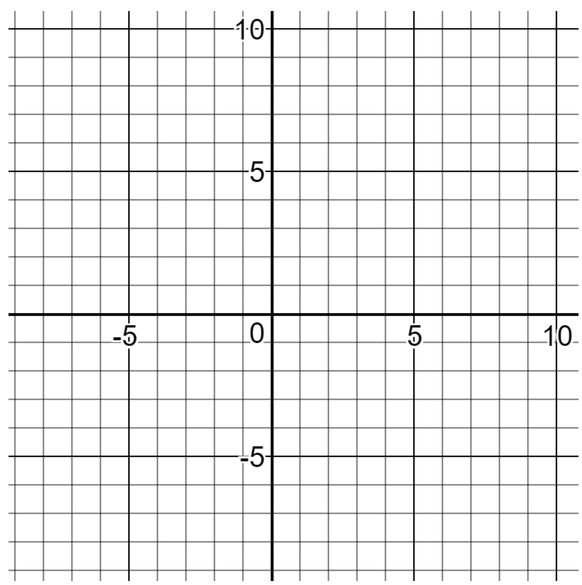
$$y = \frac{(x-3)}{(x+1)} \cdot \frac{x}{(x-4)}$$

Vertical Asymptote(s) ________________

Horizontal or Slant Asymptote ____________

Intercepts ____________________________

Graph:



3.

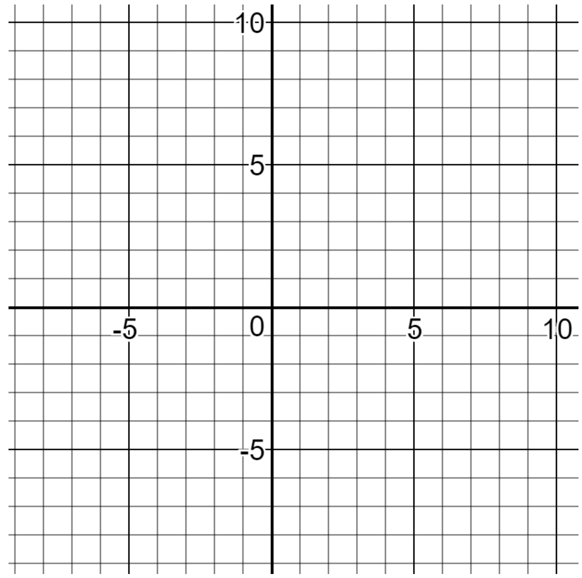
$$y = \frac{x^2 - 6x + 2}{(x - 2)}$$

Vertical Asymptote(s) ____________________

Horizontal or Slant Asymptote ______________

Intercepts ____________________________________

Graph:



4.

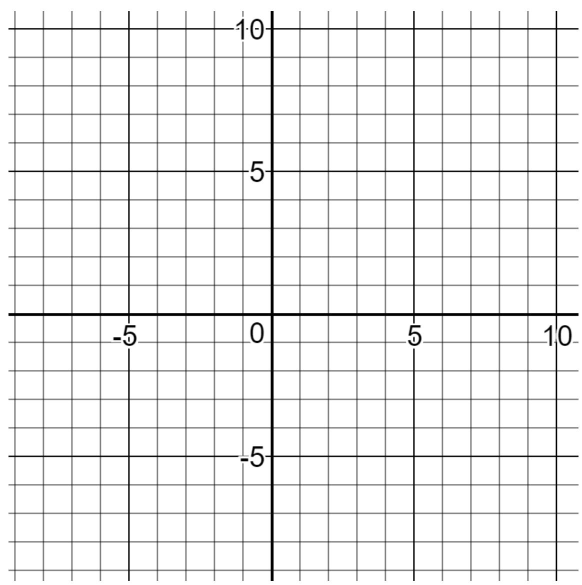
$$y = \frac{4}{(x + 1)} + \frac{(x - 5)}{(x - 3)}$$

Vertical Asymptote(s) ____________________

Horizontal or Slant Asymptote ______________

Intercepts ____________________________________

Graph:



5.

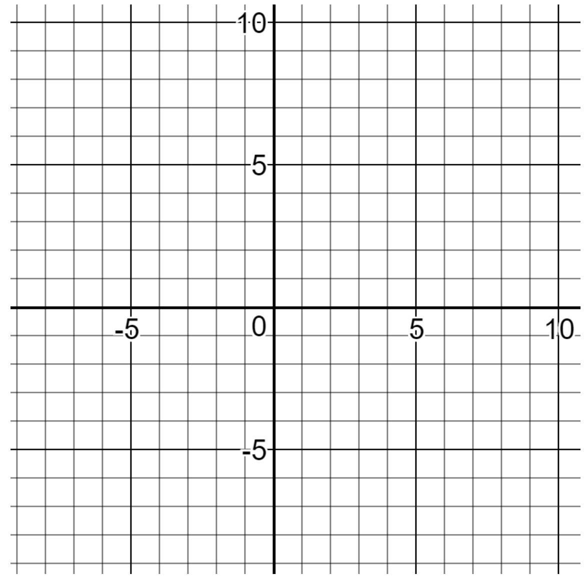
$$y = \frac{3x}{(x^2 + 2x + 1)} \div \frac{x-3}{x+1}$$

Vertical Asymptote(s) ____________________

Horizontal or Slant Asymptote ________________

Intercepts ____________________________________

Graph:



6.

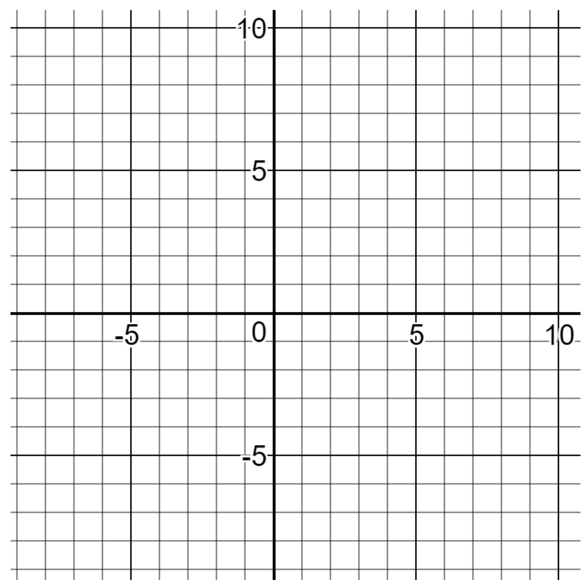
$$y = \frac{x+5}{x+4} - \frac{x+2}{x-1}$$

Vertical Asymptote(s) ____________________

Horizontal or Slant Asymptote ________________

Intercepts ____________________________________

Graph:



7.

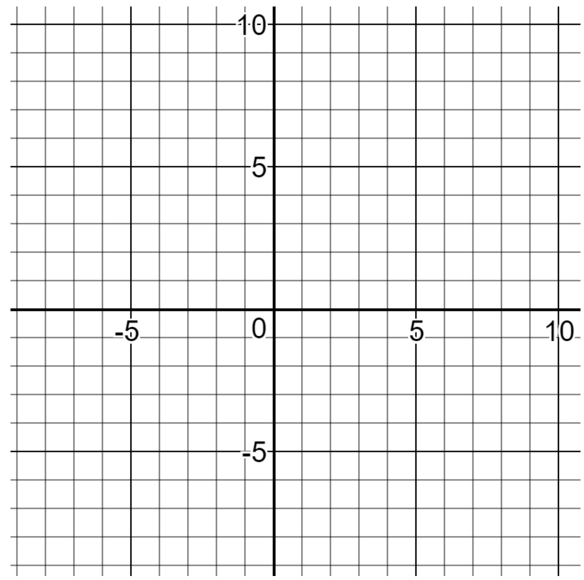
$$y = \frac{2x^2 + x - 15}{x^2 + 4x + 3}$$

Vertical Asymptote(s) ________________

Horizontal or Slant Asymptote _____________

Intercepts ______________________________

Graph:



8. Summarize your strategy for graphing rational functions with a step-by-step process.

READY, SET, GO! Name Period Date

READY

Topic: Identifying extraneous solutions

1. Below is the work done to solve a rational equation. The problem has been worked correctly. Explain why the equation has only **one** solution.

Solve: $\frac{2}{x^2-2x} - \frac{1}{x-2} = 1$	
$\frac{2}{x(x-2)} - \frac{(x)1}{(x)(x-2)} = 1$	*Write using a common denominator.*
$\frac{2-x}{(x)(x-2)} = 1$	*Subtract.*
$(x)(x-2)\frac{2-x}{(x)(x-2)} = 1(x)(x-2)$	*Multiply both sides by the common denominator.*
$2 - x = x^2 - 2x$	*Simplify.*
$x^2 - x - 2 = 0$	*Write a quadratic equation in standard form.*
$(x-2)(x+1) = 0$	*Factor*
$x = 2 \text{ or } x = -1$	*Apply the Zero-Product Property and solve for z.*
	Substitute 2 and -1 into the original equation to see if the numbers are solutions.

Substitute the given numbers into the given equation. Identify which are actual solutions and which, if any, are extraneous.

2. $a: -1 \text{ and } \frac{5}{2}$ $a - \frac{3}{2a+1} = 2$	3. d: *0 and 3* $\frac{3d}{d^2-d} - \frac{1}{d-1} = 1$	4. $m: 1$ $\frac{1}{m^2-m} - \frac{1}{m-1} = 0$
Solve 5 and 6. Watch for extraneous solutions.	5. $\frac{1}{x^2-x} - \frac{1}{x-1} = \frac{1}{2}$	6. $2x + \frac{3}{x+2} = 1$

SET

Topic: Predicting and sketching rational functions

Find the vertical asymptote(s), horizontal or slant asymptote, and intercepts. Then sketch the graph. **(**Do not use technology to get the graph. The max and mins do not need to be accurate.)

5.

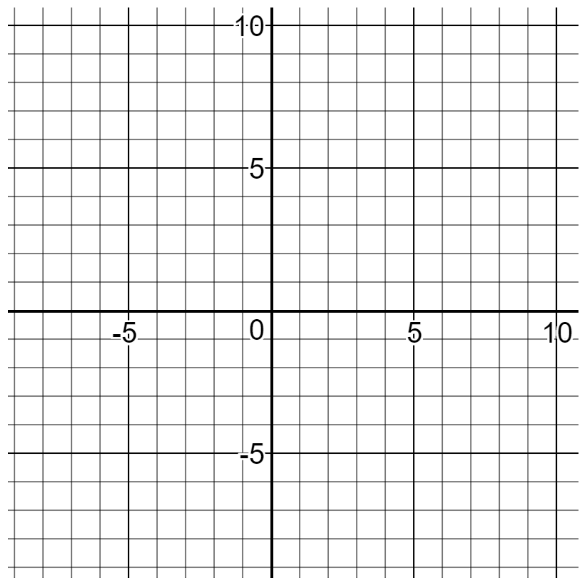
$$y = \frac{(x+4)}{(-2x-6)}$$

Vertical Asymptote(s) ___________________

Horizontal or Slant Asymptote _______________

Intercepts __________________________________

Graph:



6.

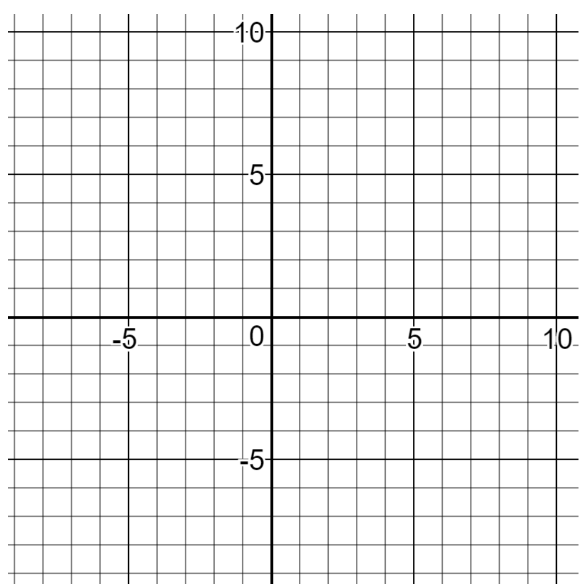
$$y = \frac{3x}{(x-3)} \cdot \frac{(x-4)}{(x+1)}$$

Vertical Asymptote(s) ___________________

Horizontal or Slant Asymptote _______________

Intercepts __________________________________

Graph:



7.

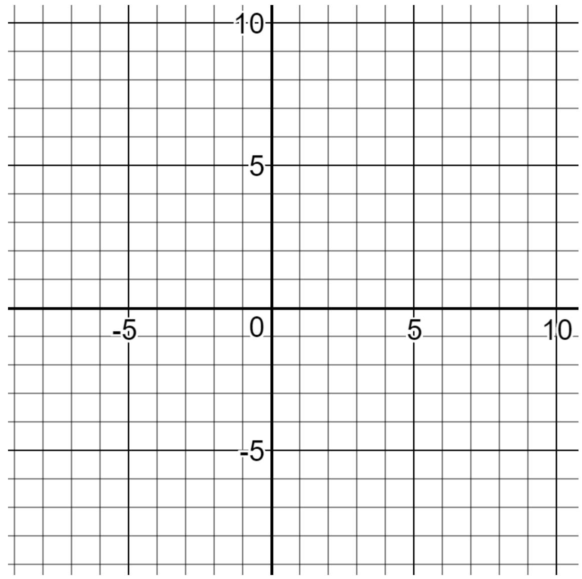
$$y = \frac{(x^2 - 4x)}{(4x - 8)} \div \frac{(x + 2)}{x + 4}$$

Vertical Asymptote(s) ____________________

Horizontal or Slant Asymptote ______________

Intercepts ___________________________________

Graph:



8.

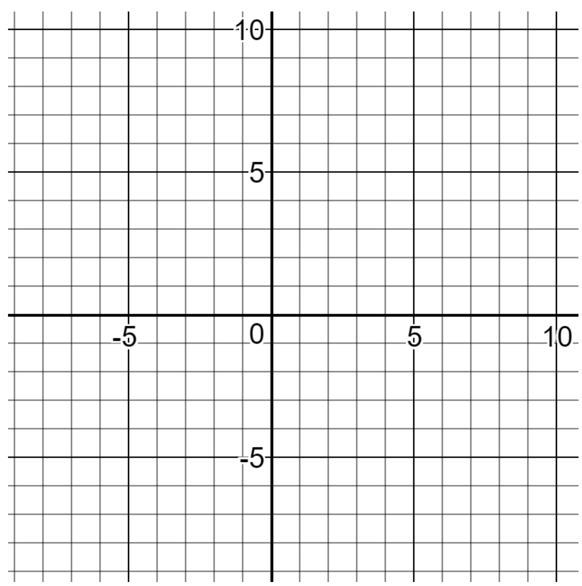
$$y = \frac{(x - 6)}{(x - 3)} + \frac{(x + 3)}{x^2 - 6x + 9}$$

Vertical Asymptote(s) ____________________

Horizontal or Slant Asymptote ______________

Intercepts ___________________________________

Graph:



GO

Topic: Exploring linear equations

9. What value of k in the equation $kx + 10 = 6y$ would give a line with slope -3?

10. What value of k in the equation $kx - 12 = -15y$ would give a line with slope $\frac{2}{5}$?

11. The standard form of a linear equation is $Ax + By = C$. Rewrite this equation in slope – intercept form. What is the slope? What is the y – intercept?

12. If b is the y – intercept of a linear function whose graph has slope m, then $y = mx + b$ describes the line. Below is an incomplete justification of this statement. Fill in the missing information.

Statements	**Reasons**
1. $m = \frac{y_2 - y_1}{x_2 - x_1}$	1. slope formula
2. $m = \frac{y-b}{x-0}$	2. By definition, if b is the y – intercept, then $(\quad, b)$ is a point on the line. (x, y) is any other point on the line.
3. $m = \frac{y-b}{x}$	3. ?
4. $m \cdot \quad = y - b$	4. Multiplication Property of Equality (Multiply both sides of the equation by x.)
5. $mx + b = y, or\ y = mx + b$	5. ?

4.7 We All Scream for Ice Cream

A Practice Understanding Task

CC BY Ruth Hartnup https://flic.kr/p/V6WqMD

The Glacier Bowl is an enormous ice cream treat sold at the neighborhood ice cream parlor. It is so large that any person who can eat it within 30 minutes gets a t-shirt and his picture posted on a wall. Because the Glacier Bowl is so big, it costs $60 and most people split the treat with a group.

Amera and some of her friends are planning to get together to share the bowl of ice cream. They plan to split the cost between them equally.

1. What is an algebraic expression for the amount that each person in the group will pay?

2. At the last minute, one of the friends couldn't go with the group. Write an expression that represents the amount that each person in the group now pays.

3. It turns out that each person in the group had to pay $2 more than they would have if everyone in the original group had shared the ice cream. How many people were in the original group?

4. Explain why your answer(s) makes sense in this situation.

This story and the problem it represents provides an opportunity to model a situation that requires a rational equation. Rational equations can take many forms, but they are solved using principles we have worked with before. Try applying some of the strategies for working with rational expressions that we have used in this module to solve these equations.

5. $\frac{2}{x+4} - \frac{1}{x} = \frac{2}{3x}$

6. $\frac{2x-3}{x+1} = \frac{x+6}{x-2}$

7. $x + \frac{20}{x-4} = \frac{5x}{x-4} - 2$

8. $\frac{x}{x+3} - \frac{4}{x-2} = \frac{-5x^2}{x^2+x-6}$

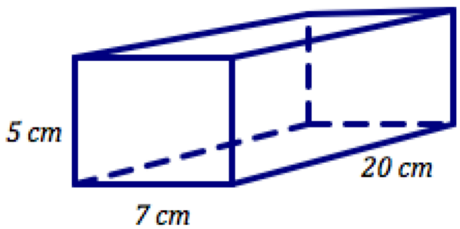
READY, SET, GO! Name Period Date

READY

Topic: Calculating volume, surface area and solving right triangles

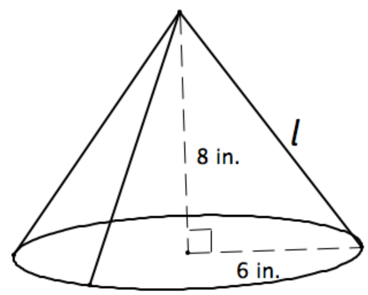
Find the indicated values for the geometric figures below.

1. Volume:
   Surface Area:



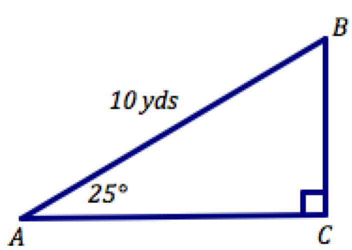
rectangular prism

2. Volume: $V = \frac{1}{3}bh$
   Surface Area: $S_A = \pi r(l + r)$
   where l is the lateral height.

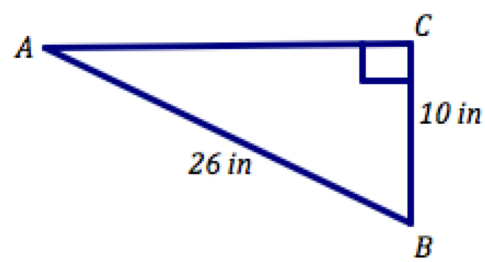


right cone

3. Solve the right triangle.
   $\angle B =$
   $\overline{AC} =$
   $\overline{BC} =$

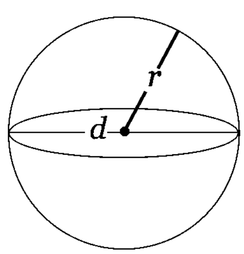


4. Solve the right triangle.
   $\angle B =$
   $\angle A =$
   $\overline{AC} =$



5. Volume: $V = \frac{4}{3}\pi r^3$
   Surface area: $S_A = 4\pi r^2$

   $r \approx 3959$ *miles*

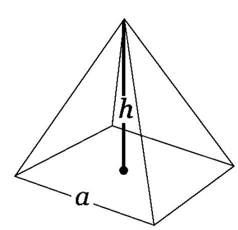


sphere
This is the radius of the earth.

6. Volume: $V = a^2 \frac{h}{3}$
   Surface area: $S_A = a^2 + 2a\sqrt{\frac{a^2}{4} + h^2}$

   $h = 147\ m$

   $a = 230m$



right square pyramid
These are the dimensions of the great pyramid of Giza.

Need help? Visit **www.rsgsupport.org**

SET

Topic: Solving rational equations

Solve each equation. Identify extraneous solutions.

7. $x + \frac{2}{x} = 3$

8. $\frac{x}{2} - \frac{1}{3x} = \frac{1}{6}$

9. $2x + \frac{3}{x+2} = 1$

10. $\frac{2}{x^2-2x} - \frac{1}{x-2} = 1$

11. $3x - \frac{1}{2x-1} = 4$

12. $\frac{2x}{x^2+3x} - \frac{2}{x+3} = \frac{2}{x}$

Topic: Using work and rate relationships to solve problems

13. Channing takes twice as long as Dakota to complete a school project. It takes them 15 hours to complete the project together. How long would it take each student to complete the project if he works alone?

14. A print shop can print the MVP math book in 24 minutes if both of their print machines are working together to do the job. If a print machine is working alone, the job takes longer. Machine A can print the book 20 minutes faster than machine B. How long does it take each machine to print the book?

15. The problem in #14 generates an extraneous solution, even though neither solution makes a denominator equal zero. What is another reason for having an extraneous solution?

GO

Topic: Simplifying rational expressions

(10 – 11) Reduce to simplest form. (12 – 15) Perform the indicated operations. Reduce each of your answers to its simplest form. (Assume all denominators ≠ 0)

16.
$$\frac{x^2+8x+12}{x^2+3x-18}$$

17.
$$\frac{x^2-3x-40}{x^2-11x+24}$$

18.
$$\frac{x^2+8x+12}{x^2+3x-18}+\frac{x^2-3x-40}{x^2-11x+24}$$

19.
$$\frac{x^2+5x-36}{(x-4)}\cdot\frac{3(x+2)}{x+9}$$

20.
$$\frac{4}{x^2-4}-\frac{1}{x-2}$$

21.
$$\frac{x^2-2x-3}{x+1}\div\frac{x-3}{5}$$