mathematics vision project

Transforming Mathematics Education

SECONDARY MATH THREE

An Integrated Approach

MODULE 5

Modeling With Geometry

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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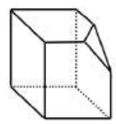
5.1 Any Way You Slice It

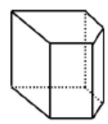
A Develop Understanding Task

Students in Mrs. Denton's class were given cubes made of clay and asked to slice off a corner of the cube with a piece of dental floss.

Jumal sliced his cube this way.



Jabari sliced his cube like this.



1. Which student, Jumal or Jabari, interpreted Mrs. Denton's instructions correctly? Why do you say so?

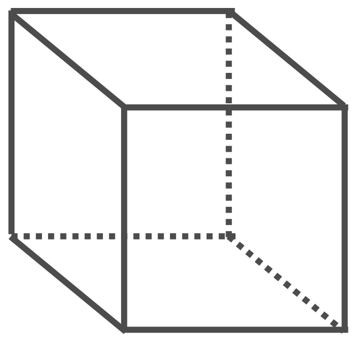
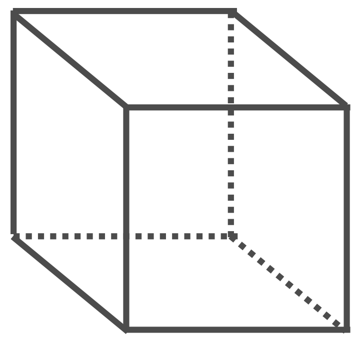
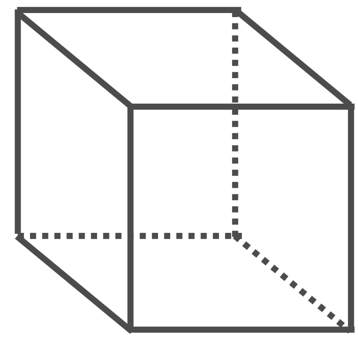
When describing three-dimensional objects such as cubes, prisms or pyramids we use precise language such as *vertex*, *edge* or *face* to refer to the parts of the object in order to avoid the confusion that words like "corner" or "side" might create.

A **cross section** is the face formed when a three-dimensional object is sliced by a plane. It can also be thought of as the intersection of a plane and a solid.

2. Draw and describe the cross section formed when Jumal sliced his cube.

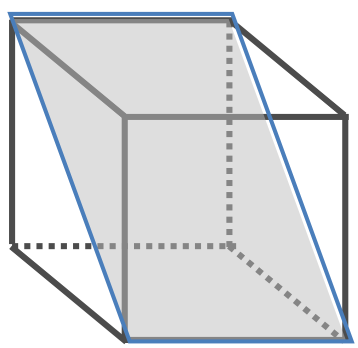
3. Draw and describe the cross section formed when Jabari sliced his cube.

4. Draw some other possible cross-sections that can be formed when a cube is sliced by a plane.

5. What type of quadrilateral is formed by the intersection of the plane that passes through diagonally opposite edges of a cube?

   Explain how you know what quadrilateral is formed by this cross section.



Cross sections can be visualized in different ways. One way is to do what Jumal and Jabari did—cut a clay model of the solid with a piece of dental floss. Another way is to partially fill a clear glass or plastic model of the three-dimensional object with colored water and tilt it in various ways to see what shapes the surface of the water can assume.

Experiment with various ways of examining the cross sections of different three-dimensional shapes.

6. Partially fill a cylindrical jar with colored water, and tilt it in various ways. Draw the cross sections formed by the surface of the water in the jar.

7. Try to imagine a cubical jar partially filled with colored water, and tilted in various ways. Which of the following cross sections can be formed by the surface of the water? Which are impossible?

- a square
- a rhombus
- a rectangle
- a parallelogram
- a trapezoid
- a triangle
- a pentagon
- a hexagon
- an octagon
- a circle

READY, SET, GO! Name Period Date

READY

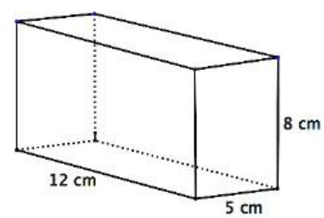
Topic: Comparing perimeter, area and volume

Solve each of the following problems. Make certain you label the units on each of your answers.

1. Calculate the perimeter of a rectangle that measures 5 cm by 12 cm.

2. Calculate the area of the same rectangle.

3. Calculate the volume of a rectangular box that measures 5 cm by 12 cm. and is 8 cm. deep.



4. Look back at problems 1 – 3. Explain how the units change for each answer.

5. Calculate the surface area for the box in problem 3. Assume it does NOT have a cover on top. Identify the units for the surface area. How do you know your units are correct?

6. Calculate the circumference of a circle if the radius measures 8 inches. (Use $\pi = 3.14$)

7. Calculate the area of the circle in problem 6.

8. Calculate the volume of a ball with a diameter of 16 inches. $\left(V = \frac{4}{3}\pi r^3\right)$

9. Calculate the surface area of the ball in problem 8. $(SA = 4\pi r^2)$

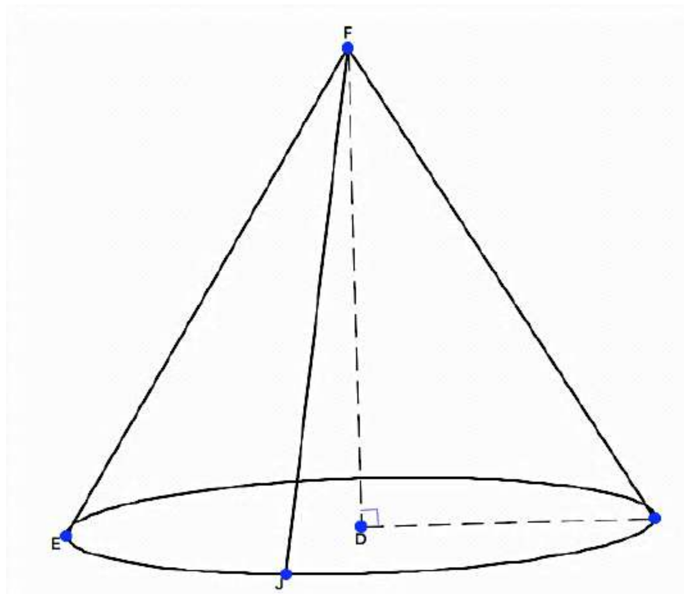
10. If a measurement were given, could you know if it represented a perimeter, an area, or a volume? Explain.

11. In the problems above, which type of measurement would be considered a "linear measurement?"

SET

Topic: Examining the cross sections of a cone

Consider the intersection of a plane and a cone.



12. If the plane were parallel to the base of the cone, what would be the shape of the cross-section?
    Can think of 2 possibilities? Explain.

13. How would a plane need to intersect the cone so that it would create a parabola?

14. Describe how the plane would need to intersect the cone in order to get a cross-section that is a triangle. Would the triangle be scalene, isosceles, or equilateral? Explain.

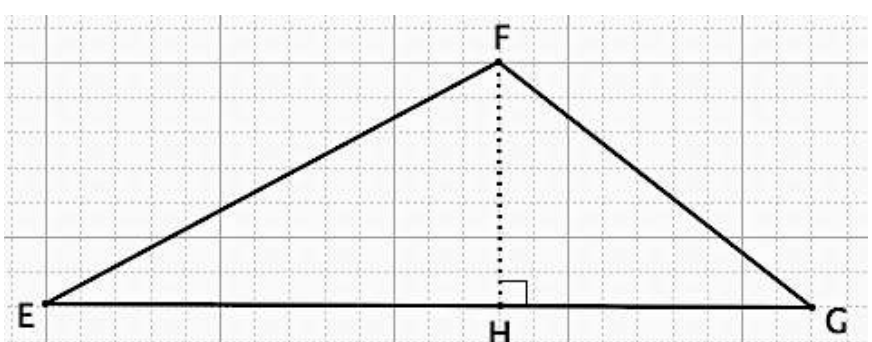
15. Would it be possible for the intersection of a plane and a cone to be a line? Explain.

GO

Topic: Finding the area of a triangle

Calculate the area of triangle EFG in each exercise below.

16.



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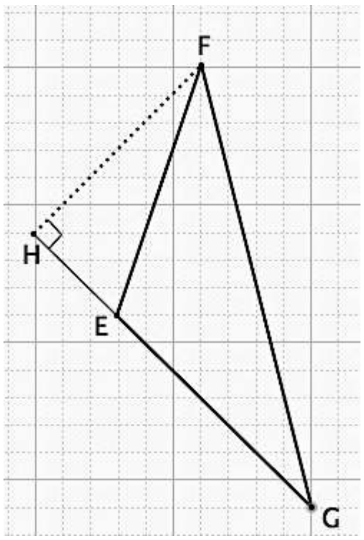

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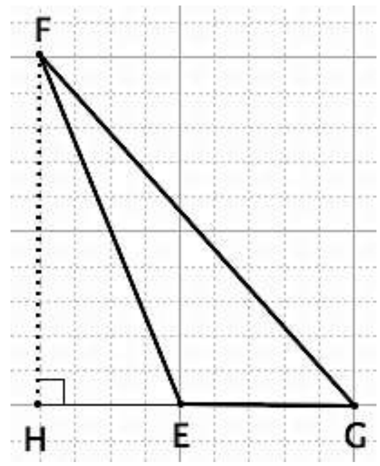
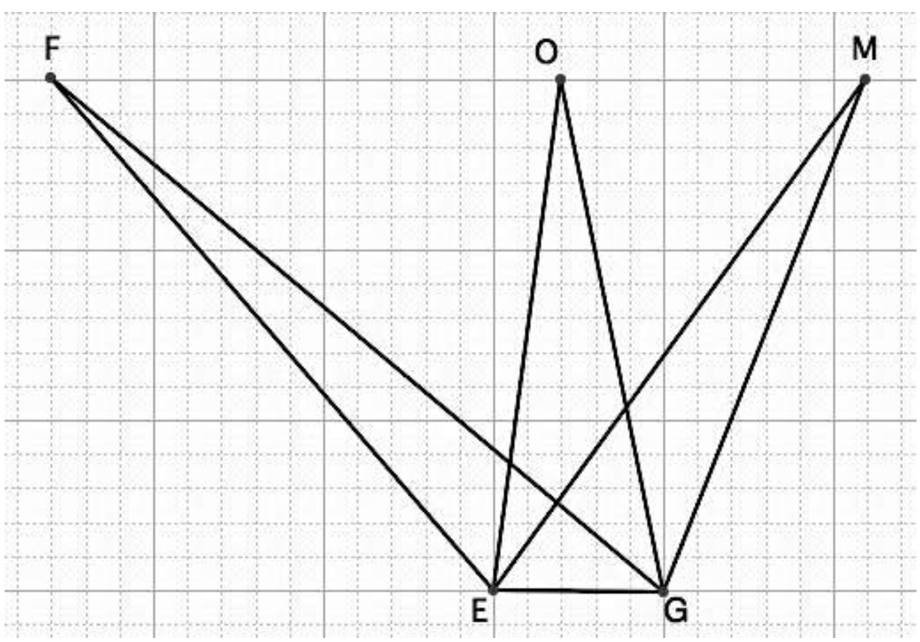


19. Calculate the areas of $\Delta EFG, \Delta EOG, and\ \Delta EMG$. Justify your answers.



5.2 Any Way You Spin It

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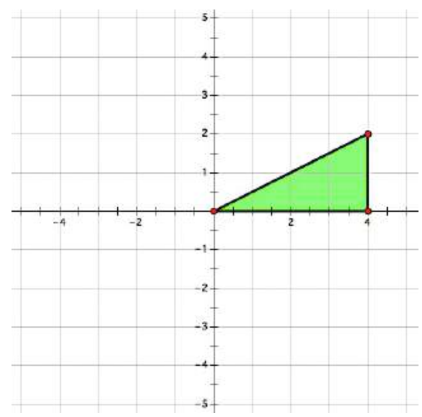
A Develop Understanding Task

Perhaps you have used a pottery wheel or a wood lathe. (A lathe is a machine that is used to shape a piece of wood by rotating it rapidly on its axis while a fixed tool is pressed against it. Table legs and wooden pedestals are carved on a wood lathe). You might have played with a spinning top or watched a figure skater spin so rapidly she looked like a solid blur. The clay bowl, the table leg, the rotating top and the spinning skater—each of these can be modeled as **solids of revolution—***a three dimensional object formed by spinning a two dimensional figure about an axis.*

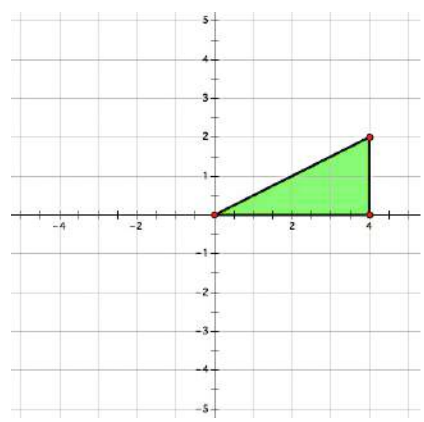
Suppose the right triangle shown below is rotating rapidly about the x-axis. Like the spinning skater, a solid image would be formed by the blur of the rotating triangle.

1. Draw and describe the solid of revolution formed by rotating this triangle about the x-axis.



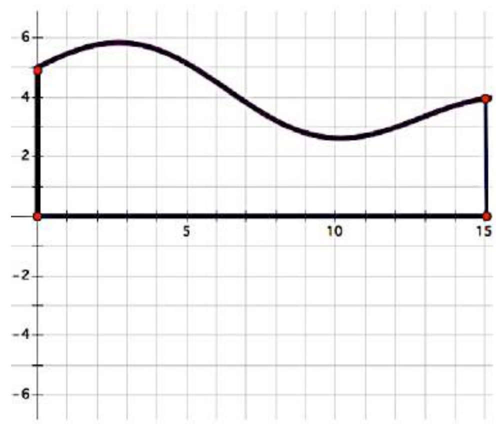
2. Find the volume of the solid formed.

3. What would this figure look like if the triangle rotates rapidly about the y-axis? Draw and describe the solid of revolution formed by rotating this triangle about the y-axis.

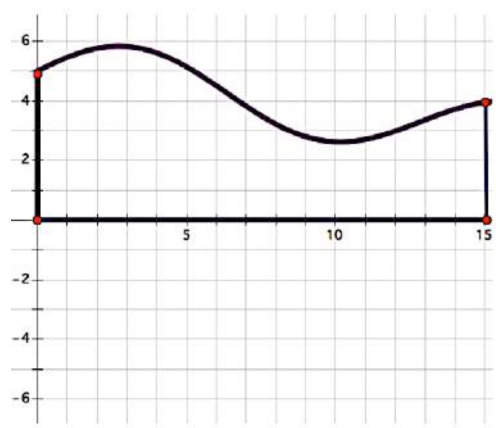


4. Find the volume of the solid formed.

5. What about the following two-dimensional figure? Draw and describe the solid of revolution formed by rotating this figure about the x-axis.



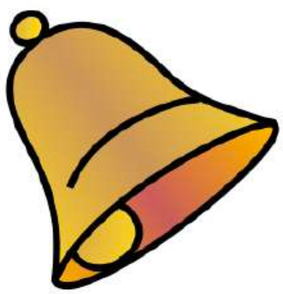
6. Draw a cross section of the solid of revolution formed by this figure if the plane cutting the solid is the plane containing the coordinate axes.



7. Draw some cross sections of the solid of revolution formed by the figure above if the planes cutting the solid are perpendicular to the plane containing the coordinate axes. Draw the cross sections when the intersecting planes are located at x = 5, x = 10 and x = 15.

So, why are we interested in solids that don't really exist—after all, they are nothing more than a blur that forms an image of a solid in our imagination. Solids of revolution are used to create mathematical models of real solids by describing the solid in terms of the two-dimensional shape that generates it.

8. For each of the following solids, draw the two-dimensional shape that would be revolved about the *x*-axis to generate it.



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http://openclipart.org/detail/139759/r-is-for-rocket-by-marauder

READY, SET, GO! Name Period Date

READY

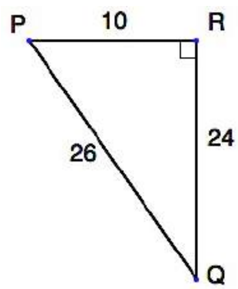
Topic: Finding the trigonometric ratios in a right triangle

Use the given measures on the triangles to write the indicated trig value.

1. $\sin P =$

   $\cos P =$

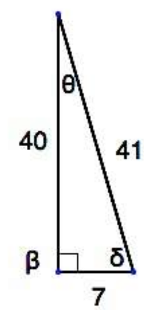
   $\tan P =$



2. $\sin \theta =$

   $\cos \theta =$

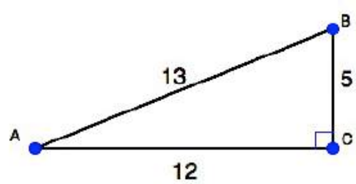
   $\tan \theta =$



3. $\sin B =$

   $\cos B =$

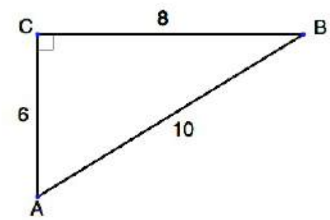
   $\tan B =$



4. $\sin A =$

   $\cos A =$

   $\tan A =$

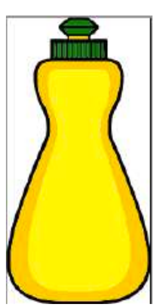


SET

Topic: Drawing solids of revolution

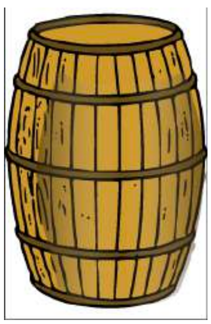
For each of the following solids, draw the two-dimensional shape that would be revolved about the x-axis to generate it.

5.



6.

7.



8.

Images used above:

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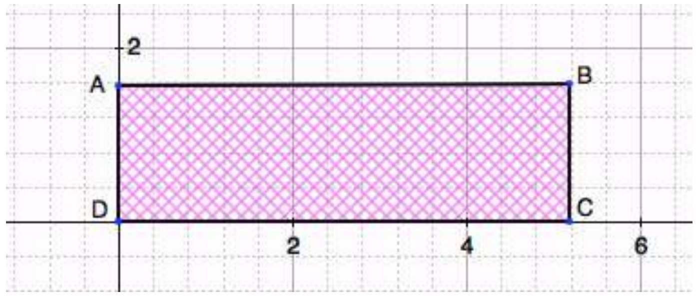
http://openclipart.org/detail/190246/snow-man-with-hat-by-tiothy-190246

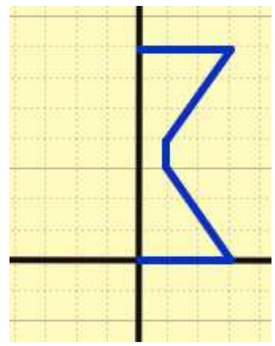
http://openclipart.org/detail/167894/barrel-rendered-by-kevie

http://openclipart.org/detail/1638/flashlight-by-johnny_automatic

9. Name something in your house that would be shaped like the solid of revolution formed, if the figure on the right were rotated about the x-axis.



10. Name something in the world that would be shaped like the solid of revolution formed if the figure on the right were rotated about the y-axis.



GO

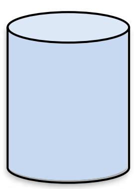
Topic: Using formulas to find the volume of a solid

Find the volume of the indicated solid.

11. $V = \pi r^2 h$ *cylinder*

$r =$ 3 inches

$h =$ 10 inches



12. $V = \frac{1}{3}BH$ *right circular cone*

$r = 8\ cm$

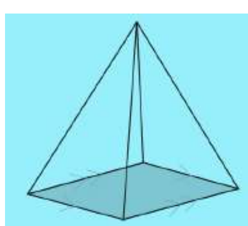
$H = 20\ cm$

13. $V = \frac{1}{3}l^2 h$ *square pyramid*

$h = \dfrac{5\sqrt{2}}{2}\ m$

$l = 3\sqrt{5}\ m$

The base is a square.



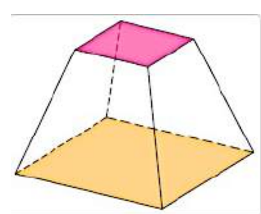
14. $V = \frac{1}{3}h(a^2 + ab + b^2)$ *square frustum*

Where a and b are the base and top side lengths and h is the height

$h = 12\ in$

$a = 5\sqrt{7}\ in$

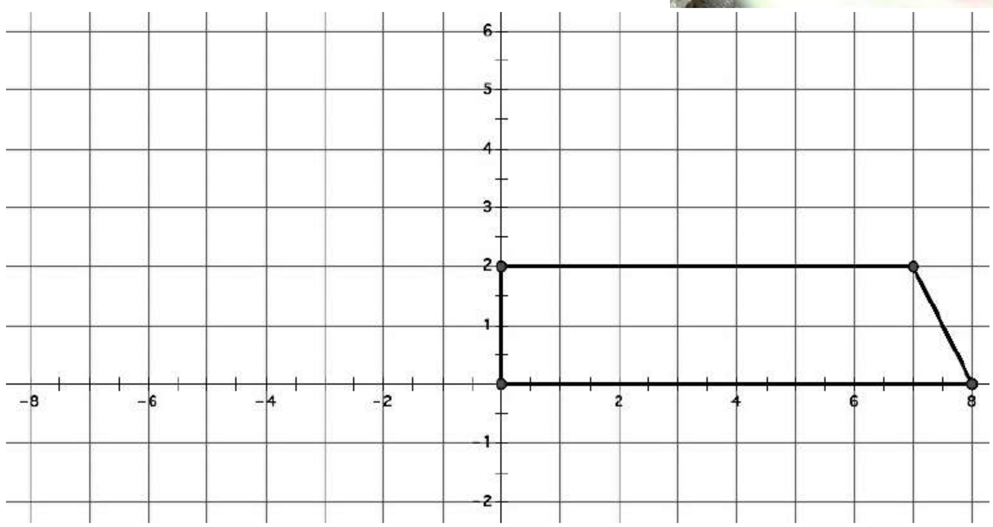
$b = 2\sqrt{7}\ in$



5.3 Take Another Spin

A Solidify Understanding Task

The trapezoid shown below is revolved about the y-axis to form a frustum (e.g., bottom slice) of a cone.

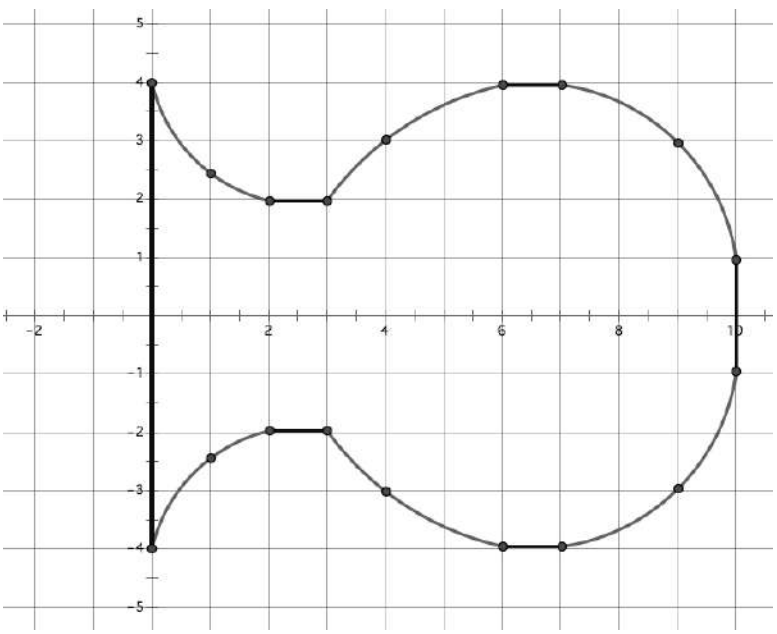


1. Draw a sketch of the three-dimensional object formed by rotating the trapezoid about the y-axis.

2. Find the volume of the object formed. Explain how you used the diagram to help you find the volume.

You have made use of the formulas for cylinders and cones in your work with solids of revolution. Sometimes a solid of revolution cannot be decomposed exactly into cylinders and cones. We can approximate the volume of solids of revolution whose cross sections include curved edges by replacing them with line segments.

3. The following diagram shows the cross section of a flower vase. Approximate the volume of the vase by using line segments to approximate the curved edges. (Show the line segments you used to approximate the figure on the diagram.)



4. Describe and carry out a strategy that will improve your approximation for the volume of the vase.

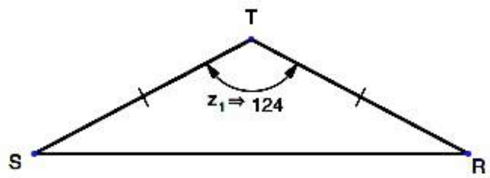
READY, SET, GO! Name Period Date

READY

Topic: Finding missing angles in a triangle

Use the given information and what you know about triangles to find the missing angles.
(All angle measures are in degrees.)

1.



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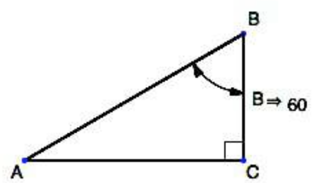

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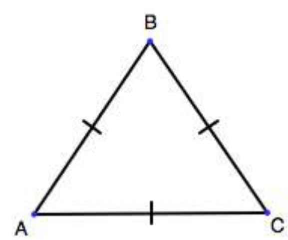

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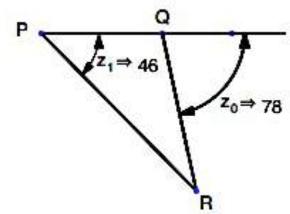

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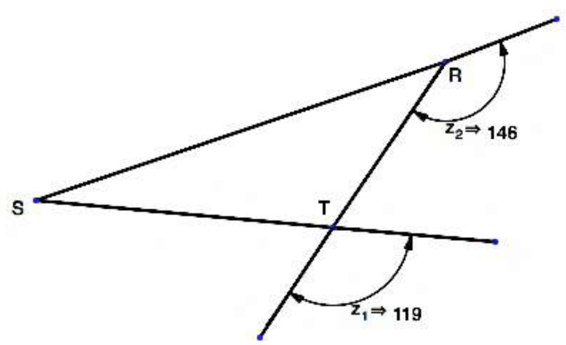
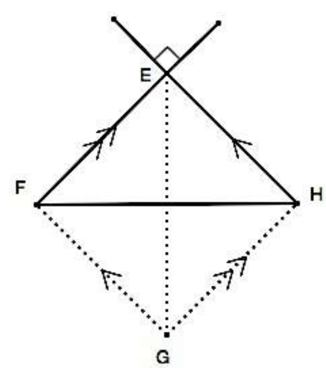


6. $\overline{EG} \cong \overline{FH}$



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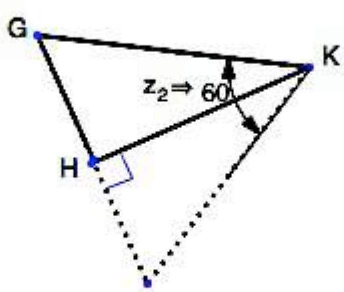

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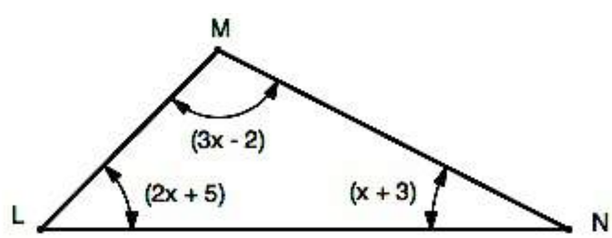


p? Visit www

SET

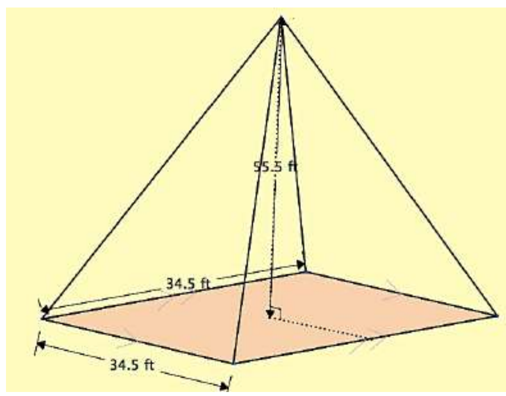
Topic: Calculating the surface area and volumes of combined shapes.

Answer the following questions about the Washington Monument.

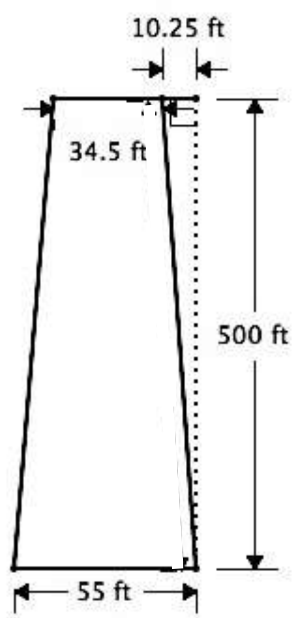
https://flic.kr/p/axqjPQ

The picture at the right is of the Washington Monument in DC. The shaft of the monument is a square frustum. The bottom square measures 55 ft. on a side and the top square measures 34.5 feet. The top is a square pyramid.

9. Find the dimensions of the 4 triangular faces of the pyramid. (Height is 55.5 ft)



10. Find the area of each face of the pyramid.



11. Find the area of the 4 trapezoids that make the faces of the frustum.

    The area of a trapezoid: $A = \frac{b_1+b_2}{2}h$

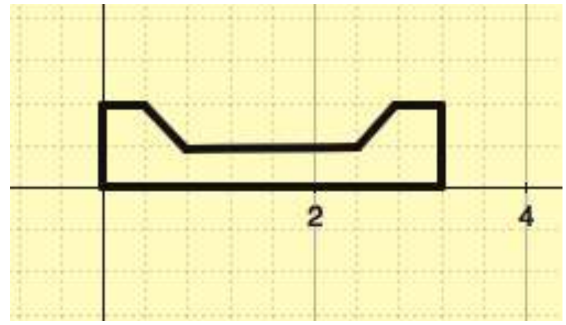
12. Find the total surface area of the Washington Monument.

13. Find the total volume of the Washington Monument.

    Volume of a square frustum: $V = \frac{1}{3}h(a^2 + ab + b^2)$ where a and b are the side lengths of each square.

    Volume of pyramid: $V = \frac{1}{3}l^2h$

14. Draw a sketch of the three-dimensional object formed by rotating the figure about the *x*-axis.

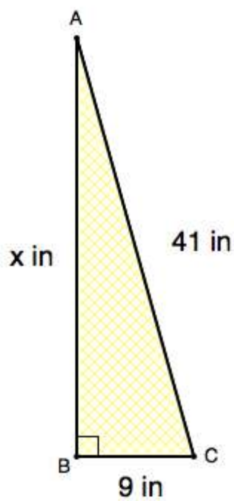


GO

Topic: Solving for missing sides in a right triangle

Calculate the missing sides in the right triangle. Give your answers in simplified radical form.

15.



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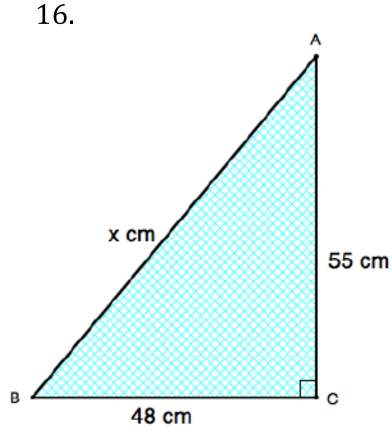

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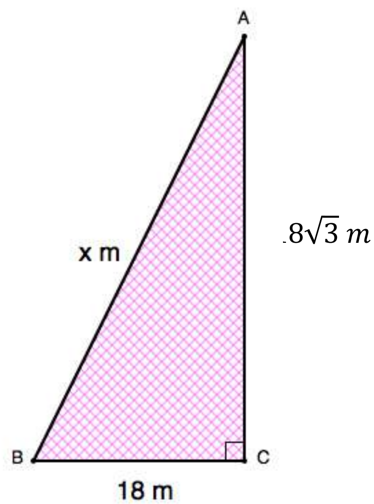

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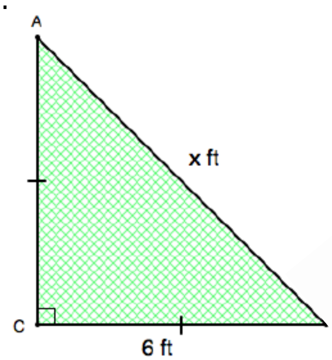

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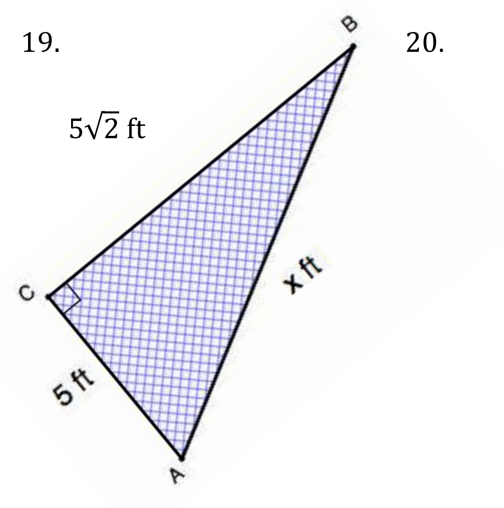

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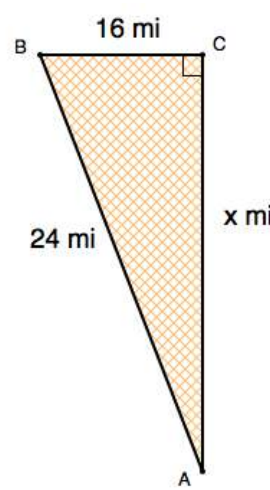

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5.4 You Nailed It!

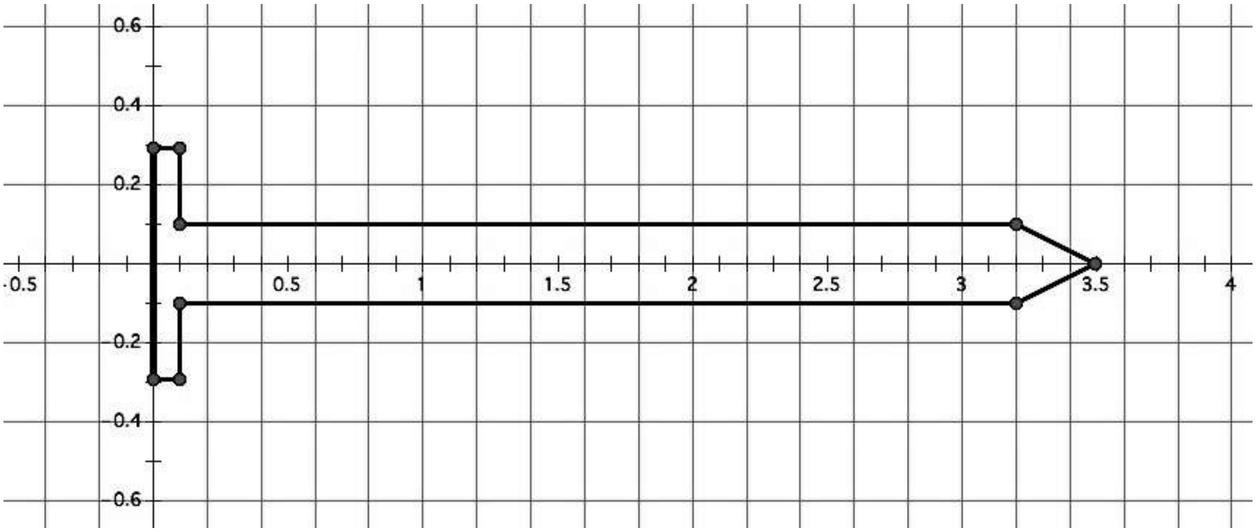
A Practice Understanding Task

Tatiana is helping her father purchase supplies for a deck he is building in their back yard. Based on her measurements for the area of the deck, she has determined that they will need to purchase 24 decking planks. These planks will be attached to the framing joists with 16d nails. (Tatiana thinks it is strange that these nails are referred to as "16 penny nails" and wonders where that way of naming nails comes from. After doing some research she has found that in the late 1700s in England the size of a nail was designated by the price of purchasing one hundred nails of that size. She doubts that her dad will be able to buy one hundred 16d nails for 16 pennies.)

Nails are sold by the pound at the local hardware store, so Tatiana needs to figure out how many pounds of 16d nails to tell her father to buy. She has gathered the following information.

- The deck requires 24 decking planks
- Each plank requires 9 nails to attach it to the framing joists
- 16d nails are made of steel that has a density of 4.67 oz/in^3
- There are 16 ounces in a pound

Tatiana has also found the following drawing of a cross section of a 16d nail. She knows she can use this drawing to help her find the volume of the nail, treating it as a solid of revolution. (Note: The scale on the *x*- and *y*-axis is in inches.)



1. Devise a plan for finding the volume of the nail based on the given drawing. Describe your plan in words, and then show the computations that support your work.

2. Devise a plan for finding the number of pounds of 16d nails Tatiana's father should buy. Describe your plan in words, and then show the computations that support your work.

READY, SET, GO! Name Period Date

READY

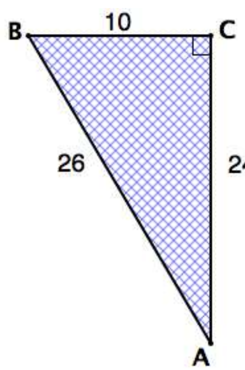
Topic: Finding the trigonometric ratios in a right triangle

Use the given measures on the triangle to write the indicated trig value. Write them as a fraction. Then write them as a decimal rounded to the thousandths place.

1. $\sin A =$

   $\cos A =$

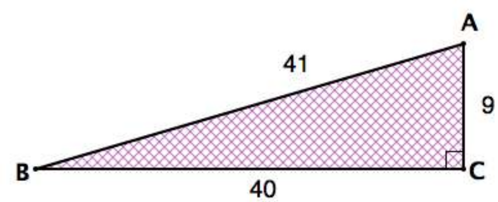
   $\tan A =$



2. $\sin B =$

   $\cos B =$

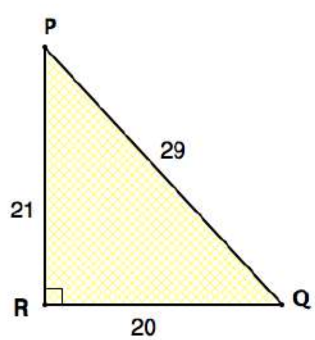
   $\tan B =$



3. $\sin P =$

   $\cos P =$

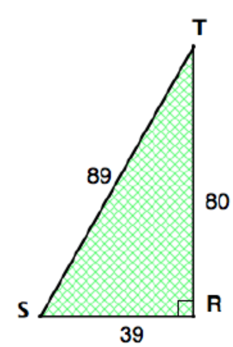
   $\tan P =$



4. $\sin S =$

   $\cos S =$

   $\tan S =$



5. Which trigonometric ratio is **exact**, the fraction or the decimal?
   Explain.

6. My calculator tells me that $\frac{\sqrt{2}}{2} = 0.7071067812$. Is one value more accurate than the other?
   Explain.

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SET

Topic: Exploring applications of volume, weight and density

Answer the following questions about the grain stored in the storage silos.

7. The figure at the right is of 2 grain, storage silos. The diameter of each measures 24 feet and the height of the cylinder measures 51 feet. The height of the cone adds an additional 12 feet. Find the total volume of one silo.

8. How many bushels of grain will each silo be able to store, if a bushel is 1.244 cubic feet? (Assume it can be filled to the top.)

9. Density relates to the degree of compactness of a substance. A cubic inch of gold weighs a great deal more than a cubic inch of wood because gold is more dense than wood. The density of grains also varies. Use the information below to calculate how many tons of each grain can be stored in one silo. (1 ton = 2000 lbs.)

   1 bushel of oats weighs 32 pounds

   1 bushel of barley weighs 48 pounds

   1 bushel of wheat weighs 60 pounds

10. A ¾ -ton pickup has the capacity to haul a little more than 1500 lbs. If the hauling bed of the pickup measures 4 ft. by 6.5 ft. by 2 ft., can a ¾ -ton pickup safely haul a full (level) load of oats, barley, or wheat? Justify your answer for each type of grain.

GO

Topic: Forms of linear and quadratic functions

Write what you know about the function (including end-behavior) and then graph it.

11. Equation: $f(x) = (x - 2)(x + 3)$

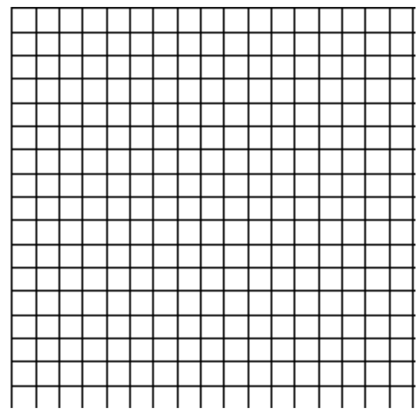
What I know about this function:

End behavior:

$as \quad x \to -\infty, \qquad f(x) \to$ ____

$as \quad x \to \infty, \qquad f(x) \to$ ____

Graph:



12. Equation: $g(x) = x^2 + 6x + 9$

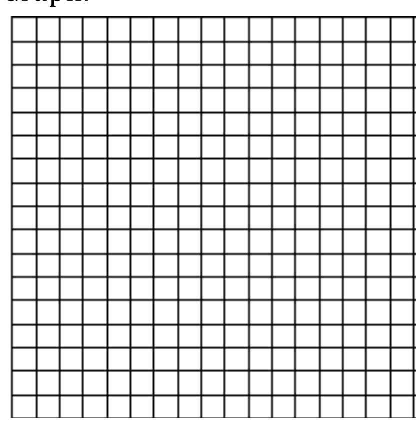
What I know about this function:

End behavior:

$as \quad x \to -\infty, \qquad f(x) \to$ ____

$as \quad x \to \infty, \qquad f(x) \to$ ____

Graph:



13. Equation: $y = -x^2 - 4$

What I know about this function:

End behavior:

$as \quad x \to -\infty, \qquad f(x) \to$ ____

$as \quad x \to \infty, \qquad f(x) \to$ ____

Graph:

14. Equation: $h(x) = 2(x - 5) + 3$

What I know about this function:

End behavior:

$as \quad x \to -\infty, \qquad f(x) \to$ ____

$as \quad x \to \infty, \qquad f(x) \to$ ____

Graph:

5.5 Special Rights

A Solidify Understanding Task

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In previous courses you have studied the Pythagorean theorem and right triangle trigonometry. Both of these mathematical tools are useful when trying to find missing sides of a right triangle.

1. What do you need to know about a right triangle in order to use the Pythagorean theorem?

2. What do you need to know about a right triangle in order to use right triangle trigonometry?

While using the Pythagorean theorem is fairly straight forward (you only have to keep track of the legs and hypotenuse of the triangle), right triangle trigonometry generally requires a calculator to look up values of different trig ratios. There are some right triangles, however, for which knowing a side length and an angle is enough to calculate the value of the other sides without using trigonometry. These are known as *special right triangles* because their side lengths can be found by relating them to another geometric figure for which we know something about its sides.

<u>One type of special right triangle is a 45°-45°-90° triangle</u>.

3. Draw a 45°-45°-90° triangle and assign a specific value to one of its sides. (For example, let one of the legs measure 5 cm, or choose to let the hypotenuse measure 8 inches. You will want to try both approaches to perfect your strategy.) Now that you have assigned a measurement to one of the sides of your triangle, find a way to calculate the measures of the other two sides. As part of your strategy, you may want to relate this triangle to another geometric figure that may be easier to think about.

4. Generalize your strategy by letting one side of the triangle measure *x.* Show how the measure of the other two sides can be represented in terms of *x.* (Make sure to consider cases where *x* is the length of a leg, as well as the case where *x* is the length of the hypotenuse.)

<u>Another type of special right triangle is a 30°-60°-90° triangle</u>.

5. Draw a 30°-60°-90° triangle and assign a specific value to one of its sides. Now that you have assigned a measurement to one of the sides of your triangle, find a way to calculate the measures of the other two sides. As part of your strategy, you may want to relate this triangle to another geometric figure that may be easier to think about.

6. Generalize your strategy by letting one side of the triangle measure *x.* Show how the measure of the other two sides can be represented in terms of *x.* (Make sure to consider cases where *x* is the length of a leg, as well as the case where *x* is the length of the hypotenuse.)

7. Can you think of any other angle measurements that will create a special right triangle?

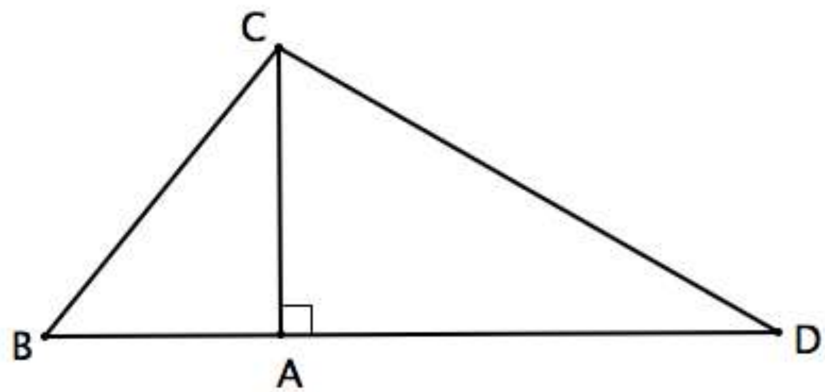
READY, SET, GO! Name Period Date

READY

Topic: Finding missing measures in triangles

Use the given figure to answer the questions. Round your answers to the hundredths place.

Given: $\boldsymbol{m\angle CBD = 51°}$
$\boldsymbol{m\angle CDA = 30°}$



1. Find $m\angle BCD$

Given : $\boldsymbol{m\angle CAD = 90°}$

2. Find $m\angle BCA$ and $m\angle ACD$

Given: ***CA = 6 ft***

3. Find BC

4. Find BA

5. Find CD

6. Find AD

7. Find BD

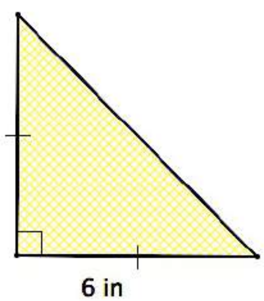
8. Find the area of $\Delta\ BCD$

SET

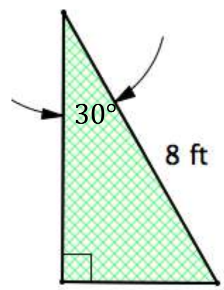
Topic: Recalling triangle relationships in Special Right Triangles

Fill in all the missing measures in the triangles.

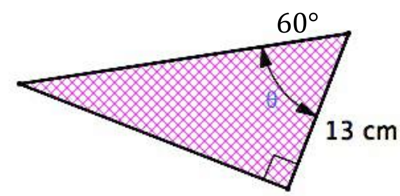
9.



10.



11.



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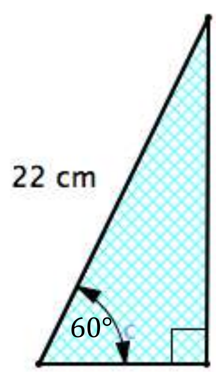
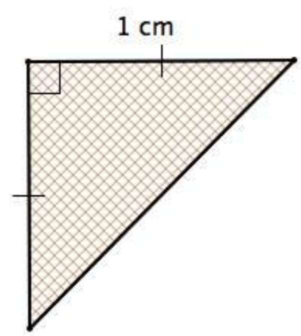
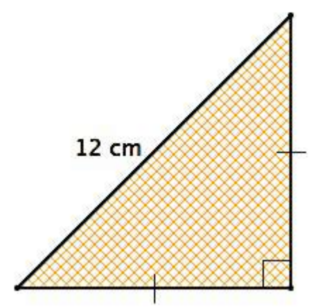


13.



14.



Use an appropriate triangle from above to fill in the function values below. No calculators.

15.

$\sin 45° =$	
$\cos 45° =$	
$\tan 45° =$	

16.

$\sin 30° =$	
$\cos 30° =$	
$\tan 30° =$	

17.

$\sin 60° =$	
$\cos 60° =$	
$\tan 60° =$	

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GO

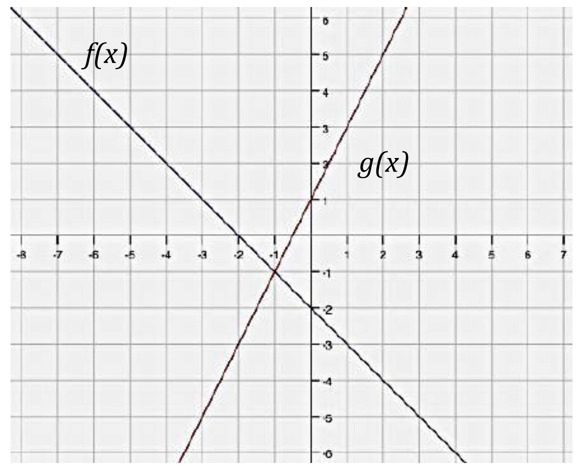
Topic: Performing function arithmetic on a graph

18. Add *f (x)* and *g(x)* using the graph at the right.

Draw the new figure on the graph and label it as *s(x), the sum of x.*

19. Subtract *f (x)* from *g(x)* using the graph at the right.

Draw the new figure on the graph and label it as *d(x), the difference of x.*

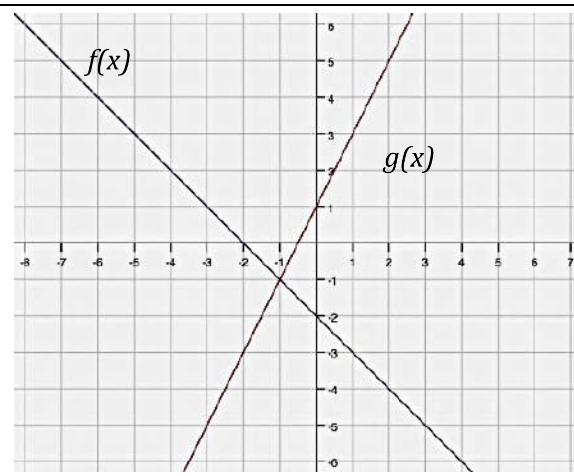


20. Multiply *f (x)* and *g(x)* on the second graph at the right.

Draw the new figure on the graph and label it as *p(x), the product of x.*

21. Divide *f (x)* by *g(x)* on the second graph at the right.

Draw the new figure on the graph and label it as *q(x), the quotient of x.*



22. Write the equations of *f (x)* and *g(x).*	
23. Write the equation of the sum of *f (x)* and *g(x).* *s(x) =*	24. Write the equation of the difference of *f (x)* and *g(x).* *d(x) =*
25. Write the equation of the product of *f (x)* and *g(x).* *p(x) =*	26. Write the equation of the quotient of *f (x)* divided by *g(x).* *q(x) =*

5.6 More Than Right

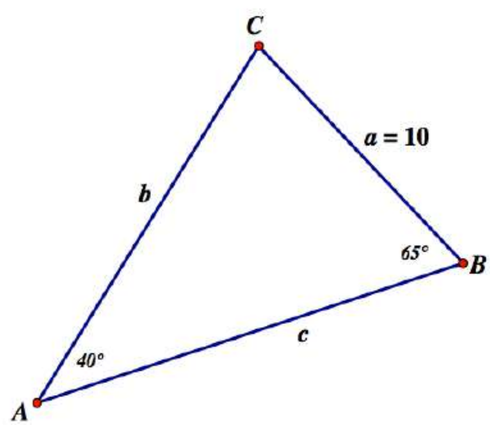
A Develop Understanding Task

We can use right triangle trigonometry and the Pythagorean theorem to solve for missing sides and angles in a right triangle. What about other triangles? How might we find unknown sides and angles in acute or obtuse triangles if we only know a few pieces of information about them?

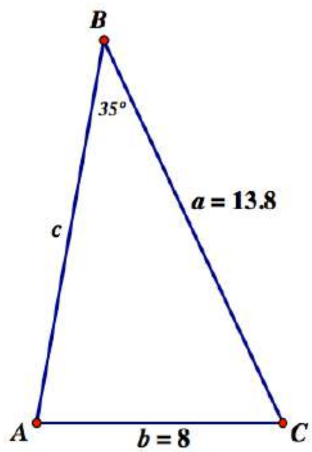
In the previous task we found it might be helpful to create right triangles by drawing an altitude in a non-right triangle. We can then apply trigonometry or the Pythagorean theorem to the smaller right triangles, which may help us learn something about the sides and angles in the larger triangle.

See if you can devise a strategy for finding the missing sides and angles of each of these triangles.

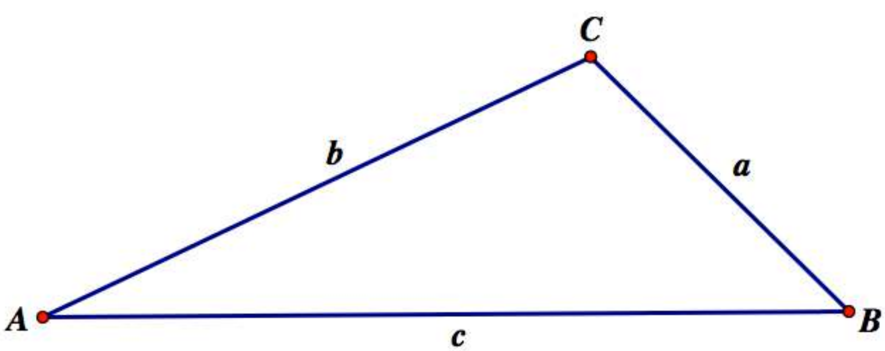
1.



2.



3. See if you can generalize the work you have done on problems 1 and 2 by finding relationships between sides and angles in the following diagram. Unlike the previous two problems, this triangle contains an obtuse angle at *C*. Find as many relationships as you can between sides *a*, *b* and *c* and the related angles *A*, *B* and *C*.



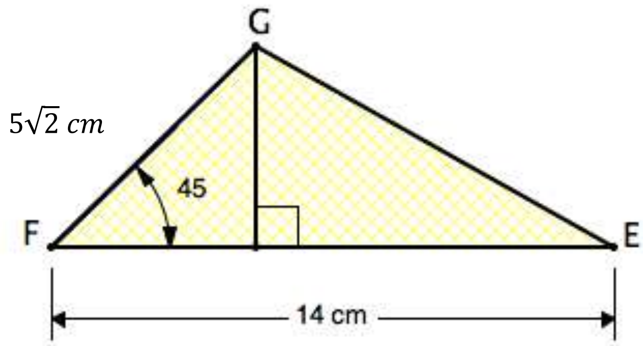
READY, SET, GO! Name Period Date

READY

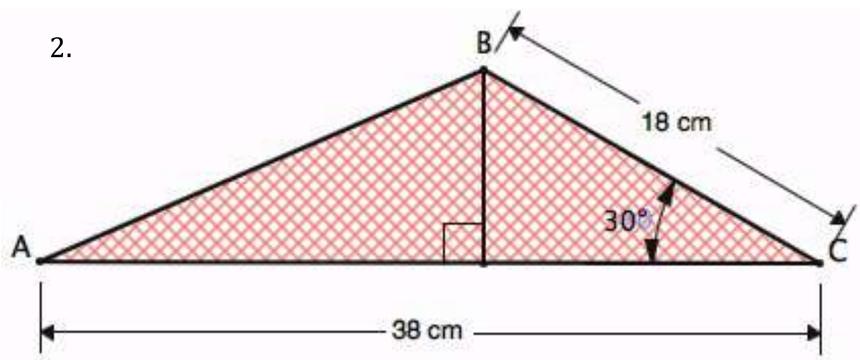
Topic: Finding area of triangles

Find the area of each triangle. $A = \frac{1}{2}bh$

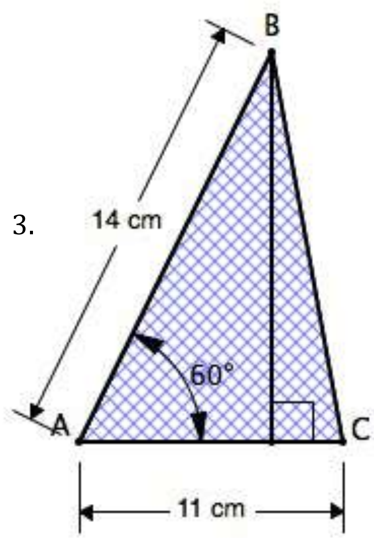
1.



2.



3.



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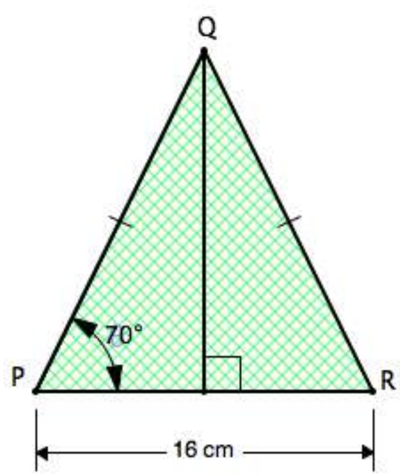
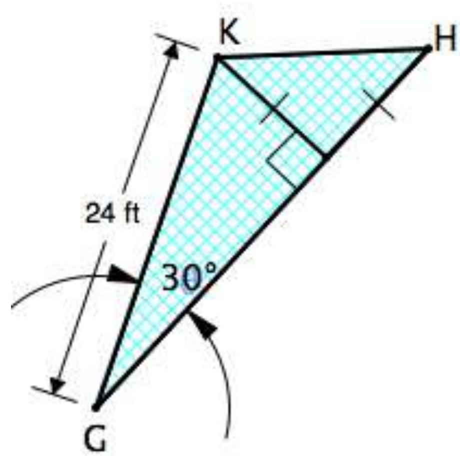


5.



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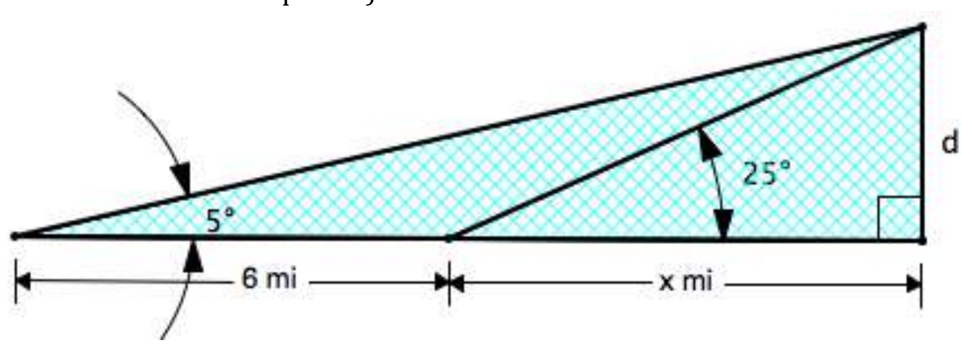
SET

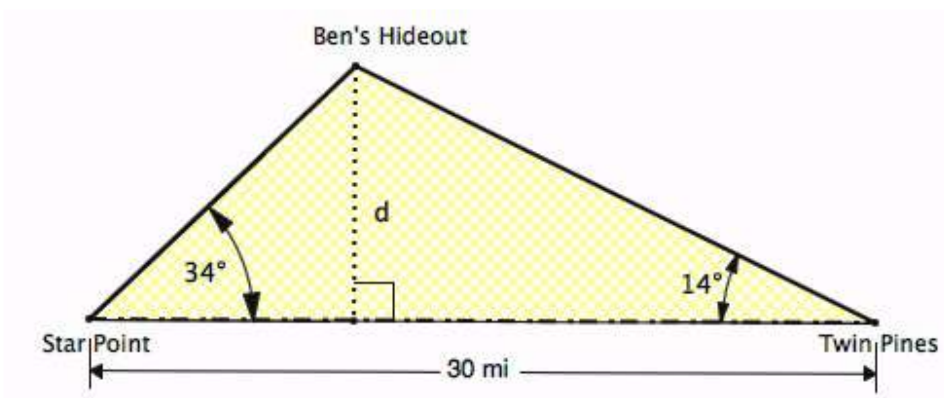
Topic: Using right triangle trig to solve triangles

Solve the following application problems using right triangle trigonometry.

6. While traveling across a flat stretch of desert, Joey and Holly make note of a mountain peak in the distance that seems to be directly in front of them. They estimate the angle of elevation to the peak as 5º. After traveling 6 miles towards the mountain the angle of elevation is 25º. Approximate the height of the mountain in miles and in feet. **5,280ft = 1 mile** (While figuring, use at least 4 decimal places.)



7. The Star Point Ranger Station and the Twin Pines Ranger Station are 30 miles apart along a straight scenic road. Each station gets word of a cabin fire in a remote area known as Ben's Hideout. A straight path from Star Point to the fire makes an angle of 34º with the road, while a straight path from Twin pines makes an angle of 14° with the road. Find the distance *d* of the fire from the road.

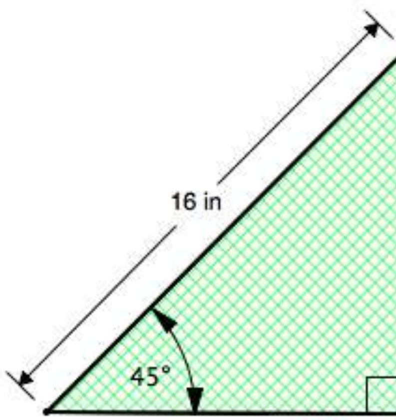


GO

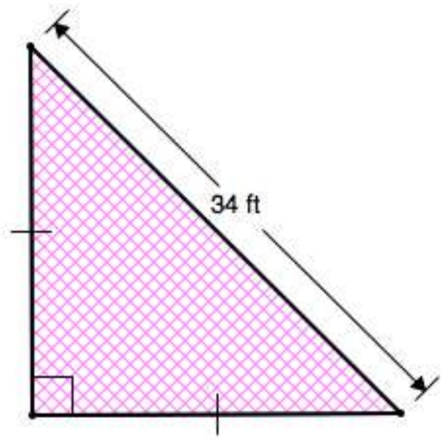
Topic: Recalling measures in special right triangles

Fill in the missing sides and angles in the right triangles. Write answers in simplified radical form. Do NOT use a calculator.

8.

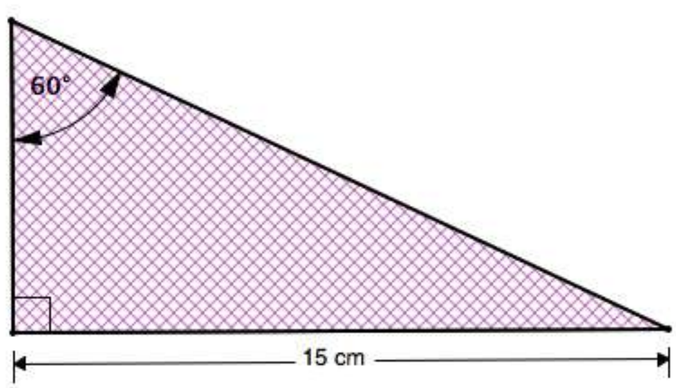


9.

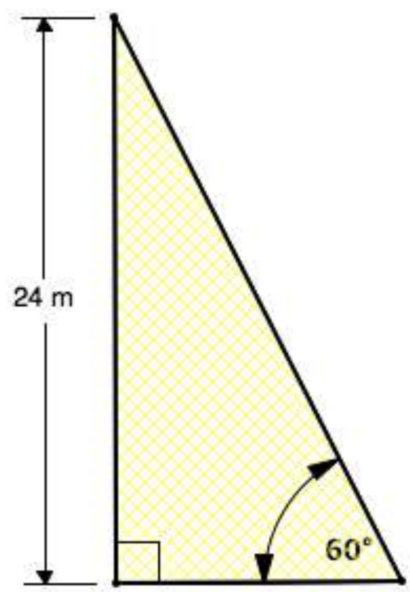


10. Write a rule for finding the sides of an isosceles right triangle when you know the hypotenuse and the measure of the hypotenuse does NOT show a $\sqrt{2}$.

11.



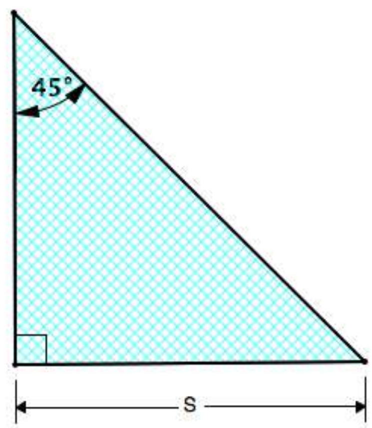
12.



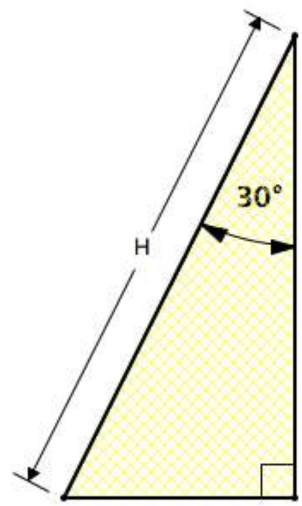
13. Write a rule for finding the missing sides in a 30º - 60º - 90º when you know the side opposite the 60º angle but the measurement doesn't show a $\sqrt{3}$.

Fill in the missing measurements.

14.



15.



Fill in the ratios for the given functions. Do not use a calculator. Answers should be in simplified radical form.

16.

$\sin 45° =$	
$\cos 45° =$	
$\tan 45° =$	

17.

$\sin 30° =$	
$\cos 30° =$	
$\tan 30° =$	

18.

$\sin 60° =$	
$\cos 60° =$	
$\tan 60° =$	

5.7 Justifying the Laws

A Solidify Understanding Task

The Pythagorean theorem makes a claim about the relationship between the areas of the three squares drawn on the sides of a right triangle: *the sum of the area of the squares on the two legs is equal to the area of the square on the hypotenuse.* We generally state this relationship algebraically as $a^2 + b^2 = c^2$, where it is understood that a and b represent the length of the two legs of the right triangle, and c represents the length of the hypotenuse.

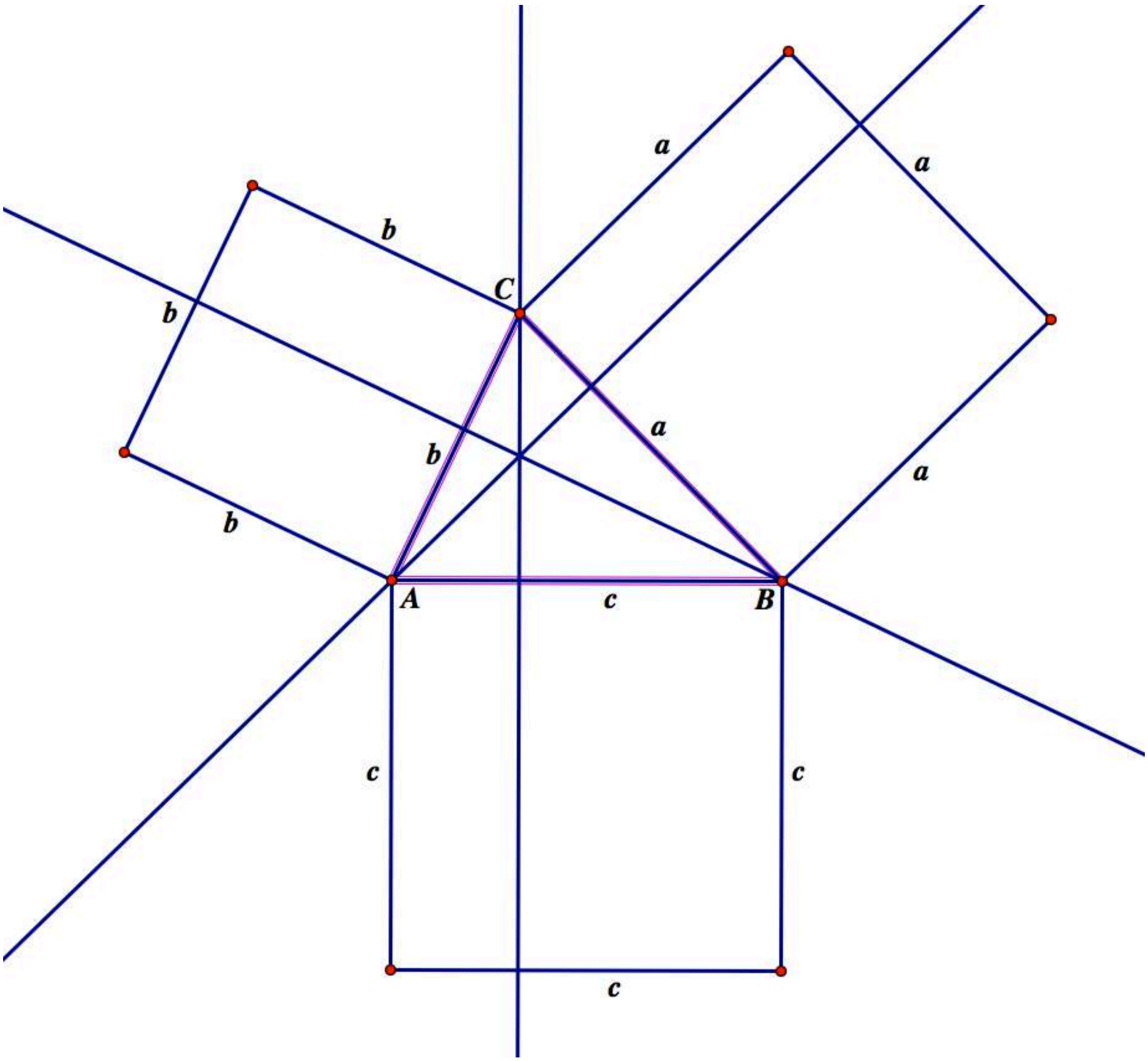
What about non-right triangles? Is there a relationship between the areas of the squares drawn on the sides of a non-right triangle? (Note: The following proof is based on *The Illustrated Law of Cosines*, by Don McConnell http://www.cut-the-knot.org/pythagoras/DonMcConnell.shtml)

The diagram on the next page shows an acute triangle with squares drawn on each of the three sides. The three altitudes of the triangle have been drawn and extended through the squares on the sides of the triangle. The altitudes divide each square into two smaller rectangles.

1. Find an expression for the areas of each of the six small rectangles formed by the altitudes. Write these expressions inside each rectangle on the diagram. (Hint: The area of each rectangle can be expressed as the product of the side length of the square and the length of a segment that is a leg of a right triangle. You can use right triangle trigonometry to express the length of this segment.)

2. Although none of the six rectangles are congruent, there are three pairs of rectangles where each rectangle in the pair has the same area. Using three different colors—red, blue and green—shade pairs of rectangles that have the same area with the same color.

3. The area of each square is composed of two smaller, rectangular areas of two different colors. Write three different "equations" to represent the areas of each of the squares. For example, you might write $a^2 = blue + red$ if those are the colors you chose for the areas of the rectangles formed in the square drawn on side a.



4. Select one of your equations from step 3, such as $a^2 = blue + red$, and use the other two squares to substitute a different expression in for each color. For example, if in your diagram

$blue = b^2 - green$ and $red = c^2 - green$, we can write this equation:
$a^2 = b^2 - green + c^2 - green$ or $a^2 = b^2 + c^2 - 2 \cdot green$.

Write your selected equation in its modified form here:

5. Since each color is actually a variable representing an area of a rectangle, replace the remaining color in your last equation with the expression that gives the area of the rectangles of that color.

   Write your final equation here:

6. Repeat steps 4 and 5 for the other two equations you wrote in step 3. You should end up with three different versions of the **Law of Cosines,** each relating the area of one of the squares drawn on a side of the triangle to the areas of the squares on the other two sides.

   $a^2 =$

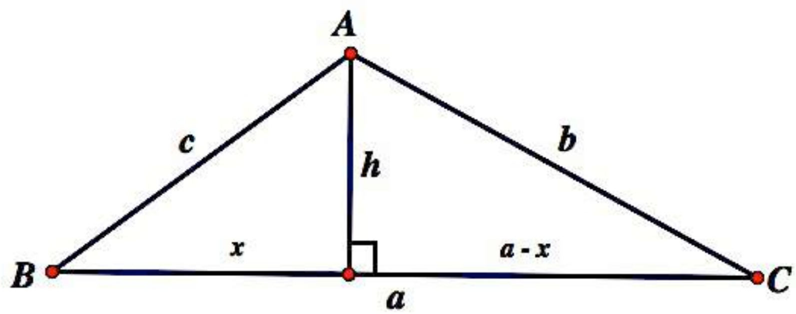
   $b^2 =$

   $c^2 =$

7. What happens to this diagram if angle C is a right angle? (Hint: Think about the altitudes in a right triangle.)

8. Why do we have to subtract some area from $a^2 + b^2$ to get c^2 when angle C is less than right?

The Law of Cosines can also be derived for an obtuse triangle by using the altitude of the triangle drawn from the vertex of the obtuse angle, as in the following diagram, where we assume that angle *A* is obtuse.



9. Use this diagram to derive one of the forms of the Law of Cosines you wrote above. (Hint: As in the previous task, *More Than Right*, the length of the altitude can be represented in two different ways, both using the Pythagorean theorem and the portions of side *a* that form the legs of two different right triangles.)

10. Use the same diagram above to derive the **Law of Sines**. (Hint: How can you represent the length of the altitude in two different ways using sides *a, b,* or *c* and right triangle trigonometry instead of the Pythagorean theorem?)

READY, SET, GO! Name Period Date

READY

Topic: Recalling circumference and area of a circle

Use the given information to find the indicated value. Leave π in your answer. Include the correct unit.

1. radius = 3 ft
circumference:
area:

2. diameter = 14 cm
circumference:
area:

3. circumference = 38π km
radius:
area:

4. area = 49π in^2
diameter:
circumference:

5. circumference = 15π mi
radius:
area:

6. area = 121π m^2
radius:
circumference:

Solve for the missing angle. Round your answers to the nearest degree.

(Hint: In problems 10, 11, and 12, get the trig function alone. Then solve for θ.)

7. $\cos\theta = \frac{1}{6}$

8. $\tan\theta = \frac{2}{3}$

9. $\sin\theta = \frac{7}{8}$

10. $5\sin\theta - 2 = 0$

11. $7\cos\theta - 6 = 0$

12. $4\tan\theta - 1 = 0$

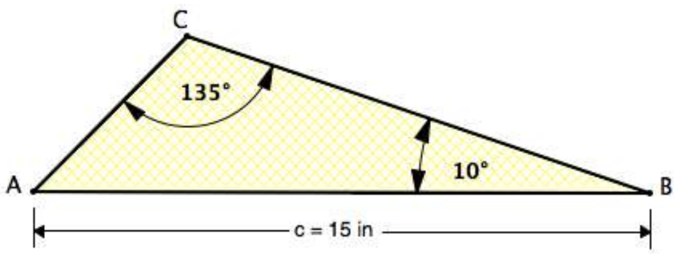
SET

Topic: Using the Laws of sine and cosine to solve triangles

Law of Sines: If ABC is a triangle with sides a, b, and c, then $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or it can be written as: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	**Law of Cosines:** If ABC is a triangle with sides a, b, and c, then $a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = a^2 + c^2 - 2ac\cos B$ $c^2 = a^2 + b^2 - 2ab\cos C$

Use the *Law of sines* to solve each triangle.

13.



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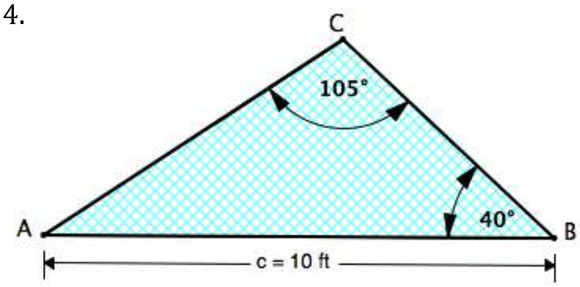

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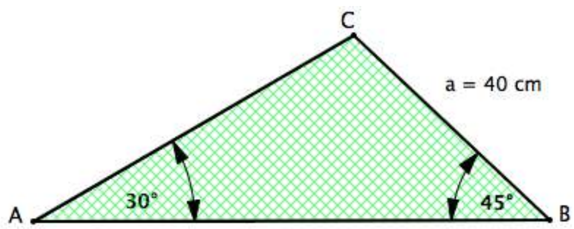

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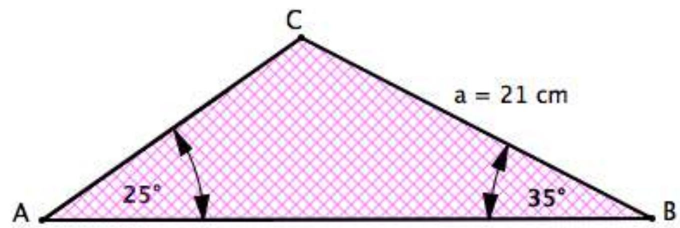
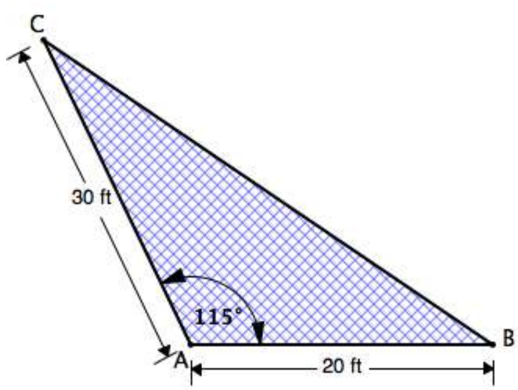
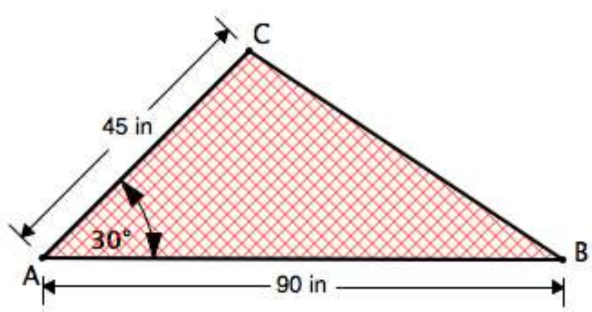


17. What information do you need in order to use the *Law of sines*?

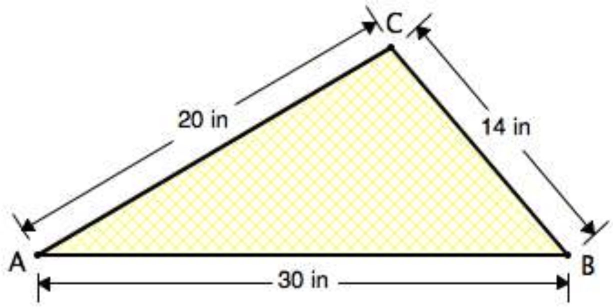
18. Use the *Law of cosines* to find the remaining angles and side of the triangle.



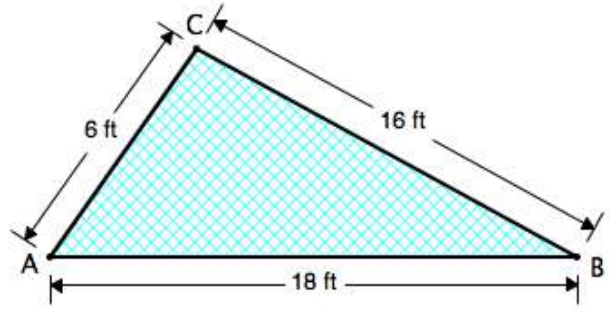
19. Use the *Law of cosines* to find the remaining angles and side of the triangle.



20. Use the *Law of cosines* to find the three angles of the triangle.



21. Use the *Law of cosines* to find the three angles of the triangle.



22. What information do you need in order to use the *Law of cosines* to solve a triangle?

GO

Topic: Recalling he trig ratios of the special right triangles

Fill in the missing angle. Do NOT use a calculator.

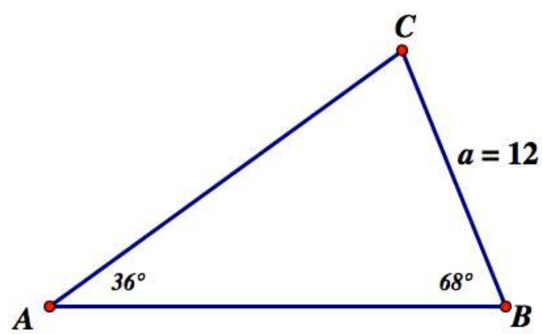
23. $\sin\theta = \frac{\sqrt{2}}{2}$	24. $\tan\theta = \sqrt{3}$	25. $\cos\theta = \frac{1}{2}$
26. $\sin\theta = \frac{\sqrt{3}}{2}$	27. $\tan\theta = 1$	28. $\tan\theta = \frac{\sqrt{3}}{3}$
29. $\sin\theta = \frac{1}{2}$	30. $\cos\theta = \frac{\sqrt{2}}{2}$	31. $\cos\theta = \frac{\sqrt{3}}{2}$

5.8 Triangle Areas by Trig

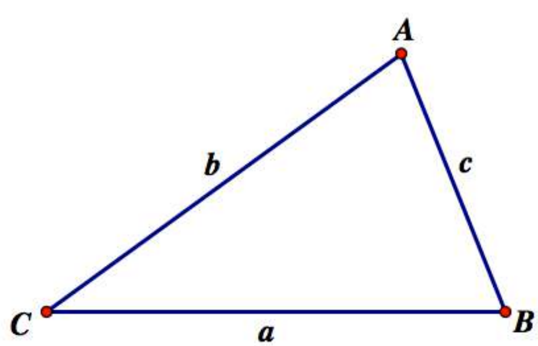
A Practice Understanding Task

Find the area of the following two triangles using the strategies and procedures you have developed in the past few tasks. For example, draw an altitude as an auxiliary line, use right triangle trigonometry, use the Pythagorean theorem, or use the Law of Sines or the Law of Cosines to find needed information.

1. Find the area of this triangle.



2. Find the area of this triangle.



Jumal and Jabari are helping Jumal's father with a construction project. He needs to build a triangular frame as a component of the project, but he has not been given all the information he needs to cut and assemble the pieces of the frame. He is even having a hard time envisioning the shape of the triangle from the information he has been given.

Here is the information about the triangle that Jumal's father has been given.

- Side a = 10.00 meters
- Side b = 15.00 meters
- Angle A = 40.0°

Jumal's father has asked Jumal and Jabari to help him find the measure of the other two angles and the missing side of this triangle.

Carry out each student's strategy as described below. Then draw a diagram showing the shape and dimensions of the triangle that Jumal's father should construct. (Note: To provide as accurate information as possible, Jumal and Jarbari decide to round all calculated sides to the nearest cm—that is, to the nearest hundredth of a meter—and all angle measures to the nearest tenth of a degree.)

<u>Jumal's Approach</u>

- Find the measure of angle B using the Law of Sines
- Find the measure of the third angle C
- Find the measure of side c using the Law of Sines
- Draw the triangle

Jabari's Approach

- Solve for c using the Law of Cosines: $a^2 = b^2 + c^2 - 2bc\cos(C)$

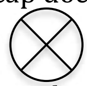
  (Jabari is surprised that this approach leads to a quadratic equation, which he solves with the quadratic formula. He is even more surprised when he finds two reasonable solutions for the length of side c.)

- Draw both possible triangles and find the two missing angles of each using the Law of Sines

READY, SET, GO! Name Period Date

READY

Topic: Rotational symmetry

Hubcaps have *rotational symmetry*. That means that a hubcap does not have to turn a full circle to appear the same. For instance, a hubcap with this pattern,  will look the same every ¼ turn. It is said to have 90º *rotational symmetry* because for each quarter turn it rotates 90º.

State the *rotational symmetry* for the following hubcaps. Focus your answer on just the spokes, not the center design.(Answers will be in degrees.

SET

Topic: Area formulas for triangles

Area of an Oblique Triangle: The area of **any** triangle is one-half the product of the lengths of two sides times the sine of their included angle. $Area = \frac{1}{2}bc\ \sin A = \frac{1}{2}ab\ \sin C = \frac{1}{2}ab \sin B$

mathematics vision project

Find the area of the triangle having the indicated sides and angle.

16. $C = 84.5^{\circ}$, $a = 32$, $b = 40$

17. $A = 29^{\circ}$, $b = 49$, $c = 50$

18. $B = 72.5^{\circ}$, $a = 105$, $c = 64$

19. $C = 31^{\circ}$, $a = 15$, $b = 14$

20. $A = 42^{\circ}$, $b = 25$, $c = 12$

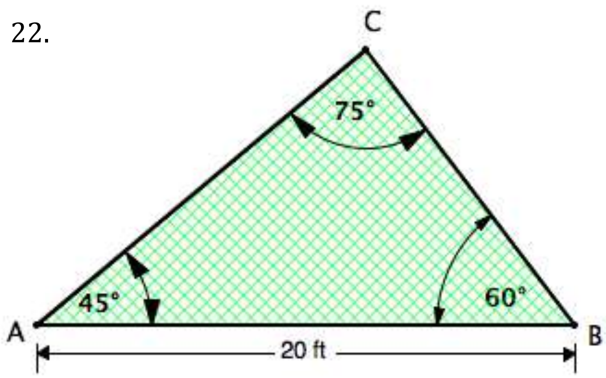
21. $B = 85^{\circ}$, $a = 15$, $c = 12$

Another formula for the area of a triangle can be derived from the *Law of Sines*.

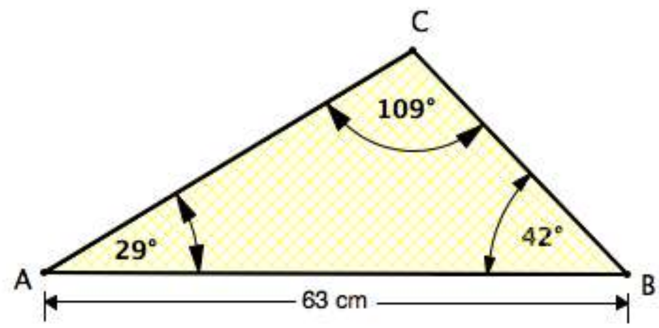
$$Area = \frac{c^2 \sin A \sin B}{2 \sin C}$$

Use this formula to find the area of the triangles.

22.



23.



Perhaps you used the *Law of Cosines* to establish the following formula for the area of a triangle. The formula was known as early as circa 100 B.C. and is attributed to the Greek mathematician, Heron.

Heron's Area Formula: Given any triangle with sides of lengths *a, b,* and *c,* the area of the triangle is:

$$Area = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{(a+b+c)}{2}.$$

Find the area of the triangle having the indicated sides.

24. $a = 11,\ b = 14,\ c = 20$

25. $a = 12,\ b = 5,\ c = 9$

26. $a = 12.32,\ b = 8.46,\ c = 15.05$

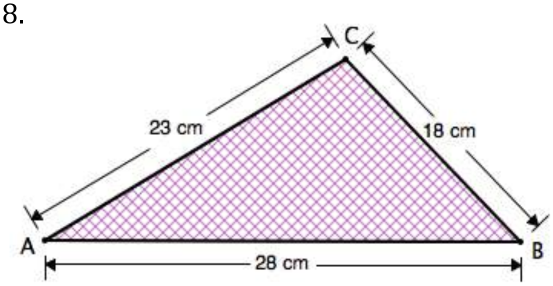
27. $a = 5,\ b = 7,\ c = 10$

GO

Topic: Distinguishing between the *law of sines* and the *law of cosines*

Indicate whether you would use the *Law of Sines* or the *Law of Cosines* to solve the triangles. Do not solve.

28.



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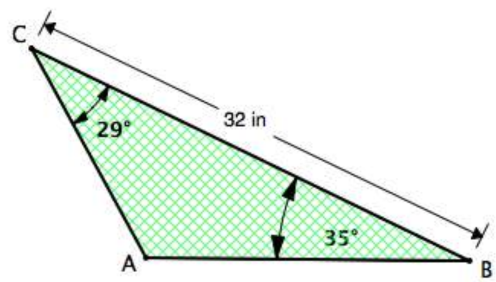

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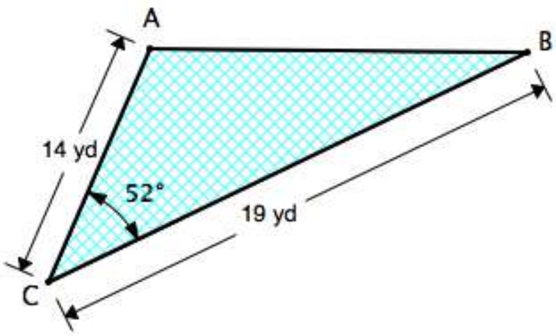

31.