Transforming Mathematics Education

SECONDARY MATH THREE

An Integrated Approach

MODULE 6

Modeling Periodic Behavior

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

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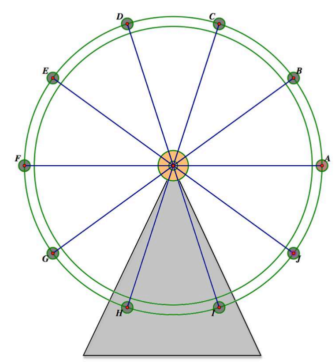
6.1 George W. Ferris' Day Off

A Develop Understanding Task

Perhaps you have enjoyed riding on a Ferris wheel at an amusement park. The Ferris wheel was invented by George Washington Ferris for the 1893 Chicago World's Fair.

Carlos, Clarita and their friends are celebrating the end of the school year at a local amusement park. Carlos has always been afraid of heights, and now his friends have talked him into taking a ride on the amusement park Ferris wheel. As Carlos waits nervously in line he has been able to gather some information about the wheel. By asking the ride operator, he found out that this wheel has a radius of 25 feet, and its center is 30 feet above the ground. With this information, Carlos is trying to figure out how high he will be at different positions on the wheel.

1. How high above the ground will Carlos be when he is at the top of the wheel? (To make things easier, think of his location as simply a point on the circumference of the wheel's circular path.)

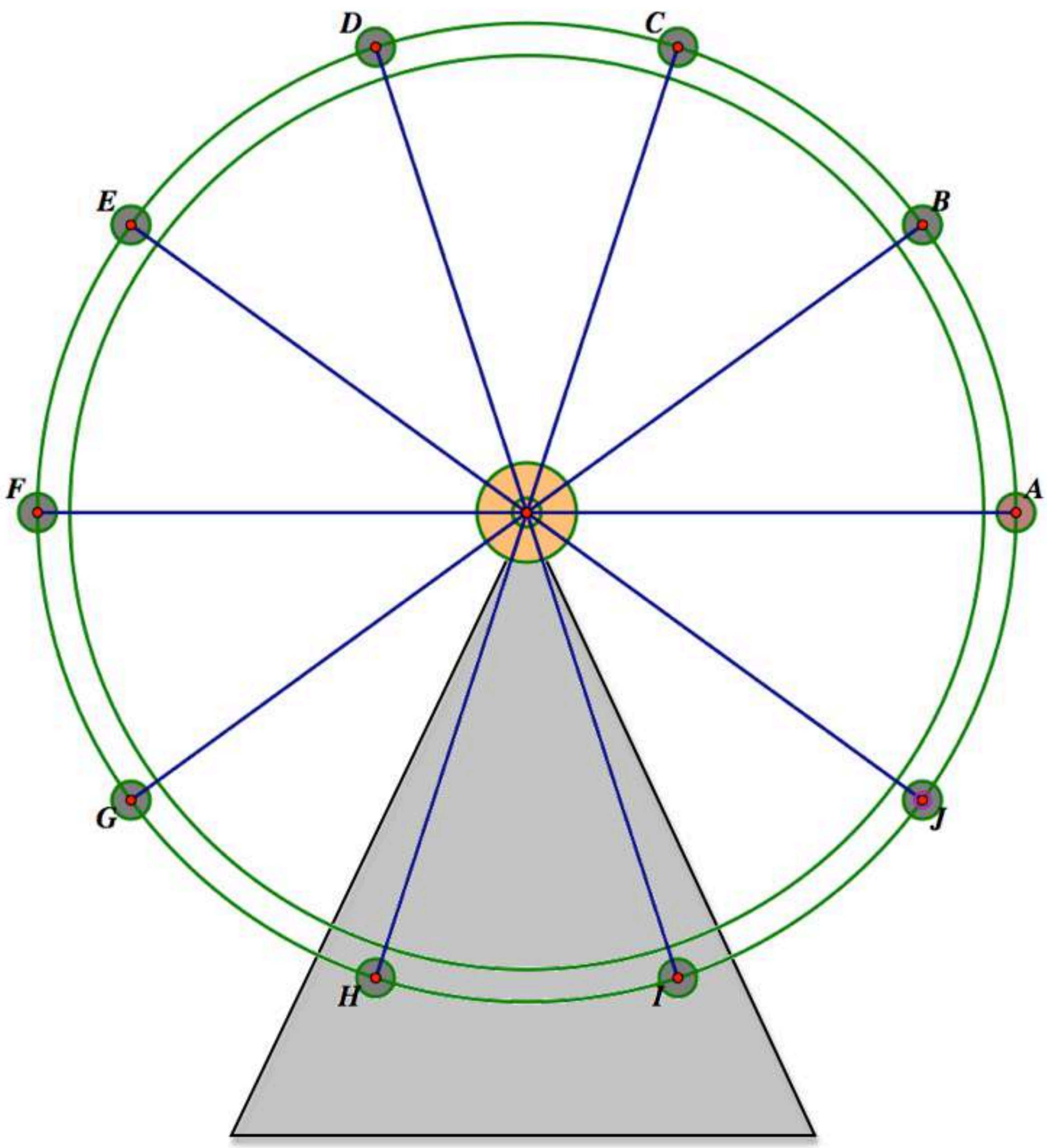


2. How high will he be when he is at the bottom of the wheel?

3. How high will he be when he is at the positions farthest to the left or the right on the wheel?

Because the wheel has ten spokes, Carlos wonders if he can determine the height of the positions at the ends of each of the spokes as shown in the diagram on the following page. Carlos has just finished studying right triangle trigonometry, and wonders if that knowledge can help him.

4. Find the height of each of the points labeled A-J on the Ferris wheel diagram below. Represent your work on the diagram so it is apparent to others how you have calculated the height at each point.



READY, SET, GO! Name Period Date

READY

Topic: Making sense of speed when distance is circular

The number of degrees an object passes through during a given amount of time is called ***angular speed.*** For instance, the second hand on a clock has an angular speed of $\frac{360°}{min}$ while the minute hand on a clock has an angular speed of $\frac{360°}{hr}$. (Remember that a revolution is a full circle or 360º.)

1. What is the angular speed of the second hand on a clock in degrees per second?
2. What is the angular speed of the minute hand on a clock in degrees per second?
3. What is the angular speed of the hour hand in degrees per hour?

Your grandparents probably enjoyed music just as much as you do, but they didn't have IPods or MP3 players. They had vinyl records and phonographs. Vinyl records came in 3 speeds. A record could be a 45, $33\frac{1}{3}$, or 78. These numbers referred to the rpms or ***revolutions per minute***.

4. Calculate the ***angular speed*** of a $45\ rpm, 33\frac{1}{3}\ rpm, and\ 78\ rpm$ record in degrees per minute.
   a) $45\ rpm$
   b) $33\frac{1}{3}\ rpm$
   c) $78\ rpm$

Angular speed describes how fast something is turning. ***Linear speed*** describes how far it travels while it is turning. ***Linear speed*** depends on the circumference of a circle ($C = 2\pi r$) and the number of revolutions per minute.

Vinyl records were not the same size. A $45\ rpm$ record had a diameter of 7 inches, a $33\frac{1}{3}$ a diameter of 12 inches and a 78 had a diameter of 10 inches.

5. a) If a fly landed on the outer edge of a $45\ rpm$ record, how far would it travel in 1 minute?
   b) How far if it was perched on a $33\frac{1}{3}\ rpm$ record?
   c) How far if it was perched on a $78\ rpm$ record?

SET

Topic: Using trigonometric ratios to solve problems

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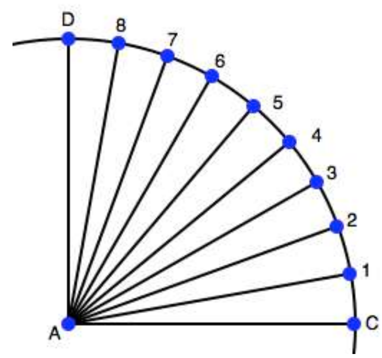
Perhaps you have seen *The London Eye* in the background of a recent James Bond movie or on a television show. When it opened in March of 2000, it was the tallest Ferris wheel in the world. The passenger capsule at the very top is 135 meters above the ground. The diameter is 120 meters.



6. How high is the center of the Ferris wheel?
7. How far from the ground is the very bottom passenger capsule?
8. Assume there are 36 passenger capsules, evenly spaced around the circumference. Find the height from the ground of each of the *numbered* passenger capsules shown in the figure. (Use the figure at the right to help you think about the problem.)

GO

Topic: Connecting the trigonometric ratios

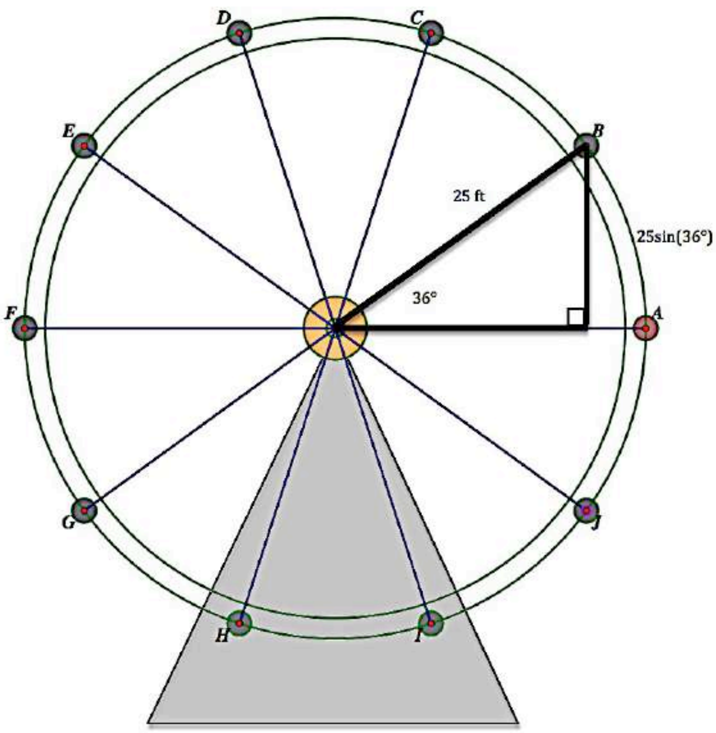
Find the other two trig ratios based on the one that is given.

9. $\sin\theta = \frac{4}{5}$	$\cos\theta =$	$\tan\theta =$
10. $\sin\theta =$	$\cos\theta = \dfrac{5}{13}$	$\tan\theta =$
11. $\sin\theta =$	$\cos\theta =$	$\tan\theta = 1$
12. $\sin\theta = \frac{1}{2}$	$\cos\theta =$	$\tan\theta =$
13. $\sin\theta =$	$\cos\theta = \dfrac{9}{41}$	$\tan\theta =$
14. $\sin\theta =$	$\cos\theta =$	$\tan\theta = \sqrt{3}$

6.2 "Sine" Language

A Solidify Understanding Task

In the previous task, *George W. Ferris' Day Off*, you probably found Carlos' height at different positions on the Ferris wheel using right triangles, as illustrated in the following diagram.



Recall the following facts from the previous task:

- The Ferris wheel has a radius of 25 feet
- The center of the Ferris wheel is 30 feet above the ground

Carlos has also been carefully timing the rotation of the wheel and has observed the following additional fact:

- The Ferris wheel makes one complete revolution counterclockwise every 20 seconds

1. How high will Carlos be 2 seconds after passing position A on the diagram?

2. Calculate the height of a rider at each of the following times *t*, where *t* represents the number of seconds since the rider passed position A on the diagram. Keep track of any regularities you notice in the ways you calculate the height. As you calculate each height, plot the position on the diagram.

Elapsed time since passing position A	Calculations	Height of the rider
1 sec		
2 sec		
2.5 sec		
3 sec		
6 sec		
8 sec		
14.5 sec		
18 sec		
23 sec		
28 sec		
36 sec		
37 sec		
40 sec		

3. Examine your calculations for finding the height of the rider during the first 5 seconds after passing position A (the first few values in the above table). During this time, the angle of rotation of the rider is somewhere between 0° and 90°. Write a general formula for finding the height of the rider during this time interval.

4. How might you find the height of the rider in other "quadrants" of the Ferris wheel, when the angle of rotation is greater than 90°?

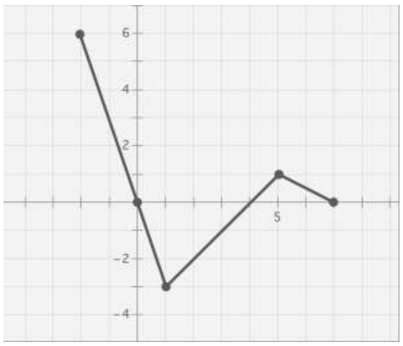
READY, SET, GO! Name Period Date

READY

Topic: Describing intervals from graphs

For each graph, write the interval(s) where $f(x)$ is positive and the interval(s) where it is negative.

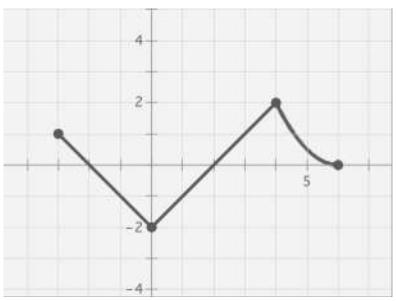
1.



Positive ___________________________________

Negative ___________________________________

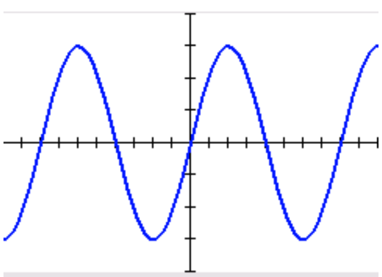
2.



Positive ___________________________________

Negative ___________________________________

3.

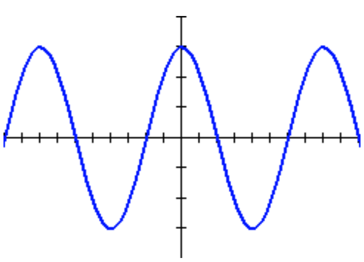


(The scale on the x-axis is in increments of 45°.)

Positive ___________________________________

Negative ___________________________________

4.



(The scale on the x-axis is in increments of 45°.)

Positive ___________________________________

Negative ___________________________________

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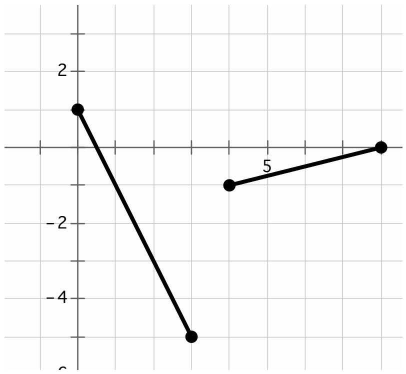
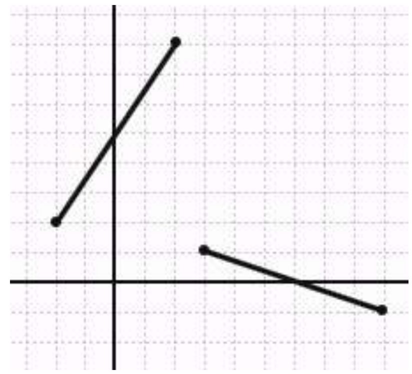
Write the piece-wise equations for the given graphs.

5.



Equation:

6.



Equation:

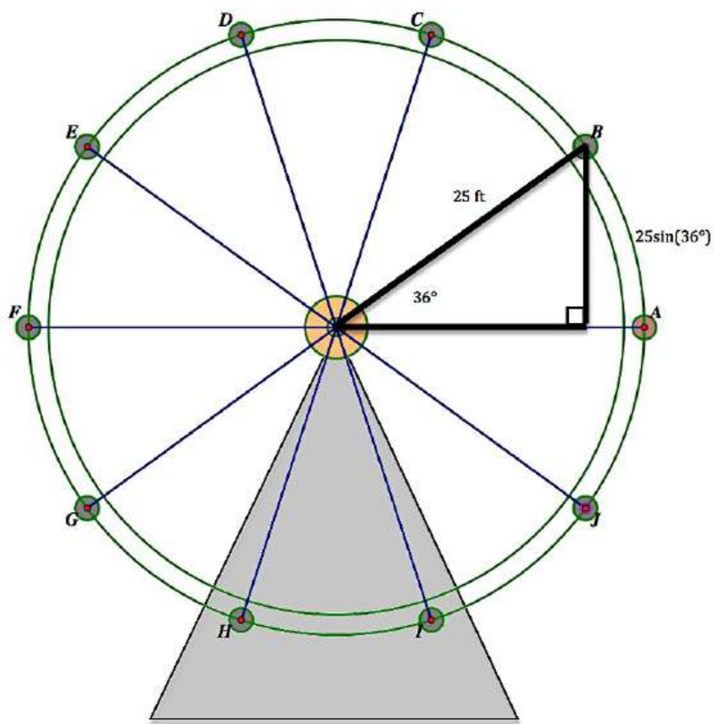
SET

Topic: Calculating sine as a function of time

Recall the following facts from the classroom task:

- The Ferris wheel has a radius of 25 feet
- The center of the Ferris wheel is 30 feet above the ground

Due to a safety concern, the management of the amusement park decides to slow the rotation of the Ferris wheel from 20 seconds for a full rotation to **30 seconds for a full rotation**.



7. Calculate how high a rider will now be 2 seconds after passing position A on the diagram.

8. Calculate the height of a rider at each of the following times t, where t represents the number of seconds since the rider passed position A on the diagram. As you calculate each height, plot the position on the diagram. Connect the center of the circle to the point you plotted. Then draw a vertical line from the plotted point on the Ferris wheel to the line segment AF in the diagram. Each time you should get a right triangle similar to the one in the figure.

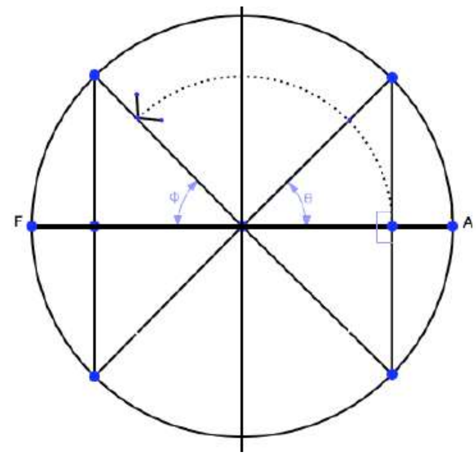
Elapsed time since passing position A	**Calculations**	**Height of the rider (in feet)**
1 sec		
3 sec		
5 sec		
7 sec		
8 sec		
11 sec		
14 sec		
15 sec		
16 sec		
20 sec		
22 sec		
23 sec		
25 sec		
27 sec		
30 sec		

9. How did the position of the triangles you drew change between 7 seconds and 8 seconds?

10. How did the triangles you drew change between 14, 15, and 16 seconds?

11. How did the triangles you drew change between 22 seconds and 23 seconds?

12. Describe a relationship between the orientation of the right triangles around the circle and the angle of rotation. Use the diagram to help you think about the question. (The dotted arc shows the angle of rotation.)



GO

Topic: Finding missing angles in triangles

Find the measure of each acute angle of right triangle ABC with $m\angle C = 90°$.

Round your answers to the nearest degree.

13. $a = 3\ in \quad c = 5\ in$

14. $a = 5\ ft \quad c = 10\ ft$

15. $a = 9.1\ cm \quad c = 12.3\ cm$

16. $a = 14.1\ cm \quad c = 18.\ cm$

17. $a = 9.7\ in \quad b = 12.7\ in$

18. $a = 14.6\ ft \quad c = 20.3\ ft$

6.3 More "Sine" Language

A Solidify Understanding Task

Clarita is helping Carlos calculate his height at different locations around a Ferris wheel. They have noticed when they use their formula $h(t) = 30 + 25\sin(\theta)$ their calculator gives them correct answers for the height even when the angle of rotation is greater than 90°. They don't understand why, since right triangle trigonometry only defines the sine for acute angles.

Carlos and Clarita are making notes of what they have observed about this new way of defining the sine that seems to be programmed into the calculator.

Carlos: "For some angles the calculator gives me positive values for the sine of the angle, and for some angles it gives me negative values."

1. Without using your calculator, list at least five angles of rotation for which the value of the sine produced by the calculator should be positive.

2. Without using your calculator, list at least five angles of rotation for which the value of the sine produced by the calculator should be negative.

Clarita: "Yeah, and sometimes we can't even draw a triangle at certain positions on the Ferris wheel, but the calculator still gives us values for the sine at those angles of rotation."

3. List possible angles of rotation that Clarita is talking about—positions for which you can't draw a reference triangle. Then, without using your calculator, give the value of the sine that the calculator should provide at those positions.

Carlos: "And, because of the symmetry of the circle, some angles of rotation should have the same values for the sine."

4. Without using your calculator, list at least five pairs of angles that should have the same sine value.

Clarita: "Right! And if we go around the circle more than once, the calculator still gives us values for the sine of the angle of rotation, and multiple angles have the same value of the sine."

5. Without using your calculator, list at least five sets of multiple angles of rotation where the calculator should produce the same value of the sine.

Carlos: "So how big can the angle of rotation be and still have a sine value?"

Clarita: "Or how small?"

6. How would you answer Carlos and Clarita's questions?

Carlos: "And while we are asking questions, I'm wondering how big or how small the value of the sine can be as the angles of rotation get larger and larger?"

7. Without using a calculator, what would your answer be to Carlos' question?

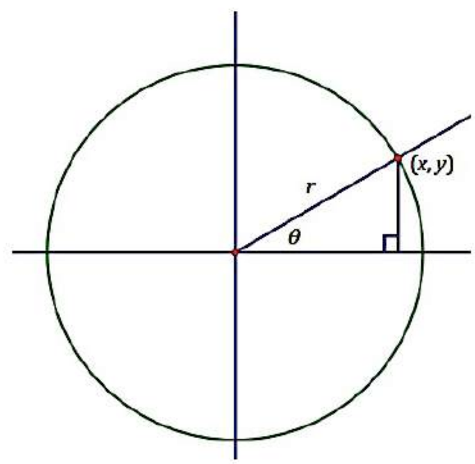
Clarita: "Well, whatever the calculator is doing, at least it's consistent with our right triangle definition of sine as the ratio of the length of the side opposite to the length of the hypotenuse for angles of rotation between 0 and 90°."

Part 2

Carlos and Clarita decide to ask their math teacher how mathematicians have defined sine for angles of rotation, since the ratio definition no longer holds when the angle isn't part of a right triangle. Here is a summary of that discussion.

We begin with a circle of radius *r* whose center is located at the origin on a rectangular coordinate grid. We represent **an angle of rotation in standard position** by placing its vertex at the origin, the *initial ray* oriented along the positive *x*-axis, and its *terminal ray* rotated θ degrees counterclockwise around the origin when θ is positive and clockwise when θ is negative. Let the ordered pair (x, y) represent the point when the terminal ray intersects the circle. (See the diagram at right, which Clarita diligently copied into her notebook.)

In this diagram, angle θ is between 0 and 90°; therefore, the terminal ray is in quadrant I. A right triangle has been drawn in quadrant I, similar to the right triangles we have drawn in the Ferris wheel tasks.

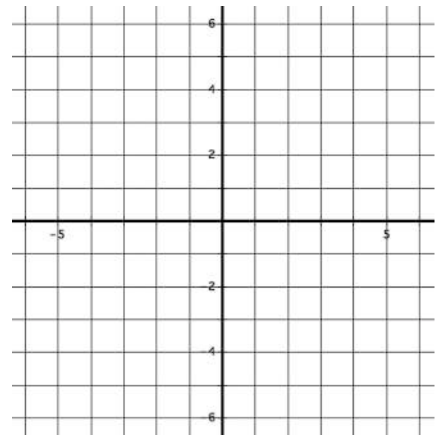


8. Based on this diagram and the right triangle definition of the sine ratio, find an expression for $sin\ \theta$ in terms of the variables *x, y* and *r.*

$sin\ \theta =$

We will use this definition for <u>any</u> angle of rotation. Let's try it out for a specific point on a particular circle.

9. Consider the point (-3, 4), which is on the circle $x^2 + y^2 = 25$.



a. What is the radius of this circle?

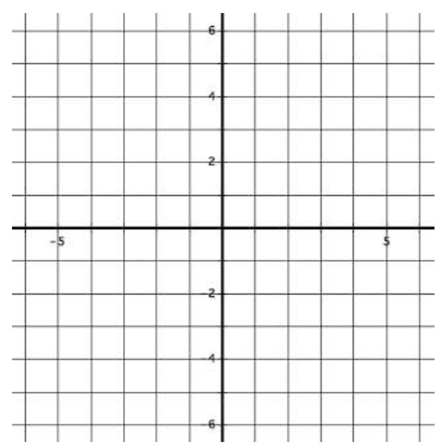
b. Draw the circle and the angle of rotation, showing the initial and terminal ray.

c. For the angle of rotation you just drew, what would the value of the sine be if we use the definition we wrote for sine in question 8?

d. What is the measure of the angle of rotation? How did you determine the size of the angle of rotation?

e. Is the calculated value based on this definition the same as the value given by the calculator for this angle of rotation?

10. Consider the point (-1, -3), which is on the circle $x^2 + y^2 = 10$.



a. What is the radius of this circle?

b. Draw the circle and the angle of rotation, showing the initial and terminal ray.

c. For the angle of rotation you just drew, what would the value of the sine be if we use the definition we wrote for sine in question 8?

d. What is the measure of the angle of rotation? How did you determine the size of the angle of rotation?

e. Is the calculated value based on this definition the same as the value given by the calculator for this angle of rotation?

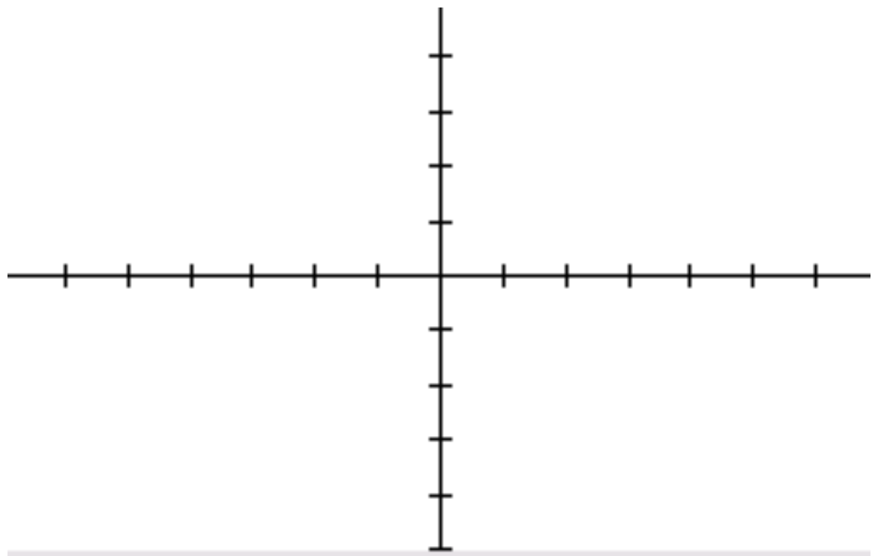
READY, SET, GO! Name Period Date

READY

Topic: Graphing a curve

1. Graph the table of values. Connect your points with a smooth curve.

x	y
-6	0
-5	-3
-4	-4
-3	-3
-2	0
-1	3
0	4
1	3
2	0
3	-3
4	-4
5	-3
6	0



2. Identify the maximum and minimum values of the curve.

3. This curve repeats itself. (It's called a *periodic function*.) Find the length of the interval that would allow you to see **exactly** one full length of the curve.

4. The curve is positive on the interval (-2, 2). Identify two more intervals where this curve will be positive.

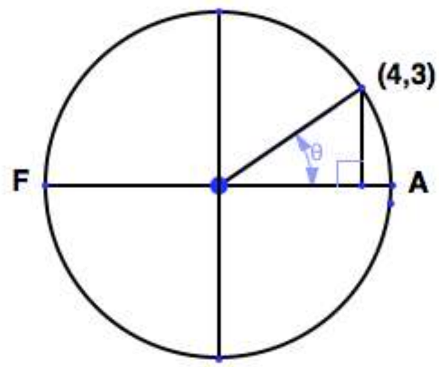
SET

Topic: Finding values of sine in the coordinate plane

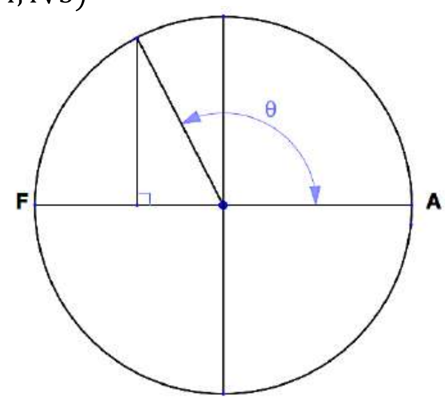
Use the given point on the circle to find the value of $sin\theta$. Then find the value of θ.

Recall $r = \sqrt{x^2 + y^2} and\ sin\theta = \frac{y}{r}.$

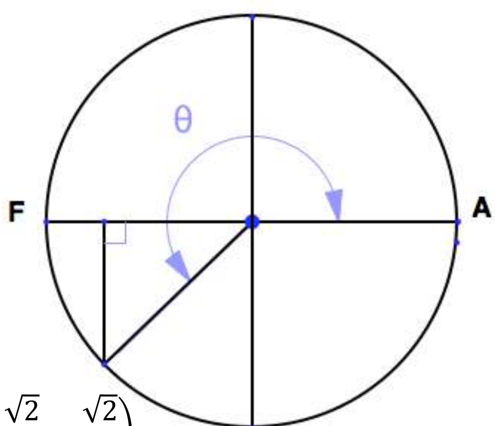
5.



6. $\left(-4, 4\sqrt{3}\right)$

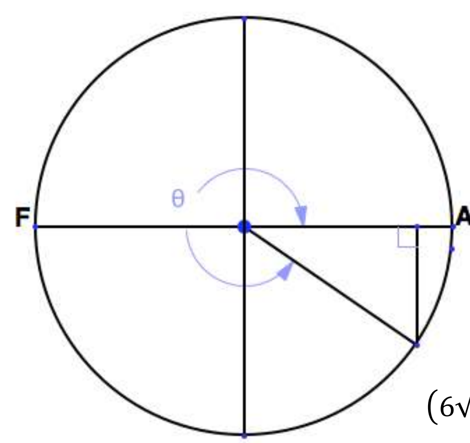


7.



$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

8.



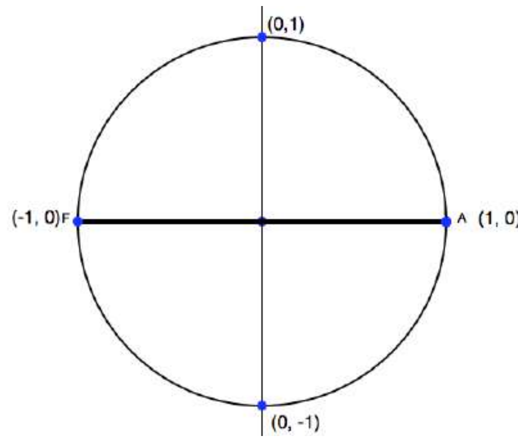
$\left(6\sqrt{3}, -6\right)$

9. In each graph above, the angle of rotation is indicated by an arc and θ. Describe the angles of rotation that make the y-values of the points be positive and the angles of rotation that make the y-values be negative.

10. What do you notice about the y-values and the value of sine in the graphs above?

11. In the graph at the right, the radius of the circle is 1 unit. The intersections of the circle and the axes are labeled. Based on your observation in #10, what do you think the value of sine might be for the following values of θ:

    90°? 180°? 270°? 360°?



GO

Topic: Solving problems using right angle trigonometry

Make a sketch of the following problems, then solve.

12. A kite is aloft at the end of a 1500 foot string. The string makes an angle of 43° with the ground. How far above the ground is the kite? (Round your answer to the nearest foot.)

13. A ladder leans against a building. The top of the ladder reaches a point on the building that is 12 feet above the ground. The foot of the ladder is 4 feet from the building. Find to the nearest degree the measure of the angle that the ladder makes with the level ground. What is the angle the ladder makes with the building?

14. The shadow of a flagpole is 40.6 meters long when the angle of elevation of the sun is 34.6°. Find the height of the flagpole.

15. The angle of depression from the top of a building to a car parked in the parking lot is 32.5°. How far from the top of the building is the car on the ground, if the building is 252 meters high?

6.4 More Ferris Wheels

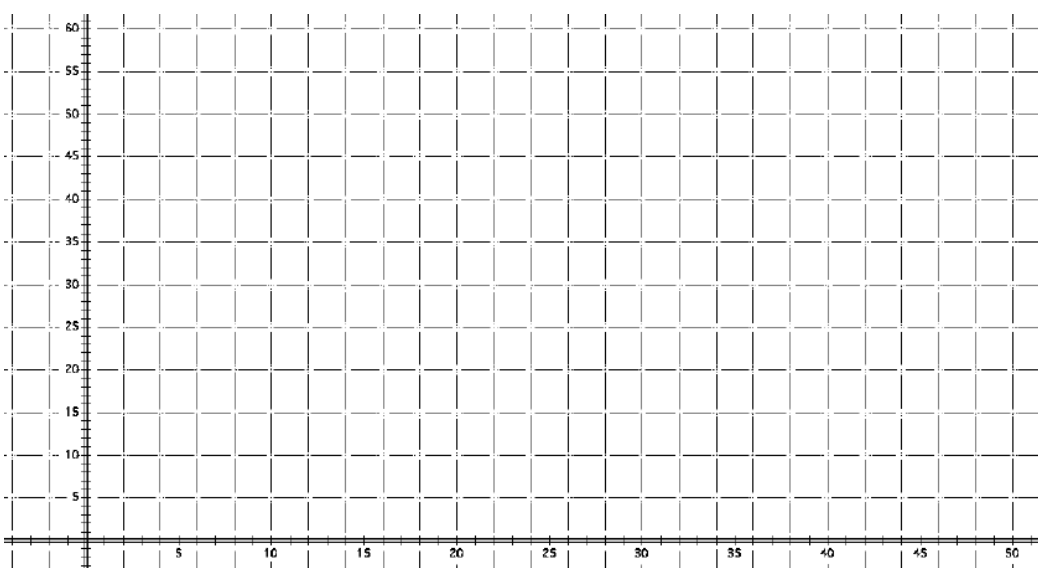
A Solidify Understanding Task

In a previous task, *"Sine" Language*, you calculated the height of a rider on a Ferris wheel at different times t, where t represented the elapsed time after the rider passed the position farthest to the right of the Ferris wheel.

Recall the following facts for the Ferris wheel in the previous tasks:

- The Ferris wheel has a radius of 25 feet
- The center of the Ferris wheel is 30 feet above the ground
- The wheel makes one complete revolution counterclockwise every 20 seconds

1. Based on the data you calculated, as well as any additional insights you might have about riding on Ferris wheels, sketch a graph of the height of a rider on this Ferris wheel as a function of the time elapsed since the rider passed the position farthest to the right of the Ferris wheel. (We can consider this position as the rider's starting position at time t = 0.)

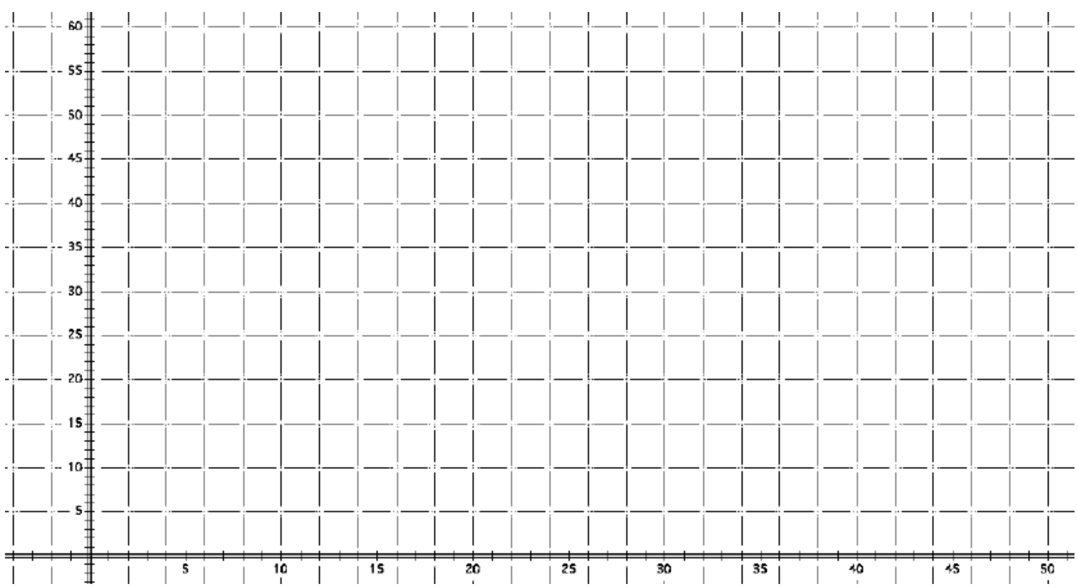


2. Write the equation of the graph you sketched in question 1.

3. Of course, Ferris wheels do not all have this same radius, center height, or time of rotation. Describe a different Ferris wheel by changing some of the facts listed above. For example, you can change the radius of the wheel, the height of the center, or the amount of time it takes to complete one revolution. You can even change the direction of rotation from counterclockwise to clockwise. If you want, you can change more than one fact. Just make sure your description seems reasonable for the motion of a Ferris wheel.

Description of my Ferris wheel:

4. Sketch a graph of the height of a rider on your Ferris wheel as a function of the time elapsed since the rider passed the position farthest to the right of the Ferris wheel.



5. Write the equation of the graph you sketched in question 4.

6. We began this task by considering the graph of the height of a rider on a Ferris wheel with a radius of 25 feet and center 30 feet off the ground, which makes one revolution counterclockwise every 20 seconds. How would your graph change if:

   - the radius of the wheel was larger or smaller?

   - the height of the center of the wheel was greater or smaller?

   - the wheel rotates faster or slower?

7. How does the equation of the rider's height change if:

   - the radius of the wheel is larger or smaller?

   - the height of the center of the wheel is greater or smaller?

   - the wheel rotates faster or slower?

8. Write the equation of the height of a rider on each of the following Ferris wheels t seconds after the rider passes the farthest right position.

   a. The radius of the wheel is 30 feet, the center of the wheel is 45 feet above the ground, and the angular speed of the wheel is 15 degrees per second counterclockwise

   b. The radius of the wheel is 50 feet, the center of the wheel is at ground level (you spend half of your time below ground), and the wheel makes one revolution *clockwise* every 15 seconds.

READY, SET, GO! Name Period Date

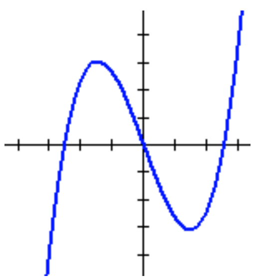
READY

Topic: Identifying *even* and *odd* functions from a graph

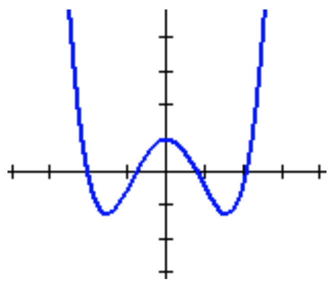
The *graphs* of even and odd functions make it easy to identify the type of function. Remember that an **even** function has a line of symmetry along the y-axis, while an **odd** function has 180° rotational symmetry.

Label the following functions as *even, odd*, or *neither.*

1.



2.



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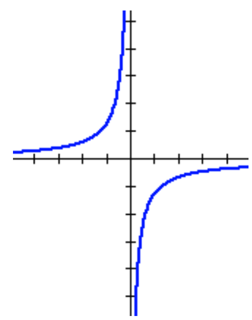

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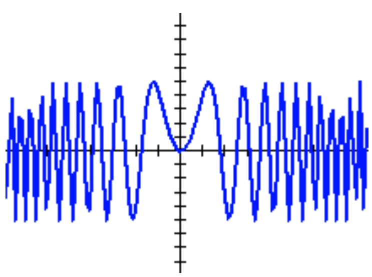

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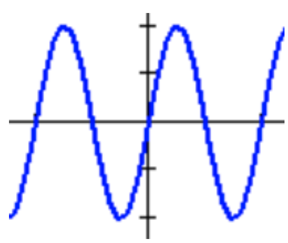

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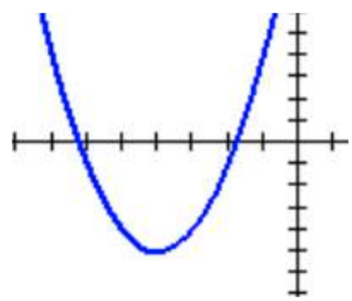

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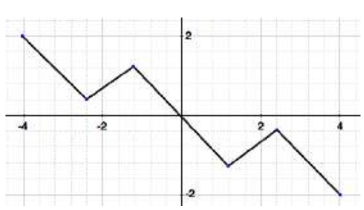

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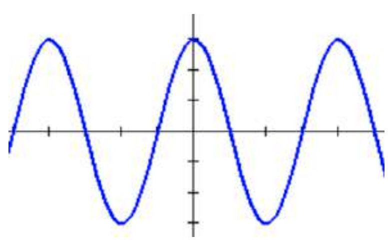

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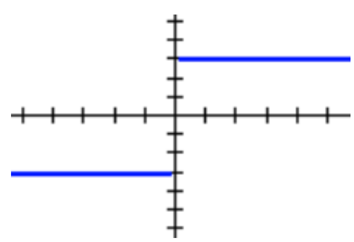


SET

Topic: Describing transformations on functions

Describe the transformation(s) on the parabola in the following equations.

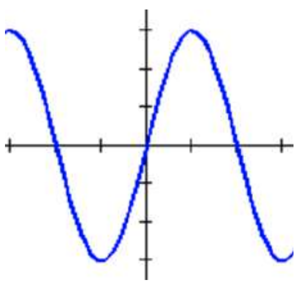
10. $y = x^2 + 5$ 11. $y = x^2 - 1$ 12. $y = -x^2$ 13. $y = 4x^2$

Match the equation with the correct graph. The scale of the x-axis is 90°. The scale of the y-axis is 1.

a. $y = \sin 2x$ b. $y = (\sin x) + 2$ c. $y = 3\sin x$

d. $y = -(\sin x) - 2$ e. $y = -2\sin x$ f. $y = 3\sin 2x$

14.



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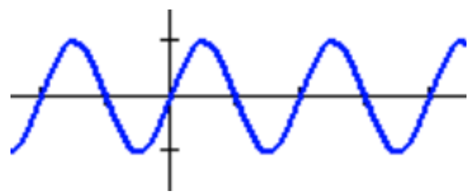

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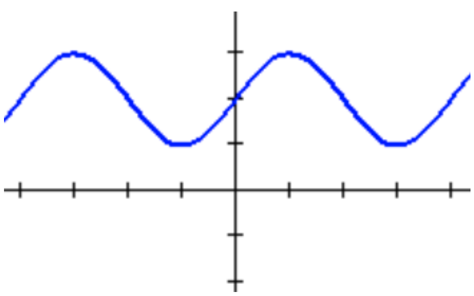

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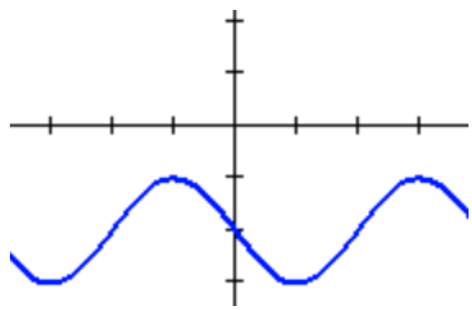

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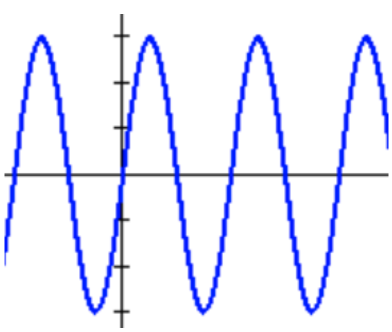

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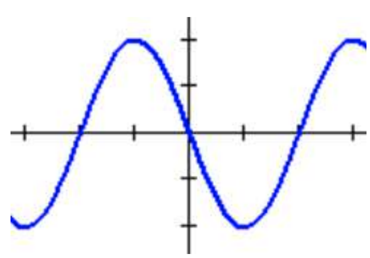


GO

Topic: Recognizing positive and negative angles of rotation

A positive angle of rotation is counter-clockwise. Let's find out why. In the following examples, indicate whether the customary direction of rotation is **counter-clockwise** by placing a (+) sign next to it or **clockwise** by placing a (–) sign next to it.

20. __________ Sprinters racing around a track

21. _________ The direction you turn the pages as you read a book

22. _________ A car in America traveling through a roundabout

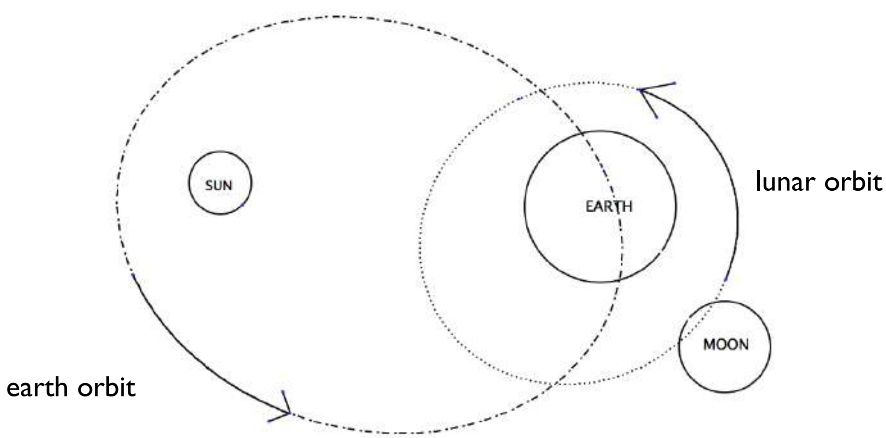
23. _________ Turning a water faucet to the on position

24. _________ A car in Australia circling in a roundabout (See sign.)

Wikimedia commons

25. ________ The rotation of the earth around the sun according to the diagram below.

26. ________ The rotation of the moon around the earth. (See diagram)



6.5 Moving Shadows

A Practice Understanding Task

In spite of his nervousness, Carlos enjoys his first ride on the amusement park Ferris wheel. He does, however, spend much of his time with his eyes fixed on the ground below him. After a while, he becomes fascinated with the fact that since the sun is directly overhead, his shadow moves back and forth across the ground beneath him as he rides around on the Ferris wheel.

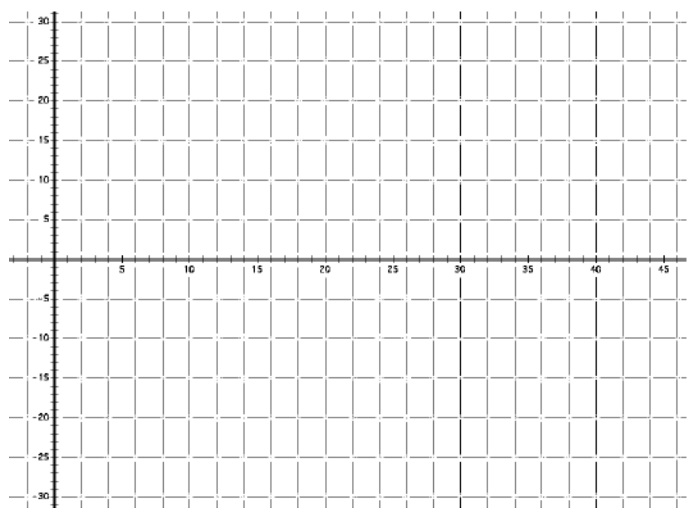
Recall the following facts for the Ferris wheel Carlos is riding:

- The Ferris wheel has a radius of 25 feet
- The center of the Ferris wheel is 30 feet above the ground
- The Ferris wheel makes one complete rotation counterclockwise every 20 seconds

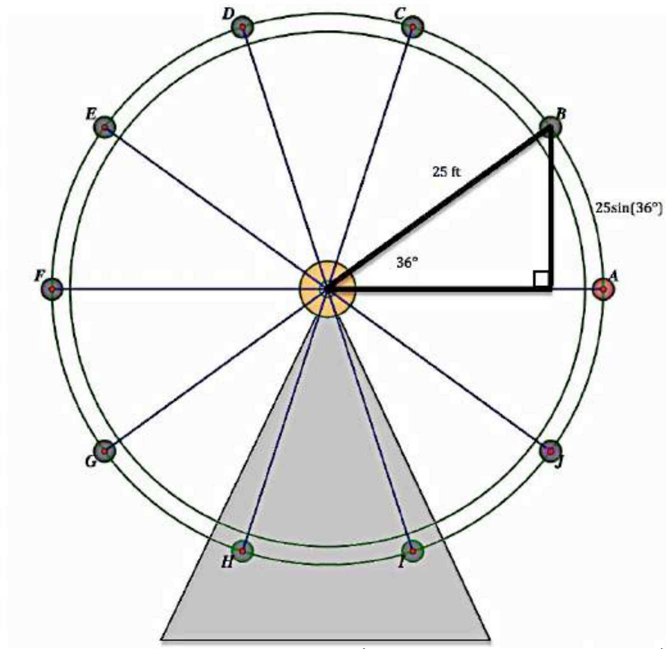
To describe the location of Carlos' shadow as it moves back and forth on the ground beneath him, we could measure the shadow's horizontal distance (in feet) to the right or left of the point directly beneath the center of the Ferris wheel, with locations to the right of the center having positive value and locations to the left of the center having negative values. For instance, in this system Carlos' shadow's location will have a value of 25 when he is at the position farthest to the right on the Ferris wheel, and a value of -25 when he is at a position farthest to the left.

1. What would Carlos' position be on the Ferris wheel when his shadow is located at 0 in this new measurement system?

2. Sketch a graph of the horizontal location of Carlos' shadow as a function of time *t*, where *t* represents the elapsed time after Carlos passes position A, the farthest right position on the Ferris wheel.



3. Calculate the location of Carlos' shadow at the times t given in the following table, where t represents the number of seconds since Carlos passed the position farthest to the right on the Ferris wheel. Keep track of any regularities you notice in the ways you calculate the location of the shadow. As you calculate each location, plot Carlos' position on the diagram at the right.



Elapsed time since passing position A	Calculations	Horizontal position of the rider
1 sec		
2 sec		
3 sec		
4 sec		
8 sec		
14 sec		
18 sec		
23 sec		
28 sec		
40 sec		

4. Write a general formula for finding the location of the shadow at any instant in time.

READY, SET, GO! Name Period Date

READY

Topic: Comparing radius and arc length

Stage coaches and wagons in the 1800s had wheels that were smaller in the front than in the back. The front wheels had 12 spokes. The top of the front wheel measured 44 inches from the ground. The rear wheels had 16 spokes. The top of the rear wheel measured 59 inches from the ground. (For these problems disregard the hub at the center of the wheel. Assume the spokes meet in the center at a point. **Leave π in your answers.**)

1. Find the area and the circumference of each wheel.

2. Determine the central angle between the spokes on each wheel.

3. Find the distance on the circumference between two consecutive spokes for each wheel.

4. The wagons could cover a distance of 15 miles per day. How many more times would the front wheel turn than the back wheel on an average day?

5. A wheel rotates r times per minute. Write a formula for how many degrees it rotates in t seconds.

SET

Topic: Determining values of cosine in a circle

Use the given point on the circle to find the value of cosine.
Recall $r = \sqrt{x^2 + y^2} and\ cos\ \theta = \frac{x}{r.}$

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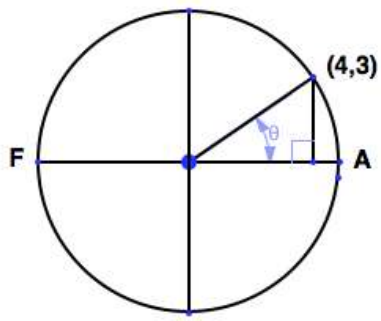

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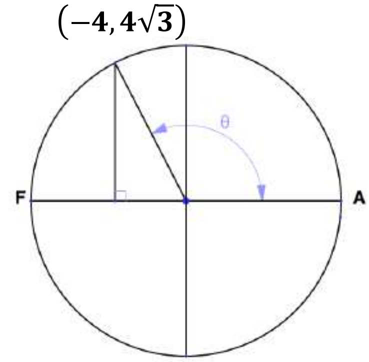

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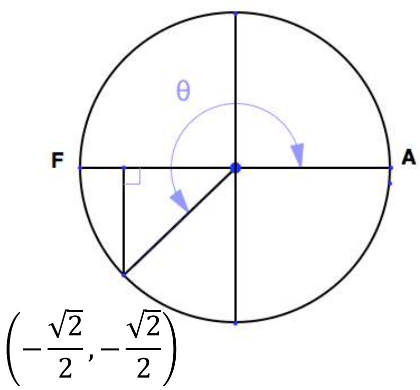

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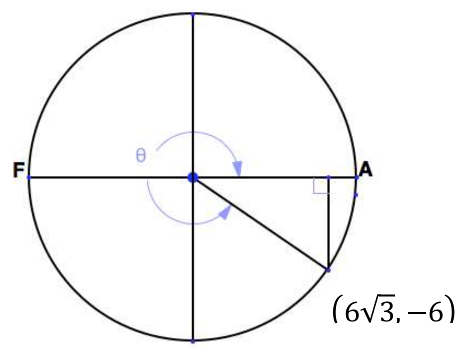
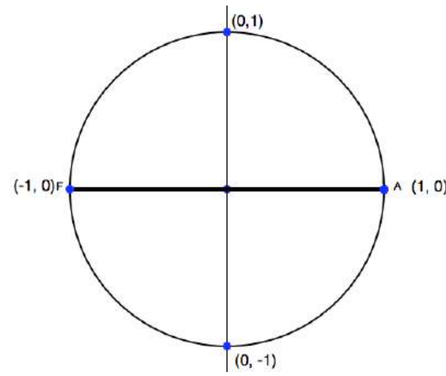


10. In each graph, the angle of rotation is indicated by an arc and θ. Describe the angles of rotation that make the x-values of the points positive and the angles of rotation that make the x-values negative.

11. What do you notice about the x-values and the value of cosine in the graphs?

12. In the graph at the right, the radius of the circle is 1. The intersections of the circle and the axes are labeled. Based on your observation in #11, what do you think the value of cosine might be for the following values of θ:

90°?   180°?   270°?   360°?



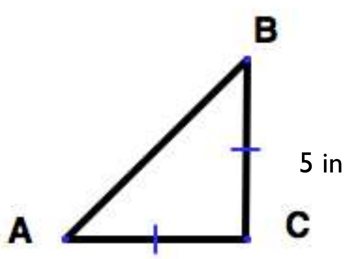
GO

Topic: Reviewing the measurements in special triangles

Use the given information to find the missing sides and the missing angles.

Triangle ABC is a right triangle. Angle C is the right angle. Write the **exact** values for the sides.

13.



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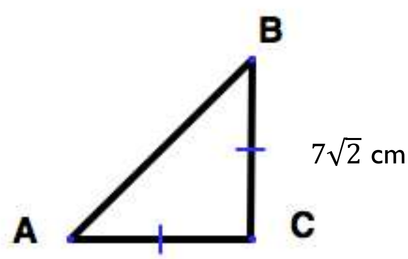

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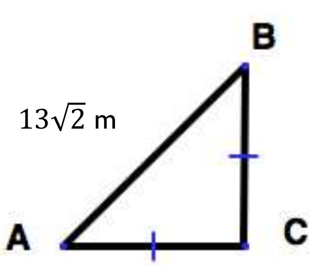

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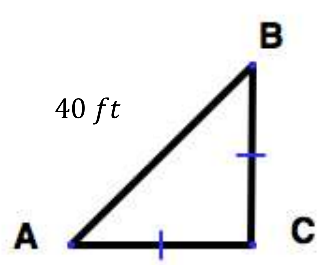

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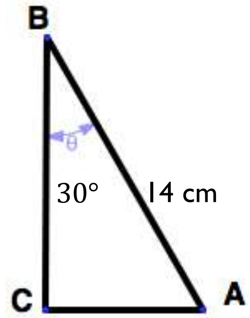

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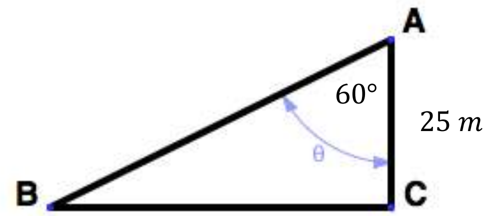

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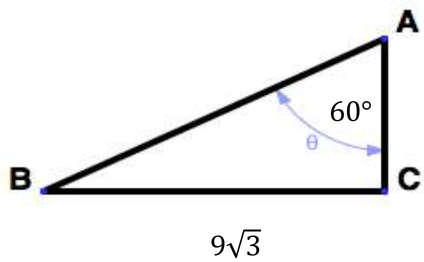

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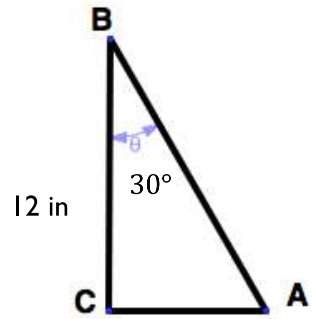
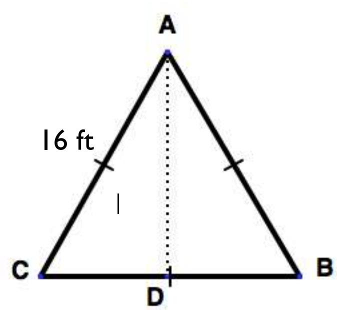


Find AD in the figures below.

21.



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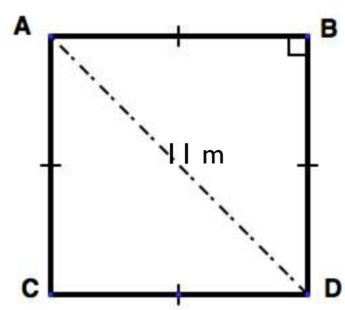


Remember that π is simply a number.

23. If you purchased π gallons of gasoline, about how many gallons of gas did you buy?

24. If you were paid 5π dollars per hour, about how many dollars would you make in 8 hours?

25. If you slept 2π hours each night, about how many hours of sleep would you get per night.

6.6 Diggin' It

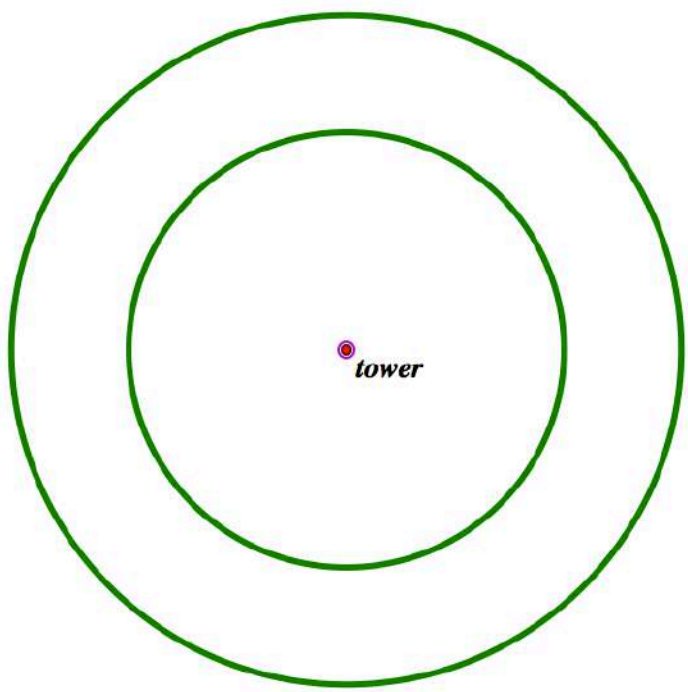
A Develop Understanding Task

Alyce, Javier, and Veronica are responsible for preparing a recently discovered archeological site. The people who used to inhabit this site built their city around a central tower. The first job of the planning team is to mark the site using stakes so they can keep track of where each discovered item was located.

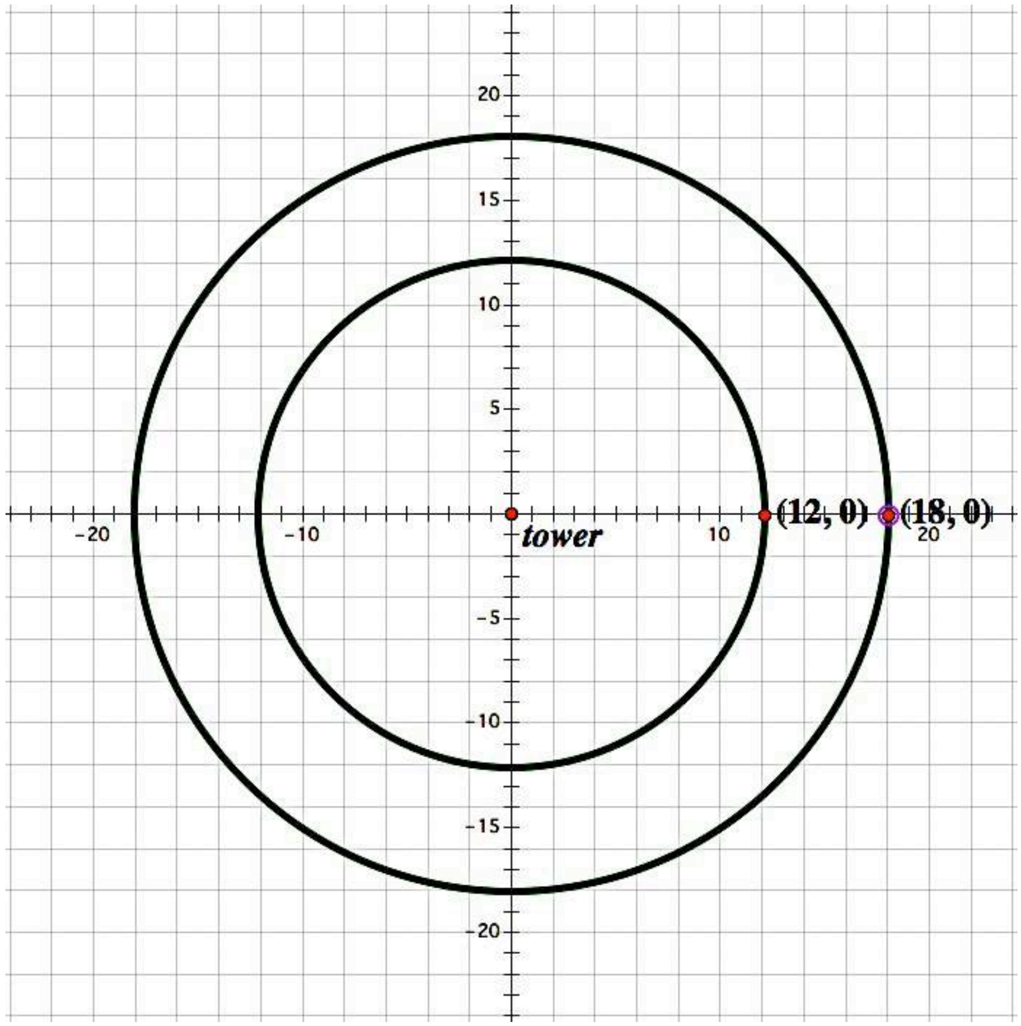
Part 1

1. Alyce suggests that the team place stakes in a circle around the tower, with the distance between the markers on each circle being equal to the radius of the circle. Javier likes this idea but says that using this strategy, the number of markers needed would depend on how far away the circle is from the center tower. Do you agree or disagree with Javier's statement? Explain.

2. Show where the stakes would be located using Alyce's method if one set of markers were to be placed on a circle 12 meters from the center and a second set on a circle 18 meters from the center.



Part 2



3. After looking at the model, Veronica says they need to have more stakes if they intend to be specific with the location of the artifacts. Since most archaeological sites use a grid to mark off sections, Veronica suggests evenly spacing 12 stakes around each circle and using the coordinate grid to label the location of these stakes. The central tower is located at the origin and the first of each set of 12 stakes for the inner and outer circles is placed at the points (12, 0) and (18, 0), respectively. Alyce also wants to make sure they record the distance around the circle to each new stake from these initial stakes. Your job is to determine the x and y-coordinates for each of the remaining stakes on each circle, as well as the arc length from the points (12, 0) or (18, 0), depending on which circle the stake is located. Keep track of the method(s) you use to find these values.

Part 3

Javier suggests they record the location of each stake and its distance around the circle for the set of stakes on each circle. Veronica suggests it might also be interesting to record the ratio of the arc length to the radius for each circle.

4. Help Javier and Veronica complete this table.

	Inner Circle: *r* = 12 meters			**Outer Circle: *r* = 18 meters**		
	Location	**Distance from (12,0) along circular path**	**Ratio of arc length to radius**	**Location**	**Distance from (18,0) along circular path**	**Ratio of arc length to radius**
Stake 1	(12, 0)	0	0	(18, 0)	0	0
Stake 2						
Stake 3						
Stake 4						
Stake 5						
Stake 6						
Stake 7						
Stake 8						
Stake 9						
Stake 10						
Stake 11						
Stake 12						

5. What interesting patterns might Alyce, Javier and Veronica notice in their work and their table? Summarize any interesting things you have noticed.

READY, SET, GO! Name Period Date

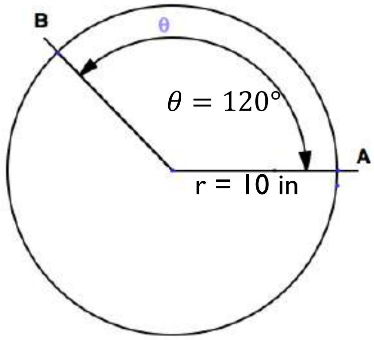
READY

Topic: Finding the length of an arc using proportions

Use the given degree measure of the central angle to set up a proportion to find the length of arc AB.

<u>Leave π in your answers.</u> Recall $s = \frac{\theta}{360°}(d\pi)$ *where s is the arc length.*

1.



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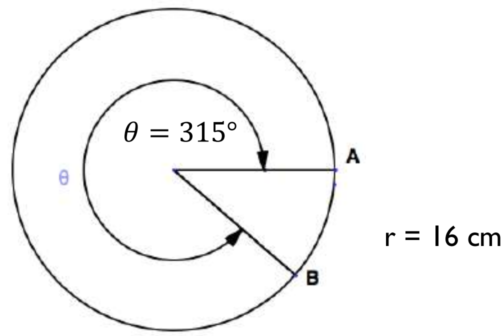

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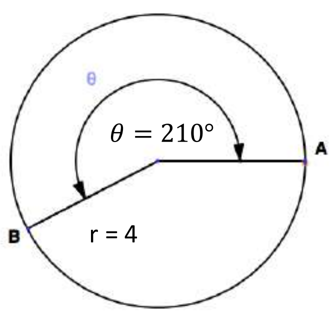

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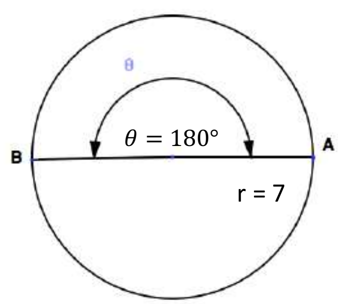

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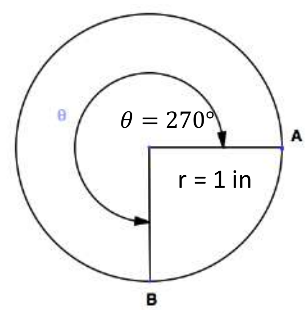


6. The circumference of circle A is 400 meters. The circumference of circle B is 800 meters. What is the relationship between the radius of circle A and the radius of circle B?

   Justify your answer.

SET

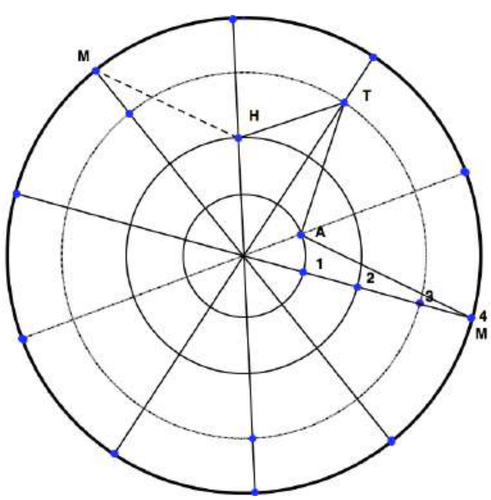
Topic: Describing the location of a point by the angle of rotation and the radius

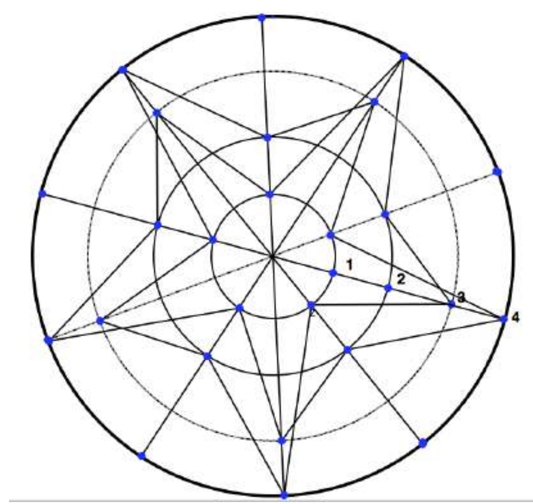
It is possible to identify the location of a point on the edge of a circle in several different ways. One way is to use rectangular coordinates (x, y). In this activity, you will be graphing "words" by using letters to identify points around a circle. The size of the rotation or θ will be the same while the length of the radius will change. First select a word. Avoid words containing 5 letters or multiples of 5. I am choosing the word MATH. Assign a number to each letter of your word according to the table below. The numbers correspond to the concentric circles. You can begin on any spoke. Move from one spoke to the next in a positive rotation. Make a dot at the intersection of the spoke and the circle corresponding with the number of the letter you are on. You will need to make more than one rotation of the circle in order to close your figure.

Circle numbers and their corresponding letters. The letters for "MATH" are high-lighted.

Circle 1: A,D,K,L,N,V,Z	**Circle 2: E, U, G, H, Q, U, X**
Circle 3: I, C, F, J, T, S, Y	**Circle 4: O, B, E, M, R, P, W**

The word MATH will use the numbered circles 4 1 3 2 in that order. You can begin on any spoke. I began on the spoke with the numbers. I made a dot on 4, rotated to the next spoke and made a dot on 1. I connected the two dots. Then I moved to circle 3, made a dot, connected the segment, and moved to circle 2. You can see MATH marked on the diagram. After marking H, I started over with M on the next spoke. (See the dotted line.) Continue spelling MATH and rotating around the circle until the figure is closed and the path repeats itself. The figure at the right is the completed graph of the word MATH. I always knew MATH was beautiful!

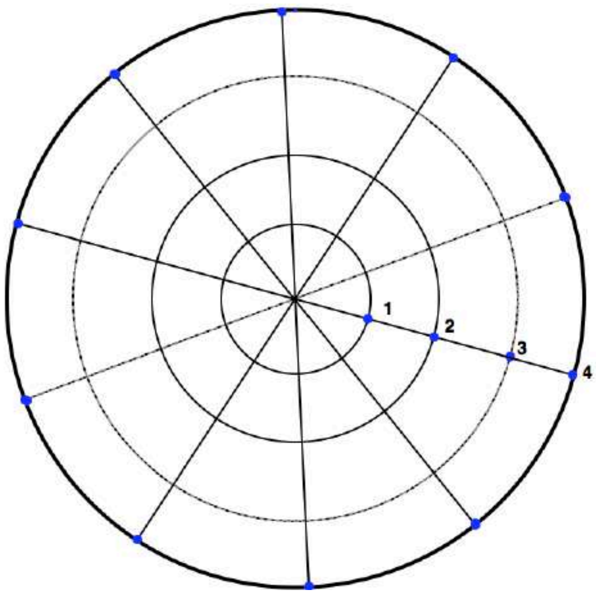




Now it's your turn. Select a word. Short ones are best. Assign the numbers and begin.

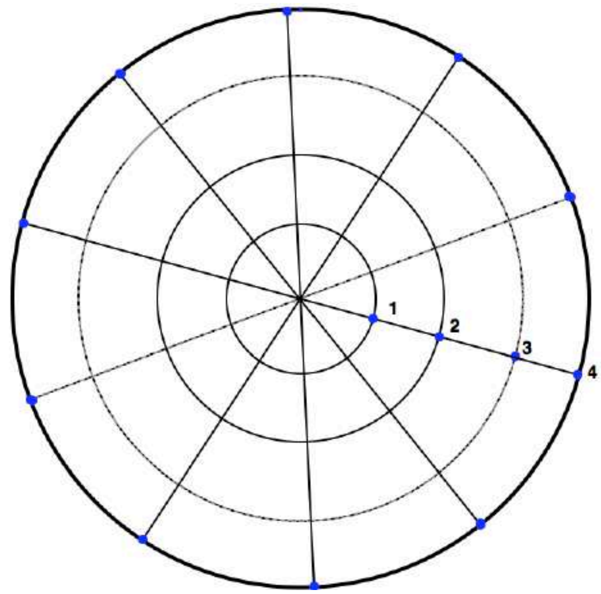
7.

word ________________________________



8.

word ________________________________



9. What is the angle between each spoke in the grid above?

10. How many degrees did it take to graph MATH once? (From M to H?)

11. How many degrees did it take to graph MATHM? (From M to the M again)

12. How many times did I need to spell the word MATH to complete the graph?

13. How many rotations did it take?

Can you figure out the answer to this question without counting? Explain.

GO

Topic: Converting angles between radians and degrees

Recall that there are 360 **degrees** in a full circle and 2π **radians** in a full circle. Therefore, $360^{\circ} = 2\pi$ radians. If we divide both sides of the equation by 2, we create another identity $180^{\circ} = 1\pi\ radians$. We can use this identity to convert degrees to radians or radians to degrees.

Since $180^{\circ} = \pi\ radians$, it follows that $\frac{\pi\ radians}{180^{\circ}} = \frac{180^{\circ}}{\pi\ radians} = 1$.

If I want to convert 72° into radian measure, then I need the unit of degrees to cancel, so I will multiply 72° by $\frac{\pi\ radians}{180^{\circ}}$, example: $72^{\circ} \cdot \frac{\pi\ radians}{180^{\circ}} = \frac{72^{\circ} \times \pi\ radians}{180^{\circ}} = \frac{2\pi\ radians}{5}$.

The unit *radians* is usually left off. Hence, an angle that measures 72° is equivalent to a radian measure of $\frac{2\pi}{5}$.

Convert the following angles from degrees to radians or radians to degrees.

14. 45°

15. 15°

16. 54°

17. 135°

18. 300°

19. 270°

20. $\frac{5\pi}{6}$

21. $\frac{\pi}{8}$

22. $\frac{3\pi}{4}$

23. $\frac{7\pi}{5}$

24. $\frac{\pi}{18}$

25. $\frac{13\pi}{12}$

6.7 Staking It

A Solidify Understanding Task

After considering different plans for laying out the archeological site described in *Diggin' It,* Alyce, Javier and Veronica have decided to make concentric circles at 10-meter intervals from the central tower. They have also decided to use 16 stakes per circle, in order to have a few more points of reference. Using ropes of different lengths to keep the radius constant, they have traced out these circles in the sand. Because they know the circles will soon be worn away by the wind and by people's footprints, they feel a sense of urgency to locate the positions of the 16 stakes that will mark each circle. The team wants to be efficient and make as few measurements as possible.

Part 1

Veronica suggests they should locate the stakes around one circle and use those positions to mark where the stakes will go on all of the other circles.

1. What do you think about Veronica's idea? How will marking stake positions on one circle help them locate the positions of the stakes on all of the other circles?

Veronica has decided they should stake out the circle with a radius of 50 meters first. She is standing at the point (50,0) and knows she needs to move 22 ½° around the circle to place her next stake. But, she wonders, "How far is that?"

- Veronica decides she will find the distance by setting up a proportion using degree measurements.
- Alyce thinks they should find the distance by taking $^1/_{16}$ of the circumference.
- Javier thinks they should use radian measurement in their calculation.

2. Show how each team member will calculate this distance.

   <u>Veronica's Strategy</u>

   <u>Alyce's Strategy</u>

   <u>Javier's Strategy</u>

Part 2

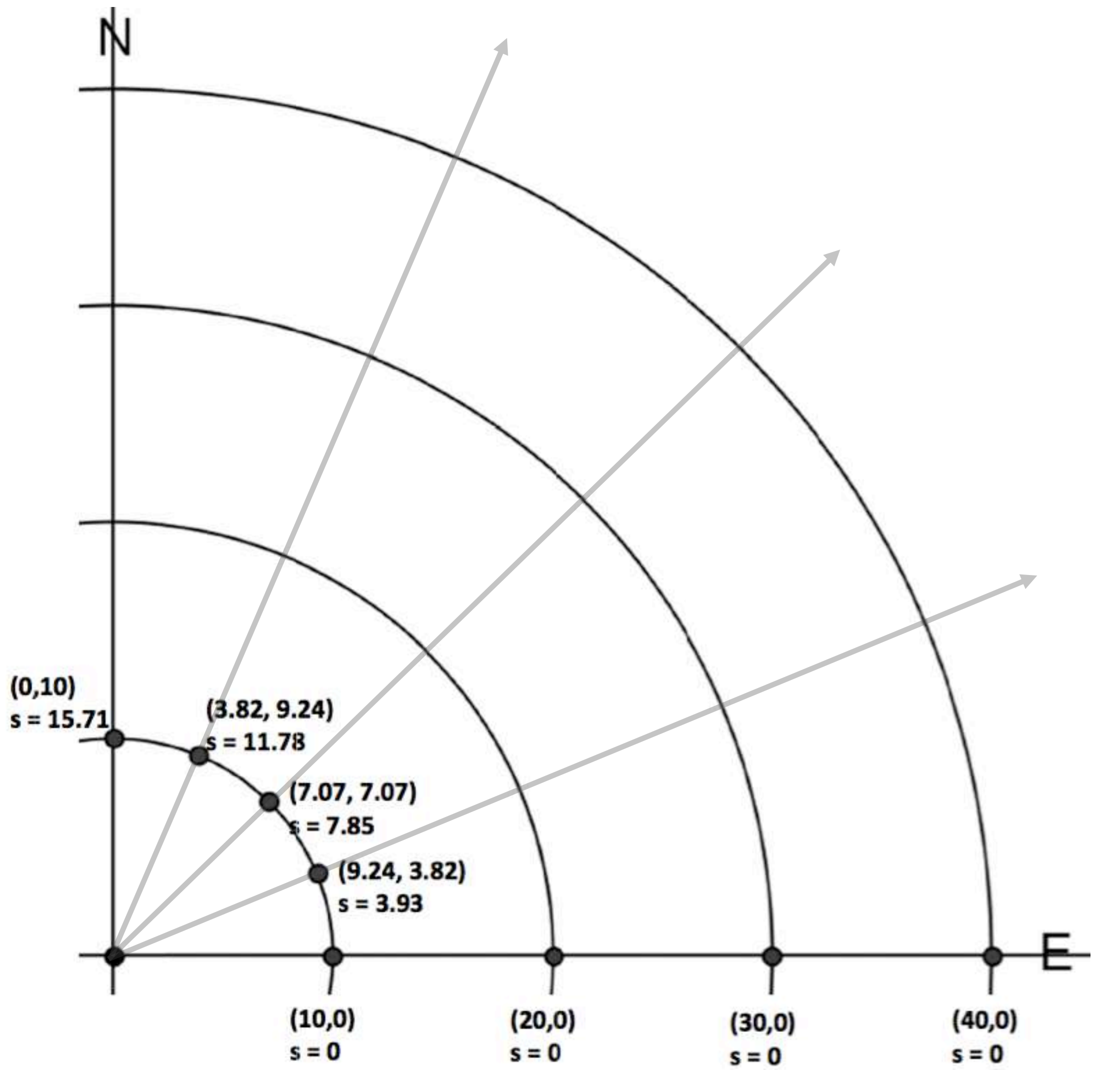
Javier has a different idea. He suggests they should figure out the locations of all of the stakes in quadrant I first, and then it would be easy to find the locations of the stakes in all the other quadrants by using the quadrant I locations.

3. What do you think about Javier's suggestion? How will marking the location of stakes in quadrant I help them figure out the location of the stakes in other quadrants?

Javier has already started working on his strategy and has completed the calculations for the 10-meter circle. (see Javier's diagram).

4. Develop a strategy to locate all of the other stakes in the first quadrant for these additional circles. Find the coordinates and arc lengths for each. Describe the strategy you used to make the fewest calculations for finding the coordinates and arc lengths for these additional stakes.

Javier's Diagram



READY, SET, GO! Name Period Date

READY

Topic: Finding the coordinates of points on a circle

Given the equation of a circle centered at (0,0), find one point in each quadrant that lies on the given circle.

1. $x^2 + y^2 = 25$ quadrant I ____________________ quadrant II ____________________
   quadrant III ____________________ quadrant IV ____________________

2. $x^2 + y^2 = 4$ quadrant I ________________ quadrant II ________________
   quadrant III ________________ quadrant IV ________________

3. $x^2 + y^2 = 36$ quadrant I ________________ quadrant II ________________
   quadrant III ________________ quadrant IV ________________

4. $x^2 + y^2 = 1$ quadrant I ________________ quadrant II ________________
   quadrant III ________________ quadrant IV ________________

5. $x^2 + y^2 = 9$ quadrant I ________________ quadrant II ________________
   quadrant III ________________ quadrant IV ________________

SET

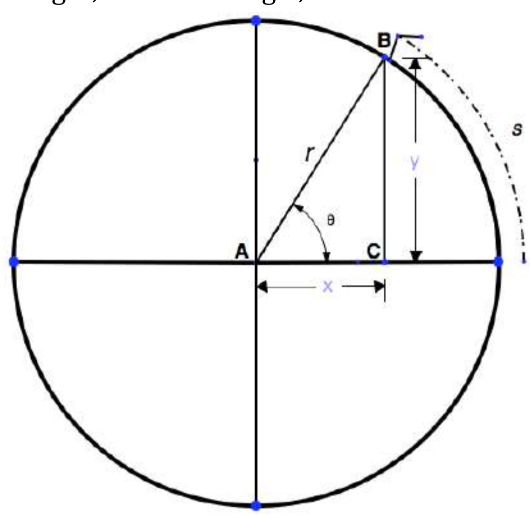
Topic: Locating points in terms of rectangular coordinates, arc length, reference angle, and radius

In the diagram triangle ABC is a right triangle.

Point B lies on the circle and is described by the rectangular coordinates (x, y).

s is the length of the arc subtended by angle θ.

r is the radius of circle A.



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Use the given information to answer the following questions.

6. B has the rectangular coordinates (5, 12).

a. Find r.

b. Find θ to the nearest tenth of a degree.

c. Find s by using the formula $s = \frac{\theta}{360^\circ}(d\pi)$.

d. Describe point B using the coordinates (r, θ).

e. Describe point B using the radius and arc length (r, s).

7. B has the rectangular coordinates (33, 56).

a. Find r.

b. Find θ to the nearest tenth of a degree.

c. Find s by using the formula $s = \frac{\theta}{360^\circ}(d\pi)$.

d. Describe point B using the coordinates (r, θ).

e. Describe point B using the radius and arc length (r, s).

8. B is described by (r, θ)

$$\text{where } \theta \approx 58.11^\circ \text{ and } r = 53.$$

a. Find (x, y) to the nearest whole number.

b. Find s by using the formula $s = \frac{\theta}{360^\circ}(d\pi)$.

c. Describe point B using the radius and arc length (r, s).

9. B is described by (r, θ)

$$\text{where } \theta \approx 25.01^\circ \text{ and } r = 85.$$

a. Find (x, y) to the nearest whole number.

b. Find s by using the formula $s = \frac{\theta}{360^\circ}(d\pi)$.

c. Describe point B using the radius and arc length (r, s).

10. B is described by (r, s)

$$\text{where } s \approx 46 \text{ and } r = 37.$$

a. Find (x, y) to the nearest whole number.

b. Find θ by using the formula $s = \frac{\theta}{360^\circ}(d\pi)$.

c. Describe point B using (r, θ)

11. B is described by (r, s)

$$\text{where } s \approx 62.26 \text{ and } r = 73.$$

a. Find (x, y) to the nearest whole number.

b. Find θ by using the formula $s = \frac{\theta}{360^\circ}(2\pi r)$.

c. Describe point B using (r, θ)

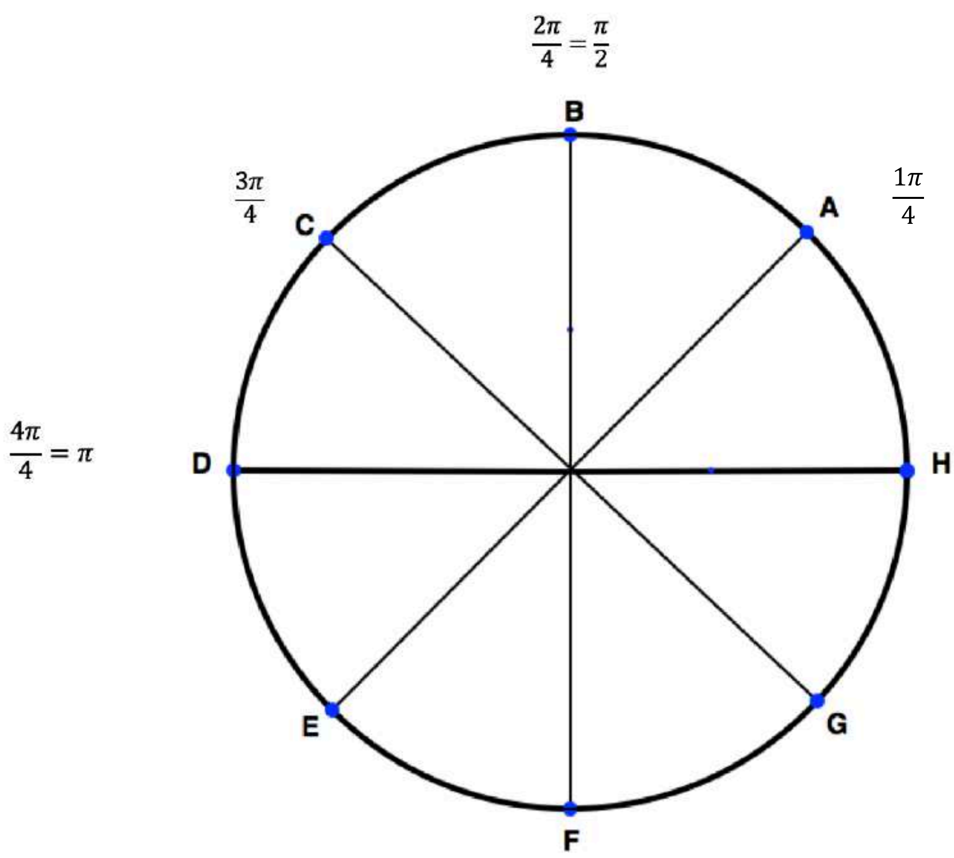
GO

Topic: Making sense of radian measure

Label each point on the circle with the measure of the angle of rotation. Angle measures should be in radians. (Recall a full rotation around the circle would be 2π radians.)

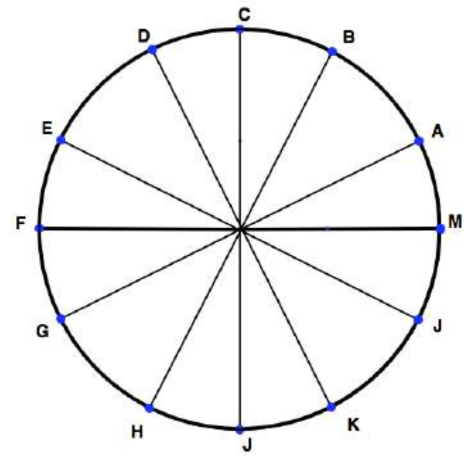
Example: The circle has been divided equally into 8 parts. Each part is equal to $\frac{2\pi}{8}$ *or* $\frac{\pi}{4}$ radians. Indicate how many parts of $\frac{\pi}{4}$ radians there are at each position around the circle.

Finish the example by writing the angle measures for points E, F, G, and H.

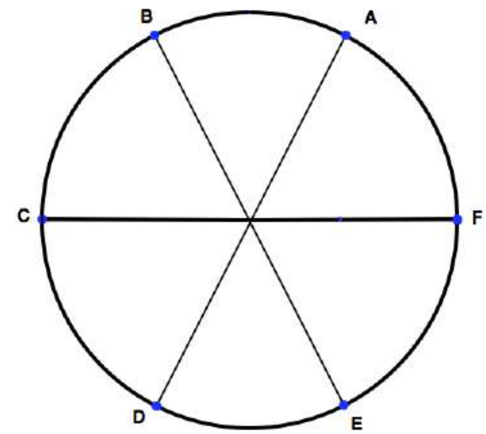


Label the figures below using a similar approach as the example.

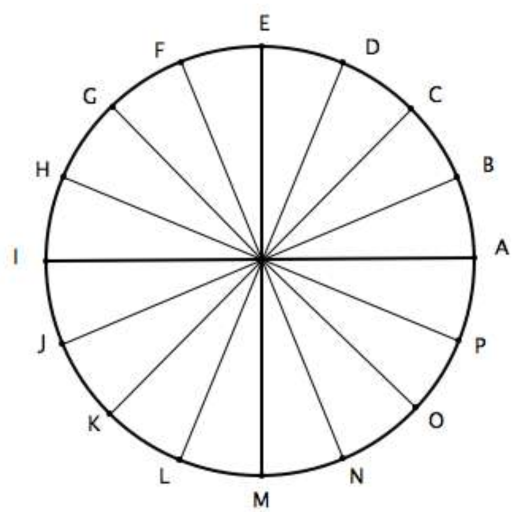
12.



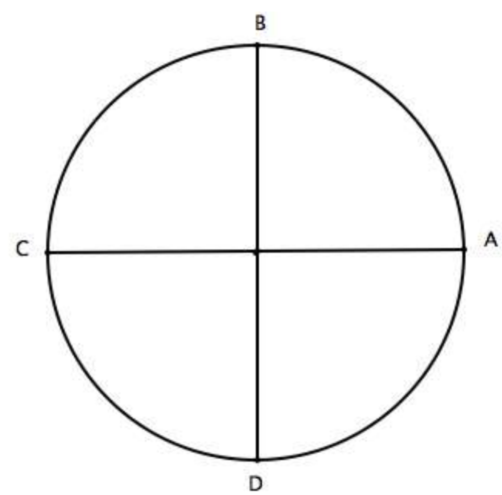
13.



14.



15.

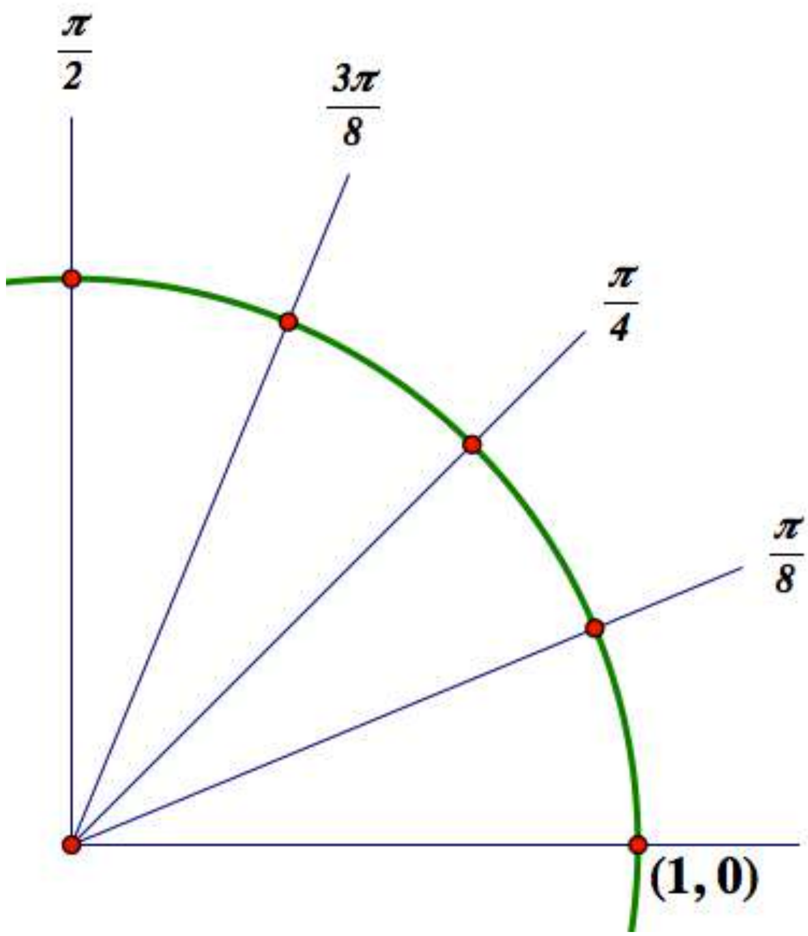


6.8 "Sine"ing and "Cosine"ing It

A Solidify Understanding Task

In the previous tasks of this module you have used the similarity of circles, the symmetry of circles, right triangle trigonometry and proportional reasoning to locate stakes on concentric circles. In this task we consider points on the simplest circle of all, the circle with a radius of 1, which is often referred to as "*the unit circle*."

Here is a portion of a unit circle—the portion lying in the first quadrant of a coordinate grid. As in the previous task, *Staking It*, this portion of the unit circle has been divided into intervals measuring $\frac{\pi}{8}$ radians. As in the previous task, find the coordinates of each of the indicated points in the diagram. Also find the arc length, *s*, from the point (1, 0) to each of the indicated points.



Javier has been wondering if his calculator will allow him to calculate trigonometric values for angles measured in radians, rather than degrees. He feels like this will simplify much of his computational work when trying to locate the coordinates of stakes on the circles surrounding the central tower of the archeological site.

After consulting his calculator's manual, Javier has learned that he can set his calculator in radian mode. After doing so, he is examining the following calculations.

1. With your calculator set in radian mode, find each of the following values. Record your answers as decimal approximations to the nearest thousandth.

$\sin\left(\frac{\pi}{8}\right) =$ $\cos\left(\frac{\pi}{8}\right) =$ $\frac{\pi}{8} =$

$\sin\left(\frac{\pi}{4}\right) =$ $\cos\left(\frac{\pi}{4}\right) =$ $\frac{\pi}{4} =$

$\sin\left(\frac{3\pi}{8}\right) =$ $\cos\left(\frac{3\pi}{8}\right) =$ $\frac{3\pi}{8} =$

$\sin\left(\frac{\pi}{2}\right) =$ $\cos\left(\frac{\pi}{2}\right) =$ $\frac{\pi}{2} =$

2. The coordinates and arc lengths you found for points on the unit circle seem to be showing up in Javier's computations. Why is that so? That is, . . .

   - explain why the radian measure of an angle on the unit circle is the same as the arc length?

   - explain why the sine of an angle measured in radians is the same as the *y*-coordinate of a point on the unit circle?

- explain why the cosine of an angle measured in radians is the same as the *x*-coordinate of a point on the unit circle?

3. Based on this work, find the following without using a calculator:

$\sin\left(\frac{5\pi}{8}\right) =$ $\cos\left(\frac{7\pi}{8}\right) =$ $\cos(\pi) =$

READY, SET, GO! Name Period Date

READY

Topic: Reducing complex fractions

Write each of the following as a simple fraction. Rationalize the denominators when appropriate.

1. $\dfrac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$

2. $\dfrac{\frac{8\sqrt{3}}{5}}{\frac{1}{5}}$

3. $\dfrac{8}{\frac{1}{2}}$

4. $\dfrac{\frac{7\sqrt{3}}{2}}{\frac{1}{2}}$

5. $\dfrac{1}{\sqrt{2}}$

6. $\dfrac{3}{\sqrt{3}}$

7. $\dfrac{4}{\sqrt{8}}$

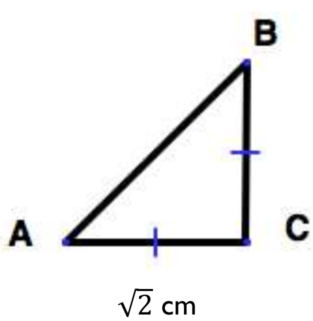
8. $\dfrac{\frac{2}{3}}{\frac{1}{2}}$

9. $\dfrac{\frac{2}{\sqrt{7}}}{\frac{5}{\sqrt{7}}}$

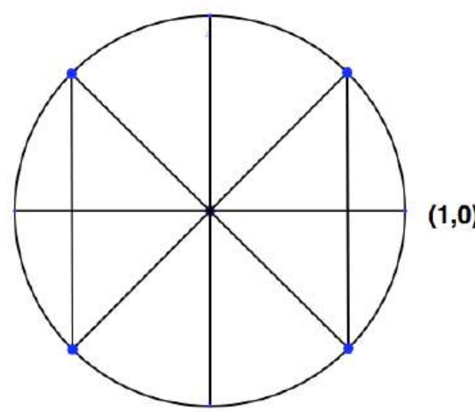
SET

Topic: Finding sine of an angle in radian measure

10. Triangle ABC is an isosceles right triangle. The length of one side is given. Fill in the values for the missing sides and angles A and B.



11. Label each point around the circle with the angle of rotation in radians starting from the point (1,0). (each section is equal)



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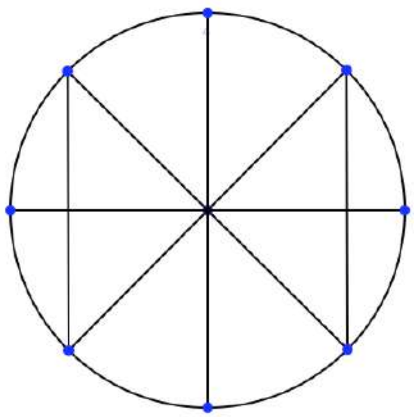
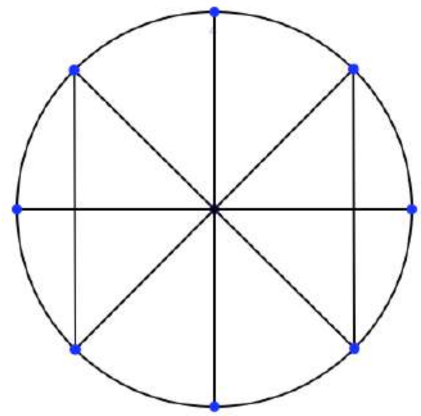
12. Use the values in #10 to write the **exact** coordinates of the 4 points on the circle below. Be mindful of the numbers that are negative.



13. Find the arc length, *s*, from the point (1,0) to each point around the circle. Record your answers as decimal approximations to the nearest thousandth.



Use your calculator to find the following values.

14. $\sin\frac{5\pi}{4} =$

15. $\sin\frac{7\pi}{4} =$

16. Why are both of your answers negative?

17. $\cos\frac{\pi}{4} =$

18. $\cos\frac{7\pi}{4} =$

19. Why are both of your answers positive?

20. $\sin\frac{3\pi}{4} =$

21. $\cos\frac{3\pi}{4} =$

22. Why is one answer positive and one answer negative?

GO

Topic: Recalling trigonometric values of special triangles

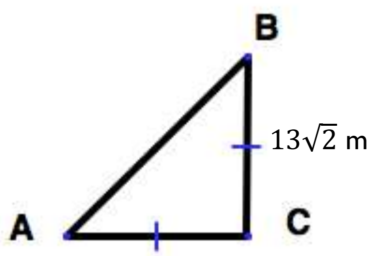
Angle C is the right angle in each of the triangles below. **Use the given information to find the missing sides and the missing angles. Then find the indicated trig values.** Rationalize denominators when appropriate. Do NOT change the values to decimals. Square roots are **exact** values. Decimal representations of the square roots are approximations.

23.

sin A =

cos A =

tan A =

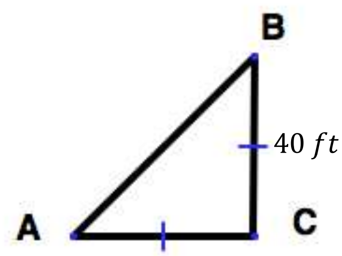


24.

sin B =

cos B =

tan B =



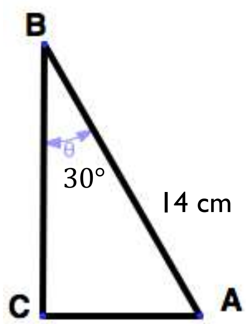
25. Explain why the trig values were the same for angle A and angle B even though the dimensions of the triangles were different.

26.

sin B =

cos B =

tan B =

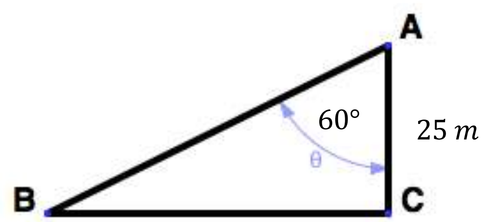


27.

sin A =

cos A =

tan A =

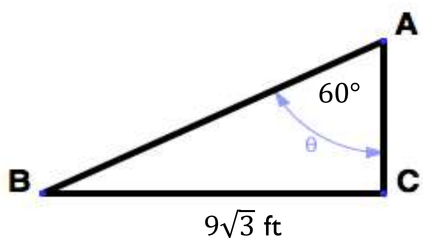


28.

sin A =

cos A =

tan A =

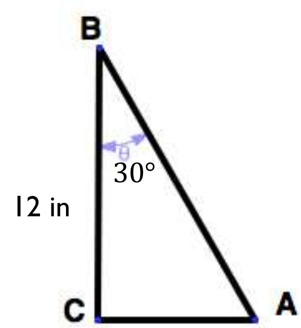


29.

sin B =

cos B =

tan B =



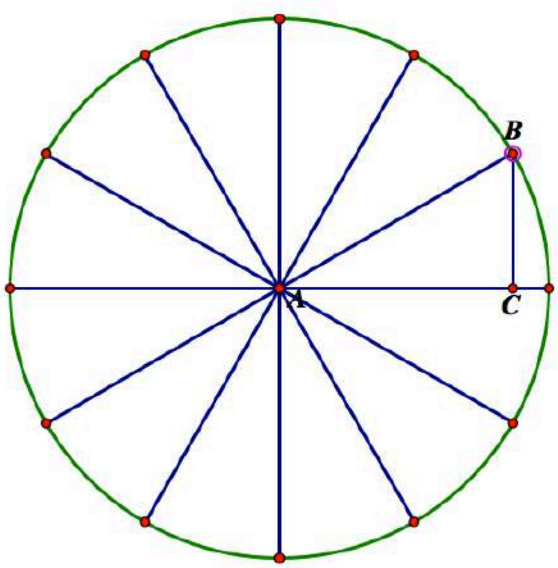
30. Explain where you see the meaning of the identity $\sin\theta = \cos(90° - \theta)$ in problems 26, 27, 28, and 29.

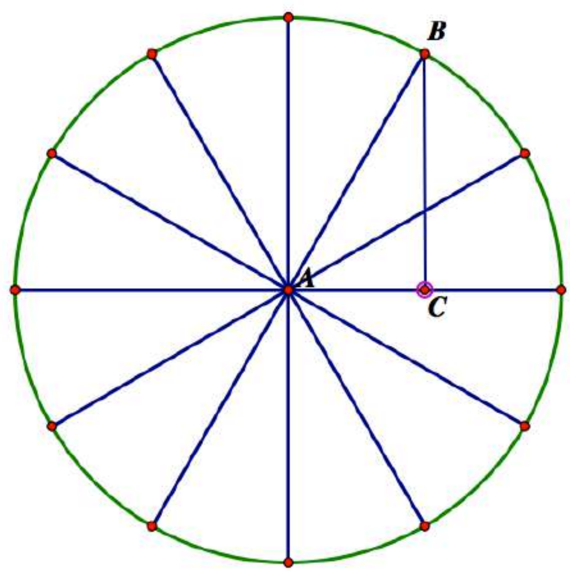
6.9 Water Wheels and the Unit Circle

A Practice Understanding Task

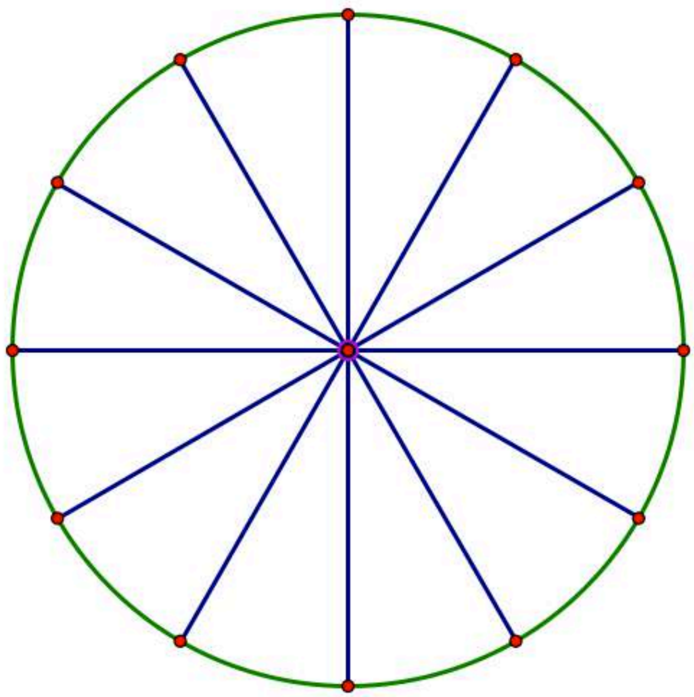
Water wheels were used to power flour mills before electricity was available to run the machinery. The water wheel turned as a stream of water pushed against the paddles of the wheel. Consequently, unlike Ferris wheels that have their centers above the ground, the center of the water wheel might be placed at ground level, so the lower half of the wheel would be immersed in the stream.

1. The following diagrams show potential designs for a water wheel. Each of the 12 spokes of the water wheel will measure 1 meter. In addition to the spokes, the designer wants to add braces to provide additional strength. Two potential placements for the braces are shown in the following diagrams. (The braces and the spoke to which they are attached form a right angle.)
   - Find the measures of $\angle BAC$ and $\angle ABC$ in each diagram.
   - Find the exact lengths of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$, not just decimal approximations. Explain how you found these lengths exactly.
   - Label the exact coordinates of point B in each diagram.





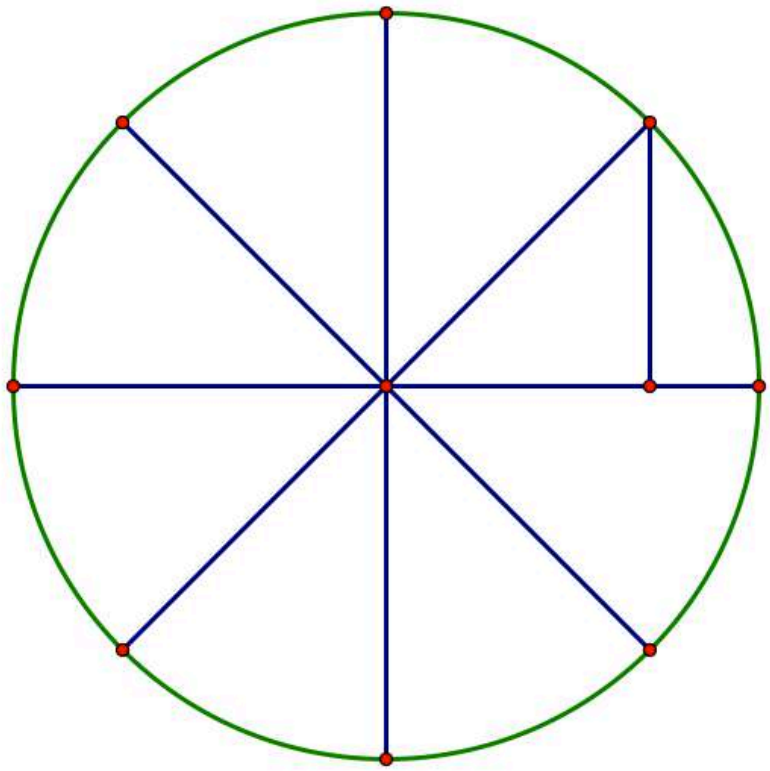
2. Based on your work above, label the exact values of the *x* and *y*-coordinates for each point on the following schematic drawing of the water wheel. Remember that the center of the wheel is at ground level, so points below the center of the wheel should be labeled with negative values. As in the Ferris wheel models, label points to the left of center with negative coordinates also.



3. Use the diagram above to give exact values for the following trigonometric expressions.

a. $\sin\left(\frac{\pi}{6}\right) =$	b. $\sin\left(\frac{5\pi}{6}\right) =$	c. $\cos\left(\frac{7\pi}{6}\right) =$
d. $\sin\left(\frac{\pi}{3}\right) =$	e. $\cos\left(\frac{\pi}{6}\right) =$	f. $\cos\left(\frac{11\pi}{6}\right) =$
g. $\sin\left(\frac{3\pi}{2}\right) =$	h. $\cos(\pi) =$	i. $\sin\left(\frac{7\pi}{3}\right) =$

4. Here is a plan for an alternative water wheel with only 8 spokes. Label the exact values of the *x* and *y*-coordinates for each point on the following schematic drawing of the water wheel. (Hint: You might want to begin this work by finding the length of the "brace" shown in the diagram.)



5. Use the diagram above to give exact values for the following trigonometric expressions.

a. $\sin\left(\frac{\pi}{4}\right) =$ b. $\sin\left(\frac{5\pi}{4}\right) =$ c. $\cos\left(\frac{3\pi}{4}\right) =$

d. $\cos\left(\frac{\pi}{4}\right) =$ e. $\cos\left(-\frac{\pi}{4}\right) =$ f. $\sin\left(\frac{7\pi}{4}\right) =$

g. $\sin\left(\frac{3\pi}{2}\right) =$ h. $\cos\left(\frac{3\pi}{2}\right) =$ i. $\sin\left(\frac{11\pi}{4}\right) =$

During the spring runoff of melting snow the stream of water powering this water wheel causes it to make one complete revolution counterclockwise every 3 seconds.

6. Write an equation to represent the height of a particular paddle of the water wheel above or below the water level at any time *t* after the paddle emerges from the water.

    - Write your equation so the height of the paddle will be graphed correctly on a calculator set in *degree* mode.

    - Revise your equation so the height of the paddle will be graphed correctly on a calculator set in *radian* mode.

During the summer months the stream of water powering this water wheel becomes a "lazy river" causing the wheel to make one complete revolution counterclockwise every 12 seconds.

7. Write an equation to represent the height of a particular paddle of the water wheel above or below the water level at any time *t* after the paddle emerges from the water.

    - Write your equation so the height of the paddle will be graphed correctly on a calculator set in *degree* mode.

    - Revise your equation so the height of the paddle will be graphed correctly on a calculator set in *radian* mode.

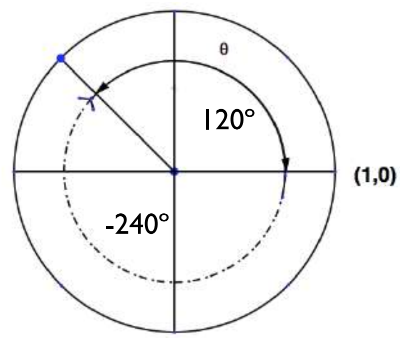
READY, SET, GO! Name Period Date

READY

Topic: Identifying coterminal angles

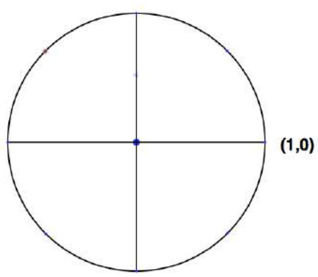
State a negative angle of rotation that is *coterminal* with the given angle of rotation. (*Coterminal* angles share the same terminal side of an angle of rotation.) Sketch and label both angles.

Example: $\theta = 120°$ is the given angle of rotation. The angle of rotation is indicated by the solid arc. The dotted angle of rotation is a coterminal angle with a rotation of $-240°$.



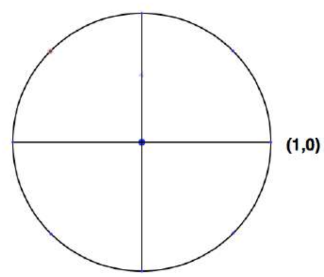
1. Given $\theta = 20°$

Coterminal angle ______________________



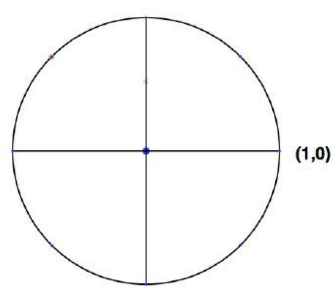
2. Given $\theta = 95°$

Coterminal angle ______________________



3. Given $\theta = 225°$

Coterminal angle ______________________



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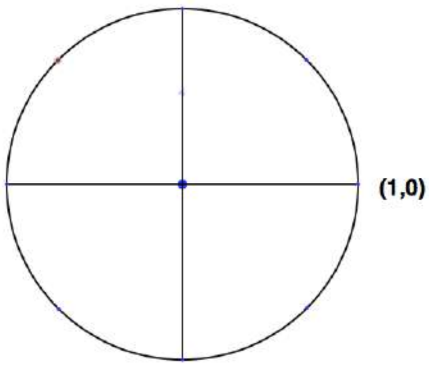
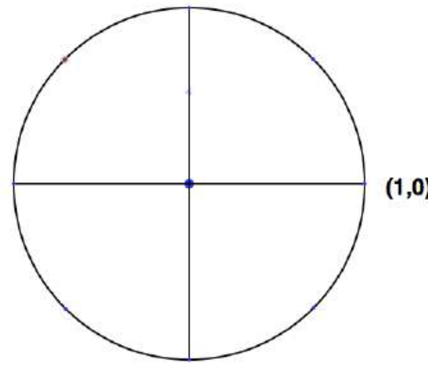
4. Given $\theta = 270°$

Coterminal angle ______________________________



5. Given $\theta = 300°$

Coterminal angle ___________________________



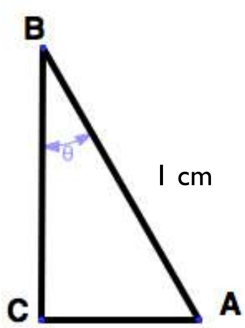
6. What is the sum of a positive angle of rotation and the absolute value of its negative coterminal angle?

7. Every angle has an infinite number of coterminal angles both positive and negative if the definition is extended to angles of rotation greater than 360°. For example: an angle of 45° is coterminal with angles of rotation measuring 405°, 765° *etc*. Given $\theta = 115°$, name 3 **positive** coterminal angles.

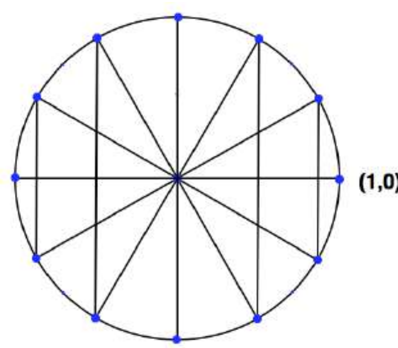
SET

Topic: Calculating sine and cosine of radian measures

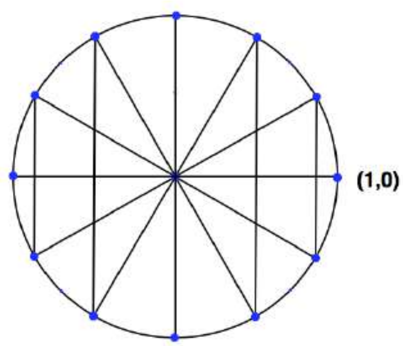
8. Triangle ABC is a 30°, 60°, 90° right triangle. The length of one side is given. Fill in the values for the missing sides. $m\angle B = 30°$.



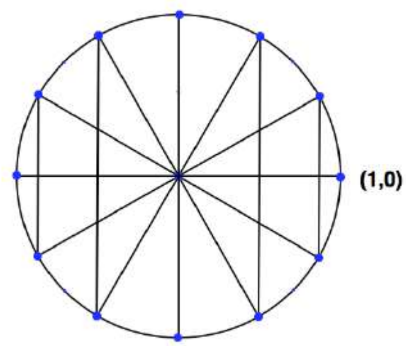
9. Label each point around the circle with the angle of rotation in radians starting from the point (1,0).



10. Use the values in #8 to write the **exact** coordinates of the points on the circle below. Be mindful of the numbers that are negative.



11. Find the arc length, *s,* from the point (1,0) to each point around the circle. Record your answers as decimal approximations to the nearest thousandth.



Use your calculator to find the following values.

12. $\sin\frac{5\pi}{6} =$

13. $\sin\frac{\pi}{3} =$

14. Why are both of your answers positive?

15. $\cos\frac{2\pi}{3} =$

16. $\cos\frac{4\pi}{3} =$

17. Why are both of your answers negative?

18. $\sin\frac{\pi}{2} =$

19. $\cos\frac{\pi}{2} =$

20. In which quadrants are sine and cosine both negative?

21. Name an angle of rotation where sine is equal to -1.

22. Name an angle of rotation where cosine is equal to -1.

GO

Topic: Finding the angle when given the trig ratio

Use your calculator to find the value of $\boldsymbol{\theta}$ *where* $\mathbf{0 \leq \theta \leq 90^\circ}$. Round your answers to the nearest degree.

23. $\sin\theta = 0.82$

24. $\cos\theta = 0.31$

25. $\cos\theta = 0.98$

26. $\sin\theta = 0.39$

27. $\sin\theta = 1$

28. $\cos\theta = 0.71$