Transforming Mathematics Education

SECONDARY MATH THREE

An Integrated Approach

MODULE 7

Trigonometric Functions, Equations & Identities

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

Scott Hendrickson, Joleigh Honey, Barbara Kuehl, Travis Lemon, Janet Sutorius

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7.1 High Noon and Sunset Shadows

A Develop Understanding Task

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In this task we revisit the amusement park Ferris wheel that caused Carlos so much anxiety. Recall the following facts from previous tasks:

- The Ferris wheel has a radius of 25 feet
- The center of the Ferris wheel is 30 feet above the ground
- The Ferris wheel makes one complete rotation counterclockwise every 20 seconds

The amusement park Ferris wheel is located next to a high-rise office complex. At sunset, the moving carts cast a shadow on the exterior wall of the high-rise building. As the Ferris wheel turns, you can watch the shadow of a rider rise and fall along the surface of the building. In fact, you know an equation that would describe the motion of this "sunset shadow."

1. Write the equation of this "sunset shadow."

At noon, when the sun is directly overhead, a rider casts a shadow that moves left and right along the ground as the Ferris wheel turns. In fact, you know an equation that would describe the motion of this "high noon shadow."

2. Write the equation of this "high noon shadow."

3. Based on your previous work, you probably wrote these equations in terms of the angle of rotation being measured in degrees. Revise you equations so the angle of rotation is measured in radians.

   a. The "sunset shadow" equation in terms of radians:

   b. The "high noon shadow" equation in terms of radians:

4. In the equations you wrote in question 3, where on the Ferris wheel was the rider located at time $t = 0$? (We will refer to the position as the rider's *initial position* on the wheel.)

5. Revise your equations from question 3 so that the rider's initial position at $t = 0$ is at the top of the wheel.

   a. The "sunset shadow" equation, initial position at the top of the wheel:

   b. The "high noon shadow" equation, initial position at the top of the wheel:

6. Revise your equations from question 3 so that the rider's initial position at $t = 0$ is at the bottom of the wheel.

   a. The "sunset shadow" equation, initial position at the bottom of the wheel:

   b. The "high noon shadow" equation, initial position at the bottom of the wheel:

7. Revise your equations from question 3 so that the rider's initial position at $t = 0$ is at the point farthest to the left of the wheel.

   a. The "sunset shadow" equation, initial position at the point farthest to the left of the wheel:

   b. The "high noon shadow" equation, initial position at the point farthest to the left of the wheel:

8. Revise your equations from question 3 so that the rider's initial position at $t = 0$ is halfway between the farthest point to the right on the wheel and the top of the wheel.

   a. The "sunset shadow" equation, initial position halfway between the farthest point to the right on the wheel and the top of the wheel:

   b. The "high noon shadow" equation, initial position halfway between the farthest point to the right on the wheel and the top of the wheel:

9. Revise your equations from question 3 so that the wheel rotates twice as fast.

   a. The "sunset shadow" equation for the wheel rotating twice as fast:

   b. The "high noon shadow" equation for the wheel rotating twice as fast:

10. Revise your equations from question 3 so that the radius of the wheel is twice as large and the center of the wheel is twice as high.

    a. The "sunset shadow" equation for a radius twice as large and the center twice as high:

    b. The "high noon shadow" equation for a radius twice as large and the center twice as high:

11. Carlos wrote his "sunset equation" for the height of the rider in question #5 as $h(t) = 50\sin\left(\frac{\pi}{10}t + \frac{\pi}{2}\right) + 30$. Clarita wrote her equation for the same problem as $h(t) = 50\sin\left(\frac{\pi}{10}(t+5)\right) + 30$.

    a. Are both of these equations equivalent? How do you know?

    b. Carlos says his equation represents starting the rider at an initial position at the top of the wheel. What does Clarita's equation represent?

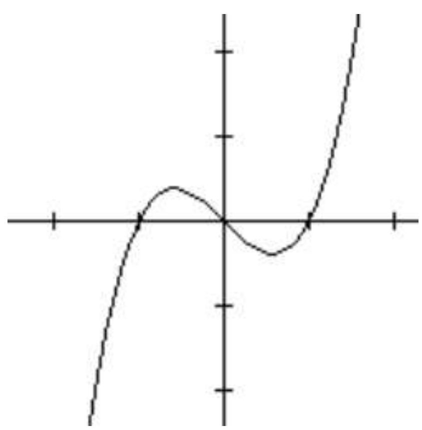
READY, SET, GO! Name Period Date

READY

Topic: Recalling invertible functions and even and odd functions

Indicate which of the following functions have an inverse that is a function. If the function has an inverse, sketch it in. (Remember, the inverse will reflect across the y = x line. Sketch that in, too.) Finally, label each one as *even, odd*, or *neither*. Recall that an *even* function is symmetric with the y-axis, while an *odd* function is symmetric with respect to the origin.

1,



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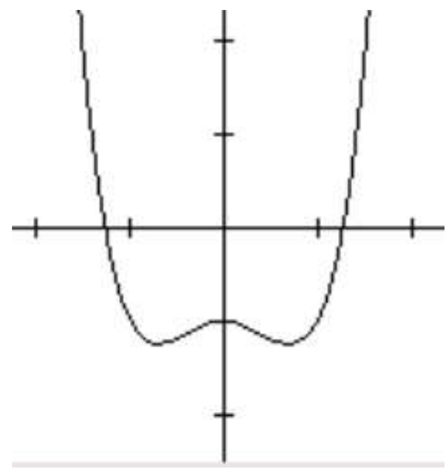

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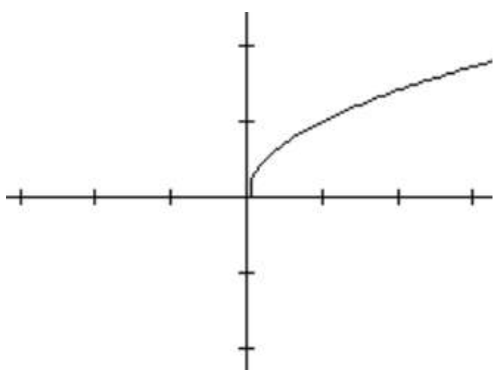

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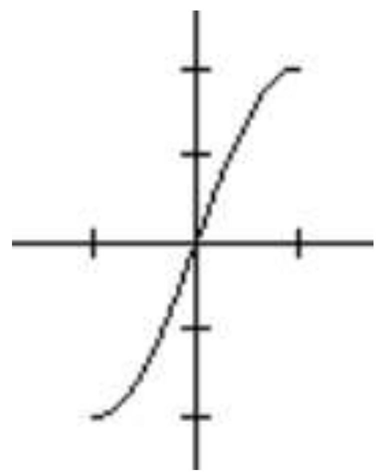
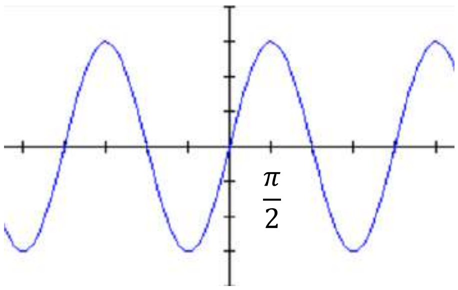


SET

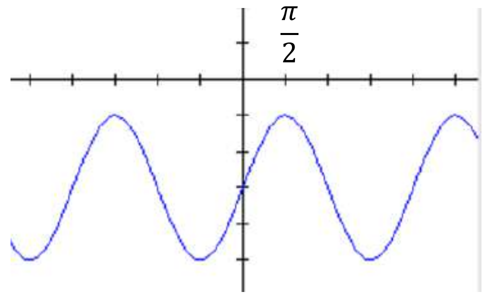
Topic: Connecting transformed trig graphs with their equations

State the period, amplitude, vertical shift, and phase shift of the function shown in the graph. Then write the equation. Use the same trigonometric function as the one that is given.

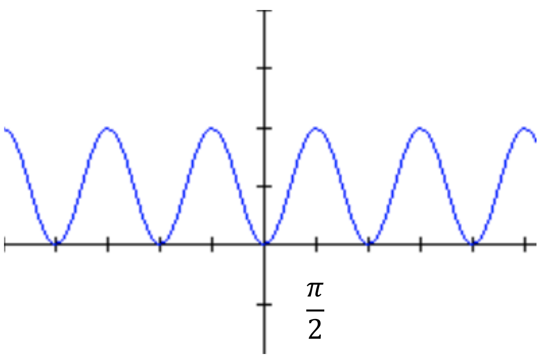
5. $y = sin\ x$



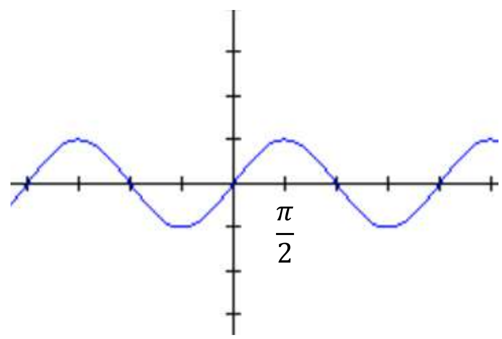
6. $y = sin\ x$



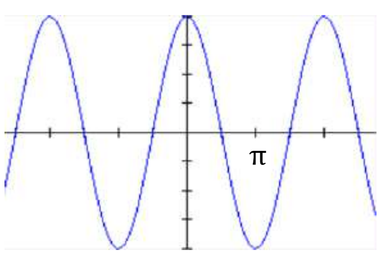
7. $y = cos\ x$



8. $y = cos\ x$



9. $y = sin\ x$



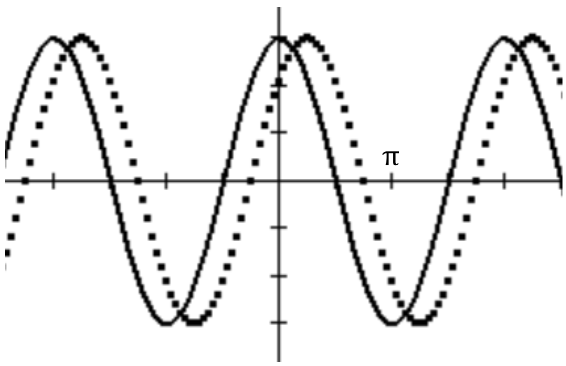
10. The cofunction identity states that $\sin\theta = \cos(90° - \theta)$ *and* $\sin(\theta - 90°) = \cos\theta$. How does this identity relate to the graph in #9?

Explain where you would see this identity in a right triangle.

Describe the relationships between the graphs of *f (x)* - solid and *g (x)* - dotted.

Then write their equations.

11.



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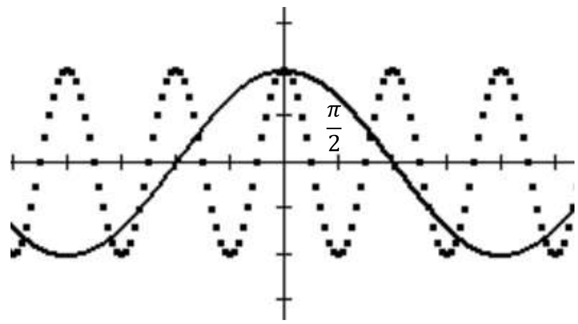
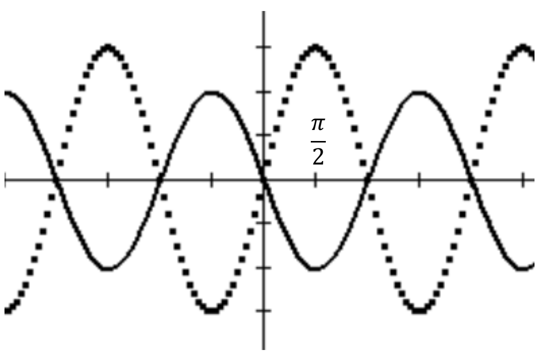


13. This graph could be interpreted as a shift or a reflection. Write the equations both ways.



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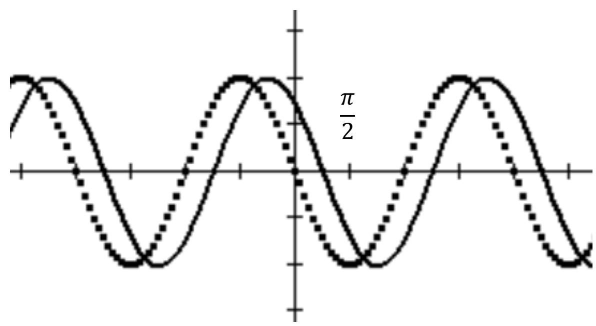
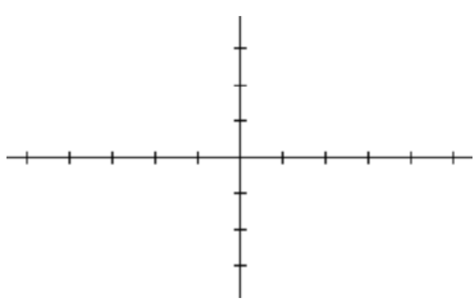
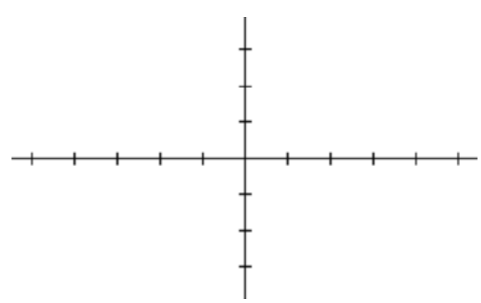


Sketch the graph of the function.
(Include 2 full periods. Label the scale of your horizontal axis.)

15. $y = 3\sin\left(x - \frac{\pi}{2}\right)$



16. $y = -2\cos(x + \pi)$



GO

Topic: Finding angles of rotation for the same trig ratio

Name two values for θ (angles of rotation) that have the given trig ratio. $0 < \theta \leq 2\pi$.

17. $\sin\theta = \frac{\sqrt{2}}{2}$

18. $\cos\theta = \frac{\sqrt{2}}{2}$

19. $\cos\theta = -\frac{1}{2}$

20. $\sin\theta = 0$

21. $\sin\theta = -\frac{\sqrt{3}}{2}$

22. $\cos\theta = -\frac{\sqrt{3}}{2}$

23. For which angles of rotation does $\sin\theta = \cos\theta$?

7.2 High Tide

A Solidify Understanding Task

Perhaps you have built an elaborate sand castle at the beach only to have it get swept away by the in-coming tide.

Spring break is next week and you are planning another trip to the beach. This time you decide to pay attention to the tides so that you can keep track of how much time you have to build and admire your sand castle.

You have a friend who is in calculus who will be going on spring break with you. You give your friend some data from the almanac about high tides along the ocean, as well as a contour map of the beach you intend to visit, and ask her to come up with an equation for the water level on the beach on the day of your trip. According to your friend's analysis, the water level on the beach will fit this equation:

$$f(t) = 20\, sin\left(\frac{\pi}{6}t\right)$$

In this equation, $f(t)$ represents how far the waterline is above or below its average position. The distance is measured in feet, and t represents the elapsed time (in hours) since midnight.

1. What is the highest up the beach (compared to its average position) that the waterline will be during the day? (This is called *high tide*.) What is the lowest that the waterline will be during the day? (This is called *low tide*.)

2. Suppose you plan to build your castle right on the average waterline just as the water has moved below that line. How much time will you have to build your castle before the incoming tide destroys your work?

3. Suppose you want to build your castle 10 feet below the average waterline to take advantage of the damp sand. What is the maximum amount of time you will have to make your castle? How can you convince your friend that your answer is correct?

4. Suppose you want to build your castle 15 feet above the average waterline to give you more time to admire your work. What is the maximum amount of time you will have to make your castle? How can you convince your friend that your answer is correct?

5. You may have answered the previous questions using a graph of the tide function. Is there a way you could use algebra and the inverse sine function to answer these questions. If so, show your work.

    a. Algebraic work for question 3:

b. Algebraic work for question 4:

6. Suppose you decide you only need two hours to build and admire your castle. What is the lowest point on the beach where you can build it? How can you convince your friend that your answer is correct?

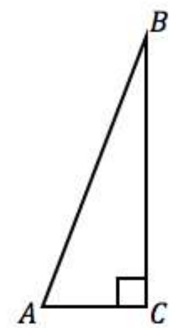
READY, SET, GO! Name Period Date

READY

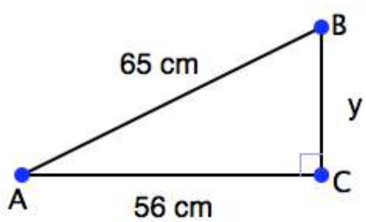
Topic: Calculating tangent in right angle trigonometry

Recall that the right triangle definition of the tangent ratio is:

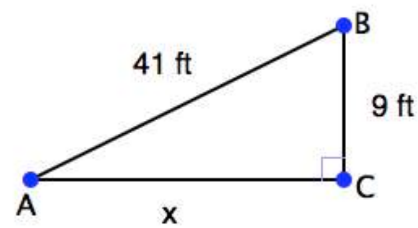
$$\tan A = \frac{\textit{length of side opposite angle } A}{\textit{length of side adjacent to angle } A}$$



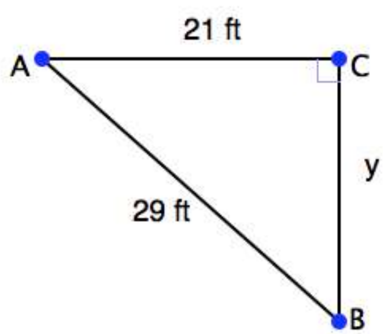
1. Find $\tan A$ *and* $\tan B$.



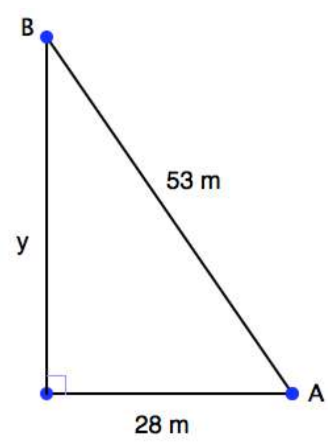
2. Find $\tan A$ *and* $\tan B$.



3. Find $\tan A$ *and* $\tan B$.



4. Find $\tan A$ *and* $\tan B$.



SET

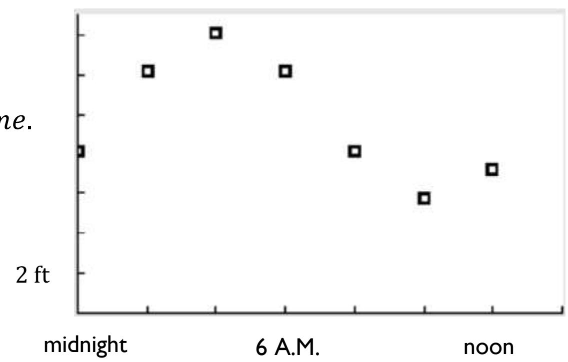
Topic: Mathematical modeling using sine and cosine functions

Many real-life situations such as sound waves, weather patterns, and electrical currents can be modeled by sine and cosine functions. The table below shows the depth of water (in feet) at the end of a wharf as it varies with the tides at various times during the morning.

t (time)	midnight	2 A.M.	4 A.M.	6 A.M.	8 A.M.	10 A.M.	noon
d (depth)	8.16	12.16	14.08	12.16	8.16	5.76	7.26

We can use a trigonometric function to model the data. Suppose you choose cosine.
$y = A\cos(bt - c) + d$, *where y is depth at any time.*

The *amplitude* will be the distance from the average of the highest and lowest values. This will be the average depth (d).



5. Sketch the line that shows the average depth.

6. Find the amplitude. $A = \frac{1}{2}(high - low)$

7. Find the period. $p = 2|low\ time - high\ time|$. Since a normal period for sine is 2π. The new period for our model will be $\frac{2\pi}{p}$ $so\ b = \frac{2\pi}{p}$. (Use the *p* you calculated, divide and turn it into a decimal.)

8. High tide occurred 4 hours after midnight. The formula for the displacement is $4 = \frac{c}{b}$. Use *b* and solve for *c.*

9. Now that you have your values for *A, b, c*, and *d*, put them into an equation.
   $y = A\cos(bt - c) + d$

10. Use your model to calculate the depth at 9 A.M. and 3 P.M.

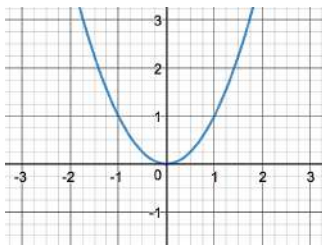
11. A boat needs at least 10 feet of water to dock at the wharf. During what interval of time in the afternoon can it safely dock?

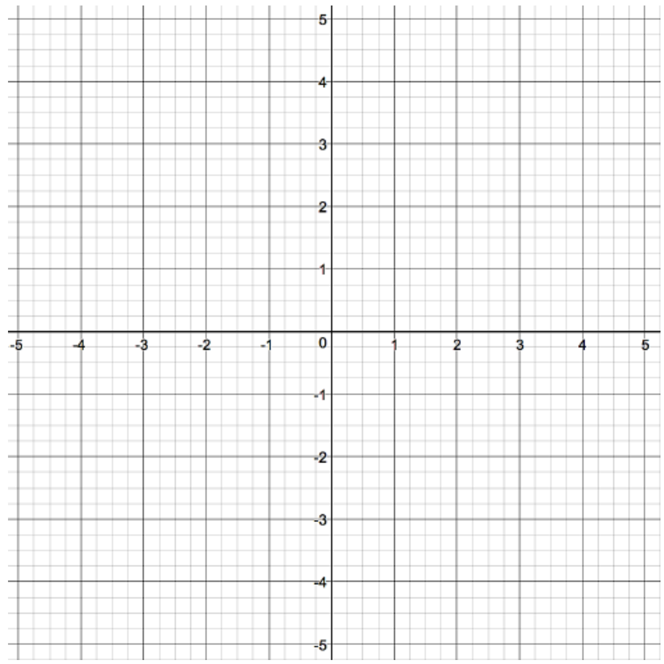
GO

Topic: Connecting transformations on functions

The equation and graph of a parent function is given. For each transformation, describe the change on the graph of the parent function. Then graph the functions on the same grid.

12. $f(x) = x^2$



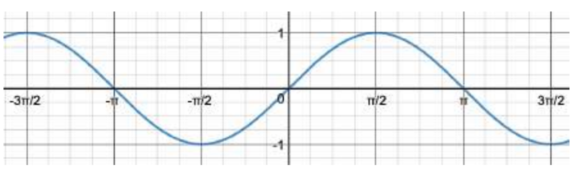


a. $f(x) = x^2 - 3$
   Description:

b. $f(x) = (x-3)^2 - 4$
   Description:

c. $f(x) = 2(x-3)^2 - 4$
   Description:

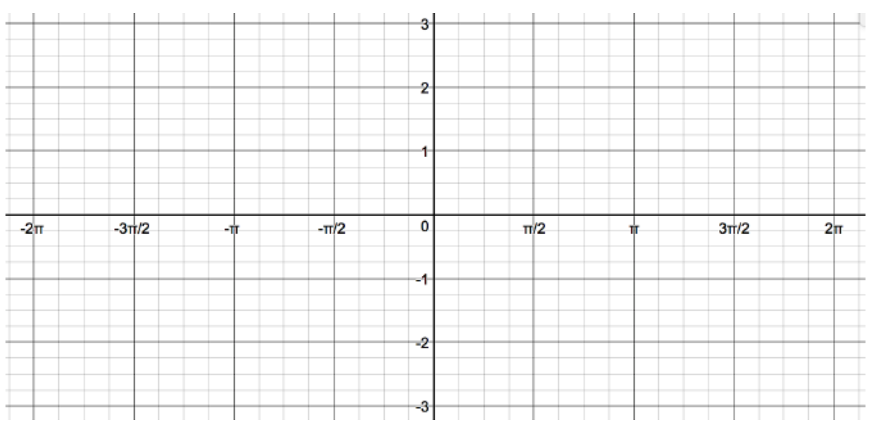
13. $g(x) = sinx$



a. $g(x) = (sinx) + 2$
   Description:

b. $g(x) = sin\left(x + \frac{\pi}{2}\right) - 1$
   Description:

c. $g(x) = 2sin\left(x + \frac{\pi}{2}\right) - 1$
   Description:



7.3 Getting on the Right Wavelength

A Practice Understanding Task

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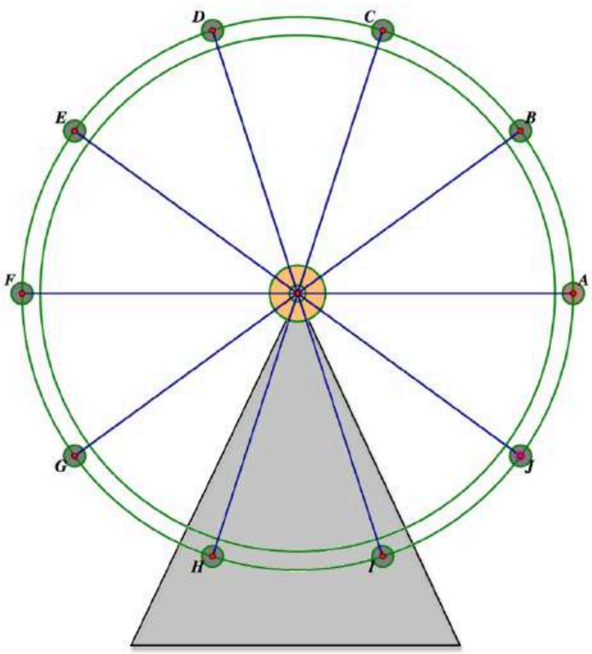
The Ferris wheel in the following diagram has a radius of 40 feet, its center is 50 feet from the ground, and it makes one revolution counterclockwise every 18 seconds.



1. Write the equation of the height of the rider at any time t, if at $t = 0$ the rider is at position A (Use radians to measure the angle of rotation).

2. At what time(s) is the rider 70 feet above the ground? Show the details of how you answered this question.

3. If you used a sine function in question 1, revise your equation to model the same motion with a cosine function. If you used a cosine function, revise your equation to model the motion with a sine function.

4. Write the equation of the height of the rider at any time t, if at t = 0 the rider is at position D (Use radians to measure the angle of rotation).

5. For the equation you wrote in question 4, at what time(s) is the rider 80 feet above the ground? Show or explain the details of how you answered this question.

6. Choose any other starting position and write the equation of the height of the rider at any time t, if at t = 0 the rider is at the position you chose. (Use radians to measure the angle of rotation). Also change other features of the Ferris wheel, such as the height of the center, the radius, the direction of rotation and/or the length of time for a single rotation. (Record your equation and description of your Ferris wheel here.)

7. Trade the equation you wrote in question 7 with a partner and see if he or she can determine the essential features of your Ferris wheel: height of center, radius, period of revolution, direction of revolution, starting position of the rider. Resolve any issues where you and your partner have differences in your descriptions of the Ferris wheel modeled by your equation.

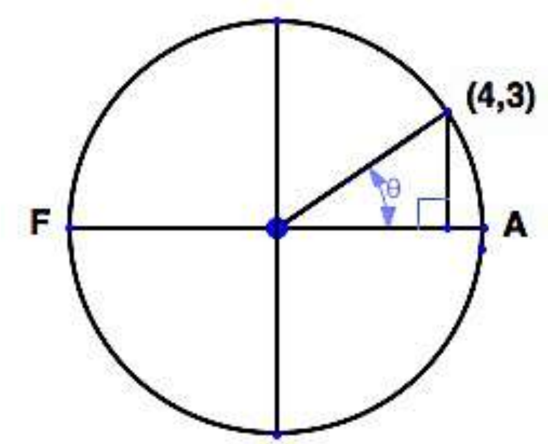
READY, SET, GO! Name Period Date

READY

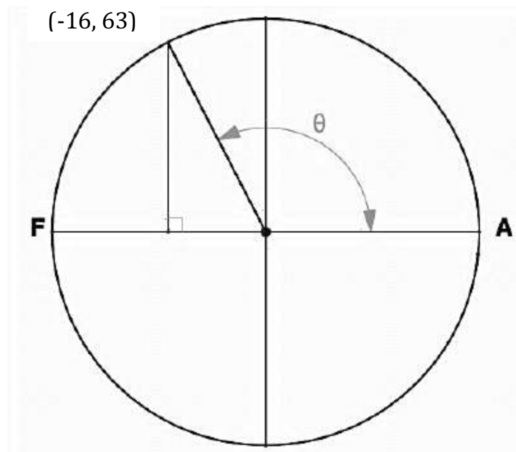
Topic: Using the definition of tangent

Use what you know about the definition of tangent in a right triangle to find the value of tangent θ for each of the right triangles below.

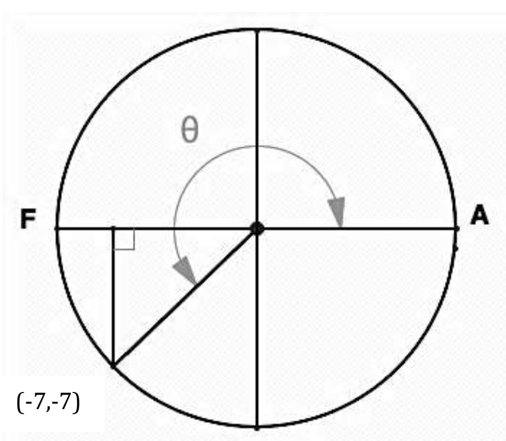
1. $\tan\theta =$



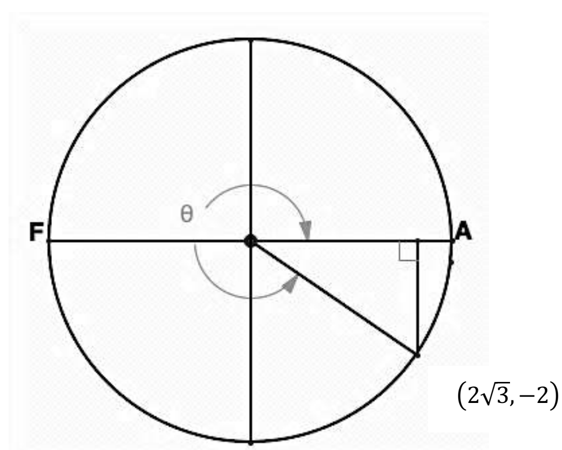
2. $\tan\theta =$



3. $\tan\theta =$



4. $\tan\theta =$



5. In each graph, the angle of rotation is indicated by an arc and θ. Describe the angles of rotation from $0 \text{ } to \text{ } 2\pi$ that make tangent θ be positive and the angles of rotation that make tangent θ be negative.

SET

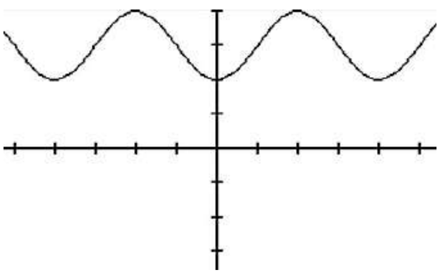
Topic: Connecting trig graphs with their equations

Match each trigonometric representation on the left with an equivalent representation on the right. Then check your answers with a graphing calculator. (The scale on the vertical axis is 1. The scale on the horizontal axis is $\left(\frac{\pi}{2}\right)$.)

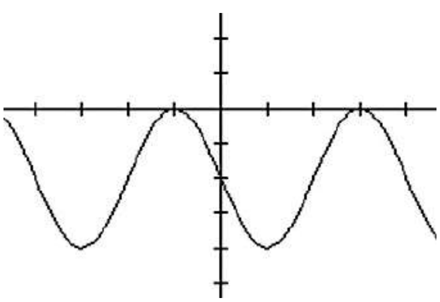
6. $y = -3sin\left(\theta + \frac{\pi}{2}\right)$

7. $y = 3\cos\left(\theta + \frac{\pi}{2}\right)$

8.



9.



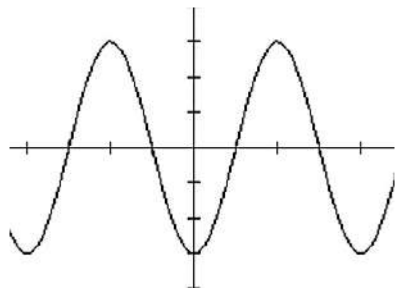
10. $y = \sin\left(2\left(\theta + \frac{\pi}{2}\right)\right) - 2$

11. $y = \sin(x + \pi)$

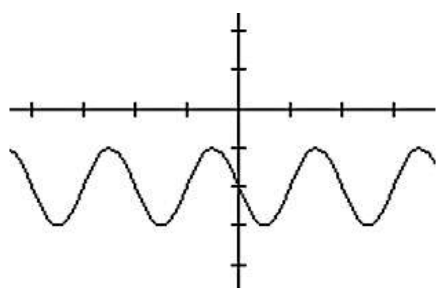
A. $y = -3\sin\theta$

B. $y = -sin\theta$

C.



D.



E. $y = 2cos\left(\theta + \frac{\pi}{2}\right) - 2$

F. $y = \cos(x + \pi) + 3$

Need help? Visit **www.rsgsupport.org**

12. Choose the equation(s) that has the same graph as $y = cos\theta$.

    a. $y = \cos(-\theta)$

    b. $y = \cos(\theta - \pi)$

    Use the unit circle to explain why they are the same.

13. Choose the equation(s) that has the same graph as $y = -sin\theta$.

    a. $y = \sin(\theta + \pi)$

    b. $y = \sin(\theta - \pi)$

    Use the unit circle to explain why they are the same.

For each function, identify the amplitude, period, horizontal shift, and vertical shift.

14. $f(t) = 150\cos\left(\frac{\pi}{6}(t-8)\right) + 80$

    amplitude:

    period:

    horizontal shift:

    vertical shift:

15. $f(t) = 4.5\sin\left(\frac{\pi}{4}t + \frac{3}{4}\right) + 8$

    amplitude:

    period:

    horizontal shift:

    vertical shift:

GO

Topic: Making sense of composite trig functions

Recall that a composite function places one function such as g(x), inside the other, f(x), by replacing the x in f(x) with the entire function g(x). In general, the notation is $f\left(g(x)\right)$. *Also, recall that inverse functions "undo" each other. Since,* $sin^{-1}\left(\frac{1}{2}\right)$ *is an angle of* $30°$ *because* $sin\,30° = \frac{1}{2}$ *the composite* $sin\left(sin^{-1}\frac{1}{2}\right)$ *is simply* $\frac{1}{2}$. *Sine function "undoes" what* $\sin^{-1}\theta$ *was does.*

Not all composite functions are inverses. Note: problems18 – 24.

Answer the following.

16. $\sin\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$

17. $\cos\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$

18. $\tan(\tan^{-1} 9.52)$

19. $\sin\left(\cos^{-1}\frac{1}{2}\right)$

20. $\cos(\tan^{-1} 1)$

21. $\sin(\tan^{-1} 2.75)$

22. $\cos(\sin^{-1} 1)$

23. $\cos(\tan^{-1} 0)$

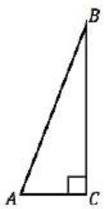
24. $\sin(\tan^{-1} undefined)$

mathematics
vision project

7.4 Off on a Tangent

A Develop and Solidify Understanding Task

Recall that the right triangle definition of the tangent ratio is:



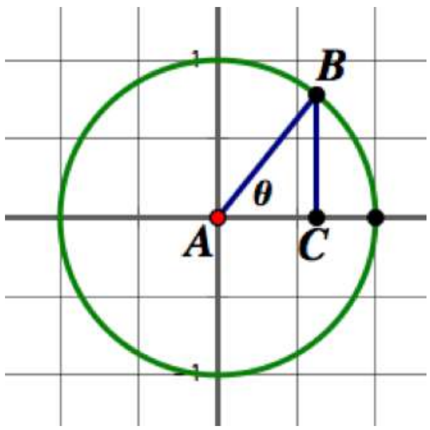
$$\tan(A) = \frac{length\ of\ side\ opposite\ angle\ A}{length\ of\ side\ adjacent\ to\ angle\ A}$$

1. Revise this definition to find the tangent of any angle of rotation, given in either radians or degrees. Explain why your definition is reasonable.

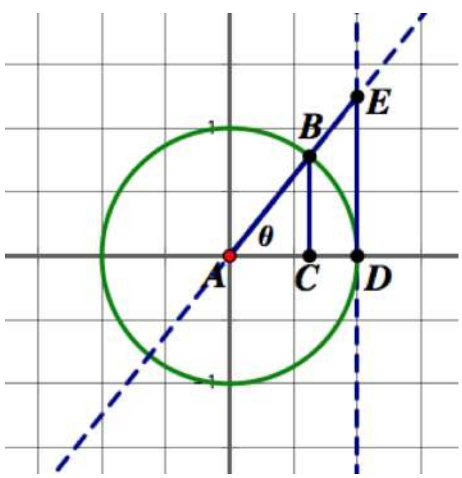
2. Revise this definition to find the tangent of any angle of rotation drawn in standard position on the unit circle. Explain why your definition is reasonable.

We have observed that on the unit circle the value of sine and cosine can be represented with the length of a line segment.

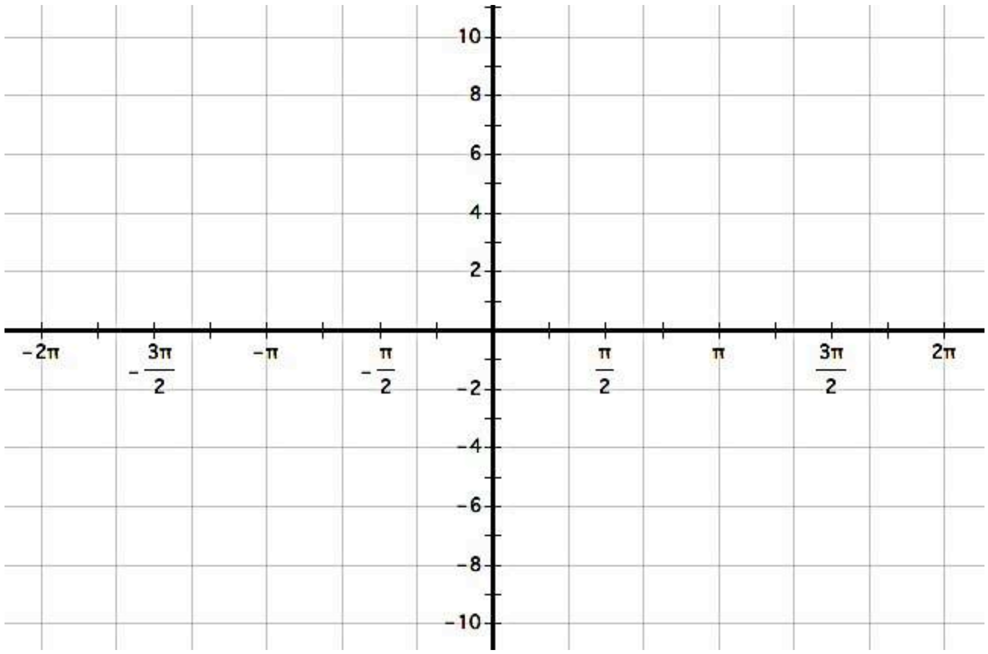
3. Indicate on the following diagram which segment's length represents the value of $\sin(\theta)$ and which represents the value of $\cos(\theta)$ for the given angle θ.



There is also a line segment that can be defined on the unit circle so that its length represents the value of $\tan(\theta)$. Consider the length of $\overline{DE}$ in the unit circle diagram below. Note that ΔADE and ΔABC are right triangles. Write a convincing argument explaining why the length of segment DE is equivalent to the value of $\tan(\theta)$ for the given angle θ.



4. On the coordinate axes below sketch the graph of $y = \tan(\theta)$ by considering the length of segment DE as θ rotates through angles from 0 radians to 2π radians. Explain any interesting features you notice in your graph.



Extend your graph of $y = \tan(\theta)$ by considering the length of segment DE as θ rotates through negative angles from 0 radians to -2π radians.

5. Using your unit circle diagrams from the task *Water Wheels and the Unit Circle*, give exact values for the following trigonometric expressions:

a. $\tan\left(\frac{\pi}{6}\right) =$

b. $\tan\left(\frac{5\pi}{6}\right) =$

c. $\tan\left(\frac{7\pi}{6}\right) =$

d. $\tan\left(\frac{\pi}{4}\right) =$

e. $\tan\left(\frac{3\pi}{4}\right) =$

f. $\tan\left(\frac{11\pi}{6}\right) =$

g. $\tan\left(\frac{\pi}{2}\right) =$

h. $\tan(\pi) =$

i. $\tan\left(\frac{7\pi}{3}\right) =$

Functions are often classified based on the following definitions:

- A function $f(x)$ is classified as an **odd function** if $f(-\theta) = -f(\theta)$
- A function $f(x)$ is classified as an **even function** if $f(-\theta) = f(\theta)$

6. Based on these definitions and your work in this module, determine how to classify each of the following trigonometric functions.

   - The function $y = sin(x)$ would be classified as an [odd function, even function, neither an odd or even function]. Give evidence for your response.

   - The function $y = cos(x)$ would be classified as an [odd function, even function, neither an odd or even function]. Give evidence for your response.

   - The function $y = tan(x)$ would be classified as an [odd function, even function, neither an odd or even function]. Give evidence for your response.

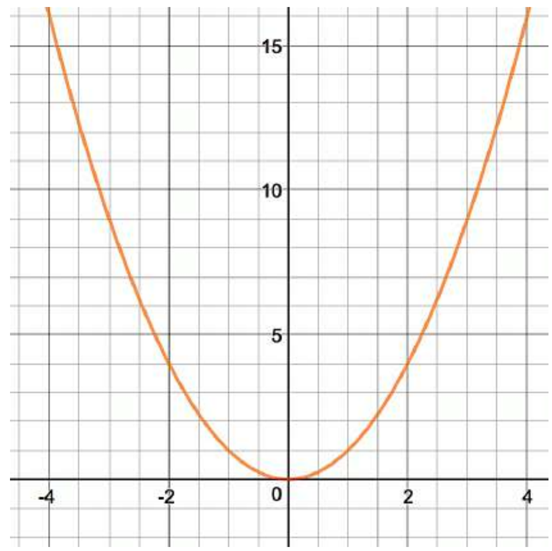
READY, SET, GO! Name Period Date

READY

Topic: Making rigid and non-rigid transformations on functions

The equation of a parent function is given. Write a new equation with the given transformations. Then sketch the new function on the same graph as the parent function. (If the function has asymptotes, sketch them in.)

1. $y = x^2$



Vertical shift: up 8

horizontal shift: left 3

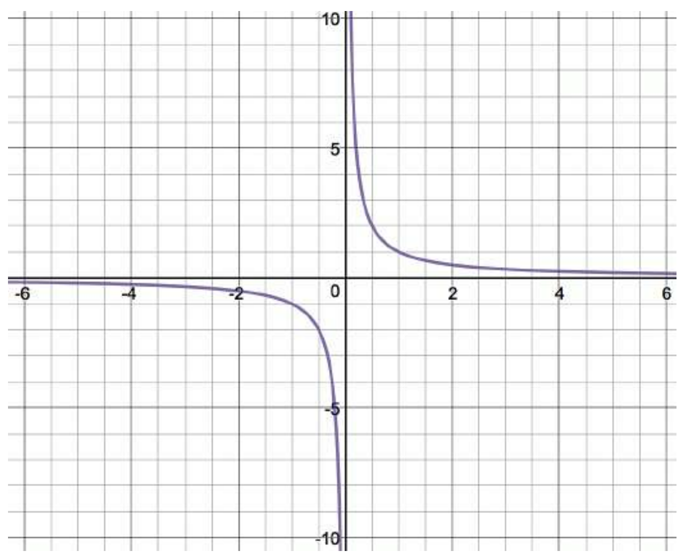
dilation: ¼

Equation:

Domain:

Range:

2. $y = \frac{1}{x}$



Vertical shift: up 4

horizontal shift: right 3

dilation: -1

Equation:

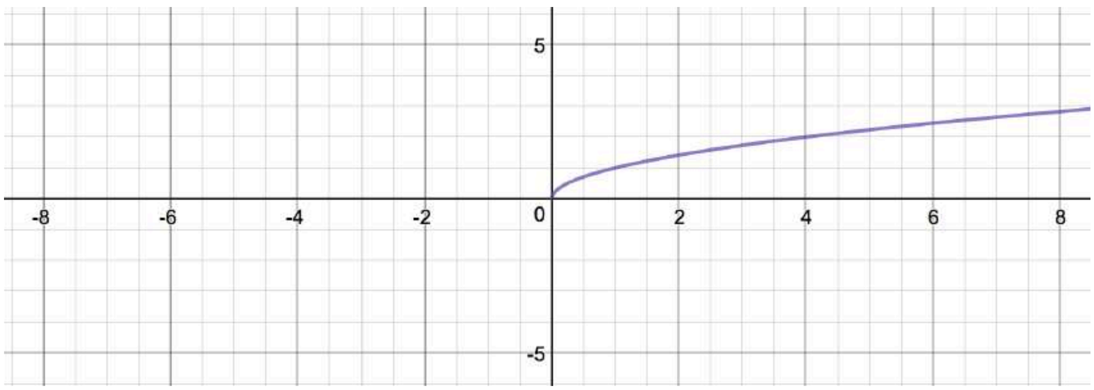
Domain:

Range:

3. $y = \sqrt{x}$

Vertical shift: none. horizontal shift: left 5 dilation: 3

Equation: Domain: Range:



4. $y = \sin x$

Vertical shift: 1

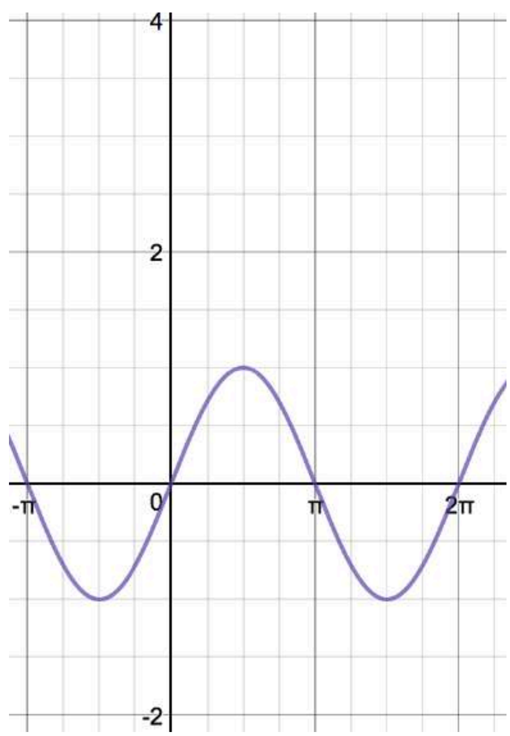
horizontal shift: left $\frac{\pi}{2}$

dilation (amplitude): 3

Equation:

Domain:

Range:

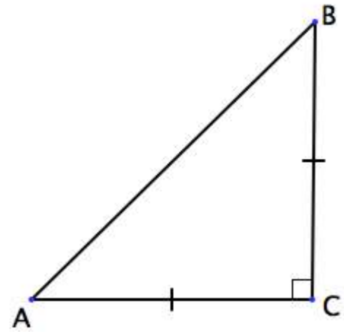


SET

Topic: Connecting values in the special triangles with radian measures

5. Triangle ABC is a right triangle. AB = 1.

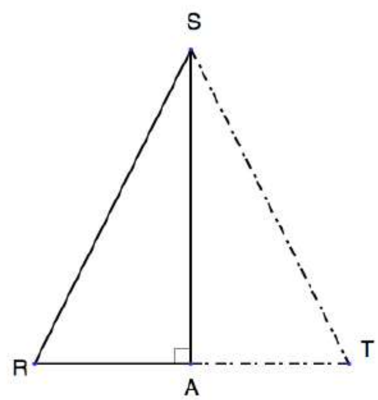
   Use the information in the figure to label the length of the sides and measure of the angles.



6. Triangle RST is an equilateral triangle. RS = 1
   $\overline{SA}$ is an altitude

   Use the information in the figure to label the length of the sides, the length of $\overline{RA}$, and the *exact* length of $\overline{SA}$.

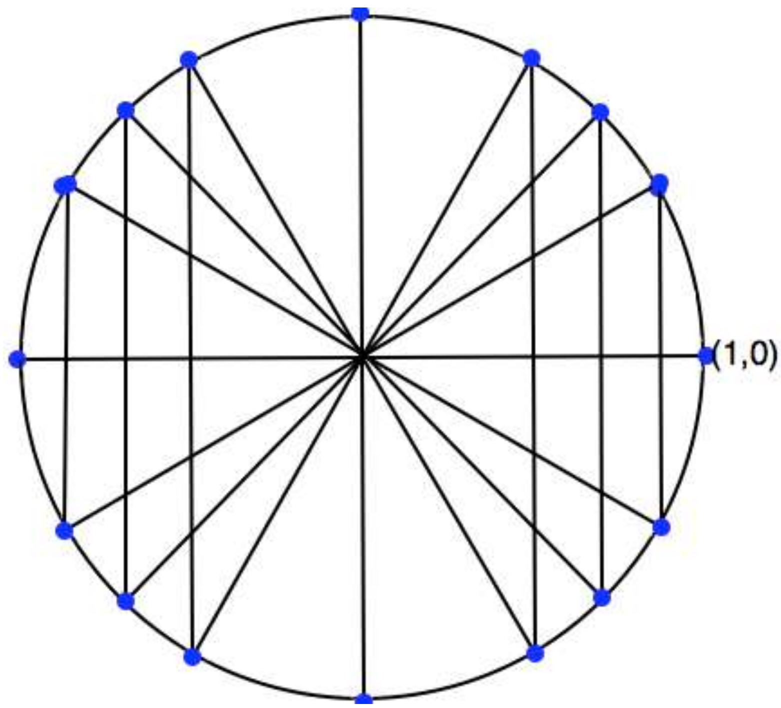
   Label the measure of angles RSA and SRA.



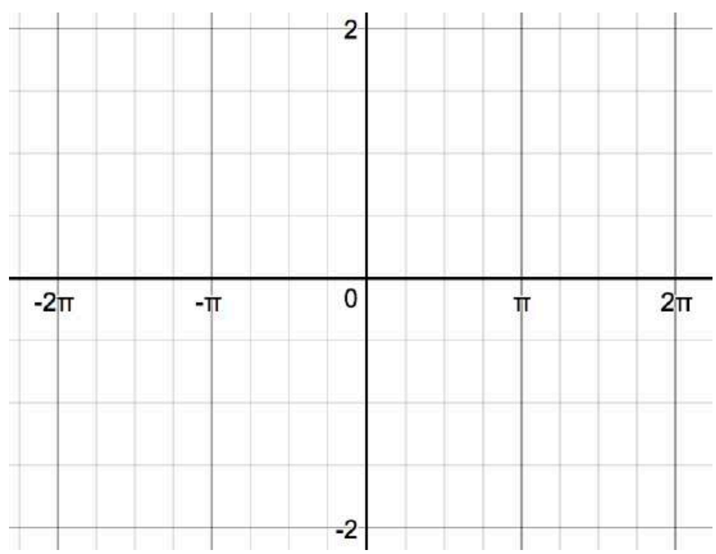
7. Use what you know about the unit circle and the information from the figures in problems 5 and 6 to fill in the table. Some values will be undefined.

function	$\theta = \frac{\pi}{6}$	$\theta = \frac{\pi}{4}$	$\theta = \frac{\pi}{3}$	$\theta = \frac{\pi}{2}$	$\theta = \pi$	$\theta = \frac{3\pi}{2}$	$\theta = 2\pi$
$\sin\theta$							
$\cos\theta$							
$\tan\theta$							

8. Label all of the points and angles of rotation in the given unit circle.



9. **Graph $f(x) = \tan\theta$.** Use your table of values above for $f(x) = \tan\theta$. Sketch your asymptotes with dotted lines.



10. Where do asymptotes always occur?

GO

Topic: Recalling trig facts

Answer the questions below. Be sure you can justify your thinking.

11. Given triangle ABC with angle C being the right angle, what is the sum of $m\angle A + m\angle B$?

12. Identify the quadrants in which $\sin\theta$ *is positive*.

13. Identify the quadrants in which $\cos\theta$ *is negative*.

14. Identify the quadrants in which $\tan\theta$ *is positive*.

15. Explain why it is impossible for $\sin\theta > 1$.

16. Name the angles of rotation (in radians) for when $\sin\theta = \cos\theta$.

17. For which trig functions do a positive rotation and a negative rotation always give the same value?

18. Explain why in the unit circle $\tan\theta = \frac{y}{x}$.

19. Which function connects with the slope of the hypotenuse in a right triangle?

20. Explain why $\sin\theta = \cos(90° - \theta)$.

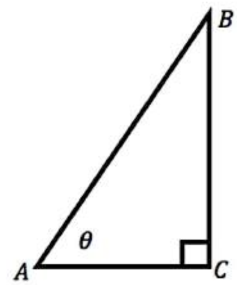
7.5 Maintaining Your Identity

A Develop Understanding Task

Right triangles and the unit circle provide images that can be used to derive, explain and justify a variety of trigonometric identities.

1. For example, how might the right triangle diagram at the right help you justify why the following identity is true for all angles θ between 0° and 90°?

   $\sin(\theta) = \cos(90° - \theta)$

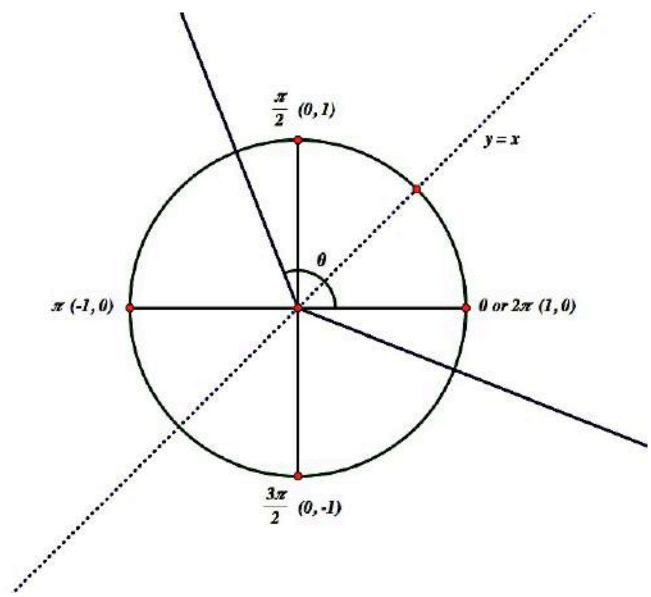


2. Since we have extended our definition of the sine to include angles of rotation, rather than just the acute angles in a right triangle, we might wonder if this identity is true for all angles θ, not just those that measure between 0° and 90°?

   A version of this identity that uses radian rather than degree measure would look like this:

   $$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$$

   How might you use this unit circle diagram to justify why this identity is true for all angles θ?



Fundamental Trig Identities

3. Here are some additional trig identities. Use either a right triangle diagram or a unit circle diagram to justify why each is true.

   a. $\sin(-\theta) = -\sin(\theta)$

   b. $\cos(-\theta) = \cos(\theta)$

   c. $\sin^2\theta + \cos^2\theta = 1$ [Note: This is the preferred notation for $(\sin\theta)^2 + (\cos\theta)^2 = 1$]

   d. $\frac{\sin\theta}{\cos\theta} = \tan\theta$

4. Use right triangles or a unit circle to help you form a conjecture for how to complete the following statements as trig identities. How might you use graphs to gain additional supporting evidence that your conjectures are true?

a. $\sin(\pi - \theta) =$ ____________________ and $\cos(\pi - \theta) =$ ____________________

b. $\sin(\pi + \theta) =$ ____________________ and $\cos(\pi + \theta) =$ ____________________

c. $\sin(2\pi - \theta) =$ __________________ and $\cos(2\pi - \theta) =$ __________________

5. We can use algebra, along with some fundamental trig identities, to prove other identities. For example, how can you use algebra and the identities listed above to prove the following identities?

a. $\tan(-\theta) = -\tan(\theta)$

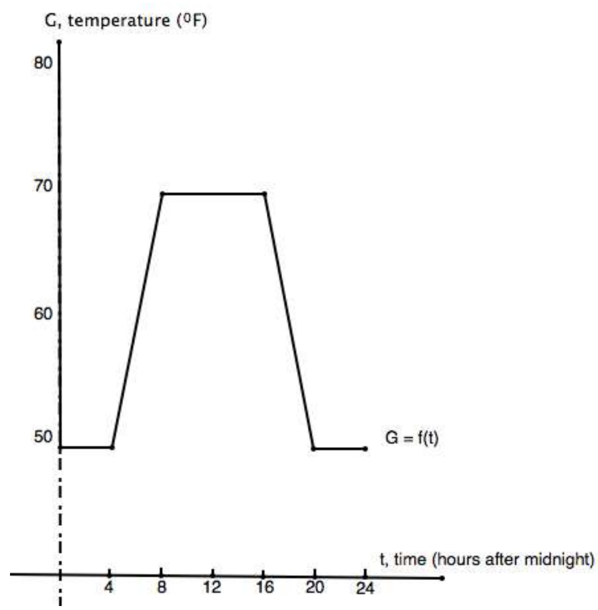
b. $\tan(\pi + \theta) = \tan(\theta)$

READY, SET, GO! Name Period Date

READY

Topic: Modeling transformations

A school building is kept warm only during school hours, in order to save money. Figure 6.1 shows a graph of the temperature, G, in °F, as a function of time, t, in hours after midnight. At midnight (t = 0), the building's temperature is 50 °F. This temperature remains the same until 4 AM. Then the heater begins to warm the building so that by 8 am the temperature is 70°F. That temperature is maintained until 4 pm, when the building begins to cool. By 8 pm, the temperature has returned to 50°F and will remain at that temperature until 4 am.



1. In January many students are sick with the flu. The custodian decides to keep the building 5 °F warmer. Sketch the graph of the new schedule on figure 6.1.

2. If $G = f(t)$ is the function that describes the original temperature setting, what would be the function for the January setting?

3. In the spring, the drill team begins early morning practice. The custodian then changes the original setting to start 2 hours earlier The building now begins to warm at 2 am instead of 4 am and reaches 70 °F at 6 am. It begins cooling off at 2 pm instead of 4 pm and returns to 50 °F at 6 pm instead of 8 pm. Sketch the graph of the new schedule on figure 6.1.

4. If $G = f(t)$ is the function that describes the original temperature setting, what would be the function for the spring setting?

SET

Topic: Using trigonometric identities to find additional trig values

The *Cofunction identity* states: $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ *and* $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

Complete the statements, using the ***Cofunction identity.***

5. $\text{Sin } 70° = \cos$ ______°
6. $\text{Sin } 28° = \cos$ ______°
7. $\text{Cos } 54° = \sin$ ______°
8. $\text{Sin } 9° = \cos$ ______°
9. $\text{Cos } 72° = \sin$ ______°
10. $\text{Cos } 45° = \sin$ ______°
11. $\text{Cos } \frac{\pi}{8} = \sin$ ——
12. $\text{Sin } \frac{5\pi}{12} = \cos$ ——
13. $\text{Sin } \frac{3\pi}{10} = \cos$ ——

14. Let $\sin\theta = \frac{3}{4}$.

    a) Use the *Pythagorean identity* $(sin^2\theta + cos^2\theta = 1)$, to find the value of $\cos\theta$.

    b) Use the *Quotient identity* $\left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$, the given information, and your answer in part (a) to calculate the value of $\tan\theta$.

15. Let $\cos\beta = \frac{12}{13}$.

    a) Find $\sin\beta$. Use the *Pythagorean identity* $(sin^2\theta + cos^2\theta = 1)$.

    b) Find $\tan\beta$. Use the *Quotient identity* $\left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$.

    c) Find $\cos\left(\frac{\pi}{2} - \theta\right)$. Use a *Cofunction identity.*

Use trigonometric definitions and identities to prove the statements below.

16. $\tan\theta\cos\theta = \sin\theta$

17. $(1+\cos\beta)(1-\cos\beta) = \sin^2\beta$

18. $(1+\sin\alpha)(1-\sin\alpha) = \cos^2\alpha$

19. $sin^2W - cos^2W = 2sin^2W - 1$

GO

Topic: Solving simple trig equations using the special angle relationships

Find <u>two</u> solutions of the equation. Give your answers in degrees $(\mathbf{0°} \le \boldsymbol{\theta} \le \mathbf{360°})$ and radians $(\mathbf{0} \le \boldsymbol{\theta} \le \mathbf{2\pi})$. Do NOT use a calculator.

20. $\sin\theta = \frac{1}{2}$

degrees: ______________________

radians: ______________________

21. $\sin\theta = -\frac{1}{2}$

degrees: ______________________

radians: ______________________

22. $\cos\theta = \frac{\sqrt{2}}{2}$

degrees: ______________________

radians: ______________________

23. $\sin\theta = -\frac{\sqrt{3}}{2}$

degrees: ______________________

radians: ______________________

24. $\tan\theta = -1$

degrees: ______________________

radians: ______________________

25. $\tan\theta = \sqrt{3}$

degrees: ______________________

radians: ______________________

7.6 Hidden Identities

A Practice Understanding Task

Note: Because trig functions are periodic, trig equations often have multiple solutions. Typically, we are only interested in the solutions that lie within a restricted interval, usually the interval from 0 to 2π. In this task you should find all solutions to the trig equations that occur on $[0, 2\pi]$.

To sharpen their trig skills, Alyce, Javier and Veronica are trying to learn how to solve some trig equations in a math refresher text that they found in an old trunk one of the adults had brought to the archeological site. Here is how each of them thought about one of the problems:

$$\textit{Solve}: \cos\left(\frac{\pi}{2} - \theta\right) = \frac{1}{2}$$

Alyce: I used the inverse cosine function.

Javier: I first used an identity, and then an inverse trig function. But it was not the same inverse trig function that Alyce used.

Veronica: I graphed $y_1 = \cos\left(\frac{\pi}{2} - \theta\right)$ and $y_2 = \frac{1}{2}$ on my calculator. I seem to have found more solutions.

1. Using their statements as clues, go back and solve the equation the way that each of the friends did.

2. How does Veronica's solutions match with Alyce and Javier's? What might be different?

Solve each of the following trig equations by adapting Alyce and Javier's strategies: that is, you may want to see if the equation can be simplified using one of the trig identities you learned in the previous task; and once you have isolated a trig function on one side of the equation, you can undo that trig function by taking the inverse trig function on both sides of the equation. Once you have a solution, you may want to check to see if you have found all possible solutions on the interval $[0, 2\pi]$ by using a graph as shown in Veronica's strategy.

3. $\sin(-\theta) = -\frac{1}{2}$

4. $\cos\theta \cdot \tan\theta = \frac{\sqrt{3}}{2}$

5. $\sin(2\theta) = \frac{\sqrt{2}}{2}$

READY, SET, GO! Name Period Date

READY

Topic: Using the calculator to find angles of rotation

Use your calculator and what you know about where sine and cosine are positive and negative in the unit circle to find the two angles that are solutions to the equation. Make sure θ *is always* $0 < \theta \leq 2\pi$. Round your answers to 4 decimals.
(Your calculator should be set in radians.)

You will notice that your calculator will sometimes give you a negative angle. That is because the calculator is programmed to restrict the angle of rotation so that the inverse of the function is also a function. Since the requested answers have been restricted to positive rotations, if the calculator gives you a negative angle of rotation, you will need to figure out the positive coterminal angle for the angle that your calculator has given you.

1. $\sin\theta = \frac{4}{5}$
2. $\sin\theta = \frac{2}{7}$
3. $\sin\theta = -\frac{1}{10}$
4. $\sin\theta = -\frac{13}{14}$

5. $\cos\theta = \frac{11}{12}$
6. $\cos\theta = \frac{1}{9}$
7. $\cos\theta = -\frac{7}{8}$
8. $\cos\theta = -\frac{2}{5}$

*Note: When you ask your calculator for the angle, you are "undoing" the trig function. Finding the angle is finding the inverse trig function. When you see "***Find** $\sin^{-1}\left(\frac{4}{5}\right)$*", you are being asked to find the angle that makes it true. The answer would be the same as the answer your calculator gave you in #1. Another notation that means the inverse sine function is* **arcsin** $\left(\frac{4}{5}\right)$.

SET

Topic: Verifying trig identities with tables, unit circles, and graphs

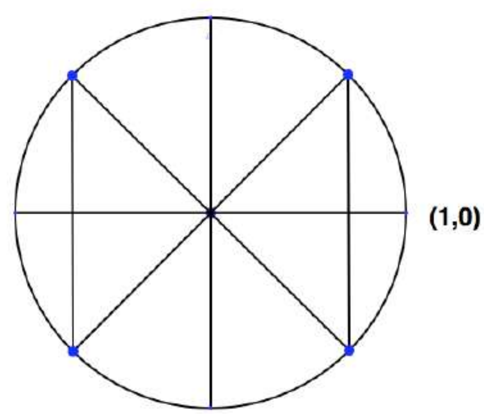
9. **Use the values in the table to verify the Pythagorean identity $\left(\sin^2\theta + \cos^2\theta = 1\right)$. Then use the Quotient Identity $\left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$ and the values in the table to write the value of tangent θ.**

9.	$sin\,\theta$	$cos\,\theta$	$sin^2\theta + cos^2\theta = 1$	$tan\theta$
a.	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$		
b.	$\frac{-3}{5}$	$\frac{4}{5}$		
c.	$-\frac{5}{13}$	$-\frac{12}{13}$		
d.	$\frac{\sqrt{14}}{7}$	$\frac{\sqrt{35}}{7}$		
e.	-1	0		
f.	0	1		

10. Label the angles of rotation and the coordinate points around the unit circle on the right. Then use these points to help you fill in the blank.

    $cos(\pi - \theta) =$ ____________________

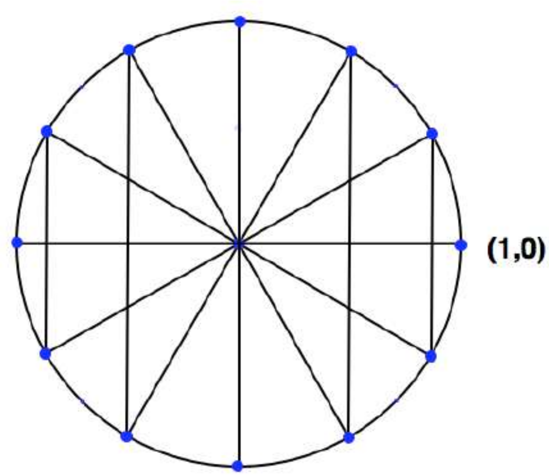
    Make your thinking visible by using the diagram. Explain your reasoning.



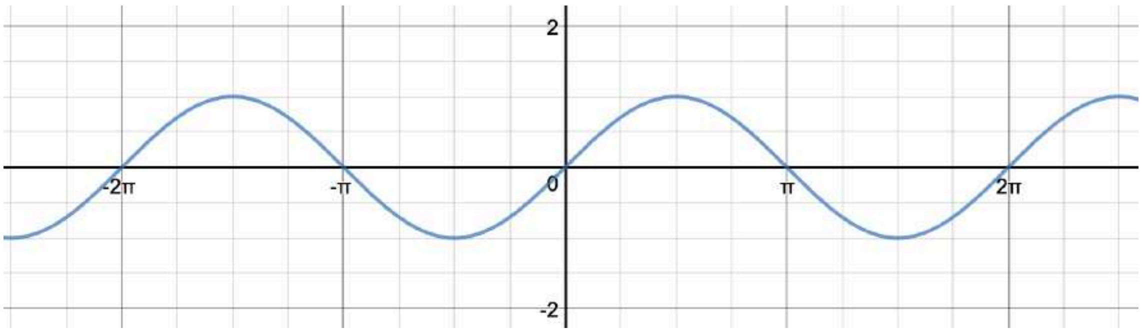
11. Label the angles of rotation and the coordinate points around the unit circle on the right. Then use these points to help you fill in the blank.

    $sin(\pi + \theta) =$ ____________________

    Make your thinking visible by using the diagram. Explain your reasoning.



12. Use the graph of $\sin\theta$ to help you fill in the blank. $sin(2\pi - \theta) =$ ________________



Make your thinking visible by using the graph. Explain your reasoning.

GO

Topic: Finding the central angle when given arc length and radius

Find the radian measure of the central angle of a circle of radius *r* that intercepts an arc of length *s*. ($s = r\theta$)

Round answers to 4 decimal places.

Radius	*Arc Length*	*Angle measure in radians*
13. 35 mm	11 mm	
14. 14 feet	9 feet	
15. 16.5 m	28 m	
16. 45 miles	90 miles	