mathematics vision project

Transforming Mathematics Education

SECONDARY MATH THREE

An Integrated Approach

Standard Teacher Notes

MODULE 3

Polynomial Functions

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The Mathematics Vision Project

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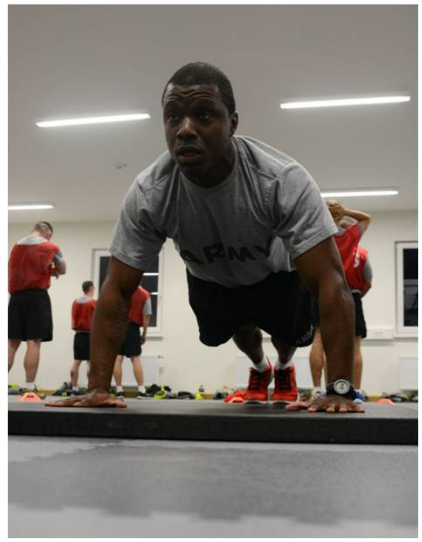
3.1 Scott's March Madness

A Develop Understanding Task



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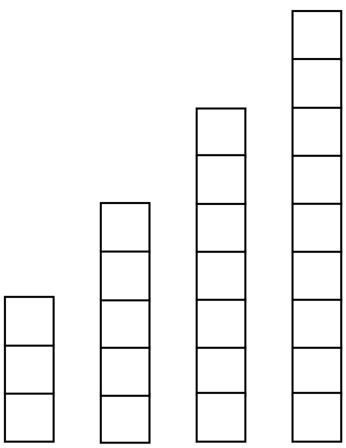
Each year, Scott participates in the "Macho March" promotion. The goal of "Macho March" is to raise money for charity by finding sponsors to donate based on the number of push-ups completed within the month. Last year, Scott was proud of the money he raised, but was also determined to increase the number of push-ups he would complete this year.

Part I: Revisiting the Past

Below is the bar graph and table Scott used last year to keep track of the number of push-ups he completed each day, showing he completed three push-ups on day one and five push-ups (for a combined **total** of eight push-ups) on day two. Scott continued this pattern throughout the month.



n Days	$f(n)$ Push-ups each **day**	$g(n)$ **Total** number of pushups in the month
1	3	3
2	5	8
3	7	15
4	9	24
5	11	35
…	…	
n		

1. Write the recursive and explicit equations for the number of push-ups Scott completed **on any given day** last year. Explain how your equations connect to the bar graph and the table above.

2. Write the recursive and explicit equation for the **accumulated total number of push-ups Scott completed by any given day** during the "Macho March" promotion last year.

Part II: March Madness

This year, Scott's plan is to look at the total number of push-ups he *completed for the month* last year

n Days	$f(n)$ Push-ups each **day last year**	$g(n)$ **Total** number of pushups in the month	$m(n)$ Push-ups each **day this year**	$T(n)$ **Total** push-ups completed for the month
1	3	3	3	
2	5	8	8	
3	7	15	15	
4	9	24		
5				
…	…			
n				

($g(n)$) and do that many push-ups each day ($m(n)$).

3. How many push-ups will Scott complete on day four? How did you come up with this number? Write the recursive equation to represent the total number of push-ups Scott will complete for the month on any given day.

4. How many **total** push-ups will Scott complete for the month on day four?

5. Without finding the explicit equation, make a conjecture as to the type of function that would represent the explicit equation for the total number of push-ups Scott would complete on any given day for this year's promotion.

6. How does the rate of change for this explicit equation compare to the rates of change for the explicit equations in questions 1 and 2?

7. Test your conjecture from question 5 and justify that it will always be true (see if you can move to a generalization for all polynomial functions).

3.1 Scott's March Madness – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is to develop student understanding of how the degree of a polynomial relates to the overall rate of change. Last year, students who did the task Scott's Macho March discovered that quadratic functions can be models for the sum of a linear function, which creates a linear rate of change. This year, students will see that cubic functions can be models for the sum of a quadratic function. Question 7 prompts students to make a conjecture and to move to the generalization that the sum of a polynomial of degree *n* will produce a polynomial of the degree *n+1.* In this task, students have the opportunity to use algebraic, numeric, and graphical representations to model a story context and make connections.

Core Standards Focus

F-BF.1 Write a function that describes a relationship between two quantities.*

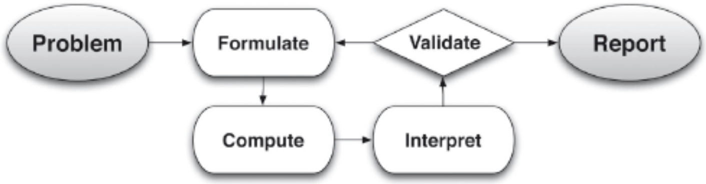
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

A.CED.2 (Create equations that describe numbers or relationships) Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Related Standards: F.IF.9, A.CED.1

This task also follows the structure suggested in the Modeling standard:



Standards for Mathematical Practice:

SMP 1 – Make sense of problems and persevere in solving them

SMP 2 – Reason abstractly and quantitatively

SMP 7 – Look for and make use of structure

SMP 8 – Look for express regularity in repeated reasoning

The Teaching Cycle:

Launch :

Begin the task by setting up the scenario and clarifying the task starts with the number of push-ups Scott completed last year during the event and then moves into the number of push-ups he intends to complete this year. Questions 1 and 2 from the task may be familiar for students from Secondary I (*Scott's Workout*) and Secondary II (*Scott's Macho March*). You may wish to have students work on question one as a warm-up to discuss prior to doing the rest of the task.

Explore :

Have students work independently for a while before they work with a partner. Monitor student thinking as they work. Since students should be familiar with representations for linear functions, do not allow any group to spend too much time on question 1 (you may even wish to make this a 'warm-up' question instead of part of the task). Question 2 is also review but is more difficult. Letting students work through this will be valuable toward making connections and answering the questions in Part II. For those students who have a hard time getting started, encourage them to use another representation (visual model, table, or graph) to help write the equations (both explicit and recursive) as they work on the task. Question 2 answers: Recursive: $g(1) = 3, g(n) = g(n-1) + (2n+1)$; Explicit equation (when simplified): $g(n) = n(n+2)$. The whole group discussion will focus on the questions from Part

II, so look for those who make conjectures that match the purpose of the task. It will be helpful to have students who share to use the visual model they created to explain how they found solutions and made conjectures. For example, a table showing the difference column similar to the one in the answer key (solution 3) to show that cubic functions can be models for the sum of a quadratic function-- just as quadratic functions can be models for the sum of a linear equation.

Discuss :

Start the whole group discussion by having a group go through their process for answering questions 2, 3 and 4 (be sure to have them show the model they used- visual diagram, table, or graph). If possible, have another group share their visual model if they used a different representation (a table highlights the difference column whereas a model or a graph visually highlights the rates of change of the explicit equations).

After students share their strategies for completing questions 2- 4, move the discussion to the last three questions in the task (questions 5-7). These questions focus on the total number of push-ups completed for the month, or in other words, the *cubic rate of change* created as a result of adding the explicit quadratic function to current number of push-ups completed on any given day. The goal for this portion of the discussion is for students to summarize the key points that reflect the purpose of this task*.

Observations to be made:

- The first difference is quadratic, the second difference is linear and the third difference is constant (a characteristic of a cubic function).
- The graph "curves" (grows at a faster rate) more than that of a quadratic (This also helps for future work regarding rates of change of polynomial functions versus that of exponential functions: while cubic functions grow at a faster rate than quadratics, they are not as fast as exponential, whose third difference is still exponential).
- Cubic functions can be models for the sum of the terms of a quadratic function in the same way that quadratic functions can be models for sum of terms of a linear function. (This is similar to the relationship from the Secondary II task *Scott's Macho March* where students discovered that quadratic functions can be models for the sum of a linear function, which creates a linear rate of change.)

- Recognize there is a relationship of the degree of a polynomial and the overall rate of change. (Generalization: when the *nth* difference is constant, then the polynomial is of degree *n*).
- Make a conjecture that the sum of a polynomial of degree *n* will produce a polynomial of the degree *n+1.*

*If students struggle to come up with the above observations, have them focus on the difference columns and compare the three recursive equations from questions 1, 2, and 3 (whose equations represent the former value plus a constant term, a linear expression, and a quadratic expression respectively). Ask students to identify similarities and differences. The important thing to notice is that *the change in the function is the expression added to the recursive formula*. If the rate of change is a constant, then the function is linear. If the change is a linear expression, then the function is quadratic. If the change is a quadratic expression, then the function is cubic.

**If time permits, it is always good to review and discuss features of functions. In this task, the function to describe the total number of push-ups Scott would complete would be a good discussion for domain/range, where the situation is increasing/decreasing, intercepts, whether the function is discrete or continuous, and the 'end behavior'. While the domain is restricted to the number of days in March, this is a good example of distinguishing between the range of the situation versus the end behavior of the actual function.

Aligned Ready, Set, Go: *Polynomials 3.1*

READY, SET, GO! Name Period Date

READY

Topic: Completing inequality statements

For each problem, place the appropriate inequality symbol between the two expressions to make the statement true.

$If\ a > b, then:$	$If\ x > 10, then:$	$If\ 0 < x < 1$
1. $3a$ ___ $3b$	4. x^2 ___ 2^x	7. x ___ x^2
2. $b - a$ ___ $a - b$	5. $\sqrt{x}$ ___ x^2	8. $\sqrt{x}$ ___ x
3. $a + x$ ___ $b + x$	6. x^2 ___ x^3	9. x ___ $3x$

SET

Topic: Classifying functions

Identify the type of function for each problem. Explain how you know.

10

x	$f(x)$
1	3
2	6
3	12
4	24
5	48

11.

x	$f(x)$
1	3
2	6
3	9
4	12
5	15

12.

x	$f(x)$
1	3
2	9
3	18
4	30
5	45

13.

x	$f(x)$
1	7
2	9
3	13
4	21
5	37

14.

x	$f(x)$
1	-26
2	-19
3	0
4	37
5	98

15.

x	$f(x)$
1	-4
2	3
3	18
4	41
5	72

16. Which of the above functions are NOT polynomials?

GO

Topic: Recalling long division and the meaning of a factor

Find the quotient without using a calculator. If you have a remainder, write the remainder as a whole number. Example: $21\overline{)149}^{\,7}$ **remainder 2**

17. $30\overline{)510}$

18. $13\overline{)8359}$

19. Is 30 a factor of 510? How do you know?

20. Is 13 a factor of 8359? How do you know?

21. $22\overline{)14857}$

22. $952\overline{)40936}$

23. Is 22 a factor of 14587? How do you know?

24. Is 952 a factor of 40936? How do you know?

25. $92\overline{)3405}$

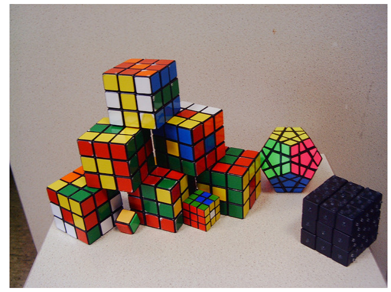
26. $27\overline{)3564}$

27. Is 92 a factor of 3405?

28. Is 27 a factor of 3564?

3.2 You-mix Cubes

A Solidify Understanding Task

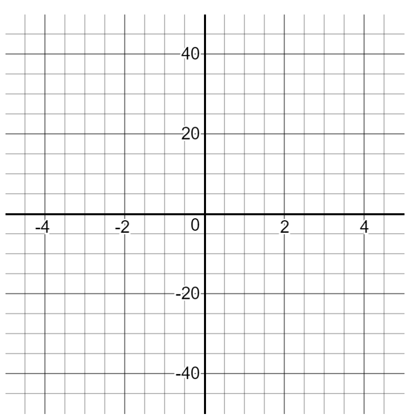


In Scott's March Madness, the function that was generated by the sum of terms in a quadratic function was called a **cubic function**. Linear functions, quadratic functions, and cubic functions are all in the family of functions called **polynomials**, which include functions of higher powers too. In this task, we will explore more about cubic functions to help us to see some of the similarities and differences between cubic functions and quadratic functions.

To begin, let's take a look at the most basic cubic function, $f(x) = x^3$. It is technically a **degree 3 polynomial** because the highest exponent is 3, but it's called a cubic function because these functions are often used to model volume. This is like quadratic functions which are **degree 2** polynomials but are called quadratic after the Latin word for square. Scott's March Madness showed that linear functions have a constant rate of change, quadratic functions have a linear rate of change, and cubic functions have a quadratic rate of change.

1. Use a table to verify that $f(x) = x^3$ has a quadratic rate of change.

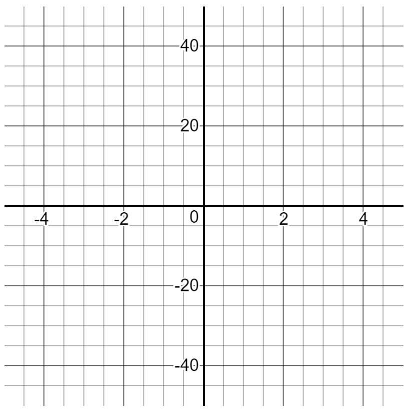
2. Graph $f(x) = x^3$.



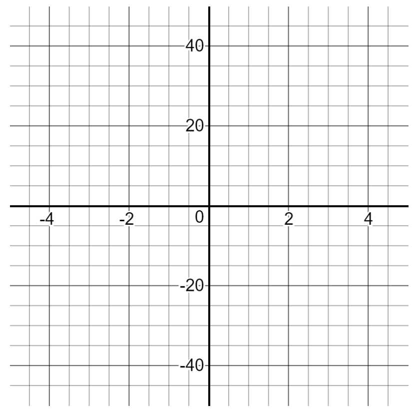
3. Describe the features of $f(x) = x^3$ including intercepts, intervals of increase or decrease, domain, range, etc.

4. Using your knowledge of transformations, graph each of the following without using technology.

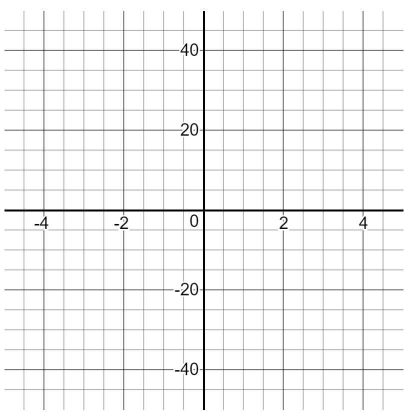
a) $f(x) = x^3 - 3$



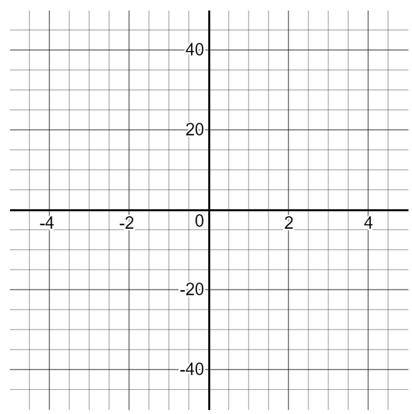
b) $f(x) = (x+3)^3$



c) $f(x) = 2x^3$



d) $f(x) = -(x-1)^3 + 2$



5. Use technology to check your graphs above. What transformations did you get right? What areas do you need to improve on so that your cubic graphs are perfect?

6. Since quadratic functions and cubic functions are both in the polynomial family of functions, we would expect them to share some common characteristics. List all the similarities between $f(x) = x^3$ and $g(x) = x^2$.

7. As you can see from the graph of $f(x) = x^3$, there are also some real differences in cubic functions and quadratic functions. Each of the following statements describe one of those differences. Explain <u>why</u> each statement is true by completing the sentence.

a) The range of $f(x) = x^3$ is $(-\infty, \infty)$, but the range of $g(x) = x^2$ is $[0, \infty)$ because:

____

b) For $x > 1, f(x) > g(x)$ because: ______________________________________

____

c) For $0 < x < 1, g(x) > f(x)$ because: ___________________________________

____

3.2 You-mix Cubes – Teacher Notes

A Solidify Understanding Task

Purpose:

The purpose of this task is to examine and extend ideas about cubic functions that were surfaced in *3.1 Scott's Macho March Madness*. Students consider the basic cubic function, $f(x) = x^3$, identifying its characteristics and graph. Students will recognize that the graph of $f(x) = x^3$ can be transformed using the same techniques as quadratic functions. Students will also make other comparisons between cubic and quadratic functions, specifically, their end behavior and the values of each function when x is a fraction.

Core Standards Focus:

F.BF.3 Identify the effect on the graph of replacing *f(x) by f(x)+k, kf(x), f(kx), and f(x+k)* for specific values of k (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from the graphs and algebraic expressions for them.

F.IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function *h(n)* gives the number of person-hours it takes to assemble *n* engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Related Standards: A.SSE.1

Standards for Mathematical Practice:

SMP 2 – Reason abstractly and quantitatively

SMP 8 – Look for express regularity in repeated reasoning

Vocabulary: cubic function, polynomial function, degree of a polynomial

The Teaching Cycle:

Launch:

Begin the task by telling students that the cubic functions introduced in task *3.1 Scott's Macho March Madness* and the quadratic and linear functions that they have previously worked with all fall into the family of polynomial functions. A technical definition of polynomial is given later, but a working definition is that a polynomial function is a sum of terms in the same variable with the natural number exponents and real coefficients. Demonstrate for students how to identify the degree of the polynomial based on the highest-powered term:

Degree 1 Polynomial (Linear Function): $y = 3x - 1$

Degree 2 Polynomial (Quadratic Function): $y = 3x^2 - 2x + 9$

Degree 3 Polynomial (Cubic Function): $y = -5x^3 + 3x^2 - 10x + 6$

Degree 4 Polynomial (Quartic Function): $y = x^4 + 7x^3 - 4x^2 - 24x + 1$

Explain that in this task, we will be introducing more of the characteristics of cubic functions and comparing them to what we know about quadratic functions.

Explore:

Monitor students as they are working on the task to see that they are creating an organized table of values that can be used throughout the task. In the table for problem #1, it is easiest to show the quadratic rate of change starting from 0 and looking at whole number values. If students are stuck, encourage them to start there. Eventually, they will need to consider both positive and negative numbers for the graph, which can be easily added to the table. Listen for students who are expressing the idea that the values on the left side of the graph are just the opposite of the values on the right side of the graph.

As students are working on the transformations, support them in connecting to their previous work with functions and identifying the transformations first, and then using anchor points from the original graph to find their new graph.

Discuss:

Begin the discussion by having a student present a table of values that shows the first, second, and third differences. Ask the class to explain how the differences relate to each other, i.e. a constant third difference means that the second difference is linear, a linear second difference means that the first difference is quadratic. Remind students that, generally, functions are defined by their rates of change, and the patterns in rates of change that we anticipated in *3.1 Scott's Macho March Madness* are consistent across polynomials.

Move the discussion to the graph in problem #2. Ask a student to share how they used the table to create the graph and how they can see the rate of change in the graph. Ask students to describe why the values to the left of zero are negative for a cubic function and reinforce the idea that when negative numbers are squared, the product is positive and when negative numbers are cubed, the product is negative. Tell students that their tables will help them with the anchor points they need to transform the graph of $f(x) = x^3$. They should be able to easily know the points (0,0), (1,1), (-1,-1), (2, 8), and (-2, -8) and use them for the transformations.

Continue the discussion by asking students to share their work on the graph in question #4. Be sure that students demonstrate how they determined the transformations and then used the anchor points to locate and graph the new function.

Finally, discuss the remaining questions. Question 7c may produce a lively discussion since students often believe that cubing a number always makes it bigger than squaring the number. After discussing how this belief is not true for fractions, project a graph of $f(x) = x^3$ and $g(x) = x^2$ together and zoom in on each of the following intervals to see which function is greater in the interval: $(-\infty, -1), (-1, 0), (0, 1), (1, \infty)$.

Aligned Ready, Set, Go: Polynomial Functions 3.2

READY, SET, GO! Name Period Date

READY

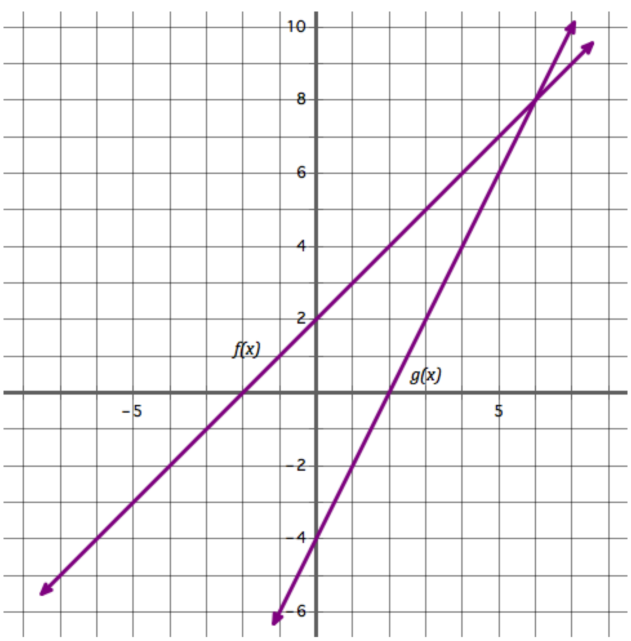
Topic: Adding and subtracting binomials

Add or subtract as indicated.

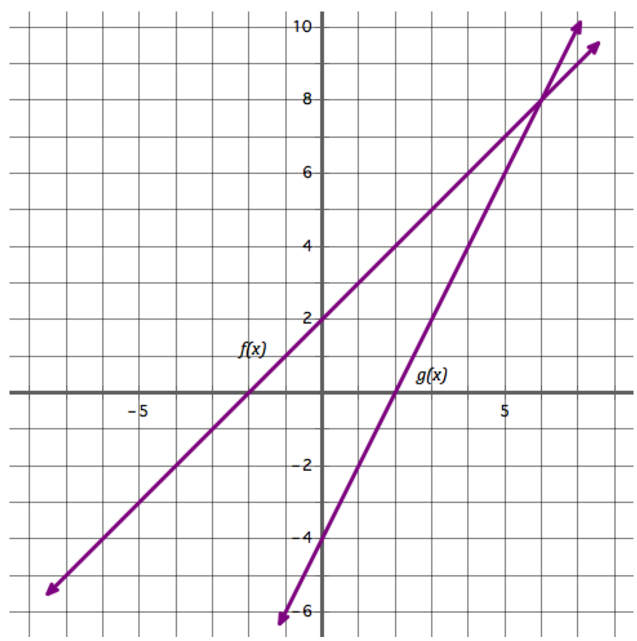
1. $(6x+3)+(4x+5)$
2. $(x+17)+(9x-13)$
3. $(7x-8)+(-2x+9)$
4. $(4x+9)-(x+2)$
5. $(-3x-1)-(2x+5)$
6. $(8x+3)-(-10x-9)$
7. $(3x-7)+(-3x-7)$
8. $(-5x+8)-(-5x+7)$
9. $(8x+9)-(7x+9)$

10. Use the graphs of $f(x)$ and $g(x)$ to sketch the graphs of $f(x)+g(x)$ and $f(x) - g(x)$.

$f(x)+g(x)$



$f(x) - g(x)$.



SET

Topic: Comparing simple polynomials

11. Complete the tables below for $y = x$ *and* $y = x^3$ *and* $y = x^5$

x	$y = x$
-1	
0	
1	

x	$y = x^3$
-1	
0	
1	

x	$y = x^5$
-1	
0	
1	

12. What assumption might you be tempted to make about the graphs of $y = x$, $y = x^3$ *and* $y = x^5$ based on the values you found in the 3 tables above?

13. What do you really know about the graphs of $y = x$ *and* $y = x^3$ *and* $y = x^5$ despite the values you found in the 3 tables above?

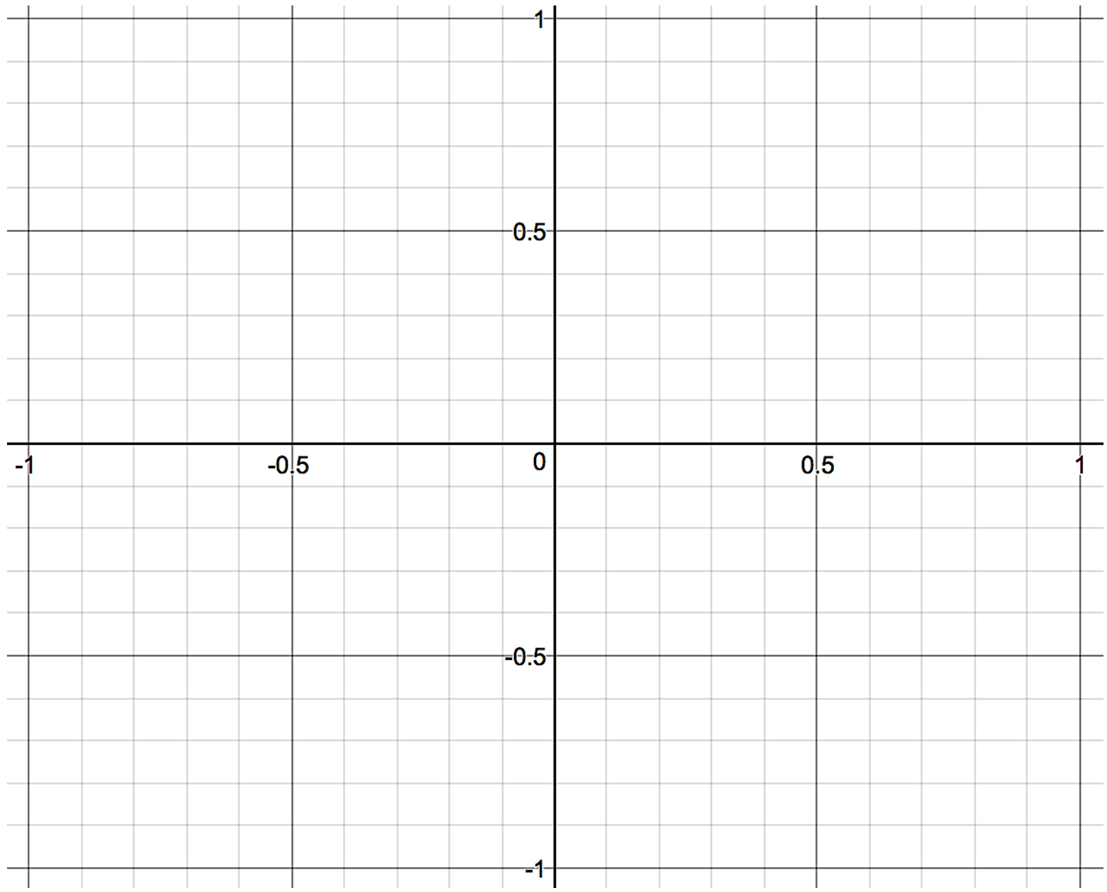
14. Complete the tables with the additional values.

x	$y = x$
-1	
$-1/2$	
0	
$1/2$	
1	

x	$y = x^3$
-1	
$-1/2$	
0	
$1/2$	
1	

x	$y = x^5$
-1	
$-1/2$	
0	
$1/2$	
1	

15. Graph $y = x$ *and* $y = x^3$ *and* $y = x^5$ on the interval $[-1, 1]$, using the same set of axes.



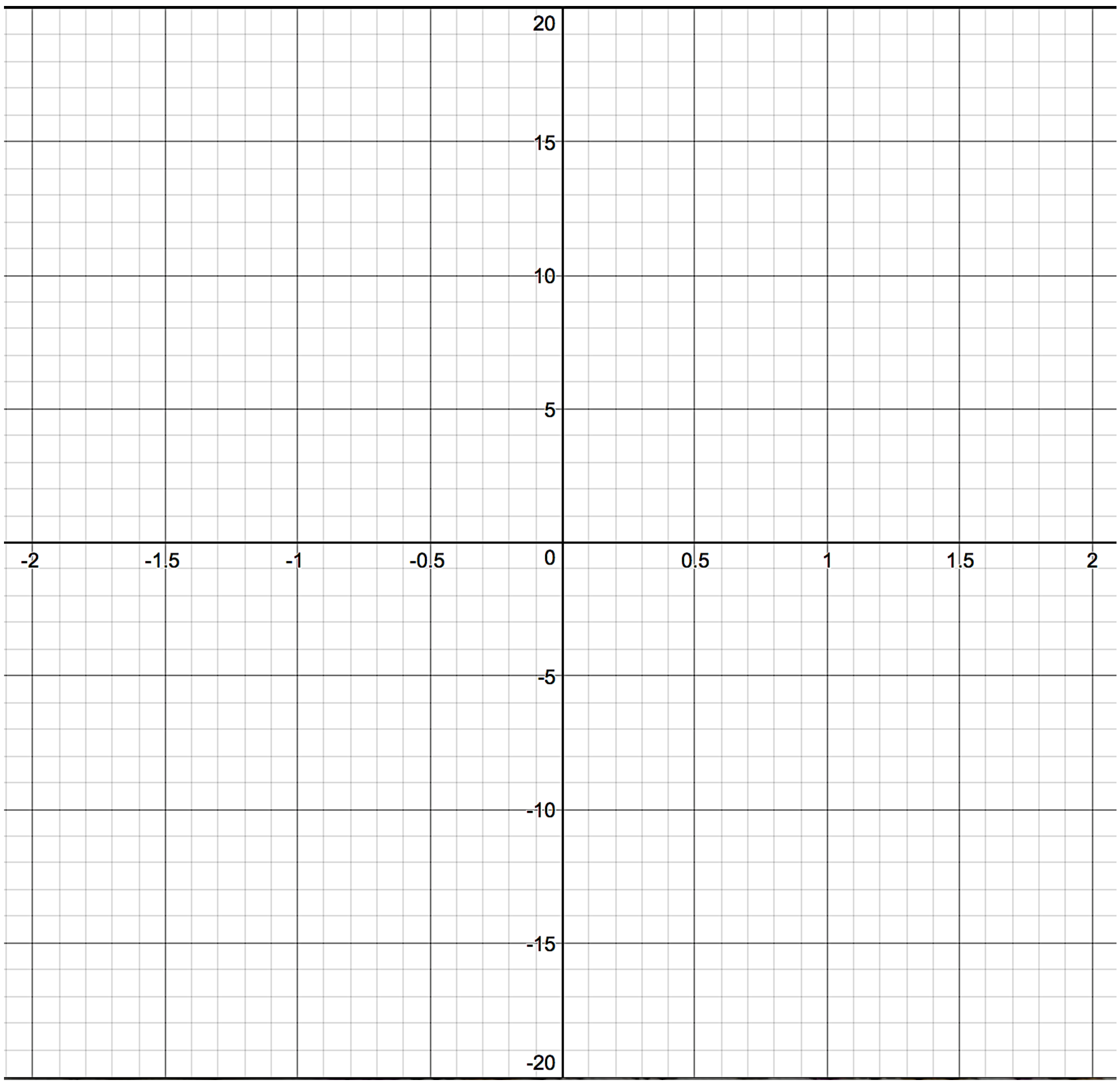
16. Complete the tables with the additional values.

x	$y = x$
-2	
-1	
$-1/2$	
0	
$1/2$	
1	
2	

x	$y = x^3$
-2	
-1	
$-1/2$	
0	
$1/2$	
1	
2	

x	$y = x^5$
-2	
-1	
$-1/2$	
0	
$1/2$	
1	
2	

17. Graph $y = x \text{ and } y = x^3 \text{ and } y = x^5$ on the interval [-2, 2], using the same set of axes.



GO

Topic: Using the exponent rules to simplify expressions

Simplify.

18. $x^{1/3} \cdot x^{1/6} \cdot x^{1/4}$

19. $a^{2/5} \cdot a^{3/10} \cdot a^{2/15}$

20. $m^{4/7} \cdot m^{3/14} \cdot m^{5/28}$

3.3 It All Adds Up

A Develop Understanding Task

Whenever we're thinking about algebra and working with variables, it is useful to consider how it relates to the number system and operations on numbers. Right now, polynomials are on our minds, so let's see if we can make some useful comparisons between whole numbers and polynomials.

Let's start by looking at the structure of numbers and polynomials. Consider the number 132. The way we write numbers is really a shortcut because:

$$132 = 100 + 30 + 2$$

1. Compare 132 to the polynomial $x^2 + 3x + 2$. How are they alike? How are they different?

2. Write a polynomial that is analogous to the number 2,675.

When two numbers are to be added together, many people use a procedure like this:

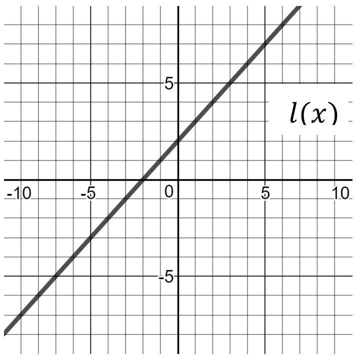
$$\begin{array}{r} 132 \\ +\ \underline{451} \\ 583 \end{array}$$

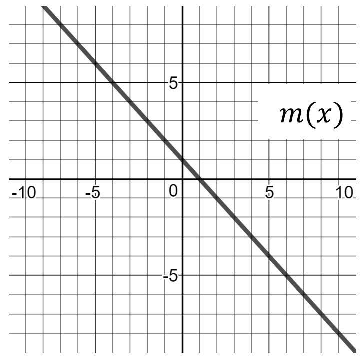
3. Write an analogous addition problem for polynomials and find the sum of the two polynomials.

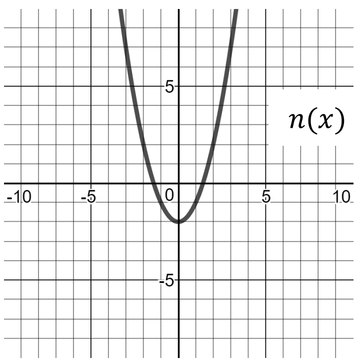
4. How does adding polynomials compare to adding whole numbers?

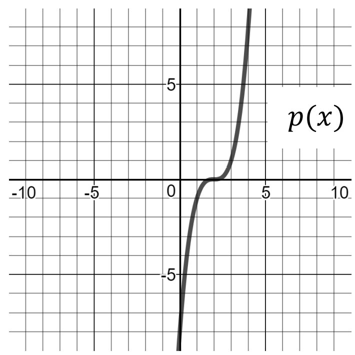
5. Use the polynomials below to find the specified sums in a-f.

$f(x) = x^3 + 3x^2 - 2x + 10$ $\quad g(x) = 2x - 1$ $\quad h(x) = 2x^2 + 5x - 12$ $\quad k(x) = -x^2 - 3x + 4$







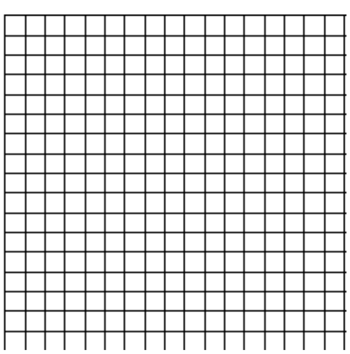


a) $h(x) + k(x)$

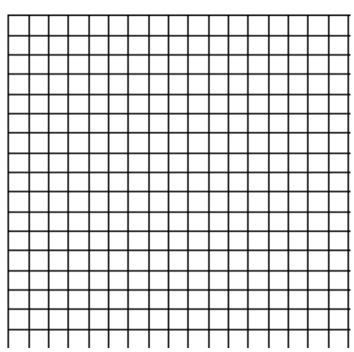
b) $g(x) + f(x)$

c) $f(x) + k(x)$

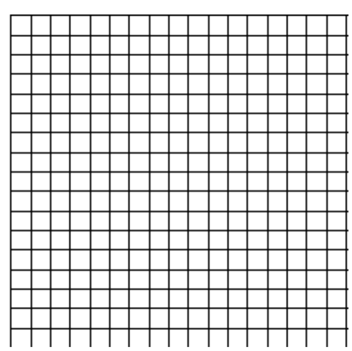
d) $l(x) + m(x)$



e) $m(x) + n(x)$



f) $l(x) + p(x)$



6. What patterns do you see when polynomials are added?

Subtraction of whole numbers works similarly to addition. Some people line up subtraction vertically and subtract the bottom number from the top, like this:

$$\begin{array}{r} 368 \\ -\underline{157} \\ 211 \end{array}$$

7. Write the analogous polynomials and subtract them.

8. Is your answer to #7 analogous to the whole number answer? If not, why not?

9. Subtracting polynomials can easily lead to errors if you don't carefully keep track of your positive and negative signs. One way that people avoid this problem is to simply change all the signs of the polynomial being subtracted and then add the two polynomials together. There are two common ways of writing this:

$$(x^3 + x^2 - 3x - 5) - (2x^3 - x^2 + 6x + 8)$$

Step 1: $= (x^3 + x^2 - 3x - 5) + (-2x^3 + x^2 - 6x - 8)$

Step 2: $= (-x^3 + 2x^2 - 9x - 13)$

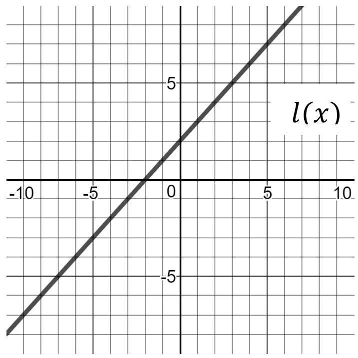
Or, you can line up the polynomials vertically so that Step 1 looks like this:

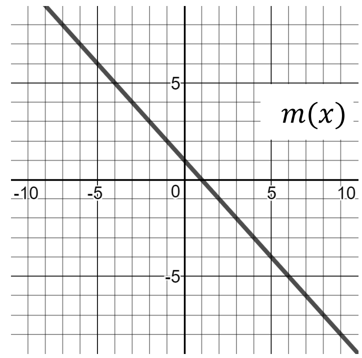
Step 1:
$$\begin{array}{r} x^3 + x^2 - 3x - 5 \\ \underline{+(-2x^3 + x^2 - 6x - 8)} \end{array}$$
Step 2:
$$-x^3 + 2x^2 - 9x - 13$$

The question for you is: Is it correct to change all the signs and add when subtracting? What mathematical property or relationship can justify this action?

10. Use the given polynomials to find the specified differences in a-d.

$f(x) = x^3 + 2x^2 - 7x - 8$ $\quad g(x) = -4x - 7$ $\quad h(x) = 4x^2 - x - 15$ $\quad k(x) = -x^2 + 7x + 4$





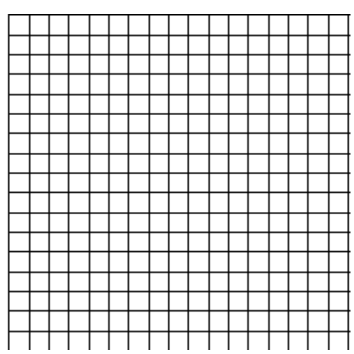
a) $h(x) - k(x)$

b) $f(x) - h(x)$

c) $f(x) - g(x)$

d) $k(x) - f(x)$

e) $l(x) - m(x)$



11. List three important things to remember when subtracting polynomials.

3.3 It All Adds Up – Teacher Notes

A Develop Understanding Task

Purpose: The purpose of this task is for students to surface comparisons between polynomials and whole numbers and use these comparisons to add and subtract polynomials algebraically. Students will also add and subtract polynomials given only graphically, adding corresponding points on the two graphs to obtain a sum. Students will make and test conjectures about the sum and differences of polynomials, such as "the sum of two quadratics is quadratic."

Core Standards Focus:

A.APR.1 Understand that polynomials for a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

F.BF.1: Write a function that describes a relationship between two quantities.
b. Combine standard function types using arithmetic operations.

F.IF.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Standards for Mathematical Practice:

SMP 7 – Look for and make use of structure

SMP 8 – Look for express regularity in repeated reasoning

The Teaching Cycle:

Launch (Whole Group):

Begin the discussion with the introduction to the task and problem 1. Help students to see that numbers are structured as the sum of powers of 10 and polynomials are structured as powers of x. Because of that, operations on polynomials and whole numbers will work similarly. Ask students to do problem 2 individually, and then check to be sure that the class understands how to write an "analogous" polynomial. Then ask students to do problem #3 and verify that they have the idea of adding like terms. They may choose to line up the polynomials as shown with whole numbers or to add them horizontally, but either way, only like terms are added. They should also notice that the exponents don't change when the terms are added. Ask if this is true of whole numbers. After discussing the first page of the task, students should be ready to work on questions 5-6.

Explore (Small Groups):

Monitor students as they are working to be sure that they are appropriately adding like terms and working with the exponents. When students are asked to add the graphs, encourage them to use the graphs without finding algebraic expressions for the graphs. It is important for students to have an understanding of how function addition works with representations other than algebraic equations.

Discuss (Whole Group):

Begin the discussion with $f(x) + k(x)$. Ask a student to share their work and explain their strategy. This is a straightforward problem which can be used to ensure that all students have the basic idea. If students are having difficulty, then ask another student to share $h(x) + k(x)$. Then, shift the discussion to adding functions graphically, using $m(x) + n(x)$. Ask a student to share how they obtained the sum by adding corresponding y-values on the two functions. Ask students why only the y-values are added, not the x-values. Reinforce the idea that $m(x) + n(x)$ means that the output values for the two functions are to be added. Ask students why the sum of the two graphs was quadratic. Would it always be true that the sum of a linear function and a quadratic function is a quadratic function?

Continue by asking a student to share his/her work on $l(x) + m(x)$, again demonstrating how to add corresponding outputs on the two graphs. Ask students why the sum turned out to be a horizontal line. Would the sum of two linear functions always be a horizontal line? Would the sum of two linear functions always be a linear function?

Ask a few students to share a conjecture on problem 6 and to explain why they believe it to be true. Ask the class to respond with support for the conjecture or counter-examples that disprove it.

This completes the discussion for the addition part of the task, so that students are ready to consider subtraction.

Re-launch (Whole Group):

Ask students to work individually to do problems #7 and 8. Briefly discuss the narrative in #9 and ask student why it is mathematically correct to change the signs of the subtrahend and then to add the two polynomials together rather than subtracting. Again, students may choose to work horizontally or vertically. Tell students to complete the task.

Explore (Small Groups):

Monitor students as they work to be sure that they have a strategy that helps them to keep the signs straight in a subtraction problem, since this is the most common error. Watch for students that are making sense of the graphs and subtracting outputs to find the difference between the two functions so that they can share during the class discussion. Encourage students that are trying to write equations to add together the graphs to try to do it without equations so that they can consider a different representation.

Discuss (Whole Group):

Ask students to demonstrate their strategies for subtracting on as many problems as time allows. Include students that have lined up the polynomials vertically and others that have worked the problems horizontally and compare the relative merits of each. Ask a student to share his/her work with subtracting the graphs and ask how this is similar to the work done with addition. End the discussion with students sharing responses to the last question about the procedure for subtracting polynomials. Highlight responses that include the following ideas:

- All of the terms in the second polynomial must be subtracted.
- Only subtract like terms.
- Subtraction can be rewritten as addition to avoid sign errors.

Aligned Ready, Set, Go: Polynomial Functions 3.3

READY, SET, GO! Name Period Date

READY

Topic: Using the distributive property

Multiply.

1. $2x(5x^2 + 7)$
2. $9x(-x^2 - 3)$
3. $5x^2(x^4 + 6x^3)$
4. $-x(x^2 - x + 1)$
5. $-3x^3(-2x^2 + x - 1)$
6. $-1(x^2 - 4x + 8)$

SET

Topic: Adding and subtracting polynomials

Add. Write your answers in descending order of the exponents. (Standard form)

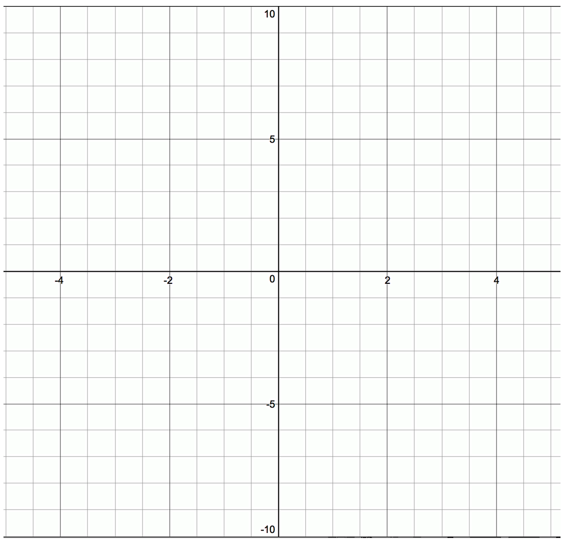
7. $(3x^4 + 5x^2 - 1) + (2x^3 + x)$
8. $(4x^2 + 7x - 4) + (x^2 - 7x + 14)$
9. $(2x^3 + 6x^2 - 5x) + (x^5 + 3x^2 + 8x + 4)$
10. $(-6x^5 - 2x + 13) + (4x^5 + 3x^2 + x - 9)$

Subtract. Write your answers in descending order of the exponents. (Standard form)

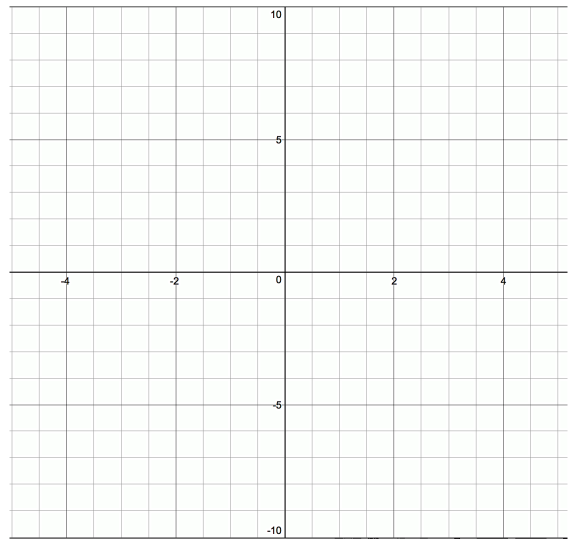
11. $(5x^2 + 7x + 2) - (3x^2 + 6x + 2)$
12. $(10x^4 + 2x^2 + 1) - (3x^4 + 3x + 11)$
13. $(7x^3 - 3x + 7) - (4x^2 - 3x - 11)$
14. $(x^4 - 1) - (x^4 + 1)$

Graph.

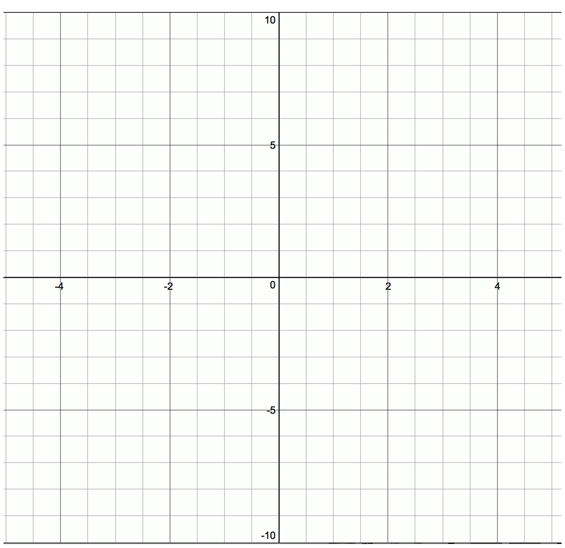
15. $y = x^3 - 2$



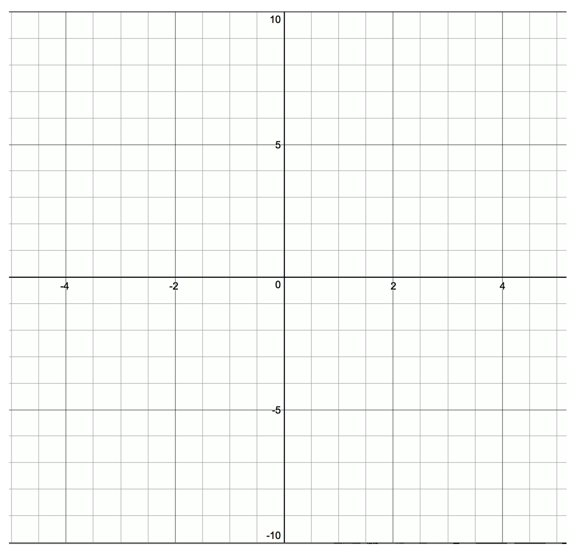
16. $y = x^3 + 1$



17. $y = (x - 3)^3$



18. $y = (x + 1)^3$



GO

Topic: Using exponent rules to combine expressions

Simplify.

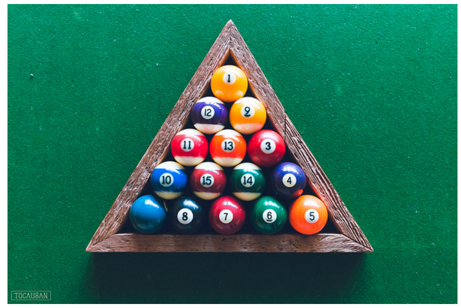
19. $x^{7/8} \cdot x^{1/4} \cdot x^{-1/2}$

20. $x^{3/16} \cdot x^{-7/8} \cdot x^{3/4}$

21. $x^{4/7} \cdot x^{2/9} \cdot x^{-1/3}$

3.4 Pascal's Pride

A Solidify Understanding Task



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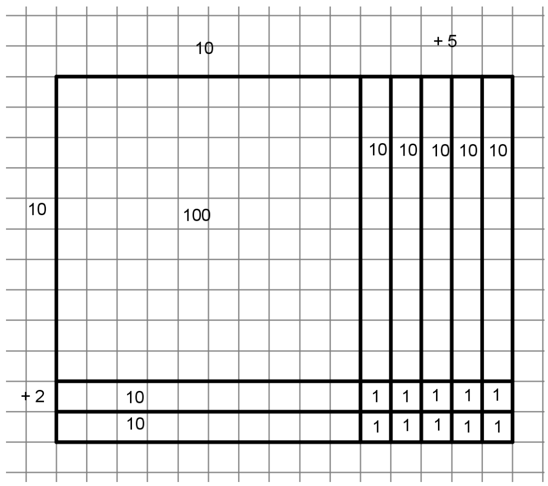
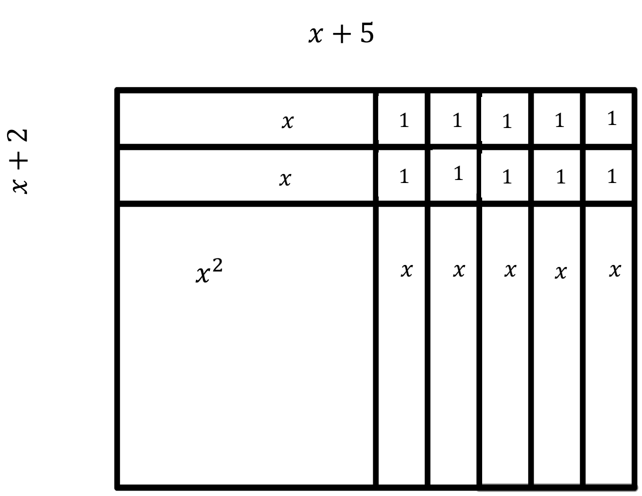
Multiplying polynomials can require a bit of skill in the algebra department, but since polynomials are structured like numbers, multiplication works very similarly. When you learned to multiply numbers, you may have learned to use an area model. To multiply 12×15 the area model and the related procedure probably looked like this:



$$\begin{array}{r} 12 \\ \times\ \underline{15} \\ 10 \\ 50 \\ +\ 20 \\ \underline{100} \\ \mathbf{180} \end{array}$$

You may have used this same idea with quadratic expressions. Area models help us think about multiplying, factoring, and completing the square to find equivalent expressions. We modeled $(x+2)(x+5) = x^2 + 7x + 10$ as the area of a rectangle with sides of length $x+2$ and $x+5$. The various parts of the rectangle are shown in the diagram below:



Some people like to shortcut the area model a little bit to just have sections of area that correspond to the lengths of the sides. In this case, they might draw the following.

	x	$+5$
x	x^2	$5x$
$+2$	$2x$	10

$$= x^2 + 7x + 10$$

1. What is the property that all of these models are based upon?

2. Now that you've been reminded of the happy past, you are ready to use the strategy of your choice to find equivalent expressions for each of the following:

a) $(x+3)(x+4)$

b) $(x+7)(x-2)$

Maybe now you remember some of the different forms for quadratic expressions—factored form and standard form. These forms exist for all polynomials, although as the powers get higher, the algebra may get a little trickier. In standard form polynomials are written so that the terms are in order with the highest-powered term first, and then the lower-powered terms. Some examples:

Quadratic: $x^2 - 3x + 8$ or $x^2 - 9$

Cubic: $2x^3 + x^2 - 7x - 10$ or $x^3 - 2x^2 + 15$

Quartic: $x^4 + x^3 + 3x^2 - 5x + 4$

Hopefully, you also remember that you need to be sure that each term in the first factor is multiplied by each term in the second factor and the like terms are combined to get to standard form. You can use area models, boxes, or mnemonics like FOIL (first, outer, inner, last) to help you organize, or you can just check every time to be sure that you've got all the combinations. It can get more challenging with higher-powered polynomials, but the principal is the same because it is based upon the mighty Distributive Property.

3. Tia's favorite strategy for multiplying polynomials is to make a box that fits the two factors. She sets it up like this: $(x+2)(x^2-3x+5)$

	x^2	$-3x$	$+5$
x			
$+2$			

Try using Tia's box method to multiply these two factors together and then combining like terms to get a polynomial in standard form.

4. Try checking your answer by graphing the original factored polynomial, $(x+2)(x^2-3x+5)$ and then graphing the polynomial that is your answer. If the graphs are the same, you are right because the two expressions are equivalent! If they are not the same, go back and check your work to make the corrections.

5. Tehani's favorite strategy is to connect the terms he needs to multiply in order like this:

$$(x-3)(x^2+4x-2)$$

Try multiplying using Tehani's strategy and then check your work by graphing. Make any corrections you need and figure out why they are needed so that you won't make the same mistake twice!

6. Use the strategy of your choice to multiply each of the following expressions. Check your work by graphing and make any needed corrections.

a) $(x+5)(x^2-x-3)$

b) $(x-2)(2x^2+6x+1)$

c) $(x+2)(x-2)(x+3)$

When graphing, it is often useful to have a perfect square quadratic or a perfect cube. Sometimes it is also useful to have these functions written in standard form. Let's try re-writing some related expressions to see if we can see some useful patterns.

7. Multiply and simplify both of the following expressions using the strategy of your choice:

a) $f(x) = (x+1)^2$ b) $f(x) = (x+1)^3$

Check your work by graphing and make any corrections needed.

8. Some enterprising young mathematician noticed a connection between the coefficients of the terms in the polynomial and the number pattern known as Pascal's Triangle. Put your answers from problem 5 into the table. Compare your answers to the numbers in Pascal's Triangle below and describe the relationship you see.

$(x+1)^0$	1	1
$(x+1)^1$	$x+1$	1 1
$(x+1)^2$		1 2 1
$(x+1)^3$		1 3 3 1
$(x+1)^4$		

9. It could save some time on multiplying the higher power polynomials if we could use Pascal's Triangle to get the coefficients. First, we would need to be able to construct our own Pascal's Triangle and add rows when we need to. Look at Pascal's Triangle and see if you can figure out how to get the next row using the terms from the previous row. Use your method to find the terms in the next row of the table above.

10. Now you can check your Pascal's Triangle by multiplying out $(x+1)^4$ and comparing the coefficients. Hint: You might want to make your job easier by using your answers from #7 in some way. Put your answer in the table above.

11. Make sure that the answer you get from multiplying $(x+1)^4$ and the numbers in Pascal's Triangle match, so that you're sure you've got both answers right. Then describe how to get the next row in Pascal's Triangle using the terms in the previous row.

12. Complete the next row of Pascal's Triangle and use it to find the standard form of $(x+1)^5$. Write your answers in the table on #6.

13. Pascal's Triangle wouldn't be very handy if it only worked to expand powers of $x+1$. There must be a way to use it for other expressions. The table below shows Pascal's Triangle and the expansion of $x+a$.

$(x+a)^0$	1	1
$(x+a)^1$	$x+a$	1 1
$(x+a)^2$	$x^2+2ax+a^2$	1 2 1
$(x+a)^3$	$x^3+\ 3ax^2+3a^2x+a^3$	1 3 3 1
$(x+a)^4$	$x^4+4ax^3+\ 6a^2x^2+3a^3x+a^4$	1 4 6 4 1

What do you notice about what happens to the a in each of the terms in a row?

14. Use the Pascal's Triangle method to find standard form for $(x+2)^3$. Check your answer by multiplying.

15. Use any method to write each of the following in standard form:

a) $(x+3)^3$ b) $(x-2)^3$ c) $(x+5)^4$

3.4 Pascal's Pride – Teacher Notes
A Solidify Understanding Task

Purpose: The purpose of this task is to sharpen students' multiplication skills for polynomials and to introduce the binomial theorem to expand polynomials using Pascal's Triangle. Students are reminded of the connections between polynomials and whole numbers, along with the area model for multiplying both polynomials and whole numbers. Standard form for polynomials is introduced and students work with polynomials in factored form and write them in standard form.

Core Standards Focus:
A-APR.5: Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Standards for Mathematical Practice:

SMP 6 – Attend to precision

SMP 8 – Look for express regularity in repeated reasoning

Vocabulary: Standard form of a polynomial

The Teaching Cycle:
Launch (Whole Group):
Begin the task by telling students that quadratics are one kind of polynomial that they have worked with previously. In this task we will be extending our work with quadratics to higher-powered polynomials, taking them from factored form to standard form. Focus students on the area models presented at the beginning of the task and ask how the large area model and the shortcut area model are related. Ask students question 1, and make sure that they know that all multiplication strategies are based upon the Distributive Property. Have students work problem 2 individually and then briefly discuss how to use the shortcut area model to do the problems. Then, ask students to work on the rest of the task.

Explore (Small Group):

Monitor students as they work to be sure that they understand both Tia and Tehani's strategy. Listen for students that have noticed that when using Tia's strategy (the box strategy), that the like terms are often lined up on the diagonal. This is a useful idea to share during the discussion to help students simplify their expressions once they have all the terms. Help students to check their work using technology as described in the task. Encourage them to not only make corrections, but to see where they are making errors so that they can avoid them in the future. As students are working, look for a student to present problem 6c that has noticed that two of the factors can be combined quickly because they are the factors that make a difference of squares.

As students work on the second part of the task with the binomial expansions, listen for students that can articulate the patterns in Pascal's Triangle that help them to construct and expand the triangle. If students are having trouble with constructing the next row of the triangle, ask them what relationships they see between numbers on one row with numbers on the next. Are there ways that they could combine numbers on one row to get the next row of numbers? Look for students to share during the discussion that can describe the pattern in the coefficients and relate it to Pascal's triangle.

Discuss (Whole Group):

Begin the discussion by having a student share their work on question #3. The student selected should be able to share the idea that the polynomial can be simplified and that the like terms are along the diagonals of the box. Ask another student to show how they used technology to check his/her answers and be sure that this strategy is understood by the class.

Next, have a student share their work with Tehani's strategy in #5. Ask the class to compare the two strategies. They should recognize that they produce the same terms and are simply two different ways to keep track of all the terms. Tell students that they may use either strategy (or both) as they choose. Ask a student to present their work in problem 6c demonstrating that two of the factors make a difference of squares, and then that result can be multiplied by the remaining factor. Point out that that the three linear factors multiplied together make a cubic. Ask students to notice what kind of polynomial is produced when a linear factor is multiplied by a quadratic factor.

Transition from problem 6c to problem 7b by asking how student used their work from #7a to help them in #7b. Ask a previously-selected student to share how they found $(x+1)^2$ and multiplied it by $(x+1)$ to find the answer. Ask students to describe how the coefficients of the terms in #7a and #7b compare to the numbers in Pascal's triangle.

Have the previously-selected student share how to construct the next row of Pascal's triangle and test their strategy by constructing the next row (problem 9). Then have a student show how to use Pascal's triangle to find the solution for problem #10. Ask a student to share their work on problem 14 and help students to see how the coefficients obtained from Pascal's triangle are multiplied by the increasing powers of 2 to get the expanded polynomial. Reinforce this idea by sharing as many of the remaining problems as time allows.

Aligned Ready, Set, Go: Polynomial Functions 3.4

READY, SET, GO! Name Period Date

READY

Topic: Recalling the meaning of division

1. Given: $f(x) = (x+7)(2x-3)$ *and* $g(x) = (x+7)$. Find $g(x)\overline{)f(x)}$.

2. Given: $f(x) = (5x+7)(-3x+11)$ *and* $g(x) = (-3x+11)$. Find $g(x)\overline{)f(x)}$

3. Given: $f(x) = (x+2)(x^2+3x+2)$ *and* $g(x) = (x+2)$ Find $g(x)\overline{)f(x)}$

4. Given: $f(x) = (5x-3)(x^2-11x-9)$ *and* $g(x) = (5x-3)$ *and* $h(x) = (x^2-11x-9)$.

   a.) Find $g(x)\overline{)f(x)}$ b.) Find $h(x)\overline{)f(x)}$

5. Given: $f(x) = (5x-6)(2x^2-5x+3)$ *and* $g(x) = (x-1)$ *and* $h(x) = (2x-3)$.

   a.) Find $g(x)\overline{)f(x)}$ b.) Find $h(x)\overline{)f(x)}$

SET

Topic: Multiplying polynomials

Multiply. Write your answers in standard form.

6. $(a+b)(a+b)$

7. $(x-3)(x^2+3x+9)$

8. $(x-5)(x^2+5x+25)$

9. $(x+1)(x^2-x+1)$

10. $(x+7)(x^2-7x+49)$

11. $(a-b)(a^2+ab+b^2)$

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$(x+a)^0$	1	1
$(x+a)^1$	$x+a$	1 1
$(x+a)^2$	$x^2+2ax+a^2$	1 2 1
$(x+a)^3$	$x^3+\ 3ax^2+3a^2x+a^3$	1 3 3 1
$(x+a)^4$	$x^4+4ax^3+\ 6a^2x^2+3a^3x+a^4$	1 4 6 4 1

Use the table above to write each of the following in standard form.

12. $(x+1)^5$

13. $(x-5)^3$

14. $(x-1)^4$

15, $(x+4)^3$

16. $(x+2)^4$

17. $(3x+1)^3$

GO

Topic: Examining transformations on different types of functions

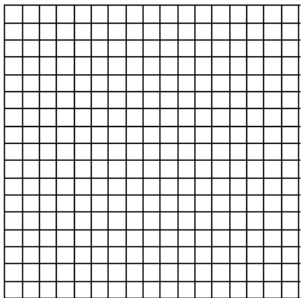
Graph the following functions.

18. $g(x)=x+2$

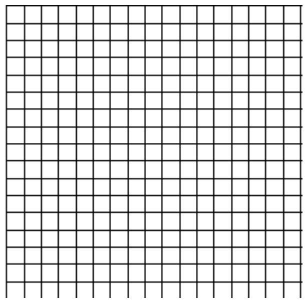
19. $h(x)=x^2+2$

20. $f(x)=2^x+2$

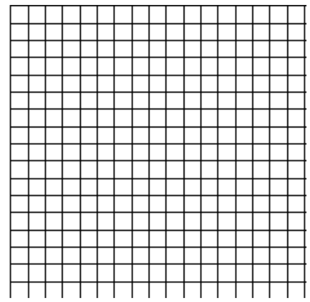
21. $g(x) = 3(x - 2)$



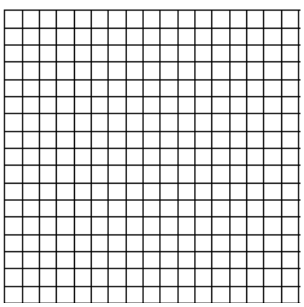
22. $h(x) = 3(x - 2)^2$



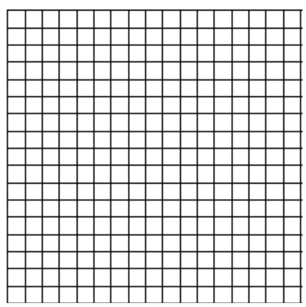
23. $f(x) = 3\sqrt{x - 2}$



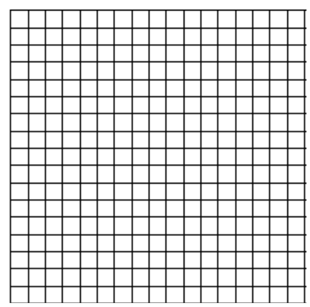
24. $g(x) = \frac{1}{2}(x - 1) - 2$



25. $h(x) = \frac{1}{2}(x - 1)^2 - 2$



26. $f(x) = |x - 1| - 2$



3.5 Divide And Conquer

A Solidify Understanding Task

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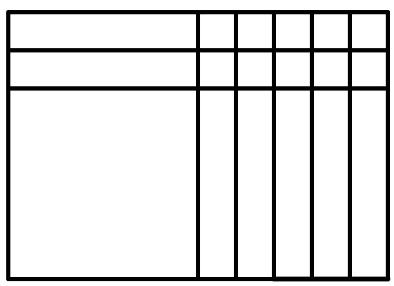
We've seen how numbers and polynomials relate in addition, subtraction, and multiplication. Now we're ready to consider division.

Division, you say? Like, long division? Yup, that's what we're talking about. Hold the judgment! It's actually pretty cool.

As usual, let's start by looking at how the operation works with numbers. Since division is the inverse operation of multiplication, the same models should be useful. The area model that we used with multiplication is also used with division. When we were using area models to factor a quadratic expression, we were actually dividing.

Let's brush up on that a bit.

1. The area model for $x^2 + 7x + 10$ is shown below:



Use the area model to write $x^2 + 7x + 10$ in factored form.

2. We also used number patterns to factor without drawing the area model. Use any strategy to factor the following quadratic polynomials:

a) $x^2 + 7x + 12$	b) $x^2 + 2x - 15$

c) $x^2 - 11x + 24$	d) $x^2 - 5x - 36$

Factoring works great for quadratics and a few special cases of other polynomials. Let's look at a more general version of division that is a lot like what we do with numbers. Let's say we want to divide 1452 by 12. If we write the analogous polynomial division problem, it would be: $(x^3 + 4x^2 + 5x + 2) \div (x + 2)$.

Let's use the division process for numbers to create a division process for polynomials. (Don't panic—in many ways it's easier with polynomials than numbers!)

Step 1: Start with writing the problem as long division. The polynomial needs to have the terms written in descending order. If there are any missing powers, it's easier if you leave a little space for them.

$$12\overline{)1452} \qquad\qquad x+2\overline{)x^3+4x^2+5x+2}$$

Step 2: Determine what you could multiply the divisor by to get the first term of the dividend.

$$\begin{array}{r} 1 \\ 12\overline{)1452} \end{array} \qquad\qquad \begin{array}{r} x^2 \\ x+2\overline{)x^3+4x^2+5x+2} \end{array}$$

Step 3: Multiply and put the result below the dividend.

$$\begin{array}{r} 1 \\ 12\overline{)1452} \\ \underline{-1200} \end{array} \qquad\qquad \begin{array}{r} x^2 \\ x+2\overline{)x^3+4x^2+5x+2} \\ -(\underline{x^3+2x^2}) \end{array}$$

Step 4: Subtract. (It helps to keep the signs straight if you change the sign on each term and add on the polynomial.)

$$\begin{array}{r} 1 \\ 12\overline{)1452} \\ \underline{-1200} \\ 252 \end{array} \qquad \begin{array}{r} x^2 \\ x+2\overline{)x^3+4x^2+5x+2} \\ +(\underline{-x^3-2x^2 }) \\ 2x^2+5x+2 \end{array}$$

Step 5: Repeat the process with the number or expression that remains in the dividend.

$$\begin{array}{r} 12 \\ 12\overline{)1452} \\ \underline{-1200} \\ 252 \\ -\underline{240} \\ 12 \end{array} \qquad \begin{array}{r} x^2+2x \\ x+2\overline{)x^3+4x^2+5x+2} \\ +(\underline{-x^3-2x^2 }) \\ 2x^2+5x+2 \\ -(\underline{2x^2+4x }) \\ x+2 \end{array}$$

Step 6: Keep going until the number or expression that remains is smaller than the divisor.

$$\begin{array}{r} 121 \\ 12\overline{)1452} \\ \underline{-1200} \\ 252 \\ -\underline{240} \\ 12 \\ -\underline{12} \\ 0 \end{array} \qquad \begin{array}{r} x^2+2x+1 \\ x+2\overline{)x^3+4x^2+5x+2} \\ +(\underline{-x^3-2x^2 }) \\ 2x^2+5x+2 \\ -(\underline{2x^2+4x }) \\ x+2 \\ -\underline{(x+2)} \\ 0 \end{array}$$

In this case, 121 divided by 12 leaves no remainder, so we would say that 12 is a factor of 121. Similarly, since $(x^3 + 4x^2 + 5x + 2)$ divided by $(x + 2)$ leaves no remainder, we would say that $(x + 2)$ is a factor of $(x^3 + 4x^2 + 5x + 2)$.

Polynomial division doesn't always match up perfectly to an analogous whole number problem, but the process is always the same. Let's try it.

3. Use long division to determine if $(x-1)$ a factor of $(x^3-3x^2-13x+15)$. Don't worry: the steps for the division process are below:

a) Write the problem as long division.
b) What do you have to multiply x by to get x^3? Write your answer above the bar.
c) Multiply your answer from step b by $(x-1)$ and write your answer below the dividend.
d) Subtract. Be careful to subtract each term. (You might want to change the signs and add.)
e) Repeat steps a-d until the expression that remains is less than $(x-1)$.

We hope you survived the division process. Is $(x-1)$ a factor of $(x^3-3x^2-13x+15)$? ________

4. Try it again. Use long division to determine if $(2x+3)$ is a factor of $2x^3+7x^2+2x+9$. No hints this time. You can do it!

When dividing numbers, there are several ways to deal with the remainder. Sometimes, we just write it as the remainder, like this:

$$3\overline{)\ 25}^{\ 8r.1}$$ because $3(8)+1=25$

You may remember also writing the remainder as a fraction like this:

$$3\overline{\big)\ 25}^{\,8\frac{1}{3}} \text{ because } 3\left(8\tfrac{1}{3}\right) = 25$$

We do the same things with polynomials.

Maybe you found that $(2x^3 + 7x^2 + 2x + 9) \div (2x + 3) = \ (x^2 + 2x - 2)\ r. 15$. (We sure hope so.) You can use it to write two multiplication statements:

$$(2x + 3)(x^2 + 2x - 2) + 15 = (2x^3 + 7x^2 + 2x + 9)$$

and

$$(2x + 3)(x^2 + 2x - 2 + \frac{15}{2x + 3}) = (2x^3 + 7x^2 + 2x + 9)$$

5. Divide each of the following polynomials. Write the two multiplication statements that go with your answers if there is a remainder. Write only one multiplication statement if the divisor is a factor. Use graphing technology to check your work and make the necessary corrections.

	a) $(x^3 + 6x^2 + 13x + 12) \div (x + 3)$	b) $(x^3 - 4x^2 + 2x + 5) \div (x - 2)$
Multiplication statements:		

	c) $(6x^3 - 11x^2 - 4x + 5) \div (2x - 1)$	d) $(x^4 - 23x^3 + 49x + 4) \div (x^2 + x + 2)$
Multiplication statements:		

3.5 Divide And Conquer – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to reinforce students' prior knowledge of factoring and to introduce polynomial long division. The task draws connections between polynomial long division and division of whole numbers to support students in understanding the procedure. Students will write remainders in two ways to create equivalent polynomial expressions and determine whether a given expression is a factor according to the Polynomial Remainder Theorem. Both factoring and long division will be used in upcoming tasks to write polynomials in factored form and to find their roots.

Core Standards Focus:

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.APR.2: Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

Standards for Mathematical Practice:

SMP 8 – Look for and express regularity in repeated reasoning

Vocabulary: Remainder Theorem

The Teaching Cycle:

Launch (Whole Group):

Introduce the task by asking students how they determine factors of numbers like 243. Without actually going through the process, students should recognize that they would divide to determine factors. Remind students that they have factored quadratic trinomials and this was a way to see what expressions had been multiplied together to give the trinomial, which is essentially a division process. Tell students that in upcoming work, they will need to find factors of higher-powered polynomials, which cannot usually be done with the same strategies as quadratics. In this task, we will be learning how to divide polynomials so that we can determine if an expression is a factor. Tell students that the task begins with brushing up on factoring. Remind students how they used area models to factor by working problem 1 together. Then ask students to work problem 2 individually. After giving students a few minutes to work, ask two students to share their work on the two problems.

Next, briefly explain how 1452 ÷12 is analogous to $(x^3 + 4x^2 + 5x + 2) \div (x + 2)$. Demonstrate each of the steps of the long division process shown in the task, with the number problem parallel to the polynomial problem so that students can see that the two processes are the same. Then, ask students to work on the remainder of the task.

Explore (Small Groups):

Support students as they are working by drawing them back to the steps in the division process for whole numbers, which they already know. Many of the problems that students typically have in long division are a result of making mistakes when subtracting, so you may wish to encourage students to subtract by consistently changing the signs and adding, as suggested in *3.3 It All Adds Up*.

Discuss (Whole Group):

Begin the discussion by asking a student to share problem 3, walking through each step carefully. Continue the discussion with as many of the rest of the problems as time allows, each time having a student share work and reinforcing the steps. At the end of each problem, ask whether the divisor is a factor. Discuss the two forms for writing polynomials that are equivalent to the dividend. Demonstrate how to use the two multiplication statements to check answers by graphing a multiplication statement and the dividend and checking to see that the two graphs coincide. Be sure to include problem 5d, which does not contain a squared term in the dividend. Most students will not anticipate leaving a space for the term, so this problem provides an opportunity to discuss why it is helpful to have all the terms in the dividend in decreasing order and to leave space for any of the terms that are missing. This problem also has a remainder that contains two terms.

Emphasize that the remainder is still handled the same way, and then ask a student to write the two multiplication statements that are equivalent to the dividend.

Aligned Ready, Set, Go: Polynomial Functions 3.5

READY, SET, GO! Name Period Date

READY

Topic: Solving linear equations

Solve for x.

1. $5x + 13 = 48$

2. $\frac{1}{3}x - 8 = 0$

3. $-4 - 9x = 0$

4. $x^2 - 16 = 0$

5. $x^2 + 4x + 3 = 0$

6. $x^2 - 5x + 6 = 0$

7. $(x + 8)(x + 11) = 0$

8. $(x - 5)(x - 7) = 0$

9. $(3x - 18)(5x - 10) = 0$

SET

Topic: Dividing polynomials

Divide each of the following polynomials. Write only one multiplication statement if the divisor is a factor. Write the two multiplication statements that go with your answers if there is a remainder.

10. $(x+1)\overline{)x^3 - 3x^2 + 6x + 11}$

11. $(x-5)\overline{)x^3 - 9x^2 + 23x - 15}$

Multiplication statement(s)

Multiplication statement(s)

12. $(2x-1)\overline{)2x^3+15x^2-34x+13}$

13. $(x+4)\overline{)x^3+13x^2+26x-25}$

Multiplication statement(s)

Multiplication statement(s)

14. $(x+7)\overline{)x^3-8x^2-111x+10}$

15. $(3x-4)\overline{)3x^3+23x^2+6x-28}$

Multiplication statement(s)

Multiplication statement(s)

GO

Topic: Describing the features of a variety of functions

Graph the following functions. Then identify the key features of the functions. Include domain, range, intervals where the function is increasing/decreasing, intercepts, maximum/minimum, and end behavior.

16. $f(x) = x^2 - 9$

domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

17. $f(n-1) = f(n) + 3;\ f(1) = 4$

domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

18. $f(x) = \sqrt{x-3} + 1$

domain: range:

increasing: decreasing:

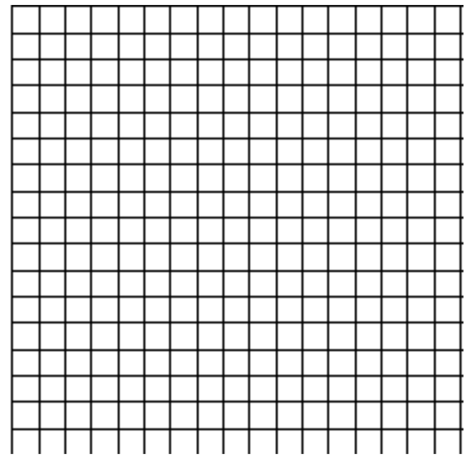
y-intercept: x-intercept(s):

19. $f(x) = log_2 x - 1$

domain: range:

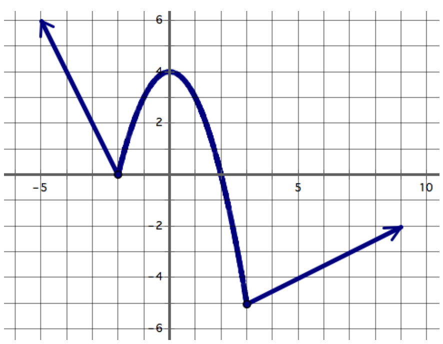
increasing: decreasing:

y-intercept: x-intercept(s):



Identify the key features of the graphed functions.

20.

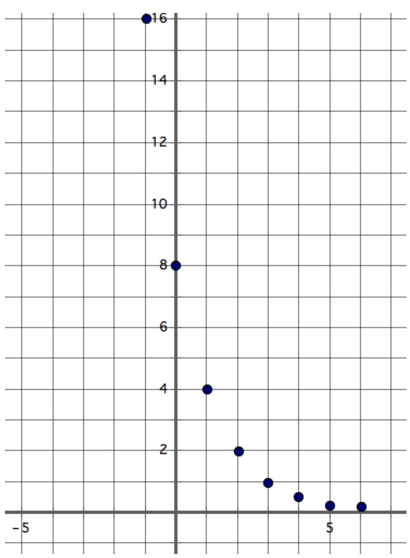


domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

21.



domain: range:

increasing: decreasing:

y-intercept: x-intercept(s):

3.6 Sorry, We're Closed

A Practice Understanding Task

Now that we have compared operations on polynomials with operations on whole numbers it's time to generalize about the results. Before we go too far, we need a technical definition of a polynomial function. Here it is:

A polynomial function has the form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

where $a_n, a_{n-1}, \ldots a_1, a_0$ are real numbers and n is a nonnegative integer. In other words, a polynomial is the sum of one or more monomials with real coefficients and nonnegative integer exponents. The degree of the polynomial function is the highest value for n where a_n is not equal to 0.

1. The following examples and non-examples will help you to see the important implications of the definition of a polynomial function. For each pair, determine what is different between the example of a polynomial and the non-example that is not a polynomial.

These **are** polynomials:	These **are not** polynomials:
a) $f(x) = x^3$	b) $g(x) = 3^x$
How are a and b different?	
c) $f(x) = 2x^2 + 5x - 12$	d) $g(x) = \frac{2x^2}{x^2 - 3x + 2}$
How are c and d different?	
e) $f(x) = -x^3 + 3x^2 - 2x - 7$	f) $g(x) = x^3 + 3x^2 - 2x + 10x^{-1} - 7$
How are e and f different?	
h) $f(x) = \frac{1}{2}x$	i) $g(x) = \frac{1}{2x}$
How are h and i different?	
j) $f(x) = x^2$	k) $g(x) = x^{\frac{1}{2}}$
How are j and k different?	

2. Based on the definition and the examples above, how can you tell if a function is a polynomial function?

Maybe you have noticed in the past that when you add two even numbers, the answer you get is always an even number. Mathematically, we say that the set of even numbers is **closed** under addition. Mathematicians are interested in results like this because it helps us to understand how numbers or functions of a particular type behave with the various operations.

3. You can try it yourself: Is the set of odd numbers closed under multiplication? In other words, if you multiply two odd numbers together will you get an odd number? Explain.

If you find any two odd numbers that have an even product, then you would say that odd numbers are not closed under multiplication. Even if you have a number of examples that support the claim, if you can find one **counterexample** that contradicts the claim, then the claim is false.

Consider the following claims and determine whether they are true or false. If a claim is true, give a reason with at least two examples that illustrate the claim. Your examples can include any representation you choose. If the claim is false, give a reason with one counterexample that proves the claim to be false.

4. The set of whole numbers is closed under addition.

5. The sum of a quadratic function and a linear function is a cubic function.

6. The sum of a linear function and an exponential function is a polynomial.

7. The set of polynomials is closed under addition.

8. The set of whole numbers is closed under subtraction.

9. The set of integers is closed under subtraction.

10. A quadratic function subtracted from a cubic function is a cubic function.

11. A linear function subtracted from a linear function is a polynomial function.

12. A cubic function subtracted from a cubic function is a cubic function.

13. The set of polynomial functions is closed under subtraction.

14. The product of two linear functions is a quadratic function.

15. The set of integers is closed under multiplication.

16. The set of polynomials is closed under multiplication.

17. The set of integers is closed under division.

18. A cubic function divided by a linear function is a quadratic function.

19. The set of polynomial functions is closed under division.

20. Write two claims of your own about polynomials and use examples to demonstrate that they are true.
Claim #1:

Claim #2:

3.6 Sorry, We're Closed – Teacher Notes
A Practice Understanding Task

Purpose: This task is designed to serve two purposes: 1) to motivate deeper thinking and develop fluency with operations on polynomials, and 2) to formalize the definition of a polynomial and understand that polynomials are closed under addition, subtraction, and multiplication. The task begins with the definition of polynomial and carefully contrasting the features of polynomials with other functions. Students are then asked to consider various conjectures about polynomials and use examples to demonstrate that a conjecture is true or provide a counterexample to prove that the conjecture is false.

Core Standards Focus:

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

F.BF.1: Write a function that describes a relationship between two quantities.
b. Combine standard function types using arithmetic operations.

Standards for Mathematical Practice:

SMP 3 – Construct viable arguments and critique the reasoning of others

SMP 6 – Attend to precision

Vocabulary: Closed

The Teaching Cycle:

Launch (Whole Group):

Begin the task by presenting the definition of a polynomial function given at the beginning of the task. Tell students that this definition describes some important features of polynomials that students may not have noticed from the definition alone. Tell students that the examples in #1 are designed to help them further differentiate between functions that are polynomials and those that are not. Go through each of the comparisons in problem 1. For each one, give students a few minutes to think about the differences and then have a short discussion about what is different about the function that is a polynomial compared to the one that is not. After completing problem 1, ask students to respond to question 2 individually. Record student responses highlighting the following ideas:

- A polynomial is a sum of terms.
- The exponents on the variables must be whole numbers.
- If the variable is in the exponent, the function is exponential, not polynomial.
- If there is division by a variable in one of the terms, then the function is not a polynomial. (It is a rational function, a function type that will be explored in Module 4.)

After discussing the definition of a polynomial, explain to students what it means for a set to be closed under a given operation. Use the example of adding two even numbers given in the task, and then check for understanding by having students work #3 and having a brief discussion to ensure that they understand the basic idea of closure. Then, give students instructions for the rest of the task and let them begin work.

Explore (Small Group):

Begin by giving students time to respond to the conjectures and come up with some examples before sharing. This will help to enhance the discussion when it is time to share. As students are working, watch for students that are thinking about counterexamples for statements that appear to be true. These students should be selected to share their ideas in the class discussion. If students need help in creating examples, some are given in the Teacher Resources section. You will probably want to keep these examples handy while you are monitoring students, so that you can offer these examples when they are stuck. Since it is sometimes hard for students to create good examples for division, some of the examples were made for this purpose. Specifically, $l(x)$ divides evenly by $f(x)$ and $j(x)$ divides evenly by $g(x)$.

Discuss:

Begin the discussion with a previously-selected student that can share a counter-example to show that #5 is false. Discuss briefly the idea that one counterexample proves a statement is false, but examples are not enough to prove a statement true. Students have encountered this idea in geometry.

Move the discussion to #7, then #9 and #13. In each case, ask selected students to share their examples, and then ask the class to use the examples to help make a general argument that might convince us that that each statement is always true.

Ask a previously-selected student to present their thinking on #19. Students may think that the statement is true because they don't recognize that when one polynomial is divided by another and there is a remainder that the quotient is not a polynomial. Remind students how to write the quotient in the form where the remainder is a fraction so that they can see that the variable in the denominator means that the quotient is not a polynomial function. Refer back to the work done in problem #1 and the definition of polynomial.

Aligned Ready, Set, Go: *Polynomial Functions 3.6*

3.6

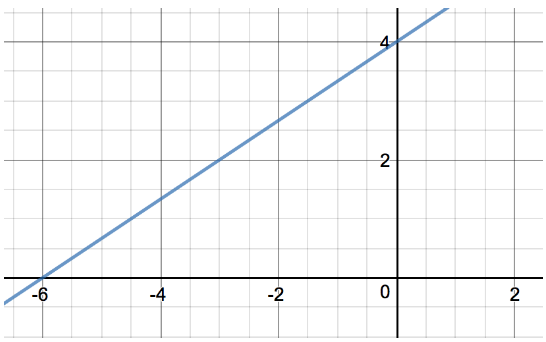
READY, SET, GO! Name Period Date

READY

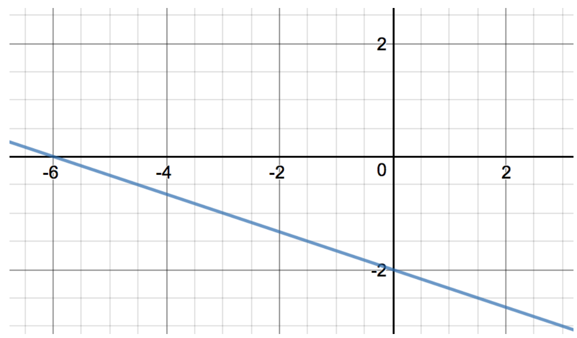
Topic: Connecting the zeros of a function to the solution of the equation

When we solve equations, we often set the equation equal to zero and then find the value of x. Another way to say this is "find when $f(x) = 0$." That's why we call solutions to equations the zeros of an equation. Find the zeros for the given equations. Then mark the solution(s) as a point on the graph of the equation.

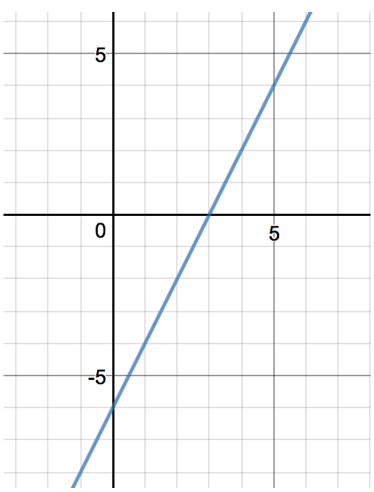
1. $f(x) = \frac{2}{3}x + 4$



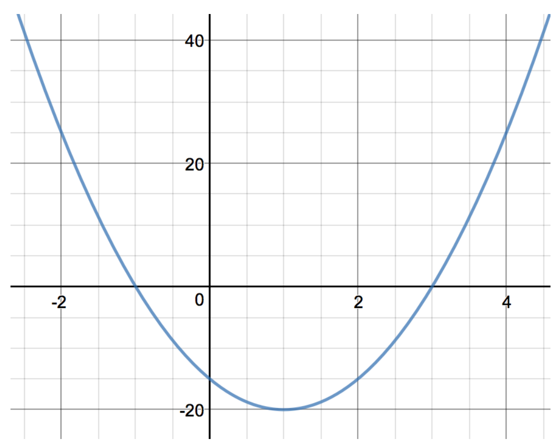
2. $g(x) = -\frac{1}{3}x - 2$



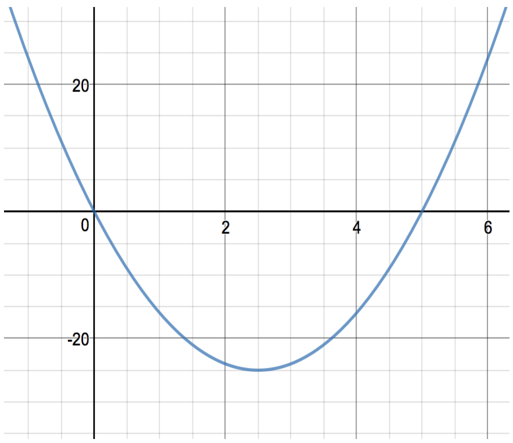
3. $h(x) = 2x - 6$



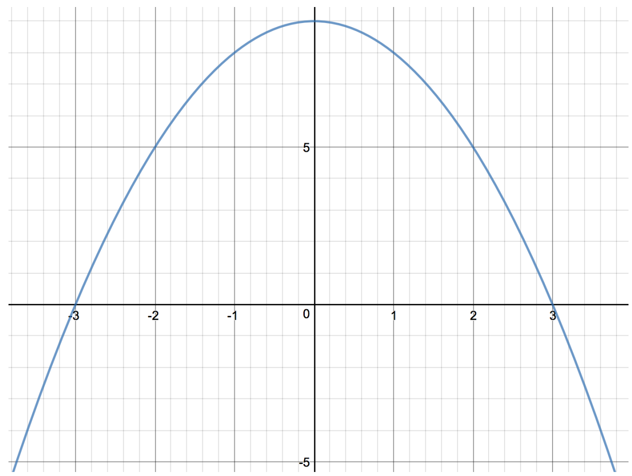
4. $p(x) = 5x^2 - 10x - 15$



5. $q(x) = 4x^2 - 20x$



6. $d(x) = -x^2 + 9$



SET

Topic: Exploring closed mathematical number sets

Identify the following statements as *sometimes* true, *always* true, or *never* true. If your answer is *sometimes* true, give an example of when it's true and an example of when it's not true. If it's *never* true, give a counter-example.

7. The product of a whole number and a whole number is an integer.

8. The quotient of a whole number divided by a whole number is a whole number.

9. The set of integers is **closed** under division.

10. The difference of a linear function and a linear function is an integer.

11. The difference of a linear function and a quadratic function is a linear function.

12. The product of a linear function and a linear function is a quadratic function.

13. The sum of a quadratic function and a quadratic function is a polynomial function.

14. The product of a linear function and a quadratic function is a cubic function.

15. The product of three linear functions is a cubic function.

16. The set of polynomial functions is **closed** under addition.

GO

Topic: Identifying conjugate pairs

A **conjugate pair** is simply a pair of binomials that have the same numbers but differ by having opposite signs between them. For example $(a + b)$ and $(a - b)$ are conjugate pairs. You've probably noticed them when you've factored a quadratic expression that is the difference of two squares. **Example:** $x^2 - 25 = (x+5)(x-5)$. The two factors $(x+5)(x-5)$ are conjugate pairs.

The quadratic formula $\boldsymbol{x} = \frac{-\boldsymbol{b} \pm \sqrt{\boldsymbol{b}^2 - 4\boldsymbol{ac}}}{2\boldsymbol{a}}$ can generate both solutions to a quadratic equation because of the $\pm$ located in the numerator of the formula. When the $\sqrt{b^2 - 4ac}$ part of the formula generates an irrational number (e.g. $\sqrt{2}$) or an imaginary number (e.g. $2i$), the formula produces a pair of numbers that are conjugates. This is important because <u>this type of solution to a quadratic always comes in pairs.</u> **Example:** The conjugate of $(3+\sqrt{2})$ *is* $(3-\sqrt{2})$. The conjugate of $(-2i)$ is $(+2i)$. Think of it as $(0-2i)$ *and* $(0+2i)$. <u>Change only the sign between the two numbers.</u>

Write the conjugate of the given value.

17. $(8+\sqrt{5})$

18. $(11+4i)$

19. $9i$

20. $-5\sqrt{7}$

21. $(2-13i)$

22. $(-1-2i)$

23. $(-3+5\sqrt{2})$

24. $-4i$

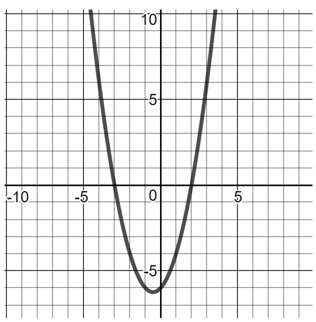
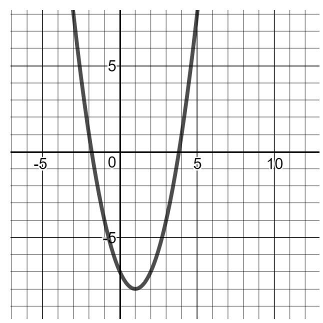
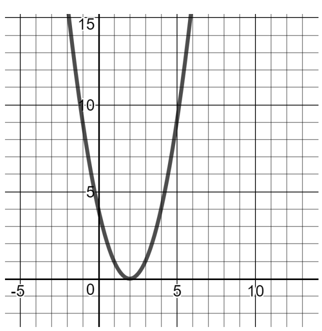
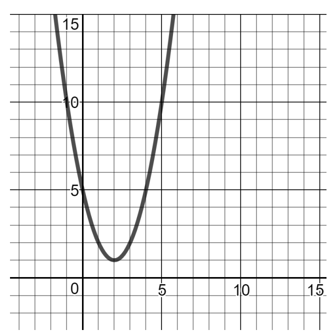
3.7 Building Strong Roots

A Solidify Understanding Task

When working with quadratic functions, we learned the Fundamental Theorem of Algebra:
An n^{th} degree polynomial function has n roots.
In this task, we will be exploring this idea further with other polynomial functions.

First, let's brush up on what we learned about quadratics. The equations and graphs of four different quadratic equations are given below. Find the roots for each and identify whether the roots are real or imaginary.

1.

a) $f(x) = x^2 + x - 6$ 	b) $g(x) = x^2 - 2x - 7$ 
Roots:	Roots:
Type of roots:	Type of roots:
c) $h(x) = x^2 - 4x + 4$ 	d) $k(x) = x^2 - 4x + 5$ 
Roots:	Roots:
Type of roots:	Type of roots:

2. Did all of the quadratic functions have 2 roots, as predicted by the Fundamental Theorem of Algebra? Explain.

3. It's always important to keep what you've previously learned in your mathematical bag of tricks so that you can pull it out when you need it. What strategies did you use to find the roots of the quadratic equations?

4. Using your work from problem 1, write each of the quadratic equations in factored form. When you finish, check your answers by graphing, when possible, and make any corrections necessary.

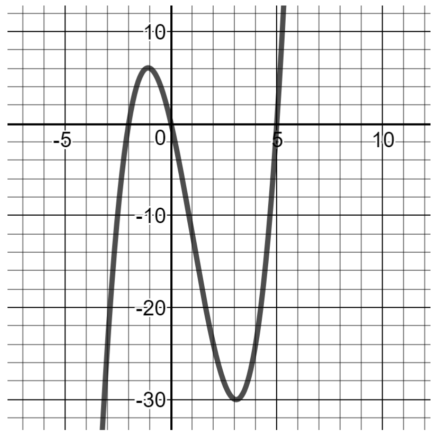
a) $f(x) = x^2 + x - 6$	b) $g(x) = x^2 - 2x - 7$
Factored form:	Factored form:
c) $h(x) = x^2 - 4x + 4$	d) $k(x) = x^2 - 4x + 5$
Factored form:	Factored form:

5. Based on your work in problem 1, would you say that roots are the same as x-intercepts? Explain.

6. Based on your work in problem 4, what is the relationship between roots and factors?

Now let's take a closer look at cubic functions. We've worked with transformations of $f(x) = x^3$, but what we've seen so far is just the tip of the iceberg. For instance, consider:

$g(x) = x^3 - 3x^2 - 10x$



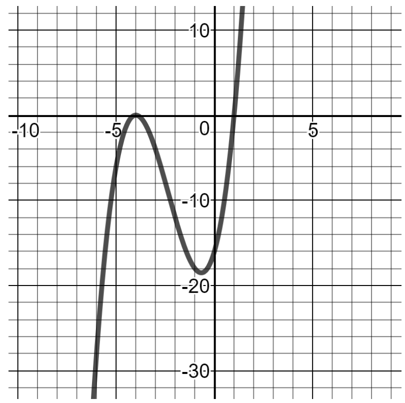
7. Use the graph to find the roots of the cubic function. Use the equation to verify that you are correct. Show how you have verified each root.

8. Write $g(x)$ in factored form. Verify that the factored form is equivalent to the standard form.

9. Are the results you found in #7 consistent with the Fundamental Theorem of Algebra? Explain.

Here's another example of a cubic function.

$f(x) = x^3 + 7x^2 + 8x - 16$

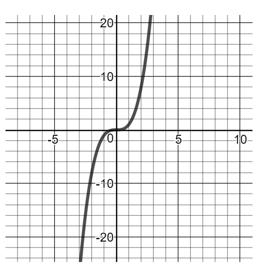


10. Use the graph to find the roots of the cubic function.

11. Write $f(x)$ in factored form. Verify that the factored form is equivalent to the standard form. Make any corrections needed.

12. Are the results you found in #10 consistent with the Fundamental Theorem of Algebra? Explain.

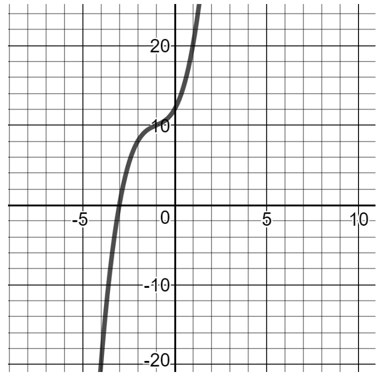
13. We've seen the most basic cubic polynomial function, $h(x) = x^3$ and we know its graph looks like this:



Explain how $h(x) = x^3$ is consistent with the Fundamental Theorem of Algebra.

14. Here is one more cubic polynomial function for your consideration. You will notice that it is given to you in factored form. Use the equation and the graph to find the roots of $p(x)$.

$$p(x) = (x + 3)(x^2 + 4)$$



15. Use the equation to verify each root. Show your work below.

16. Are the results you found in #14 consistent with the Fundamental Theorem of Algebra? Explain.

17. Explain how to find the factored form of a polynomial, given the roots.

18. Explain how to find the roots of a polynomial, given the factored form.

3.7 Building Strong Roots – Teacher Notes
A Solidify Understanding Task

Purpose: The purpose of this task is to extend the Fundamental Theorem of Algebra from quadratic functions to cubic functions. The task asks students to use graphs and equations to find roots and factors and to consider the relationship between them. Students will also consider quadratic and cubic functions with multiple real roots and imaginary roots.

Core Standards Focus:

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

N.CN.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (For Secondary III, limit to polynomials with real coefficients).

Standards for Mathematical Practice:

SMP 7 – Look for and make use of structure

Vocabulary: roots, factors, x-intercepts, multiplicity

The Teaching Cycle:

Launch (Whole Group):

Begin the task by introducing the Fundamental Theorem of Algebra. Students were introduced to this theorem in relation to quadratic functions in previous courses, so the first part of this task is to activate background knowledge so that their understanding and strategies for quadratic functions can be extended to cubic functions. Tell students that today's work will involve determining the number of roots for functions as well as the relationship between roots and factors. Students may need to be reminded that roots are the values of x that make $f(x) = 0$. Then, ask students to work on questions 1-6.

Explore (Small Group):

As students are working, support them in using the graphs to find roots. Ask questions to help them to make connections between the equations and the graphs. Listen for students who are using quadratic formula to find the exact roots when they are not obvious on the graph. Some students may try to estimate the roots; nudge them to think about what they could do to find exact values. In the case where the roots are not evident from the graph, ask students how they could use the equation to find roots. When students are working on finding factors from roots, you may need to remind them of how they solved equations in factored form, so that they think about how to work backwards.

Discuss (Whole Group):

Begin the discussion with the function in #1b. Ask a previously-selected student to share their work in finding the exact values of the roots using quadratic formula. Ask students to find the approximate values and ask where those values can be seen on the graph. Ask another student to show how they wrote the factors for the roots in #1b.

Next, ask a student to share how they found the roots for #1c and wrote it in factored form (question 2c). Ask the class if this function has the number of roots predicted by the Fundamental Theorem of Algebra. This may generate some controversy. After allowing students to share arguments, tell them that the factorization helps us to see that this function has a multiple root. Explain that it is said that this is a "root with multiplicity of 2". Remind students that for quadratics with multiple roots, the vertex is generally on the x-axis.

Have a student share their work finding the roots and factors for question 2d. This will serve as a brief reminder of imaginary numbers, which will be used in this task and upcoming tasks.

Discuss questions 5 and 6. Many students may argue that roots and x-intercepts are the same thing. If so, ask students if the roots were x-intercepts in question 2d. Explain that a root is defined as the value of x that makes $f(x) = 0$. An x-intercept is defined to be the point where a graph crosses the x-axis. If a function has imaginary roots, it will not cross the x-axis at the roots, so the roots are not x-intercepts. Conventionally, a root is written as $x = a$, and x-intercepts are written as points, e.g. $(a, 0)$. Remind students of the relationship between roots and factors, then ask them to work on the remainder of the task.

Explore (Small Groups):

Support students as they are working to apply some of the techniques that they learned with quadratic functions to cubic functions. They should start with looking for roots that can be easily identified from the graph or a factor, and then use the remaining factor to find the roots. As students are working on questions 10 and 11, look for a student that initially thought that $(x + 4)$ is a single root, and then discovered that it must be a multiple root to make the equation cubic. Also look for a student that has found the imaginary roots in #14.

Discuss (Whole Group):

Start the second discussion with questions #10 and #11. Have a student selected during the *Explore* time share his/her work in finding the roots and how they determined that $(x + 4)$ must be a multiple root. Ask the class to compare how the cubic graph looks when there is a multiple root and how the quadratic graph looked with the multiple root. This will help students to recognize that when a polynomial touches the x-axis without crossing through, it probably contains a multiple root of even multiplicity at that point.

Ask students to respond to #13. There may be some disagreement about this question. Again, have students share ideas and come to the conclusion that it must have a multiple root at $x = 0$. Point out that the graph crosses the x-axis at this multiple root, which makes it harder to identify as a multiple root than when the graph touches the axis and bounces off, as it does with an even number of multiple roots.

Ask previously-selected students to share their work on #14 and #15 that demonstrates one method for solving a quadratic to find imaginary roots. Point out that there are two imaginary roots and ask if that would always happen. Students may recognize that using the quadratic formula or square-rooting both sides of the equation would always lead to two imaginary roots in similar cases.

Wrap the discussion up with the last two questions (17 and 18), demonstrating how to use the roots to get factors and how to use factored form of a polynomial to find roots.

Aligned Ready, Set, Go: Polynomial Functions 3.7

READY, SET, GO! Name Period Date

READY

Topic: Practicing long division on polynomials

Divide using long division. (These problems have no remainders. If you get one, try again.)

1. $(x+3)\overline{)5x^3+2x^2-45x-18}$

2. $(x-6)\overline{)x^3-x^2-44x+84}$

3. $(x-5)\overline{)3x^3-15x^2+12x-60}$

4. $(x+2)\overline{)x^4+6x^3+7x^2-6x-8}$

SET

Topic: Applying the Fundamental Theorem of Algebra

Predict the number of roots for each of the given polynomial equations. (Remember that the Fundamental Theorem of Algebra states: An n^{th} degree polynomial function has n roots.)

5. $a(x)=x^2+3x-10$

6. $b(x)=x^3+x^2-9x-9$

7. $c(x)=-2x-4$

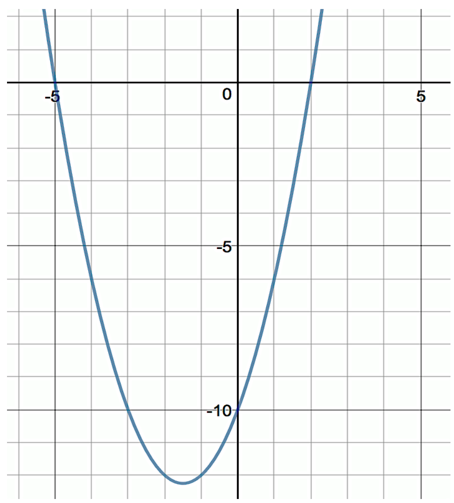
8. $d(x)=x^4-x^3-4x^2+4x$

9. $f(x)=-x^2+6x-9$

10. $g(x)=x^6-5x^4+4x^2$

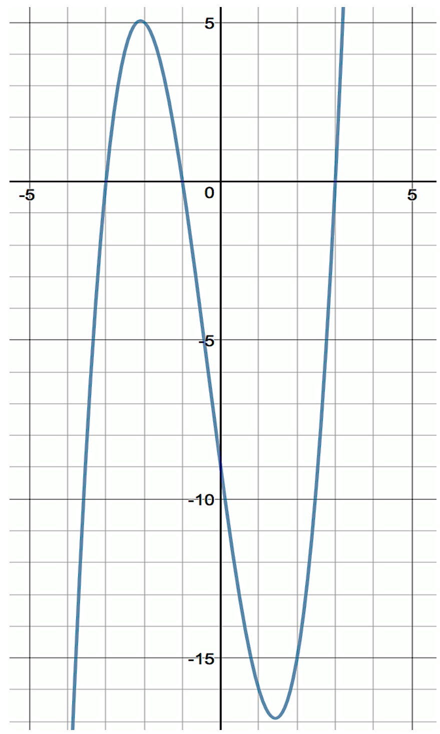
Below are the graphs of the polynomials from the previous page. Check your predictions. Then use the graph to help you write the polynomial in factored form.

11. $a(x) = x^2 + 3x - 10$



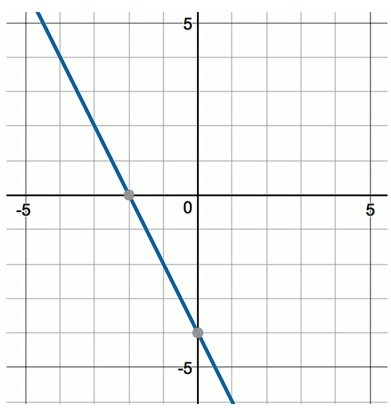
Factored form:

12. $b(x) = x^3 + x^2 - 9x - 9$



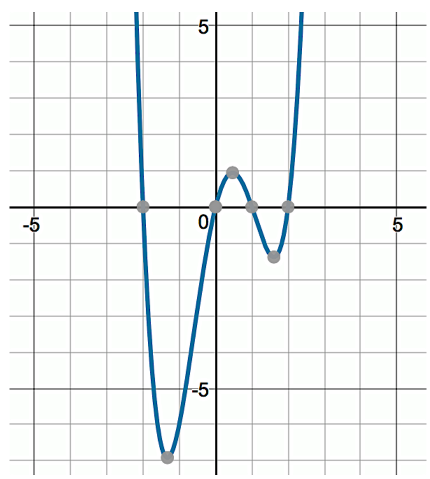
Factored form:

13. $c(x) = -2x - 4$



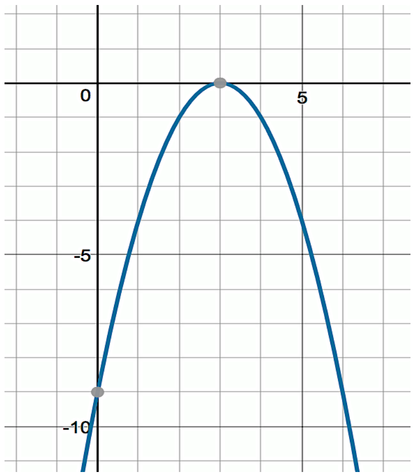
Factored form:

14. $d(x) = x^4 - x^3 - 4x^2 + 4x$



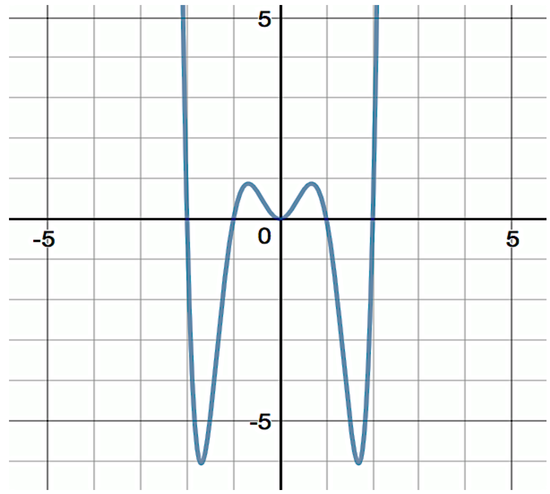
Factored form:

15. $f(x) = -x^2 + 6x - 9$



Factored form:

16. $g(x) = x^6 - 5x^4 + 4x^2$



Factored form:

17. The graphs of #15 and #16 don't seem to follow the Fundamental Theorem of Algebra, but there is something similar about each of the graphs. Explain what is happening at the point (3, 0) in #15 and at the point (0,0) in #16.

GO

Topic: Solving quadratic equations

Find the zeros for each equation using the quadratic formula.

18. $f(x) = x^2 + 20x + 51$

19. $f(x) = x^2 + 10x + 25$

20. $f(x) = 3x^2 + 12x$

21. $f(x) = x^2 - 11$

22. $f(x) = x^2 + x - 1$

23. $f(x) = x^2 + 2x + 3$

3.8 Getting to the Root of the Problem

A Solidify Understanding Task

In *3.7 Building Strong Roots,* we learned to predict the number of roots of a polynomial using the Fundamental Theorem of Algebra and the relationship between roots and factors. In this task, we will be working on how to find all the roots of a polynomial given in standard form.

Let's start by thinking again about numbers and factors.

1. If you know that 7 is a factor of 147, what would you do to find the prime factorization of 147? Explain your answer and show your process here:

2. How is your answer like a polynomial written in the form: $P(x) = (x-7)^2(x-3)$?

The process for finding factors of polynomials is exactly like the process for finding factors of numbers. We start by dividing by a factor we know and keep dividing until we have all the factors. When we get the polynomial broken down to a quadratic, sometimes we can factor it by inspection, and sometimes we can use our other quadratic tools like the quadratic formula.

Let's try it! For each of the following functions, you have been given one factor. Use that factor to find the remaining factors, the roots of the function, and write the function in factored form.

3. Function: $f(x) = x^3 + 3x^2 - 4x - 12$ Factor: $(x+3)$ Roots of function:

Factored form:

4. Function: $f(x) = x^3 + 6x^2 + 11x + 6$ Factor: $(x + 1)$ Roots of function:

Factored form:

5. Function: $f(x) = x^3 - 5x^2 - 3x + 15$ Factor: $(x - 5)$ Roots of function:

Factored form:

6. Function: $f(x) = x^3 + 3x^2 - 12x - 18$ Factor: $(x - 3)$ Roots of function:

Factored form:

7. Function: $f(x) = x^4 - 16$ Factor: $(x - 2)$ Roots of function:

Factored form:

8. Function: $f(x) = x^3 - x^2 + 4x - 4$ Factor: $(x - 2i)$ Roots of function:

Factored form:

9. Is it possible for a polynomial with real coefficients to have only one imaginary root? Explain.

10. Based on the Fundamental Theorem of Algebra and the polynomials that you have seen, make a table that shows all the number of roots and the possible combinations of real and imaginary roots for linear, quadratic, cubic, and quartic polynomials.

3.8 Getting To The Root Of The Problem – Teacher Notes

A Solidify Understanding Task

Purpose:

The purpose of this task is for students to find roots of polynomials and write the polynomials in factored form. This task builds on previous algebraic work, including factoring, polynomial long division, and quadratic formula. Students also use their knowledge of the Fundamental Theorem of Algebra to know how many roots a function has and to consider the possible combinations of real and imaginary roots for polynomials of degree 1-4. Students learn that imaginary roots occur in conjugate pairs and use this knowledge to both find roots and know the possible combinations of roots for polynomials.

Core Standards Focus:

A.APR.3: Identify zeros polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

N.CN.8: Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4\ as\ (x + 2i)(x - 2i)$.

N.CN.9: Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (In Secondary III, limit to polynomials with real coefficients).

Standards for Mathematical Practice:

SMP 1 – Make sense of problems and persevere in solving them

SMP 8 – Look for express regularity in repeated reasoning

Vocabulary: conjugate pairs

The Teaching Cycle:

Launch:

Begin the task by asking students about question 1, what is the prime factorization of 147 and how did you find it? Students will describe a process of dividing by the known factor and then breaking it down until all the factors are prime. This is analogous to the work that will be done in this task. We'll be given a polynomial and a known factor. We'll divide by the known factor and then break down the result until all the factors are linear. Question 2 illustrates that both polynomials and numbers can be written in factored form or standard form. Tell students that they will need to use everything they know about polynomials and all the tools in their toolkit, like quadratic formula, to break down the polynomials and find their roots and factors.

Explore:

As students begin working on problems #3 and #4, you may encounter some students who have graphed the function and seen that they both have integer roots that are easily identified from the graph. If so, ask them to verify the roots, either by long division or substitution into the function. If this occurs, you may wish to ask one of these students to present their work to the class, since connecting the graph to the roots is a very useful strategy.

Support students as they are working to use factoring, quadratic formula, taking the square root of both sides, etc. to solve the equations and find the roots. Look for students that have selected efficient strategies, rather than just one consistent strategy, to share. Students should be exposed to the idea that the quadratic formula is a powerful tool, but not always the most efficient tool.

Students are likely to have some trouble with #8. They can try to use long division, although it is algebraically more complicated than what they have seen previously. They may not notice that the given factor is the same as one of the factors in #7, and they may or may not have noticed that imaginary roots occur in pairs. This problem is designed to motivate the discussion about this idea. Let students work on the problem, but once they start becoming frustrated, tell them to move on and finish the task. If you have a student that is able to solve #8, he/she should be asked to share during the class discussion.

Discuss:

Begin the discussion with question 3. Ask a student to share that has used long division to divide by the given factor $(x + 3)$ and then factored the quadratic that is left to get the remaining factors. If a student initially graphed the function and then verified the roots, as discussed in the Explore narrative, then ask them to share. Compare the two approaches and discuss how a graph can support the algebraic work. Remind students that if they use a graph to find roots, they must verify them algebraically.

Next, discuss question 5. Ask a previously-selected student to share that has divided by the given factor and then set the quadratic that is left equal to zero and solved by square rooting both sides of the equation. It is a common error to forget to use both the positive and negative roots. If this has happened in class, remind students that this error will cause them to miss one of the roots predicted by the Fundamental Theorem of Algebra.

Discuss #6. The presenting student should have used the quadratic formula to solve the polynomial that remained after dividing by the given factor. This problem provides the opportunity to reinforce how to simplify expressions that result from the quadratic formula.

Briefly discuss question 7. If possible, have a student that has factored the initial polynomial share their work to demonstrate that there is more than one way to solve these types of problems. Be sure that students notice that problem #6 produced a pair of imaginary roots using the quadratic formula, and problem 7 produced a pair of imaginary roots by square rooting both sides of the equation. Tell students these are called **conjugate pairs.**

Move to question 8. If your classroom climate allows students to feel comfortable sharing work that isn't entirely successful, you may want to have a student share their attempt at long division. Ask students if what they have noticed about complex roots in previous problems may be useful in this problem. Use the graph of the function and the Fundamental Theorem of Algebra to argue that there must be another imaginary root. Then show students how to use the conjugate of the given root to find the remaining roots. Verify your work for the class by graphing both the standard form and the factored form to show they are equivalent. Make explicit to students that imaginary roots occur in conjugate pairs.

Finally, wrap up the discussion by completing the table for question 10, showing the possible combinations of real and imaginary roots for each of the function types.

Aligned Ready, Set, Go: *Polynomial Functions 3.8*

READY, SET, GO! Name Period Date

READY

Topic: Ordering numbers from least to greatest

Order the numbers from least to greatest.

1.	100^3	$\sqrt{100}$	$log_2 100$	100	2^{10}
2.	2^{-1}	$-\sqrt{100}$	$log_2\left(\frac{1}{8}\right)$	0	$(-2)^1$
3.	2^0	$\sqrt{25}$	$log_2 8$	$2(x^0), x \neq 0$	$(2)^{-\frac{1}{2}}$
4.	$log_3 3^3$	$log_5 5^{-2}$	$log_6 6^0$	$log_4 4^{-1}$	$log_2 2^3$

Refer to the given graph to answer the questions.

Insert $>, <, or =$ in each statement to make it true.

5. $f(0)$ ________ $g(0)$

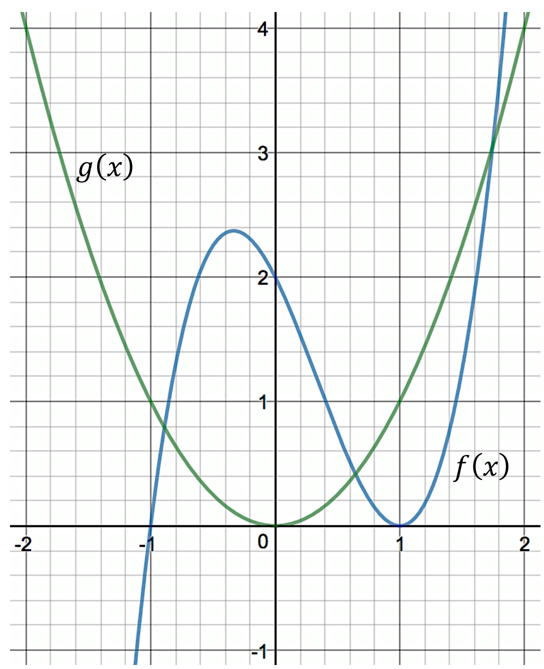
6. $f(2)$ ________ $g(2)$

7. $f(-1)$ ________ $g(-1)$

8. $f(1)$ ________ $g(-1)$

9. $f(5)$ ________ $g(5)$

10. $f(-2)$ ________ $g(-2)$



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SET

Topic: Finding the roots and factors of a polynomial

Use the given root to find the remaining roots. Then write the function in factored form.

Function	Roots	Factored form
11. $f(x) = x^3 - 13x^2 + 52x - 60$	$x = 5$	
12. $g(x) = x^3 + 6x^2 - 11x - 66$	$x = -6$	
13. $p(x) = x^3 + 17x^2 + 92x + 150$	$x = -3$	
14. $q(x) = x^4 - 6x^3 + 3x^2 + 12x - 10$	$x = \sqrt{2}$	

GO

Topic: Using the distributive property to multiply complex expressions

Multiply using the distributive property. Simplify. Write answers in standard form.

15. $(x-\sqrt{13})(x+\sqrt{13})$

16. $(x-3\sqrt{2})(x+3\sqrt{2})$

17. $(x-4+2i)(x-4-2i)$

18. $(x+5+3i)(x+5-3i)$

19. $(x-1+i)(x-1-i)$

20. $(x+10-\sqrt{2}\,i)(x+10+\sqrt{2}\,i)$

3.9 Is This The End?

A Solidify Understanding Task

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In previous mathematics courses, you have compared and analyzed growth rates of polynomial (mostly linear and quadratic) and exponential functions. In this task, we are going to analyze rates of change and end behavior by comparing various expressions.

Part I: Seeing patterns in **end behavior**

1. In as many ways as possible, compare and contrast linear, quadratic, cubic, and exponential functions.

2. Using the graph provided, write the following functions vertically, from **greatest to least for $\boldsymbol{x = 0}$**. Put the function with the greatest value on top and the function with the smallest value on the bottom. Put functions with the same values at the same level. An example, $l(x) = x^7$, has been placed on the graph to get you started.

$f(x) = 2^x$ $\qquad$ $p(x) = x^3 + x^2 - 4$ $\qquad$ $g(x) = x^2 - 20$

$h(x) = x^5 - 4x^2 + 1$ $\qquad$ $k(x) = x + 30$ $\qquad$ $m(x) = x^4 - 1$

$r(x) = x^5$ $\qquad$ $n(x) = \left(\frac{1}{2}\right)^x$ $\qquad$ $q(x) = x^6$

3. What determines the value of a polynomial function at $x = 0$? Is this true for other types of functions?

4. Write the same expressions on the graph in order from **greatest to least** when x represents a very large number (this number is so large, so we say that it is approaching positive infinity). If the value of the function is positive, put the function in quadrant 1. If the value of the function is negative, put the function in quadrant IV. An example has been placed for you.

5. What determines the end behavior of a polynomial function for very large values of x?

6. Write the same functions in order from **greatest to least** when x represents a number that is approaching negative infinity. If the value of the function is positive, place it in Quadrant II, if the value of the function is negative, place it in Quadrant III. An example is shown on the graph.

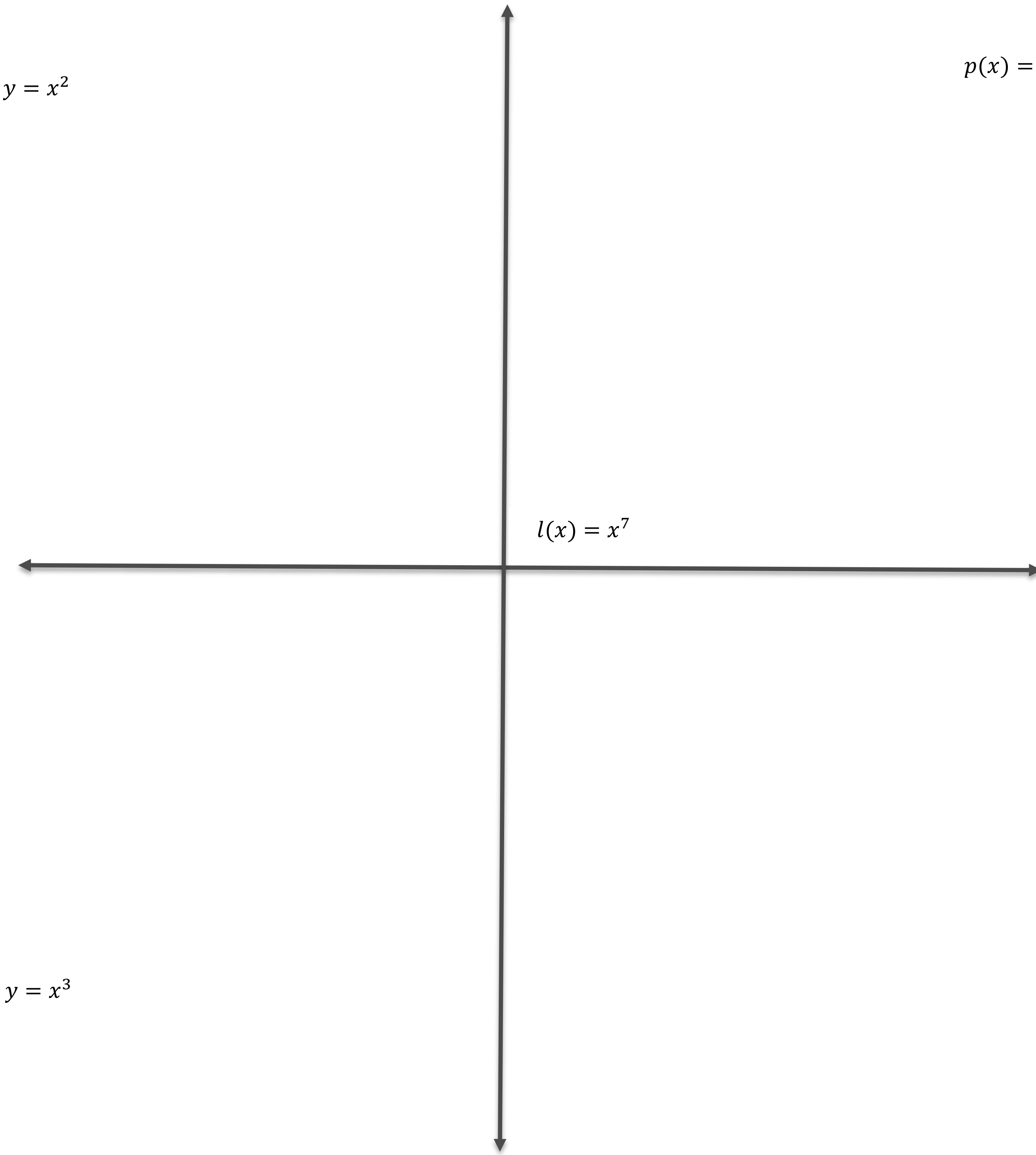
7. What patterns do you see in the polynomial functions for x values approaching negative infinity? What patterns do you see for exponential functions? Use graphing technology to test these patterns with a few more examples of your choice.

8. How would the end behavior of the polynomial functions change if the lead terms were changed from positive to negative?

$\boldsymbol{x \to -\infty}$ $\boldsymbol{x = 0}$ $\boldsymbol{x \to \infty}$

$p(x) = x^7$

$y = x^2$



$l(x) = x^7$

$y = x^3$

Part II: Using end behavior patterns

For each situation:

- Determine the function type. If it is a polynomial, state the degree of the polynomial and whether it is an even degree polynomial or an odd degree polynomial.
- Describe the end behavior based on your knowledge of the function. Use the format:
  As $x \to -\infty, f(x) \to$ _____ *and* *as* $x \to \infty\ f(x) \to$ _____

1. $f(x) = 3 + 2x$

   Function type:

   End behavior: As $x \to -\infty, f(x) \to$ _____

   End behavior: As $x \to \infty, f(x) \to$ _____

2. $f(x) = x^4 - 16$

   Function type:

   End behavior: As $x \to -\infty, f(x) \to$ _____

   End behavior: As $x \to \infty, f(x) \to$ _____

3. $f(x) = 3^x$

   Function type:

   End behavior: As $x \to -\infty, f(x) \to$ _____

   End behavior: As $x \to \infty, f(x) \to$ _____

4. $f(x) = x^3 + 2x^2 - x + 5$

   Function type:

   End behavior: As $x \to -\infty, f(x) \to$ _____

   End behavior: As $x \to \infty, f(x) \to$ _____

5. $f(x) = -2x^3 + 2x^2 - x + 5$

   Function type:

   End behavior: As $x \to -\infty, f(x) \to$ _____

   End behavior: As $x \to \infty, f(x) \to$ _____

6. $f(x) = log_2 x$

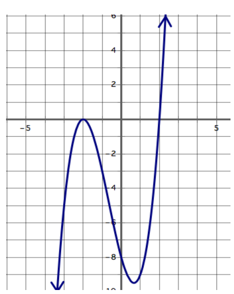
   Function type:

   End behavior: As $x \to -\infty, f(x) \to$ _____

   End behavior: As $x \to \infty, f(x) \to$ _____

Use the graphs below to describe the end behavior of each function by completing the statements.

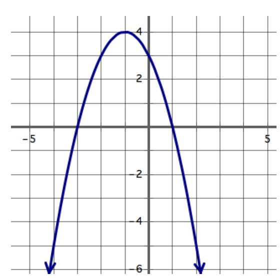
7.



End behavior: As $x \to -\infty, f(x) \to$ _____

End behavior: As $x \to \infty, f(x) \to$ _____

8.



End behavior: As $x \to -\infty, f(x) \to$ _____

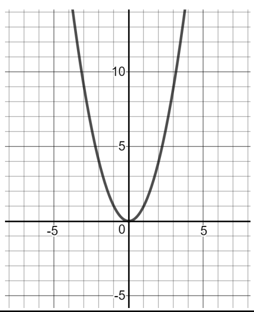
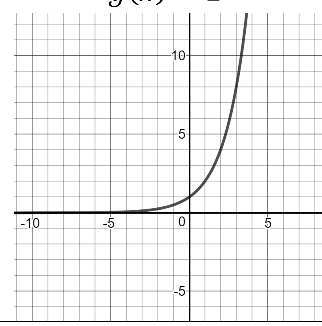
End behavior: As $x \to \infty, f(x) \to$ _____

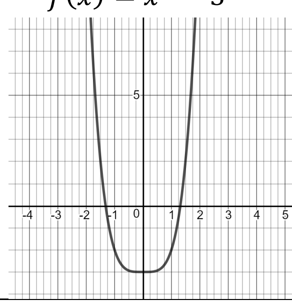
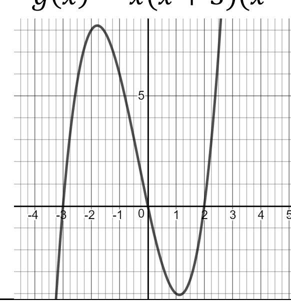
9. How does the end behavior for quadratic functions connect with the number and type of roots for these functions? How does the end behavior for cubic functions connect with the number and type of roots for cubic functions?

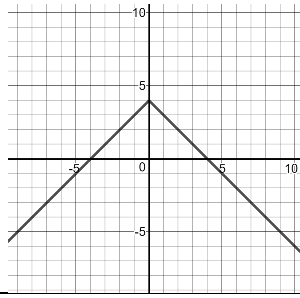
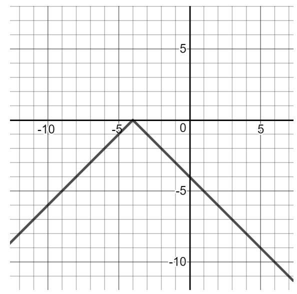
Part III: Even and Odd Functions

Some functions that are not polynomials may be categorized as even functions or odd functions. When mathematicians say that a function is an even function, they mean something very specific.

1. Let's see if you can figure out what the definition of an even function is with these examples:

<table>
<tr>
<td>Even function:
$f(x) = x^2$
</td>
<td>Not an even function:
$g(x) = 2^x$
</td>
</tr>
<tr>
<td colspan="2">Differences:</td>
</tr>
</table>

<table>
<tr>
<td>Even function:
$f(x) = x^4 - 3$
</td>
<td>Not an even function:
$g(x) = x(x + 3)(x - 2)$
</td>
</tr>
<tr>
<td colspan="2">Differences:</td>
</tr>
</table>

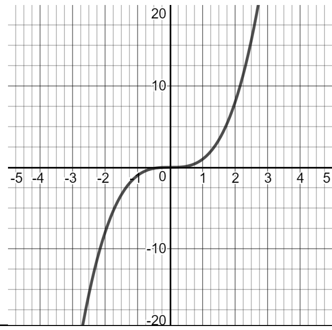
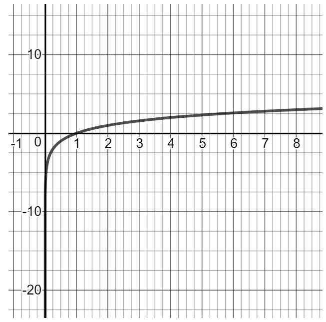
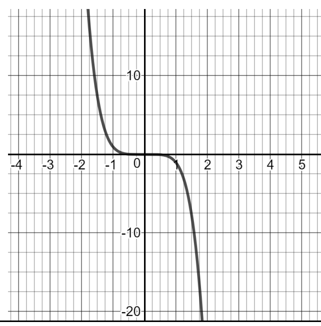
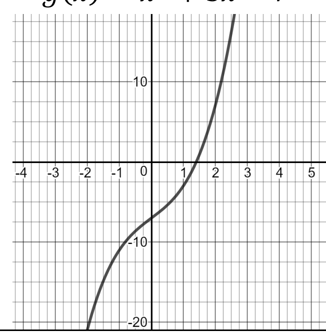
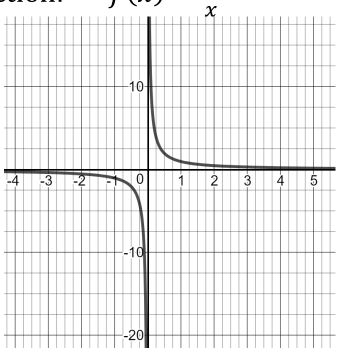
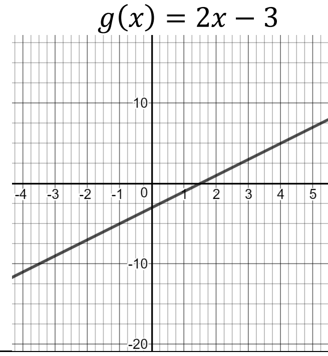
<table>
<tr><td>Even function:
$f(x) = -|x| + 4$
</td><td>Not an even function:
$g(x) = -|x + 4|$
</td></tr>
<tr><td colspan="2">Differences:</td></tr>
<tr><td>Even function:
$f(2) = 5 \ and \ f(-2) = 5$</td><td>Not an even function:
$g(2) = 3 \ and \ g(-2) = 5$</td></tr>
<tr><td colspan="2">Differences:</td></tr>
</table>

2. What do you observe about the characteristics of an even function?

3. The algebraic definition of an even function is:
$f(x)$ is an even function if and only if $f(x) = f(-x)$ for all values of x in the domain of f.
What are the implications of the definition for the graph of an even function?

4. Are all even-degree polynomials even functions? Use examples to explain your answer.

5. Let's try the same approach to figure out a definition for odd functions.

<table>
<tr><td>Odd function:
$f(x) = x^3$
</td><td>Not an odd function:
$g(x) = \log_2 x$
</td></tr>
<tr><td colspan="2">Differences:</td></tr>
<tr><td>Odd function:
$f(x) = -x^5$
</td><td>Not an odd function:
$g(x) = x^3 + 3x - 7$
</td></tr>
<tr><td colspan="2">Differences:</td></tr>
<tr><td>Odd function: $f(x) = \frac{1}{x}$
</td><td>Not an odd function:
$g(x) = 2x - 3$
</td></tr>
<tr><td colspan="2">Differences:</td></tr>
<tr><td>Odd function:
$f(2) = 3 \; and \; f(-2) = -3$</td><td>Not an odd function:
$g(2) = 3 \; and \; g(-2) = 5$</td></tr>
<tr><td colspan="2">Differences:</td></tr>
</table>

6. What do you observe about the characteristics of an odd function?

7. The algebraic definition of an odd function is:

   $f(x)$ is an odd function if and only if $f(-x) = -f(x)$ for all values of x in the domain of f.

Explain how each of the examples of odd functions above meet this definition.

8. How can you tell if an odd-degree polynomial is an odd function?

9. Are all functions either odd or even?

3.9 Is This The End? – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to solidify student understanding of how the degree of the polynomial function impacts the rate of change and end behavior. By comparing the values of expressions with 'extreme' values, students will be able to:

- Understand that the degree of the polynomial is the highest valued whole number exponent and that this term determines the end behavior (regardless of the other terms in the expression).
- Determine that the higher the degree of a polynomial, the greater the value as x approaches infinity. Understand that while the highest degree polynomial has the greatest value as $x \to \infty$, that exponential functions have the greatest rate of change, and therefore the greatest value when x becomes very large.
- Identify differences between even and odd degree functions. In this task, students will know the end behavior for odd degree functions (as $x \to -\infty, f(x) \to -\infty$ and that both even and odd degree polynomial functions as $x \to \infty, f(x) \to \infty$) as long as the co-efficient is positive and realize that the opposite is true if the co-efficient is negative.

Teacher Note: Make copies of the expressions and have them cut out in advance. To make the most of the whole group discussion, be prepared by having several 'movable copies' of each expression (large cutouts are attached and being able to move expressions on a Smartboard or using a document camera will be handy). You will want more than one set so that everyone can visually see the change in order depending on the values chosen for x. See Discuss section for more details.

Core Standards Focus:

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F.BF.3 Identify the effect on the graph of replacing *f(x) by f(x)+k, kf(x), f(kx), and f(x+k)* for specific values of k (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from the graphs and algebraic expressions for them.

Related Standards: F.IF.7

Standards for Mathematical Practice:

- **SMP 1 – Make sense of problems and persevere in solving them**
- **SMP 3 – Construct viable arguments and critique the reasoning of others**
- **SMP 5 – Use appropriate tools strategically**
- **SMP 7 – Look for and make use of structure**

Vocabulary: **end behavior**

The Teaching Cycle:

Launch (Whole Class):

Begin by reading the introduction in the task and then explain to students that while they will be comparing and ordering expressions, they should also pay attention to *why* the expressions are in a particular order. Tell students that they should be looking for patterns that they can generalize and use.

Explore (Small Group):

Start the task by having students work individually and then work together in small groups on the rest of the task. As you monitor, listen for student reasoning throughout, paying particular attention when they are discussing the order of the expressions in questions 3 and 4 (connecting to end behavior). Take note of how different groups are determining order. Are they plugging in values? Are they looking at a graphical representation? Are they making assumptions based on their knowledge of exponents? As students answer question 5, press for them to explain their reasoning. Listen for answers that include comparing polynomials to exponential functions (and that exponential functions with a base greater than one will always outgrow any polynomial function). Also listen for answers that distinguish between when the degree of the polynomial is even versus when it is odd (and how this impacts the values as $x \rightarrow -\infty$).

Discuss (Whole Class):

When all students have had the opportunity to answer question 6 (some may have already completed all questions), move to the whole group discussion. Begin with a student that shares their ordering for question 2. Ask the student to explain how they figured out the order. During this explanation, there should be some discussion of substituting in 0 for x and finding that the only term left in a polynomial is the constant term. Ask the class if this will be the case for every polynomial function. Ask the class if this is true of all functions. They should recognize that some functions, like logarithms are undefined at $x = 0$, and some functions, like exponentials, depend on more than just a constant term.

Next, ask a student to share their ordering of the functions for #4. Use either the axes provided in the task or post a larger set of axes and use the large function cards, that are provided following the teacher notes, so that the order is very visual for students. Focus first on the order of the polynomial functions. Be sure that students notice that the highest-powered term essentially determines the end behavior for large values of x. Look specifically at h(x) and r(x). Since these two functions are both degree 5 polynomials, then they are more difficult to order. Ask for students to share different ideas for how to compare these two functions, including using technology to graph both functions and comparing them for large values, substituting large numbers into both functions and comparing, and reasoning about the effect of the additional terms in h(x). It is likely that there will be some controversy around whether or not the exponential function is the greatest as $x \rightarrow \infty$. Look for similar strategies to compare the exponential with the highest-powered polynomial. Then, ask students to consider the rates of change of the two functions to think about which one is the greatest for large values of x. Have students calculate the average rate of change of both functions from [15, 20] to see that the rapid rate of change of the exponential function will eventually make it exceed any polynomial.

Next, ask a student to share their ordering of the functions for question #6. Again, make the visual available so that all students can easily see the pattern that even-powered polynomials approach infinity as $x \rightarrow -\infty$ and odd-powered polynomials approach negative infinity as $x \rightarrow -\infty$. Ask students if this is consistent with their experience of quadratic and cubic functions, and why it would be so. Ask how this connects to the number and types of roots that were found for polynomial functions in the previous task. (Be prepared to pull the chart out as part of the discussion.) Also discuss question #8 and the effect of changing the sign of the lead coefficient in a polynomial.

Ask students to share as many of the problems in Part II as time allows. Be sure that they are using the results of Part I and the correct notation to record their results.

Finally ask students for their observations about even functions. Highlight observations that include the idea that the graphs are symmetric about the y-axis. Ask students how the definition given relates to the symmetry of the graphs and help them to articulate that the output of the function is the same for both positive and negative values of x in even functions. This is true of the parent function for all even-degree polynomials and if they have a vertical translation and/or a vertical stretch.

Repeat the process by asking students for their observations about odd functions. It may be harder to recognize the 180˚ rotation about the origin. Demonstrating this idea with a piece of patty paper may help students to see it more easily. Reinforce how the definition of an odd function creates this pattern in the graph of odd functions. Ask students how to tell if an odd-degree polynomial will be an odd function. Support them to think about which transformations of an odd-degree parent function like $f(x) = x^3$ will result in an odd function. Close the discussion by asking students to share a couple of examples of functions that are neither odd nor even. Many of the examples from earlier in the task can be used for this purpose.

Aligned Ready, Set, Go: Polynomial Functions 3.9

x^5	$x^2 - 20$
$x + 30$	$x^4 - 1$
2^x	$\left(\frac{1}{2}\right)^x$
x^7	x^6
$x^3 + x^2 - 4$	
$x^5 - 4x^2 + 1$	

READY, SET, GO! Name Period Date

READY

Topic: Recognizing special products

Multiply.

1. $(x+5)(x+5)$
2. $(x-3)(x-3)$
3. $(a+b)(a+b)$

4. In problems 1 – 3 the answers are called **perfect square trinomials**. What about these answers makes them be a **perfect square trinomial**?

5. $(x+8)(x-8)$
6. $(x+\sqrt{3})(x-\sqrt{3})$
7. $(x+b)(x-b)$

8. The products in problems 5 – 7 end up being binomials, and they are called the **difference of two squares**. What about these answers makes them be the **difference of two squares**?

   Why don't they have a middle term like the problems in 1 – 3?

9. $(x-3)(x^2+3x+9)$
10. $(x+10)(x^2-10x+100)$
11. $(a+b)(a^2-ab+b^2)$

12. The work in problems 9 – 11 makes them feel like the answers are going to have a lot of terms. What happens in the work of the problem that makes the answers be binomials?

    These answers are called the **difference of two cubes** (#9) and the **sum of two cubes** (#10 and #11.) What about these answers makes them be the **sum or difference of two cubes**?

SET

Topic: Determining values of polynomials at zero and at $\pm\infty$. (End behavior)

State the y-intercept, the degree, and the end behavior for each of the given polynomials.

13. $f(x) = x^5 + 7x^4 - 9x^3 + x^2 - 13x + 8$

y- intercept:

Degree:

End behavior:

As $x \to -\infty,\ f(x) \to$ ________

As $x \to +\infty,\ f(x) \to$ ________

14. $g(x) = 3x^4 + x^3 + 5x^2 - x - 15$

y- intercept:

Degree:

End behavior:

As $x \to -\infty,\ g(x) \to$ ________

As $x \to +\infty,\ g(x) \to$ ________

15. $h(x) = -7x^9 + x^2$

y- intercept:

Degree:

End behavior:

As $x \to -\infty,\ h(x) \to$ ________

As $x \to +\infty,\ h(x) \to$ ________

16. $p(x) = 5x^2 - 18x + 4$

y- intercept:

Degree:

End behavior:

As $x \to -\infty,\ p(x) \to$ ________

As $x \to +\infty,\ p(x) \to$ ________

17. $q(x) = x^3 - 94x^2 - x - 20$

y- intercept:

Degree:

End behavior:

As $x \to -\infty,\ q(x) \to$ ________

As $x \to +\infty,\ q(x) \to$ ________

18. $y = -4x + 12$

y- intercept:

Degree:

End behavior:

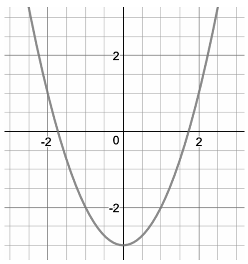
As $x \to -\infty,\ y \to$ ________

As $x \to +\infty,\ y \to$ ________

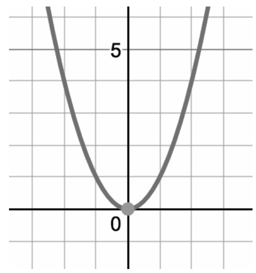
Topic: Identifying even and odd functions

19. Identify each function as even, odd, or neither.

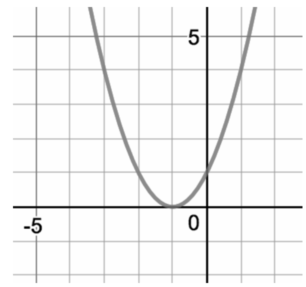
a) $f(x) = x^2 - 3$



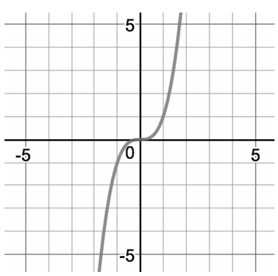
b) $f(x) = x^2$



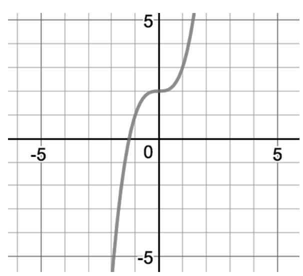
c) $f(x) = (x+1)^2$



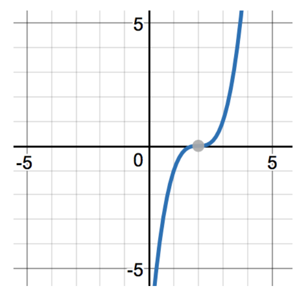
d) $f(x) = x^3$



e) $f(x) = x^3 + 2$



f) $f(x) = (x-2)^3$



GO

Topic: Factoring special products

Fill in the blanks on the sentences below.

20. The expression $\boldsymbol{a^2 + 2ab + b^2}$ is called a **perfect square trinomial**. I can recognize it because the first and last terms will always be perfect ______________________________.
The middle term will be 2 times the _____________________ and ______________________.
There will always be a ____________ sign before the last term.
It factors as (____________)(____________).

21. The expression $\boldsymbol{a^2 - b^2}$ is called the **difference of 2 squares**. I can recognize it because it's a binomial and the first and last terms are perfect ______________________________.
The sign between the first term and the last term is always a ______________________ .
It factors as (______________)(______________).

22. The expression $\boldsymbol{a^3 + b^3}$ is called the **sum of 2 cubes**. I can recognize it because it's a binomial and the first and last terms are ____________________________. The expression $a^3 + b^3$ factors into a binomial and a trinomial. I can remember it as a *short* (____) and a *long* (____________).
The sign between the terms in the binomial is the ________________ as the sign in the expression. The first sign in the trinomial is the ___________________ of the sign in the binomial. That's why all of the middle terms cancel when multiplying.
The last sign in the trinomial is always _________.
It factors as (______________)(___________________________).

Factor using what you know about special products.

23. $25x^2 + 30 + 9$

24. $x^2 - 16$

25. $x^3 + 27$

26. $49x^2 - 36$

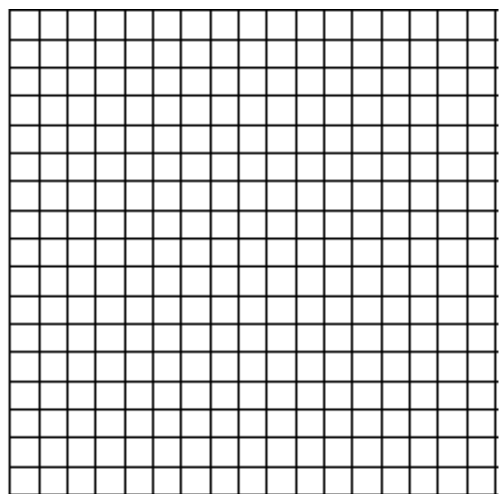
27. $x^3 - 1$

28. $64x^2 - 240 + 225$

3.10 Puzzling Over Polynomials

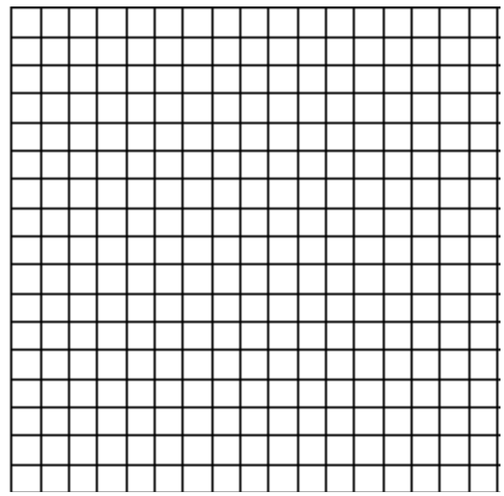
A Practice Understanding Task

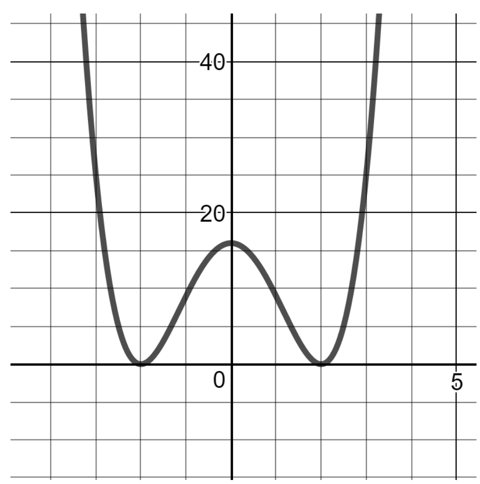
For each of the polynomial puzzles below, a few pieces of information have been given. Your job is to use those pieces of information to complete the puzzle. Occasionally, you may find a missing piece that you can fill in yourself. For instance, although some of the roots are given, you may decide that there are others that you can fill in.

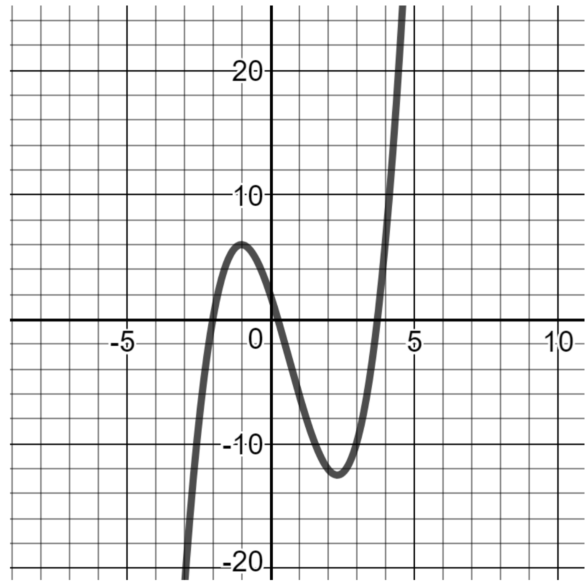
1.	**Function (in factored form)** **Function (in standard form)** **End behavior:** $as\ \ x \to -\infty, \quad f(x) \to$ ____ $as\ \ x \to \infty, \quad f(x) \to$ ____ **Roots (with multiplicity):** -2, 1, and 1 **Value of leading co-efficient:** -2 **Degree:** 3	Graph: 

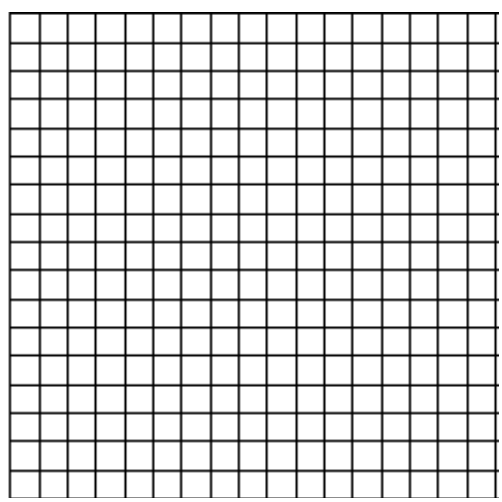
2.	**Function (in factored form)** **Function (in standard form)** **End behavior:** $as \quad x \to -\infty, \qquad f(x) \ \to$ ____ $as \quad x \to \ \ \infty, \qquad f(x) \ \to$ ____ **Roots (with multiplicity):** $2+i, 4, 0$ **Value of leading co-efficient:** 1 **Degree:** 4	Graph:

3.	**Function:** $f(x) = 2(x-1)(x+3)^2$ **End behavior:** $as \quad x \to -\infty, \qquad f(x) \ \to$ ____ $as \quad x \to \ \ \infty, \qquad f(x) \ \to$ ____ **Roots (with multiplicity):** **Value of leading co-efficient:** **Domain:** **Range:** All Real numbers	Graph:

4.	**Function:** **End behavior:** $as\ \ x \to -\infty, \qquad f(x)\ \to \infty$ $as\ \ x \to \ \ \infty, \qquad f(x)\ \to$ ____ **Roots (with multiplicity):** (3,0) m: 1; (-1,0) m: 2 (0,0) m: 2 **Value of leading co-efficient:** -1 **Domain:** **Range:**	Graph: 

5.	**Function:** **End behavior:** $as\ \ x \to -\infty, \qquad f(x)\ \to$ ____ $as\ \ x \to \ \ \infty, \qquad f(x)\ \to$ ____ **Roots (with multiplicity):** **Value of leading co-efficient:** 1 **Domain:** **Range:** **Other:** $f(0) = 16$	Graph: 

6.	**Function (in standard form):** $f(x) = x^3 - 2x^2 - 7x + 2$ **Function (in factored form):** **End behavior:** $as\ \ x \to -\infty, \quad f(x) \to$ ____ $as\ \ x \to \ \infty, \quad f(x) \to$ ____ **Roots (with multiplicity):** -2 **Domain:** **Range:**	Graph: 

7.	**Function (in standard form):** $f(x) = x^3 - 2x$ **Function (in factored form):** **End behavior:** $as\ \ x \to -\infty, \quad f(x) \to$ ____ $as\ \ x \to \ \infty, \quad f(x) \to$ ____ **Roots (with multiplicity):** **Domain:** **Range:**	Graph: 

3.10 Puzzling Over Polynomials – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is tie together what students have learned about roots, end behavior, operations, and graphs of polynomials. Each problem gives a few features of a polynomial and asks students to find other features, sometimes including the graphs. This will require them to know the end behavior of a polynomial, based on the degree, and to find all the roots, given some of the roots. In most cases, students are asked to write the equation of the polynomial or to write the equation in a different form than is given.

Core Standards Focus:

F-IF.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. (3.2,

A-APR.3: Identify zeros polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

N-CN.8: Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

N-CN.9: Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (In Secondary III, limit to polynomials with real coefficients).

A-CED: Create equations that describe numbers or relationships

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Standards for Mathematical Practice:

SMP 1 – Make sense of problems and persevere in solving them

SMP 6 – Attend to precision

The Teaching Cycle:

Launch:

Launch the task by telling students that they are given several polynomial puzzles that will help them to connect all the work that they have done in the module so far. In each puzzle they are given some information and they need to use that information to find the other information. Sometimes the puzzles are little sneaky because information that they need is not given directly, but if they think about what they know, they can find it.

Explore:

Monitor students as they work to see which problems are causing difficulty so that they can be used in the class discussion. You may need to support students in solving problem 2. One imaginary root is given, and they will need to know that the other root is the conjugate. If students are stuck, ask questions to help remind them of the relationship between imaginary roots of a polynomial. As students work on problems like #5 and #6, allow them to use the given graph to find the integer roots and then require them to use those roots to find the remaining irrational or imaginary roots using long division.

Discuss:

Discuss as many of the problems as time allows, with previously-selected students sharing their work for each problem. Start with the problems that were more difficult so that students have a chance to see how to work through some of the difficulties. After a student presents their work, ask the rest of the class to summarize the process that the presenting student used with the following sentence frame:

In the problem, we were given ______________________________ and so we knew ___________________.

Then we were able to find _____________________________ by ____.

An example of using the sentence frame with problem #2 is:

In the problem, we were given **2 real roots and 1 imaginary root** and so we knew **there was another imaginary root, $2 - i$.**

Then we were able to find **the equation** by **writing each root as a factor and multiplying them together.**

Aligned Ready, Set, Go: ***Polynomial Functions 3.10***

READY, SET, GO! Name Period Date

READY

Topic: Reducing rational numbers and expressions

Reduce the expressions to lowest terms. (Assume no denominator equals 0.)

1. $\frac{3x}{6x^2}$

2. $\frac{2 \cdot 5 \cdot x \cdot x \cdot x \cdot y}{3 \cdot 5 \cdot x \cdot y \cdot y}$

3. $\frac{7ab^2}{7ab^2}$

4. $\frac{(x+2)(x-9)}{(x+2)(x-9)}$

5. $\frac{(3x-5)(x+4)}{(x-1)(3x-5)}$

6. $\frac{(2x-11)(3x+17)}{(2x-11)(3x-5)}$

7. $\frac{(8x-7)(x+3)}{8x(x+3)(2x-3)}$

8. $\frac{3x(2x+7)(x-1)(6x-5)}{x(2x+7)(x-1)(6x-5)}$

9. Why is it important that the instructions say to assume that no denominator equals 0?

SET

Topic: Reviewing features of polynomials

Some information has been given for each polynomial. Fill in the missing information.

10.

Function: $f(x) = x^3$

Function in factored form:

End behavior:

As $x \to -\infty, f(x) \to$ ______ As $x \to \infty, f(x) \to$ _____

Roots (with multiplicity):

Degree:

Value of leading co-efficient:

Graph:

11. **Graph:**

Function in standard form:

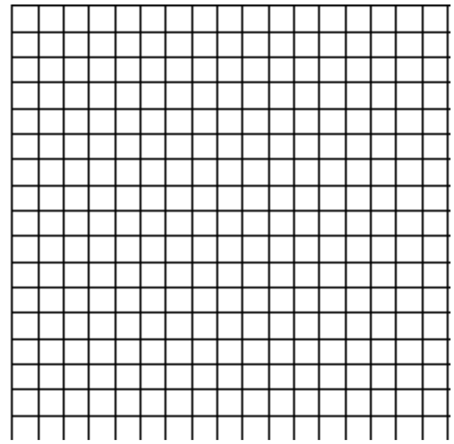
Function in factored form: $g(x) = -x(x-2)(x-4)$

End behavior:
As $x \to -\infty, g(x) \to$ ______ As $x \to \infty, g(x) \to$ _____

Roots (with multiplicity):

Degree:

Value of leading co-efficient:



12. **Graph:**

Function in standard form: $h(x) = x^3 - 2x^2 - 3x$

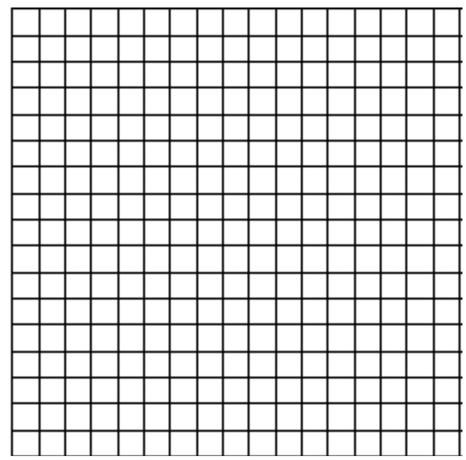
Function in factored form:

End behavior:
As $x \to -\infty, h(x) \to$ ______ As $x \to \infty, h(x) \to$ _____

Roots (with multiplicity):

Degree:

Value of $\boldsymbol{h(2)}$:



13. **Graph:**

Function in standard form:

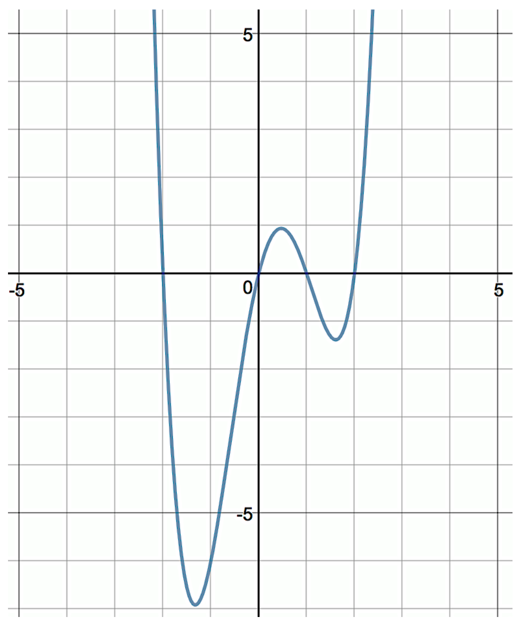
Function in factored form:

End behavior:
As $x \to -\infty, f(x) \to$ ______ As $x \to \infty, f(x) \to$ _____

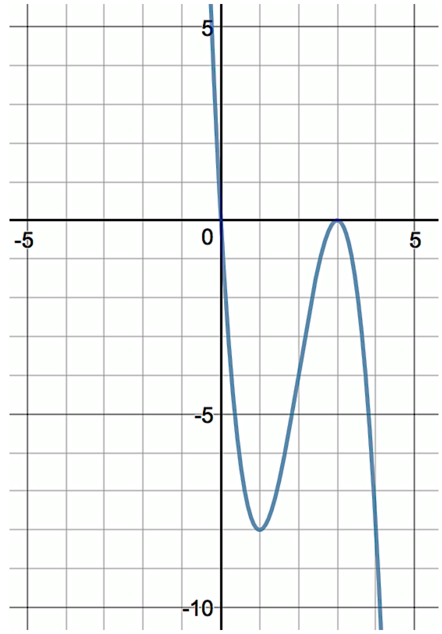
Roots (with multiplicity):

Degree:

y-intercept:



14. **Graph:**



Function in standard form:

Function in factored form:

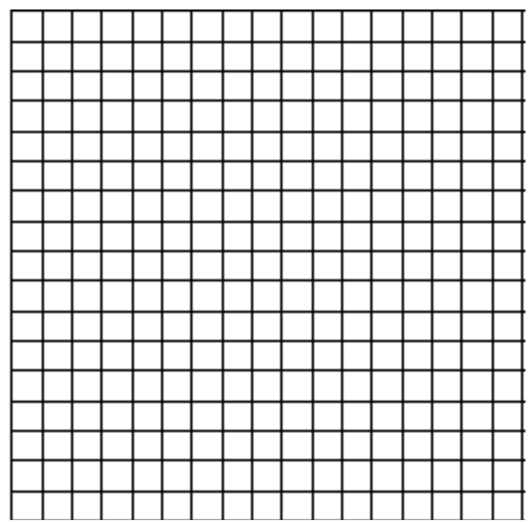
End behavior:
As $x \to -\infty, p(x) \ \to$ ______ As $x \to \infty, p(x) \ \to$ _____

Roots (with multiplicity):

Degree:

Value of leading coefficient:

15. **Graph:**



Function in standard form: $q(x) = x^3 + 2x^2 + x + 2$

Function in factored form:

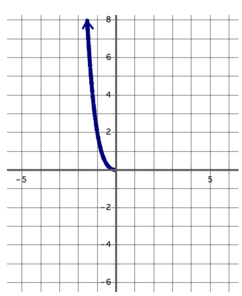
End behavior:
As $x \to -\infty, q(x) \ \to$ ______ As $x \to \infty, q(x) \ \to$ _____

Roots (with multiplicity):
$x = i$

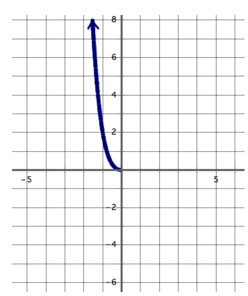
Degree:

y-intercept:

16. Finish the graph if it is an even function.



17. Finish the graph if it is an odd function.



GO

Topic: Writing polynomials given the zeros and the leading coefficient

Write the polynomial function in standard form given the leading coefficient and the zeros of the function.

18. Leading coefficient: 2; $roots$: $2, \sqrt{2}, -\sqrt{2}$

19. Leading coefficient: -1; $roots$: $1, 1+\sqrt{3},\ 1-\sqrt{3}$

20. Leading coefficient: 2; $roots$: $4i, -4i$

Fill in the blanks to make a true statement.

21. If $f(b) = 0$, then a factor of $f(b)$ must be ______________________________.

22. The rate of change in a linear function is always a ________________________________.

23. The rate of change of a quadratic function is _______________________________________.

24. The rate of change of a cubic function is ____.

25. The rate of change of a polynomial function of degree n can be described by a function of degree ____________________.