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Chapter 10 Geometry: Angles, Triangles, and Distance (3 weeks)

Utah Core Standard(s):

- Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. (8.G.5)
- Explain a proof of the Pythagorean Theorem and its converse. (8.G.6)
- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (8.G.7)
- Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (8.G.8)

Academic Vocabulary:

right triangle, right angle, congruent, leg, hypotenuse, Pythagorean Theorem, converse of Pythagorean Theorem, simplest radical form, Pythagorean triple, rectangular prism, cube, unit cube, distance formula, vertical angles, adjacent angles, straight angles, supplementary, congruent, parallel lines, ∥, transversal, vertex, point of intersection, corresponding angles, alternate interior angles, alternate exterior angles, similar, angle-angle criterion for triangles

Chapter Overview:

This chapter centers around several concepts and ideas related to angles and triangles. In the first section, students will study theorems about the angles in a triangle, the special angles formed when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. They will apply these theorems to solve problems. In Sections 2 and 3, students will study the Pythagorean Theorem and its converse and realize the usefulness of the Pythagorean Theorem in solving many real-world problems. In this chapter, we are referring to these theorems as a collection of facts. The focus in 8^{th} grade is that students are able to observe these facts through examples, exploration, and concrete models. Students will explain why the theorems are true by constructing mathematical arguments, relying on knowledge acquired throughout the year, particularly the properties of rigid motion and dilations and the understanding of congruence and similarity. The explanations and arguments made by students will come in many different forms, including a bulleted list, a narrative paragraph, a diagram without words, and proof by example. They should give their arguments and explanations within their writing and speaking. The emphasis is on students starting to gain an understanding of what makes a good argument or explanation. Can they explain things in a number of different ways? Can they critique the reasoning of others? They should be asking themselves questions such as: What do I know? What is the question asking? Can I draw a model of the situation? Does my argument/explanation have a claim, evidence, and warrant? What is the connection? These practices engaged in by students set the foundation for a more formal study of proof in Secondary II.

Connections to Content:

Prior Knowledge: In elementary grades, students have worked with geometric objects such as points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. They have also studied the different types of triangles (right, acute, and obtuse and equilateral, scalene, and isosceles). They have also learned and used facts about supplementary, complementary, vertical, and adjacent angles. In Chapter 9 of this text, students studied rigid motions and dilations and the definition of congruence and similarity.

Future Knowledge: In Secondary II, students will formally prove many of the theorems studied in this chapter about lines, angles, triangles, and similarity. They will also define trigonometric ratios and solve problems involving right triangles.

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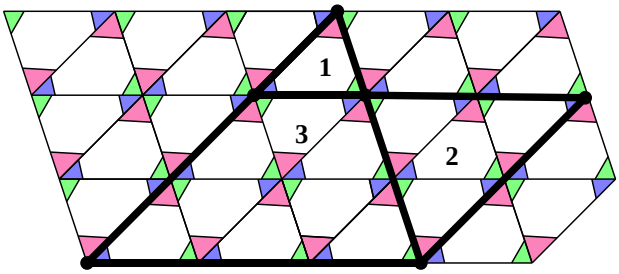
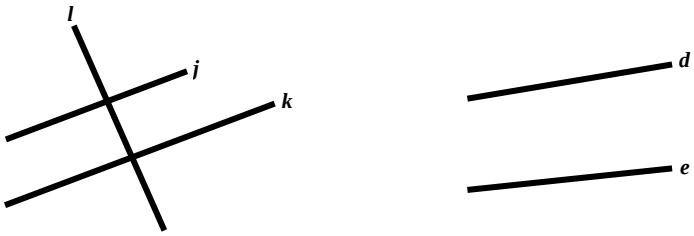
MATHEMATICAL PRACTICE STANDARDS (emphasized):

	Make sense of problems and persevere in solving them.	 What is the relationship between the triangles formed by the dark lines? Justify your answer. *Students will use the concrete model shown above in order to make arguments about several of the theorems studied in this chapter. They will also rely on their knowledge of rigid motions and dilations.*
	Model with mathematics.	A new restaurant is putting in a wheelchair ramp. The landing from which people enter the restaurant is 2 feet higher than street level. Safety standards require that for every 1 foot of rise on a wheelchair ramp there must be a run of 12 feet. How long will the ramp have to be to meet safety standards? Round your answer to the nearest tenth. *Students will apply the Pythagorean Theorem in order to solve many real-world problems. They will have to analyze the situation to determine if the Pythagorean Theorem can be used to solve the problem, draw a picture of the situation, analyze givens and constraints, and understand what they are solving for.*
	Construct viable arguments and critique the reasoning of others.	Suppose you are given two lines *j* and *k* in the picture below. You have been asked to determine whether the two lines are parallel. You start by drawing the transversal *l* through the two lines as shown below. Devise a strategy to determine whether the two lines are parallel using what you know about the properties of rigid motion. Next, use your strategy to determine whether lines *d* and *e* are parallel. Just saying they do not look parallel, is not a justification.  *Throughout the chapter, students will observe theorems about angles and triangles by example, exploration, and concrete models. Students will construct mathematical arguments as to why the theorems are true, relying on knowledge acquired throughout the year, particularly the properties of rigid motion and dilations and the understanding of congruence and similarity. Students will begin to understand the necessary elements of what makes a good proof as outlined in the chapter overview.*

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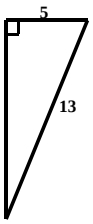
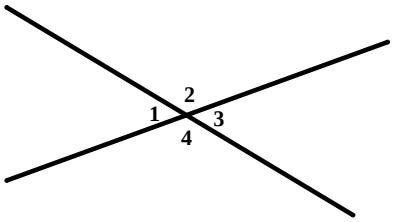
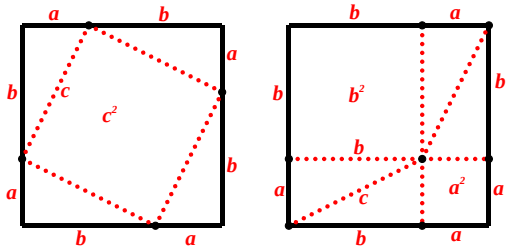
	Attend to Precision	**Find, Fix, and Justify:** Megan was asked to solve for the unknown side length in the triangle below. Her work is shown below. She made a mistake when solving. Explain the mistake she made and then solve the problem correctly.  **Megan's Solution:** $a^2 + b^2 = c^2$ $5^2 + 13^2 = c^2$ $25 + 169 = c^2$ $194 = c^2$ $\sqrt{194} = c$ *This problem requires that students are clear in their understanding of the Pythagorean Theorem and how to use it to solve for missing side lengths.*
	Use appropriate tools strategically.	 Use ideas of rigid motion to prove that $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. *Students will rely heavily on the knowledge learned in Chapter 9 about rigid motions and congruence and dilations and similarity. This knowledge will be a tool they apply to understand and informally prove many of the theorems about angles, triangles, and similarity in this chapter.*
	Reason abstractly and quantitatively.	 Using the picture above, prove that **the sum of the areas of the squares along the two smaller sides of the right triangle equals the area of the square along the larger side of the triangle for *any*** right triangle. *Students first begin to study and understand the Pythagorean Theorem using concrete examples. Then, they move to an abstract proof of the Pythagorean Theorem to show that it holds true for any right triangle.*

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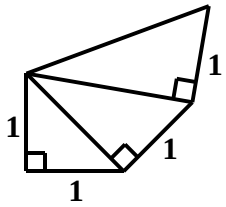
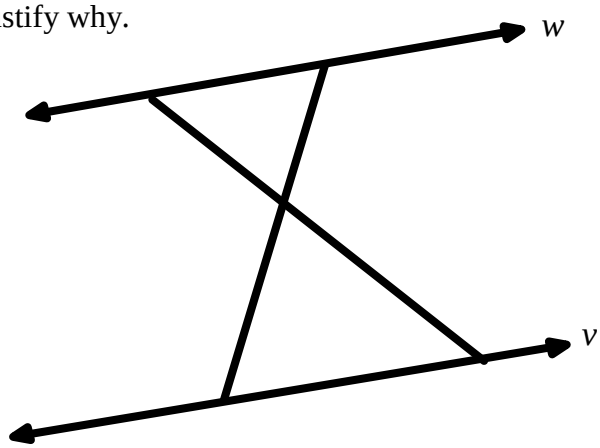
	Look for and express regularity in repeated reasoning.	Use the picture below to answer questions a) and b).  a. Find all the missing side lengths and label the picture with the answers. b. Using the picture above, devise a strategy for constructing a segment with a length of $\sqrt{5}$. Explain your strategy below. *In this problem, students should start to notice that the hypotenuse of the new triangle will follow a pattern. This observation gives them a process for constructing any segment of length $\sqrt{n}$ where n* is a whole number.
	Look for and make use of structure.	Given that line *w* ‖ line *v*, determine if the triangles given below are similar. If they are similar justify why.  *In the problem above students must look at the geometric figure above and evaluate the information given to them. They are given that line w ‖ line v. They must recognize that the two intersecting lines that form the triangles are transversals of the parallel lines. Students might do this by extending the transversals beyond the interior of the two parallel lines or by drawing an auxiliary line over these lines that extends beyond the parallel lines. Once they look at these lines as transversals they can use what they know about special angle relationships to determine congruence amongst angles within the triangles. As students view the structure of the intersecting lines their perspective shifts and they are able to derive more information about the figure.*

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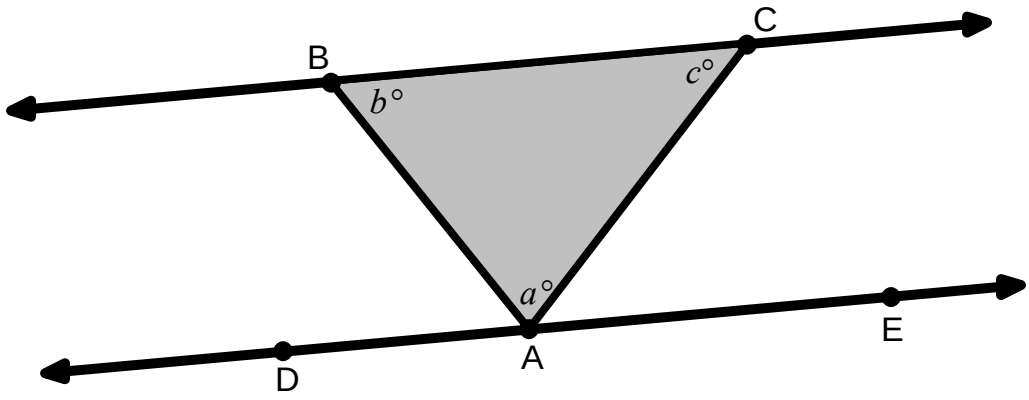
10.0 Anchor Problem: Reasoning with Angles of a Triangle and Rectangles

Part I

Given that $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE}$ in the picture below, show that $a + b + c = 180°$.



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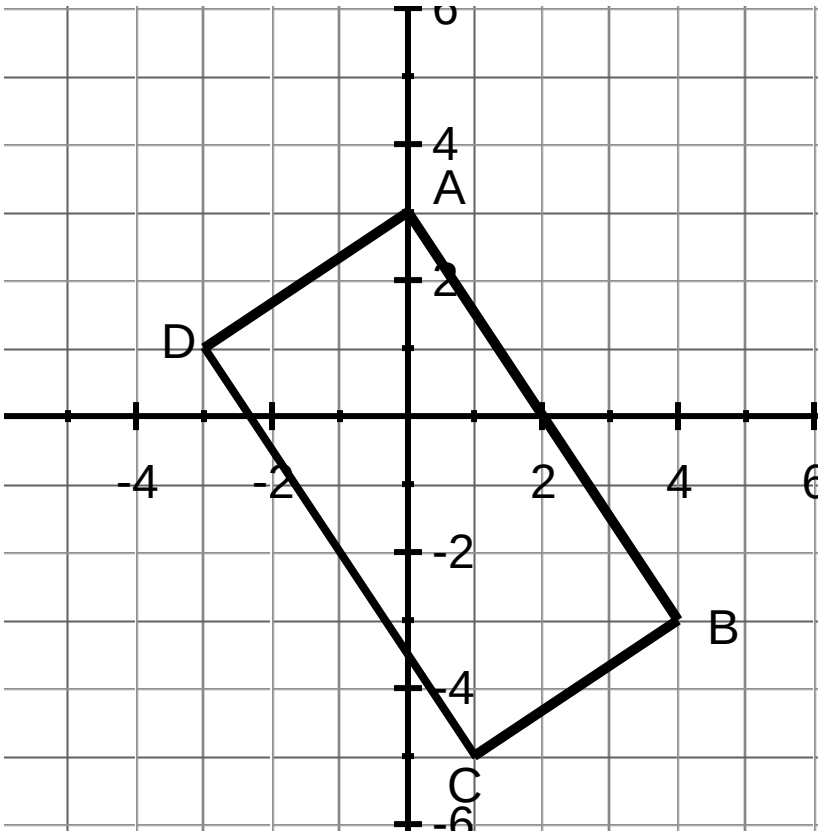
Part II

Pedro's teacher asks him to classify the quadrilateral below. He claims it is a rectangle. His teacher tells him to give a good argument and explanation. Help Pedro to support his claim using mathematical evidence.

Remember: Opposite sides of a rectangle have the same length and are parallel and the sides of a rectangle meet at right angles.



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10.1 Angles and Triangles

Section Overview:
The focus of this section is on the development of geometric intuition through exploration with rigid motions and dilations. Through exploration, observation, and the use of concrete models, students will analyze facts about triangles and angles and use these facts to describe relationships in geometric figures. There will also be a focus on making sound mathematical explanations and arguments in order to verify theorems about angles and triangles and when explaining and justifying solutions to problems throughout the section.

Concepts and Skills to Master:
By the end of this section, students should be able to:
1. Know that straight angles sum to 180° and that vertical angles are congruent.
2. Know that the sum of the angles in a triangle is 180°. Understand that the measure of an exterior angle of a triangle is equal to the sum of the measures of the non-adjacent angles. Use these properties to find missing angle measures related to a triangle.
3. Determine the relationship between angles formed when a transversal intersects parallel lines. Use these relationships to find missing angle measures.
4. Determine whether two lines are parallel based on the angle measures when a transversal intersects the lines.
5. Understand and apply the angle-angle criterion to determine whether two triangles are similar.

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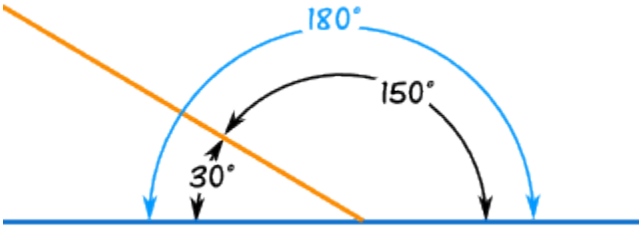
10.1a Class Activity: Straight and Vertical Angles Review

In this section, you will observe and use several different geometric facts learned in previous grades. They will be denoted using bullets.

- **Angles that lie on the same line (straight angles) are supplementary.**

In 7th grade, you learned that a straight angle has a measure of 180˚ as shown below. Angles that sum to 180˚ are **supplementary**. In the picture below, 30˚ and 150˚ are supplementary and together they form a straight angle.

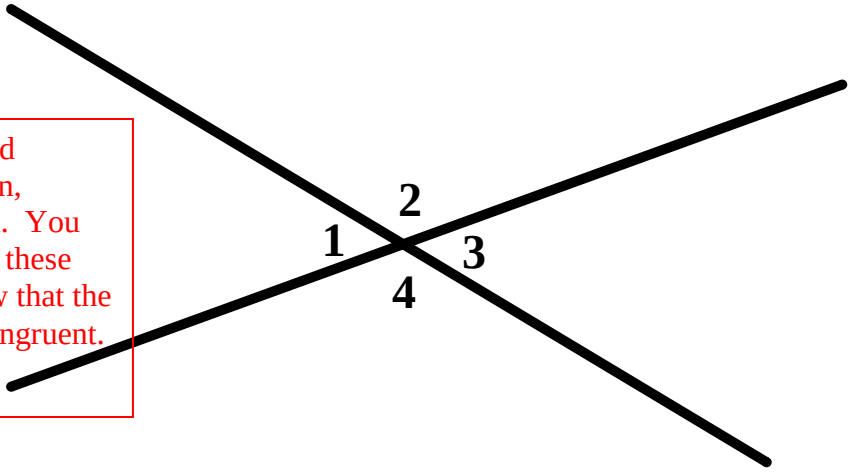


- **Vertical angles have the same measure.**

Vertical angles are the opposing angles formed by two intersecting lines.

In the picture below, ∠1 and ∠3 are vertical angles and ∠2 and ∠4 are vertical angles.

Remember that a rigid motion is a translation, reflection, or rotation. You may use one or all of these rigid motions to show that the vertical angles are congruent.



1. Show that ∠1 ≅ ∠3 and ∠2 ≅ ∠4. (Hint: Think about ideas of rigid motion and straight angles.)
   Students may trace the angles on patty paper and fold, reflecting angle 1 so that it sits on angle 3. They can use a similar method to map angle 2 to angle 4. Alternatively, students can copy angle 1 and rotate it 180˚ about the vertex to map it to angle 3. The same motion will map angle 2 to angle 4. Since one angle maps to the other using rigid motion, the angles are congruent. Alternatively, they can give an explanation without using rigid motions by saying, that since angles 1 and 3 are both supplementary to angle 2, they must have the same measure. They can set this up as an equation: $\angle 1 + \angle 2 = 180$ and $\angle 2 + \angle 3 = 180$; therefore $\angle 1 + \angle 2 = \angle 2 + \angle 3$. When we solve, we see that $\angle 1 = \angle 3$. They can use a similar process to prove $\angle 2 \cong \angle 4$.
2. Which pairs of angles are supplementary in the picture above?
   ∠1 and ∠4; ∠1 and ∠2; ∠2 and ∠3; ∠3 and ∠4

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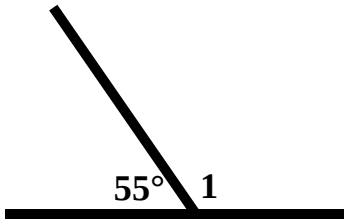
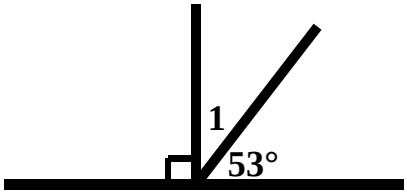
Review: Find the missing angle measures without the use of a protractor.

3.



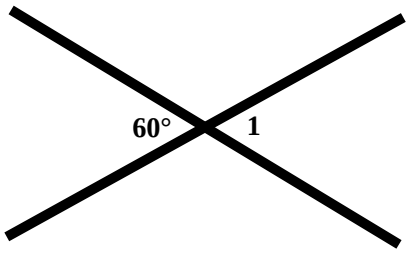
$m\angle 1 =$ ___125˚_____

4.



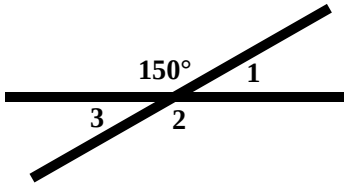
$m\angle 1 =$ ___37˚_____

5.



$m\angle 1 =$ ___60˚_____

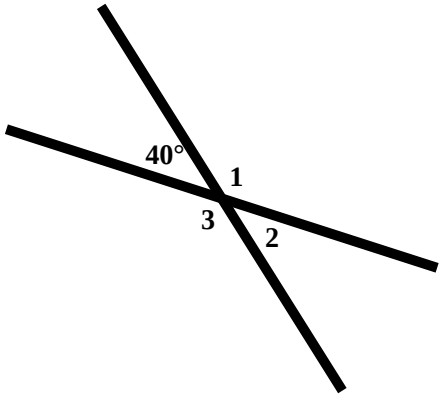
6.



$m\angle 1 =$ _____30˚_____ $m\angle 2 =$ _____

$m\angle 3 =$ _____30˚_____

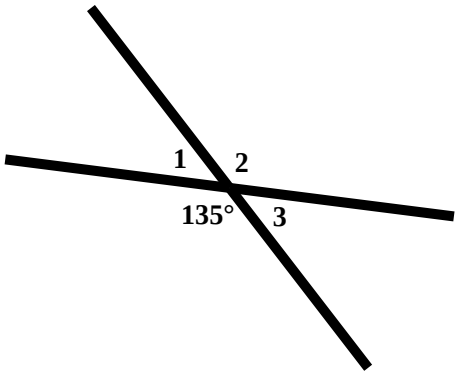
7.



$m\angle 1 =$ _____ $m\angle 2 =$ _____

$m\angle 3 =$ _____

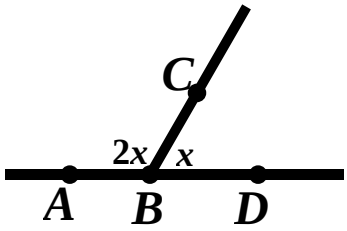
8.



$m\angle 1 =$ _____ $m\angle 2 =$ _____

$m\angle 3 =$ _____

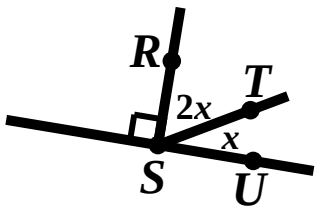
9.



$m\angle ABC =$ _____120˚_____

$m\angle CBD =$ _____60˚_____

10.



$m\angle RST =$ ___60˚_ $m\angle RSU =$ _____90˚____

$m\angle TSU =$ _____30˚____

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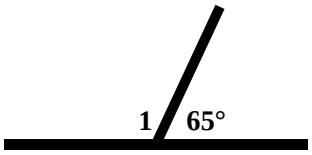
10.1a Homework: Straight and Vertical Angles Review

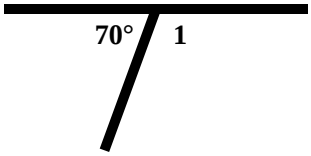
Review: Find the missing angle measures without the use of a protractor.

1.



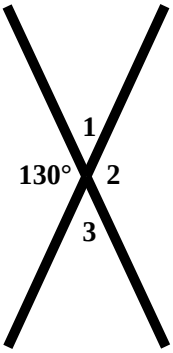
$m\angle 1 =$ _____115˚_____

2.



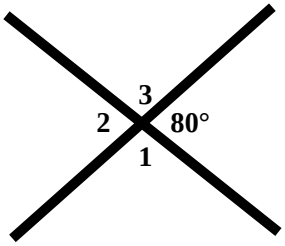
$m\angle 1 =$ _____

3.



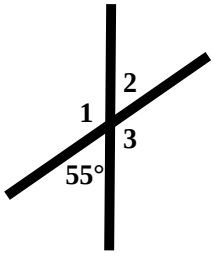
$m\angle 1 =$ __50˚__ $m\angle 2 =$ __130˚__ $m\angle 3 =$ _50˚

4.



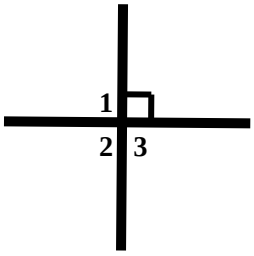
$m\angle 1 =$ ____ $m\angle 2 =$ ___ $m\angle 3 =$ ____

5.



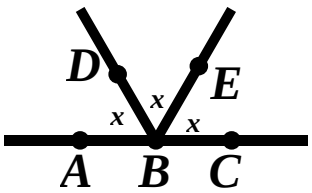
$m\angle 1 =$ _____ $m\angle 2 =$ _____ $m\angle 3 =$ _____

6.



$m\angle 1 =$ _____ $m\angle 2 =$ _____ $m\angle 3 =$ _____

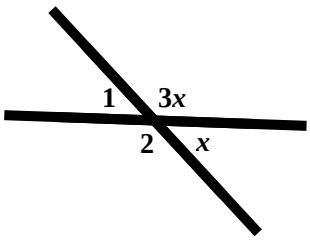
7.



$m\angle ABD =$ ___60˚__ $m\angle DBE =$ __60˚__

$m\angle ABE =$ __120˚__ $m\angle ABC =$ __180˚____

8.



$m\angle 1 =$ _____ $m\angle 2 =$ _____

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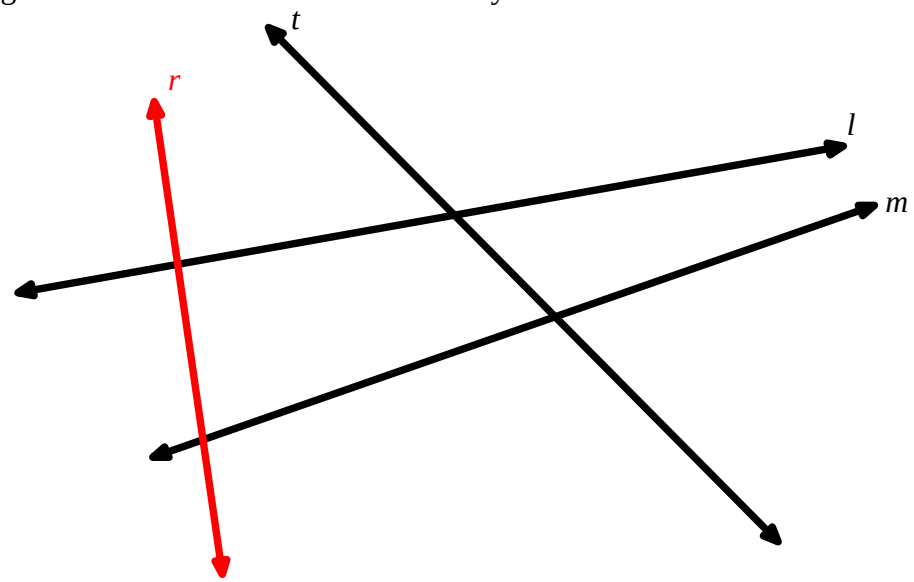
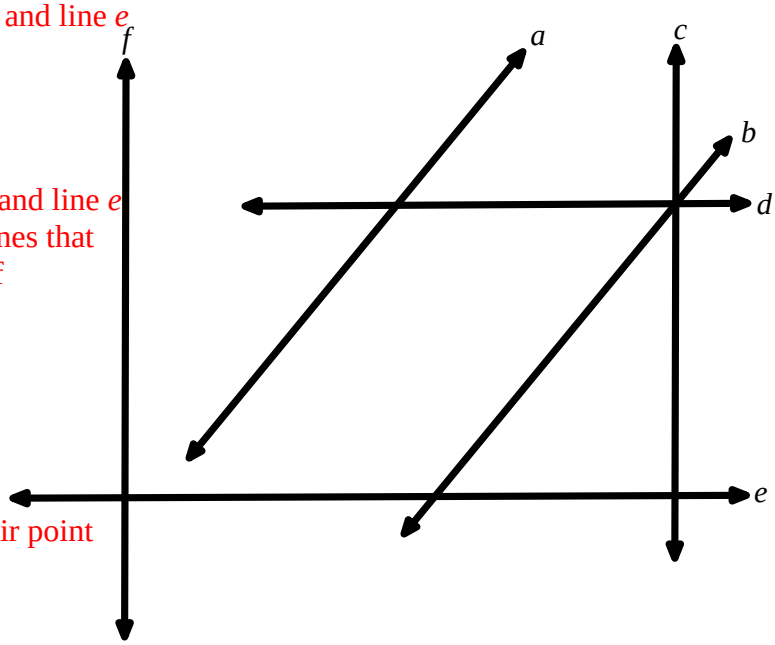
10.1b Class Activity: Special Angles Formed by Transversals

1. In the picture given below line *l* and line *m* are cut by a transversal line called *t*.



2. Define transversal in your own words. Draw another transversal for the two lines above and label it line *r*. A transversal is a line that intersects two or more lines at different points.

3. Some of the runways at a major airport are shown in the drawing below. Identify at least 2 sets of lines to which each line is a transversal.



   a. line *a* answers may include;
   line *d* and line *e*, line *f* and line *c*, line *f* and line *e*,
   line *c* and line *d*, line *d* and line *f* , line *c* and line *e*

   b. line *b* answers may include;
   line *d* and line *e*, line *f* and line *c*,
   line *f* and line *e*, line *d* and line *f* , line *c* and line *e*
   (line *c* and line *d* do not work because lines that intersect two other lines at their point of intersection are not transversals.)

   c. line *c* answers may include;
   line *a* and line *b*, line *a* and line *e*
   line *a* and line *d*, line *b* and line *e*
   line *d* and line *e*
   (line *b* and line *d* do not work because lines that intersect two other lines at their point of intersection are not transversals.)

   d. line *e* answers may include;
   line *b* and line *a*, line *b* and line *f*
   line *b* and line *c*, line *c* and line *a*
   line *c* and line *f*, line *a* and line *f*

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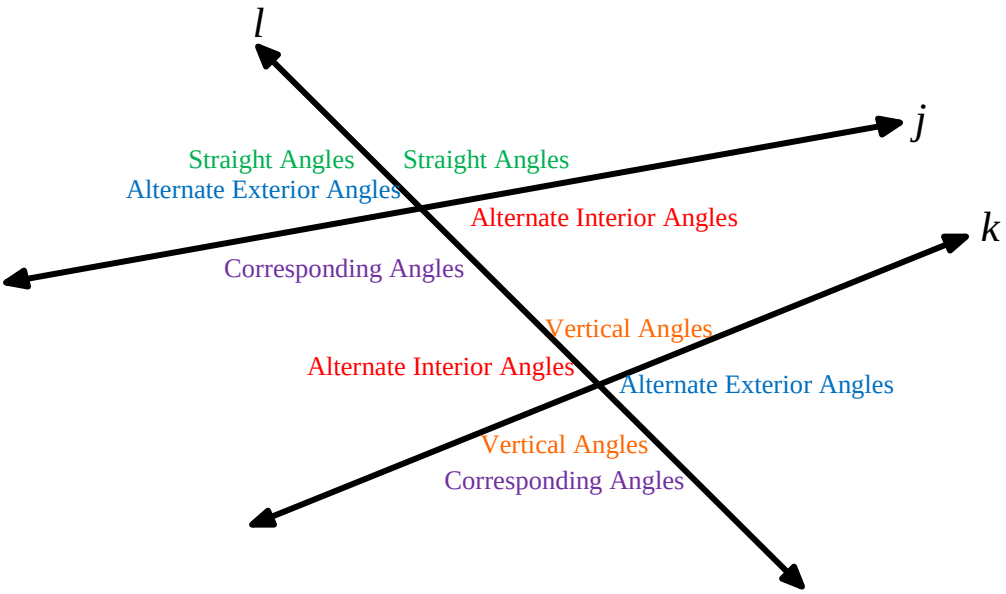
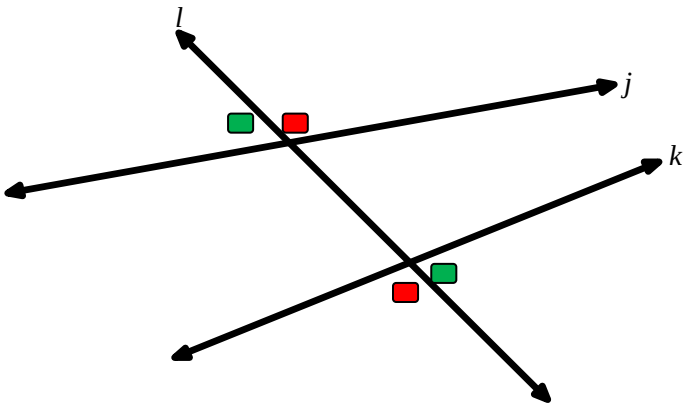
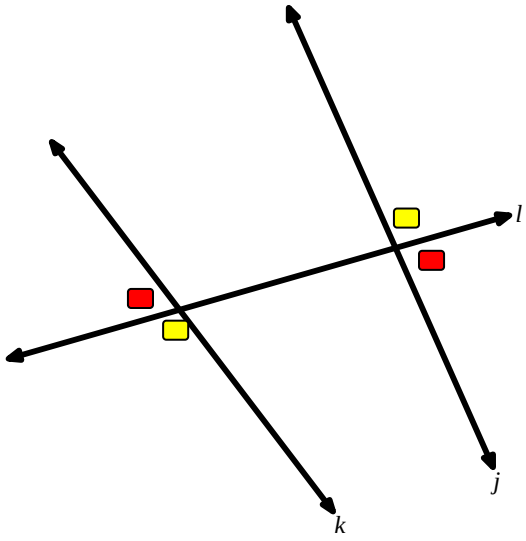
When two lines are intersected by a transversal there are special angle pairs that are formed. Use the angle names provided by your teacher to move the angle names around the picture below until you think you have found its correct location. Be ready to justify your reasoning. There will be several correct locations for each set of angle pairs and more than one term may fit at an angle.
Sample answers are shown.



Directions: Color code the following sets of angles by coloring each set of angle pairs the same color. Find at least two sets of the special angles for each drawing.
Sample color coding is given.

4. **Alternate Exterior Angle Pairs**





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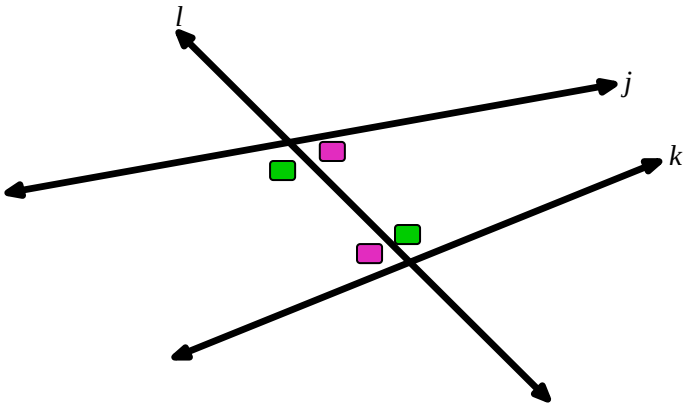
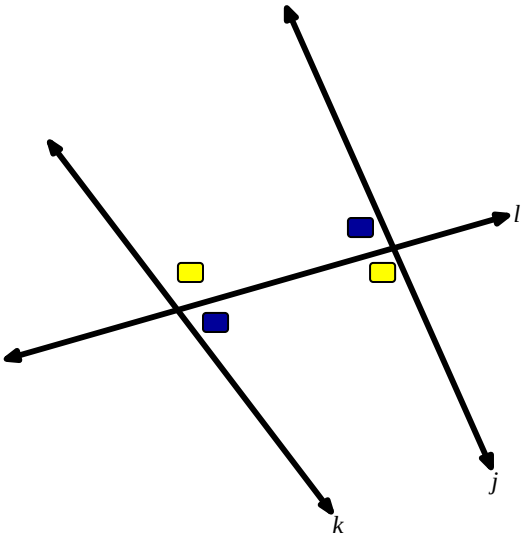
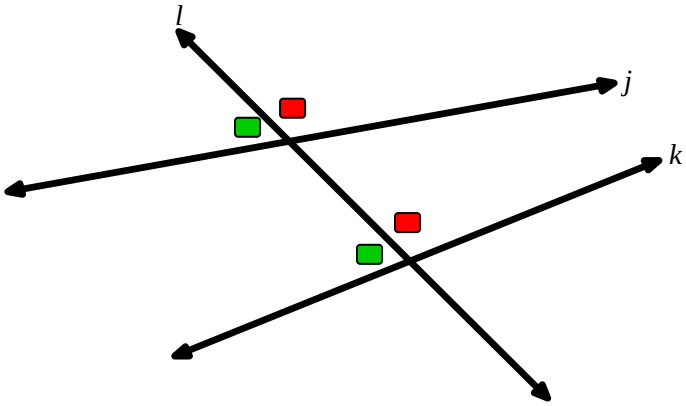
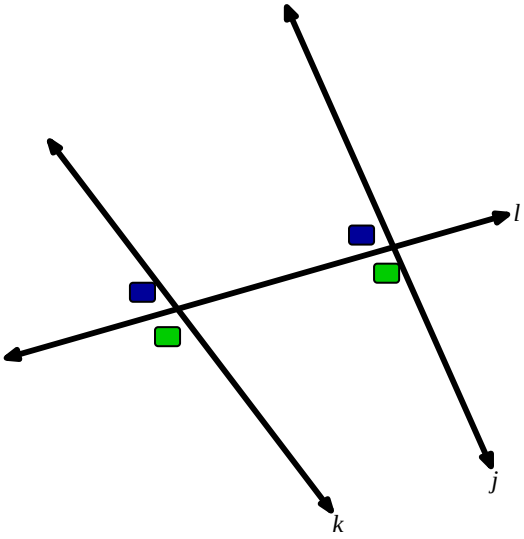
5. **Alternate Interior Angle Pairs**



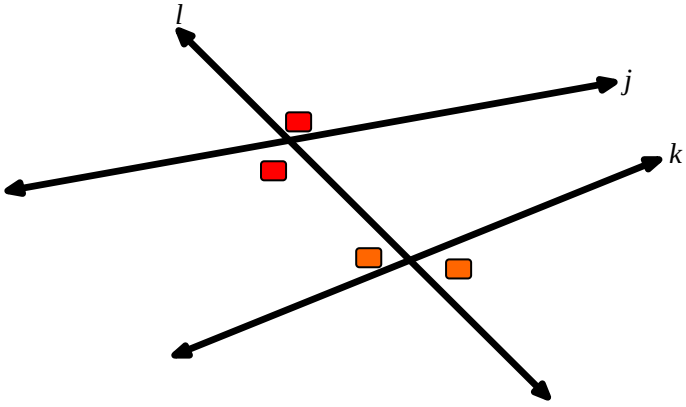


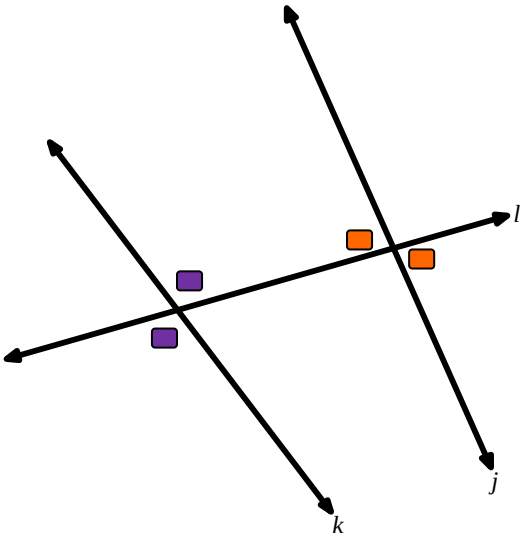
6. **Corresponding Angle Pairs**





7. **Vertical Angle Pairs**





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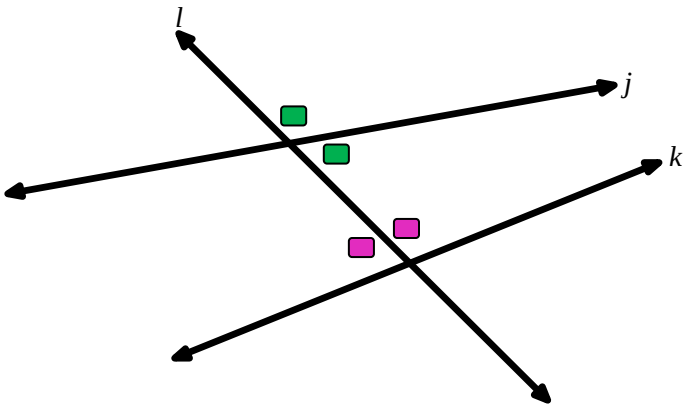
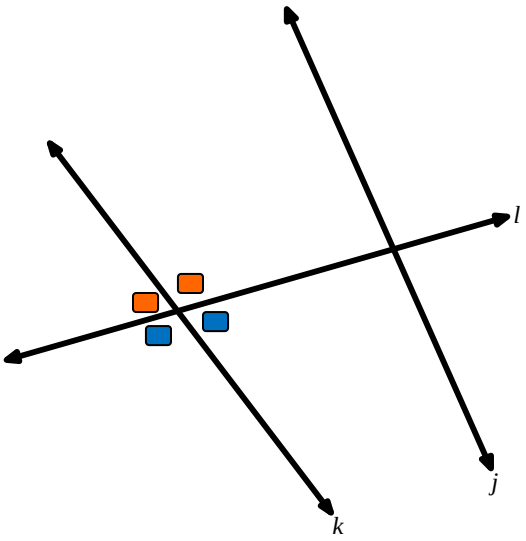
8. **Straight Angle Pairs**





9. Refer to the figure below; identify the following pairs of angles as alternate exterior angles, alternate interior angles, corresponding angles, vertical angles, or straight angles.

a. ∠1 and ∠8 alternate exterior angles

b. ∠12 and ∠11 straight angles

c. ∠13 and ∠21 corresponding angles

d. ∠14 and ∠15 vertical angles

e. ∠7 and ∠14 alternate interior angles

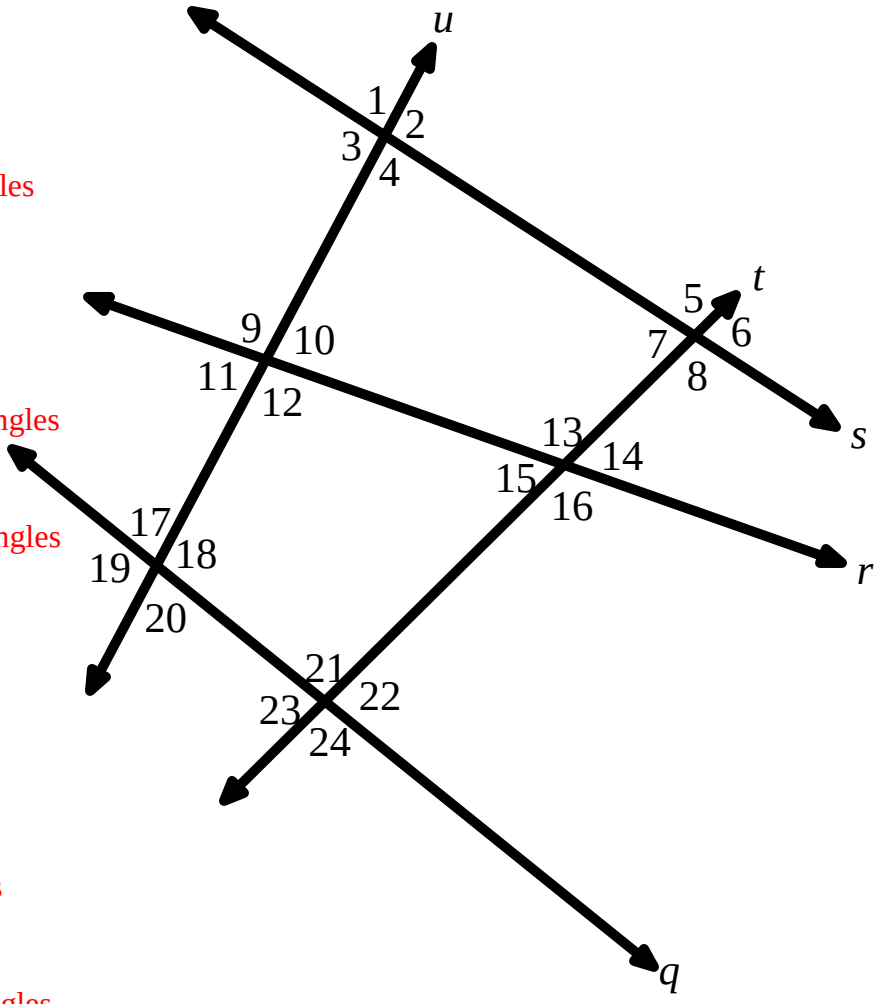
f. ∠9 and ∠20 alternate exterior angles

g. ∠5 and ∠7 straight angles

h. ∠22 and ∠23 vertical angles

i. ∠1 and ∠5 corresponding angles

j. ∠21 and ∠8 alternate interior angles



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10.1b Homework: Special Angles Formed by Transversals

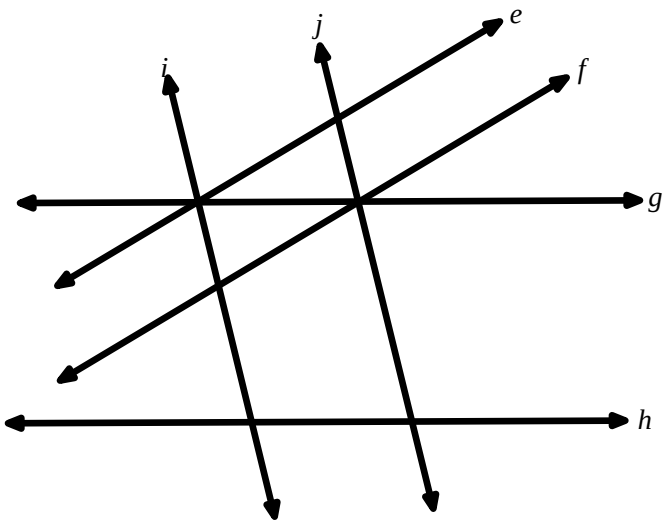
1. Identify the sets of given lines to which each line is a transversal.

   a. line *e*
   line *i*, line *j*, line *g*, line *h*

   b. line *g*

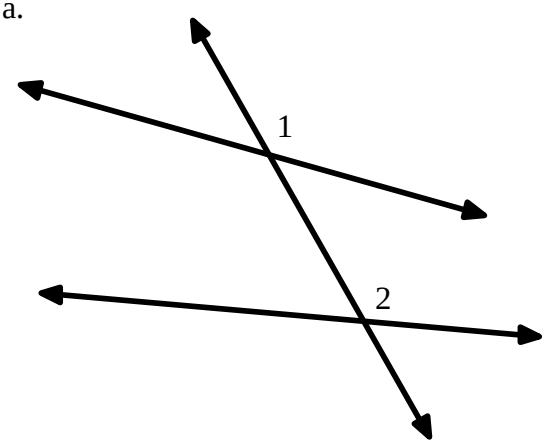
   c. line *h*
   line *i*, line *j*, line *e*, line *f*

   d. line *j*



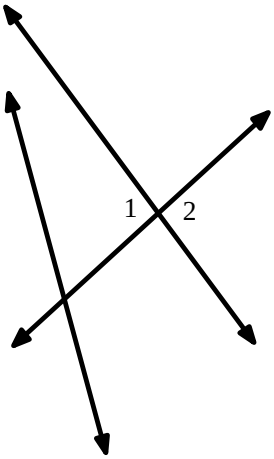
2. Refer to the figures below. State if $\angle 1 \text{ and } \angle 2$ are alternate exterior angles, alternate interior angles, corresponding angles, vertical angles, or straight angles.

a.

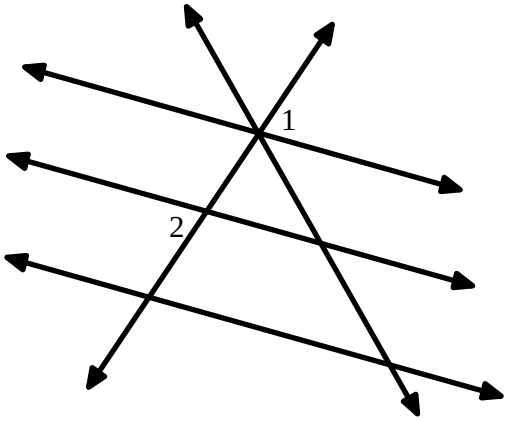


Corresponding Angles

b.

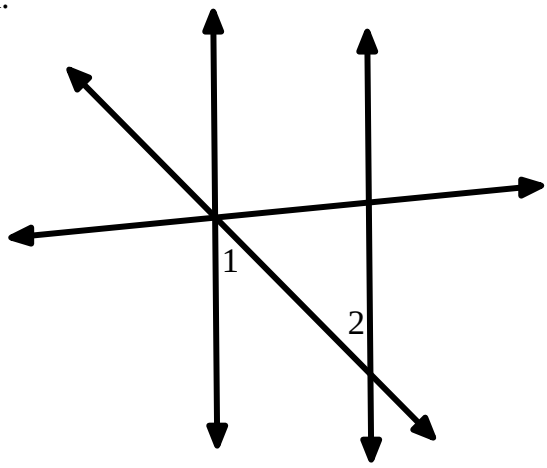


c.



Alternate Exterior Angles

d.



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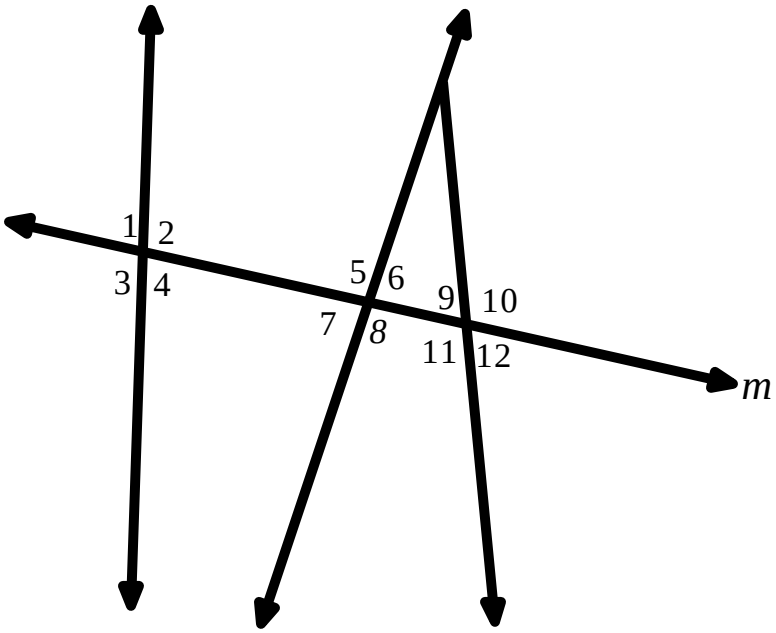
3. Refer to the figure below; state if the following pairs of angles are alternate exterior angles, alternate interior angles, corresponding angles, vertical angles, or straight angles.



a. ∠4 and ∠9 alternate interior angles

b. ∠12 and ∠11

c. ∠1 and ∠5 corresponding angles

d. ∠1 and ∠8

e. ∠6 and ∠7 vertical angles

f. ∠1 and ∠3 straight angles

g. ∠8 and ∠9

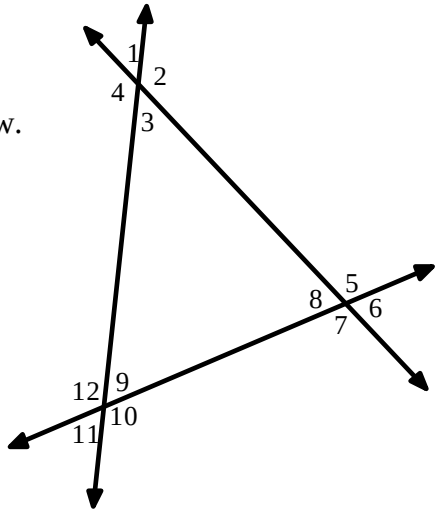
h. ∠7 and ∠11

i. ∠3 and ∠10 alternate exterior angles

j. ∠10 and ∠11

Find, Fix, and Justify

4. Patel and Ari are naming alternate interior angles for the figure below. They are listing alternate interior angle pairs for angle 3. Their work is shown below.



Patel	Ari
∠3 and ∠12	∠3 and ∠9
∠3 and ∠5	∠3 and ∠5

Who is correct? Explain your reasoning.

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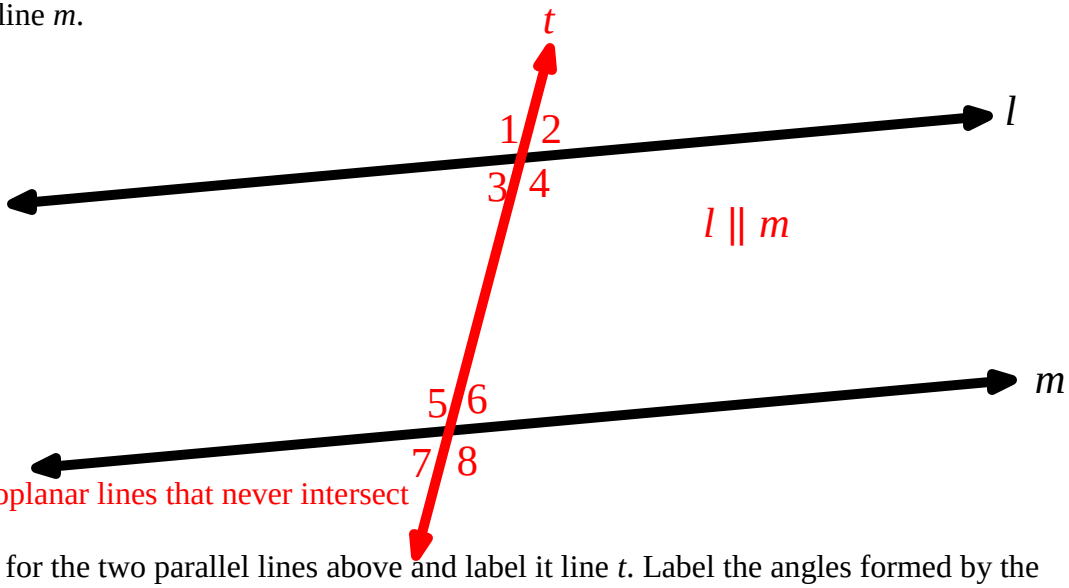
10.1c Class Activity: Parallel Lines and Transversals

1. Use the picture given below to describe what parallel lines are. Use the correct notation to denote that line *l* is parallel to line *m*.



Parallel lines are coplanar lines that never intersect

.

2. Draw a transversal for the two parallel lines above and label it line *t*. Label the angles formed by the transversal and the parallel lines with numbers 1 through 6. *Be sure to number in the same order as your teacher.

Note: A transversal can intersect any two lines, but we are now particularly interested in transversals that intersect parallel lines.

Transversals that intersect two or more parallel lines create angle pairs that have special properties. Use what you know about rigid motions to discover some of these relationships.

3. What type of angle pair is ∠2 and ∠6?
Corresponding angles
4. Copy ∠2 on a piece of tracing paper (or patty paper). Describe the rigid motion that will carry ∠2 to ∠6. Determine the relationship between ∠2 and ∠6.
Students will see that angle 2 can be carried onto angle 6 by a translation. Thus ∠2 and ∠6 are congruent.
5. Use a similar process to see if the same outcome holds true for all of the corresponding angles in the figure. Start by listing the remaining pairs of corresponding angles and then state the relationship.
∠1 and ∠5 Congruent ∠3 and ∠7 Congruent

∠4 and ∠8 Congruent
The other angles can be mapped to their corresponding angles using a similar process. Thus all of the corresponding angles in the figure are congruent to each other. They can also use the fact that vertical angles are congruent and once they know that angle 2 is congruent to angle 6, they also know that angle 3 and angle 7 are also congruent. As students are investigating these angles and making arguments be sure to ask them to give support and warrants for their claims.

6. List the pairs of angles that are vertical angles, what do you know about vertical angles?
∠1 and ∠4, ∠2 and ∠3, ∠5 and ∠8, ∠6 and ∠7
Vertical angles are congruent.

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7. Continue to use rigid motions and what you know about vertical angles to discover other relationships that exist between alternate interior angles and alternate exterior angles. Be sure to provide justification for your claims. Students may argue that ∠2 is congruent to ∠6 because they are corresponding angles, also ∠2 is congruent to ∠3 because they are vertical angles. Therefore ∠3 and ∠6 are congruent. Similar justification can be made to show that every pair of alternate interior angles are congruent and every pair of alternate exterior angles are congruent.

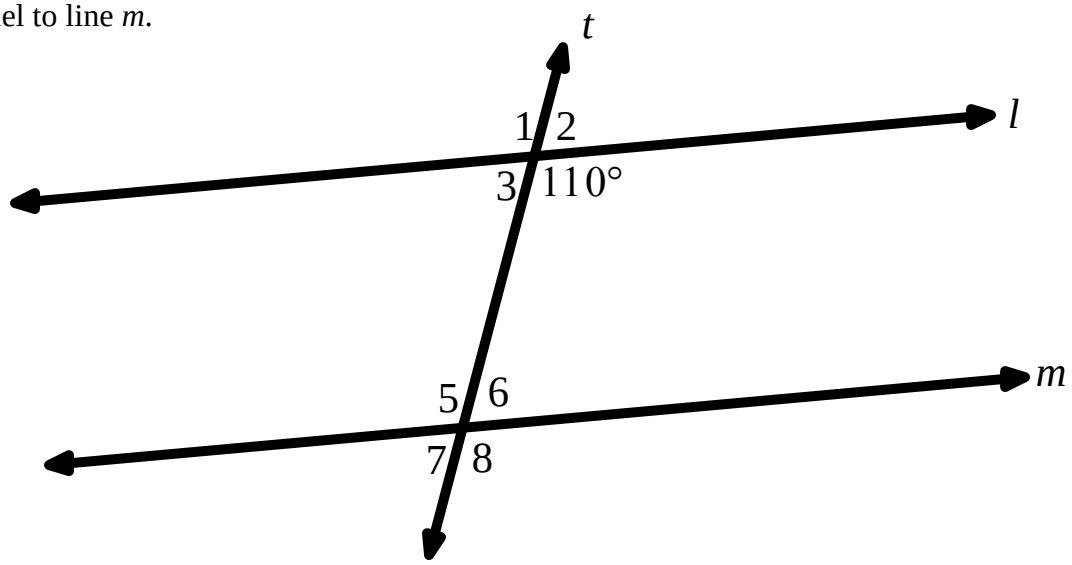
8. Complete the following statements in the box below.

Properties of Transversals to Parallel Lines

If two parallel lines are intersected by a transversal,

- Corresponding angles are ____congruent_____.
- Alternate interior angles are ___congruent_____.
- Alternate exterior angles are __congruent_____.

9. In the diagram below one angle measure is given. Find the measure of each remaining angle if line *l* is parallel to line *m*.



a. $m\angle 1 =$ __110°__

b. $m\angle 2 =$ _70°__

c. $m\angle 3 =$ __70°_

d. $m\angle 5 =$ __110°__

e. $m\angle 6 =$ _70°_

f. $m\angle 7 =$ __70°_

g. $m\angle 8 =$ _110°_

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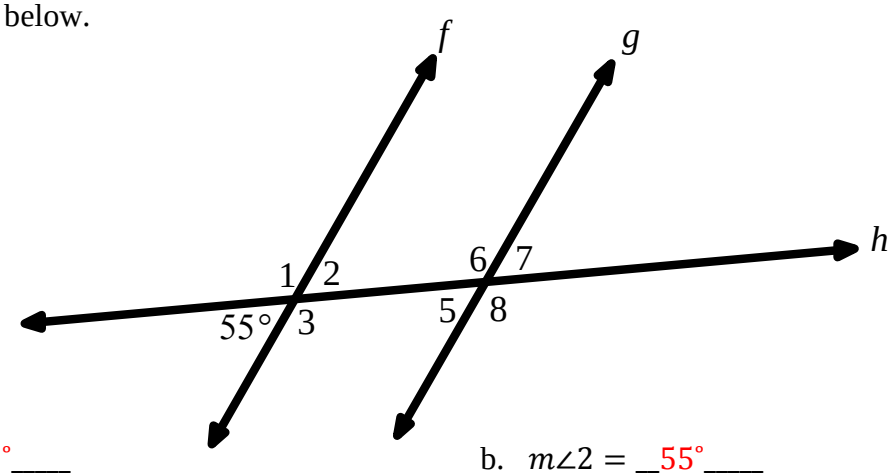
10. Line f ∥ line g and one angle measure is given in the diagram. Determine the measures of the remaining angles in the diagram below.



a. $m\angle 1 =$ __125°____

b. $m\angle 2 =$ __55°____

c. $m\angle 3 =$ ____

d. $m\angle 5 =$ _____

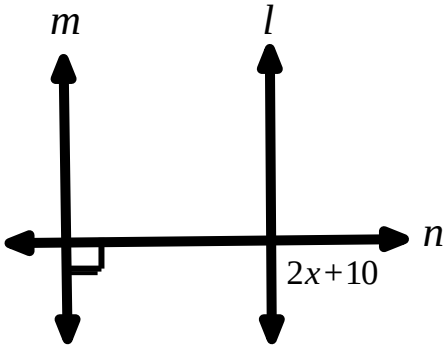
e. $m\angle 6 =$ __125°___

f. $m\angle 7 =$ _____

g. $m\angle 8 =$ _____

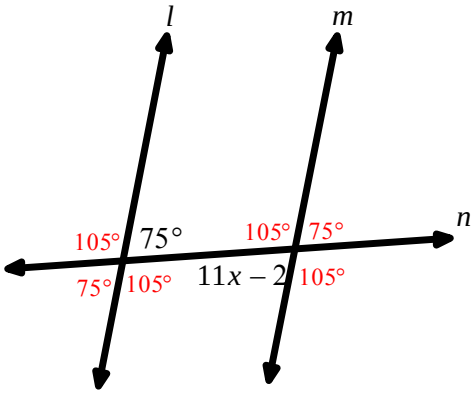
11. Given that line l ∥ line m solve for x and then find the measure of all the remaining angles. Write the angle measures on the picture.

a.



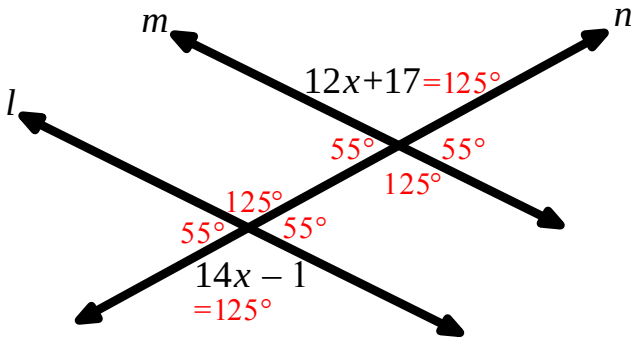
$x =$

b.



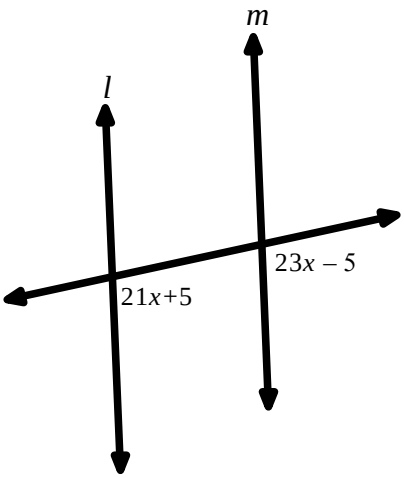
$x = 7$

c.



$x = 9$

d.



$x =$

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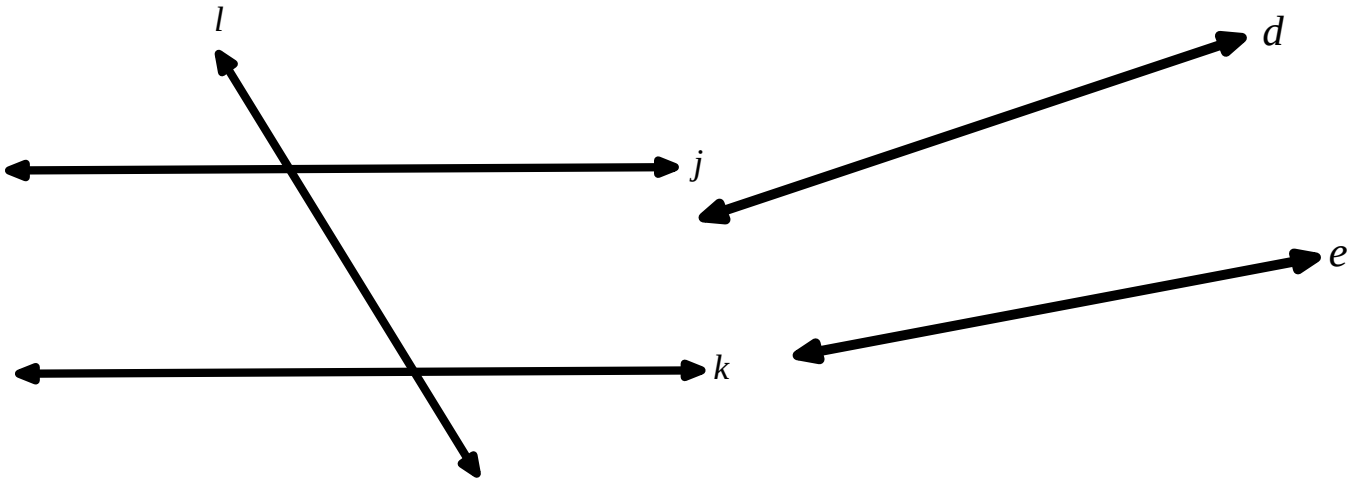
12. Given two lines *j* and *k* in a picture below with transversal *l* devise a strategy to determine whether the two lines are parallel using what you know about the properties of rigid motion. Also use your strategy to determine whether lines *d* and *e* are parallel. Stating that the lines do not look parallel, is not a justification.

Copy one angle formed by the transversal and line *j*. Does it map to its corresponding angle formed by the transversal and line *k* using rigid motion? Yes, it maps using a translation. Under a translation, corresponding segments are parallel so the segment that is part of the first angle will be parallel to the segment that is part of the second angle. Since the segments sit on lines *j* and *k*, lines *j* and *k* are also parallel. For lines *d* and *e*, we see that if we draw a transversal through the lines and copy one angle formed by the transversal and line *d*, we cannot map it to its corresponding angle formed by the transversal and line *e*; therefore the lines are not parallel.

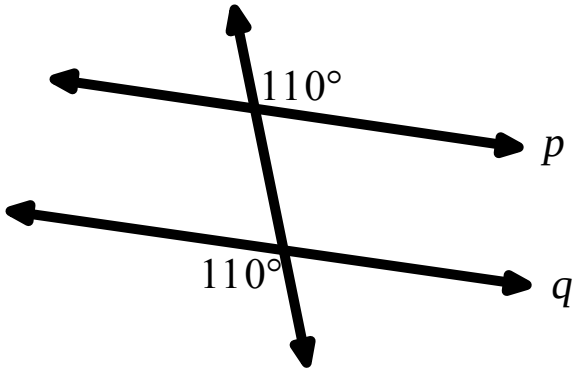


13. Complete the statement below.

Given two lines, if a transversal cuts through both lines so that corresponding angles are_____congruent_____, then the two lines are _____parallel_____.

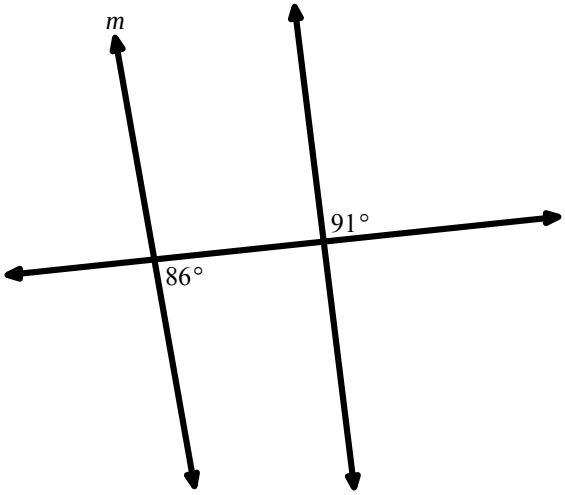
14. Determine whether the following sets of lines are parallel or not. Provide a justification for your response.

a. Is *p* parallel to *q*? Why or why not?



Yes, corresponding angles are congruent.

b. Is *m* parallel to *n*? Why or why not?



No, corresponding angles are not congruent.

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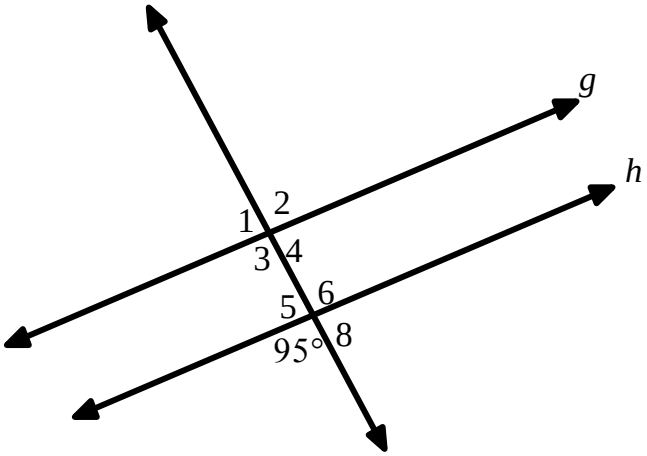
10.1c Homework: Parallel Lines and Transversals

Directions: Use the diagram below to answer questions #1 and 2 given that $g \parallel h$.



1. For each of the following pairs of angles, describe the relationship between the two angles (corresponding angles, alternate interior angles, alternate exterior angles, or vertical angles).

   a. $\angle 3$ and $\angle 6$ alt int

   b. $\angle 4$ and $\angle 8$

   c. $\angle 1$ and $\angle 8$

   d. $\angle 1$ and $\angle 5$ corresponding

2. One angle measure is given in the diagram. Determine the measures of the remaining angles in the diagram.

   a. $m\angle 1 =$ _____

   b. $m\angle 2 =$ _95°___

   c. $m\angle 3 =$ _____

   d. $m\angle 4 =$ __85°___

   e. $m\angle 5 =$ _____

   f. $m\angle 6 =$ __95°___

   g. $m\angle 8 =$ _____

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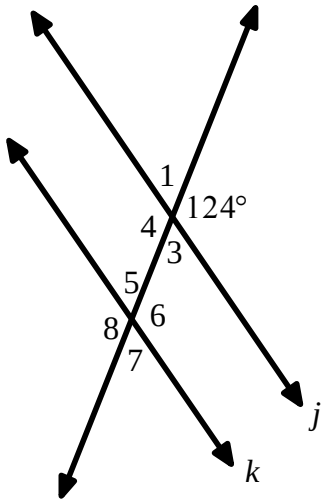
Directions: Use the diagram below to answer question #3 given that line $j \parallel$ line k.



3. One angle measure is given in the diagram. Determine the measures of the remaining angles in the diagram.

   a. $m\angle 1 =$ _____

   b. $m\angle 3 =$ _56°___

   c. $m\angle 4 =$ _124°___

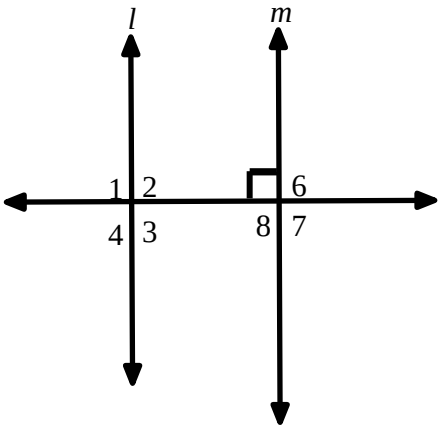
   d. $m\angle 5 =$ _____

   e. $m\angle 6 =$ _____

   f. $m\angle 7 =$ _56°___

   g. $m\angle 8 =$ _____

Directions: Use the diagram below to answer question #4 given that line $l \parallel$ line m.



4. One angle measure is given in the diagram. Determine the measures of the remaining angles in the diagram.

   a. $m\angle 1 =$ _____

   b. $m\angle 2 =$ _____

   c. $m\angle 3 =$ _____

   d. $m\angle 4 =$ _____

   e. $m\angle 6 =$ _____

   f. $m\angle 8 =$ _____

   g. $m\angle 7 =$ _____

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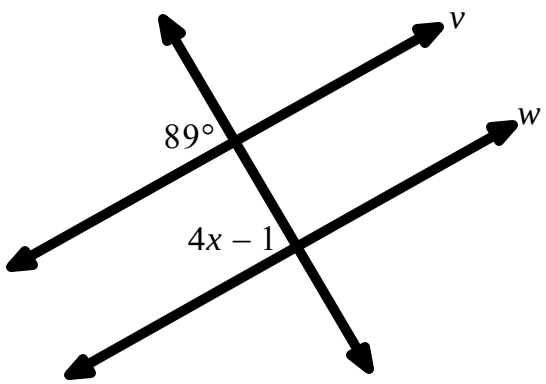
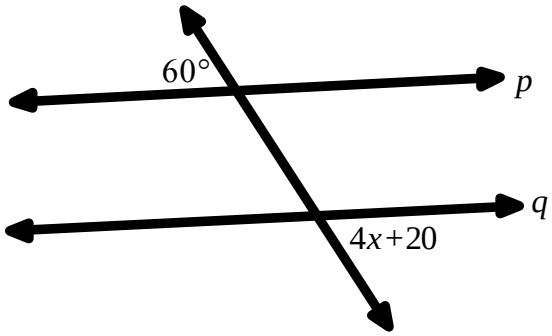
5. Given line $v \parallel$ line w, solve for x.

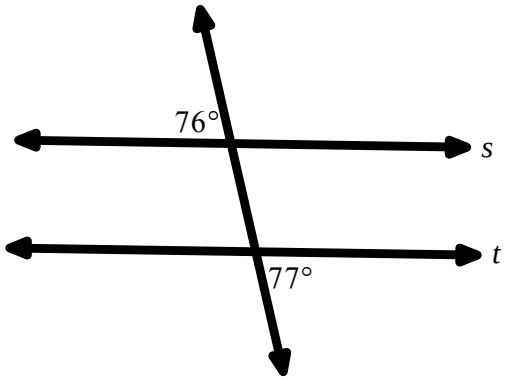


$x = 15$

6. Given line $p \parallel$ line q, solve for x.

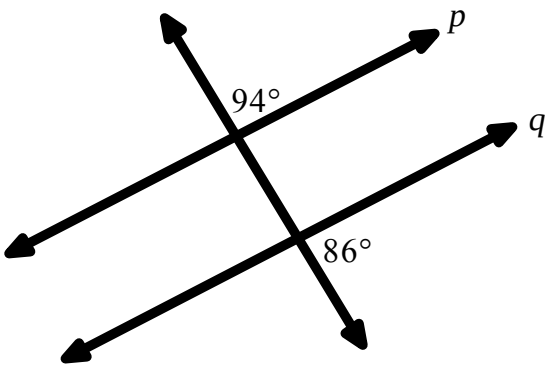


7. Determine whether lines s and t are parallel. Provide a justification for your response.

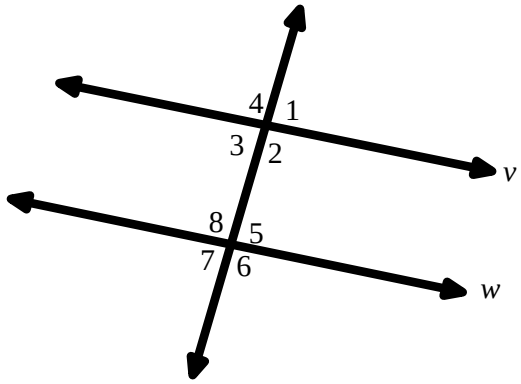


No, alternate exterior angles are not congruent

8. Determine whether lines p and q are parallel. Provide a justification for your response.



9. Given: line $v \parallel$ line w.

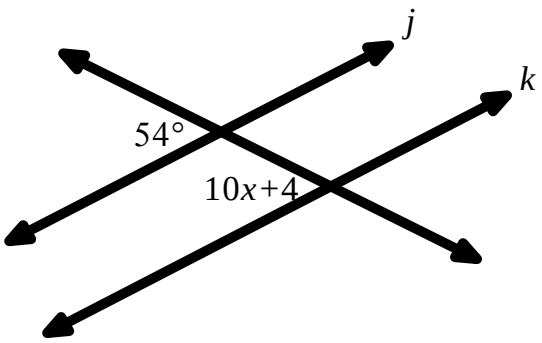


a. Which angles are congruent to $\angle 1$?

b. Which angles are congruent to $\angle 8$?

c. Name **three** pairs of supplementary angles.

10. What value of x will make line j parallel to line k?



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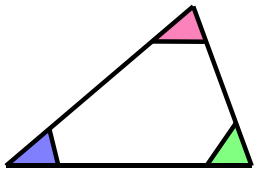
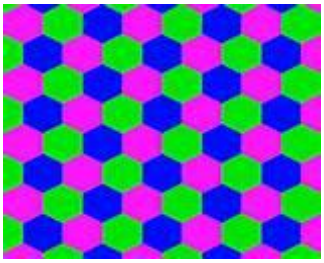
10.1d Class Activity: Tesselating Triangles

1. Take the index card that has been given to you and using a ruler draw an obtuse scalene triangle or an acute scalene triangle. Remember, in a scalene triangle, the side lengths of the triangle are all different. If the triangle has to be acute or obtuse, that means it can't have a right angle.

2. Cut out the triangle and color the angles each a different color as shown below.



3. Tessellate an 8 ½" x 11" white piece of paper with copies of your triangle. A **tessellation** is when you cover a surface with one or more geometric shapes, called tiles, with no overlaps or gaps. A tessellation by regular hexagons is shown below.



After each tessellation of your triangle, color each angle with its corresponding color.

4. What types of motion did you use to tessellate the plane with your triangle?
   Rigid motions (i.e. translations and rotations of 180˚)

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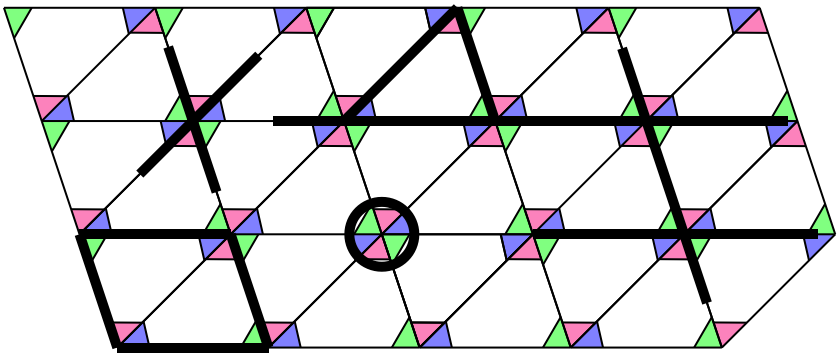
5. Look back at some of the facts we have studied so far in this section. How does your tessellation support these facts?

- **Angles that lie on the same line are supplementary and have a common vertex.**
- **Vertical angles have the same measure.**
- **If two lines are parallel and they are intersected by a transversal, then corresponding angles at the points of intersection have the same measure.**
- **Given two lines, if a third line cuts through both lines so that corresponding angles are congruent, then the two lines are parallel.**

The following tessellation is similar to the tessellation that will be created by students. In this tessellation, they can observe many of the theorems we have studied so far.



6. The following bolded bullets are additional facts we can observe in our tessellation. Use your tessellation to obverse each fact and then provide a mathematical explanation as to why each fact is true.

- **The sum of the interior angles of a triangle is a straight angle (180˚).**

- **The sum of the interior angles of a quadrilateral is 360˚.**

- **The measure of an exterior angle of a triangle is equal to the sum of the measures of the non-adjacent angles.**

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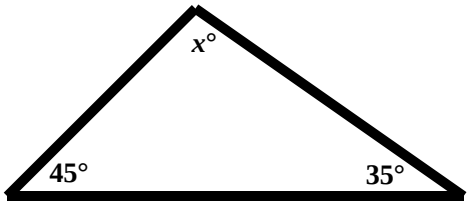
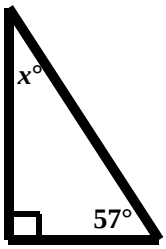
Directions: In the following problems, solve for the missing angle(s).

7.



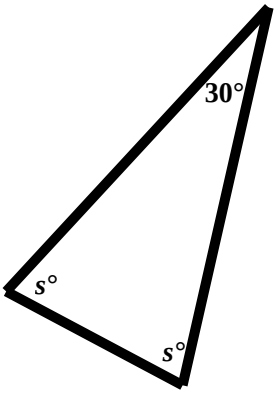
x = ____100_____

8.



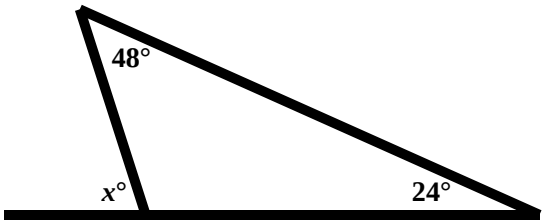
x = ___33_____

9.



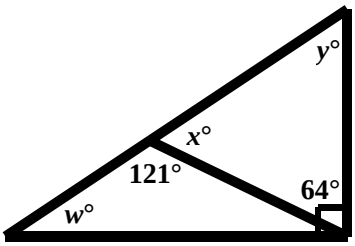
s = ___75_____

10.



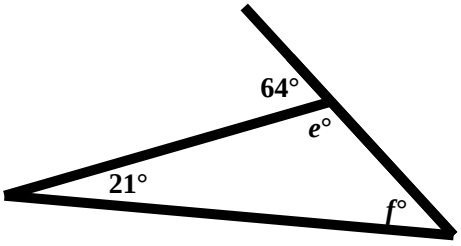
x = ____72_____

11.



w __33_____ x = __59_____ y = __57__

12.



e = __116_____ f = ___43_____

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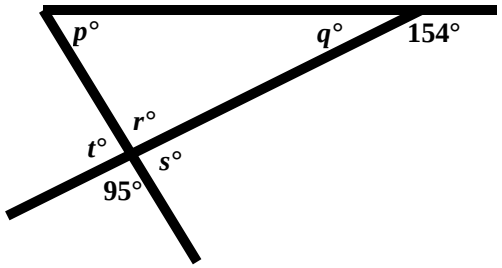
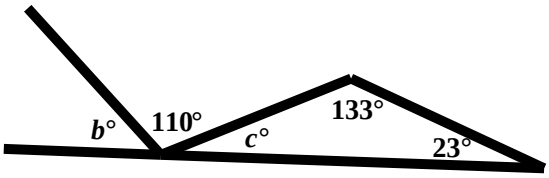
13.



p = __59____ q = __26___ r = __95_____

s = __85____ t = __85____

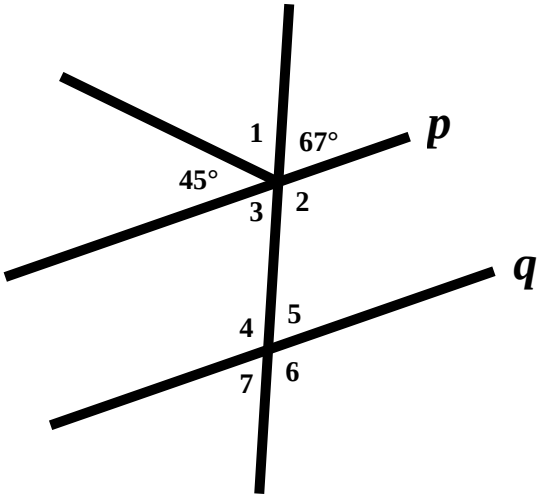
14.



b = ___46_____

c = ___24_____

15. Given: line p ∥ line q

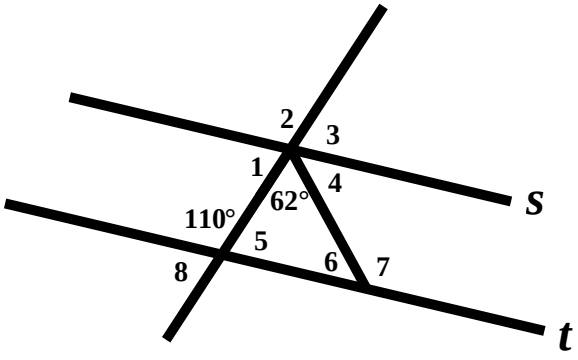


∠1 = __68___ ∠2 = _113___ ∠3 = __67

∠4 = _113___ ∠5 = _67__ ∠6 = _113_

∠7 = __67___

16. Given line s ∥ line t



∠1 = _70_ ∠2 = _110_ ∠3 = _70___

∠4 = _48_ ∠5 = __70__ ∠6 = __48__

∠7 = _132____ ∠8 = _70____

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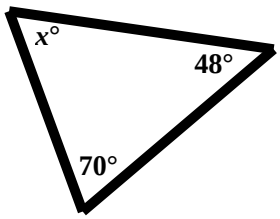
10.1d Homework: Finding Angle Measures in Triangles

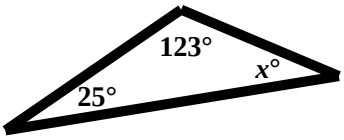
Directions: In the following problems, solve for the missing angle(s).

1.



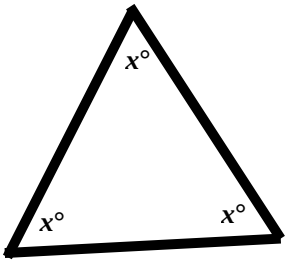
$x =$ _____

2.



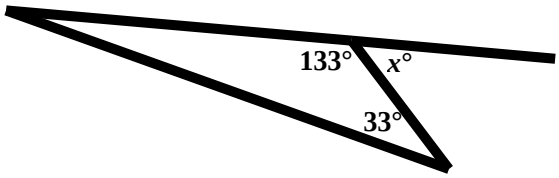
$x =$ ___32_____

3.



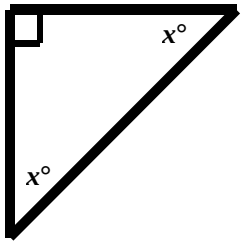
$x =$ _____

4.



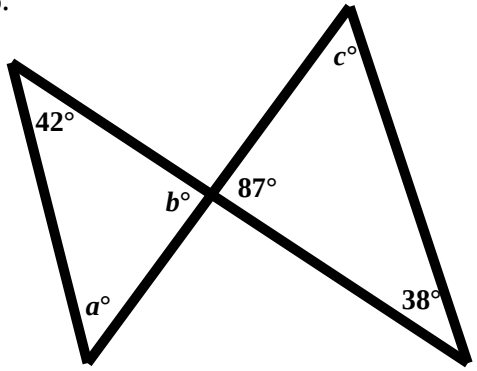
$x =$ ____47_____

5.



$x =$ _____

6.



$a =$ _____ $b =$ ____ $c =$ _____

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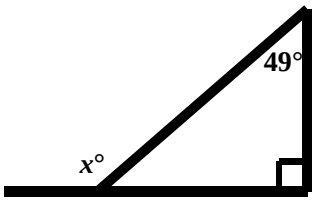
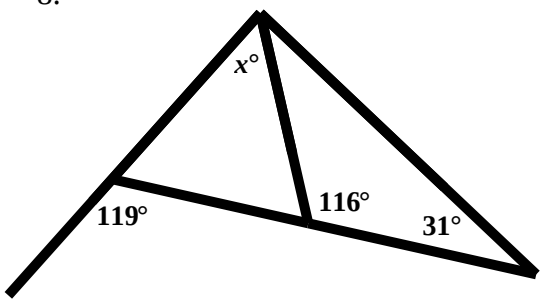
7.



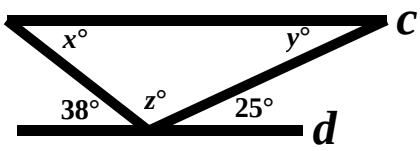
$x =$ _____

8.



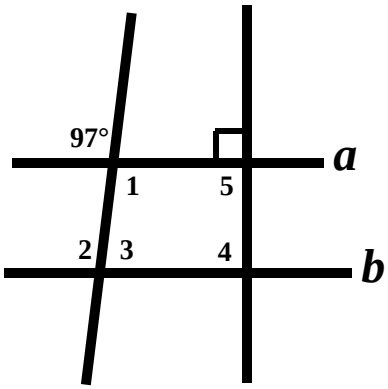
$x =$ ____55_____

9. Given: line $c \parallel$ line d



$x =$ __38___ $y =$ __25_____ $z =$ __117_____

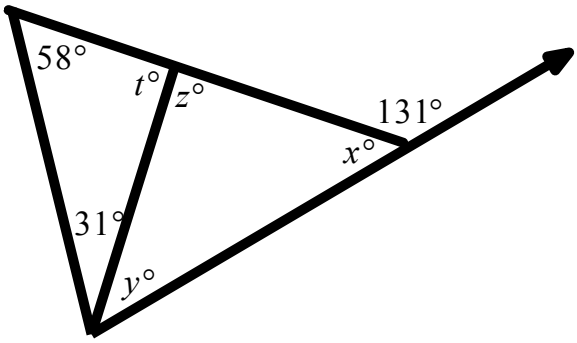
10. Given: line $a \parallel$ line b



$m\angle 1 =$ ___ $m\angle 2 =$ ____ $m\angle 3 =$ ____

$m\angle 4 =$ _____ $m\angle 5 =$ _____

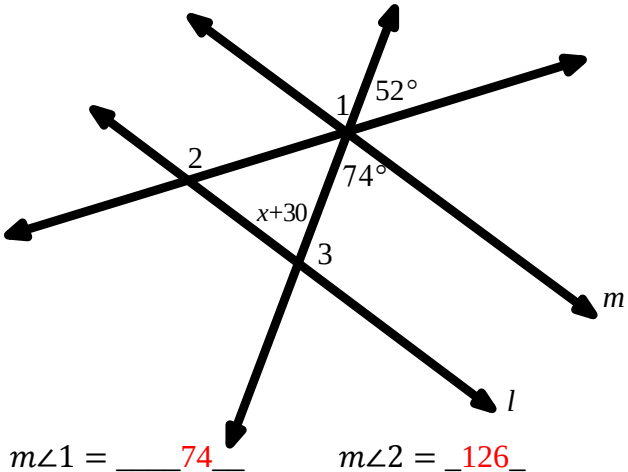
11.



$t =$_____ $y =$_____

$z =$ _____ $x =$ _____

12. Given: line $l \parallel$ line m



$m\angle 1 =$ ____74__ $m\angle 2 =$ _126_

$m\angle 3 =$ ____106___ $x =$ _44____

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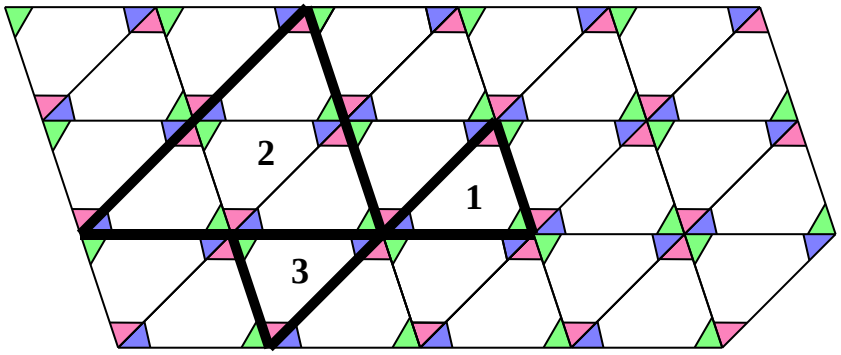
10.1e Class Activity: Similar Triangles

Revisit some of the following facts about similar triangles from Chapter 9.

- **If two triangles are similar, then the ratios of the lengths of corresponding sides are the same.**
- **If two triangles are similar, then corresponding angles have the same measure.**

Use the tessellation you made to continue your study of triangles.



1. In Chapter 9 we learned that if one figure can be carried onto another by a series of rigid motions and dilations, then the two figures are similar.
   a. In the picture above triangle 1 is similar to triangle 2. Describe the sequence of transformations that will carry triangle 1 onto triangle 2. What is the scale factor?
   Answers may vary: A translation of two to the left followed by a dilation with a scale factor of 2 and center at the lower left vertex of triangle 1 or a dilation with factor 2 with center at the vertex just one to the right of the rightmost vertex of triangle 1.

   b. In the picture above triangle 2 is similar to triangle 3. Describe the sequence of transformations that will carry triangle 2 onto triangle 3. What is the scale factor?
   Answers may vary but will likely be some combination of a rotation of 180˚ and a translation and a dilation with a scale factor of one-half.

   c. What do you notice about the corresponding angles of similar triangles?
   They are congruent

2. Can you find a triangle that is a dilation of triangle 1 with a scale factor of 3? Trace the triangle. What do you notice about the angle measures in the new triangle you created?
   Triangles will vary. The angle measures are congruent.

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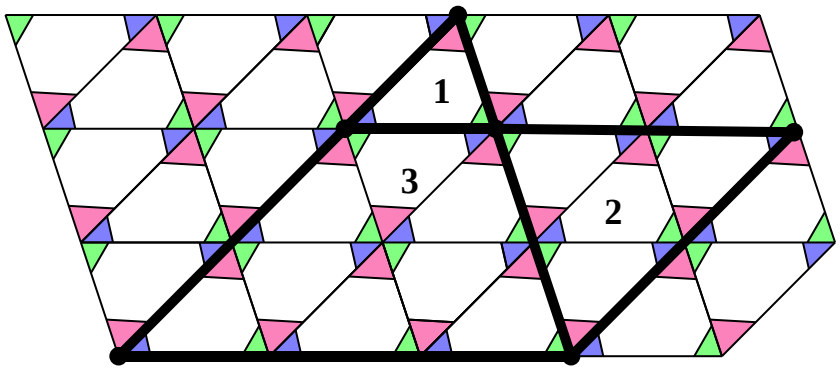
Below is another fact about similar triangles.

- **Given two triangles, if the corresponding angles have the same measure, then the triangles are similar.**

3. We will be using the tessellation you made to explore the proposition above. Find and highlight in black two triangles that have the same angle measures but are a different size.



4. What is the relationship between the triangles formed by the dark lines? Justify your answer.
   The triangles are similar. One can be mapped to the other through a series of rigid motions and dilations.

5. Find a third triangle that is a different size than the other two you highlighted. Highlight the third triangle. What is the relationship of this triangle to the other triangles? Justify your answer.
   The triangles are similar. One can be mapped to the other through a series of rigid motions and dilations.

6. Complete the following statement. It two triangles have corresponding angles that are the same measure, then one triangle can be mapped to the other using __rigid motions and dilations_____; therefore the triangles are ___similar_____.

7. Do all 3 pairs of corresponding angles have to be congruent in order to say that the two triangles are similar? What if only 2 pairs of corresponding angles are congruent? Would the triangles still be similar? Why or why not?
   If will suffice to know that two angles have the same measure. Due to the triangle sum theorem, we know that the angles sum to 180˚ so the remaining angle in both triangles will have to be the same in order to sum to 180˚.

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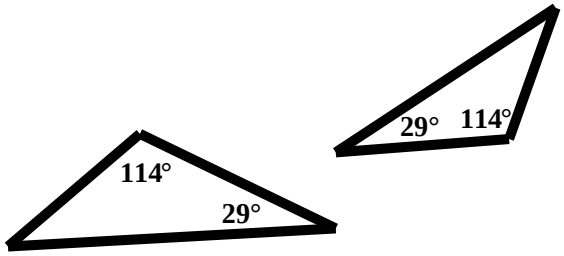
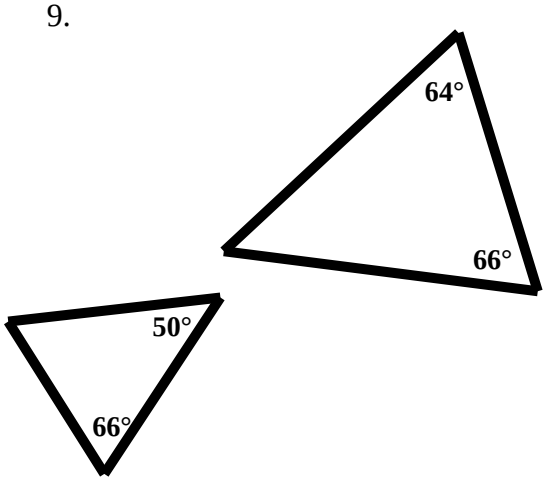
Directions: Are the triangles similar? If they are similar justify why.

8.

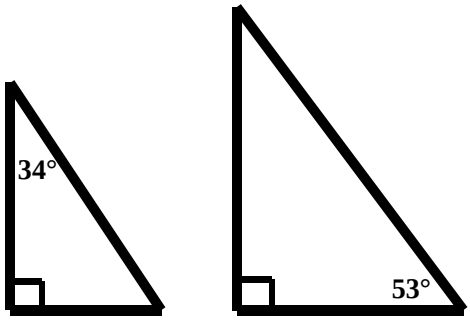


Yes, AA similarity

9.

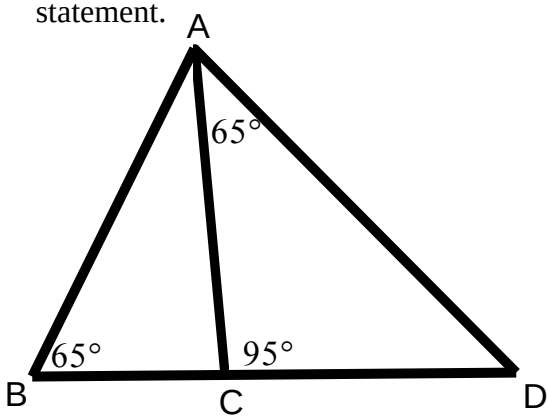


10.



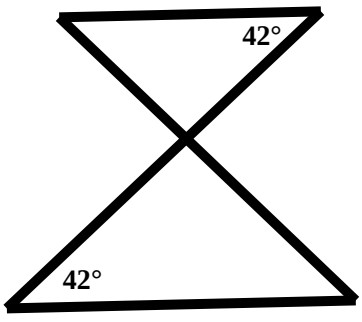
No

11. In the picture below be sure to consider all three triangles shown. If any of the triangles are similar write a similarity statement.

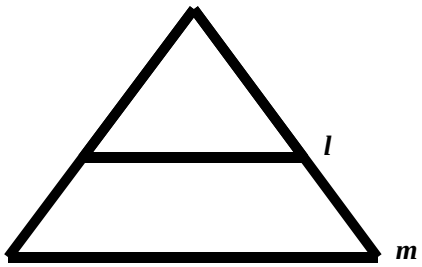


Yes, by AA similarity $\triangle ABD \sim \triangle CAD$.

12.



13. Given line $l \parallel$ line m



Yes, we know the corresponding angles at the bases of both triangles are congruent and they share the third angle.

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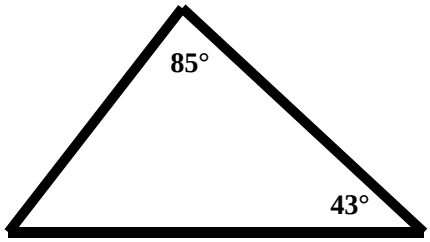
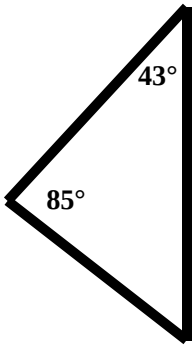
10.1e Homework: Similar Triangles

Directions: Are the triangles similar? If they are similar justify why.

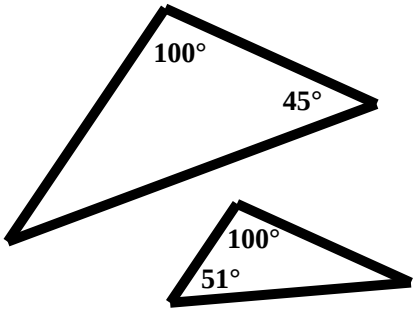
1.



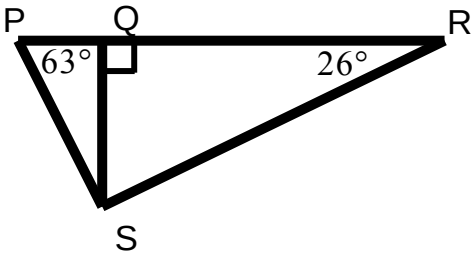


Yes, AA similarity

2.

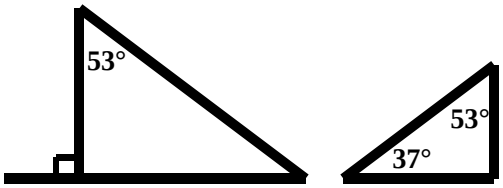


3. In the picture below be sure to consider all three triangles shown. If any of the triangles are similar write a similarity statement.



None of the triangles are similar.

4.



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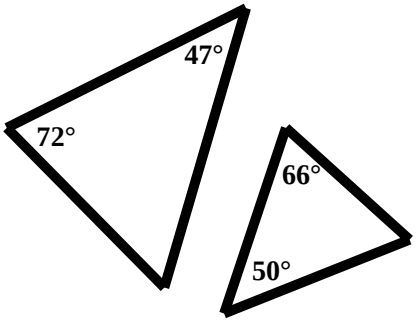
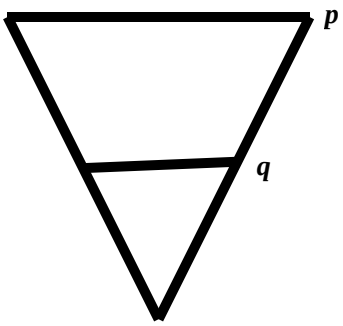
5.

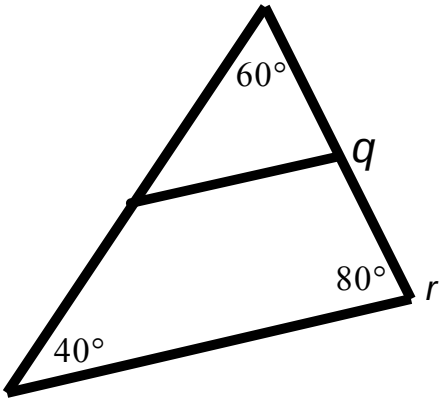


No

6. In the picture below p is not parallel to q.

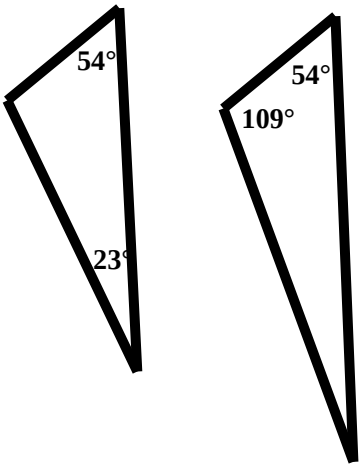


7. In the picture below line $q \parallel$ line r.



Yes, AA similarity. The corresponding angles at the base of both triangles are congruent and they share the third triangle.

8.



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10.1f Self-Assessment: Section 10.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provide on the next page.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Know that straight angles sum to 180˚ and that vertical angles are congruent. *See sample problem #1*				
2. Know that the sum of the angles in a triangle is 180˚. Understand that the measure of an exterior angle of a triangle is equal to the sum of the measures of the non-adjacent angles. Use these properties to find missing angle measures related to a triangle. *See sample problem #2*				
3. Determine the relationship between angles formed when a transversal intersects parallel lines. Use these relationships to find missing angle measures. *See sample problem #3*				
4. Determine whether two lines are parallel based on the angle measures when a transversal intersects the lines. *See sample problem #4*				
5. Understand and apply the angle-angle criterion to determine whether two triangles are similar. *See sample problem #5*				

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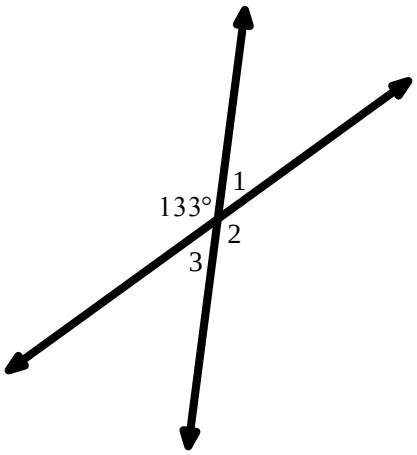
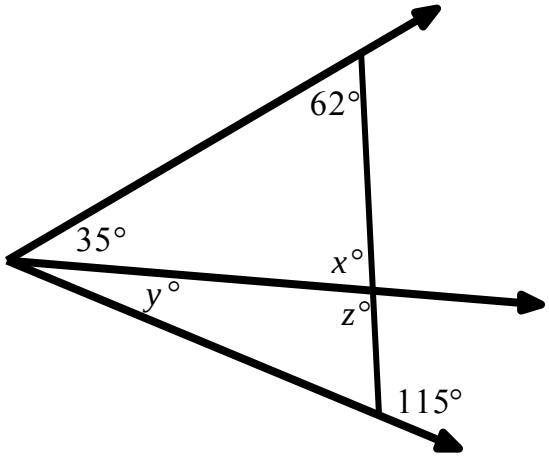
10.1f Sample Problems: Section 10.1

Sample Problem #1
Determine the measures of angles 1, 2, and 3. Justify your answers.



Sample Problem #2
In the figure to the right find the value for *x*, *y*, and *z*. Justify your answers.

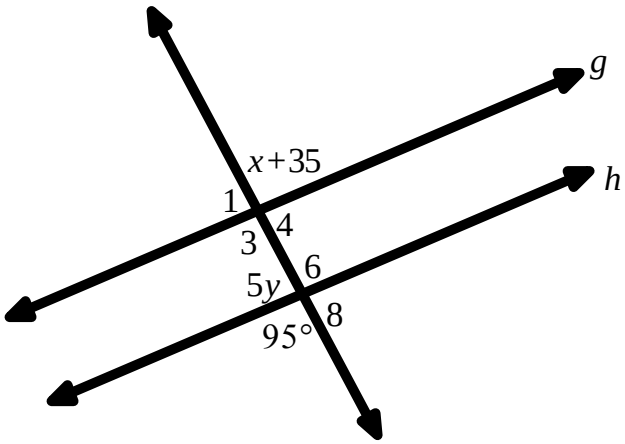


Sample Problem #3
Use the figure to the right to answer each question given that line *g* is parallel to line *h*

a. State the relationship between the following pairs of angles.
$\angle 1$ and $\angle 8$

$\angle 4$ and $\angle 8$

$\angle 3$ and $\angle 6$



b. Find the measure of the angles given below.

$m\angle 1 =$ _____ $m\angle 3 =$ _____

$m\angle 4 =$ _____ $m\angle 6 =$ _____

$m\angle 8 =$ _____

c. Find the value of *x* and *y*.
$x =$ _____ $y =$ _____

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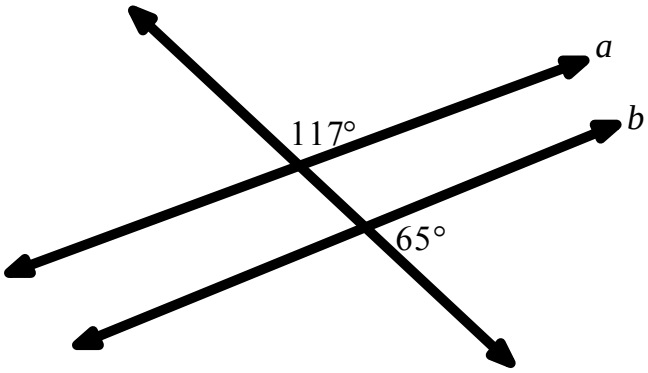
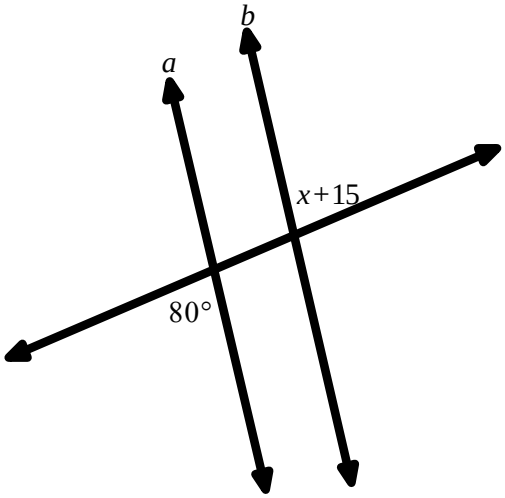
Sample Problem #4

a. Determine if line *a* || line *b*. Justify your answer.



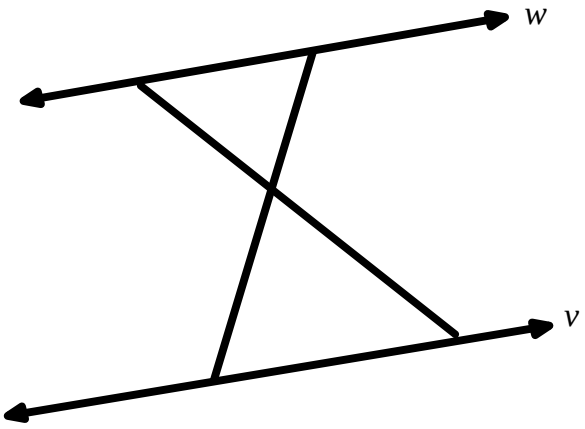
b. Find the value of *x* that will make line *a* || line *b*.



$x =$ _____

Sample Problem #5

Given that line *w* || line *v*, determine if the triangles formed below are similar. If they are similar justify why.



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Section 10.2 The Pythagorean Theorem

Section Overview:
In this section students begin to formalize many of the ideas learned in Chapter 7. They will transition from using the area of a square to find the length of a segment to generalizing the relationship between the side lengths of a right triangle, i.e. the Pythagorean Theorem, to find the length of a segment. They begin this transition by finding the areas of the squares adjacent to a given right triangle. Using these concrete examples, students describe the relationship between the sides of a right triangle. From here, students work to explain a proof by picture and subsequently a paragraph proof of the Pythagorean Theorem, starting first with a right triangle of side lengths 3, 4, and 5. Students then use a similar process to explain a proof of the Pythagorean Theorem for any right triangle with side lengths *a*, *b*, and *c* where *a* and *b* are the legs of the right triangle and *c* is the hypotenuse. Students arrive at the Pythagorean Theorem: $a^2 + b^2 = c^2$ where *a* and *b* are the legs of the right triangle and *c* is the hypotenuse. Throughout the section, students are connecting the Pythagorean Theorem to work done in Chapter 7. Next, students use the Pythagorean Theorem to solve for the missing side length of a right triangle given the measurements of the other two sides, relying on skills learned in Chapters 7 and 8. This is followed by explaining a proof of the converse of the Pythagorean Theorem: For a triangle with side lengths *a*, *b*, and *c* if $a^2 + b^2 = c^2$, then the triangle is a right triangle. Using this theorem, students determine whether three given side lengths form a right triangle. Throughout this section emphasis is placed on creating good arguments and explanation. Students are not formally proving the Pythagorean Theorem and its converse but explaining why the theorems are true by learning how to provide sufficient explanations and arguments. In addition students are providing evidence and warrants for claims that they make. At the end of the section is an optional exploration on Pythagorean triples.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Know that in a right triangle $a^2 + b^2 = c^2$, where *a* and *b* are the legs of the right triangle and *c* is the hypotenuse.
2. Understand and explain a proof of the Pythagorean Theorem.
3. Use the Pythagorean Theorem to solve for the missing side length of a right triangle given the measurements of the other two sides.
4. Understand and explain a proof of the converse of the Pythagorean Theorem. That is, for a triangle with side lengths *a*, *b*, and *c* if $a^2 + b^2 = c^2$, then the triangle is a right triangle.
5. Use the converse of the Pythagorean Theorem to determine whether three given measurements are possible side lengths of a right triangle.

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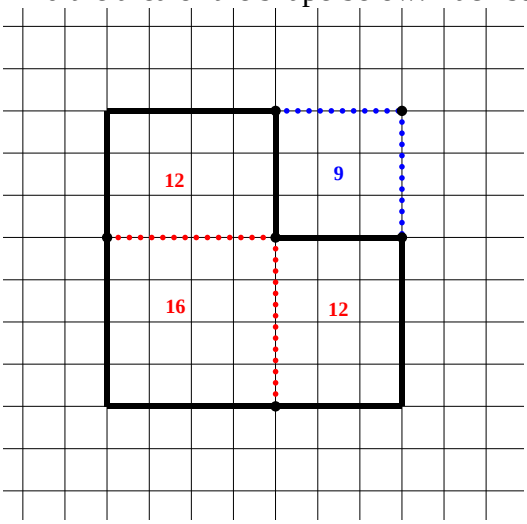
10.2a Class Activity: A Proof of the Pythagorean Theorem

1. Find the area of the shape below. Each square on the grid has a side length of 1 unit.

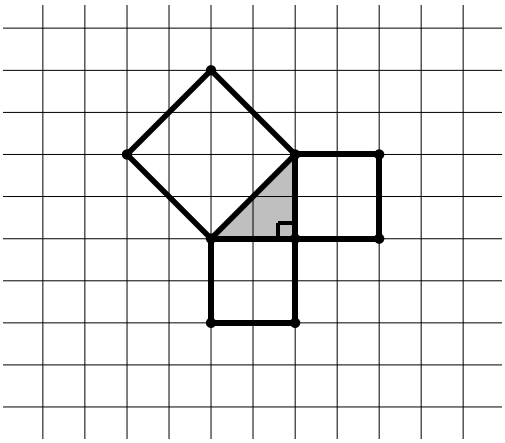


The area of the shape is 40 square units. Students may find the area by breaking the shape into smaller shapes and adding up the areas (shown in red) or finding the area of the large square (49) and subtracting out the area of the small square (9) (shown in blue).

In numbers 2 and 3, a right triangle is shown in gray. The shorter sides of a right triangle are referred to as **legs**. The longer side of the right triangle (the side opposite of the right angle) is called the **hypotenuse**.

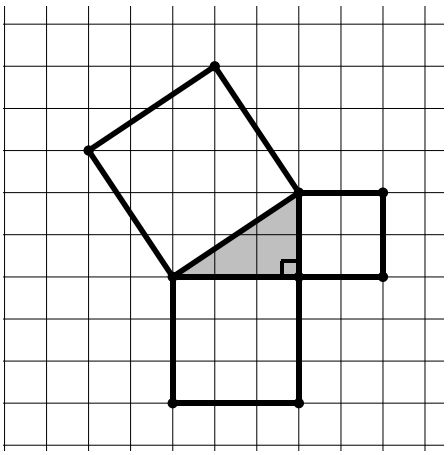
Directions: Squares have been drawn adjacent to the sides of the right triangle. Find the area of each of the squares. Assuming each square on the grid has a side length of 1 unit. Write the areas inside each of the squares. Students can count squares, find the area using the subtraction method for the tilted squares, or use the area formula learned in Chapter 7 ($A = s^2$).

2.



Small squares both have an area of 4. Large square has an area of 8.

3.



Small squares have areas of 9 and 4. Large square has an area of 13.

4. What do you notice about the relationship between the areas of the squares formed adjacent to the legs of a right triangle?

At this point, students may notice that the sum of the areas of the squares adjacent to the legs of the right triangle equals the area of the square adjacent to the hypotenuse of the triangle.

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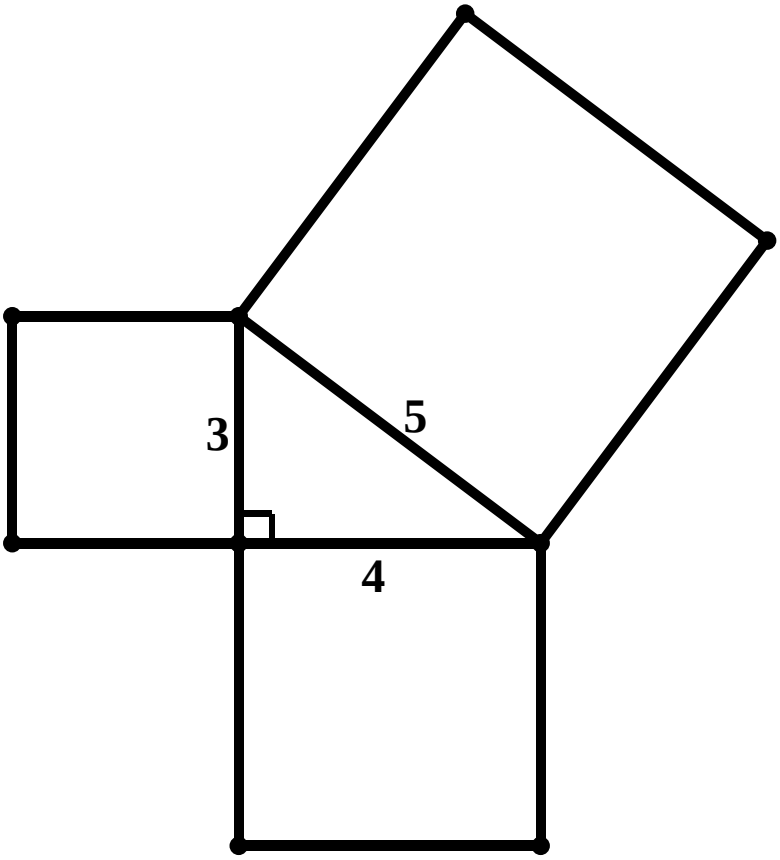
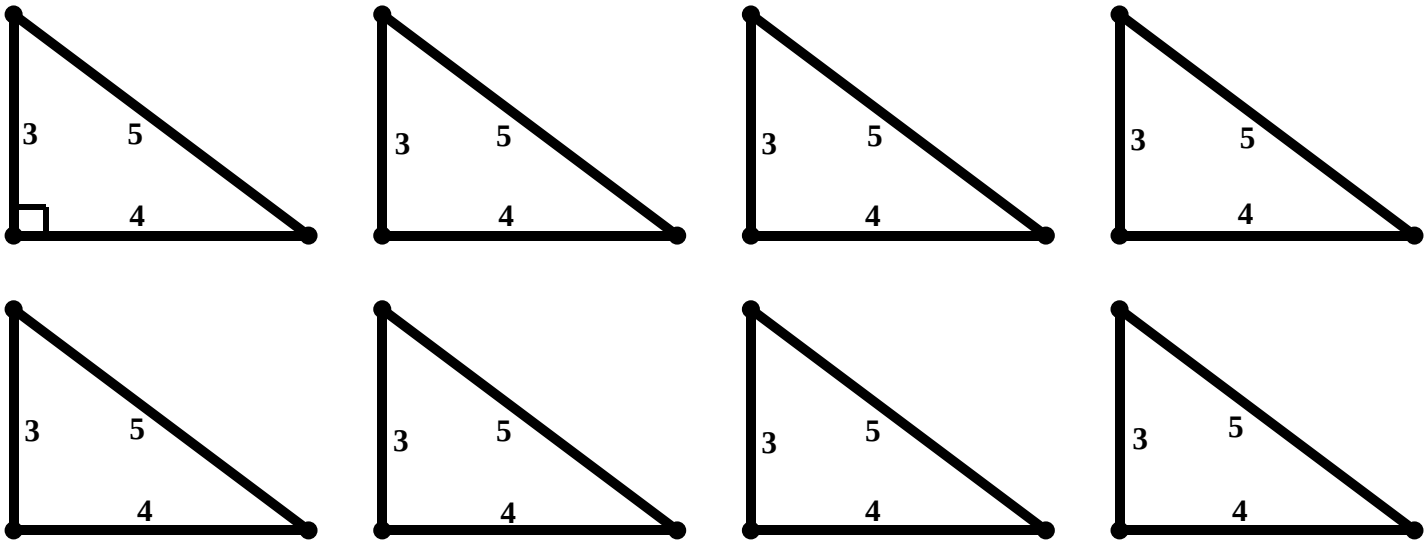
5. Below is a right triangle with side lengths 3, 4, and 5. Squares have been drawn adjacent to the sides of the right triangle.



a. Find the area of each of the squares. Write the area inside each of the squares. Then, cut out the three squares very carefully. Students write areas of 9, 16, and 25 inside each square above.

b. Below are 8 copies of the original right triangle. Cut out the 8 triangles very carefully.



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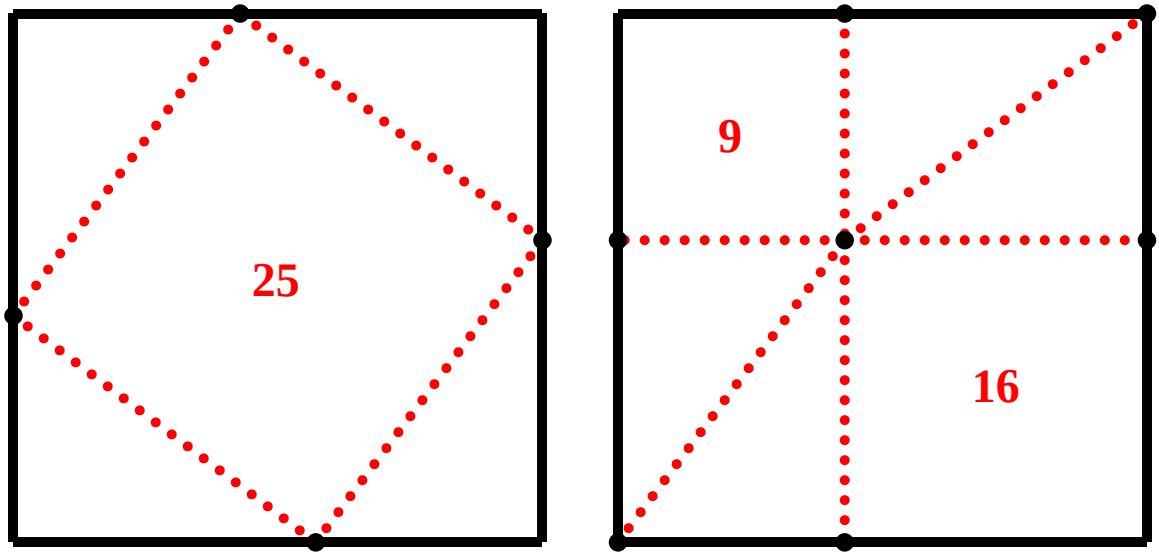
c. Below are two **congruent squares**. Since the squares are congruent, we know that their sides have the same length and subsequently they have the same area. Use your square with an area of 25 and four of the triangles from the previous page to cover one of the squares. Use your squares with areas 9 and 16 and four of the triangles from the previous page to cover the other square. Tape the pieces into place. Possible configurations shown.



Make sure that students are clear that the squares shown above (25, 9, and 16) are squares formed adjacent to the sides of the right triangle. They can see the original 3, 4, 5 right triangle in the puzzle above as well as the squares adjacent to the sides of the right triangle. In addition, once they have the puzzle pieces in place, have them find the area of the large square to explain that the areas of the squares shown above are congruent.

d. Use the large squares in part c) to explain the relationship you discovered in #2 – 4 between the squares formed adjacent to the sides of a right triangle.

Logic:

If the areas of the large squares are the same, I don't need to worry about the area covered by the triangles because the triangles are covering the same amount of area on both large squares. That leaves me with the large square adjacent to the right triangle which is equal in area to the squares adjacent to the two smaller sides of the right triangle.

4 triangles and the large square formed adjacent to the right triangle cover the same area as 4 triangles and the small squares adjacent to the right triangle.

Each triangle has an area of 6 square units. That means, on each large square, the triangles have an area of 24. I know that the large square has an area of 49. If I take the difference between 49 and 24, I am left with 25. On the square on the left, our large square formed adjacent to the right triangle has an area of 25 and on the square on the right, our two smaller squares formed adjacent to the right triangle have areas that sum to 25.

Equation:

Area of Square 1 = Area of Square 2

5 squared (25) + area of 4 triangles = 3 squared (9) + 4 squared (16) + area of 4 triangles

25 = 9 + 16

25 = 25

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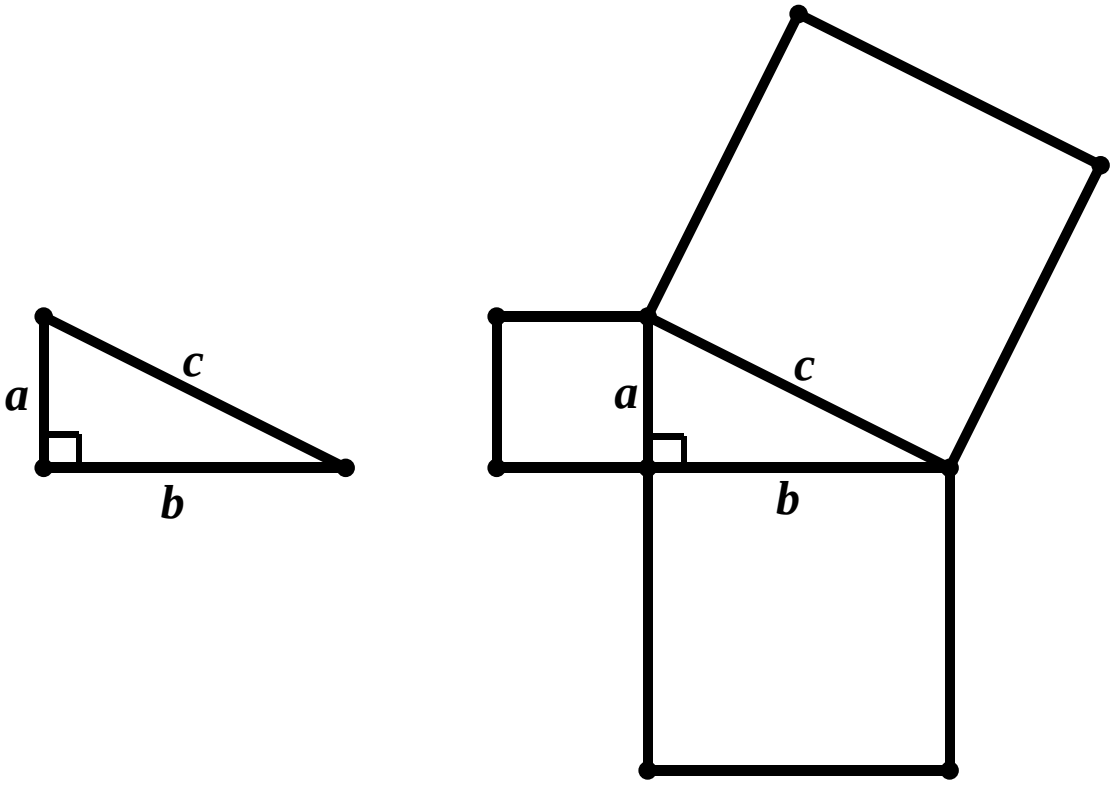
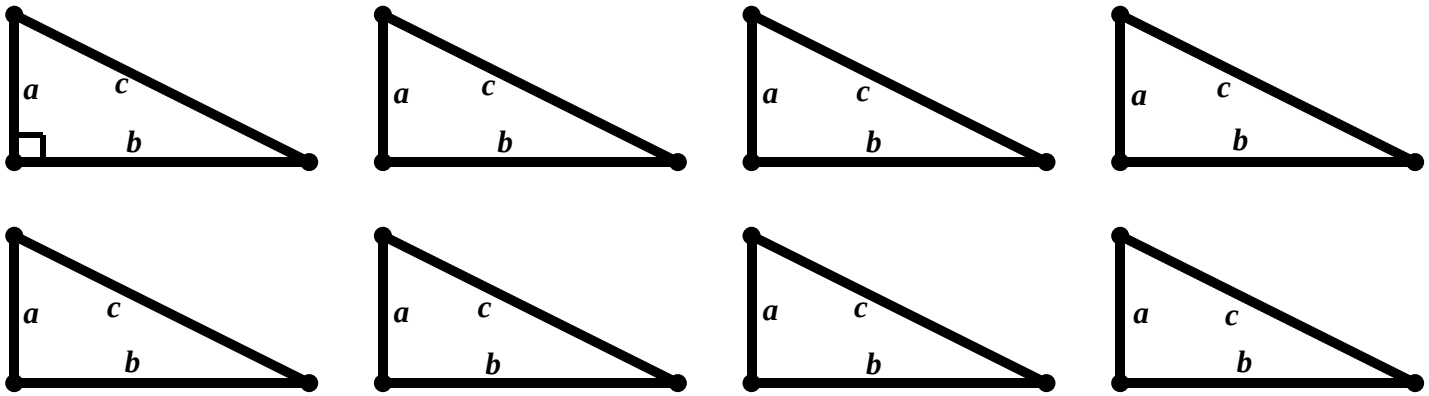
6. In the previous problems, we saw that for specific triangles **the sum of the areas of the squares along the legs of the right triangle equals the area of the square along the hypotenuse of the triangle** by looking at several examples. Now, we want to show that this relationship holds true for ***any*** right triangle.

   Suppose you have a right triangle with any side lengths *a*, *b*, and *c* where *a* and *b* are the legs of the triangle and *c* is the hypotenuse of the right triangle as shown below. The squares have been drawn along the sides of the right triangle. Our goal is to show that $a^2 + b^2 = c^2$ is always true.



   a. Find the area of each of the squares adjacent to the sides of the right triangle. Write the areas inside each square.

   b. Cut out the squares formed on the sides of the triangle above as well as the 8 copies of the triangle with side lengths *a*, *b*, and *c* below.



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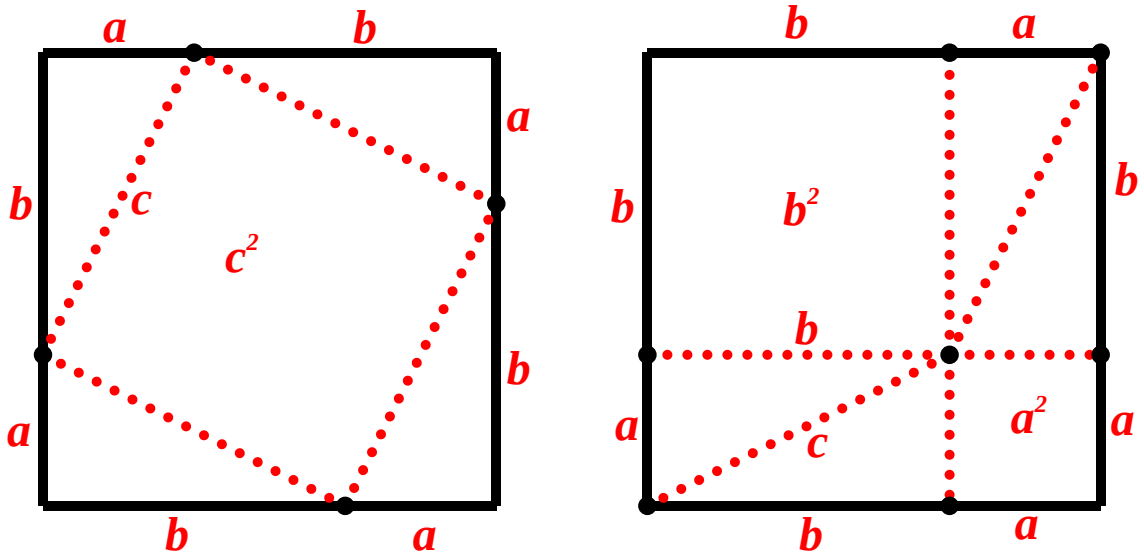
c. Arrange the 3 squares and 8 triangles to cover the 2 squares shown below.



d. Using the picture above, show that **the sum of the areas of the squares adjacent to the legs of the right triangle equals the area of the square adjacent to the hypotenuse of the triangle for *any*** right triangle.

Area of Square 1 = Area of Square 2 (congruent by rigid motion)
area of 4 triangles + c^2 = area of 4 triangles + $a^2 + b^2$
$c^2 = a^2 + b^2$

Area of Square 1 = Area of Square 2 (both squares have side lengths of $(a + b)$)
area of 4 triangles + c^2 = area of 4 triangles + $a^2 + b^2$
$4\left(\frac{1}{2}ab\right) + c^2 = 4\left(\frac{1}{2}ab\right) + a^2 + b^2$
$2ab + c^2 = 2ab + a^2 + b^2$
$c^2 = a^2 + b^2$

e. Conventionally, the leg lengths of a right triangle are denoted using the variables a and b and the hypotenuse of a right triangle is denoted using the variable c. State the relationship between the side lengths of a right triangle using the words **legs** and **hypotenuse**.
The sum of the areas of the squares adjacent to the legs of a right triangle equals the area of the square adjacent to the hypotenuse of a right triangle.

f. Write an **equation** that shows the relationship between the side lengths of a right triangle using a and b for the lengths of the legs and c for the length of the hypotenuse.
The Pythagorean Theorem: $a^2 + b^2 = c^2$ for a right triangle whose leg lengths are a and b and whose hypotenuse is of length c.

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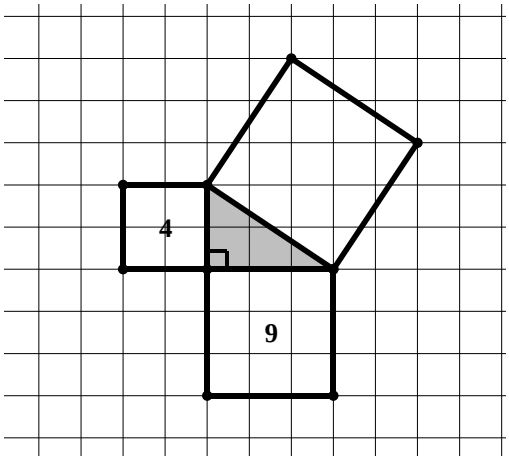
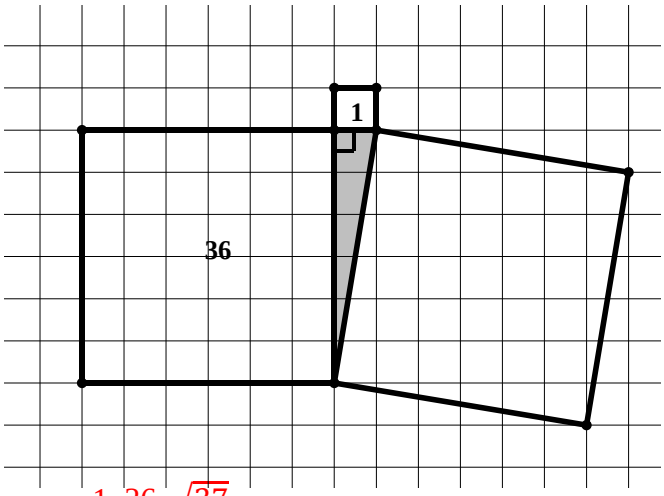
Directions: In each of the problems below, a right triangle is shown in gray. The squares along each of the three sides of the triangles have been drawn. The area of two of the squares is given. Determine the area of the third square. Write your answer in the square. Also find the side length of each square, write the sides lengths below each picture.

1. 13



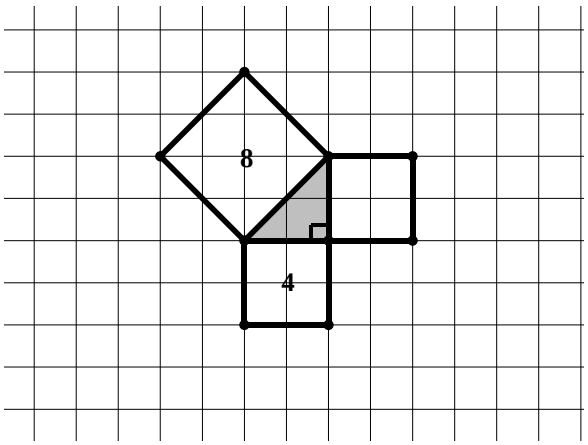
2, 3, $\sqrt{13}$

2. 37



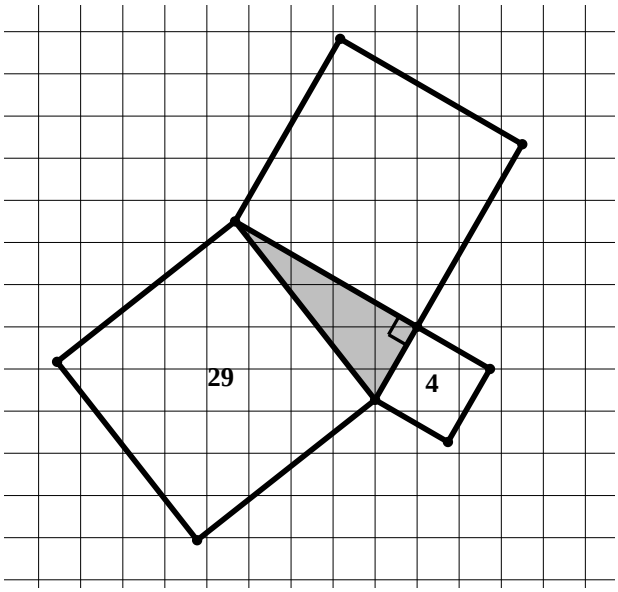
1, 36, $\sqrt{37}$

3. 4



2, 2, $\sqrt{8} = 2\sqrt{2}$

4. 25



2, 5, $\sqrt{29}$

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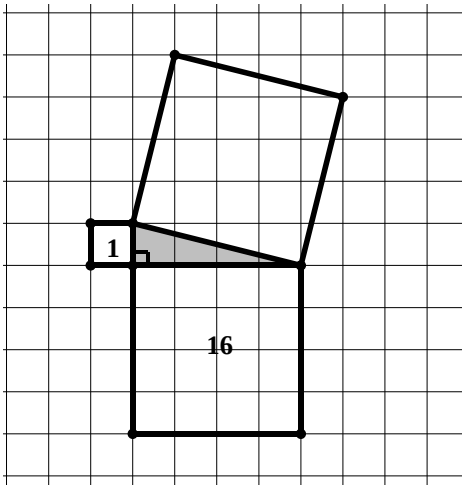
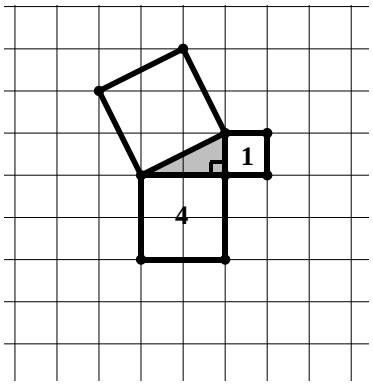
10.2a Homework: A Proof of the Pythagorean Theorem

Directions: In each of the problems below, a right triangle is shown in gray. The squares along each of the three sides of the triangles have been drawn. The area of two of the squares is given. Determine the area of the third square. Write your answer in the square.

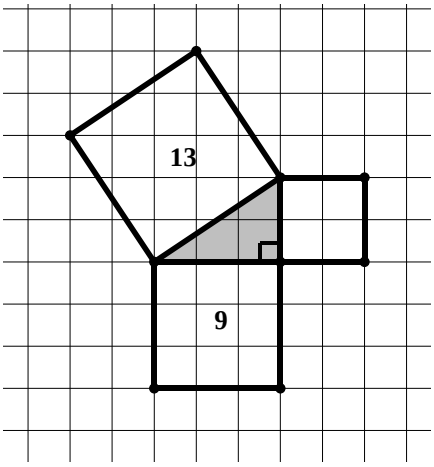
1. 17



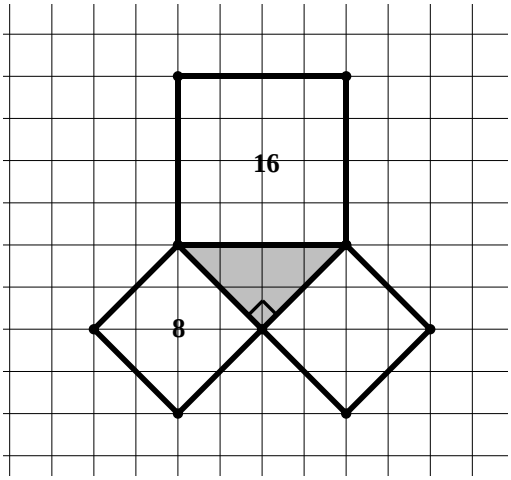
2.



3. 4



4.



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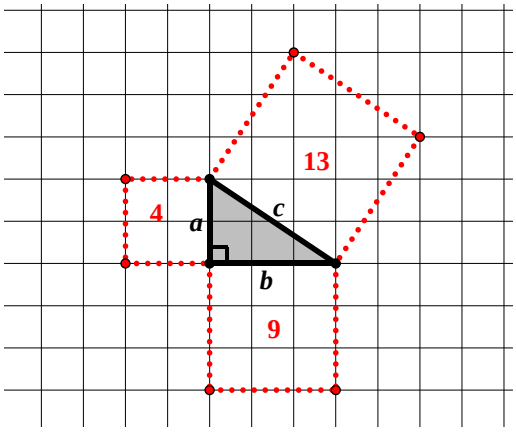
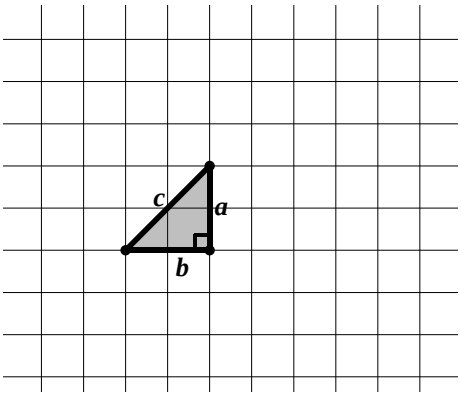
Directions: For each of the following problems, the gray triangle is a right triangle. Draw the squares adjacent to each of the three sides of the triangles. Find the area of each square and write the area in each square. Then, find the side lengths *a*, *b*, *c* of each triangle.

5.



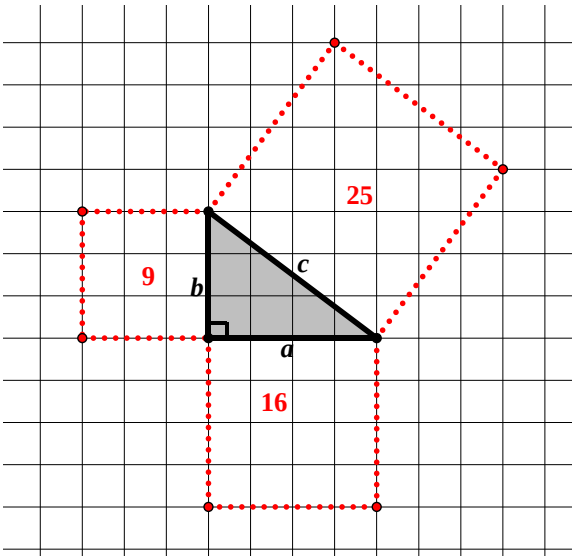
a = ___2____ b = ___3____ c = __$\sqrt{13}$____

6.



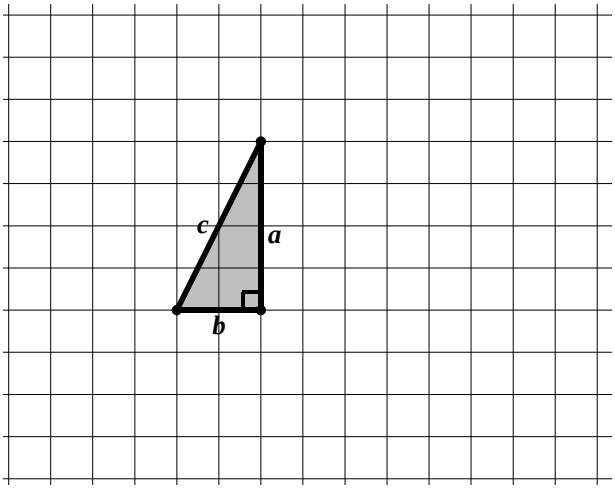
a = _____ b = _____ c = _____

7.



a = ___4____ b = ___3____ c = ___5____

8.



a = _____ b = _____ c = _____

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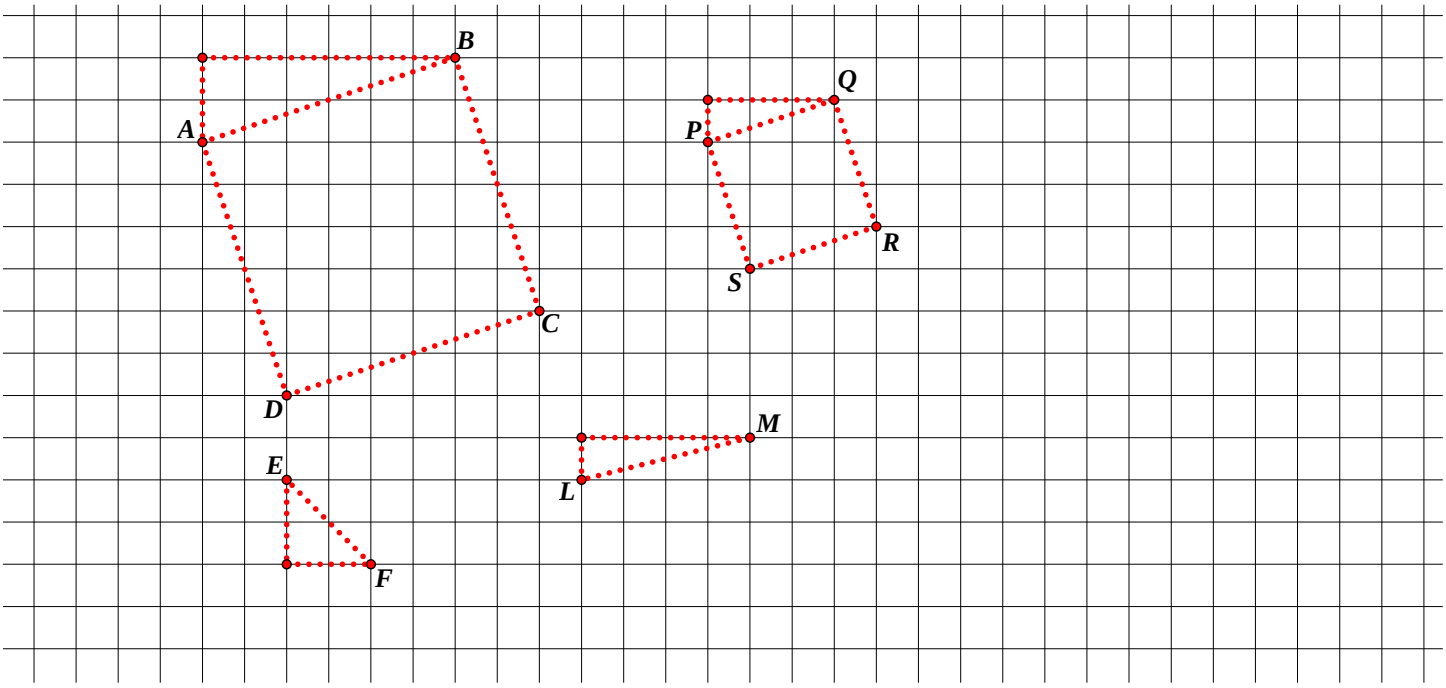
10.2b Class Activity: The Pythagorean Theorem and Tilted Squares

There are many questions you can ask to get them thinking about how to construct the objects below if they are struggling. For example, on question a) you may ask, if I need a square with an area of 40, can I make two smaller squares that add to 40? It will obviously be easier for students to choose perfect squares. In this case, students may choose 36 and 4. Elicit ideas about the Pythagorean Theorem – if the 36 and 4 were squares adjacent to the sides of a right triangle, what would be the measures of the sides of the triangle? Would these be the legs of the right triangle or the hypotenuse?

Alternatively, you may ask, if I need a square with an area of 40 square units, what does the side length of the square need to be. $\sqrt{40}$. Can you construct a right triangle that has a side length of $\sqrt{40}$ or $2\sqrt{10}$? What about using ideas about scaling, can you create a right triangle with a side length of $\sqrt{10}$ and then double all of the lengths of the sides? You can ask similar probing questions for b – c.

1. On the grids below, construct the following and clearly label each object:
   a. Square $ABCD$ that has an area of 40 square units
   b. Square $PQRS$ that has an area of 10 square units
   c. $\overline{EF}$ that has a length of $\sqrt{8}$ units
   d. $\overline{LM}$ that has a length of $\sqrt{17}$ units



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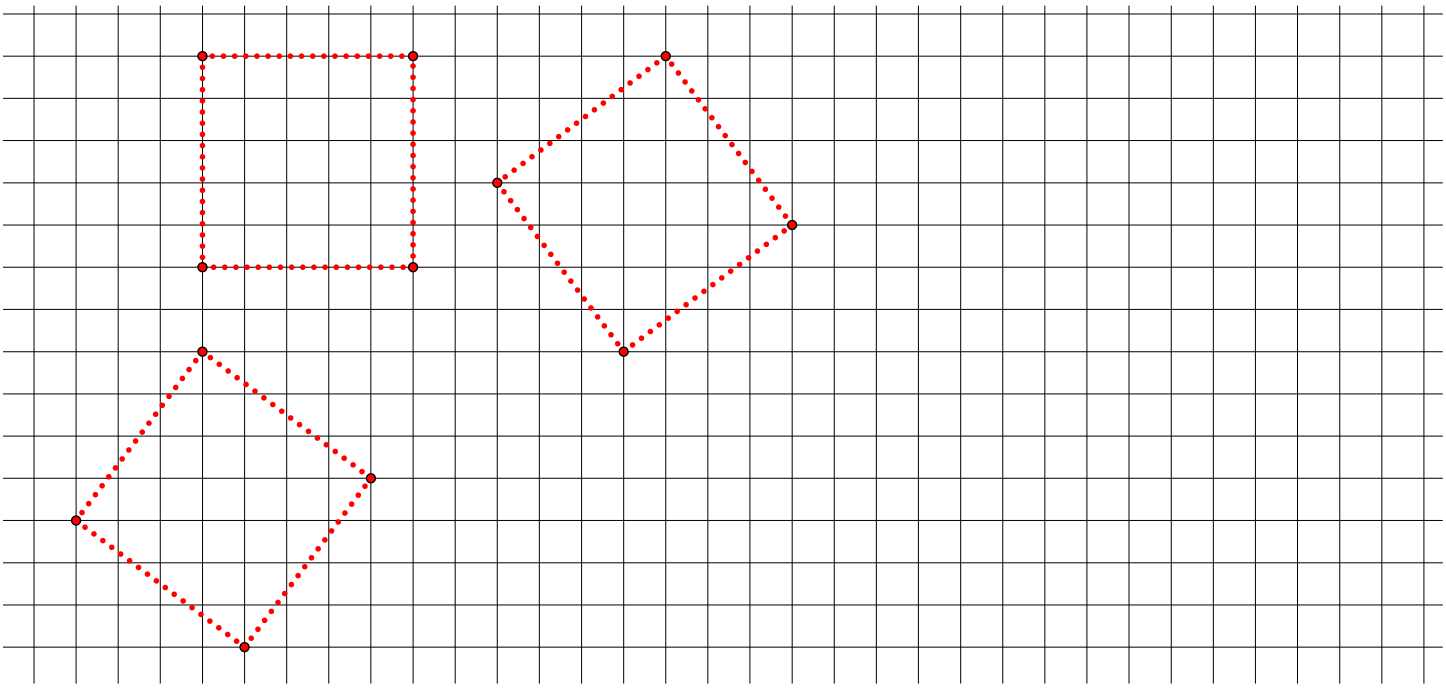
2. Draw as many different squares as you can with an area of 25 square units on the grids below. In this problem, different means that the squares are not tilted the same way. Students should think about their 3, 4, 5 right triangles. If they need a square with an area of 25, they need a side length of 5. This can be constructed with 3, 4, 5 triangles. Changing the rise and run of the right triangles will change the tilt of the square. Again, you can ask probing questions like, "If I want a square with an area of 25, what does the side length need to be? How can I create different segments with lengths of 5?"



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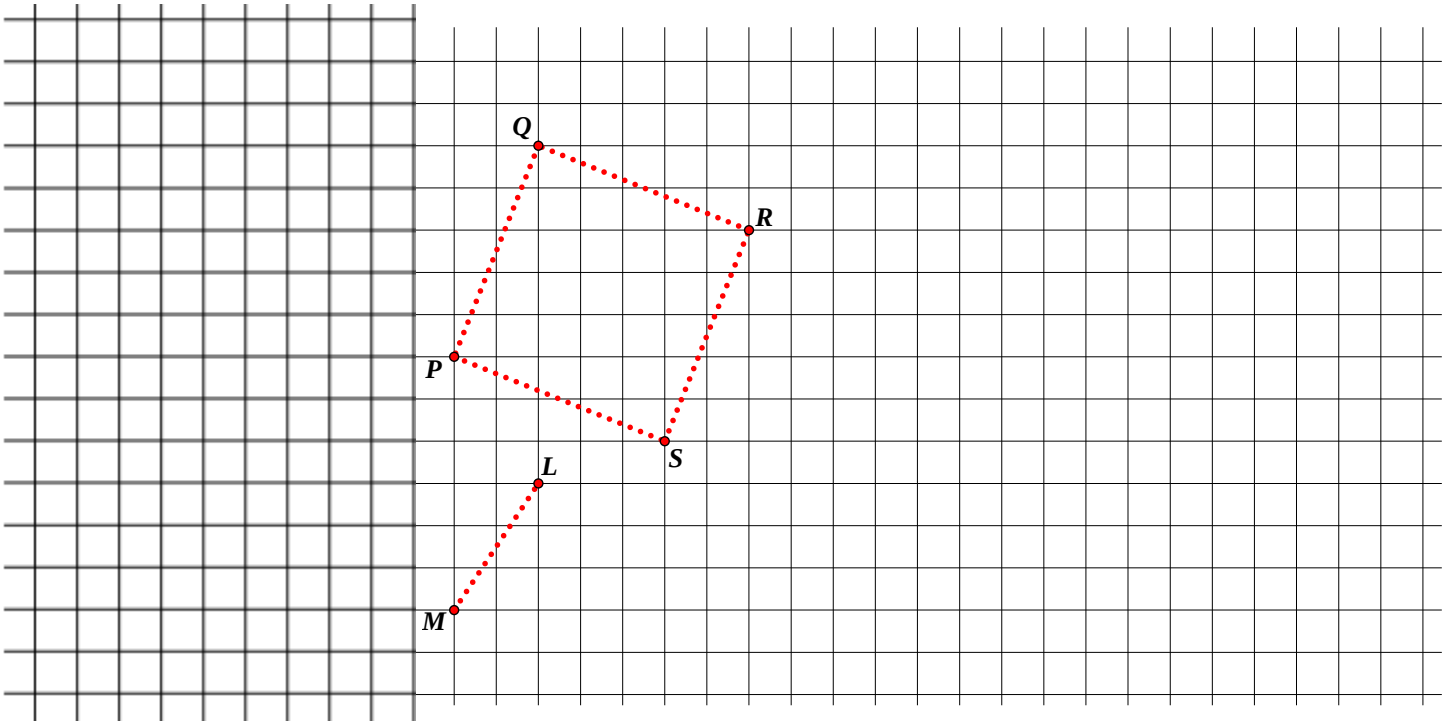
10.2b Homework: The Pythagorean Theorem and Tilted Squares

1. On the grids below, construct the following and clearly label each object:
   a. Square $ABCD$ that has an area of 5 square units
   b. Square $PQRS$ that has an area of 29 square units
   c. $\overline{EF}$ that has a length of $\sqrt{18}$ units
   d. $\overline{LM}$ that has a length of $\sqrt{13}$ units



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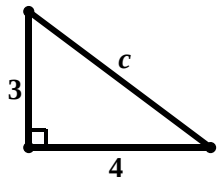
10.2c Class Activity: The Pythagorean Theorem and Unknown Side Lengths

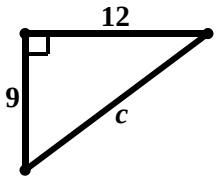
Directions: Find the length of the hypotenuse of each right triangle shown below using the Pythagorean Theorem. Leave your answer in simplest radical form.

1.



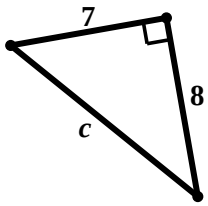
c = __5___

2.



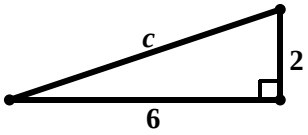
c = _____

3.



c = _____

4.



$6^2 + 2^2 = c^2$

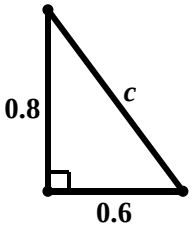
$36 + 4 = c^2$

$40 = c^2$

$\sqrt{40} = c \Rightarrow \sqrt{4 \cdot 10} = \sqrt{2 \cdot 2 \cdot 10} = 2\sqrt{10}$

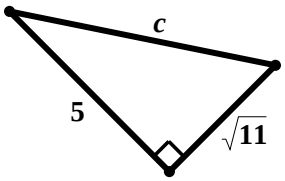
c = __$2\sqrt{10}$____

5.



c = _____

6.



$5^2 + \sqrt{11}^2 = c^2$

$25 + 11 = c^2$

$36 = c^2$

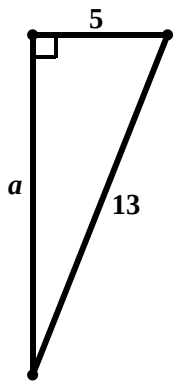
$\sqrt{36} = c$

$6 = c$

c = __6___

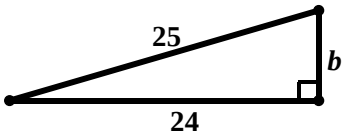
Directions: Find the length of the leg of each right triangle shown below using the Pythagorean Theorem. Leave your answer in simplest radical form.

7.



a = _____

8.



b = __7___

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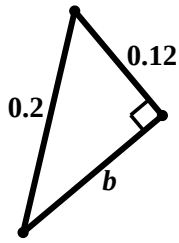
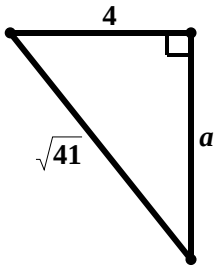
9.



b = _____

10.



$a^2 + 4^2 = \sqrt{41}^2$

$a^2 + 16 = 41$

$a^2 = 41 - 16$

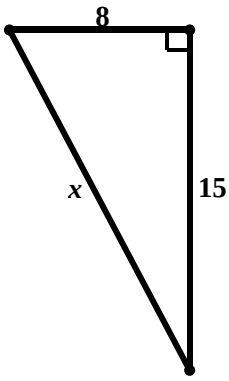
$a^2 = 25$

$a = 5$

a = _5_____

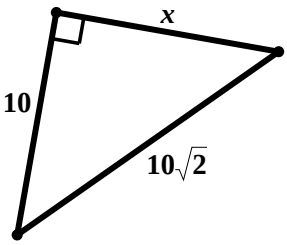
Directions: Find the value of x using the Pythagorean Theorem. Leave your answer in simplest radical form.

11.



x = _____

12.



$x^2 + 10^2 = \left(10\sqrt{2}\right)^2$

$x^2 + 100 = 100 \cdot 2$

$x^2 + 100 = 200$

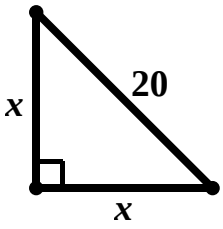
$x^2 = 200 - 100$

$x^2 = 100$

$x = 10$

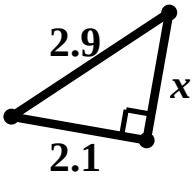
x = _10____

13.



x = _$10\sqrt{2}$____

14.



x = _____

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10.2c Homework: The Pythagorean Theorem and Unknown Side Lengths

Directions: Two side lengths of a right triangle have been given. Solve for the missing side length if a and b are leg lengths and c is the length of the hypotenuse. Leave your answer in simplest radical form.

1. $a = 16$, $b = 30$, $c = ?$

2. $a = 2$, $b = 2$, $c = ?$

3. $a = 40$, $b = ?$, $c = 50$

   30

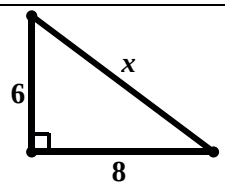
4. $a = ?$, $b = 4\sqrt{3}$, $c = 8$

   $a^2 + \left(4\sqrt{3}\right)^2 = 8^2 \Rightarrow a^2 + 16 \cdot 3 = 64$

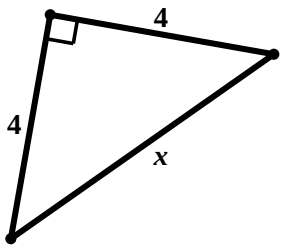
   $a^2 + 48 = 64 \Rightarrow a^2 = 64 - 48$

   $a^2 = 16 \Rightarrow a = 4$

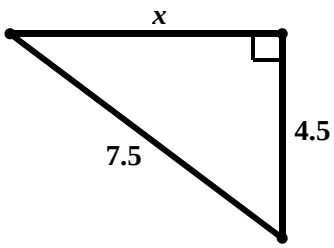
Directions: Find the value of x using the Pythagorean Theorem. Leave your answer in simplest radical form.

5. 

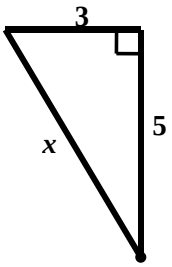
$x =$ _____

6. 

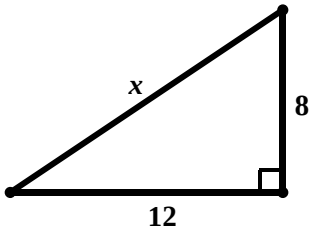
$x =$ $4\sqrt{2}$

7. 

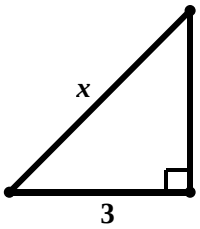
$x =$ _____

8. 

$x =$ $\sqrt{34}$

9. 

$x =$ _____

10. 

$3^2 + 3^2 = x^2$

$9 + 9 = x^2$

$18 = x^2 =$

$\sqrt{18} = x^2$

$\sqrt{2 \cdot 9} \Rightarrow \sqrt{2 \cdot 3 \cdot 3} = x^2$

$3\sqrt{2} = x$

$x =$ $3\sqrt{2}$

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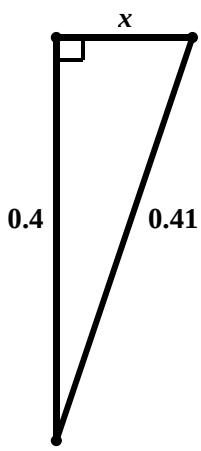
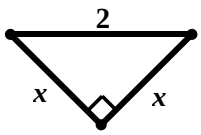
11.



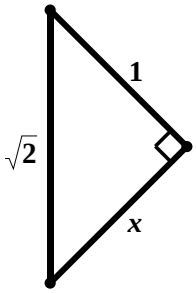
x = _____

12.



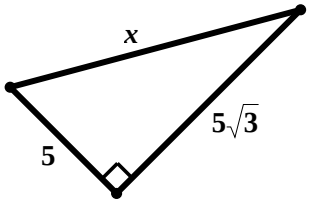
x = __$\sqrt{2}$_____

13.



x = _____

14.



$5^2 + \left(5\sqrt{3}\right)^2 = x^2$

$25 + 25 \cdot 3 = x^2$

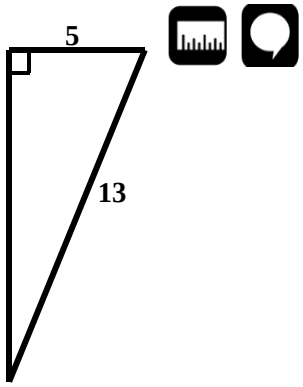
$25 + 75 = x^2$

$100 = x^2$

$10 = x$

x = ___10____

15. **Find, Fix, and Justify:** Megan was asked to solve for the unknown side length in the triangle below. Her work is shown below. She made a mistake when solving. Explain the mistake she made and then solve the problem correctly.



Megan's Solution:

$a^2 + b^2 = c^2$

$5^2 + 13^2 = c^2$

$25 + 169 = c^2$

$194 = c^2$

$\sqrt{194} = c$

Correct Solution:

$a^2 + b^2 = c^2$

$5^2 + b^2 = 13^2$

$25 + b^2 = 169$

$b^2 = 144$

$b = 12$

Explain Mistake:

a and *b* are legs of a right triangle. Megan substituted in the length of the hypotenuse for one of the legs.

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16. **Find, Fix, and Justify:** Raphael was asked to solve for the length of the hypotenuse in a right traingle with legs that have side lengths of 4 and 5. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.

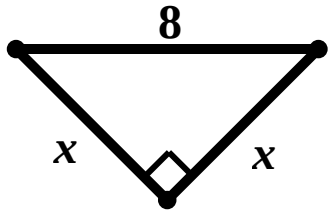
Raphael's Solution:

$a^2 + b^2 = c^2$
$4^2 + 5^2 = c^2$
$16 + 25 = c^2$
$41 = c$

Correct Solution:

Explain Mistake:

17. **Find, Fix, and Justify:** Nataani was asked to solve for the unknown side length in the triangle below. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.



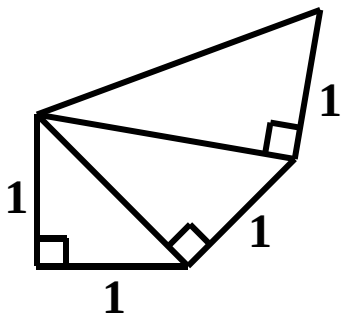
Nataani's Solution:

$a^2 + b^2 = c^2$
$x^2 + x^2 = 8$
$2x^2 = 8$
$x^2 = 4$
$x = 2$

Correct Solution:

Explain Mistake:

Extra for Experts: Use the picture below to answer questions a) and b).



a. Find all the missing side lengths and label the picture with the answers.
b. Using the picture above, devise a strategy for constructing a segment with a length of $\sqrt{5}$. Explain your strategy below.

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10.2d Class Activity: The Converse of the Pythagorean Theorem

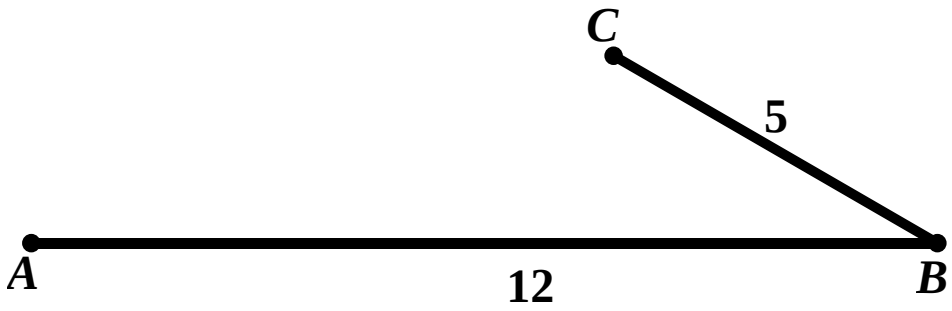
1. Mr. Riley's 8th grade class has been studying the Pythagorean Theorem. One day, he asked his class to find numbers a, b, and c where $a^2 + b^2 = c^2$, and draw triangles with those side lengths.

   Oscar determined that the numbers 5, 12, and 13 satisfy the Pythagorean Theorem as shown below:

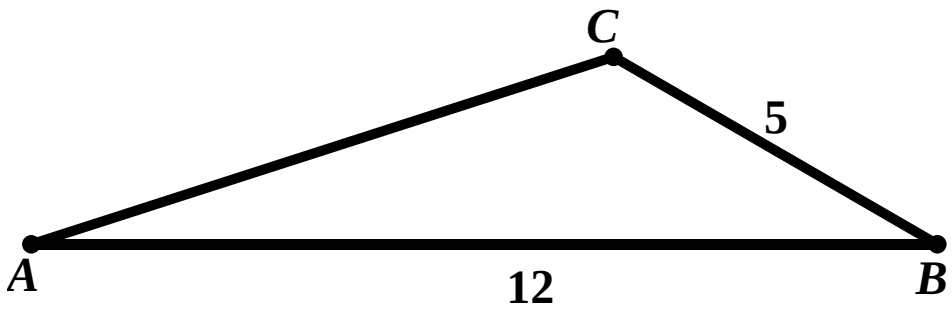
   $a^2 + b^2 = c^2$
   $5^2 + 12^2 = 13^2$
   $25 + 144 = 169$
   $169 = 169$

   Mr. Riley then said, "OK, so you have found three numbers that satisfy the Pythagorean Theorem. Now, show me that the triangle formed with these side lengths is a right triangle."

   a. Oscar continued working on the problem. He constructed a segment with a length of 12 cm and labeled the segment *AB*. From the endpoint *B*, he constructed a segment with a length of 5 cm and labeled the segment *BC* as shown in the picture below. Using a ruler, verify the lengths of the segments below.



   b. Then, he thought to himself, "I need to make the third side length AC equal to 13 because I know the triple 5, 12, 13 satisfies the Pythagorean Theorem." He connected A and C as shown below. He measured the length of AC and determined it did not measure 13 cm. Using a ruler, verify that AC does not measure 13 cm.



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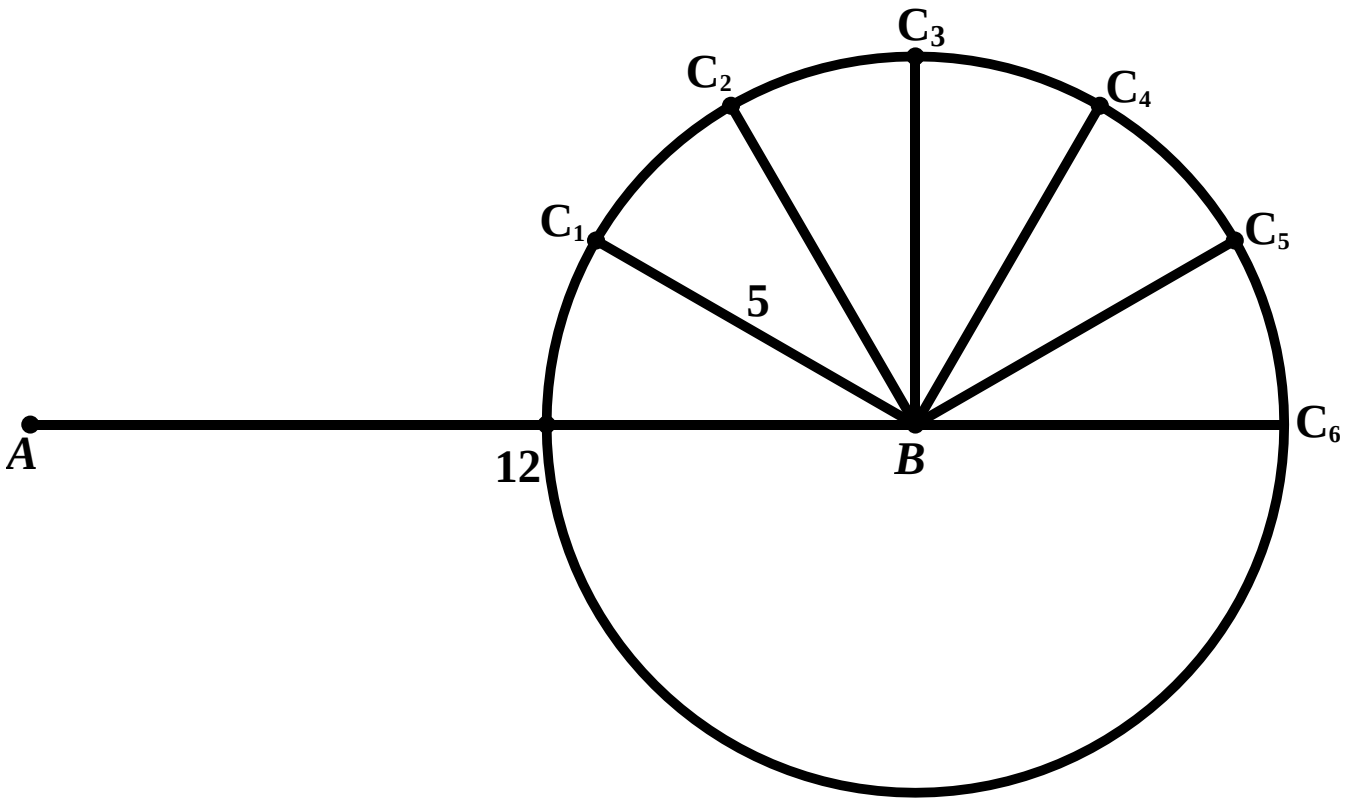
c. Then, he thought to himself, "What if I rotate $\overline{BC}$ around point B until AC measures 13 cm?" He began to rotate $\overline{BC}$ clockwise about B in increments as shown below. Help Oscar to find the location of C on the circle below that will give him a triangle with side lengths 5, 12, and 13.



d. What type of triangle is formed when AC equals 13 cm?
Have them measure $\angle ABC$ to see that it is a 90° angle.

2. Lucy also found a set of numbers that satisfy the Pythagorean Theorem: 3, 4 and 5. Verify in the space below that Lucy's numbers satisfy the Pythagorean Theorem.

$a^2 + b^2 = c^2$
$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
$25 = 25$

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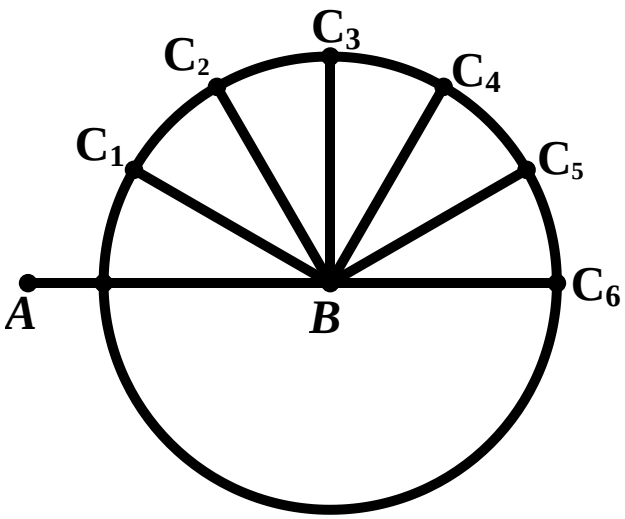
3. Using a process similar to Oscar's, Lucy set out to prove that a triangle with side lengths 3, 4 and 5 is in fact a right triangle. In the picture below $AB = 4$ cm and $CB = 3$ cm. Help Lucy determine the location of C that will create a triangle with side lengths 3 cm, 4 cm, and 5 cm.



4. What type of triangle is formed when AC equals 5 cm?
   A right triangle

5. Based on the problems above, what type of triangle is formed with side lengths that satisfy the Pythagorean Theorem? Write down the Converse of the Pythagorean Theorem.
   A right triangle will be formed.
   Converse of Pythagorean Theorem: For a triangle with side lengths a, b, and c if $a^2 + b^2 = c^2$, then the triangle is a right triangle.

6. Do the side lengths given below satisfy the Pythagorean Theorem? Remember to distinguish between legs (shorter sides) and the hypotenuse (longest side) and enter them into the equation correctly.

a. 11, 60, 61 Yes	b. 2, 4, 6
c. 14, 50, 48	d. 1, 3, $\sqrt{10}$; Yes
e. 2, 4, and $2\sqrt{5}$. Yes,	f. 5, 6, 8 No

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7. Mr. Garcia then asks the class, "What if the tick marks in Lucy's picture are each 2 cm instead of 1 cm? What are the measures of the side lengths that form the right triangle? Do they satisfy the Pythagorean Theorem?"

6, 8, and 10 – yes they do satisfy the Pythagorean Theorem.

8. What if the tick marks in Lucy's picture are each 3 cm? 0.1 cm? 10 cm? What are the measures of the side lengths that form the right triangles given these different scales and do they satisfy the Pythagorean Theorem?

3 cm: 9, 12, 15 0.1 cm: 0.3, 0.4, 0.5 10 cm: 30, 40, 50

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10.2d Homework: The Converse of the Pythagorean Theorem

Directions: Determine whether the three side lengths form a right triangle. Write yes or no on the line provided.

1. 9, 12, 15 __Y____
2. 18, 36, 45 _____
3. 12, 37, 35 _____
4. 8, 15, 16 __ N ____
5. $\sqrt{6}$, $\sqrt{10}$, 4 __ Y ____
6. 6.4, 12, 12.2 _____
7. 8.6, 14.7, 11.9 __ N ____
8. 8, $8\sqrt{3}$, 16 _____
9. 8, 8, $8\sqrt{2}$ ___ Y ___
10. 7, 9, 11.4 _____

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10.2e Class Activity: Exploration with Pythagorean Triples Extension

While we have seen several different sets of numbers that form a right triangle, there are special sets of numbers that form right triangles called Pythagorean triples. A **Pythagorean triple** is a set of nonzero **whole numbers** *a*, *b*, and *c* that can be put together to form the side lengths of a right triangle. 3, 4, 5 and 5, 12, 13 are examples of Pythagorean triples. We have seen many other sets of numbers that form a right triangle such as 0.09, 0.4, 0.41 that are not Pythagorean triples because their side lengths are not whole numbers.

a. The chart below shows some sets of numbers *a*, *b*, and *c* that are Pythagorean triples. Verify that the sets satisfy the equation $a^2 + b^2 = c^2$.
b. Can you find additional Pythagorean triples? Explain the method you used.

a	b	c	a^2	b^2	c^2
3	4	5			
5	12	13			
7	24	25			

c. The chart above starts with values for *a* that are odd numbers. Why didn't the chart start with a value of 1 for *a*.

d. Can you find Pythagorean triples where *a* is even? What is the smallest Pythagorean triple you can find with *a* being an even number?

e. Design a method to confirm that these numbers actually form right triangles. Write a short paragraph describing the method you used, and the results you obtained.

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10.2f Self-Assessment: Section 10.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provided on the next page that match each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Know that in a right triangle $a^2 + b^2 = c^2$, where *a* and *b* are the legs of the right triangle and *c* is the hypotenuse. *See sample problem #1*				
2. Understand and explain a proof of the Pythagorean Theorem. *See sample problem #2*				
3. Use the Pythagorean Theorem to solve for the missing side length of a right triangle given the measurements of the other two sides. *See sample problem #3*				
4. Understand and explain a proof of the converse of the Pythagorean Theorem. That is, for a triangle with side lengths *a*, *b*, and *c* if $a^2 + b^2 = c^2$, then the triangle is a right triangle. *See sample problem #4*				
5. Use the converse of the Pythagorean Theorem to determine whether three given measurements are possible side lengths of a right triangle. *See sample problem #5*				

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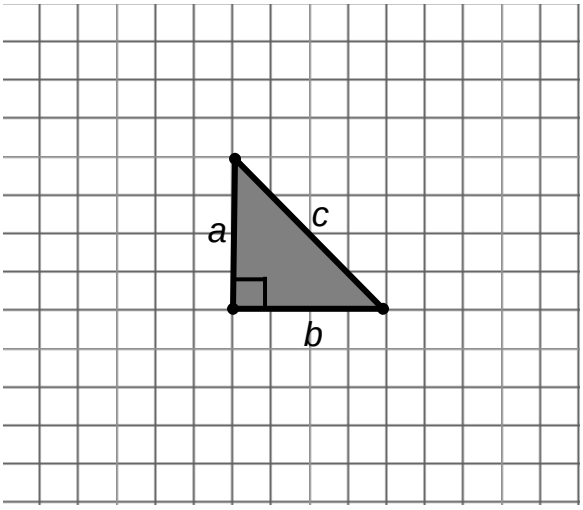
10.2f Sample Problems: Section 10.2

Sample Problem #1

In the picture below the gray triangle is a right triangle. Draw the squares along each of the three sides of the triangles. Find the area of each square and write the area in each square. Then, find the side lengths *a*, *b*, *c* of the triangle.



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Sample Problem #2

Below is a geometric explanation for a proof of the Pythagorean Theorem: Given a right triangle with side lengths *a* and *b* and a hypotenuse of *c*, then $a^2 + b^2 = c^2$. The figures for the proof are given in order. Choose the explanation that provides a sound argument accompanied with reasoning and warrants to support the claims given for each figure. Write the letter that matches each explanation in the space provided.

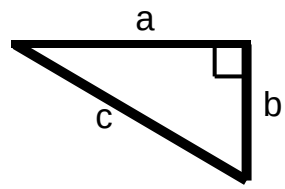

Figure 1_____

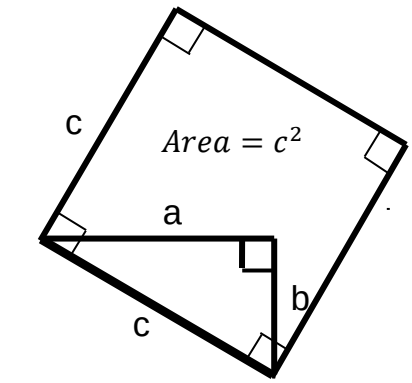

Figure 2_____

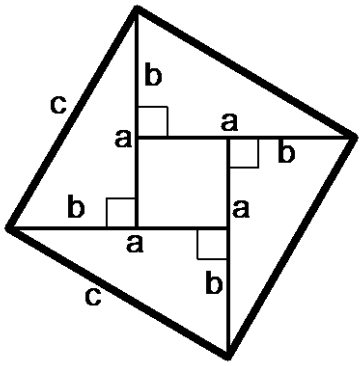

Figure 3____

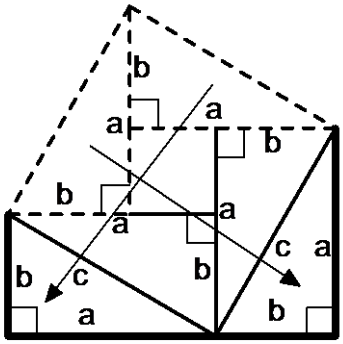

Figure 4_____

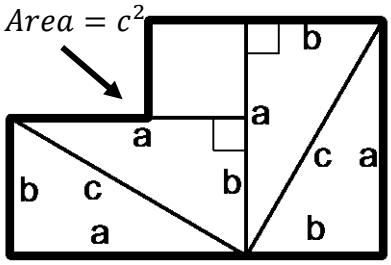

Figure 5_____

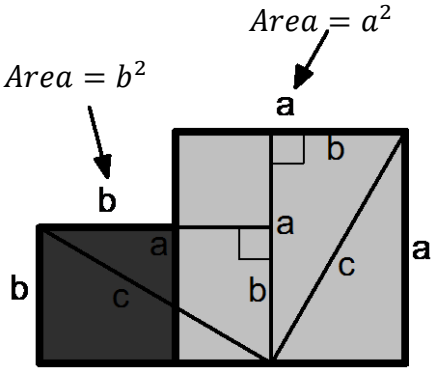

Figure 6 _____

A. Inside of the square draw 4 congruent right triangles with side length *a* and *b* and a hypotenuse of *c*.

B. Draw a square with off of this triangle with a side length of *c*. The area of square this square is c^2. This is because the area of a square is the side length squared

C. You can view the area of this figure as the composition of two squares with sides length *a* and *b*. The area of the darker square is b^2 and the area of the lighter shaded square is a^2. Thus the area of the whole figure is a^2+b^2. As stated above this is the same as the area of the original square with side length *c*. Thus $a^2 + b^2 = c^2$.

D. Rearrange the square by translating the top two triangles to the bottom of the figure.

E. The area of this figure is the same as the area of the original square because we have not added or removed any of the pieces.

F. Begin with right triangle with a horizontal side length of *a* and a vertical side length of *b* and a hypotenuse of *c*.

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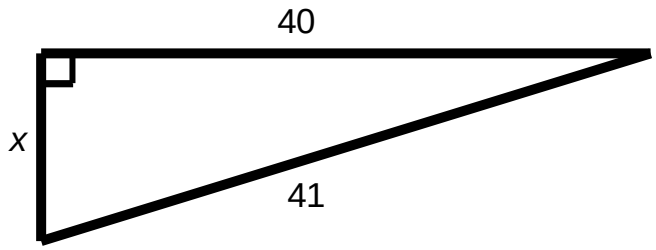
Sample Problem #3

Find the value of *x* using the Pythagorean Theorem. Leave your answer in simplest radical form.



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Sample Problem #4

The Converse of the Pythagorean Theorem states that given a triangle with side lengths *a*, *b*, and *c*, if $a^2 + b^2 = c^2$, then the triangle is a right triangle. Explain the proof of the Converse of the Pythagorean Theorem that your teacher provides for you.

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Sample Problem #5

Determine whether the three side lengths form a right triangle. Show your work to verify your answer.

5.5, 12.5, 13.5

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Section 10.3 Applications of the Pythagorean Theorem

Section Overview:
In this section, students apply the Pythagorean Theorem to solve real-world problems in two- and three-dimensions. Then, students use the Pythagorean Theorem to find the distance between two points. After the students gain an understanding of the process being used to find the distance between two points in a coordinate system, students have the opportunity to derive the distance formula from the Pythagorean Theorem and the process being used. Rather than memorizing the distance formula, the emphasis is placed on the process used to find the distance between two points in a coordinate system and the connection between the Pythagorean Theorem and the distance formula.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Use the Pythagorean Theorem to solve problems in real-world contexts, including three-dimensional contexts.
2. Find the distance between two points in a coordinate system.

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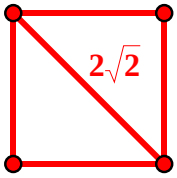
10.3a Class Activity: Applications of the Pythagorean Theorem

Directions: For each problem, first draw a picture if one is not provided and then solve the problem.

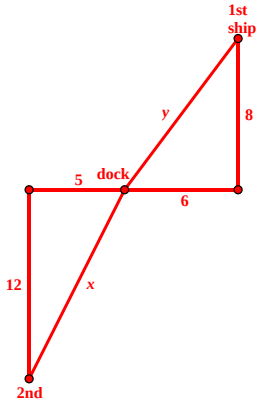
1. What is the length of the diagonal of a rectangle of side lengths 1 inch and 4 inches?

2. A square has a diagonal with a length of $2\sqrt{2}$ inches. What is the side length of the square?



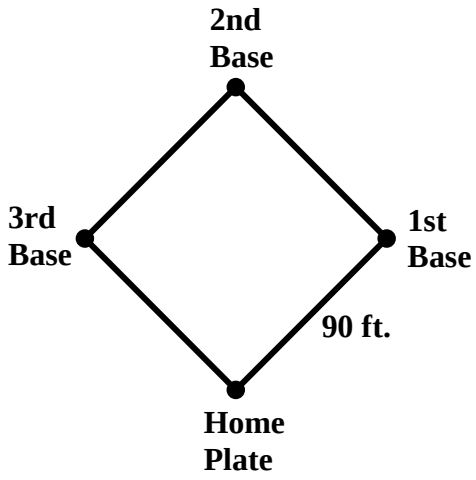
$x = 2$

3. Two ships leave a dock. The first ship travels 6 miles east and then 8 miles north and anchors for the night. The second ship travels 5 miles west and then 12 miles south and anchors for the night. How far are each of the ships from the dock when they anchor for the night?



Ship 1 is 10 miles from the dock and ship 2 is 13 miles from the dock.

4. A baseball diamond is in the shape of a square. The distance between each of the consecutive bases is 90 feet. What is the distance from Home Plate to 2nd Base?



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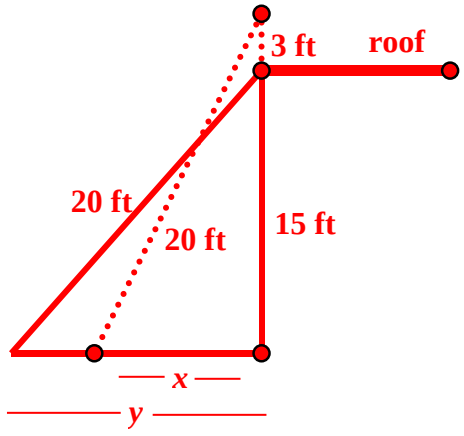
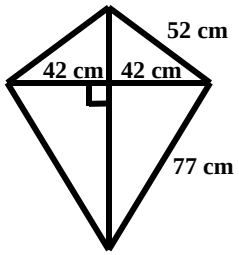
5. Ray is a contractor that needs to access his client's roof in order to assess whether the roof needs to be replaced. He sees that he can access a portion of the roof that is 15 feet from the ground. He has a ladder that is 20 feet long.



   a. How far from the base of the house should Ray place the ladder so that it just hits the top of the roof? Round your answer to the nearest tenth of a foot.
   $y \approx 13.2$ feet
   b. How far should he place the ladder from the base of the house if he wants it to sit 3 feet higher than the top of the roof? Round your answer to the nearest tenth of a foot.
   $x \approx 8.7$ feet

6. The dimensions of a kite sail are shown below. The support rod that runs from the top of the kite to the bottom of the kite has been broken and needs to be replaced. What length of rod is needed to replace the broken piece? Round your answer to the nearest tenth.



7. A new restaurant is putting in a wheelchair ramp. The landing that people enter the restaurant from is 2 feet higher than street level. Safety standards require that for every 1 foot of rise on a wheelchair ramp there must be a run of 12 feet. How long will the ramp have to be to meet safety standards? Round your answer to the nearest tenth.

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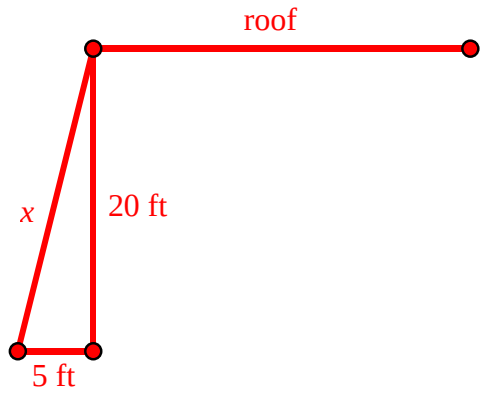
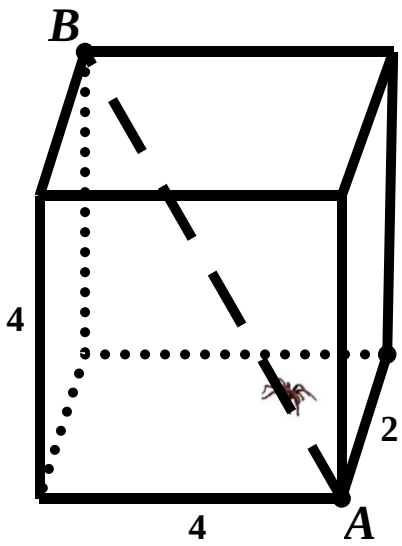
8. Melanie is having a rectangular-shaped patio built in her backyard. It is very important to Melanie that the corners of the patio are right angles. The contractor built a patio with a width of 10 feet and a length of 15 feet. The diagonal measures 20 feet. Does the patio have the right angles that Melanie requested?

9. Fred is safety conscious. He knows that to be safe, the distance between the foot of the ladder and the wall should be $\frac{1}{4}$ the height of the wall. Fred needs to get on the roof of the school building which is 20 ft. tall. How long should the ladder be if he wants it to rest on the edge of the roof and meet safety standards? Round your answer to the nearest tenth. $x \approx 20.6$ or students may also state that as long as the ladder is longer than this, they will meet safety standards, the ladder will just have to be placed further from the base of the building in order to rest on the edge of the roof



10. A spider has taken up residence in a small cardboard box which measures 2 inches by 4 inches by 4 inches. What is the length, in inches, of a straight spider web that will carry the spider from the lower right front corner of the box to the upper left back corner of the box?



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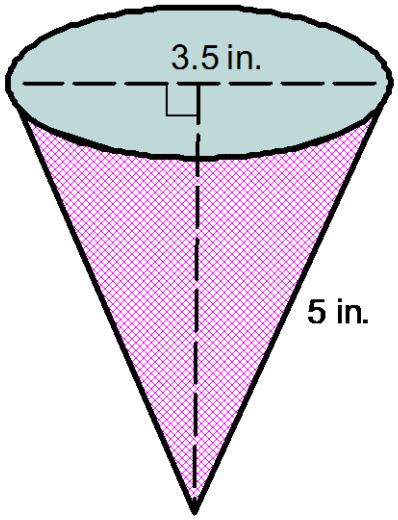
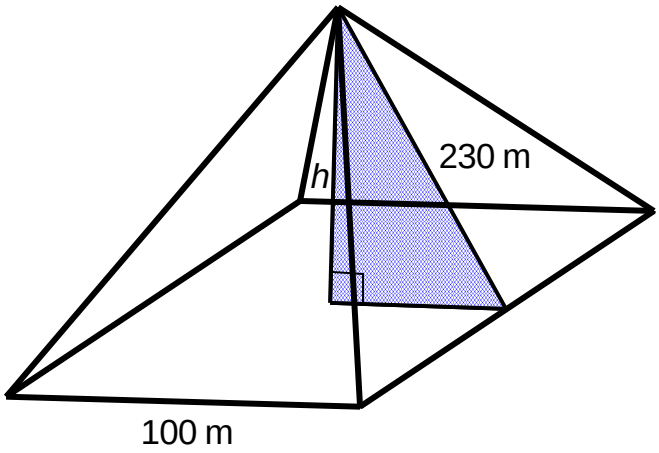
11. Sunny made a paper cone to hold candy for favors for a baby shower. After making the cones she measures the slant height of the cone and the diameter of the base of the cone. Her measurements are shown in the picture below. Find the volume of the cone.



12. In the movie Despicable Me, an inflatable model of The Great Pyramid of Giza in Egypt was created by Vector to trick people into thinking that the actual pyramid had not been stolen. When inflated, the false Great Pyramid had a square base of side length 100 m. and the height of one of the side triangles was 230 meters. This is also called the slant height of the pyramid. What is the volume of gas that was used to fully inflate the fake Pyramid? (Hint: Recall the formula for the volume of a pyramid is $\frac{1}{3}Bh$ where B is the area of the base and h is the height of the pyramid (the distance from the base to the apex).



The volume of the pyramid is approximately 748,333 m^3.

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10.3a Homework: Applications of the Pythagorean Theorem

1. What is the length of the diagonal of a square with a side length of 4 cm?

   $x = 4\sqrt{2}$

2. One side length of a rectangle is 2 inches. The diagonal of the rectangle has a length of $2\sqrt{5}$ inches. What is the length of the other side of the rectangle?

3. A football field is 360 feet long and 160 feet wide. What is the length of the diagonal of a football field assuming the field is in the shape of a rectangle?

4. The length of an Olympic-size swimming pool is 55 meters. The width of the pool is 25 meters. What is the length of the diagonal of the pool assuming the pool is in the shape of a rectangle?

   $x \approx 60.4$ m

5. You are locked out of your house. You can see that there is a window on the second floor that is open so you plan to go and ask your neighbor for a ladder long enough to reach the window. The window is 20 feet off the ground. There is a vegetable garden on the ground below the window that extends 10 ft. from the side of the house that you can't put the ladder in. What size ladder should you ask your neighbor for?

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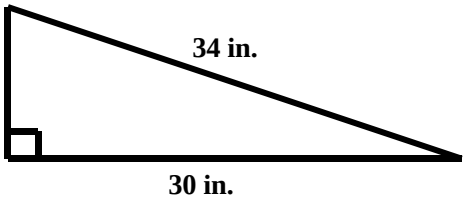
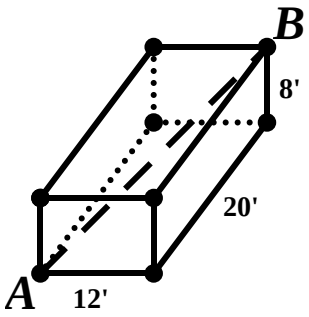
6. Kanye just purchased a skateboarding ramp. The ramp is 34 inches long and the length of the base of the ramp is 30 inches as shown below. What is the height of the ramp?



7. A rectangular-shaped room has a width of 12 feet, a length of 20 feet, and a height of 8 feet. What is the approximate distance from one corner on the floor (Point A in the figure) to the opposite corner on the ceiling (Point B in the figure)? $AB \approx 24.6$ feet or 25 feet



8. A large pile of sand has been dumped into a conical pile in a warehouse. The slant height of the pile is 20 feet. The diameter of the base of the sand pile is 32 ft. Find the volume of the pile of sand.

Hint: Recall that the slant height is not the height of the cone.

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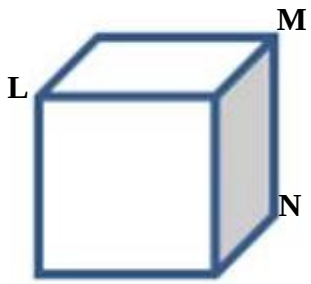
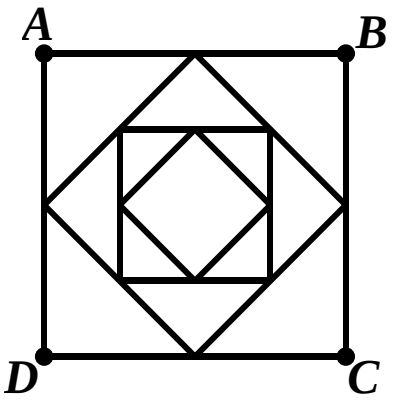
9. The cube below is a unit cube. A unit cube is a cube of side length 1.



a. What is the length of $\overline{LM}$? Leave your answer in simplest radical form.
$\sqrt{2}$

b. What is the length of $\overline{LN}$? Leave your answer in simplest radical form.
$\sqrt{3}$

Extra for Experts: Square *ABCD* has side lengths equal to 4 inches. Connecting the midpoints of each side forms the next square inside *ABCD*. This pattern of connecting the midpoints to form a new square is repeated.



a. What is the side length of the inner-most square?

b. What is the area of the inner-most square?

c. What is the ratio of the area of each square to the area of the next square created?

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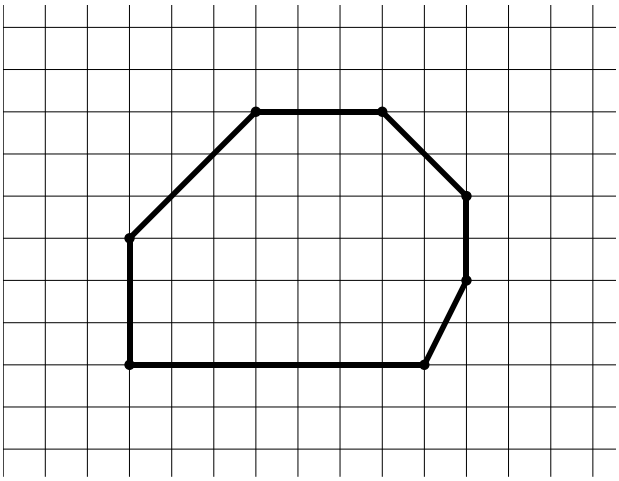
Extra for Experts: The following is a scale drawing of a patio that Mr. Davis plans to build in his backyard. Each box in the scale drawing represents 1 unit.



a. Find the exact value of the perimeter of the scale drawing of the patio. Show all work and thinking.

b. Find the area of the scale drawing of the patio. Show all work and thinking.

c. If the scale on the drawing above is 1 unit = 3 feet, what is the actual measure of the perimeter of the patio? The area? Show all work.

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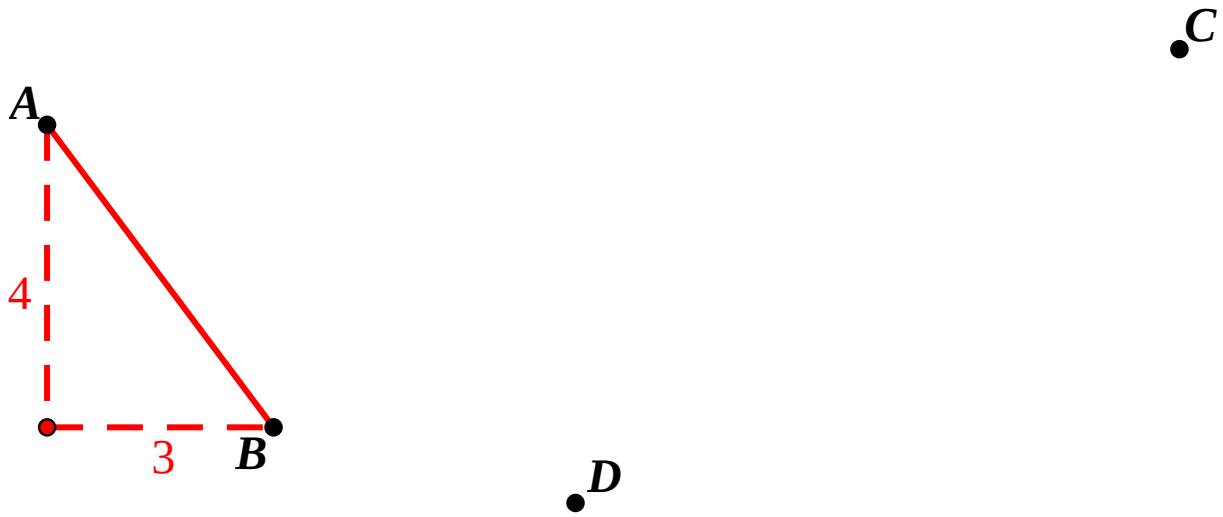
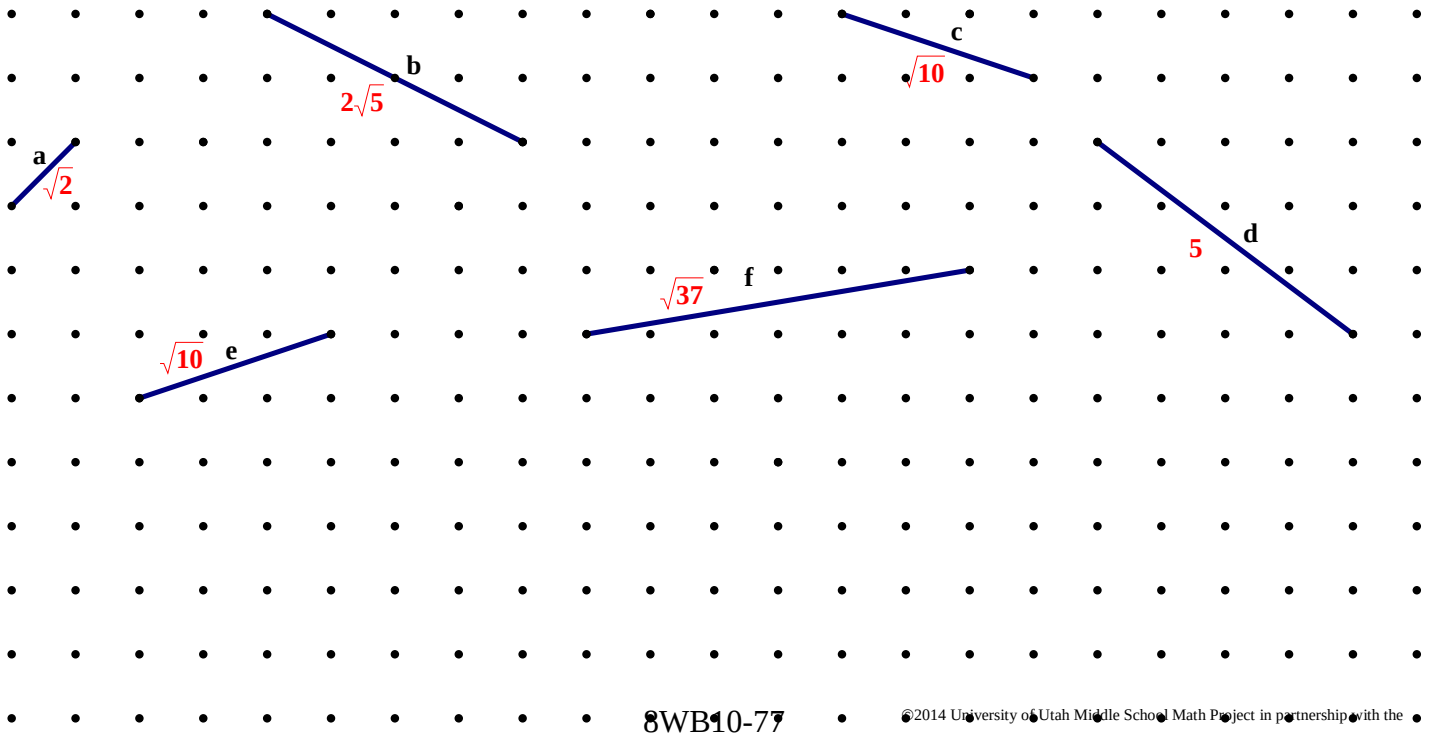
10.3b Class Activity: Finding Distance Between Two Points

1. Using a centimeter ruler, find the distance between the following sets of points shown below. Then draw the slope triangle of each segment, measure the lengths of the rise and run, and verify that the Pythagorean Theorem holds true.
   a. *A* to *B* 5 cm
   b. *B* to *C* 13 cm
   c. *C* to *D* 10 cm



2. Find the lengths of the segments below. Assume that each horizontal and vertical segment connecting the dots has a length of 1 unit.



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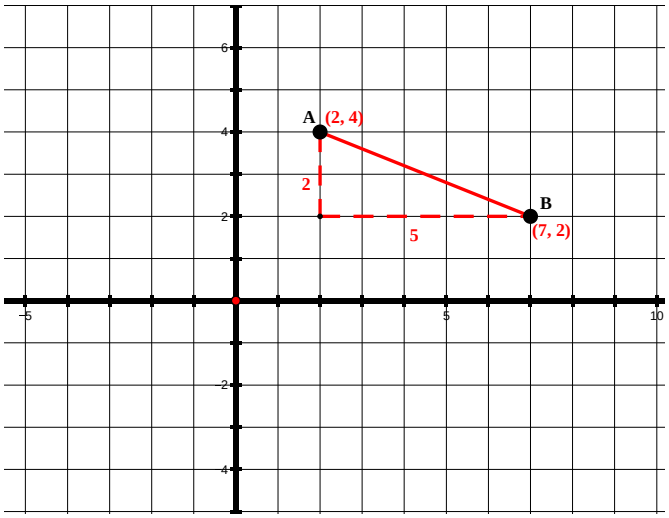
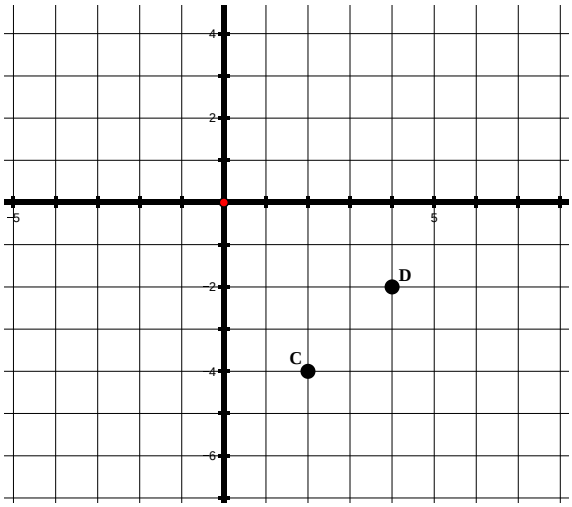
Directions: Label the coordinates of each point. Then, find the distance between the two points shown on each grid below.

3.



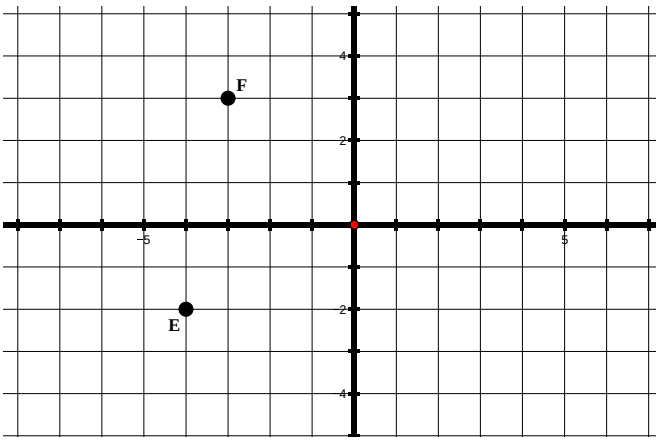
If we draw the slope triangle of $\overline{AB}$, as shown above, we can use the PT to find the distance from point A to B which is $\sqrt{29}$.

4.



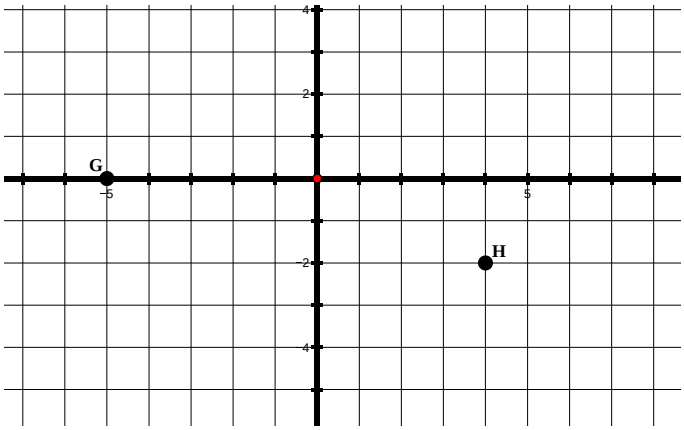
$CD = 2\sqrt{2}$

5.



$EF = \sqrt{26}$

6.



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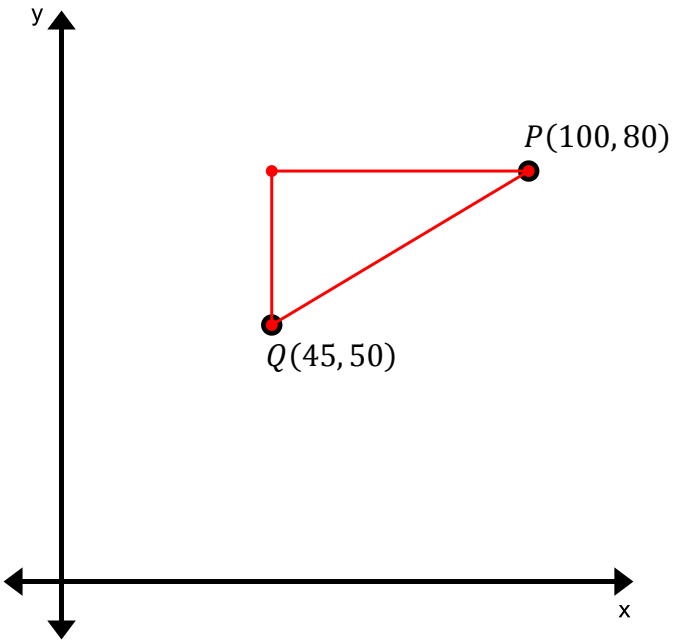
The Coordinate Distance Formula

7. Find the distance between the two points given on the graph below.



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8. Find the distance between the two points given below. Leave your answers in simplest radical form.

a. $A: (3, 5)$ $B: (6, 9)$
5

b. $R: (-1, 4)$ $S: (3, 8)$
$4\sqrt{2}$

c. $C: (0, 5)$ $D: (2, -3)$
$2\sqrt{17}$

d. $S: (-3, -5)$ $T: (2, -7)$
$\sqrt{29}$

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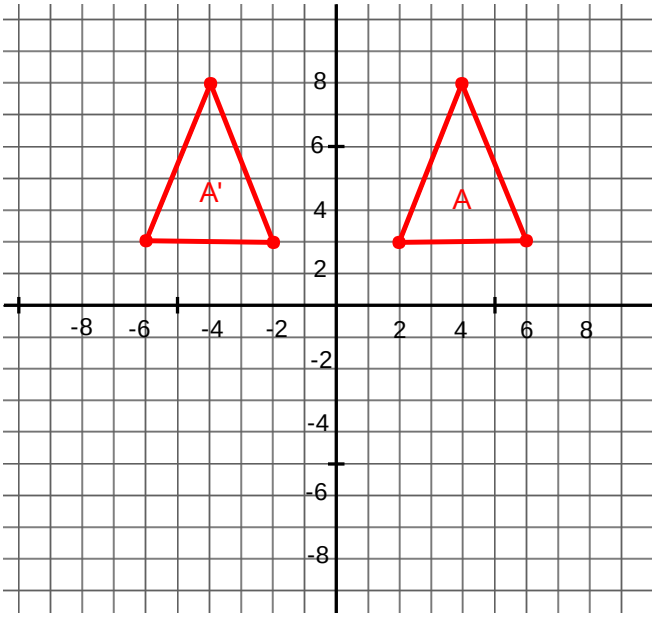
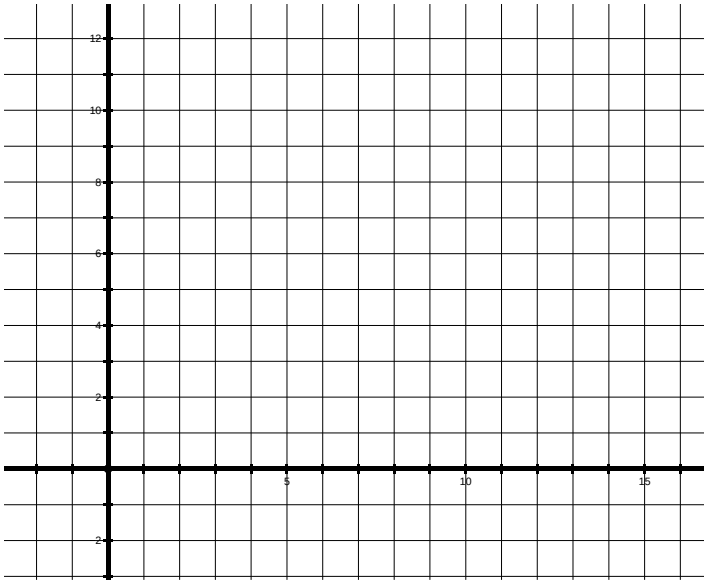
9. A triangle has vertices at the points (2,3) and (4,8), and (6,3) on the coordinate plane.
   a. Find the perimeter of the triangle. Use the grid below if needed.
      The perimeter of the triangle is approximately 14.8 units. Noticing that the triangle is isosceles with make this computation easier.
   b. Find the area of the triangle.
      10 units^2
   c. If the triangle is dilated by a scale factor of 3 what will the new perimeter be?
      The new perimeter will be approximately 44.3 units. This is 3 times the old perimeter.
   d. If the triangle is dilated by a scale factor of 3 what will the new area be?
      The new area will be 90 units^2. This is 3^2 times the old area.
   e. Plot the original triangle, label it triangle A. Then reflect the triangle over the *y*-axis, label the new triangle A'. Does this transformation change the perimeter of the triangle? Explain your answer.



Reflecting the triangle will not change its perimeter because reflections are a rigid motion. Congruence is maintained under a rigid motion so the triangle side lengths remain the same.

10. List **three** coordinate pairs that are 5 units away from the origin in the first quadrant. Describe how to find the points and justify your reasoning. The grid has been provided to help you.
(**Note:** Points on the axes are not in the first quadrant).

There are many possible answers: (3, 4); (4, 3); $(\sqrt{5}, 2\sqrt{5})$; $(2, \sqrt{21})$



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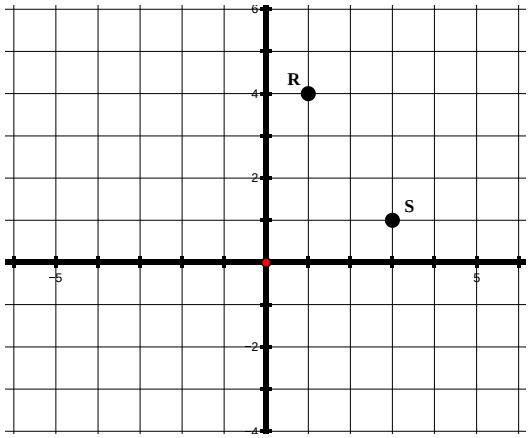
10.3b Homework: Finding Distance Between Two Points

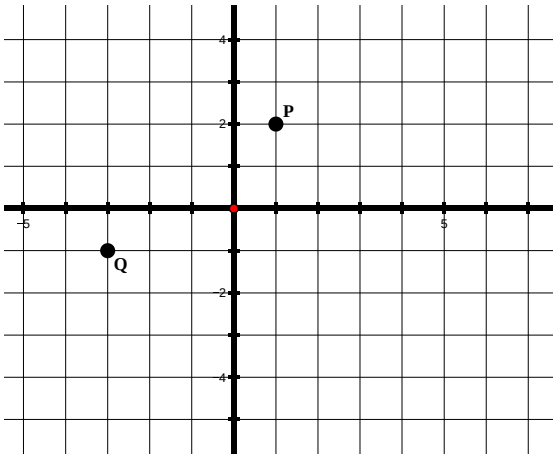
Directions: Find the distance between the two points shown on each grid below. Leave your answers in simplest radical form.

1.

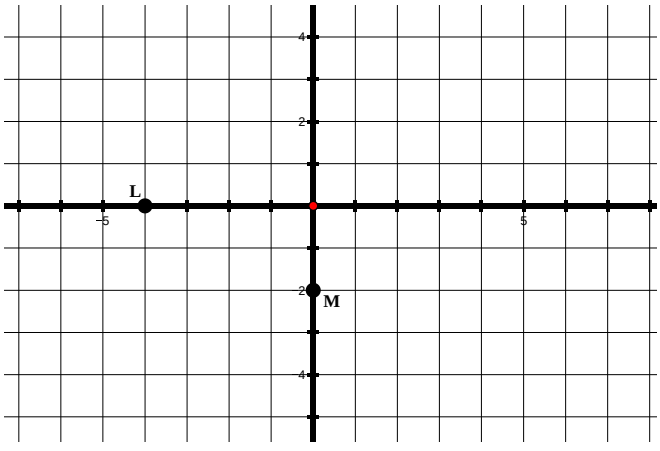


$\sqrt{13}$

2.

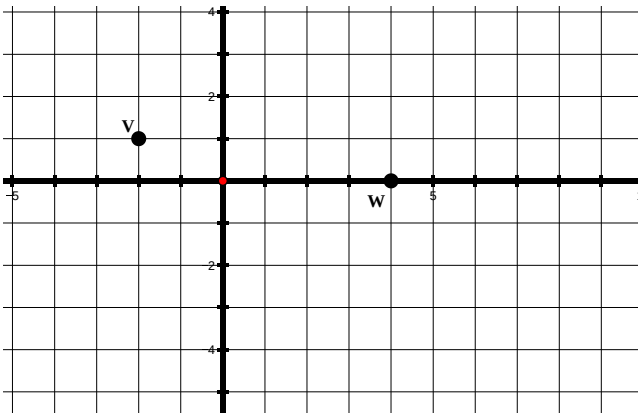


3.



$2\sqrt{5}$

4.



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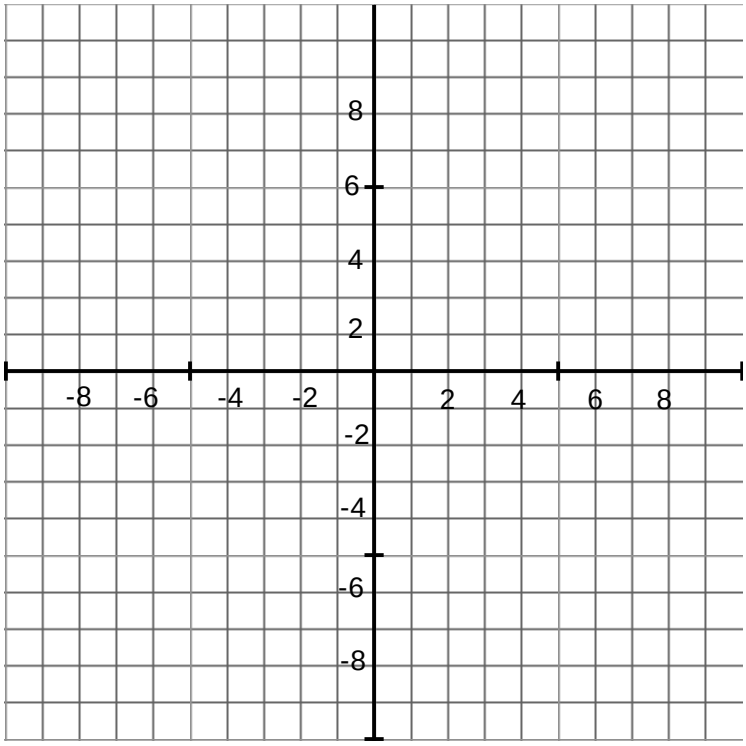
5. Find the distance between the two points given below. Leave your answers in simplest radical form.
   a. $A: (2, 1)$ $B: (4, 7)$ $2\sqrt{10}$

   b. $R: (2, -1)$ $S: (8, -7)$

   c. $C: (1, 0)$ $D: (2, -3)$ $\sqrt{10}$

   d. $S: (-2, -4)$ $T: (2, -5)$

6. Plot any letter of the alphabet that is made up of segments that are straight lines on the coordinate plane given below. For example you can plot the letter A, E, F, etc. but not the letter B,C, D, etc



   a. Find the total distance for the segments that make up this letter.

   b. If you dilated this letter by a scale factor of 4 what is the total distance of the segments that make up your letter?

   c. If you dilated this letter by a scale factor of $\frac{1}{5}$ what is the total distance of the segments that make up your letter?

   d. Rotate your letter 180 degrees about the origin. Does this transformation change the size or shape of the letter? Explain your answer.

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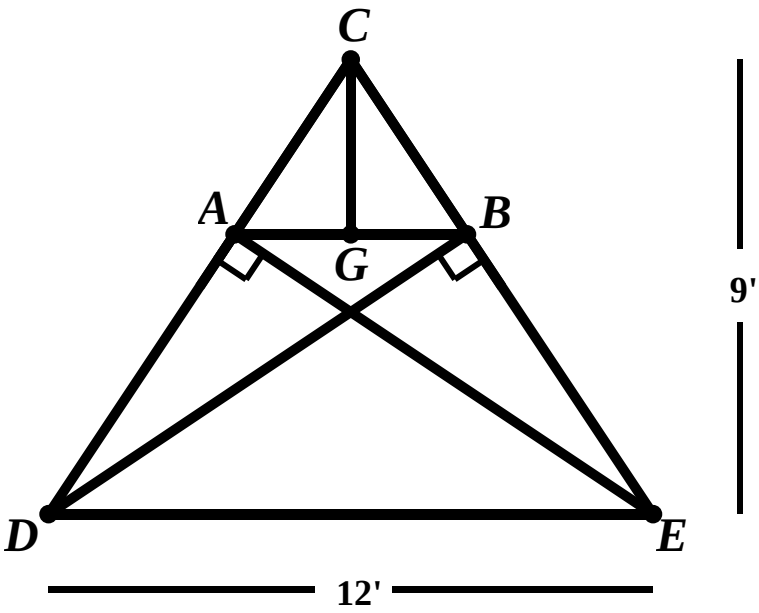
10.3c Extension: Construction

Mario is designing an A-frame for the lodge of a ski resort. Below is a scale drawing of his design.
Given: C lies over the center of the building

$\overline{AB} \parallel \overline{DE}$

$\angle DAE$ and $\angle EBD$ are right angles.



What are the lengths of all segments in the diagram?

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10.3d Self-Assessment: Section 10.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provided on the next page that match each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Use the Pythagorean Theorem to solve problems in real-world contexts, including three-dimensional contexts. *See sample problem #1*				
2. Find the distance between two points in a coordinate system. *See sample problem #2*				

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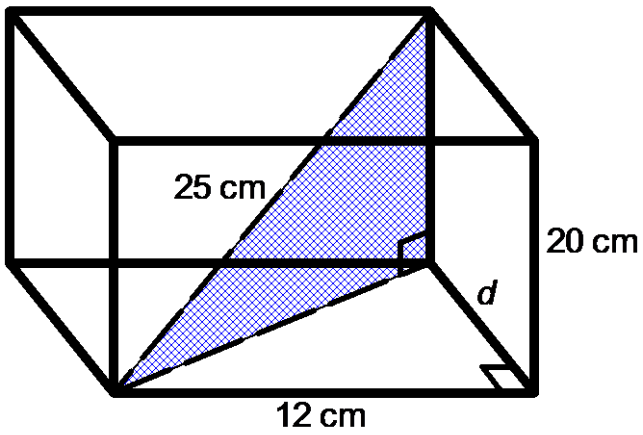
10.3d Sample Problems: Section 10.3

Sample Problem #1

a. A park is 6 miles east of your home. The bakery is 4 miles north of the park. How far is your home from the bakery as the crow flies?

b. Find the volume of the rectangular prism given below.



Sample Problem #2

Find the distance between each set of points.

a. $A(-10,2)$ and $B(-7,6)$

b. $C(-2,-6)$ and $D(6,9)$

c. $E(3,5)$ and $F(7,9)$

d. $G(3,4)$ and $H(-2,-2)$

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