

First published in 2013 by Math Materials Access Improvement for Seventh and Eighth Grades (MMAISE), Salt Lake City, USA.

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Chapter 1: Probability, Percent, Rational Number Equivalence (3-4 weeks)

UTAH CORE Standard(s)

Number Sense:

1. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.2d**
2. Solve real-world and mathematical problems involving the four operations with rational numbers. **7.NS.3**

Probability and Statistics:

1. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **7.SP.5**
2. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. **7.SP.6**

Equations and Expressions:

1. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **7.EE.3**

Chapter 1 Summary:

Chapter 1 begins with a brief introduction to probability as a means of reviewing and applying arithmetic with whole numbers and fractions. In addition to covering basic counting techniques and listing outcomes in a sample space, students distinguish theoretical probabilities from experimental approaches to estimate probabilities. One reason for starting the year with probability activities is to develop a culture of thinking about mathematics as a way to investigate real world situations. A second reason is that activities at the beginning of the year can help foster a classroom culture of discussion and collaboration.

Throughout the chapter students are provided with opportunities to review and build fluency with fractions, percents, and decimals from previous grades. Students should understand that fractions, percents and decimals are all relative to a whole. Students will also compare and order fractions (both positive and negative.) This chapter concludes with a section specifically about solving percent and fraction problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease.

VOCABULARY: chance, decimal, experimental probability, fraction, frequency, outcome, percent, probability, ratio, theoretical probability

CONNECTIONS TO CONTENT:

Prior Knowledge

In previous coursework, students developed the concept of a ratio. Though students should be familiar with the idea of part:whole and part:part relationships, they have likely not completely solidified it yet. It is important to emphasize that in this chapter only part:whole relationships are discussed (probability is a part:whole relationship as are fractions, decimals and percents.) In Chapter 4 students will practice differentiating between part:part and part:whole relationships and then examine “odds” which are part:part relationships.

Students have used all four operations (addition, subtraction, multiplication, and division) when working with fractions and decimals in prior grades. They should have used both number line and bar/tape models to represent fractions, percents, and decimals.

In 6th grade students placed both positive and negative numbers on a number line, however they do not operate with negative numbers until 7th grade (this will take place in Chapter 2).









Future Knowledge

As students move through this chapter, they will begin by studying probability (this chapter is only an introduction to probability, students will work more with probability in Chapter 7). The concepts learned in 7th grade regarding chance processes as well as theoretical and experimental probabilities will be extended in later courses when students study conditional probability, compound events, evaluate outcomes of decisions, use probabilities to make fair decisions, etc.

While studying probability students will continue their study of rational numbers. They will convert rational numbers to decimals and percents and will look at their placement on the number line. This lays the foundation for 8th grade where students study irrational numbers to complete the Real Number system.

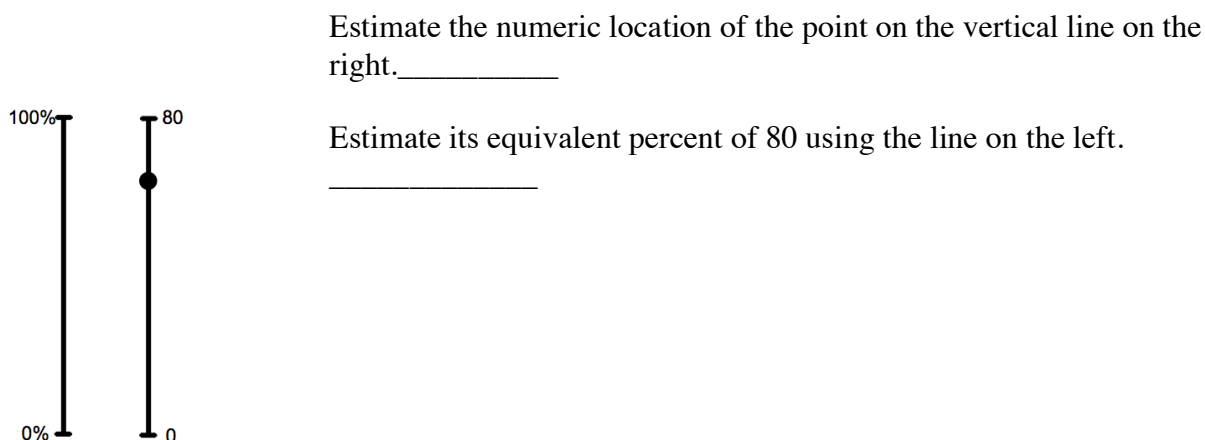
Another important concept that will be extended is the notion of “unit.” Throughout the year students will: a) clearly distinguish between kinds of units (e.g. linear, square or cubic) and b) consider units of quantities. For example a “unit” of consideration might be a distance of 5 miles as in walking 5 miles per hour. So, if they want to know how far one walks in 20 minutes, $\frac{1}{3}$ of an hour, then one simply divides the 5 mile unit into 3 parts. Thus in 20 minutes one has walked $\frac{5}{3}$ miles.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

	Make sense of problems and persevere in solving them	Students explain and demonstrate rational number operations by using symbols, models, words, and real life contexts. Students demonstrate perseverance while using a variety of strategies (number lines, manipulatives, drawings, etc.). Students make sense of probability situations by creating visual models to represent situations.
	Reason abstractly and quantitatively	Students connect ideas of models to ideas of numbers. For example, students should reason that one can partition a whole into any number n equal pieces and then represent a portion of the whole as m/n . The number m/n can then be located on the real line.
	Construct viable arguments and critique the reasoning of others	Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, or models. Students discuss rules for operations with rational numbers using appropriate terminology and tools/visuals. Students approximate probabilities, create probability models and explain reasoning for their approximations. They also question each other about the representations they create to represent probabilities.
	Model with mathematics	Students model understanding of rational number operations using tools such as tiles, counters, visuals, tape/bar models, and number lines and connect these models to solve problems involving real-world situations. Students model problem situations symbolically, graphically, and contextually. Students use experiments or simulations to create probability models.
	Attend to precision	Students should use appropriate terminology when referring to ratios, probability models, rational numbers, and equations. Students also use appropriate forms of a number to fit a context, e.g. one would not say $\$3/2$, rather they would say $\$1.50$.
	Look for and make use of structure	Students look for structure in rational numbers when they place them appropriately on the number line. Students should connect the structure of fractions, percents, and decimals to the idea of part:whole relationships to missing finding values (e.g. students should use structure, not algorithms, to solve problems). Students recognize that probability can be represented in tables, visual models, or as a rational number.
	Use appropriate tools strategically	Students demonstrate their ability to select and use the most appropriate tool (paper/pencil, manipulatives, pictorial models, and calculators) while solving problems with rational numbers. Students might use physical objects or applets to generate probability.
	Look for and express regularity in repeated reasoning	Students use repeated reasoning to understand algorithms and make generalizations about patterns. They extend their thinking to include complex fractions and rational numbers. They create, explain, evaluate, and modify probability models to describe simple events.

1.0 Anchor Problem: Percent Estimation Game

Below are two vertical lines on which you will make a guess for both a number and a percent.



2) You and five of your friends like to go to McDonalds once a week and get Happy Meals for the prize inside. McDonalds has just started a new “dinosaur toy” promotion for their Happy Meals with six different dinosaurs you can collect: Brachiosaurus, Brontosaurus, Diplodocus, Tyrannosaurus, Plesiosaurus, and Allosaurus. You each want to collect at least one of all six dinosaurs, but the prizes are randomly placed in Happy Meals, and there is no way to know which dinosaur you’re getting until you open the Happy Meal bag.

How might you design a simulation experiment to find the likelihood (experimental probability) of getting all six toys after one, two, three, etc. weeks?

Section 1.1: Investigate Chance Processes. Develop/Use Probability Models.

Section Overview:

This is students' first formal introduction to probability. In this section students will study chance processes, which concern experiments or situations where they know which outcomes are possible, but they do not know precisely which outcome will occur at a given time. They will look at probabilities as part:whole ratios expressed as fractions, decimals, or percents. Probabilities will be determined by considering the results or outcomes of experiments. Students will learn that the set of all possible outcomes for an experiment is a sample space. They will recognize that the probability of any single event can be expressed in terms of impossible, unlikely, equally likely, likely, or certain or as a number between 0 and 1, inclusive. Students will focus on two concepts in probability of an event: *experimental* and *theoretical*. They will understand the commonalities and differences between experimental and theoretical probability in given situations.

Concepts and Skills to be Mastered (from standards)

1. Understand and apply likelihood of a chance event as between 0 and 1.
2. Approximate probability by collecting data on a chance process (experimental probability).
3. Calculate theoretical probabilities on a chance process for simple events.
4. Given the probabilities (different scenarios in a chance process), predict the approximate frequencies for those scenarios (if experimenting on a chance process).
5. Use appropriate fractions, decimals and percents to express the probabilities.

REVIEW FROM EARLIER GRADES:

$$1 = 1.0 = 100\%$$



$$\frac{1}{2} = 0.5 = 50\%$$



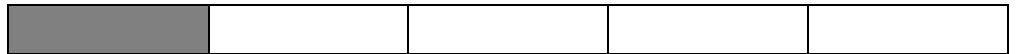
$$\frac{1}{4} = 0.25 = 25\%$$



$$\frac{1}{3} = 0.3\bar{3} = 33\frac{1}{3}\% \text{ or } 33.\bar{3}\%$$



$$\frac{1}{5} = 0.2 = 20\%$$



$$\frac{1}{10} = 0.1 = 10\%$$



1. Use a bar model to represent $\frac{3}{4}$ of a whole.



2. Use a bar model to represent $\frac{3}{5}$ of a whole.



3. Use a bar model to represent $\frac{3}{10}$ of a whole.



4. What do you notice about the fractions in #1-3?

1.1a Class Activity: Using Data to Make Predictions (Probability)

1. In your own words, what do you think these terms mean?

“Experimental” probability:

“Theoretical” probability:

HOW MANY GREEN TILES ARE IN YOUR BAG?

We will examine *experimental* and *theoretical* probability in this activity. You have been given a bag with a total of 12 tiles, some are green and the rest are blue. You will do the experiment described below 6, 12, 18, 24 and then 30 times to try to figure out how many green tiles are in your bag. **DO NOT LOOK IN YOUR BAG!** Each group has a different number of green tiles in their bag.

Instructions: a) without looking in the bag, draw ONE tile record the color (G or B) in the table, b) put the tile back into the bag and shake it to mix up the tiles, c) repeat the indicated number of trials (notice that there are a – e experiments), d) based on each experiment guess how many GREEN tiles are in the bag.

- a. Draw a marble/tile (without looking in the bag), record the color, replace it, mix them by shaking, redraw, record, replace. Do this 6 times.

Draw #	1	2	3	4	5	6
Color						

Based on your experiment, how many of the 6 draws were GREEN? _____

Based on these 6 draws, how many of the 12 marbles/tiles in the bag do you think are green? _____

- b. Repeat the experiment in “a” but now do it 12 times.

Draw #											
Color											

Based on your experiment, how many of the 12 draws were GREEN? _____

Based on these 12 draws, how many of the 12 marbles/tiles in the bag do you think are green? _____

- c. Repeat the experiment in “a” but now do it 18 times.

Draw #																	
Color																	

Based on your experiment, how many of the 18 draws were GREEN? _____

Based on these 18 draws, how many of the 12 marbles/tiles in the bag do you think are green? _____

- d. Repeat the experiment in “a” but now do it 24 times.

Draw #																	
Color																	
Draw #																	
Color																	

Based on your experiment, how many of the 24 draws were GREEN? _____

Based on these 24 draws, how many of the 12 marbles/tiles in the bag do you think are green? _____

- e. Repeat the experiment in “a” but now do it 30 times.

Draw #																	
Color																	
Draw #																	
Color																	

Based on your experiment, how many of the 30 draws were GREEN? _____

Based on these 30 draws, how many of the 12 marbles/tiles in the bag do you think are green? _____

Probability has standard notation. We write $P(G)$ to mean the probability of drawing a green tile. To write $P(G)$ we need to know the “observed frequency” and the “total number of trials.” Probability is the ratio of the observed frequency to the total number of trials:

$$P(G) = \frac{\text{observed frequency}}{\text{total number of trials}}$$



Note: probability is a part:whole relationship.

Write your group’s $P(G)$ as both a fraction and decimal for each of the experiments you did on the previous page.

a. $P(G) =$ _____

b. $P(G) =$ _____

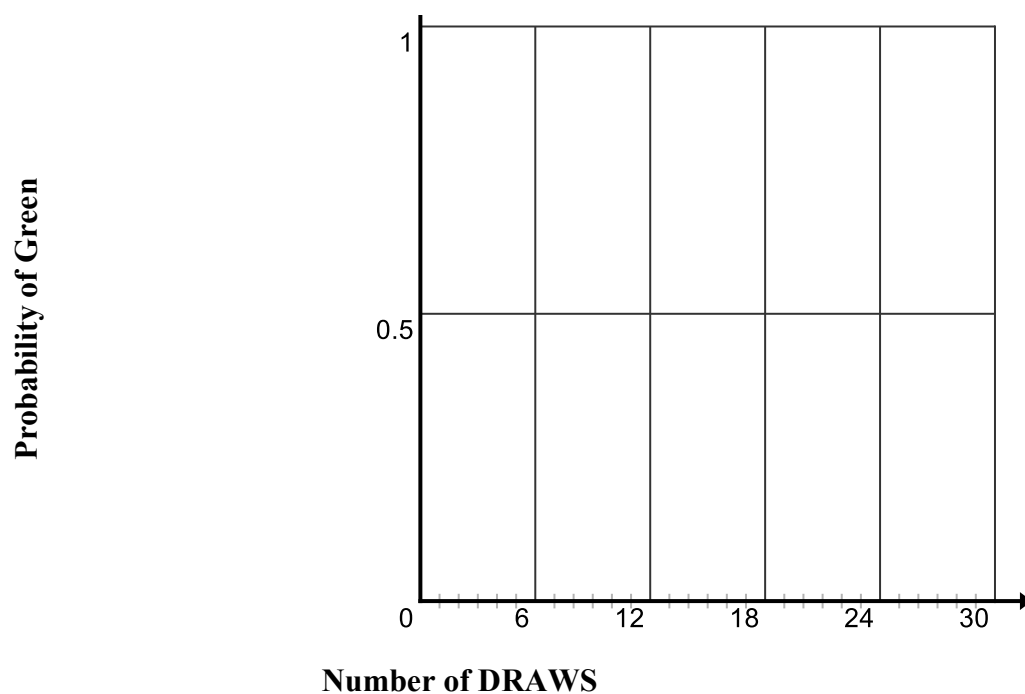
c. $P(G) =$ _____

d. $P(G) =$ _____

e. $P(G) =$ _____

3. Make a graph of your group’s samplings.

Sampling $P(G)$



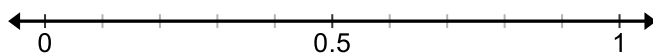
4. Explain what your graph shows about the probability of GREEN tiles.
5. Explain why your probability of GREEN tiles is a fraction (between 0 and 1).
6. How does knowing the probability of GREEN tiles help you know the probability of BLUE tiles?
7. Make a conjecture about how many GREEN tiles are in your bag if the bag contains 12 total tiles.
8. Count how many blue and green marbles/tiles are actually in your bag. Based on this information, what is the “theoretical” probability of drawing a green tile from your bag?
9. How did your group’s “experimental” $P(G)$ compare with the “theoretical” $P(G)$?
10. In your own words, what do you think:

“Experimental” probability means?



“Theoretical” probability means?

11. Place the theoretical probability of drawing a green for each group in the class on the number line below.



12. Which groups are **most** and **least** likely to have an outcome of drawing a green out of the bag? Justify your answer.



As groups discuss:

13. Suppose you had a bag of 1000 blue and green tiles, how many times do you think you would need to draw tiles to make an accurate prediction of the number of blue and green tiles are actually in the bag? Explain.



14. You're a teacher in a 7th grade math class and you want to create an experiment for your class with red, yellow and purple marbles in a bag. You want the theoretical probability of drawing a red marble to be $\frac{1}{4}$, the theoretical probability of drawing a yellow to be $\frac{1}{4}$ and the theoretical probability of drawing a purple to also be $\frac{1}{2}$. If you want a total of 120 marbles in the bag:

- a. How many red marbles should you put in the bag? _____
- b. How many yellow marbles should you put in the bag? _____
- c. How many purple marbles should you put in the bag? _____

Review exercises:

15. Without using a calculator, determine which fraction is larger. Justify your answer with an explanation and model.

a. $\frac{1}{8}$ — $\frac{1}{9}$

b. $\frac{7}{8}$ — $\frac{8}{9}$

c. $\frac{8}{15}$ — $\frac{8}{17}$

d. $\frac{9}{21}$ — $\frac{13}{22}$

1.1a Homework: Probability Predictions

1. You flipped a coin 50 times and got 23 heads. What is the experimental probability of getting a head? Write your answer as a fraction, decimal and percent.

2. If you flipped the coin 100 times, how many heads would you expect to get? Explain your answer.

3. A coin is tossed 20 times. It lands on heads 9 times. What is $P(H)$ according to your experiment? Write your answer as a fraction, decimal and percent.

4. You're a teacher in a 7th grade math class and you want to create an experiment for your class with red, yellow and purple marbles in a bag. You want the theoretical probability of drawing a red marble to be $\frac{1}{2}$, the theoretical probability of drawing a yellow to be $\frac{1}{2}$, and the theoretical probability of drawing a purple to also be $\frac{1}{6}$. If you want a total of 1260 marbles in the bag:
 - a. How many red marbles are you going to put in the bag? Why?
 - b. How many yellow marbles are you going to put in the bag? Why?
 - c. How many purple marbles are you going to put in the bag? Why?

4. **Challenge:** You've decided you want to make the marble experiment a little more difficult. You want to use 400 marbles and you want six different colors—blue, red, green, yellow, purple, and pink. You also do not want more than two colors to have the same probability. State the number of each color you are going to put in the bag and what the theoretical probability of drawing the color will be (answers will vary.)
 - a. Blue: $P(B)$ _____ and actual number of blue _____
 - b. Red: $P(R)$ _____ and actual number of red _____
 - c. Green: $P(G)$ _____ and actual number of green _____
 - d. Yellow: $P(Y)$ _____ and actual number of yellow _____
 - e. Purple: $P(Purple)$ _____ and actual number of purple _____
 - f. Pink: $P(Pink)$ _____ and actual number of pink _____

Answer the following:

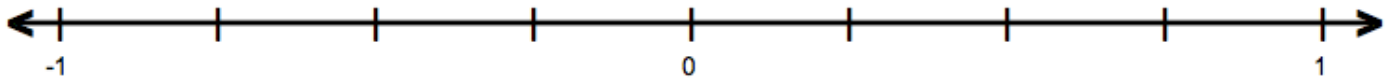
5. Without using a calculator, determine which fraction is bigger in each pair. Justify your answer with picture and words.

a. $\frac{1}{3}$ or $\frac{1}{2}$

b. $\frac{3}{7}$ or $\frac{3}{5}$

6. Place the fractions on the number line below.

$$\frac{1}{2}, \quad \frac{3}{5}, \quad \frac{1}{3}, \quad \frac{2}{5}$$



7. Order the following fractions from *least to greatest*.

$$\frac{1}{2}, \quad \frac{3}{5}, \quad \frac{1}{3}, \quad \frac{2}{5}$$

Spiral Review

1. Use a bar model to represent $\frac{3}{4}$ of a whole.

2. Solve using bar model $\frac{1}{2} + \frac{3}{7} = \underline{\hspace{2cm}}$

1.1b Class Activity: Probability—Race to the Top

- For the horse race experiment you will need two dice. For each roll of the dice record the sum in the appropriate column below by shading in the box. For example, a roll of 2,5 means you will shade one box in the 7 column, a roll of 1,4 means you will shade one box in the 5 column. Do this 30 times. ***BEFORE you start, predict which horse will win (2 through 12).***

2	3	4	5	6	7	8	9	10	11	12

- Which horse won (had the most rolls) in your group? _____
- List your group's experimental probability for each outcome:

a. $P(2)$ _____	e. $P(6)$ _____	i. $P(10)$ _____
b. $P(3)$ _____	f. $P(7)$ _____	j. $P(11)$ _____
c. $P(4)$ _____	g. $P(8)$ _____	k. $P(12)$ _____
d. $P(5)$ _____	h. $P(9)$ _____	
- List the horse that won for each of the groups in your class. Which horse won the most often?
- Create a class histogram that combines the data from all of the groups' histograms. What do you notice about the histogram?

2	3	4	5	6	7	8	9	10	11	12

6. Which horse won the most often for all the groups? Why?



7. Do you think that this game is fair? Why or why not?



9. What are all the possible outcomes when you roll two dice? In your group, organize these possible outcomes on a chart of your choosing.

10. How many total outcomes did you get? Explain the system you used to get all those outcomes.

11. Use the above information to determine the probability for each outcome:

$P(1)$ _____

$P(2)$ _____

$P(3)$ _____

$P(4)$ _____

$P(5)$ _____

$P(6)$ _____

$P(7)$ _____

$P(8)$ _____

$P(9)$ _____

$P(10)$ _____

$P(11)$ _____

$P(12)$ _____

12. How do you think the individual probabilities relate to the probabilities of the sums? For example,

$P(1, 6) = \frac{1}{36}$ but $P(7) = \frac{1}{6}$, why is this true?

1.1b Homework: Probability

Write all probabilities as a fraction, decimal and percent.

1. Find all the possible outcomes of flipping a coin (heads or tails) FOUR times. In other words, how many different ways are there to get heads and/or tails in four flips?

2. Based on #1, what is the theoretical probability that you:
 - a. Get HEADS for *all* four flips?

 - b. Get HEADS *at least* once in four flips?

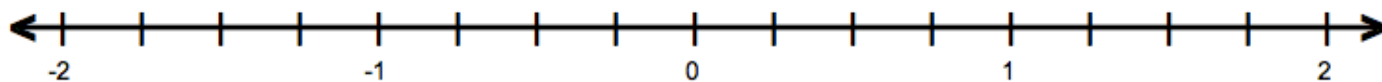
 - c. Get HEADS *exactly* three times in four flips?

3. Without using a calculator, determine which fraction is bigger in each pair. Justify your answer.
 - a. $\frac{3}{7}$ or $\frac{3}{8}$

 - b. $\frac{7}{13}$ or $\frac{9}{20}$

4. Place the fractions on the number line below.

$$\frac{6}{5}, \quad \frac{3}{10}, \quad \frac{1}{5}, \quad \frac{3}{2}$$



5. Order the following fractions from *least to greatest*.

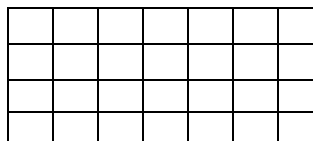
$$\frac{6}{5}, \quad \frac{3}{10}, \quad \frac{1}{5}, \quad \frac{3}{2}$$

Spiral Review

1. Write $\frac{1}{3}$ as a percent.

2. What is the area of the following figure?

4 in.




7 in.

3. Write one fourth as a fraction, decimal, and percent.

1.1c Class Activity: Probability Continued and Fair Game?

Use the data from problem 1 on Homework 1.1.b to answer the following:

1. What is the probability of getting exactly one HEAD on the first flip? _____
2. What is the probability of getting one HEAD on the first *and* second flips? _____
3. What is the probability of getting one HEAD on the first, second *and* third flips? _____
4. What is the probability of getting one HEAD on *all four* flips? _____

5. What pattern do you see emerging? 

6. After two flips of the coin, what is the probability of getting *at least* one HEAD? _____

7. After three flips of the coin, what is the probability of getting *at least* one HEAD? _____

8. After four flips of the coin, what is the probability of getting *at least* one HEAD? _____

9. What do you notice about the probabilities in questions 6-8?



10. Explain why the numerator is always one less than the denominator when finding the probability for at least one HEAD.

Fair Games:

Games 1 and 2: Use two dice OR an electronic random number generator on a calculator (on the TI-73 go to Math, PRB, dice, enter, type in 2 for two dice, then hit enter to roll the dice.) Play the game in pairs. One person will be “even” the other will be “odd”. For the “Addition Game” the “even” person earns a point if the sum of the two dice is even and the “odd” person earns a point if the sum is odd. For the “Multiplication Game” the “even” person gets a point if the product of the dice is even and the “odd” person gets a point if the product is odd.

Game 1: The Addition Game

1. Do you think the Addition Game will be fair—do “odd” and “even” have the same chance at winning? Explain.
2. Play the game—36 rolls of the dice. Based on your data, what is the experimental probability of rolling an odd sum? Probability of rolling an even sum?
 $P(\text{odd}) = \underline{\hspace{2cm}}$ $P(\text{even}) = \underline{\hspace{2cm}}$
3. Look back at the data you gathered from the probability experiment in 1.1b Class Activity (Race to the Top game). What was the theoretical probability of rolling an odd sum or even sum from that data?
 $P(\text{odd}) = \underline{\hspace{2cm}}$ $P(\text{even}) = \underline{\hspace{2cm}}$
4. Do you think the addition game is a fair game? Explain.

Game 2: The Multiplication Game

1. Do you think the multiplication game will be fair—do “odd” and “even” have the same chance at winning? Explain.
2. Play the game—36 rolls of the dice. Based on your data, what is the experimental probability of rolling an odd product? Probability of rolling an even product?
 $P(\text{odd}) = \underline{\hspace{2cm}}$ $P(\text{even}) = \underline{\hspace{2cm}}$
3. How will the sample space for the multiplication game be the same and/or different from the addition game?
4. Find the theoretical probability of rolling an odd product and even product?
 $P(\text{odd}) = \underline{\hspace{2cm}}$ $P(\text{even}) = \underline{\hspace{2cm}}$
5. Do you think the multiplication game is a fair game? Explain why or why not.

1.1c Homework: Probability Continued

Write all probabilities as fractions, decimals and percents.

1. Without using a calculator, for each pair, determine which fraction is bigger. Justify your answer.

a. $\frac{1}{9}$ or $\frac{1}{10}$

b. $\frac{8}{9}$ or $\frac{9}{10}$

c. $\frac{5}{9}$ or $\frac{5}{11}$

d. $\frac{5}{12}$ or $\frac{5}{14}$

2. A bag of marbles contains 3 red marbles, 5 blue marbles, and 2 yellow marbles.

a. What is $P(\text{red})$?

b. What is $P(\text{blue})$?

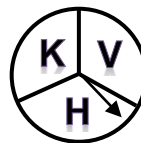
c. What is $P(\text{yellow})$?

d. What is the most likely outcome when drawing a marble out of the bag? Explain.

e. What is the least likely outcome when drawing a marble out of the bag? Explain.

f. Have you been computing theoretical or experimental probabilities? Explain.

3. A spinner contains three letters of the alphabet.



a. How many outcomes are possible if the spinner is spun three times?

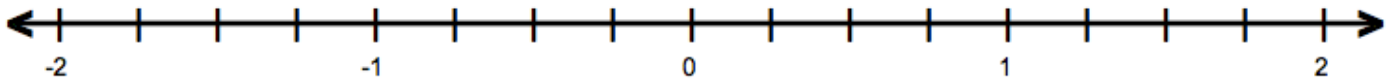
b. List all of the outcomes for spinning three times.

c. What is the probability of getting exactly one H in three spins?

- d. What is the probability of getting two V's on three spins?
- e. What is the probability of getting three K's on three spins?
- f. What outcome(s) is/are most likely for three spins?
- g. What is the probability of getting a constant in three spins?

4. Place the fractions on the number line below.

$$\frac{2}{3}, \quad \frac{3}{10}, \quad \frac{1}{2}, \quad \frac{3}{2}, \quad \frac{4}{3}$$

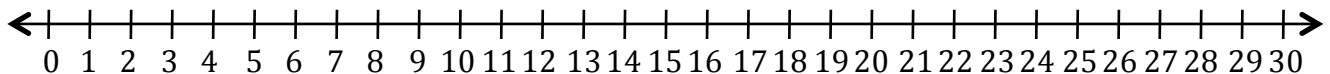
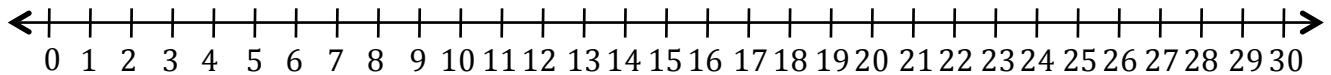


5. Order the following fractions from *least to greatest*.

Spiral Review

1. Model $\frac{5}{9}$

2. Use a number line to solve $24 - 7$.



3. Simplify: $\frac{2}{3} + \frac{3}{6}$ Draw a model if necessary.

1.1d Self-Assessment: Section 1.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Understand and apply likelihood of a chance event as between 0 and 1. [1a, 2a]	I'm not sure how to express a probability.	I know a probability is expressed as the relationship between possible outcomes and total outcomes, but I'm not sure how exactly to write that number.	I know that a probability is expressed as the quotient between possible outcomes and total outcomes, but I struggle with transitioning between fraction, decimal and percent forms.	I can easily express the probability of an event as a decimal, fraction, and percent.
2. Approximate probability by collecting data on a chance process (experimental probability).	I don't understand how to collect and record data in a probability experiment.	I can collect data on a chance process, but I don't know how to use that data to calculate experimental probability.	I can collect data on a chance process and calculate experimental probability using the data. However, I have a hard time explaining how the data relates to the theoretical probability of the event.	I can collect data on a chance process and calculate the experimental probability using the data. I can also explain how the experimental probability is different from theoretical probability.
3. Calculate theoretical probabilities on a chance process for simple events. [2a]	I don't understand the difference between a theoretical probability and an experimental probability so I'm not sure how to find it.	I can calculate theoretical probabilities for simple events like those in Problem 2a. However, I have a hard time explaining how to find it or why a theoretical probability isn't always the same as the experimental probability.	I can calculate theoretical probabilities for simple events like those in Problem 2a. However I sometimes struggle with either explaining how to find it and/or why an experimental probability may not have the same result as the theoretical probability.	I can calculate theoretical probabilities for simple events as in Problem 2a. I can easily explain why the process works and why a theoretical probability may not be the same as the experimental probability.
4. Given the theoretical probabilities of a chance event, predict the approximate frequencies for a given number of trials in an experiment of the event. [3a, 3b]	I don't understand how a theoretical probability helps me predict the approximate frequencies of an experiment.	When given a theoretical probability, I know the process for predicting the frequency of a scenario such as Problem 3a on the following page, but sometimes I struggle.	When given a theoretical probability, I know the process for predicting the frequency of a scenario such as Problem 3a on the following page and can usually find an approximation for the frequencies.	When given a theoretical probability, I know and can explain the process for predicting the frequency of a scenario such as Problem 3a on the following page.
5. Use appropriate fractions, decimals and percents to express the probabilities. [1a, 2a]	Sometimes I can express probabilities as either a fraction, decimal, or percent, but not all three.	I can express probabilities as two of the following: fraction, decimal, or percent.	I can express probabilities as a fraction, decimal, and percent.	I can express and understand probabilities as a fraction, decimal, and percent. I can also explain when each is most appropriate.

Sample Problems for Section 1.1

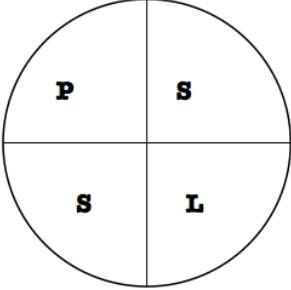
Express all probabilities as fractions, decimals, and percents.

1. Scarlet had a bag with red, green, and blue marbles. The following table shows what color she drew each time.

Draw	1	2	3	4	5	6	7	8	9	10
Result	red	green	blue	blue	red	blue	red	green	green	blue

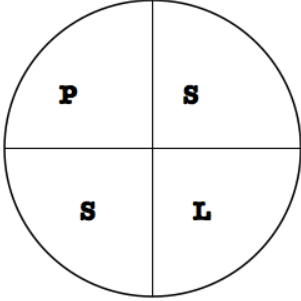
- a) Find the experimental probability of drawing a red marble.
- b) If there are 100 marbles in the bag, how many of them do you think are red? Justify your answer.
2. Find the theoretical probabilities for each of the following:

a.

If you flip a penny once, what is the probability of getting heads?	If you spin the following spinner once, what is the probability of spinning an S? 	If you roll a six-sided die once, what is the probability of rolling a 5?
---	---	---

Explain how you know your answer to each of the above is correct.

b.

<p>If you flip a penny three times, what is the probability of getting heads for all three flips?</p>	<p>If you spin the following spinner twice, what is the probability of spinning a P and an S in any order?</p> 	<p>If you roll a six-sided die twice, what is the probability of rolling a 5 (on either roll)?</p>
---	--	--

Explain how you know your answer to each of the above is correct.

3. Tommy has 30 marbles in a bag. If $\frac{1}{2}$ are blue, $\frac{1}{3}$ are red, and $\frac{1}{6}$ are yellow and he draws out a marble, records the color, returns it to the bag and repeats the process 10 times,
- a) approximately how many times should he expect to draw a blue marble?
- b) Suppose Tommy did the same experiment 1000 times, approximately how many times would Tommy expect to draw a blue marble?
- c) Which of the two approximations is likely to be the most accurate? Justify your answer.

Section 1.2: Understand/Apply Equivalence in Rational Number Forms. Convert Between Forms (Fraction, Decimal, Percent).

Section Overview:

In this section students solidify and practice rational number sense through the careful *review* of fractions, decimals and percents in this section. The two key objectives of this section are: a) students should be confidently able to articulate with words, models and symbols the relationship among equivalent fractions, decimals, and percents and b) students should understand and use models to find portions of different wholes.

The concept of equivalent fractions naturally leads students to the issues of ordering and estimation. Students will represent order of fractions on the real number line. It is important that students develop estimation skills in conjunction with both ordering and operating on positive and negative rational numbers.

Lastly, students look at percent as being a fraction with a denominator of 100. Percent and fraction contexts in this section should be approached intuitively with models. In section 1.3 students will begin to transition to writing numeric expressions.

Concepts and Skills to be Mastered (from standards)

1. Express probability using appropriate fractions, decimals, and percents.
2. Express and convert between rational numbers in different forms.
3. Draw models to show equivalence among fractions and rational numbers.
4. Compare rational numbers in different forms.
5. Find the percent of a quantity using a model.
6. Solve problems with rational numbers using models.

1.2a Class Activity: “10 × 10 Grids” & Conversion

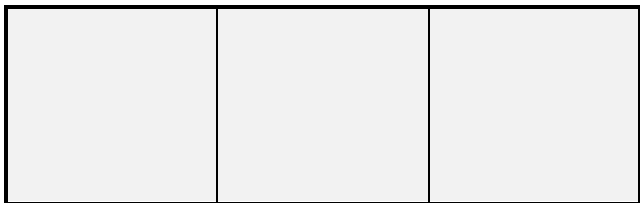
Write the equivalent values for the following parts of a candy bar.



The bar on the left is divided into two parts.

1 part = _____ (fraction) = _____ (decimal) = _____ (percent)

2 parts = _____ (fraction) = _____ (decimal)
= _____ (percent)



The bar on the left is divided into three parts.

1 part = _____ (fraction) = _____ (decimal) = _____ (percent)

2 parts = _____ (fraction) = _____ (decimal) = _____ (percent)

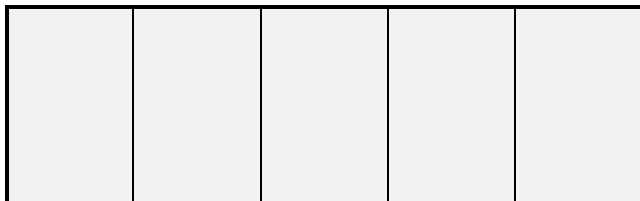


The bar on the left is divided into four parts.

1 part = _____ (fraction) = _____ (decimal) = _____ (percent)

2 parts = _____ (fraction) = _____ (decimal) = _____ (percent)

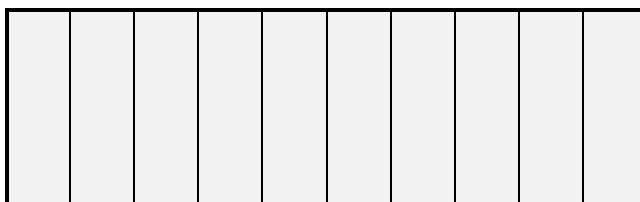
3 parts = _____ (fraction) = _____ (decimal) = _____ (percent)



The bar on the left is divided into five parts.

1 part = _____ (fraction) = _____ (decimal) = _____ (percent)

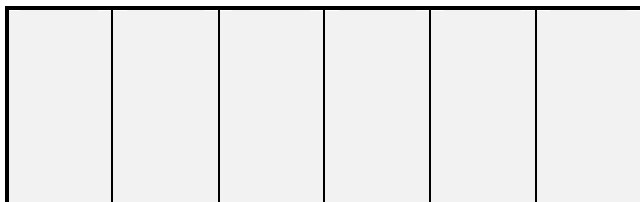
3 parts = _____ (fraction) = _____ (decimal) = _____ (percent)



The bar on the left is divided into ten parts.

3 parts = _____ (fraction) = _____ (decimal) = _____ (percent)

8 parts = _____ (fraction) = _____ (decimal) = _____ (percent)

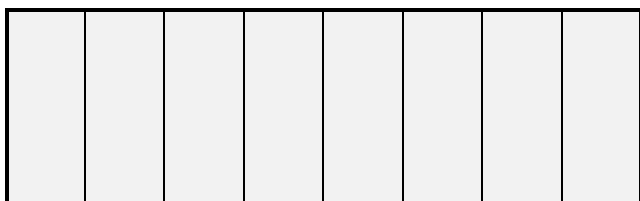


The bar on the left is divided into six parts.

1 part = _____ (fraction) = _____ (decimal) = _____ (percent)

2 parts = _____ (fraction) = _____ (decimal) = _____ (percent)

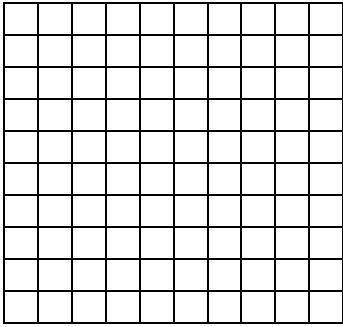
4 parts = _____ (fraction) = _____ (decimal) = _____ (percent)



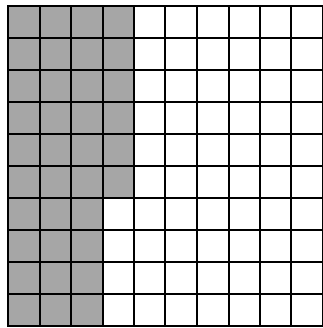
The bar on the left is divided into eight parts.

2 parts = _____ (fraction) = _____ (decimal) = _____ (percent)

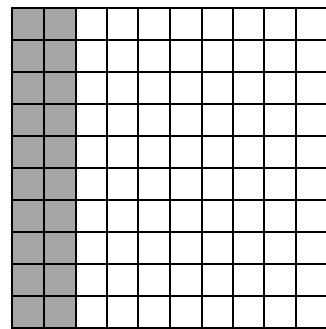
7 parts = _____ (fraction) = _____ (decimal) = _____ (percent)



1. To the left is a 10×10 Grid. Why do you think it is called a 10×10 grid?
2. Use the grid to show the fraction $\frac{6}{100}$. Explain why this model is correct.



3. What fraction is shown in this 10×10 grid? Explain
4. What is the decimal equivalent for this fraction?



5. What fraction is shown in this 10×10 grid?
6. What is the decimal equivalent for this fraction?



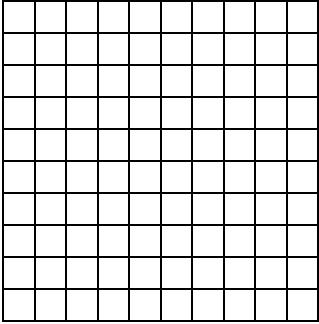
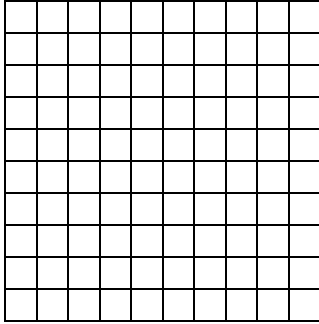
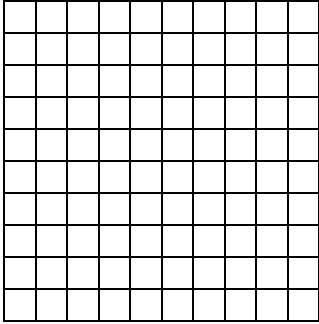
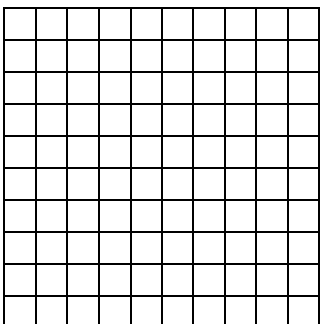
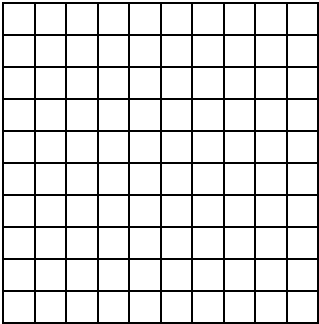
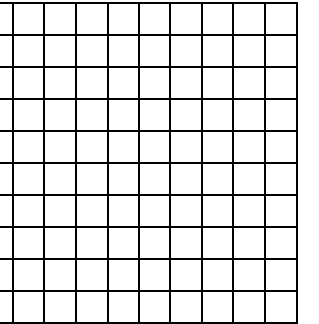
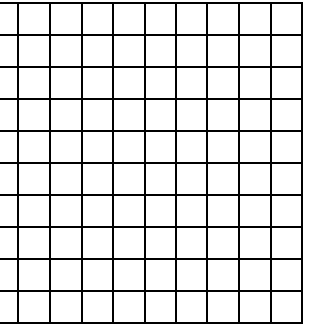
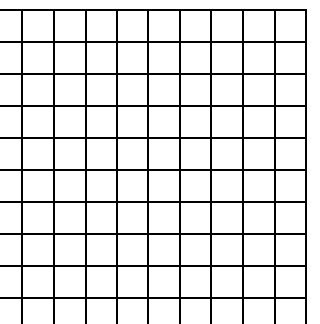
7. Shade the given decimal in each grid below:

a. 0.27	b. 0.35	c. 0.4	d. 0.125

8. Write the fraction equivalent for each decimal in #7 (above), in **simplest form**, here.

- a. _____ b. _____ c. _____ d. _____

9. Shade the fractional part of each grid. Then write the fraction as a decimal and a percent.

<p>a. $\frac{1}{2}$</p>  <p>decimal: _____</p> <p>percent: _____</p>	<p>b. $\frac{1}{4}$</p>  <p>decimal: _____</p> <p>percent: _____</p>	<p>c. $\frac{1}{10}$</p>  <p>decimal: _____</p> <p>percent: _____</p>	<p>d. $\frac{1}{5}$</p>  <p>decimal: _____</p> <p>percent: _____</p>
<p>e. $\frac{1}{100}$</p>  <p>decimal: _____</p> <p>percent: _____</p>	<p>f. $\frac{1}{8}$</p>  <p>decimal: _____</p> <p>percent: _____</p>	<p>g. $\frac{1}{3}$</p>  <p>decimal: _____</p> <p>percent: _____</p>	<p>h. $\frac{1}{9}$</p>  <p>decimal: _____</p> <p>percent: _____</p>

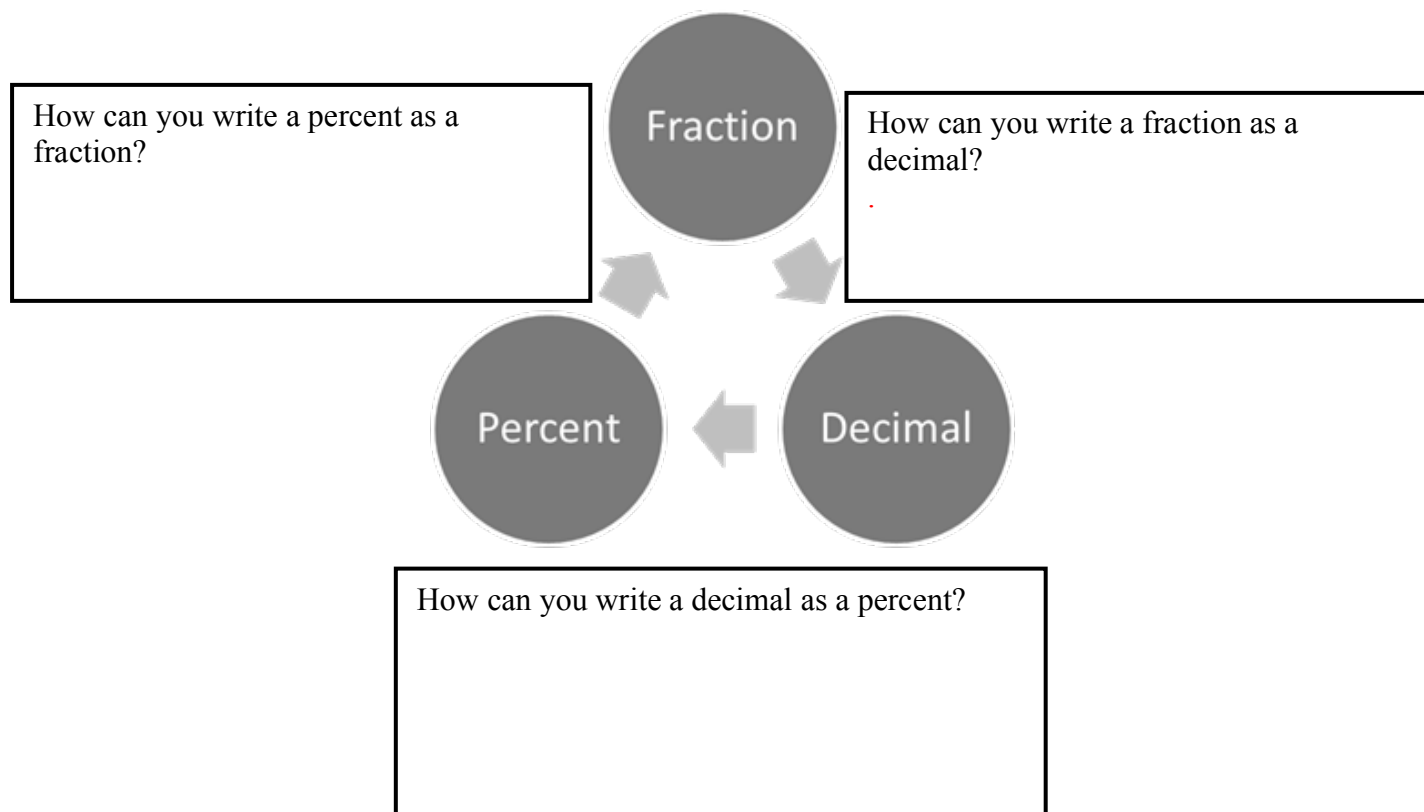
10. Use long division to show how you can convert each fraction to a decimal and then a percent. Then use equivalent fractions to do the same.

- a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{1}{10}$ d. $\frac{1}{5}$ e. $\frac{1}{100}$ f. $\frac{1}{8}$ g. $\frac{1}{3}$ h. $\frac{1}{9}$

$$\begin{array}{r} 0.5 \\ 2 \overline{)1.0} \end{array}$$

1.2a Homework: Converting Between Fractions, Decimals, and Percents

1. Fill in the boxes below to show how you can convert between fractions, decimals, and percentages.



Show how to use the steps you described above to complete the following problems.

2. Write 45% as a fraction.
3. Write $\frac{3}{5}$ as a decimal.
4. Write $\frac{1}{9}$ as a decimal.
5. Write 0.45 as a percent.
6. Write 1 as a percent.

Fill in each blank with the equivalent fraction, decimal or percent. Use bar notation for repeated decimals.

Show your work over here!

	Fraction	Decimal	Percent
7.	$\frac{6}{10}$		
8.	$\frac{4}{25}$		
9.		0.42	
10.		0.8	
11.			32%
12.	$\frac{9}{20}$		
13.			21%
14.		0.06	
15.			7%
16.	$\frac{1}{8}$		
17.		0.99	
18.			80%
19.	$\frac{1}{4}$		
20.			20%
21.	$\frac{6}{15}$		
22.		1.5	
23.			250 %
24.		3.0	
25.	$\frac{8}{11}$		
26.	$\frac{2}{3}$		

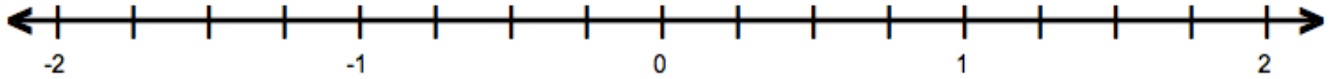
Spiral Review

1. Order the following fractions from *least to greatest*.

$$\frac{1}{2}, \frac{4}{5}, \frac{1}{3}, \frac{2}{5}$$

2. Write three-fourth as a fraction, decimal, and percent.

3. Place each fraction on the number line below $\frac{6}{5}$, $\frac{3}{10}$, $\frac{1}{5}$, $\frac{3}{2}$

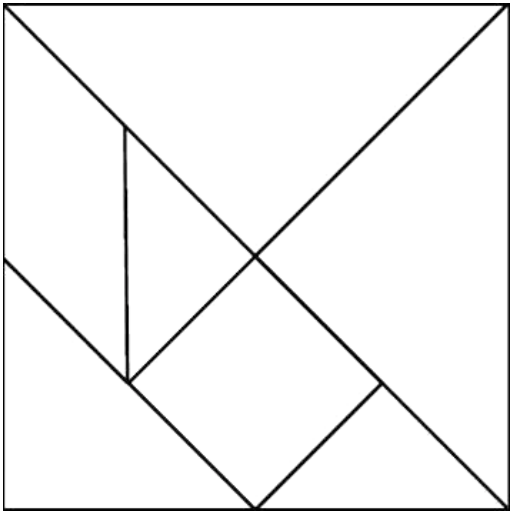


4. Without using a calculator, determine which fraction is bigger in the pair. Justify your answer.

$$\frac{3}{7} \quad \text{or} \quad \frac{3}{8}$$

1.2b Class Activity: Using Tangrams to Understand Fractions and Decimals

Below is a square created out of tangrams.



- 1. With a partner, take two different colored tangram sets and recreate the square shown above.
- 2. Using your different colored shapes, compare the small, medium, and large triangles to each other. How do they compare in size?
- 3. Now look at the small square and parallelogram. What do you observe?
- 4. Complete each equation. Find as many solutions as possible. The first one is started for you.

a. one small square =

one small square = 2 small triangles	one small square = $\frac{1}{2}$ of the large triangle

b. one large triangle=

c. one parallelogram =

d. one medium triangle =

5. Find the value of each shape relative to the entire square. Remember the square represents 1 whole. Record your findings in the table below.

Name of Shape	Fraction	Decimal	Percent
Large triangle			
Medium triangle			
Small triangle			
Small Square			
Parallelogram			

Reduce/simplify each fraction. Draw a model to show the equivalence between the original fraction and the reduced one:

6. $\frac{4}{10} =$

7. $\frac{12}{18} =$

8. $\frac{4}{12} =$

Find an equivalent fraction for each. Draw a model to show the equivalence between the original fraction and the new one.

9. $\frac{1}{3} = \frac{?}{9}$

10. $\frac{3}{7} = \frac{18}{?}$

11. $\frac{4}{5} = \frac{?}{25}$

Change each to a mixed number. Draw a bar model to show the equivalence between the original fraction and the new one.

12. $\frac{7}{5} =$

13. $\frac{25}{3} =$

14. $\frac{18}{4} =$

1.2b Homework: Equivalent Fractions

1. Simplify each fraction. Draw a model for questions “a” and “b” to show equivalent fractions.

a. $\frac{4}{6} =$	b. $\frac{3}{9} =$	c. $\frac{10}{18} =$
d. $\frac{14}{21} =$	e. $\frac{9}{21} =$	f. $\frac{7}{35} =$

2. Write an equivalent fraction. Draw a bar model for questions “a” and “b” to show the equivalent fractions.

a. $\frac{1}{2} = \frac{\quad}{6}$	b. $\frac{2}{5} = \frac{\quad}{15}$	c. $\frac{2}{3} = \frac{\quad}{15}$
d. $\frac{4}{7} = \frac{\quad}{14}$	e. $\frac{5}{8} = \frac{\quad}{24}$	f. $\frac{3}{4} = \frac{\quad}{24}$

3. Change each fraction to a mixed number. Draw a bar model for questions “a” and “d”.

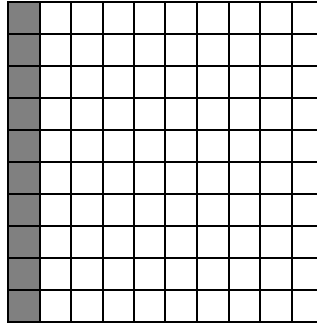
a. $\frac{10}{3} =$	b. $\frac{29}{4} =$	c. $\frac{25}{9} =$
d. $\frac{25}{12} =$	e. $\frac{20}{7} =$	f. $\frac{19}{5} =$

Spiral Review

1. Write $\frac{2}{3}$ as a decimal and percent.

$\frac{2}{3}$		
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2. Write this model in fraction form then simplify.



Use long division to show how you can convert this fraction to a decimal and then a percent

a) $\frac{1}{5}$

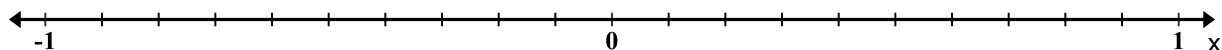
b) $\frac{3}{8}$

1.2c Class Activity: Rational Number Ordering and Estimation

1. Plot each fraction on the number line below.

a. $\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, \frac{2}{5}$

b. $-\frac{1}{2}, -\frac{1}{3}, -\frac{3}{4}, -\frac{2}{5}$



c. Compare the fractions using $<$, $>$, or $=$.

$$\frac{1}{2} \text{ — } \frac{1}{3}$$

$$-\frac{1}{2} \text{ — } -\frac{1}{3}$$

$$\frac{2}{5} \text{ — } \frac{3}{4}$$

$$-\frac{2}{5} \text{ — } -\frac{3}{4}$$

d. What differences do you observe when comparing the fractions in part c?



e. How does the number line help you determine which number is larger?

2. Classify these fractions as close to 0, close to $\frac{1}{2}$, or close to 1.

$$\frac{1}{2}, \frac{1}{9}, \frac{5}{8}, \frac{5}{6}, \frac{7}{8}, \frac{1}{5}, \frac{5}{9}, \frac{3}{8}, \frac{1}{7}$$

Close to 0	Close to $\frac{1}{2}$	Close to 1
------------	------------------------	------------

3. Order the fractions from least to greatest.

Fractions: _____, _____, _____, _____, _____, _____, _____, _____, _____

4. Classify these fractions as close to 0, close to $-\frac{1}{2}$, close to -1 .

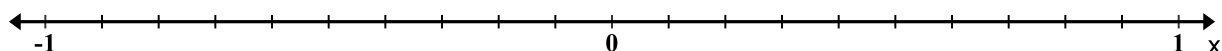
$$-\frac{1}{2}, -\frac{4}{28}, -\frac{3}{5}, -\frac{2}{7}, -\frac{5}{14}, -\frac{2}{11}, -\frac{4}{5}, -\frac{7}{9}$$

Close to 0	Close to $-\frac{1}{2}$	Close to -1

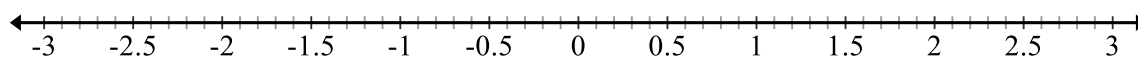
5. Order the above numbers from least to greatest.

Fractions: _____, _____, _____, _____, _____, _____, _____, _____

Now approximate their location on the number line below:



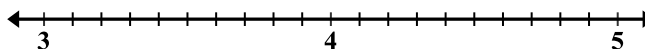
6. Approximate where each value is located on the number line below. State a “common” fraction and decimal you can use to help you find the approximate location for each. The first one is done for you.



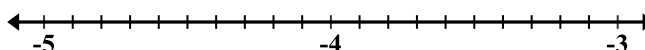
a. $0.32 \approx 0.333 \approx \frac{1}{3}$	b. 0.67	c. 0.76	d. 0.98
e. -0.24	f. 2.32	g. -3.38	h. 1.76

Plot each fraction on the number line. Fill in the blank using $<$, $>$, or $=$.

7. $4\frac{3}{10}$ _____ $4\frac{2}{7}$

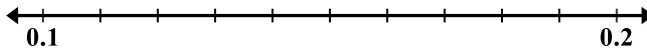


8. $-4\frac{3}{10}$ _____ $-4\frac{2}{7}$



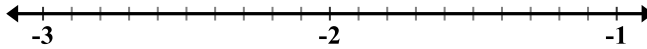
Plot each fraction on the number line. Fill in the blank with $<$, $>$, or $=$. How do you know your answer is correct? Justify your answer.

9. $0.\overline{14}$ 0.14



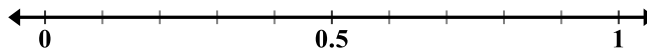
Justification:

10. -2.15 -2.13



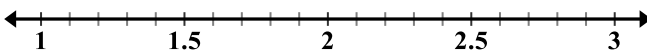
Justification:

11. 0.15 $\frac{3}{20}$



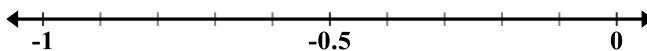
Justification:

12. $2\frac{2}{3}$ 2.6



Justification:

13. -0.3 $-\frac{1}{3}$



Justification:

14. Is 0.74 to the left or right of $\frac{3}{4}$? Explain.

15. Is 1.26 to the left or right of $1\frac{1}{4}$? Explain.

16. Give a fraction and decimal approximation of $\frac{71}{102}$.

17. Give a fraction and decimal approximation of $\frac{9}{23}$.

1.2c Homework: Rational Number Ordering and Estimation

ESTIMATE each of the following. Justify your answer with either words or a model. Indicate if your estimate is larger or smaller than the exact answer.

	Fraction	≈ Decimal	≈ Percent	Justification
1.	$\frac{11}{21}$			
2.	$\frac{16}{27}$			
3.	$\frac{89}{99}$			
4.	$\frac{13}{17}$			
5.	$\frac{5}{23}$			

Fill in the blank with $<$, $>$, or $=$. Justify each with a picture or number-line or explanation.

6. $1\frac{4}{25}$ 1.2 Justification:

7. $-1\frac{4}{25}$ -1.2 Justification:

8. -2.34 $-2\frac{4}{11}$ Justification:

9. $\frac{5}{11}$ $0.\overline{45}$ Justification:

10. $4\frac{5}{6}$ $4\frac{10}{13}$ Justification:

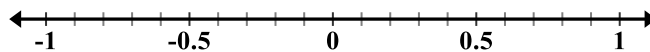
11. $-3\frac{19}{20}$ -3.94 Justification:

12. Order the following set of numbers from least to greatest: $1\frac{3}{4}$, 1.73, $1\frac{3}{5}$, 1.78

13. Order the following set of numbers from least to greatest: -0.26 , $-\frac{3}{10}$, -0.35 , $-\frac{6}{25}$

14. Plot each rational number on the number line. Write them in order from least to greatest:

$$\frac{3}{8}, -0.38, \frac{3}{7}, -0.43$$



Spiral Review

Change each fraction to a mixed number. Draw a bar model for each.

1. $\frac{7}{3}$

2. $\frac{11}{3}$

3. Compare these two fractions: $\frac{6}{12}$ $\frac{8}{14}$

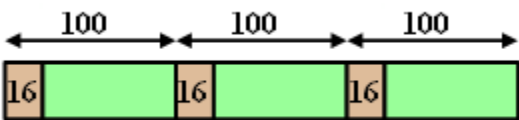
1.2d Class Activity: Probability, Fractions, and Percentage

1. A bag contains 100 marbles. The table below shows how many red, blue, green and yellow marbles are in the bag. Use that data to complete the table below. The first row is completed for you.

Color	Number of Marbles
Red	16
Blue	24
Green	45
Yellow	15

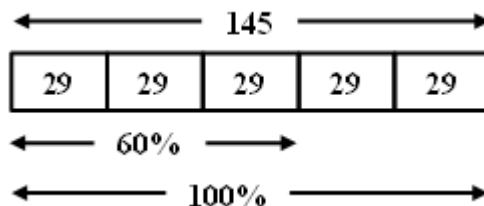
Color of Marble	<u>Probability of drawing the colored marble</u>	<u>Ratio of colored marble to all marbles</u>	<u>Percentage of colored marble within all marbles</u>
1. Red	$\frac{16}{100} = \frac{4}{25}$	$\frac{16}{100} = \frac{4}{25}$	16%
2. Blue			
3. Green			
4. Yellow			
5. Orange			
6. Red OR blue			
7. Red, blue, green OR yellow			

2. Suppose you want to create a bag with 300 red, blue, green and yellow marbles that has the same probability for drawing each color as #1 above. Find how many of each color you will need.

Color	Probability→Amount	Percent→Amount	Model
Red	$\frac{4}{25} = \frac{4}{25} \times \frac{12}{12} = \frac{48}{300}$ or $\frac{4}{25} = \frac{16}{100} = \frac{3 \times 16}{3 \times 100} = \frac{48}{300}$	$16\% = \frac{16}{100}$ $\frac{16 \cdot 3}{100 \cdot 3} = \frac{48}{300}$ 48 red marbles	
Blue			
Green			
Yellow			

Modeling to find the part, whole or percent

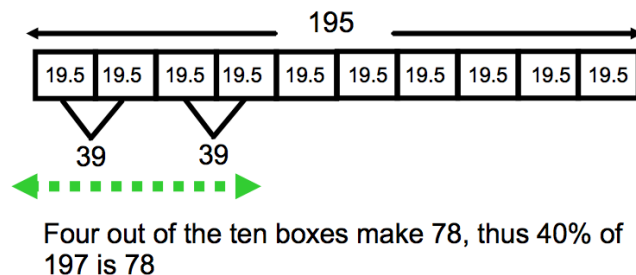
Example 1: Sally wants to compute 60% of 145. She knows that $60\% = \frac{60}{100} = \frac{6}{10} = \frac{3}{5}$. This means that 145 should be divided into either 10, 10 or 5 parts. Dividing 145 into 10 parts would mean 14.5 in each part, dividing into 10 parts would mean 14.5 in each part; dividing into 5 parts would mean 29 in each part. She decided to use 5:



Sixty percent of 145 is the same as $\frac{3}{5}$ of 145; thus 60% of 145 is $29 + 29 + 29$ or 87.

Example 2: What percent of 195 is 78?

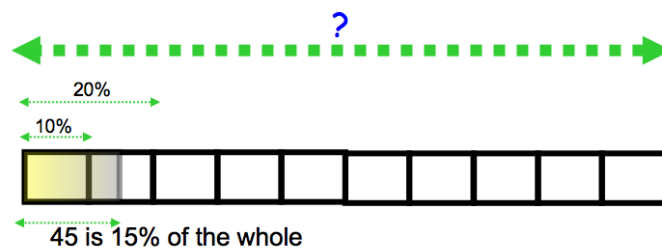
We know that 10% of 195 is 19.5. So one way to think about this is to divide 195 into ten parts:



Above we can see that two 10% portions make 39; thus four 10% portions make 78. So, 78 is 40% of 195.

Example 3: 15% of what is 45?

We know that percents are relationships of 100 and that if we can find 10%, we can compute anything fairly easily. If 45 is 15% of the whole, then 15 is 5% of the whole (15% divided into 3 is 5%, and 45 divided into 3 is 15.) That means that 5% of the whole is 15, so 10% is 30:



If each 10% portion is 30, then the whole is 30×10 or 300.

Use a model like the one shown previously to answer the following questions.

3. Find 60% of 180.

7. What percent of 120 is 48?

8. What percent of 95 is 19?

4. Find 40% of 80.

9. 12 is 10 percent of what number?

5. Find 25% of 324.

10. 45 is 15 percent of what number?

6. What percent of 80 is 60?

11. 30 is 25 percent of what number?

1.2d Homework: Probability, Fractions, and Percentage

Express each fraction as a percent.

1. $\frac{1}{5} =$

3. $\frac{3}{20} =$

2. $\frac{9}{10} =$

4. $\frac{2}{5} =$

Express each percent as a fraction in simplest form.

5. $60\% =$

8. $10\% =$

6. $80\% =$

9. $5\% =$

7. 75%

10. 25%

Solve each using a model and mental math. Show your work:

11. Find 80% of 150.

14. What percent of 240 is 60?

12. Find 40% of 40.

15. 40 is 8% of what number?

13. What percent of 30 is 15?

16. 60 is 30% of what number?

Use a model to ESTIMATE each. Indicate if your estimate is slightly higher or lower than the exact answer.

17. Estimate 74% of 24.

18. Makayla got a score of 77% on her English final. If there were 48 questions on the test, approximately how many questions did she get right?

19. Milo can run 10 miles in 60 minutes. He needs to reduce his time by 18%, approximately how many minutes does he have to take off his time?

20. The cycle store has a bike regularly priced at \$660, Tom negotiated a 32 % discount. What fraction can you use to estimate the money he saved? Approximately how much did he pay for the bike?

Spiral Review

1. Circle the number that's greater: 0.17 0.107 Justification:

2. Circle the number that's greater: 2.65 2.63 Justification:

3. Is 0.77 to the left or right of $\frac{3}{4}$ on a number line? Explain.

4. Write 32% as a decimal and fraction.

1.2e Class Activity: Applications with Models, Multi-Step Problems

Percent and fraction questions: Use a model to find the solutions.

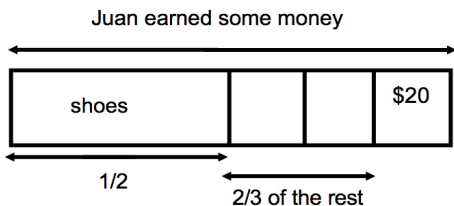


1. 60 is 40% of what number?

2. There are 36 students in a math class. $\frac{3}{4}$ of the students take an art class after their math class, the rest take a social studies class. How many students take art after math?

3. You get 80% correct on a history quiz with 150 questions. How many questions did you get correct?

4. Juan earned money for creating a webpage for a local business. He used $\frac{1}{2}$ of the money he earned for new shoes and $\frac{2}{3}$ of the rest for music. He has \$20 left. How much money did he earn for his work?



5. Lydia volunteers with an organization that helps older citizens take care of their yards. 75% of the volunteers in the organization are 20-30 years old. Of the remaining portion, 75% are over 30 and 25% are under 20. If there are 15 people under 20, how many people are in the organization?

6. There are 360 7th grade students at Eisenhower Middle School. One-fourth of the students went to Clermont Elementary. Of the rest, half went to Central Elementary and the others came from a variety of other elementary schools. How many students came from Central Elementary?

7. A snowboard at a local shop normally costs \$450. Over Labor Day weekend, the snowboard is on sale for 50% off. Customers who make their purchase before 8:00 AM earn an additional 10% off of the sale price. If Mia buys the snowboard before 8:00 AM, how much will she pay?
8. A local business is reviewing their expenditures. They found that they spend $\frac{1}{3}$ of their income on payroll, another $\frac{1}{2}$ goes back into the business to purchase inventory and pay for the facility, $\frac{1}{3}$ of the remainder goes to paying off their original small business loan. If they have \$100,000 left to reinvest in their company, what are their total expenditures?
9. At a Monument Valley High School, three fourths of the 7th grade students went to Salt Lake City on a field trip. Half of the rest of the students went to Monticello for a different field trip. If there were 12 students that did not go on either trip, how many 7th grade students are there in all?
10. Camilla earned \$160 over the summer. If she put 80% of her earnings into her savings account and spent 75% of the rest on a gift for her mother, how much money did she have left over?
11. Mila rode in a bike tour across Utah. On one particular day, 40% percent of her ride was uphill of the rest. Of her ride $\frac{1}{3}$ was down hill and $\frac{2}{3}$ was flat. If the flat portion of her ride was 36 miles, how far did she ride that day?
12. Marco's football team was 20 yards from the goal when they got possession of the football. At the end of one play, they got half way to the goal. After the second play, they made half that distance closer to the goal. After the third play, they got half the remaining distance. How far were they from the goal line before the fourth play?

1.2e Homework: Rational Numbers with Models, Multi-Step Problems

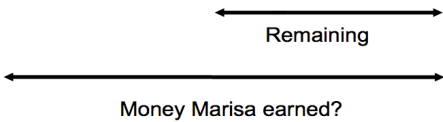
Use a model to fill in the table below:

	Fraction	Decimal	Percent	MODEL
1.	$\frac{3}{5}$			
2.	$\frac{4}{25}$			
3.		0.15		
4.	$\frac{2}{3}$			
5.		0.45		

Use a model to solve each problem.

6. Marisa earned money for helping a local business with organizing their inventory. She spent half of her earnings on accessories for her phone, and half of the remaining money on a gift for her mother. If she has \$15 left, how much did she make organizing the inventory?

Spent $\frac{1}{2}$ on phone accessories	$\frac{1}{2}$ the remaining on mom	\$15 left
--	------------------------------------	-----------



7. Pedro has \$80 left from the money he earned during the summer helping at his father’s business. He had spent $\frac{1}{3}$ of his earnings on climbing equipment, $\frac{1}{3}$ on camping gear and $\frac{1}{2}$ of the remainder on entertainment during the summer. How much did he earn during the summer?

8. Jose estimates that 50% of his income goes toward living expenses (rent, utilities and food). Of the rest, 50% goes to paying for his car and 25% to other expenses. If Jose has \$300 left at the end of the month, how much does he earn?
9. Marco earned \$360 helping his grandfather at his business. He spent $\frac{1}{4}$ of his earnings on a gift for his mother and put $\frac{2}{3}$ of the rest into a savings account. How much does he have left over for fun?
10. Julia found a great pair of boots for \$240, but that was more than she wanted to spend. A few months later they were on sale for 40% off. She searched online and found a coupon for 25% off the sale price. How much will she pay for the boots?
11. Jose earned money over the summer working at his family's store. He put three-fourths of the money he earned in his savings account and spent half of the rest. If he has \$120 left over, how much did he earn?
12. Jessi bought 18 gallons of paint to paint her house and garage. If she used 75% of the paint on the house and half of the rest on the garage, how much paint did she have left over?

Spiral Review

Express each fraction as a percent.

1. $\frac{1}{5} =$

3. $\frac{2}{5} =$

Express each percent as a fraction in simplest form.

3. 40%

4. 25%

1.2f Self-Assessment: Section 1.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Express probability using appropriate fractions, decimals, and percents. [1]	I sometimes can express probabilities as either a fraction, decimal, or percent, but not all three.	Most of the time I can express probabilities as two of the following: fraction, decimal, or percent.	I can express probabilities as a fraction, decimal, and percent.	I can express probabilities as a fraction, decimal, and percent. I can also justify the most appropriate form(s) of a number for a given context.
2. Express and convert between rational numbers in different forms. [2]	I struggle to convert between rational numbers in different forms and/or I'm not sure what a rational number is.	I can convert between rational numbers in two of the different forms: fraction, decimal, or percent.	I can convert between rational numbers in all three different forms (fraction, decimal, and percent).	I can convert between rational numbers in all three different forms. I can also explain when each may be more appropriate.
3. Draw models to show equivalence among fractions and rational numbers.	I struggle to draw models of rational numbers.	I can draw models of some, but not all forms of rational numbers: fractions, decimals, and percents.	I can draw models of all forms of rational numbers. I can use the models to show equivalence.	I can draw models of all forms of rational numbers. I can explain how the models show equivalence in different forms.
4. Compare rational numbers in different forms. [3]	I can order most sets of positive rational numbers like in #3 Set A on the following page. I have trouble with other forms	I can order most sets of positive and negative rational numbers if they are in the same form like in #3 Set A or B on the following page.	I can order sets of positive and negative rational numbers in two forms like in #3 Set A, B or C on the following page.	I can compare rational numbers in different forms such as #3 Sets A, B, C and D. I can also justify my comparison.
5. Find the percent of a quantity using a model. [4]	I can draw a model of a percent. I struggle to use that model to find percent of a quantity.	I can use a model to correctly find the percent of a quantity such as Problem 4b on the following page.	I can use a model to find an unknown quantity given a percent and can find the percent of a quantity such as Problem 4a and 4b.	I can use a model to find an unknown quantity given a percent and the percent of a quantity such as Problem 4a and 4b. I can explain how the model is related to the "mathematical" process for finding percents values.
6. Solve problems with rational numbers using models. [5]	I can draw a model of a rational number. I struggle to use that model to solve problems.	I can use a model to solve single-step problems with rational numbers such as Problem 5a or 5b on the following page.	I can use a model to solve one-step and multi-step problems with rational numbers such as Problem 5a – d on the following page.	I can use a model to solve one-step and multi-step problems with rational numbers such as Problem 5a – d. I can also explain how the model is related to the computational process.

Sample Problems for Section 1.2

- Rufina has a bag of marbles. She has 30 striped marbles, 4 cat's eye marbles and 10 solid color marbles. Rufina draws one marble out of her bag. Express each of the following probabilities as a fraction, decimal and percent.
 - P(striped)_____
 - P(solid)_____
- Complete the following chart of equivalent rational numbers.

Fraction	Decimal	Percent
$\frac{13}{25}$		
$\frac{84}{300}$		
	0.74	
	0.1325	
		81%
		1.5%

- Order the numbers in each box from least to greatest.

Set A

Set B

$\frac{1}{2}, \frac{3}{4}, \frac{4}{9}$	5.15, 5.002, 4.7, 5.1	$\frac{2}{3}, \frac{3}{10}, \frac{1}{4}, \frac{4}{9}$
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$-\frac{1}{2}, \frac{2}{7}, \frac{1}{2}, -\frac{8}{9}$	-7.34, -7.002, -7.7, -7.2	$\frac{3}{7}, -\frac{10}{2}, \frac{4}{3}, -\frac{9}{8}$
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Set C

Set D

$\frac{2}{7}, 0.24, \frac{1}{2}, 0.8$	2.54, $\frac{5}{2}, 2.7, \frac{15}{7}$	$0.1, \frac{10}{99}, \frac{4}{36}, 0.\overline{66}$
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-2.15, $\frac{17}{7}, 2.7, -2.105$	$-\frac{4}{5}, -\frac{5}{8}, 0.4, \frac{3}{8}$	$1.4, -\frac{10}{8}, -\frac{4}{3}, 1.08$
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4. Find each.
- a. 40% of what number is 50?
- b. What is 85% of 420?

5.

<p>a.</p> <p>Shauna wants to buy a sweater at the store. The sweater is normally \$28, but today it is on sale for 15% off. How much is the sweater?</p>	<p>b.</p> <p>In Mr. Garcia's 7th Grade Math class, $\frac{4}{5}$ of the students brought a pencil to class. If 6 people did not have pencils, how many students are in Mr. Garcia's class?</p>
<p>c.</p> <p>Uzumati has \$80. He spends $\frac{1}{5}$ on groceries. Then he spends 75% of what's left on two new Blu-ray's. How much money does he have left?</p>	<p>d.</p> <p>Ms. Wells surveyed her 205 students. She found that 80% of the students were wearing blue jeans. One student was wearing shorts. Of the remaining students, $\frac{3}{8}$ were wearing skirts. How many students were wearing skirts?</p>

Section 1.3: Solve Percent Problems Including Discounts, Interest, Taxes, Tips, and Percent Increase or Decrease.

Section Overview:

In this section, students continue to solve contextual problems with fractions, decimals and percent but begin to transition from relying solely on models to writing numeric expressions. In future chapters, students will extend their understanding by writing equations and proportional equations using variables.

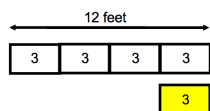
Concepts and Skills to be Mastered (from standards)

1. Find a percent or fractional portion of a whole with or without a model.
2. Solve percent problems involving percent increase and decrease (including discounts, interest, taxes, tips, etc.).
3. Develop numeric expressions from percent and fraction models.

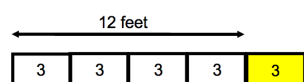
1.3a Class Activity: Model Percent and Fraction Problems

Use a model to solve each of the following multi-step problems. Then write a number sentence that reflects your model and answer.

1. Larry has a piece of rope that's 12 feet long.
 - a. He cuts off 25% of the rope off. How long is the rope now?



- b. Joe has a rope that is 25% longer than Larry's 12 foot long rope. How long is Joe's rope?



2. Lydia invested \$150. If Lydia earned 10% on her investment. How much money would she now have?

How much money would Lydia have if instead she lost 10% on her original investment?

3. A refrigerator costs \$1200 wholesale. If the mark-up on the refrigerator is 20%, what is the new price?
4. Rico's resting heart rate is 50 beats per minute. His target exercise rate is 350% of his resting rate. What is his target rate?
5. A pair of boots was originally priced at \$200. The store put them on sale for 25% off. A month later, the boots were reduced an additional 50% off the previous sale price. What is the price now?
6. Marie went out for dinner with her friend. The dinner cost \$24. Tax is 5% and Marcie wants to leave a 15% tip (she computed tip on the cost of dinner and tax.) How much will Marcie pay all together for dinner?

1.3a Homework: Model Percent and Fraction Problems

Use a model to answer each of the questions below. Then write a number sentence that reflects your model and answer.

1. Last year Cory harvested 42 tomatoes from his backyard garden. This year, his harvest increased by $\frac{1}{3}$. How many tomatoes did he harvest this year?
2. Jerry is taking care of a vacant lot in his neighborhood. There are approximately 64 thistle weeds in the lot. He decided to try a homemade weed killer his grandmother suggested. Five weeks later, the thistles have decreased by 75%. Approximately how many thistles are in the vacant lot now?
3. Maria is learning to play golf. She has been working particularly hard on driving. Before lessons, her drives average 240 yards. After her first lesson, her drives increase 25%. After her second lesson, they increase another 25%. How far are her average drives after two lessons?

4. Two stores have the same skateboard on sale. The original price of the skateboard is \$200. At store AAA, it's on sale for 30% off with a rewards coupon that allows the purchaser to take an *additional* 20% off the sale price at the time of purchase. At store BBB, the skateboard is on sale for 50% off. Will the price for the skateboard be the same at both stores? If not, which store has the better deal?
5. Two schools start with 1000 students. The first school's enrollment increase 20% in 2012 and then decreases 20% in 2013; the second school's enrollment stays constant in both 2012 and 2013. Which school has the most students now?

Review questions. Write all probabilities as fractions, decimals and percentages.

6. Suppose you were to roll a fair 6-sided number cube once, then flip a coin. List all the possible outcomes.
7. What is the probability of getting a 2 and heads?
8. What is the probability that you would roll an even number and flip heads?
9. What is the probability that you would roll an even number or flip heads?

Spiral Review

1. What is 85% of 420?
2. Fill in the equivalent decimal and percent for this fraction:

Fraction	Decimal	Percent
$\frac{9}{15}$		

3. Jana wants to buy a shirt at the store. The shirt is normally \$24, but today it is on sale for 15% off.
 - a. How much will she save?
 - b. How much will Jana pay for the shirt?

4. Change this fraction to a mixed number. Draw a bar model.

$$\frac{8}{3}$$

5. What is the greatest common factor of 12 and 18?

1.3b Class Activity: Percent and Fraction Problems Transition to Numeric Expressions

Part A: For each problem below, use a model to answer the question.

Context	Model	Fraction Change	Percent Change	Fractional Portion of the original	Percent of the original
1a. Li planted a 12 foot tall tree in her yard. 10 years later it is 18 feet tall.		$\frac{1}{2}$ factor increase of the original.	Grew by 50% of the original.	$\frac{3}{2}$ of the original height	150% of the original height
2a. Pedro had \$600 in his savings account. Five years later he has \$800.					
3a. It used to take Naya 10 minutes to walk to school. Now it takes her 8 minutes.					
4a. Mario's old car got 20 miles to the gallon. His new car gets 24 miles to the gallon.					
5a. The Castro's old dishwasher used 12 gallons of water per load. Their new dishwasher uses 8 gallons per load.					
6a. A pair of running shoes was originally \$75, they are on sale for \$60.					
7a. The pair of running shoes was originally \$75, they are on sale for \$50.					

Part B: These are the same questions as above in Part A. Use the model you created for each problem to write a numeric expression to answer the question.

Context	Fraction Change Expression	Percent Change Expression
1b. Li planted a 12 foot tall tree in her yard. 10 years later it is 18 feet tall.	$12(1) + 12\left(\frac{1}{2}\right)$ $= 12\left(\frac{3}{2}\right)$ $= 18$	$12(1) + 12(.5)$ $= 12(1.5)$ $= 18$
2b. Pedro had \$600 in his savings account. Five years later he has \$800.	$600() + 600()$ $= 600()$ $= 800$	$600() + 600()$ $= 600()$ $= 800$
3b. It used to take Naya 10 minutes to walk to school. Now it takes her 8 minutes.	$10() - 10()$ $= 10()$ $= 8$	$10() - 10()$ $= 10()$ $= 8$
4b. Mario's old car got 20 miles to the gallon. His new car gets 24 miles to the gallon.		
5b. The Castro's old dishwasher used 12 gallons of water per load. Their new dishwasher uses 8 gallons per load.		
6b. A pair of running shoes was originally \$75, they are on sale for \$60.		
7b. The pair of running shoes was originally \$75, they are on sale for \$50.		

1.3b Homework: Percent and Fraction Problems Transition to Numeric Expressions

For each problem below, use a model to answer the question.

Context	Model	Fraction Change	Percent Change	Fraction	Percent
1. At the beginning of the year there were 32 students in Ms. Herrera's class. There are now 36 students in her class.					
2. At Lincoln Middle School 300 students participated in Science Fair in 2011. In 2012 400 students participated.					
3. Celesta used to spend 90 minutes a day on the phone. She now spends 60 minutes a day.					
4. During the school year, Jose worked 15 hours a week. During the summer he works 30 hours a week.					
5. A local business used to spend \$800 a month on employee rewards. They now spend \$1000.					

For each context, write a fraction change and percent change expression.

Context	Fraction Change Expression	Percent Change Expression
6. Tio's mom used to drink 4 diet sodas a day. Now she drinks 1 a day.		
7. In a small town in Utah, 40,000 homes used to have land-line phones. Now 35,000 homes have land-line phones.		
8. A small business's profits in 2011 were \$120,000. In 2012 they were \$150,000		
9. Nina and her friend went to dinner. Their bill came to \$40. Nina paid \$60 to cover the bill, tax and tip.		
10. A local business used to spend \$800 a month on employee rewards. They now spend \$1000.		

Spiral Review

1. Alyssa's resting heart rate is 50 beats per minute. Her target exercise rate is 325% of her resting rate. What is her target rate?

2. Samantha has a bag of marbles. She has 21 striped marbles, 5 cat's eye marbles and 10 solid color marbles. Samantha draws one marble out of her bag. Express each of the following probabilities as a fraction, decimal and percent.

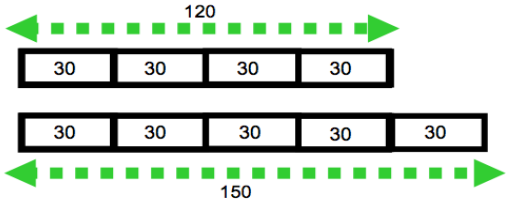
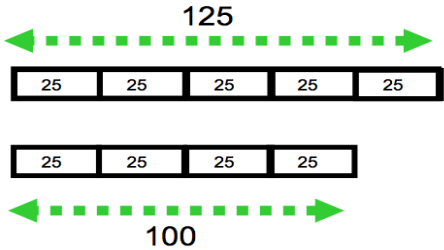
a. P(striped) _____

b. P(solid) _____

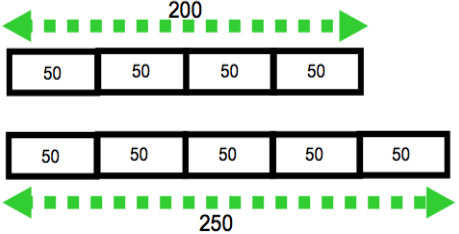
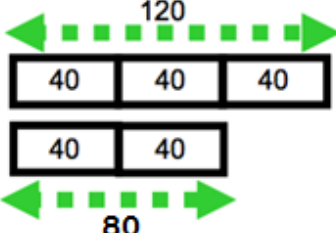
3. Put in order from least to greatest: $2.54, \frac{5}{2}, 2.7, \frac{15}{7}$

4. Write $\frac{23}{7}$ as a mixed number. Draw a bar model as needed.

1.3c Class Activity: Create a Context for Each Model or Numeric Representation

Context	Model or numeric representation
1.	
2.	$300(1) - 300(1/5) = 240$
3.	$48(1) + 48(.25) = 48(1.25) = 60$
4.	
5.	$60(1) + 60(1/3) = 60(4/3) = 80$
6.	$80(1) - 80(.25) = 80(.75) = 60$

1.3c Homework: Create a Context for Each Model or Numeric Representation

Context	Model or numeric representation
1.	
2.	$500(1) - 500(1/5) = 400$
3.	$60(1) + 60(.75) = 60(1.75) = 105$
4.	
5.	$80(1) + 80(.40) = 80(1.40) = 112$
6.	$120(1) - 120(1/4) = 120(3/4) = 90$

Spiral Review

1. The downtown bakery normally sells one dozen muffins for \$12. Today, the muffins are 25% off.
 - a. What is the discount for one dozen muffins?
 - b. How much will **two** dozen muffins cost with the discount?
2. Find 25% of \$500.00.
3. What is the greatest common factor of 30 and 75?
4. Matt earned \$800 helping his grandfather at his business. If he spent $\frac{1}{5}$ of his earnings on a gift for his mom, how much money did he have left over?
5. Write 3 as a fraction and percent:

	3.0	
Fraction	Decimal	Percent

1.3d Self-Assessment: Section 1.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

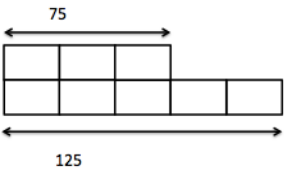
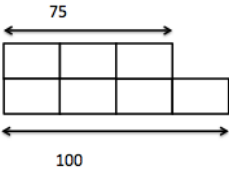
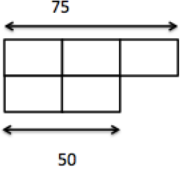
Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Find a percent or fractional portion of a whole with or without a model. [1a, 1bi, 1ci]	I can sometimes find the percent or fractional portion of a whole.	I can find the percent or fractional portion of a whole with a calculator accurately.	I can find the percent or fractional portion of a whole with a calculator and with mental math strategies accurately.	I can find the percent or fractional portion of a whole with a calculator or with mental math strategies accurately. I can explain how a model and my strategy are related to the question and answer.
2. Solve percent problems involving percent increase and decrease (including discounts, interest, taxes, tips, etc.). [1b, 1c, 2]	I sometimes have a hard time getting started, but once I know what to do, I can do the problem.	I know how to find the percent of the whole, but I sometimes mess up on the next step.	I can solve percent problems involving percent increase and decrease.	I can solve percent problems involving percent increase and decrease accurately with either a model or numeric expression. I can also explain how the model and expression are related to the answer.
3. Develop numeric expressions from percent and fraction models. [3]	I don't know how to write a numeric expression from percent and fraction models.	I can match a numeric expression to the correct percent or fraction model.	I can write a numeric expression from percent and fraction models.	I can write a numeric expression from percent and fraction models. I can also draw percent and fraction models from a numeric expression.

Sample Problems for Section 1.3

1. Solve each of the following problems with or without a model.
 - a. Ana has 75 necklaces. She sells $\frac{1}{3}$ of the necklaces. How many necklaces did she sell?
 - b. Bubba invested \$760. If he earned 20% on his investment,
 - i. How much interest did he earn?
 - ii. How much money does he have now?
 - c. The bakery normally sells one dozen cupcakes for \$24. Today, the cupcakes are 25% off.
 - i. What is the discount for one dozen cupcakes?
 - ii. If 5% tax is added to the total after the discount, how much will two dozen cupcakes cost?
2. Solve the following problems with or without a model.
 - a. Daniel left a \$9 tip for the waiter at a restaurant. If the tip was 15% of the bill, how much was the bill?
 - b. Ellie borrowed money from a company that charges 20% interest for loans. If she repaid them a total of \$420, how much was her original loan?
 - c. Flo sold 14 cars last year. This year, she sold 35 cars. What was the percent change?
 - d. Gabrielle had \$90 in her savings account last month. Now she has \$63 in her account. What was the percent change?

3.

- a. Match the context with the correct model, and numeric expression.

Context			
1. Hans had \$75 in his bank account. Two years later, he had \$100.	2. Isi could do 75 sit-ups last year. Now she can only do 50 sit-ups.	3. Jenna's family used 75 gallons of water per day last month. This month, her family used 125 gallons of water per day.	
A.	B.	C.	
			
X.	Y.	Z.	
$75(1) + 75\left(\frac{1}{3}\right)$	$75(1) - 75\left(\frac{1}{3}\right)$	$75(1) + 75\left(\frac{2}{3}\right)$	

Context 1, Model _____, Numeric Expression _____

Context 2, Model _____, Numeric Expression _____

Context 3, Model _____, Numeric Expression _____

- b. For Context 1: What percent of Hans' original amount of money does he have 2 years later?
- c. For Context 2: What is the percent change in the number of sit-ups Isi can do?
- d. For Context 3: What is the percent change in the amount of water Jana's family now uses compared to last month?

4. Write a context to match the following numeric expressions:

	$24(1) + 24\left(\frac{1}{4}\right)$
	$700(1) - 700(0.3)$

First published in 2013 by Math Materials Access Improvement for Seventh and Eighth Grades (MMAISE), Salt Lake City, USA.

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Chapter 2 Operations with Rational Numbers

(4 weeks)

UTAH CORE Standard(s)

Number Sense:

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **7.NS.1**
 - a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* **7.NS.1a**
 - b. Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **7.NS.1b**
 - c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **7.NS.1c**
 - d. Apply properties of operations as strategies to add and subtract rational numbers. **7.NS.1d**
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **7.NS.2**
 - a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world concepts. **7.NS.2a**
 - b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. **7.NS.2b**
 - c. Apply properties of operations as strategies to multiply and divide rational numbers. **7.NS.2c**
 - d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.2d**
3. Solve real-world and mathematical problems involving the four operations with rational numbers. **7.NS.3**

CHAPTER OVERVIEW:

In Chapter 2, students extend and formalize their understanding of the number system, including arithmetic with negative rational numbers. In the first two sections of the chapter, students extend their understanding of the properties of arithmetic to operating with integers. Section 1 focuses on adding and subtracting integers, while section 2 focuses on multiplication and division with integers. By section 3, students operate with all rational numbers by applying the rules they learned in the previous two sections for working with integers. Throughout all three sections, students will increase their proficiency with mental arithmetic by articulating strategies based on properties of operations.

VOCABULARY:

additive inverse, difference, integers, multiplicative inverse, opposites, product, quotient, rational numbers, repeating decimal, sum, terminating decimal, zero pairs.

CONNECTIONS TO CONTENT:

Prior Knowledge

In 6th grade students understand that positive and negative numbers are used together to describe quantities having “opposite” directions or values. They use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. Students recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line and recognize that the opposite of the opposite of a number is the number itself. Student should come to 7th grade knowing how to locate integers on the real line as well as positive rational numbers. In Chapter 1, students have placed negative rational numbers on the number line and this chapter will expand on that knowledge.

Students also build on their prior understanding of addition (joining), subtraction (take-away or comparison), multiplication (repeated addition) and division (the inverse of multiplication). In previous grades students modeled these operations with manipulatives and on a number line (positive side only). They will now extend these strategies to working with integers and negative rational numbers. Note that 7th Grade will be students’ first experience operating with negative numbers.








Future Knowledge

The development of rational numbers in 7th grade is a progression in the development of the real number system that continues through 8th grade. In high school students will move to extending their understanding of number into the complex number system. Note that through 7th grade students find points on the real line that correspond to quantities, e.g. students will locate $-3/8$ on the real line in 7th grade. In 8th grade, students do something different; they start with lengths and try to identify the location that corresponds to the length. For example, students might find that the length of the hypotenuse of a right triangle is $\sqrt{5}$, they then try to identify the number that corresponds to that length.

In later courses, students will also know and apply the properties of integer exponents to generate equivalent numerical expressions, for example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. Students will use the rules for operations on integers when solving linear equations with integer coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

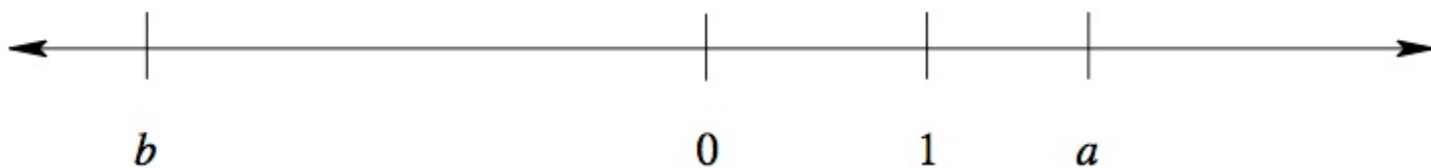
In this chapter, attention is paid to understanding subtraction as the signed distance between numbers. This idea extends into geometric ideas in later mathematics such as subtraction of vectors.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

	Make sense of problems and persevere in solving them	Students explain and demonstrate operations on integers using symbols, visuals, words, and real life contexts. Students demonstrate perseverance while using a variety of strategies (number lines, chips/tiles, drawings, etc.)
	Reason abstractly and quantitatively	Students demonstrate quantitative reasoning with integers by representing and solving real world situations using visuals, numbers, and symbols. They demonstrate abstract reasoning by translating numerical sentences into real world situations or for example reasoning that $a - (-b)$ is the same as $a + b$.
	Construct viable arguments and critique the reasoning of others	Students justify rules for operations with integers using appropriate terminology and tools/visuals. Students apply properties to support their arguments and constructively critique the reasoning of others while supporting their own position with models and/or properties of arithmetic.
	Model with mathematics	Students model understanding of integer operations using tools such as chips, tiles, and number lines and connect these models to solve problems involving real-world situations and rules of arithmetic. For example, students will be able to use a model to explain why the product of two negative integers is positive.
	Attend to precision	Students demonstrate precision by using correct terminology and symbols when working with integers. Students use precision in calculation by checking the reasonableness of their answers. Additionally, students will use properties of arithmetic to justify their work with integers, expressions and equations.
	Look for and make use of structure	Students look for structure when operating with positive and negative numbers in order to find algorithms to work efficiently. For example, students will notice that subtracting a negative integer is the same as adding its opposite or that when a product involves an even number of negative integers, the product is always positive.
	Use appropriate tools strategically	Students demonstrate their ability to select and use the most appropriate tool (paper/pencil, manipulatives, or calculators) while solving problems with integers. They also learn to estimate answers before using a calculators. Students should recognize that the most powerful tool they possess is their ability to reason and make sense of problems.
	Look for and express regularity in repeated reasoning	Students will use manipulatives to explore the patterns of operations with integers. Students will use these patterns to develop algorithms. They can use these algorithms to solve problems with a variety of problem solving structures.

2.0 Anchor Problem: Operations on the Number Line

A number line is shown below. The numbers 0 and 1 are marked on the line, as are two other numbers a and b .



Which of the following numbers is negative? Choose all that apply. Explain your reasoning.

1. $a - 1$
2. $a - 2$
3. $-b$
4. $a + b$
5. $a - b$
6. $ab + 1$

Section 2.1: Add and Subtract Integers; Represent Using Chip/Tile Model and Number Line Model.

Section Overview:

Section 2.1 is the first time students add and subtract integers. For this reason, it is important to begin with hands-on manipulatives and number lines, gradually working towards developing the rules of arithmetic with integers and then ultimately adding and subtracting integers without models. The goal in this section is to build intuition and comfort with integer addition and subtraction so that by the end of the section students can *reason* through addition and subtraction of integers without a model.

Students start by working with “opposites” (additive inverses) to notice that pairs of positives and negatives result in “zero pairs.” They then move to adding integers. Students know from previous grades that the fundamental idea of addition is “joining.” Students should notice that joining positive and negative numbers results in one or more “zero pairs” and that the “left over” is the final sum. Students will develop this idea first with a chip or tile model and the number line.

Next, students move to subtraction. Students begin by reviewing from previous grades that there are two ways to think concretely about subtraction: i) “take-away;” Anna Maria has 5 gummy bears. Jose eats 3. How many gummy bears does she now have? $5 - 3$, thus 2 gummy bears. ii) “Comparison;” Anna Maria has 5 gummy bears, Jose has 3 gummy bears, how many more gummy bears does Anna Maria have than Jose? Again, the operation is $5 - 3$ resulting in 2 gummy bears. We build on the comparison conceptualization for a concrete way to think about subtraction with integers on the real line. We start by locating integers on the line and note that when comparing two integers, there is a directional or signed distance between them, e.g. when comparing 5 and 3 we can think “5 is two units to the right of 3,” ($5 - 3$ is 2), or “3 is two units to the left of 5,” ($3 - 5$ is -2). Another example, when comparing 5 and -3 and think, “5 is eight units to the right of -3 ,” ($5 - (-3)$ is 8), but “ -3 is eight units to the left of 5,” ($(-3) - 5$ is -8 .) In section 2.1e, students will examine subtraction exercises and notice that $a - b$ can be written as $a + (-b)$ and that $a - (-b)$ can be written as $a + b$. This is an essential understanding and one on which students can build fluency for working with integers.

Adding and subtracting integers can cause students a great deal of trouble particularly when students first confront exercises that have both addition and subtraction with integers. For example, in problems like $-5 - 7$ students are often unsure if they should treat the “7” as negative or positive integer and if they should add or subtract. For this reason, it is *extremely* helpful to make it a norm from the beginning of this chapter that students circle the operation before they perform it.

Rules for operating with integers come from extending rules of arithmetic. Students should understand that arithmetic with negative numbers is consistent with arithmetic with positive numbers. Students need to remember that subtraction in the set of integers is neither commutative nor associative. In other words, in general, $a - b \neq b - a$ and $(a - b) - c \neq a - (b - c)$. However, $a - b = a + (-b) = (-b) + a$.


Concepts and Skills to be Mastered (from standards)

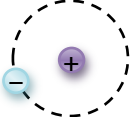

By the end of this section, students should be able to:

1. Use a concrete model (chips/tiles or number line) to add integers.
2. Use a concrete model (chips/tiles or number line) to subtract integers.
3. Find the sums of integers accurately without a model.
4. Find the differences of integers accurately without a model.
5. Solve contextual problems involving adding or subtracting integers.

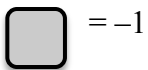
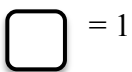
2.1a Class Activity: Additive Inverse (Zero Pairs) in Context and Chip/Tile Model

An **additive inverse** is an important mathematical idea that we will begin to informally explore in this activity. For now, we will use the informal term “zero pair” to refer to two things that are “opposite” or “undo each other.” The sum of the two elements of a zero pair is zero. We see zero pairs in many contexts. For example, an atom with 5 protons and 5 electrons has a neutral charge, or a net charge of zero. Use the idea of “zero pairs”




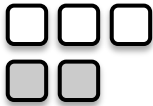
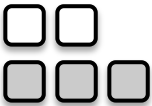

to complete the worksheet. 

Context	Model/Picture	Net Result in Words	How Many Zero Pairs?
1. A Hydrogen atom has one proton and one electron.		The atom has a neutral charge.	1
2. I took 4 steps forward and 3 steps back.		I am one step ahead of where I started.	3
3. I got a check in the mail for \$1,000,000, but then I got a bill that says I owe \$1,000,000.			
4. I ate one candy bar with 200 calories. Then I ate four apples with 50 calories each.			
5. Joe found four quarters, but he had a hole in his pocket and lost one quarter.			
6. Lisa earned \$8 then spent \$6.			
7. George gained ten pounds, but then he went on a diet and lost twelve pounds.			
8. John took \$15 out of his bank account then deposited \$10.			

For the following exercises, use the key below:



In pairs, look at the following models. Make a conjecture for what you think the model represents. Construct an argument to support your claim.

<p>9.</p>  <p>This model represents:</p> <p>Argument:</p>	<p>10.</p>  <p>This model represents:</p> <p>Argument:</p>
<p>11.</p>  <p>This model represents:</p> <p>Argument:</p>	<p>12.</p>  <p>This model represents:</p> <p>Argument:</p>
<p>13.</p>  <p>This model represents:</p> <p>Argument:</p>	<p>14.</p>  <p>This model represents:</p> <p>Argument:</p>

15. In general, explain how you can use a model to represent positive and negative numbers:

Model the following expressions using integer chips/tiles. Draw pictures of your models using the above key to show what you did, then explain in words how you arrived at your answer.

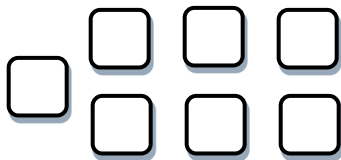
16. Represent -5 with chips/tiles. Draw your representation in the space below:



Explain your representation. Can you represent -5 in another way? Explain.



17. Represent 7 with chips/tiles. Draw your representation in the space below:



Explain your representation. Can you represent 7 in another way? Explain.

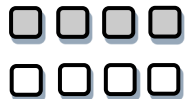
18. Explain what it means to “add” numbers:

For #19 – 27, circle the operation, draw a representation, and then find the sum, explaining your reasoning.

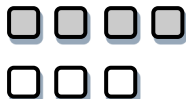
19. Represent $5 + 6$ with chips/tiles. Draw your representation and its sum in the space below:

<p>Representation:</p> <div style="text-align: center; margin-top: 20px;"> </div>	<p>Explanation:</p>
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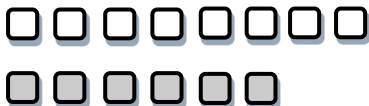
20. Represent $(-4) + 4$ with chips/tiles. Draw your representation in the space below:

Representation:	Explanation:
	

21. Represent $-4 + 3$ with chips/tiles. Draw your representation in the space below:


Representation:	Explanation:
	

22. Represent $8 + (-6)$ with chips/tiles. Draw your representation in the space below:


Representation:	Explanation:
	

23. Why do you think #20 and #22 used “()” around one of the numbers, but #21 did not?

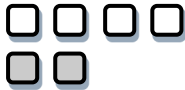
24. Represent $-5 + (-8)$ with chips/tiles. Draw your representation and its sum in the space below:

Representation:	Explanation:
	


25. Represent $(-3) + (8)$ with chips/tiles. Draw your representation and its sum in the space below:

Representation:	Explanation:
	

26. Represent $4 + -2$ with chips/tiles. Draw your representation and its sum in the space below:

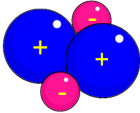
Representation:	Explanation:
	

27. Represent $-5 + (-3)$ with chips/tiles. Draw your representation and its sum in the space below:

Representation:	Explanation:
	

2.1a Homework: Additive Inverse (Zero Pairs) in Context and Chip/Tile Model

Fill in the table below.

Context	Model/Picture	Net Result in Words	How Many Zero Pairs?
1. The Helium atom had two protons and two electrons.			
2. The treasure map said to take ten steps north, then eight steps south.			
3. Jen earned \$6 working in the garden, and then she spent \$5 on a toy.			
4. The elevator started on the 8 th floor, went up five floors, and then went down two floors.			
5. Seven students were added to Ms. Romero's class and then four students dropped it.			
6. The temperature fell twelve degrees between midnight and 6 a.m. The temperature rose twelve degrees between 6 a.m. and 8 a.m.			
7. The home team scored a goal, which was answered with a goal from the visiting team.			
8. The football team gained seven yards on the first down, and lost nine yards on the second down.			

Circle the operation, draw a chip/tile model for each sum, and then state the sum. The first one is done for you.

For the following exercises, use the key below:

= 1

= -1

n#

<div> <div>9. 7 + (-4)</div> <div> <div>3</div> <div> <div></div><div></div><div></div><div></div><div></div><div></div><div></div> <div></div><div></div><div></div><div></div> </div> </div> </div>	10. -3 + 8
11. 4 + 6	12. -5 + 8
<div> <div>13. -4 + (-2)</div> <div> <div>-</div> </div> </div>	14. -6 + (-3)
15. 7 + (-6)	16. 4 + (-8)
<div> <div>Extension Questions:</div> <div>Suppose $a + b = c$ and $c < 0$, give an example when this might happen.</div> <div>What do you know about a and b relative to each other for $c < 0$?</div> </div>	

Spiral Review

1. Order the following rational numbers from least to greatest. $\frac{27}{3}$, 6.5, $\frac{18}{3}$, 0.99,
2. What is the opposite of -15 ?
3. What is 40% of 220? Use a bar model.
4. Without using a calculator, determine which fraction is bigger in each pair. Justify your answer with a picture *and* words.
 - a. $\frac{1}{3}$ or $\frac{1}{2}$
 - b. $\frac{3}{7}$ or $\frac{3}{5}$
5. Solve using a bar model. $\frac{1}{2} + \frac{3}{7} = \underline{\hspace{2cm}}$

2.1b Class Activity: Add Integers Using a Number Line

Explore:

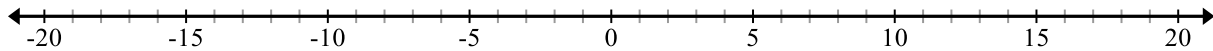
Answer each of the following questions. Using pictures and words, explain how you arrived at your answer.

1. An osprey flies off the ground and reaches 35 feet above a river when he sees a trout. He then dives 37 feet down to get the trout. How many feet below the water does he end up?
2. Zach's football team moves the football 35 yards forward on the first down. On the next down, they lose 12 yards. On the down after that they go forward 8 yards. How many yards from the starting point did they move the football in the three down?
3. You walk 3 miles from your house to the store. At the store you meet up with a friend and walk with her 1 mile back towards your house. How far are you from your house now?
4. You ride your bike 12 miles and then get a flat tire! You turn around and walk the bike 4 miles before you mom is able to pick you up. How far are you from the house when your mom picks you up?

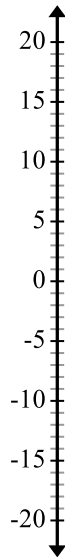
Review:

Place each of the following integers on the number line below. Label each point:

5. **A** = 4 **B** = -4 **C** = -15 **D** = 7 **E** = 18 **F** = -19



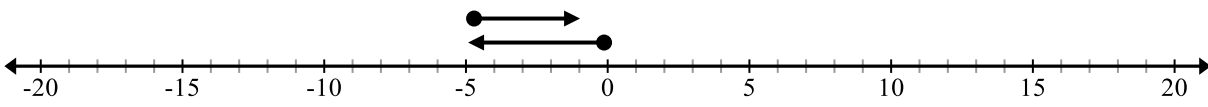
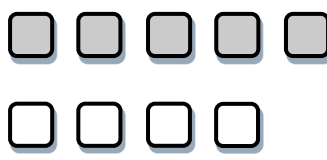
6. **A** = -20 **B** = -17 **C** = 7 **D** = 13 **E** = -6 **F** = 19



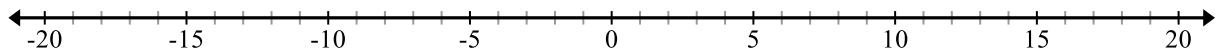
7. How did you locate 7 on the number line?
8. How did you locate -15 on the number line?
9. In general, how do you locate a positive or negative number on a number line?
10. Brainstorm similarities and differences between a chip model and number line model for representing integers.

Previously in this section, you used a chip model for addition of integers. In this activity you will explore a number line model of addition of integers. Model each of the following with both a number line and chips. Start by circling the operation

Example: $-5 + 4$

<p>Number line:</p>  <p>$-5 + 4 = -1$</p>
<p>Chips</p>  <p>$-5 + 4 = -1$</p>

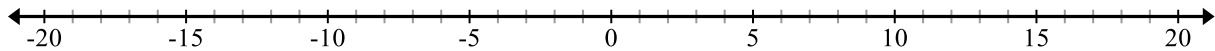
11. Model $7 + -3$ on a number line and with chips:

<p>Number line</p> 
<p>Chips</p>

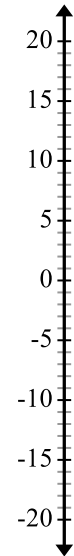
12. Explain how the number line and chip models are related:

Circle the operation you are going to perform. Find the sum using a number line for the following addition exercises.

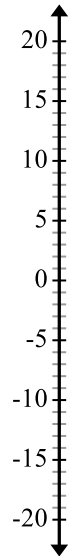
13. $6 + (-3)$



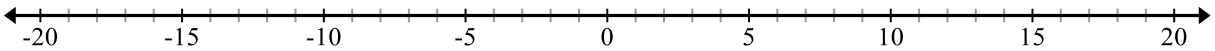
14. $-3 + (-9)$



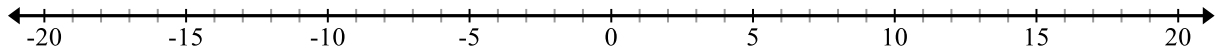
15. $-4 + (-4)$



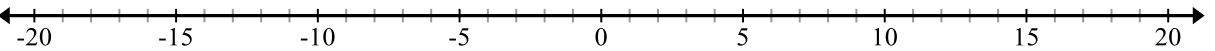
15. $6 + (-6)$



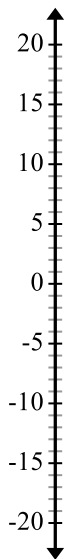
16. $-9 + 8$



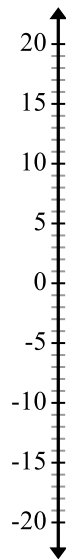
17. $20 + (-8)$



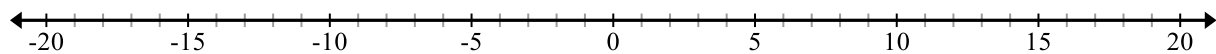
18. $-12 + 14$



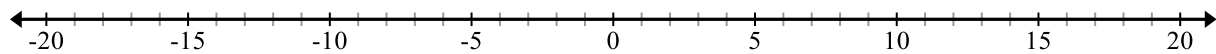
19. $9 + (-25)$



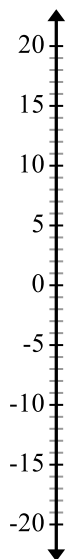
20. $12 + (-9)$



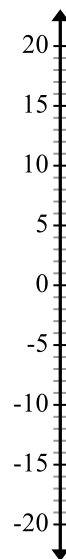
21. $-8 + (-7)$



22. $-7 + (-7)$



23. $13 + (-13)$

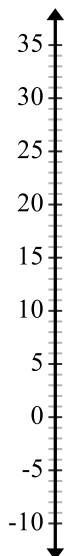


Below are the questions from the exploration at the beginning of Class Activity 2.1b. For each context, do the following:

- use a number line to model the problem situation
- answer the question in the context
- write an addition equation to show the sum
- explain how the model is related to how you found the answer when you answered the question

24. An osprey flies off the ground and reaches 35 feet above a river when he sees a trout. He then dives 37 feet down to get the trout. How many feet below the water does he end up?

a.



b.

c.

d.

25. Zach's football team moves the football 35 yards forward on the first down. On the next down, they lose 12 yards. On the down after that they go forward 8 yards. How many yards from the starting point did they move the football in the three downs?

a.



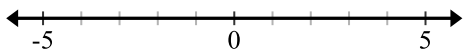
b.

c.

d.

26. You walk 3 miles from your house to the store. At the store you meet up with a friend and walk with her 2 mile back towards your house. How far are you from your house now?

a.



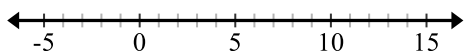
b.

c.

d.

27. You ride your bike 12 miles and then get a flat tire! You turn around and walk the bike 8 miles before you mom is able to pick you up. How far are you from the house when your mom picks you up?

a.

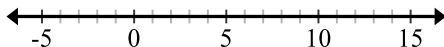


b.

c.

d.

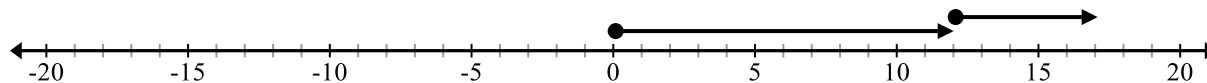
28. For the model below, write a numeric expression and then create a context.



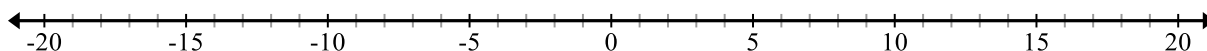
2.1b Homework: Add Integers Using a Number Line

Circle the operation. Model the expressions using a number line and write the answer. The first one is done for you.

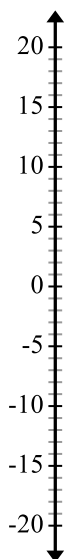
1. $12 + 5$ 17



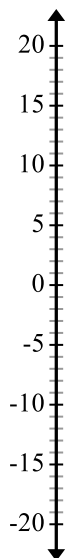
2. $-7 + 5$



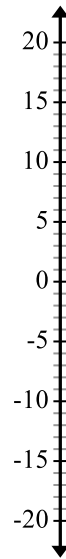
3. $9 + (-9)$



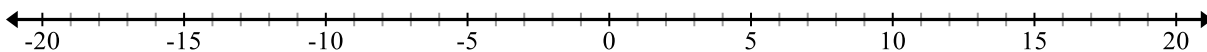
4. $-11 + (-5)$



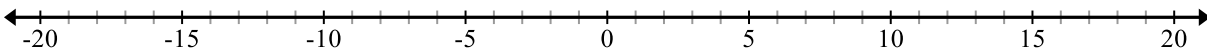
5. $8 + (-13)$



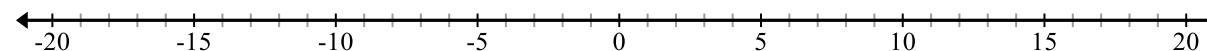
6. $-15 + (-4)$



7. $11 + 9$



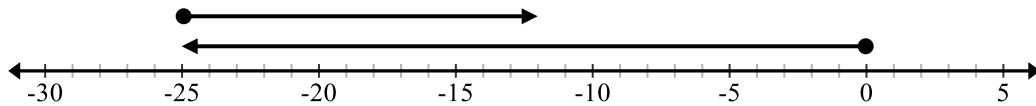
8. $7 + (-16)$



9. You borrowed \$25 from your Mom to buy clothes. A week later you repaid her \$13.

a. Write an addition expression to show how much money you owe.

b. Label the number line and model the expression. Circle the answer to the expression.



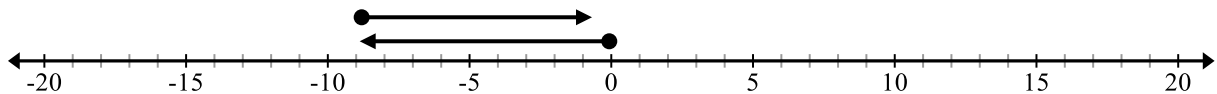
c. Explain your expression and model.

10. Write a context for the following the addition expression: $-15 + 25$

a. Model the context using the number line.



11. Write a context and expression for the model; when state the answer.

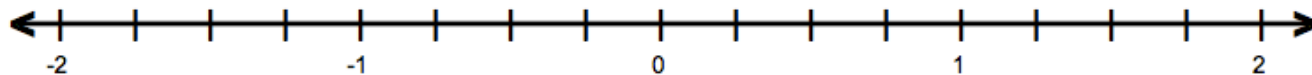


Spiral Review

1. Using bar notation, show $4\frac{1}{4} - 2\frac{1}{2}$.

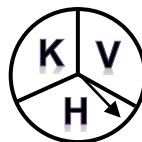
2. Place the fractions on the number line below.

$$\frac{6}{5}, \quad \frac{3}{10}, \quad \frac{1}{5}, \quad \frac{3}{2}$$



3. Write $\frac{1}{3}$ as a percent and decimal.

4. A spinner contains three letters of the alphabet.



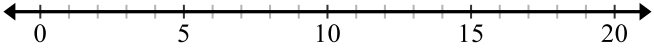
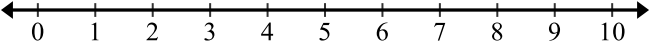
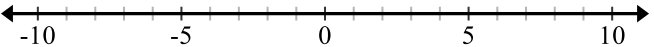
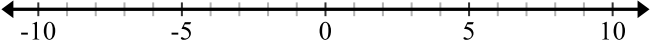
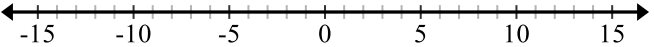
a. How many outcomes are possible if the spinner is spun three times?

b. List all of the outcomes for spinning three times.

c. What is the probability of getting exactly one H in three spins?

2.1c Class Activity: Model Integer Addition with Chips/Tiles and Number Lines

Circle the operation you are going to perform. Model each integer addition exercise with chips/tiles (on the left) and with a number line (on the right). Record the sum.

Chips/Tiles	Number Line
1. $8 + 9$	$8 + 9$ 
2. $7 + (-2)$	$7 + (-2)$ 
3. $-5 + 6$	$-5 + 6$ 
4. $-9 + 4$	$-9 + 4$ 
5. $-6 + (-8)$	$-6 + (-8)$ 

6. How are the models related? What do you notice about “zero pairs?”

7. Which model do you prefer and why?

Circle the operation you are going to perform. Use a model to find the sum for each problem # 8 - 15.

8. $5 + (-4)$

9. $-5 + 9$

10. $-2 + (-3)$

11. $0 + (-9)$

12. $-1 + (-9)$

13. $-1 + 1$

14. $-2 + 6$

15. $-9 + (-7)$

Circle the operation you are going to perform. Find the sum for each. Justify your answer.

16. $12 + (-9)$

17. $-4 + (-7)$

18. $-3 + 8$

19. $10 + (-11)$

20. $7 + (-12)$

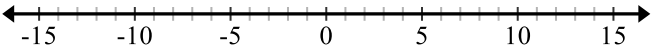
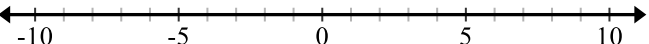
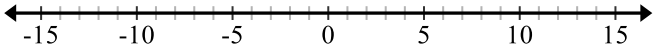
21. $-4 + (-9)$

22. $-5 + 9$

23. $13 + 8$

2.1c Homework: Compare Chips/Tiles and Number Line Models for Addition of Integers

Circle the operation you are going to perform. Model each integer addition problem with chips/tiles and with a number line. Record the sum.

Chips/Tiles	Number Line
1. $5 + 8$	 <p>A horizontal number line with arrows at both ends. It has major tick marks labeled -15, -10, -5, 0, 5, 10, and 15. There are also minor tick marks between the major ones, representing increments of 1.</p>
2. $-4 + (-3)$	 <p>A horizontal number line with arrows at both ends. It has major tick marks labeled -10, -5, 0, 5, and 10. There are also minor tick marks between the major ones, representing increments of 1.</p>
3. $7 + (-9)$	 <p>A horizontal number line with arrows at both ends. It has major tick marks labeled -15, -10, -5, 0, 5, 10, and 15. There are also minor tick marks between the major ones, representing increments of 1.</p>

Circle the operation you are going to perform. Use the chip/tile or number line model to find the sum for each.

4. $4 + (-4)$

5. $-3 + 11$

6. $-3 + (-3)$

7. $0 + (-5)$

8. $3 + 2$

9. $-7 + (-4)$

10. $4 + (-3)$

11. $7 + (-3)$

Circle the operation you are going to perform. Find the sum for each. Justify your answer.

12. $2 + (-7)$

13. $-2 + (-5)$

14. $-5 + 8$

15. $6 + (-12)$

16. $-5 + (-11)$

17. $3 + (-5)$

18. $-4 + 6$

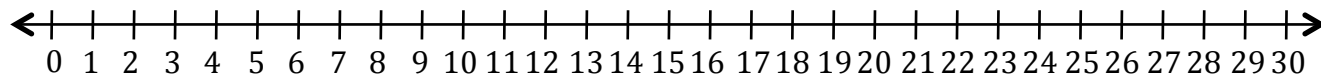
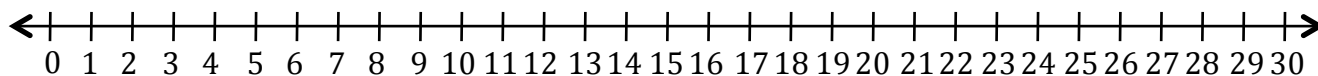
19. $-9 + (-9)$

Spiral Review

1. Order the following fractions from *least to greatest*.

$$\frac{2}{3}, \quad \frac{3}{10}, \quad \frac{1}{2}, \quad \frac{3}{2}, \quad \frac{4}{3}$$

2. Use a number line to find $27 - 7$.



3. Write $\frac{27}{4}$ as a mixed number. Model to solidify understanding.

4. Write the fraction equivalent (in simplest form) for each decimal.

a. 0.27

b. 0.35

c. 0.4

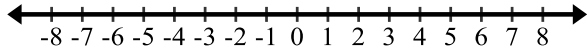
d. 0.125

5. Write 0.452 as a percent.

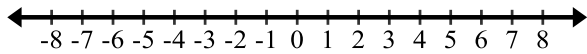
2.1d Class Activity: Number Line Model for Subtraction

Plot each pair of points on the same number line.

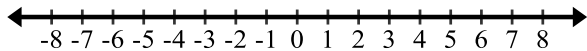
1. 3 and 8



2. -4 and 3



3. -1 and -7



4. Circle the operation. Write a context for $8 - 3$ and then state the difference.

5. Circle the operation. Make a conjecture for $3 - 8$.

6. Circle the operation. Make a conjecture for the two differences below. Refer to the # 2 to help you.

$$3 - (-4)$$

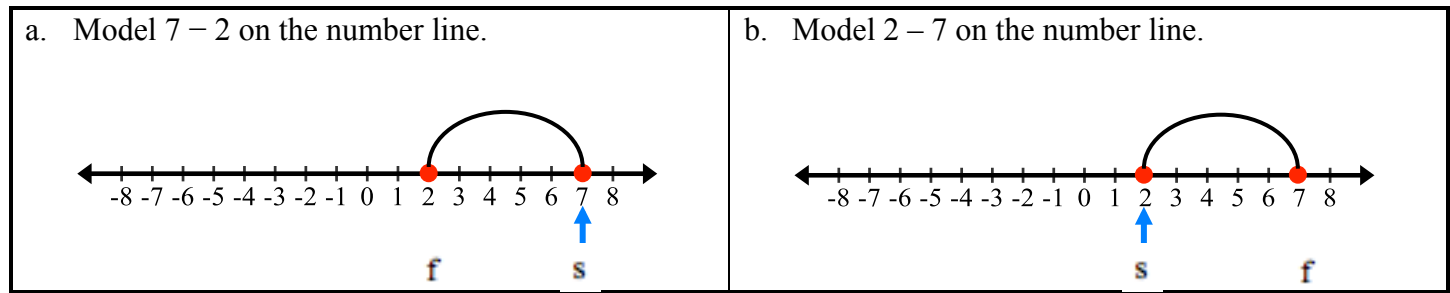
$$(-4) - 3$$

7. Circle the operation. Make a conjecture for the following two differences below. Refer to #3 to help you.

$$-1 - (-7)$$

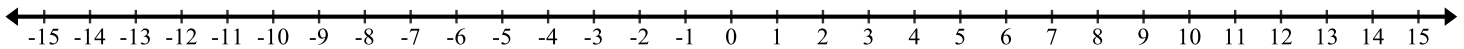
$$(-7) - (-1)$$

Subtraction can be thought of as the “*signed distance*” or “*directional distance*” between two points.

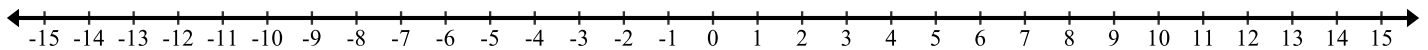


For both **a** and **b** the difference is $|5|$. How can you use a number line to know if the difference is represented with a positive or negative integer?

8. Circle the operation. Model $8 - 7$ on the number line.



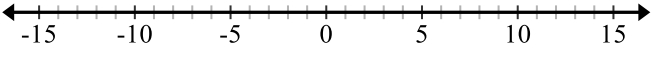
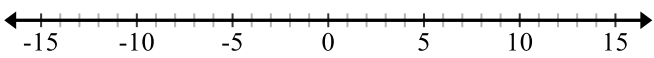
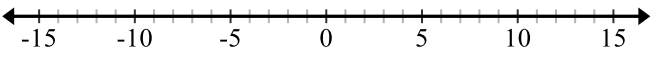
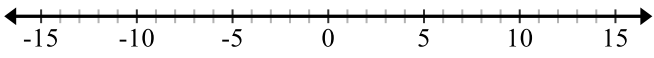
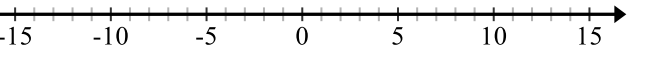
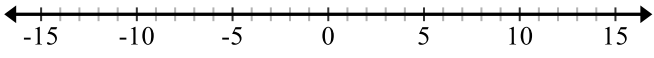
9. Circle the operation. Model $8 - (-7)$ on the number line.



10. Explain why the answers for the two problems above are so different.

11. What is $-8 - 7$? Explain.

Circle the operation. Locate each value in the difference expression on the number line and compute the difference.

<p>12. $-3 - (-10)$</p> 	<p>13. $-3 - 10$</p> 
<p>14. $3 - 10$</p> 	<p>15. $14 - 20$</p> 
<p>16. $6 - (-16)$</p> 	<p>17. $-15 - (-25)$</p> 

Circle the operation. Draw a number line, locate each value in the difference expression on the number line, and compute the difference.

18. $-14 - 1$

22. $-13 - (-13)$

19. $9 - 15$

23. $13 - 13$

20. $14 - 18$

24. $15 - (-8)$

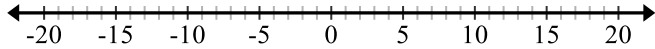
21. $-12 - (-9)$

25. $-10 - (-6)$

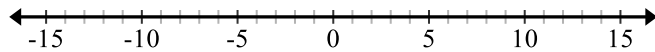
2.1d Homework: Number Line Model for Subtraction

Circle the operation. The model is provided. Find the difference. The first one is done for you.

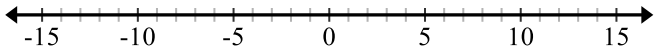
1. $4 - 17$ Answer: -13



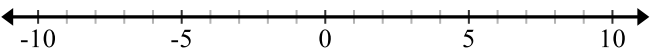
2. $5 - (-10)$



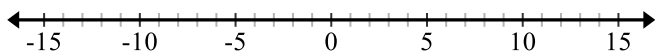
3. $-7 - 6$



4. $-8 - (-9)$



5. $9 - 4$



Circle the operation. Draw your own number line. Find the difference for each.

6. $7 - (-3)$

7. $8 - (-4)$

8. $5 - 12$

9. $6 - 18$

10. $14 - 3$

11. $21 - 5$

12. $-13 - 17$

13. $-8 - 3$

14. $-9 - (-4)$

15. $-2 - (-6)$

Spiral Review

1. Use the chip model to find $-12 + 5$.
2. What percent of 80 is 60?
3. Camilla earned \$160 over the summer. If she put 80% of her earnings into her savings account how much money did she have left over?
4. Order the numbers from least to greatest.
 $17/7$, 2.7 , 2.15 , 2.105

2.1e Class Activity: More Subtraction with Integers

1 – 20 Addition and Subtraction exercises. CIRCLE the operation. State the sum or difference.

1. $3 - 14$	2. $3 + (-14)$
3. $5 + (-13)$	4. $5 - 13$
5. $6 - 13$	6. $6 + (-13)$
7. $9 - 8$	8. $9 + (-8)$
9. $-6 - (14)$	10. $14 + 6$
11. $8 + 3$	12. $8 - (-3)$
13. $5 + 12$	14. $5 - (-12)$
15. $-3 + 18$	16. $-3 - (-18)$
17. $-7 + 4$	18. $-7 - (-4)$
19. $-15 - (-20)$	20. $-15 + 20$

Look back at the previous exercises. Which is easier to think about, addition or subtraction? What pattern do you notice with the subtraction exercises?

Write an equivalent addition expression for each subtraction expression and then simplify the expression.

Subtraction Expression	Addition Expression	Simplified
Example: $3 - 5$	$3 + (-5)$	-2
21. $-2 - 6$		
22. $9 - 14$		
23. $7 - (-4)$		
24. $-10 - (-7)$		
25. $12 - 8$		
26. $-5 - 3$		
27. $-1 - (-15)$		
28. $6 - (-8)$		

Write each term as two equivalent expressions both involving subtraction.

Example: -4 ; $-7 - (-3)$ or $3 - 7$

29. -2	30. -4	31. -1
----------	----------	----------

Write each quantity below as two equivalent numeric expressions, one involving addition and the other involving subtraction. **Example:** -3 ; -3 can be written as $4 - 7$ or $4 + -7$; 5 can be written as $8 - 3$ or $8 + (-3)$

32. 4	33. -2	34. -4
---------	----------	----------

35. How will you determine if “ $-$ ” means subtract or negative in a numeric expression?

Answer the two questions below. Draw a model to justify your answer.

36. Louis picks up a football that has been fumbled by his quarter back 12 yards behind the line of scrimmage. He runs forward 15 yards before he is tackled. Where is he relative to the line of scrimmage?

37. Leah owes her friend seven dollars. Leah's friend forgives three dollars of her debt. How much does Leah now owe her friend?

Circle the operation. Find the indicated sum or difference. Draw a number line or chip/tile model when necessary. You may change the expression to addition when applicable.

38. $2 - 8$	39. $-3 + 7$	40. $3 - (-2)$
41. $-2 - (-5)$	42. $-4 - 5$	43. $-1 + -4$
44. $7 - 3$	45. $5 + -3$	46. $6 - 9$

2.1e Homework: More Subtraction and Integers

Write an equivalent addition expression for each subtraction expression and then simplify the expression.

Subtraction Expression	Addition Expression	Simplify
1. $-7 - 3$		
2. $6 - 11$		
3. $12 - (-3)$		
4. $-6 - (-8)$		
5. $14 - 10$		
6. $-2 - 7$		
7. $-8 - (-10)$		
8. $5 - (-9)$		

Circle the operation. You may change the expression to addition when applicable. Find the indicated sum or difference. Draw a number line or chip/tile model when necessary.

9. $3 - (-9)$	10. $-4 + (-1)$
11. $-3 - (-7)$	12. $8 - 5$
13. $4 - 9$	14. $5 - (-2)$
15. $-3 - 4$ -	16. $-5 + (-2)$
17. $3 - (-1)$	18. $2 - 6$
19. $-5 + 10$	20. $-2 - (-9)$

Spiral Review

1. What is 80% of 520?
2. Suppose you were to roll a fair 6-sided number cube once, then flip a coin. List all the possible outcomes.

Using the information in #2, answer the questions below. Write all probabilities as fractions, decimals and percentages.

3. What is the probability of getting a 2 and heads from the experiment above?
4. What is the probability that you would roll an even number and flip heads?
5. What is the probability that you would roll an even number or flip heads?

2.1f Class Activity: Applying Integer Operations

Use a chip or number line model to solve each of the following. Write a numerical expression that models your solution and then write a complete sentence stating your answer.

1. At the Masters Golf Tournament, Tiger Woods' score is -3 and Phil Mickelson's score is -5 . Who is in the lead and by how many shots? (Note: In golf, the lower score wins.)
2. Steve found a treasure map that said to take ten steps north, then ten steps south. Where is Steve in relationship to where he started?
3. Tom left school and walked 5 blocks east. Charlie left school and walked the same distance west. How far apart did the two friends end up?
4. Ricardo's grandmother flew from Buenos Aires, Argentina to Minnesota to visit some friends. When Ricardo's grandmother left Buenos Aires the temperature was 84° . When she arrived in Minnesota it was -7° . What was the temperature change for Ricardo's grandmother?
5. The elevator started on the 8th floor, went up 5 floors, and then went down 2 floors. What floor is it on now?
6. The temperature rose 13°F between noon and 5 p.m. and then fell 7°F from 5 p.m. to 10 p.m. If the temperature at noon is 75°F , what would the temperature be at 10 p.m.?
7. Paula was standing on top of a cliff 35 feet above sea level. She watched her friend Juan jump from the cliff to a depth of 12 feet into the water. How far apart are the two friends?
8. The carbon atom had 6 protons and 6 electrons. What is the charge of the atom?
9. The Broncos got possession of the football on their own 20 yard line. They ran for an 8 yard gain. The next play was a 3 yard loss. What is their field position after the two plays?
10. Samantha earned \$14 for mowing the lawn. She then spent \$6 on a new shirt. How much money does she have now?

2.1f Homework: Applying Integer Operations

Use a chip or number line model to solve each of the following. Write a numerical expression that models your solution and then write a complete sentence stating your answer.

- | | |
|--|--|
| 1. Chloe has \$45 in her bank account. After she went shopping, she looked at her account again and she had $-\$12$. How much did she spend shopping? | 4. Mitchell owes his mom \$18. He's been helping around the house a lot, so his mom decided to forgive \$12 of his debt. How much does he owe now? |
| 2. There is a Shaved Ice Shack on 600 South. Both Mary and Lina live on 600 South. Mary lives 5 blocks west of the shack and Lina lives 3 blocks east of it. How far apart do they live? | 5. On the first play of a possession Drew Brees was sacked 4 yards behind the line of scrimmage. He then threw a pass for a 12 yard gain. Did the Saints get a first down? |
| 3. Eva went miniature golfing with Taylor. She shot 4 under par and Taylor shot 2 under par. By how many strokes did Eva beat Taylor? | 6. Joe had \$2. He found a quarter, but he lost a dollar in the vending machine. How much money does he have now? |

Write two stories that you could model with positive and negative numbers, then model and solve both.

Spiral Review

1. What is the greatest common factor of 48 and 36?

2. Fill in the equivalent fraction and percent for this decimal:

Fraction	Decimal	Percent
	0.8	

3. Solve each of the following problems with or without a model.

a. Wanda has 90 bracelets. She sells $\frac{1}{3}$ of the bracelets. How many bracelets did she sell?

b. Chad invested \$840. If he earned 20% on his investment, how much interest did he earn?

How much does he have now?

4. Write a context for the following numeric expression: $24(1) + 24\left(\frac{1}{4}\right)$

2.1f Integer Addition and Subtraction Practice

Solve.

1. $-4 + 9 =$

11. $14 + 18 =$

21. $9 - (-11) =$

2. $8 + (-8) =$

12. $-4 - 4 =$

22. $8 - (-2) =$

3. $3 - 4 =$

13. $-3 + (0 - 9) =$

23. $-7 - 8 =$

4. $-7 + 8 =$

14. $5 - (-6) =$

24. $-8 + 10 =$

5. $4 - 2 =$

15. $4 + (-10) =$

25. $-5 - 21 =$

6. $-5 - 13 =$

16. $6 - 9 =$

26. $8 + (-10) =$

7. $8 + 12 =$

17. $8 + 3 =$

27. $9 + (-1) =$

8. $6 - (-7) =$

18. $10 - 11 =$

28. $8 - (-11) =$

9. $-9 + (-10) =$

19. $-12 + 7 =$

29. $9 - 6 =$

10. $-6 - (-8) =$

20. $3 - 8 =$

30. $7 - (-4) =$

2.1g Self-Assessment: Section 2.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Use a concrete model (chips/tiles or number line) to add integers. [1]	I can draw a model of integer addition. I struggle to use that model to add integers.	Most of the time when I use a model to add integers I get the right answer.	I always get the right answer when I add integers with a model.	I always get the right answer when I add integers with a model. I can explain how the model is related to the sum.
2. Use a concrete model (chips/tiles or number line) to subtract integers. [2]	I can draw a model of integer subtraction. I struggle to use that model to subtract integers.	Most of the time when I use a model to subtract integers I get the right answer.	I always get the right answer when I subtract integers with a model.	I always get the right answer when I subtract integers with a model. I can explain how the model is related to the difference.
3. Find the sums of integers accurately without a model. [3]	I struggle to add integers without a model.	Most of the time when I add integers without a model I get the right answer.	I always get the right answer when I add integers without a model.	I always get the right answer when I add integers without a model. I can explain how I got my answer.
4. Find the differences of integers accurately without a model. [4]	I struggle to subtract integers without a model.	Most of the time when I subtract integers without a model I get the right answer.	I always get the right answer when I subtract integers without a model.	I always get the right answer when I subtract integers without a model. I can explain how I got my answer.
5. Solve contextual problems involving adding or subtracting integers. [5]	I struggle to solve contextual problems involving integers.	I can usually write an expression to solve a contextual problem involving integers, but I struggle using that expression to get an answer.	I can solve contextual problems involving integers.	I can solve contextual problems involving integers. I can explain the solution in context.

Sample Problems for Section 2.1

1. Draw a model (chips/tiles or number line) to solve each of the following addition problems.
 - a. $4 + 7$
 - b. $-4 + (-6)$
 - c. $1 + (-7)$
 - d. $-3 + 7$

2. Draw a model (chips/tiles or number line) to solve each of the following subtraction problems.
 - a. $7 - 9$
 - b. $-6 - (-3)$
 - c. $9 - (-2)$
 - d. $-4 - 7$

3. Find each sum without a model.
 - a. $3 + 6$
 - b. $-7 + (-5)$
 - c. $98 + (-1)$
 - d. $-6 + 7$

4. Find each difference without a model.
 - a. $16 - 29$
 - b. $-2 - (-8)$
 - c. $5 - (-3)$
 - d. $-90 - 87$

5. Solve each of the following contextual problems involving integers.
 - a. Juan's football team gains 3 yards on one play. On the next play, the quarterback is sacked for a loss of 10 yards. What was the net change in their position?
 - b. Kathryn, Loralie, and Madison are playing golf. Kathryn ends with a score of -8 . Loralie's score is -4 . Madison scores $+5$. What is the difference between the scores of Madison and Kathryn?
 - c. Nantai is hiking in Death Valley. He starts out at Badwater Basin, the lowest point of Death Valley at an elevation of 282 feet below sea level. He walks northward towards Telescope Peak in the Panamints and reaches an elevation of 2400 feet. How much did his altitude change?
 - d. Mr. O'Connor lives in North Dakota. When he leaves for work one wintry morning, the temperature is -4°C . By the time he comes home, the temperature has increased 25° . What is the temperature when he comes home?

Section 2.2: Multiply and Divide Integers; Represented with Number Line Model

Section Overview:

In Section 2.2 students encounter multiplication with integers. The goal for students in this section is two-fold: 1) fluency with multiplication and division of integers and 2) understanding how multiplication and division with integers is an extension of the rules of arithmetic as learned in previous grades.

This section starts with a review of concepts from elementary mathematics. The mathematical foundation more formally explains multiplication in this manner: “if a and b are non-negative numbers, then $a \cdot b$ is read as ‘ a times b ,’ and means the total number of objects in a groups if there are b objects in each group.” Students will apply this concept to integers. For example, students understand 3×5 as 3 groups of 5 or $5 + 5 + 5$ or 15. With the same logic students move to finding $3 \times (-5)$. Here they should reason: “this mean 3 groups of (-5) or $(-5) + (-5) + (-5)$ or -15 .” Students will model these types of products on the real line. Next they move to products like: $(-3) \times 5$. There are two ways a student may attack this using rules of arithmetic: one way is to recognize that $(-3) \times 5$ is the same as $5 \times (-3)$ (e.g. the commutative property) and then apply the logic above: $-3 + -3 + -3 + -3 + -3$ or -15 . The other is to rely on their understanding of integers from 6th grade. There they developed the idea of -3 by recognizing that it is simply the “opposite” of 3. Thus, $(-3) \times 5$ is the same as “the opposite of 3 times 5” or $-(3 \times 5)$ or $-(15)$ or -15 (here they are applying the associative property). Students then find products such as $(-3) \times (-5)$. Using the logic they developed with the previous example, student should reason: $(-3) \times (-5)$ can be thought of as “the opposite of 3 times -5 ” or $-(3 \times (-5))$ or $-(-15)$ or 15. This will all be modeled on the real line. Before students move to division, they will formalize that a product involving an even number of negatives is positive while a product involving an odd number of negatives is negative (assuming none of the factors is 0). There is a more thorough discussion of these ideas in the mathematical foundation.

Students begin the transition to division of integers by reviewing the relationship between multiplication and division:

$$(3)(5) = 15, \text{ so } \frac{15}{5} = 3 \text{ and } \frac{15}{3} = 5$$

The mathematical foundation describes this relationship as follows, “If a and b are any integer, where $b \neq 0$ then $a \div b$ is the unique (one and only one) integer (or rational number) c , such that $a = bc$.” Since division and multiplication are inverse operations, it comes as no surprise that the rules for dividing integers are the same for multiplication.

Concepts and Skills to be Mastered (from standards)

By the end of this section, students should be able to:

1. Use a model to multiply integers
2. Explain why the product of a positive and negative integer is negative or the product of a negative and negative integer is positive.
3. Use rules for multiplying and dividing integers accurately.
4. Add, subtract, multiply and divide integers fluently.
5. Solve contextual problems involving integers.

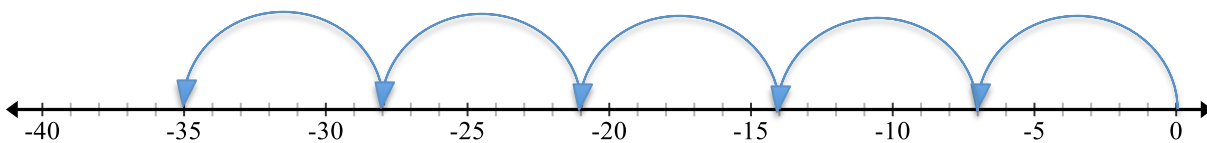
2.2 Anchor Problem

In groups of 2-3 answer the following questions. Use words and pictures to provide evidence for your answer.

1. Miguel has \$500 in the bank. Each week for 12 week he puts \$50 in to his account. How much money will he have at the end of 12 weeks?



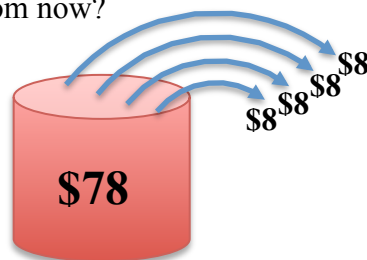
2. Rick borrowed \$7 from his brother every week for 5 weeks. What is Rick's financial situation with his brother at the end of the 5 weeks?



3. Lana wants to take a picture of her school. She stands on the sidewalk in front of her school, but she is too close to the school for her camera to capture the entire school in the frame. She takes 10 steps backwards. If each step is about 1.5 feet in length, where is she relative to where she started?

$$1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 1.5$$

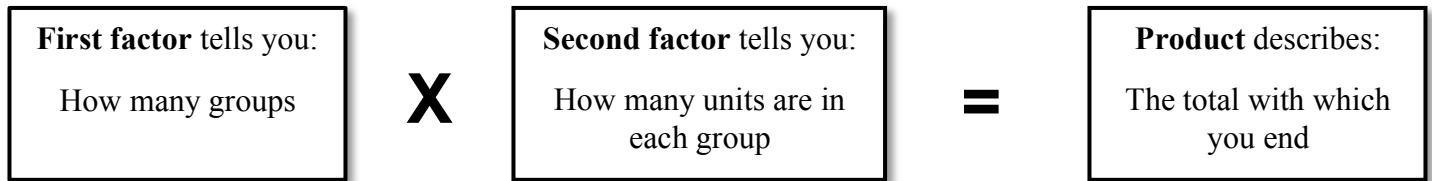
4. Lisa owes her mom \$78. Lisa's mom removed 4 debts of \$8 each when Lisa did some work for her at her office. What is Lisa's financial situation with her mom now?



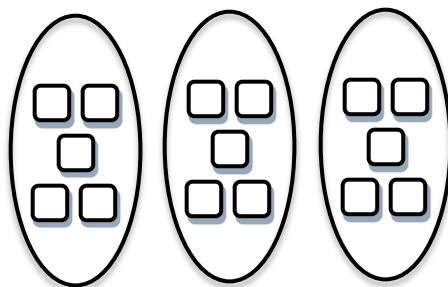
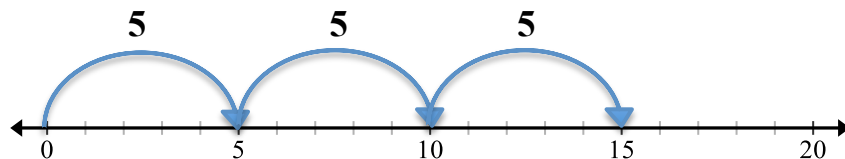
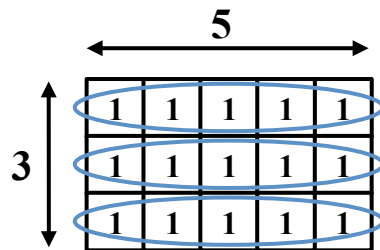
2.2a Class Activity: Multiply Integers Using a Number Line

Review:

In elementary school you modeled multiplication in several ways: arrays, skip counting, number lines, blocks, etc. You learned:



For example, you learned that 3×5 means three groups of 5 units in each group. The “product” is the total number of object. Three representations for 3×5 are:



In this section we will apply this understanding to multiplication of integers

1. Show how you might use a model to do the following:

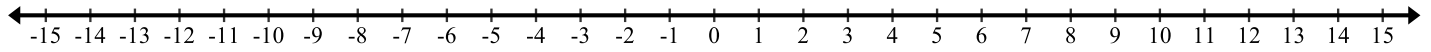
<p>a.</p> <p>$3 \times (-5)$</p>	<p>b.</p> <p>$(-3) \times 5$</p>	<p>c.</p> <p>$(-3) \times (-5)$</p>
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2. How are integers (positive and negative numbers) related to the models for multiplication?

Write the meaning of each multiplication problem in terms of a number line model and then do the multiplication on the number line provided.

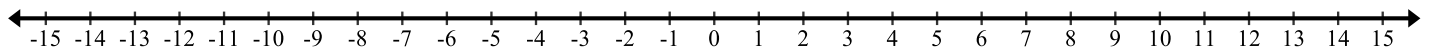
3. 2×3

Means:



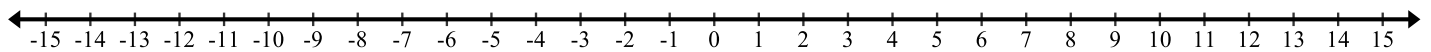
4. $2 \times (-3)$

Means:



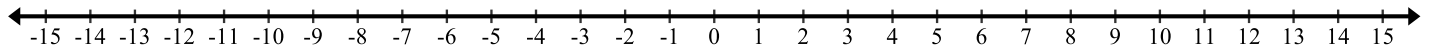
5. -2×3

Means:



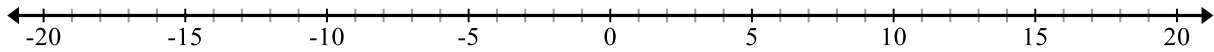
6. $-2 \times (-3)$

Means:

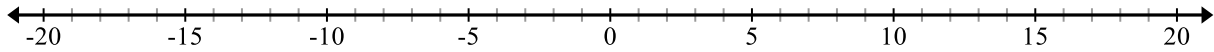


Model each multiplication problem using a number line. Write the product in the space provided.

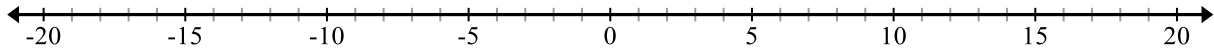
7. $6 \times (-3) =$



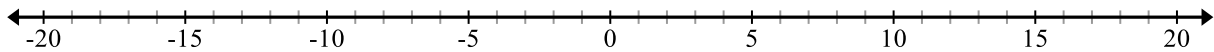
8. $6 \times 3 =$



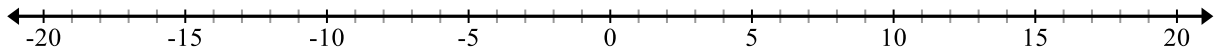
9. $-6 \times (-3) =$



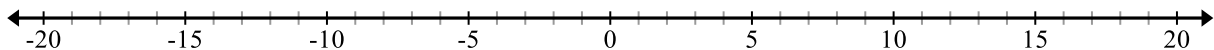
10. $-6 \times 3 =$



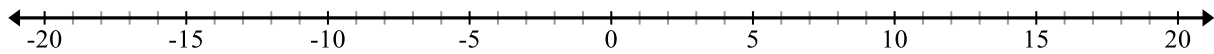
11. $2 \times (-5) =$



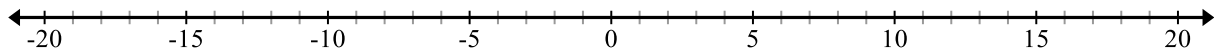
12. $-5 \times 2 =$



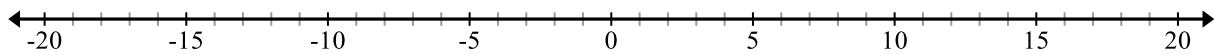
13. $-4 \times (-3) =$



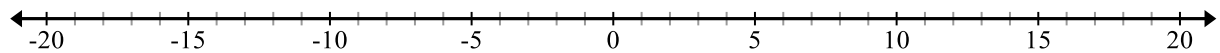
14. $-4 \times 3 =$



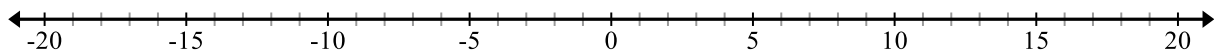
15. $-6 \times (3) =$



16. $5 \times (-4) =$



17. $0 \times (-4) =$



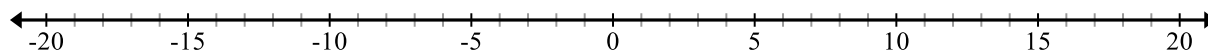
2.2a Homework: Multiply Integers

Write the meaning of each multiplication problem using a number line model. Then model the problem on the number line provided. Record the product. Look back at 2.2a Classroom Activity for assistance.

1. $3 \times 4 =$

Means:

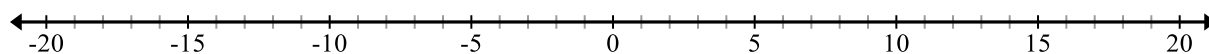
Model:



2. $3 \times (-6) =$

Means:

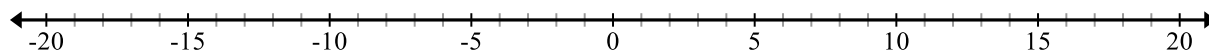
Model:



3. $-2 \times 6 =$

Means:

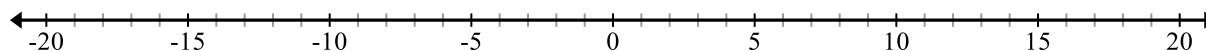
Model:



4. $-5 \times (-3) =$

Means:

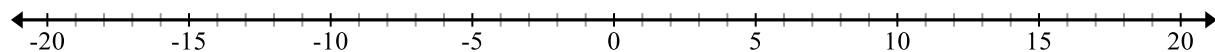
Model:



5. $-7 \times (-2) =$

Means:

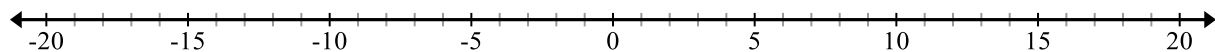
Model:



6. $-3 \times 4 =$

Means:

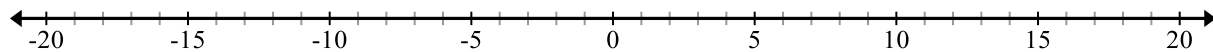
Model:



7. $-1 \times (-1) =$

Means:

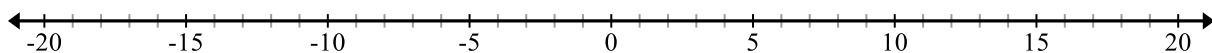
Model:



8. $-5 \times 0 =$

Means:

Model:



Spiral Review

1. Daniel left a \$9 tip for the waiter at a restaurant. If the tip was 15% of the bill, how much was the bill? Solve with or without a model.
2. What is the greatest common factor of 65 and 39?
3. In Mr. Garcia's 7th Grade Math class, $\frac{4}{5}$ of the students brought a pencil to class. If 6 people did not have pencils, how many students are in Mr. Garcia's class?
4. Order the numbers from least to greatest. $10/8$, $4/3$, 1.4, 1.08
5. Express each fraction as a percent.
 - a. $3/20 =$
 - b. $7/25 =$

2.2b Class Activity: Rules and Structure for Multiplying Integers

- 1. Complete the multiplication chart below. Be very careful and pay attention to signs.
- 2. Describe two patterns that you notice in the chart.
- 3. Create a color code for positive and negative numbers (for example, you could choose yellow for positive and red for negative.) Shade the chart according to your color code.

	-5	-4	-3	-2	-1	0	1	2	3	4	5	
5												5
4												4
3												3
2												2
1												1
0												0
-1												-1
-2												-2
-3												-3
-4												-4
-5												-5
	-5	-4	-3	-2	-1	0	1	2	3	4	5	

- 4. What do you notice? Do you think this pattern is a rule?

Review concept:

In 6th grade, you learned that opposite signs on a number indicate locations on opposite sides of 0 on the number line. For example, -3 is three units from 0, but on the opposite side of 0 than 3.

5. Use that logic to answer the following questions:

- a. What is the opposite of -1 ?
- b. Why is $1(-1) = (-1)(1) = -1$ a true statement?
- c. What is the opposite of the opposite of -1 ?
- d. Why is $(-1)(-1) = 1$ a true statement?
- e.

6. Use your experiences above to generalize some rules for multiplication with integers:

- a. A positive times a positive is a
- b. A positive times a negative is a
- c. A negative times a positive is a
- d. A negative times a negative is a

7. In groups, find each product. Be able to justify your answers for a – o.

a. 2×8	b. $5 \times (-9)$	c. -7×7
d. $-5(-3)$	e. $-5(3)$	f. 11×7

g. $(4)(-3)$	h. $(9)(-4)$	i. $(-6)(-4)$
j. $0(-3)$	k. $(-1)(1)(-3)$	l. $(4)(-2)(5)$
m. $(-2)(-2)(-2)(-2)$	n. $(5)(-3)(-2)$	o. $(-1)(-2)(3)(2)(-1)(2)$

8. In k – o you multiplied more than two factors together. Can you extend the rules you developed in #6 for this type of situation?

9. Look at the following expressions. What is the operation for each? Explain how you identified the operation and then simplify each.

a. $2(3)$
b. $2(-3)$
c. $(2) - (3)$
d. $-2(3)$
e. $-2 - 3$
f. $-2 - (-3)$
g. $2 - (-3)$

2.2b Homework: Rules and Structure for Multiplying Integers

Find each product.

1. $-2(3)$

7. $15(-3)$

13. -24×3

2. $-4(-7)$

8. $15(4)$

14. $14(-12)$

3. $8(-4)$

9. -1×4

15. $3(-8)(-4)$

4. $9(7)$

10. $15(-8)$

16. $-2 \times 5 \times 10$

5. $-3(7)$

11. -4×13

17. $-1 \times -3 \times -6$

6. $-12(-3)$

12. $-13(-3)$

18. $-8 \times 7 \times -3$

Write a multiplication problem for each situation. Write the answer in a complete sentence.

19. Karla borrowed \$5 each from 4 different friends. How much money does Karla owe her friends altogether?

Problem:

Answer:

20. The temperature increased 2° per hour for six hours. How many degrees did the temperature raise after six hours?

Problem:

Answer:

21. Jim was deep sea diving last week. He descends 3 feet every minute. How many feet will he descend in 10 minutes?

Problem:

Answer:

Spiral Review

1. Express each percent as a fraction in simplest form.

35%

22%

2. Solve the following expression by modeling with tiles, chips, or a number line $-6 + (-3)$

3. A snowboard at a local shop normally costs \$400. Over Labor Day weekend, the snowboard is on sale for 60% off. What is the sale price of the snowboard?

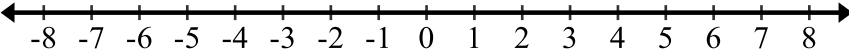
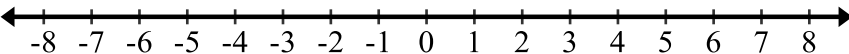
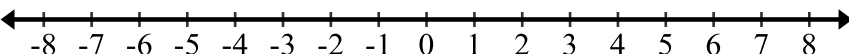
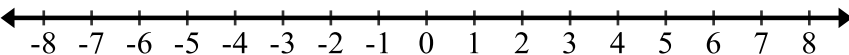
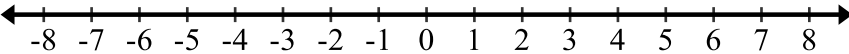
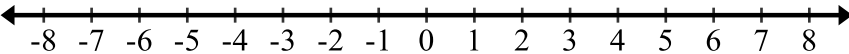
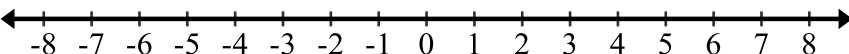
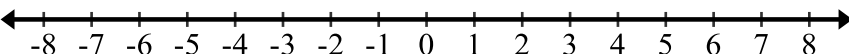
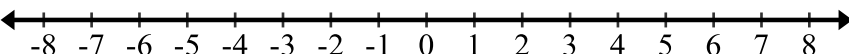
4. Place each of the following integers on the number line below. Label each point:

A = 4 **B** = -4 **C** = -15 **D** = 7 **E** = 18 **F** = -19



5. $8 - (-11) =$

Review: Operations with models. Draw models to find the answer to each problem.

Chips/Tiles	Number Lines
22. $2 + -3$	$2 + -3$ 
23. $2 - (-3)$	$2 - (-3)$ 
25. $2(-3)$	$2(-3)$ 
26. $-4 + (-2)$	$-4 + (-2)$ 
27. $-4 - (-2)$	$-4 - (-2)$ 
28. $-4(-2)$	$-4(-2)$ 
29. $-2 + 2$	$-2 + 2$ 
30. $-2 - 2$	$-2 - 2$ 
31. $-2(2)$	$-2(2)$ 

2.2c Class Activity: Integer Division

Review concept:

In previous grades you learned that division is the inverse of multiplication.

Write the missing-factor in the multiplication equation. Then write the related division equation.

Missing Factor Multiplication Problems	can be thought of as ...	Related Division Problems
1. $3 \times \underline{\quad} = 12$	\longleftrightarrow	$12 \div 3 = \underline{\quad}$
2. $\underline{\quad} \times 6 = 42$	\longleftrightarrow	$42 \div 6 = \underline{\quad}$
3. $-9 \times \underline{\quad} = -72$	\longleftrightarrow	$-72 \div -9 = \underline{\quad}$
4. $-7 \times \underline{\quad} = 56$	\longleftrightarrow	$56 \div -7 = \underline{\quad}$
5. $\underline{\quad} \times -3 = 15$	\longleftrightarrow	$15 \div -3 = \underline{\quad}$
6. $\underline{\quad} \times 4 = -48$	\longleftrightarrow	$-48 \div 4 = \underline{\quad}$
7. $-9 \times \underline{\quad} = -36$	\longleftrightarrow	$\underline{\quad} \div -9 = 4$
8. $\underline{\quad} \times -8 = 88$	\longleftrightarrow	$88 \div -8 = \underline{\quad}$

Make use of structure: How could you use related multiplication problems to find each quotient? Explain and then state the quotient.

9. $12 \div (-3)$

10. $-8 \div (-2)$

11. $40 \div (-10)$

12. $-96 \div (-12)$

13. What are the rules for dividing integers?

2.2c Homework: Rules and Structure for Division of Integers/Application

Find each quotient.

1. $12 \div (-4)$

8. $39 \div (-13)$

2. $-36 \div (-6)$

9. $-28 \div (-4)$

3. $21 \div 3$

10. $-63 \div 9$

4. $-45 \div 9$

11. $-36 \div (-3)$

5. $4 \div (-1)$

12. $60 \div (-15)$

6. $-52 \div (-4)$

13. $90 \div (-5)$

7. $72 \div (-3)$

14. $160 \div 20$

Write a division expression for each situation. Answer the question in a complete sentence.

15. Keith borrowed a total of \$30 by borrowing the same amount of money from 5 different friends. How much money does Keith owe each friend?

Expression:

Answer:

16. The temperature fell 12° over 4 hours. What was the average change in temperature per hour?

Expression:

Answer:

17. Max lost 24 pounds in 8 weeks on his new weight-loss plan. What was his average change in weight per week?

Expression:

Answer:

18. Siegfried borrowed \$4 a day until he had borrowed a total of \$88. For how many days did he borrow money?

Expression:

Answer:

Spiral Review

1. Juan's football team gains 3 yards on one play. On the next play, the quarterback is sacked for a loss of 10 yards. What was the overall change in their position for the two plays?
2. Lisa owes her mom \$78. Lisa made four payments of \$8 to her mom. How much does Lisa now owe her mother?
3. Write $\frac{26}{4}$ as a mixed number. Model to solidify understanding
4. What is the greatest common factor of 24 and 56?
5. $5 \times (-9)$

2.2c Extra Integer Multiplication and Division Practice

Solve.

1. $3 \times 5 =$

11. $-24 \div 12 =$

2. $96 \div (-12) =$

12. $33 \div (-3) =$

3. $-40 \div (-10) =$

13. $8 \times (-2) =$

4. $-6 \times 0 =$

14. $-21 \div (-3) =$

5. $6 \times (-10) =$

15. $9 \times (-4) =$

6. $-55 \div 5 =$

16. $20 \times 4 =$

7. $63 \div 9 =$

17. $10 \times (-4) =$

8. $-6 \div (-1) =$

18. $-5 \times (-9) =$

9. $-4 \times (-10) =$

19. $-40 \div (-5) =$

10. $10 \times (-3) =$

20. $1 \div (-1) =$

Write an equation and then solve. Write a sentence representing your answer.

11. Alicia owes \$6 to each of 4 friends. How much money does she owe?

12. A video game player receives \$50 for every correct answer and pays \$45 every time he gets a question incorrect. After a new game of 30 questions, he misses 16. How much money did he end up with?

13. An oven temperature dropped 135° in 15 minutes. If the temperature dropped at a constant rate, how many degrees per minute did the temperature drop?

2.2d Extra Practice: Multiple Operations Review

1. The coldest place in Utah is Middlesink, near Logan. One day the temperature there was -17 degrees. You drive from Middlesink to Salt Lake where the temperature is 30 degrees. What is the change in temperature? Explain.
2. You are playing a board game and you have to pay \$50. Then you win \$200. After that, you have to pay \$800.
 - a. Write the expression that represents this situation.
 - b. What integer represents the amount of money you end up with?
3. You go into a business partnership with three other friends. Your business loses \$65,000. You agree to share the loss equally. How much money has each of the four people lost?

Integers Review: Perform the indicated operation.

- | | | |
|-------------------|--------------------|---------------------|
| 4. $5 + (-3)$ | 9. $-5(3)$ | 14. $15 \div (-3)$ |
| 5. $3 - 5$ | 10. $13 - (-5)$ | 15. $-4 + 12$ |
| 6. -15×2 | 11. $-2(-3)$ | 16. $(-5) + (-4)$ |
| 7. $-6 \div (-2)$ | 12. $12 \div 12$ | 17. $-9(2)$ |
| 8. $-14 - (-6)$ | 13. $(-6) - (-10)$ | 18. $\frac{-20}{4}$ |

2.2e Self-Assessment: Section 2.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Use a model to multiply integers. [1]	I can draw a model for thinking about integers but I struggle to use models to multiply integers.	I can usually get the right answer when I multiply integers using a model.	I always get the right answer when I multiply integers using a model.	I always get the right answer when I multiply integers using a model. I can explain the relationship between the model and the product.
2. Explain why the product of a positive and negative integer is negative or the product of a negative and negative integer is positive. [2]	I don't know why the product of a positive and negative integer is negative or the product of a negative and negative integer is positive.	I know that product of a positive and negative integer is negative or the product of a negative and negative integer is positive, but I have a hard time showing why that's true with models and words.	I can <i>show</i> why the product of a positive and negative integer is negative or the product of a negative and negative integer is positive using models and words. But struggle to explain it with only words and symbols.	I can explain with models and words <i>and</i> with words alone why the product of a positive and negative integer is negative or the product of a negative and negative integer is positive.
3. Use rules for multiplying and dividing integers accurately. [3]	I know the rules for multiplying and dividing integers.	I know the rules to multiply and divide integers, but struggle to explain them with models and words.	I can accurately multiply and divide any integers using the rules and can show the rules with models and words.	I can accurately multiply and divide any integers using the rules. I can explain with models and words <i>and</i> with words alone why the rules are true.
4. Add, subtract, multiply and divide integers fluently. [4]	I sometimes get confused recognizing if the expression is asking me to add, subtract, multiply or divide.	I always know what operation I'm to perform, and most of the time I get the right answer when I add, subtract, multiply, and divide integers.	I always know what operation I'm to perform, and I always get the right answer when I add, subtract, multiply or divide integers.	I always know what operation I'm to perform, and I always get the right answer when I add, subtract, multiply or divide integers. I can explain my answer.
5. Solve contextual problems involving integers. [5]	I struggle to solve contextual problems involving integers.	I can usually write an expression to solve a contextual problem involving integers.	I can always solve contextual problems involving integers.	I can always solve contextual problems involving integers. I can explain the solution in context.

Sample Problems for Section 2.2

1. Draw a model (chips, array or number line) to solve each of the following multiplication problems.

a. $-1(7)$

c. $8(-8)$

b. $-3(-6)$

d. $-2 \cdot 7$

2. Explain why the product of a positive and negative integer is negative or the product of a negative and negative integer is positive, using pictures, models, and words for your explanation.

3. Find each product or quotient.

a. $9 \cdot 4$

d. $\frac{-80}{8}$

b. $\frac{-14}{-7}$

e. $-9 \cdot -2$

c. $5(-2)$

f. $\frac{-48}{2}$

4. Simplify each.

a. $-6(-5)$

e. $-10 \cdot 31$

b. $-16(-3)$

f. $-89 + (-6)$

c. $\frac{-81}{-9}$

g. $6 - 9$

d. $-4 + 21$

h. $\frac{-40}{8}$

5. Solve each of the following contextual problems involving integers.

a. David is in a submarine at 200 feet below sea level. Casey is above him in a helicopter at 5,900 feet altitude. How far apart are David and Casey?

b. Scuba diving, Elisa descends at a rate of 30 feet per minute. How long will it take her to reach a depth of 90 feet?

c. Rome was founded in 753 BCE. Rome fell in 476 ACE. How many years passed between Rome's founding and fall?

Section 2.3: Add, Subtract, Multiply, Divide Positive and Negative Rational Numbers (all forms)

Section Overview:

In this section students extend their understanding of operating with integers to operating with all rational numbers. Students start with an activity to help them think about: 1) a line as a way to represent numbers, 2) rational numbers as numbers that can be written in the form a/b , where $b \neq 0$, and 3) locating any number that can be written as a/b (any rational number) on the number line. Students then move to operating with rational numbers in all forms by applying the rules of arithmetic that they should have solidified in the previous two sections.

A central idea throughout this section is the importance of estimating quantities before one computes. This is not only an important life skill; it's also an important skill in making sense of answers and working with technology. Students should get in the habit of estimating answers before they compute them so that they can check the reasonableness of their results.

Concepts and Skills to be Mastered (from standards)

By the end of this section, students should be able to:

1. Estimate the sum, difference, product or quotient of positive and negative rational numbers.
2. Find the sum or difference of rational numbers fluently.
3. Find the product or quotient of rational numbers fluently.
4. Solve contextual problems involving positive and negative rational numbers.
5. Explain why division by 0 is undefined.

2.3a Class Activity: Rational Numbers/Number Line City—Add, Subtract Rational Numbers

Activity: Create your own number line with the length of string your teacher gave you. Pick a spot in the middle of your line for “0”. Your “units” will be the length of your string. You will also find other points on your line. The mathematical ideas on which this activity is based can be found in the mathematical foundation page

In Number Line School, there is one straight hallway. The numbers on the classrooms indicate the distance from the main office, which is at zero. Mark the location of the following classrooms on the map of Number Line School below.

Mr. Bowen: $-3\frac{1}{4}$

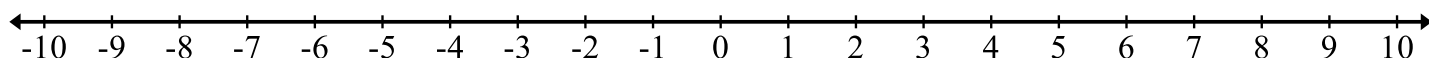
Ms. England: $-6\frac{1}{4}$

Mr. Francks: $-8\frac{1}{3}$

Ms. Abe: 3.5

Mrs. Chidester: 5.25

Mr. David: -5.2



Work with your group to answer the following questions. Be sure to explain your reasoning!

Whose classroom is further from the Main Office, Ms. Abe, or Mrs. Chidester?

1. a. *Approximately* how far apart are Ms. Abe and Mrs. Chidester?

b. *Exactly* how far apart are Ms. Abe and Mrs. Chidester?
2. Joe has Mr. Bowen's class first hour and Ms. England's class second hour. How far must he walk to get from Mr. Bowen's class to Ms. England's class? Estimate your answer before you compute it.
3. What is the distance between Mrs. Chidester's class and Mr. David's class? Estimate your answer before you compute it.
4. What is the distance between Mr. Franck's class and Ms. England's class? Estimate your answer before you compute it.

2.3a Homework: Rational Numbers/Number Line City

In Number Line City, there is one straight road. All the residents of number line city live either East (+) or West (–) of the Origin (0). All addresses are given as positive or negative rational numbers. Label the location of the following addresses on the map of Number Line City below with a point and the first letter of the resident (for example put a “B” above the point for Betsy.):

● Betsy: $-1\frac{3}{4}$

● Frank: $\frac{10}{4}$

● Number Line School: $1\frac{1}{2}$

● Allyson: -2.5

● Emily: 7.25

● Town Hall: 0

● Chris: $5\frac{1}{2}$

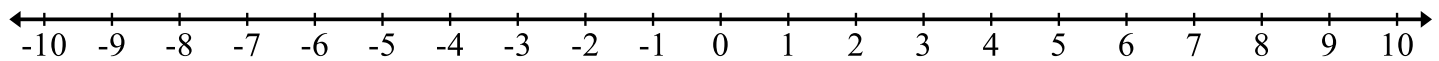
● Hannah: $6\frac{1}{5}$

● Grocery Store: -4.8

● David: 6.0

● Jamee: -6.2

● Post Office: -10



Now answer the following questions. Estimate your answer first, then compute it.

1. How far is it from Allyson’s house to Betsy’s house?
2. How far is it from Chris’ house to Betsy’s house?
3. How far is it from David’s house to Chris’ house?
4. How far is it from Emily’s house to Jamee’s house?
5. Who lives closer to the Grocery Store, Allyson or Jamee? Explain.
6. Who lives closer to Town Hall, Jamee or Hannah? Explain.

7. The mailman leaves the post office and makes a delivery to Hannah, picks up a package from Jamee, and then delivers the package to Frank before returning to the post office. How far did the mailman travel?
8. Every morning, Betsy walks to Allyson's house and they walk together to school. How far does Betsy walk each morning?
9. Iris moves into Number Line City. She finds out that she lives exactly 3.45 units from Emily. Where does Iris live?
10. Logan moves into Number Line City. He moves into a home $2\frac{1}{4}$ units from Chris' home. Where could Logan live? Where do you suggest he live and why?

Spiral Review

1. Solve
 - a. $16 - 25 =$
 - b. $-2 - (-8) =$
 - c. $5 + -11 =$
 - d. $-81 - 19 =$
2. Model $\frac{4}{5} - \frac{2}{3} =$
3. Jack and Jill went on their first date to Chili's. Their food bill totaled \$32.00 plus the 15% tip they need to pay the waitress.
 - a. How much will their 15% tip be?
 - b. How much will Jack pay in all?

2.3b More Operations with Rational Numbers

Review: Addition/Subtraction

In elementary school, you learned the following:

Example 1: Adding is a way to simplify or collect together things that are the same units.

$$3 \text{ apples} + 4 \text{ apples} = 7 \text{ apples}$$

$$3 \text{ apples} + 4 \text{ bananas} = 3 \text{ apples} + 4 \text{ bananas}$$

Example 2: Place value matters when you add whole numbers because you want to add units that are the same.

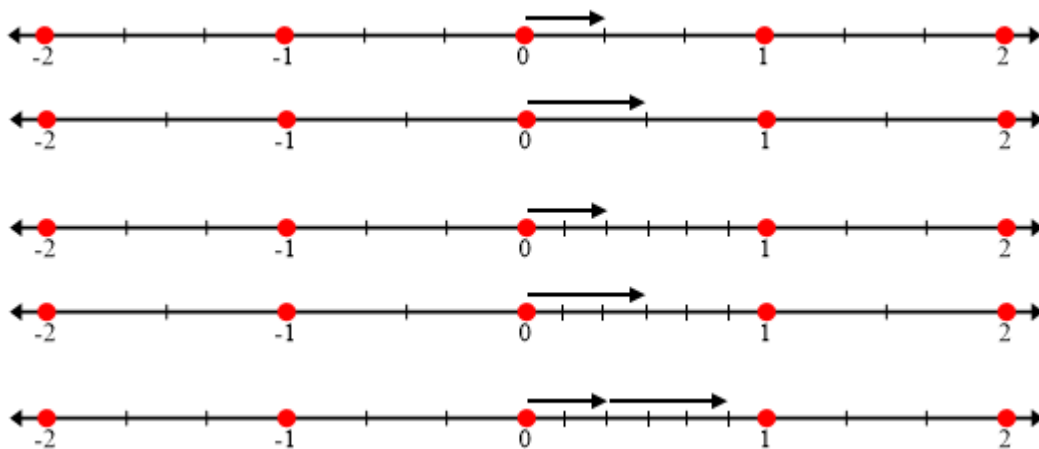
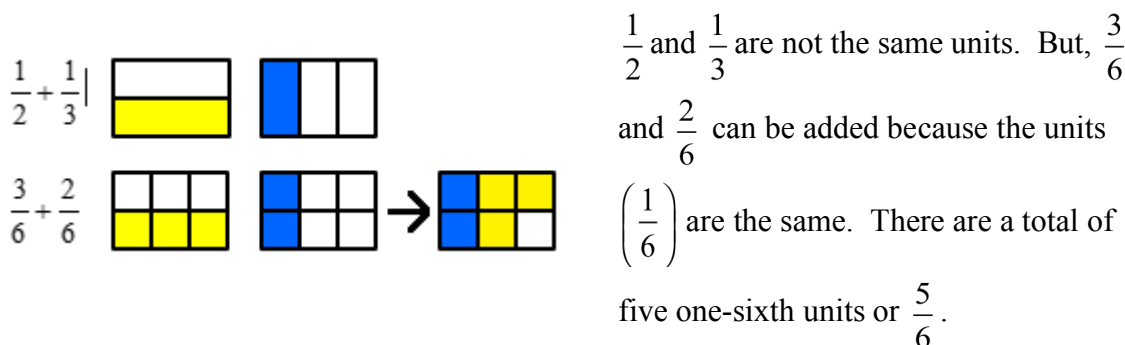
$24 + 7 \neq 94$ (you can't add the 2 to the 7 because the "2" represents the number of tens and the "7" represents the number of ones—they are not the same kinds of units.) $24 + 7 = 31$.

Example 3: Place value matters when you add decimals, again you want to add the same kinds of units.

$$3.2 + 4.01 \neq 4.3$$

$$3.2 + 4.01 = 7.21$$

Example 4: To add or subtract fractions you must have the same units, thus you must have a common denominator.



In this chapter we have learned how to add and subtract with integers first with models and then by applying rules. We will now apply these skills to all rational numbers.

1. Think back to the activity you did in 2.3a; what is a “rational number”?

Find the sum or difference for each.

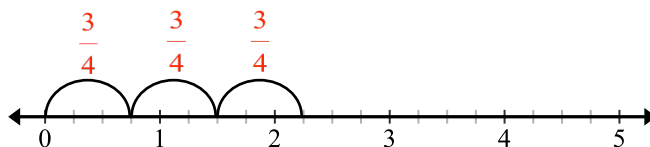
2. $-\frac{2}{3} + \frac{1}{4}$	3. $\frac{5}{3} - \frac{3}{4}$	4. $\frac{2}{3} - \left(-\frac{3}{2}\right)$
5. $\frac{3}{4} - 1\frac{2}{3}$	6. $-\frac{5}{4} - \left(-\frac{1}{3}\right)$	7. $-\frac{4}{3} + \left(-\frac{3}{4}\right)$
8. $-0.5 - \frac{2}{3}$	9. $\frac{7}{4} + (-0.\overline{6})$	10. $-\frac{7}{3} - (-0.75)$
11. $-0.\overline{3} + 1.4$	12. $-1.6 - (-1.\overline{6})$	13. $0 - (-1.75)$

Review: Multiplication and Division

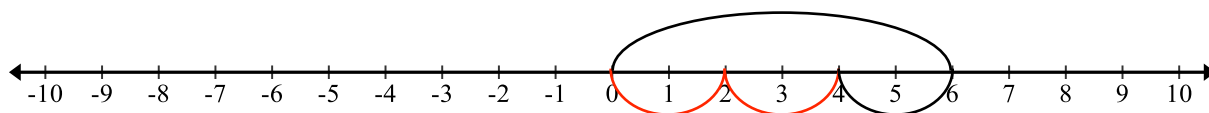
Review the general multiplication models in 2.2a Class Activity. Remember that multiplying means you have a certain number of groups with a certain number of units per group. You do NOT need a common denominator for multiplication.

Example 5: Whole number by fraction or fraction by whole number

$3 \times \frac{3}{4}$ means three groups of $\frac{3}{4}$, so $3 \times \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}$



$\frac{2}{3} \times 6$ means two-thirds of a group of six, so $\frac{2}{3} \times 6 = 4$



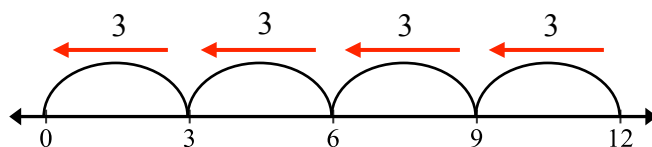
Example 6: Fraction by a fraction

$\frac{1}{2} \times \frac{1}{3}$ means “one-half groups of one-third.”

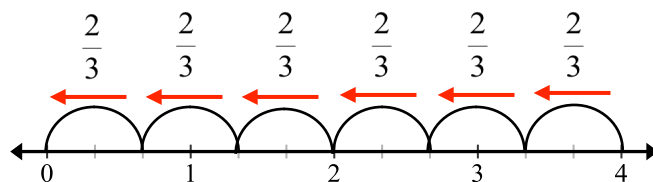


Example 7: Division, whole number by a fraction

Recall that division is the inverse of multiplication. In other words, you’re looking for a missing factor when you divide. $12 \div 3$ can mean that you’re starting with 12 and you want to know how many groups of 3 units are in 12. Similarly, $4 \div \frac{2}{3}$ can mean starting with 4 and want to know how many groups of $\frac{2}{3}$ are in 4.

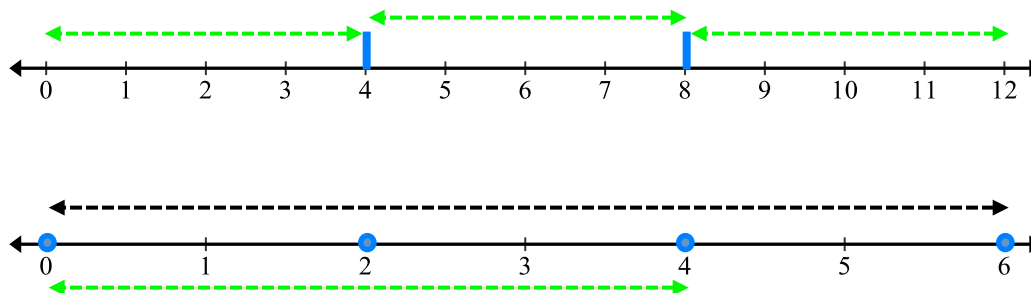


$12 \div 3 = 4$ because there are four groups of 3 units in 12.



Similarly, $4 \div \frac{2}{3} = 6$ because there are 6 groups of $\frac{2}{3}$ in 4.

The other way to think about $12 \div 3$ is, you're starting with 12 and you want to know how many units are in each **whole** group if you have 3 groups. Similarly, $4 \div \frac{2}{3}$ can be thought of as starting with 4 and you want to know how many are in a **whole** group if you have $\frac{2}{3}$ of a group:



Use your understanding of multiplication of fractions and integers to find the following products:

13. $\left(-\frac{1}{2}\right)\left(-\frac{4}{5}\right)$	14. $\left(-1\frac{1}{2}\right)(-6)$	15. $\left(-\frac{3}{4}\right)(12)$
16. $(-3)\left(\frac{5}{4}\right)$	17. $\left(-\frac{2}{3}\right)\left(-\frac{1}{4}\right)$	18. $\left(\frac{2}{3}\right)\left(-\frac{3}{5}\right)$

Use your understanding of division of fractions and integers to find the following quotients:

19. $(3) \div \left(-\frac{1}{4}\right)$	20. $(-2) \div \left(-\frac{2}{3}\right)$	21. $\left(-\frac{2}{3}\right) \div \left(\frac{1}{2}\right)$
22. $\left(-\frac{3}{5}\right) \div \left(-\frac{3}{2}\right)$	23. $\left(\frac{2}{5}\right) \div (-2)$	24. $\left(-2\frac{3}{4}\right) \div \left(-2\frac{3}{4}\right)$

2.3c Class Activity: Multiply and Divide Rational Numbers

Work in groups to extend your understanding of multiplying and dividing rational numbers.

1. Will the product of $\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)$ be bigger or smaller than each of the factors? Explain

Find the product to check your explanation.

2. Explain the value of the product $\left(\frac{3}{4}\right)(2)$. Will it be bigger or smaller than each of the factors?

Find the product to check your explanation.

3. Explain the value of the product $\left(\frac{3}{4}\right)(0.5)$. Will it be bigger or smaller than each factor?

Find the product to check your explanation.

4. Will the quotient of $\frac{3}{4} \div \frac{1}{4}$ be bigger or smaller than each term? Explain.

Find the quotient to check your explanation.

5. Explain the quotient $\frac{3}{4} \div 2$. Will it be bigger or smaller than each term?

Find the quotient to check your explanation.

6. Explain the quotient $\frac{3}{4} \div 0.5$. Will it be bigger or smaller than each term?

Find the quotient to check your explanation.

Estimate by rounding to the nearest integer. Discuss in your group if your answer is an over estimate or an under estimate of the actual value. Be able to justify your answer.

8. $3\frac{1}{3} \div \frac{7}{9} \approx \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} \approx \underline{\hspace{1cm}}$

9. $\left(6\frac{1}{3}\right)(2.2) \approx (\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) \approx \underline{\hspace{1cm}}$

10. $4\frac{1}{9} \div 7.891 \approx \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} \approx \underline{\hspace{1cm}}$

11. $\left(\frac{4}{5}\right)(-5.9)\left(4\frac{1}{8}\right) \approx (\underline{\hspace{1cm}})(\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) \approx \underline{\hspace{1cm}}$

Estimate and then confirm your estimation by finding the exact value with a calculator.

12. $-0.875 \div \frac{2}{7} \approx \underline{\hspace{1cm}}$ Exact value: $\underline{\hspace{1cm}}$

13. $\left(3\frac{3}{5}\right)\left(2\frac{1}{10}\right) \approx \underline{\hspace{1cm}}$ Exact value: $\underline{\hspace{1cm}}$

14. $0.75\left(\frac{2}{11}\right)\left(\frac{11}{12}\right) \approx \underline{\hspace{1cm}}$ Exact value: $\underline{\hspace{1cm}}$

15. $3\frac{4}{5} \div -19 \approx \underline{\hspace{1cm}}$ Exact value: $\underline{\hspace{1cm}}$

16. $(-9)\left(-8\frac{1}{4}\right) \approx \underline{\hspace{1cm}}$ Exact value: $\underline{\hspace{1cm}}$

17. $9\frac{3}{4} \div 12\frac{1}{4} \approx \underline{\hspace{1cm}}$ Exact value: $\underline{\hspace{1cm}}$

Exploration Activity:

Divide each

a. $12 \div 12 =$

g. $12 \div 1/2 =$

b. $12 \div 6 =$

h. $12 \div 1/3 =$

c. $12 \div 4 =$

i. $12 \div 1/4 =$

d. $12 \div 3 =$

j. $12 \div 1/6 =$

e. $12 \div 2 =$

k. $12 \div 1/12 =$

f. $12 \div 1 = 1$

What do you notice?

Divide each:

l. $1/2 \div 12 =$

q. $1/2 \div 1/2 =$

m. $1/2 \div 6 =$

r. $1/2 \div 1/3 =$

n. $1/2 \div 4 =$

s. $1/2 \div 1/4 =$

o. $1/2 \div 2 =$

t. $1/2 \div 1/6 =$

p. $1/2 \div 1 =$

u. $1/2 \div 1/12 =$

What do you notice?

What is your conjecture about $12 \div 0$ or $1/2 \div 0$? Justify your conjecture.

What do you think would happen if we took any (nonzero) number, b , and divided by 0?

2.3c Homework: Multiply and Divide Rational Numbers

Multiplying and dividing rational numbers. For #5 - 13 estimate first, then compute.

1. Will the product of $(2)\left(\frac{1}{3}\right)$ be bigger or smaller than each of the factors? Explain

5. $(2)\left(\frac{1}{3}\right) =$

6. $5 \times \frac{1}{4} =$

7. $\frac{1}{2} \div 0 =$

2. Will the product of 2 and 3 be bigger or smaller than each factor? Explain.

8. $\frac{2}{3} \times 5 =$

3. Explain the quotient $2 \div \frac{1}{3}$: will it be bigger or smaller than each term?

9. $\left(2\frac{1}{2}\right)(3.5) =$

10. $\frac{2}{7} \div -0.9 =$

4. Will the quotient of $\frac{1}{3} \div 2$ be bigger or smaller than each term?

11. $3 \div \frac{1}{2} =$

12. $\frac{5}{8} \div 3.7 =$

13. $\frac{1}{2} \div 4 =$

Spiral Review

Solve the following with or without a model

1. $(-12)(-10) =$

2. $23 + (-23) =$

3. $\frac{1}{6} + \frac{3}{7} =$

4. $0.85 + \frac{1}{2} =$

5. The temperature increased 2° per hour for six hours. How many degrees did the temperature raise after six hours?

2.3c Extra Practice: Multiply and Divide Rational Numbers

Estimate each product or quotient and then compute.

1. $-0.5 \times \frac{1}{3} =$

11.

2. $\frac{5}{9} \div \left(-\frac{5}{6}\right) =$

12. $\left(-\frac{2}{3}\right) \div 0.24 =$

3. $-0.375 \times \frac{2}{9} =$

13. $0.25 \div \frac{1}{3} =$

4. $\left(-\frac{2}{7}\right) \times \left(-\frac{1}{3}\right) =$

14. $0.4 \times \left(-\frac{1}{2}\right) =$

5. $\frac{8}{9} \div 0.2 =$

15. $-0.75 \div \left(-\frac{3}{8}\right) =$

6. $-0.75 \times \frac{1}{3} =$

16. $\frac{3}{4} \div 0.125 =$

7. $\frac{4}{5} \times \frac{5}{8} =$

17. $\left(-\frac{1}{7}\right) \div \left(-\frac{1}{5}\right) =$

8. $\frac{7}{8} \div \left(-\frac{2}{3}\right) =$

18. $(-0.2) \div \frac{3}{7} =$

9. $\frac{2}{7} \div \frac{1}{7} =$

19. $\frac{3}{7} \times \frac{7}{12} =$

10. $\left(-\frac{2}{3}\right) \times -0.4 =$

20. $\left(-\frac{4}{11}\right) \div \left(-\frac{8}{11}\right) =$

2.3d Class Activity: Multiply and Divide continued

For questions #1 – 5, do each of the following:

- a. Write an equation or draw model.
- b. Estimate an answer.
- c. Find the exact answer.

1. Dory can run 3.25 miles every hour. How long will it take her to run a race that is $16\frac{1}{4}$ miles long?

2. From 6 p.m. to 6 a.m. the temperature dropped 1.4 degrees each hour. What was the total change in temperature between 6 p.m. and 6 a.m.?

3. Bridgette wants to make 14 matching hair bows for her friends. Each hair bow requires $\frac{3}{4}$ of a yard of ribbon. How many yards of ribbon will Bridgette need to make all the hair bows?

4. There are 18 apple trees in my orchard. If I can harvest the fruit from $1\frac{2}{3}$ trees each day, how many days will it take to harvest the entire orchard?

5. I want to use a recipe for muffins that calls for 3 cups of flour and 2 eggs. If I only have 1 egg, how much flour should I use?

For problems # 6 – 15, first estimate, then compute.

6. $(-0.5)\left(\frac{5}{3}\right) =$

7. $(-0.75)\left(\frac{5}{7}\right) =$

8. $-0.23 \times \left(-\frac{2}{5}\right) =$

9. $\frac{1}{2}\left(-\frac{1}{3}\right) =$

10. $\left(\frac{2}{3}\right)(-0.8) =$

11. $\frac{3}{4} \div 0.25 =$

13. $\frac{1}{4} \div \left(-\frac{1}{3}\right) =$

14. $-\frac{2}{5} \div (-0.86) =$

15. $-\frac{1}{5} \div \left(-\frac{2}{3}\right) =$

16. Write a context (story) for the given problem:
 $5 \times \frac{3}{4} =$

17. Write a context (story) for the given problem:
 $2 \div \frac{1}{3} =$

2.3d Homework: Problem Solve with Rational Numbers

Problem	Estimate the quantity, show how you arrived at your estimate	Calculate (show your work)
1. $2\frac{1}{3} - 4.8$	$2 - 5 \approx -3$	
2. $(8.6)\left(-3\frac{3}{4}\right)$		
3. $-4.7 \div \frac{1}{4}$		
4. $2\frac{1}{2} + \left(-\frac{3}{4}\right) + 0.75$		
5. $\frac{5}{8} - (-0.125)$		
6. $\frac{1}{2}\left(-\frac{3}{7}\right)(1.75)$		
7. $\frac{1}{6} + \frac{1}{12} + (-0.25) + 1.5$		
8. $-7\frac{1}{2} \div -2.5$		

Estimate first, then calculate. Finally, evaluate whether your answers are reasonable by comparing the estimate with the calculated answer. Correct your answers when appropriate.

9. Janice is ordering a new pair of eyeglasses. The frames she wants cost \$98.90. High index lenses cost \$76.00. Her insurance says that they will pay \$100 of the cost, and 20% of the remaining amount. How much will Janice have to pay?

10. Fernando entered a weight-loss contest. The first week he lost $5\frac{1}{5}$ pounds, the next week he lost 4.59 pounds and the last week he lost 2.31 pounds. What was his total change in weight?
11. A marathon is 26.2 miles. June wants to run the distance of a marathon in a month, but she thinks that she can only run $\frac{3}{4}$ of a mile each day. How many days will she need to run? Will she be able to finish in a month?
12. Reynold just got hired to paint fences this summer. He gets paid \$0.60 for every foot of fencing that he paints. If Reynold paints an average of 45 yards of fencing a week, how much can he earn in $8\frac{2}{5}$ weeks?
13. A super freezer can change the internal temperature of a 10 pound turkey by -2.4°F every 10 minutes. How much can the freezer change the temperature of the turkey in $3\frac{3}{4}$ hours?
14. Dylan is ordering 5 dozen donuts for his grandmother's birthday party. He wants $\frac{1}{4}$ of the donuts to be chocolate frosting with chocolate sprinkles, and $\frac{1}{3}$ of the donuts to have caramel frosting with nuts. The rest of the donuts will just have a sugar glaze. The donuts with the sugar glaze cost \$0.12 each, and the other donuts cost \$0.18 each. How much will Dylan have to pay for all the donuts?
15. Penny and Ben want to buy new carpet for their living room. They measured the dimensions of the living room and found that it was $12\frac{1}{4}$ feet by $8\frac{5}{8}$ feet. They know that installation will cost \$37.50. If Penny and Ben want to spend no more than \$500 on carpet, how much can they afford to pay for each square foot?

Spiral Review

1. Use the chip model to solve $-14 + 5$.
2. What percent of 80 is 60?
3. Parker earned \$450 over the summer. If he put 70% of his earnings into his savings, how much money did he have left over?
4. Order the numbers from least to greatest.
 $-2.15, \frac{17}{7}, 2.7, -2.105$
5. Max has \$150. He spends $\frac{2}{3}$ of his money on groceries. How much money did he have left over?

2.3e Self-Assessment: Section 2.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Estimate the sum, difference, product or quotient of positive and negative rational numbers. [1, 2]	I'm not very good at estimating the sum, difference, products or quotients with rational numbers.	Most of the time I can estimate the sum, difference, product or quotient of rational numbers.	I can always estimate the sum, difference, product or quotient of rational numbers.	I can always estimate the sum, difference, product or quotient of rational numbers. I can explain why my estimate is a reasonable.
2. Find the sum or difference of rational numbers fluently. [1]	I struggle to add and subtract rational numbers.	Most of the time I can find the sum or difference of rational numbers.	I can always find the sum or difference of rational numbers.	I can always find the sum or difference of rational numbers. I can explain why my answer is correct.
3. Find the product or quotient of rational numbers fluently. [2]	I struggle to multiply and divide rational numbers.	I usually get the right answer when I multiply and divide rational numbers.	I always get the right answer when I multiply and divide rational numbers.	I can multiply and divide positive and negative rational numbers fluently (quickly and accurately). I do not need to refer back to any rules or draw a model to find the answer.
4. Solve contextual problems involving positive and negative rational numbers. [3]	I struggle to solve contextual problems involving rational numbers.	I can usually solve contextual problems involving rational numbers.	I can always solve contextual problems involving rational numbers.	I can always solve contextual problems involving rational numbers. I can explain the solution in context.
5. Explain why division by 0 is undefined.	I forget if $0 \div 12$ is undefined or if $12 \div 0$ undefined.	I know which of these two statements is false: a) $0 \div 12 = 0$ b) $12 \div 0 = 0$	I can explain with examples and words why division by 0 is undefined.	I can explain with examples and words why division by 0 is undefined. This idea makes perfect sense to me.

Sample Problems for Section 2.3

1. Estimate sum or difference then find the actual value.

a. $-0.9 - \frac{4}{9}$

e. $\frac{-5}{8} - \frac{5}{6}$

b. $-0.2 + \left(-\frac{2}{5}\right)$

f. $-0.9 - \frac{4}{9}$

c. $8 - (-0.8)$

g. $\frac{-1}{2} + 4$

d. $-2.5 - \frac{3}{4}$

2. Estimate each product or quotient. Then find the actual product or quotient.

a. $-6\left(\frac{1}{6}\right)$

e. $-10 \div \frac{1}{2}$

b. $-1.6(-0.4)$

f. $-\frac{8}{9} \div \left(-\frac{3}{4}\right)$

c. $\frac{-8}{9}(0.5)$

g. $\frac{6.3}{-0.7}$

d. $-4(-0.2)$

h. $\frac{-3}{8} \div -0.9$

3. Solve each of the following contextual problems involving rational numbers. Express answers as fractions or decimals when appropriate.

- Viviana buys a 500 mL bottle of water. She knows that she usually drinks 80 mL each hour. How many hours will it take her to finish her bottle of water?
- The temperature at midnight was -2°C . By 8 am, it had risen 1.5° . By noon, it had risen another 2.7° . Then a storm blew in, causing it to drop 4.7° by 6 pm. What was the temperature at 6 pm?
- Xochitl has saved \$201.60. Yamka has saved $\frac{2}{3}$ the amount Xochitl saved. Zack has saved 1.5 times the amount Yamka saved. How much have they save all together?

First published in 2013 by the University of Utah in association with the Utah State Office of Education.

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CHAPTER 3: Expressions and Equations Part 1

(4-5 weeks)

UTAH CORE Standard(s): Expressions and Equations

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 7.EE.1
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. 7.EE.2
3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. 7.EE.3
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations to solve problems by reasoning about the quantities. 7.EE.4
 - a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. 7.EE.4a

CHAPTER OVERVIEW:

The goal of chapter 3 is to facilitate students' transition from concrete representations and manipulations of arithmetic and algebraic thinking to abstract representations. Each section supports this transition by asking students to model problem situations, construct arguments, look for and make sense of structure, and reason abstractly as they explore various representations of situations. Throughout this chapter students work with fairly simple expressions and equations to build a strong intuitive understanding of structure (for example, students should understand the difference between $2x$ and x^2 or why $3(2x - 1)$ is equivalent to $6x - 3$ and $6x + -3$). Students will continue to practice skills manipulating algebraic expressions and equations throughout Chapters 4 and 5. In Chapter 6, students will revisit ideas in this chapter to extend to more complicated contexts and manipulate with less reliance on concrete models. Another major theme throughout this chapter is the identification and use in argument of the arithmetic properties. The goal is for students to understand that they have used the commutative, associative, additive and multiplicative inverse, and distributive properties informally throughout their education. They are merely naming and more formally defining them now for use in justification of mathematical (quantitative) arguments. See the Mathematical Foundation for more information.

Section 3.1 reviews and builds on students' skills with arithmetic from previous courses to write basic numerical and algebraic expressions in various ways. In this section students should understand the difference between an expression and an equation. Further, they should understand how to represent an unknown in either an expression or equation. Students will connect manipulations with numeric expressions to manipulations with algebraic expressions. In connecting the way arithmetic works with integers to working with algebraic expressions, students name and formalize the properties of arithmetic. By the end of this section students should be proficient at simplifying expressions and justifying their work with properties of arithmetic.

Section 3.2 uses the skills developed in the previous section to solve equations. Students will need to distribute and combine like terms to solve equation. In 7th grade, Students only solve linear equations in the form of

$ax + b = c$ or $a(x + b) = c$, where a , b , and c are rational numbers. This section will rely heavily on the use of models to solve equations, but students are encouraged to move to abstract representation when they are ready and fluent with the concrete models.

Section 3.3 ends the chapter with application contexts. Contexts involve simple equations with rational numbers. Percent increase and decrease is revisited here. Time should be spent understanding the meaning of each part of equations and how the equation is related to the problem context. Note that the use of models is to develop an intuitive understanding and to transition students to abstract representations of thinking.

VOCABULARY: coefficient, constant, expression, equation, factor, integer, like terms, product, rational number, simplify, term, unknown, variable.

CONNECTIONS TO CONTENT:

Prior Knowledge:

Students extend the skills they learned for operations with whole numbers, integers and rational numbers to algebraic expressions in a variety of ways. For example, in elementary school students modeled 4×5 as four “jumps” of five on a number line. They should connect this thinking to the meaning of “ $4x$ ” or “ $4(x + 1)$.” Students also modeled multiplication of whole numbers using arrays in earlier grades. In this chapter they will use that logic to multiply using unknowns. Additionally, in previous grades, students explored and solidified the idea that when adding/subtracting one must have “like units.” Thus, when adding $123 + 14$, we add the “ones” with the “ones,” the “tens” with the “tens” and the “hundreds” with the “hundreds”. Similarly, we cannot add $1/2$ and $1/3$ without a common denominator because the unit of $1/2$ is not the same as a unit of $1/3$. Students should extend this idea to adding variables. In other words, $2x + 3x$ is $5x$ because the unit is x , but $3x + 2y$ cannot be simplified further because the units are not the same (three units of x and two units of y .)









In 6th grade, students solved one-step equations. Students will use those skills to solve equations with more than one-step in this chapter. Earlier in this course, students developed skills with rational number operations. In this chapter, students will be using those skills to solve equations that include rational numbers.

Future Knowledge:

As students move on in this course, they will continue to use their skills in working with expressions and equations in more complicated situations. The idea of inverse operations will be extended in later grades to inverse functions of various types. Additionally, students in later grades will return to the idea of field axioms.

A strong foundation in simplifying expressions and solving equations is fundamental to later grades. Students will also need to be proficient at translating contexts to algebraic expressions and equations AND at looking at expressions and equations and making sense of them relative to contexts.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

	Make sense of problems and persevere in solving them	Students will make sense of expressions and equations by creating models and connecting intuitive ideas to properties of arithmetic. Properties of arithmetic should be understood beyond memorization of rules.
	Reason abstractly and quantitatively	Students will, for example, note that $x + x + x + x + x$ is the same as $5x$. Students should extend this type of understanding to $5(x + 1)$ meaning five groups of $(x + 1)$ added together, thus simplifying to $5x + 5$. For each of the properties of arithmetic, students should connect concrete understanding to abstract representations.
	Construct viable arguments and critique the reasoning of others	Students should be able to explain and justify any step in simplifying an expression or solving an equation first in words and/or pictures and then with properties of arithmetic. Further, students should be able to evaluate the work of others to determine the accuracy of that work and then construct a logical argument for their thinking that involves properties of arithmetic.
	Model with mathematics	Students should be able to model situations with expressions and equations AND they should be able to translate expressions and equations to contexts. Further, they should be able to interchange models with abstract representations.
	Attend to precision	Students demonstrate precision by using correct terminology and symbols when working with expressions and equations. Students use precision in calculation by checking the reasonableness of their answers and making adjustments accordingly.
	Look for and make use of structure	Using models, students develop an understanding of algebraic structures. For example, in section 3.2 students should understand the structure of an equation like $3x + 4 = 5$ as meaning the same thing as $3x = 1$ or $x = 1/3$ when it is “reduced.” Another example, in section 3.3, students will write equations showing that a 20% increase is the original amount plus 0.2 of the original amount or 1.2 of the original amount. They will note that in an expression that can be written in multiple ways: $x + 0.2x$ or $x(1 + 0.2)$.
	Use appropriate tools strategically	Students demonstrate their ability to select and use the most appropriate tool (paper/pencil, manipulatives, and calculators) while solving problems. Students should recognize that the most powerful tool they possess is their ability to reason and make sense of problems.
	Look for and express regularity in repeated reasoning	Students will study patterns throughout this chapter and connect them to both their intuitive understanding and the properties of arithmetic. Students will move to expressing patterns they notice to general forms.

3.0 Anchor Problem: Decorating a Patio.

Below are the first three steps of a pattern; on the next page we will investigate steps 10 and “x.” Each square in the pattern is one unit.

Step 1



How many units are in this step of the pattern? Write down at least two methods that you could use to quickly “add up” the units.

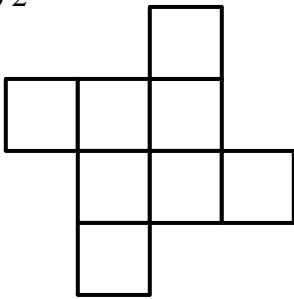
Method 1:

Method 2:

Method 3:

Method 4:

Step 2



How many units are in this step of the pattern? Write down at least two methods that you could use to quickly “add up” the units.

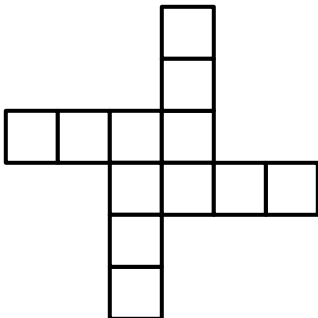
Method 1:

Method 2:

Method 3:

Method 4:

Step 3



How many units are in this step of the pattern? Write down at least two methods that you could use to quickly “add up” the units.

Method 1:

Method 2:

Method 3:

Method 4:

Step 10

Draw what you think step 10 would look like.

How many units are in this step of the pattern? Write down at least two methods that you could use to quickly “add up” the units.

Method 1:

Method 2:

Method 3:

Method 4:

Step x

Use this space to draw a model to help you think about the pattern.

How could you adapt the methods you used above to find the number of units for ***any*** pattern.

Method 1:

Method 2:

Method 3:

Section 3.1: Communicate Numeric Ideas and Contexts Using Mathematical Expressions and Equations

Section Overview: This section contains a brief review of numerical expressions. Students will recognize that a variety of expressions can represent the same situation. Models are encouraged to help students connect properties of arithmetic in working with numeric expressions to working with algebraic expressions. These models, particularly algebra tiles, aid students in the transition to abstract thinking and representation. Students extend knowledge of mathematical properties (commutative property, associative property, etc.) from purely numerical problems to expressions and equations. The distributive property is emphasized and factoring, “backwards distribution,” is introduced. Work on naming and formally defining properties appears at the beginning of the section so that students can attend to precision as they verbalize their thinking when working with expressions. Through the section, students should be encouraged to explain their logic and critique the logic of others.

Concepts and Skills to be Mastered (from standards)

By the end of this section, students should be able to:

1. Recognize and explain the meaning of a given expression and its component parts.
2. Recognize that different forms of an expression may reveal different attributes of the context.
3. Combine like terms with rational coefficients.
4. Use the Distributive Property to expand and factor linear expressions with rational numbers.
5. Recognize properties of arithmetic and use them in justifying work when manipulating expressions.
6. Write numeric and algebraic expressions to represent contexts.

3.1a Class Activity: Naming Properties of Arithmetic

Naming Properties of Arithmetic

In mathematics, there are things called “properties;” you may think of them as “rules.” There is nothing new in the properties discussed in this section. Everything you expect to work still works. We are just giving vocabulary to what you’ve been doing so that when you construct a mathematical argument, you’ll be able to use language with precision.

Commutative Property

Examples:

The sum of both $8 + 7 + 2$ and $8 + 2 + 7$ is 17.

The sum of both $13 + 14 + (-3)$ and $13 + (-3) + 14$ is 24.

The product of both $\left(\frac{1}{2}\right)(7)(8)$ and $\left(\frac{1}{2}\right)(8)(7)$ is 28.

The product of both $(9)\left(-\frac{1}{3}\right)(7)$ and $\left(-\frac{1}{3}\right)(9)(7)$ is -21.

The word “commute” means “to travel” or “change.” It’s most often used in association with a location. For example, we say people *commute* to work.

Which pairs of expressions are equivalent?

1. $12 + 4$ $4 + 12$	2. $9.8 - 3.4$ $3.4 - 9.8$
3. $12 - 4$ $4 - 12$	4. $5 \cdot 4$ $4 \cdot 5$
5. $3 \cdot 0.9$ $0.9 \cdot 3$	6. $18 \div 6$ $6 \div 18$

7. What pattern are you noticing?

8. In your own words, what is the Commutative Property?

Associative Property

The word “associate” means “partner” or “connect.” Most often we use the word to describe groups. For example, if a person goes to Eastmont Middle School and not Indian Hills Middle School, we would say that person is *associated* with Eastmont Middle School.

Examples:

The sum of both $3 + (17 + 4) + 16$ and $(3 + 17) + (4 + 16)$ is 40

The product of both $(2 \times 5)(3)$ and $(2)(5 \times 3)$ is 30

For each of the following pairs of expressions, the operations are the same, but the constants have been associated (grouped) in different ways. Determine if the pairs are equivalent; be able to justify your answer.

9. $(12 + 4) + 6$ $12 + (4 + 6)$	10. $(12 - 4) - 3$ $12 - (4 - 3)$
11. $(3 + 5) + 7.4$ $3 + (5 + 7.4)$	12. $(20.9 - 8) - 2$ $20.9 - (8 - 2)$
13. $(5 \cdot 4) \cdot \left(\frac{1}{2}\right)$ $5 \cdot \left(4 \cdot \left(\frac{1}{2}\right)\right)$	14. $(18 \div 6) \div 3$ $18 \div (6 \div 3)$
15. $(6 \cdot 2) \cdot 5$ $6 \cdot (2 \cdot 5)$	16. $(24 \div 12) \div 3$ $24 \div (12 \div 3)$

17. What patterns do you notice about the problems that were given?

18. In your own words, what is the Associative Property?

Identity Property

The word “identity” has to do with “sameness.” We use this word when we recognize the sameness between things. For example, you might say that a Halloween costume cannot really hide a person’s true *identity*.

Above we defined the Associative and Commutative Properties for both addition and multiplication. We need to do the same thing for the Identity Property.

19. What do you think the Identity Property for Addition should mean?

20. Give examples of what you mean:

21. In your own words, what is the Identity Property of Addition?

22. What do you think the Identity Property for Multiplication should mean?

23. Give examples of what you mean:

24. In your own words, what is the Identity Property of Multiplication:

Inverse Properties

The word “inverse” means “opposite” or “reverse.” You might say, forward is the *inverse* of backward. There is an inverse for both addition and multiplication.

25. What do you think should be the additive inverse of 3?

26. What do you think would be the additive inverse of -3 ?

27. What do you think would be the multiplicative inverse of 3?

28. What do you think would be the multiplicative inverse of $\frac{1}{3}$?

29. In your own words, what is the Inverse Property of Addition?

30. In your own words, what is Inverse Property of Multiplication?

Generalizing Properties

Properties of Mathematics:

Name Property	Algebraic Statement	Meaning	Numeric Examples
Identity Property of Addition		.	
Identity Property of Multiplication			
Multiplicative Property of Zero			
Commutative Property of Addition			
Commutative Property of Multiplication			
Associative Property of Addition			
Associative Property of Multiplication			
Additive Inverse			
Multiplicative Inverse			

3.1a Homework: Naming Properties of Arithmetic

Complete the table below:

Identity Property of Addition: $a + 0 = a$	1a. Show the Identity Property of Addition with 2.17	1b. Show the Identity Property of Addition with -3
Identity Property of Multiplication: $a \cdot 1 = a$	2a. Show the Identity Property of Multiplication with 23	2b. Show the Identity Property of Multiplication with $-3b$
Multiplicative Property of Zero: $a \cdot 0 = 0$	3a. Show Multiplicative Property of Zero with 43.581	3b. Show the Multiplicative Property of Zero with $-4xy$
Commutative Property of Addition: $a + b = b + a$	4a. $4.38 + 2.01$ is the same as:	
	4b. $x + z$ is the same as:	
Commutative Property of Multiplication: $ab = ba$	5a. $\frac{5}{7} \cdot \frac{3}{8}$ is the same as:	
	5b. $6k$ is the same as:	
Associative Property of Addition: $(a + b) + c = a + (b + c)$	6a. $(1.8 + 3.2) + 9.5$ is the same as:	
	6b. $(x + 1) + 9$ is the same as:	
Associative Property of Multiplication: $(ab)c = a(bc)$	7a. $(2.6 \cdot 5.4) \cdot 3.7$ is the same as:	
	7b. $(wh)l$ is the same as:	
Use the listed property to fill in the blank.		
Multiplicative Inverse: $a \left(\frac{1}{a}\right) = 1$	8a. $3 (\quad) = 1$	
	8b. $\frac{1}{4} (\quad) = 1$	
Additive Inverse: $a + (-a) = 0$	9a. $\frac{5}{9} + \quad = 0$	
	9b. $\quad + -x = 0$	

Name the property demonstrated by each statement.

10.	$3 + -2 + 7 = 3 + 7 + -2$	
11.	$5 + (-5 + 4) + 6 = (5 + -5) + (4 + 6)$	
12.	$25 + (-25) = 0$	
13.	$(2/5)(5/2) = 1$	
14.	$(x + 3) + y = x + (3 + y)$	
15.	$2.37 \times 1.5 = 1.5 \times 2.37$	
16.	$1 \cdot mp = mp$	
17.	$9 + (5 + 35) = (9 + 5) + 35$	
18.	$0 + 6b = 6b$	
19.	$xy = yx$	
20.	$7x \cdot 0 = 0$	
21.	$4(3 \cdot z) = (4 \cdot 3)z$	
22.	$\frac{2}{3} \cdot 4.9 = 4.9 \cdot \frac{2}{3}$	
23.	$x + 4 = 4 + x$	

Spiral Review

1. Order the following rational numbers from least to greatest. $\frac{27}{3}$, 6.5, $\frac{18}{3}$, .99,
2. What is the opposite of -19 ?
3. What is 10% of 90? Use bar model to help you.
4. Without using a calculator, determine which fraction is bigger in each pair. Justify your answer with a picture *and* words.
 - a. $\frac{1}{3}$ or $\frac{1}{2}$
 - b. $\frac{3}{7}$ or $\frac{3}{5}$
5. Solve: $\frac{3}{4} + \frac{1}{8} =$

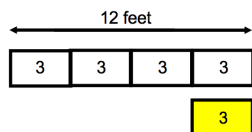
3.1b Class Activity: Translating Contexts to Numeric Expressions (Equivalent Expressions)

Review from Chapter 1: In 1.3, you drew models and then wrote arithmetic expressions to find fraction and percent decreases and increases. For example:

1.3a Class Activity 1a was a percent decrease of the original amount:

Larry has a piece of rope that's 12 feet long.

He cuts off 25% of the rope off. How long is the rope now?

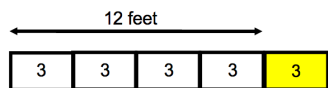


To write an expression for the length of the rope with the portion cut off, we can think about it two ways:

... we started with the 12 foot section and then subtracted 25% of 12: $12 - (0.25)12$ $12 - 3$ 9 feet of rope	...we can recognize that when we remove 25% of the rope, we're left with 75% ($100\% - 25\%$) of the rope: $(1 - 0.25)12$ $(0.75)12$ 9 feet of rope
---	--

1.3 Class Activity 1b was a percent increase of the original amount:

Joe has a rope that is 25% longer than Larry's 12-foot long rope. How long is Joe's rope?



To write an expression for the percent increase we think about it two ways:

...we start with the 12 foot rope and then add 25% of 12 to the length: $12 + (0.25) 12$ $12 + 3$ 15 feet of rope	...we recognize that a 25% increase means we start with the whole length and then add 25%; thus we now have 125% ($100\% + 25\%$) of the original: $(1 + 0.25) 12$ $(1.25) 12$ 15 feet of rope
--	---

In both examples above, we wrote *equivalent expressions*. What do you think it means to say “two expressions are equivalent”?

For each context: a) write a numeric expression, b) justify an estimate for the answer, and then c) find the answer (a calculator will be helpful for these.)

1. The tree in Maria's yard has grown 27% in the last two years. If it was originally 12 feet tall two years ago, how tall is it now?
2. Paulo can run a marathon in four hours and 20 minutes. If he's able to cut 18% off his time, how long will it take him to run a marathon?
3. Joe's snowboard is 140 centimeters; Carly's is 14% longer. How long is Carly's snowboard?
4. Write a context involving a percent for the following numeric expression: $300 + 0.27(300)$
5. Write a context involving a percent for the following numeric expression: $0.78(144)$
6. Juan, Calista, and Angelo are working together on the following percent problem:
There are 400 fish in a large tank. Write a numeric expression to find the number of fish in the tank if the number of fish increases by 23%.
Juan says the expression is $1.23(400)$; Calista says it's $400 + 0.23$; and Angelo says it's $(1 + 0.23)(400)$. Are they all correct? Explain why each expression is correct or incorrect.

7. Below are 4 expressions, circle the three that are equivalent.

- a $(1 - 0.35)(423)$
- b $(0.65)(423)$
- c $423 - 0.35(423)$
- d $423 - 0.35$

For each of the five contexts below, there are four numeric expressions offered. Look at each expression offered and determine whether or not it is appropriate for the given context. Explain why the expression “works” or “doesn’t work.”

8. Josh made five 3-pointers and four 2-pointers at his basketball game. Write a numeric expression to represent how many points Josh scored.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$3 + 3 + 3 + 3 + 3 + 2 + 2 + 2 + 2$			
b.	$5 + 3 + 4 + 2$			
c.	$(5 + 3)(4 + 2)$			
d.	$5(3) + 4(2)$			

9. Carlo bought two apples for \$0.30 each and three pounds of cherries for \$1.75 a pound. Write a numeric expression for how much money Carlo spent.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$2(1.75) + 3(0.30)$			
b.	$2(0.30) + 3(1.75)$			
c.	$0.30 + 0.30 + 1.75 + 1.75 + 1.75$			
d.	$(2 + 3)(0.30 + 1.75)$			

10. Inez bought two apples for \$0.30 each and three oranges for \$0.30 each. Write a numeric expression for how much money Inez spent.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$(0.30 + 0.30 + 0.30) + (0.30 + 0.30)$			
b.	$3(0.30) + 2(0.30)$			
c.	$5(0.30)$			
d.	$0.60 + 0.90$			

11. Aunt Nancy gave her favorite niece 3 dollars, 3 dimes, and 3 pennies. Write a numeric expression for how much money Nancy gave her favorite niece.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$3(1.00) + 3(0.10) + 3(0.01)$			
b.	$3(1.00 + 0.10 + 0.01)$			
c.	$3 + 1.00 + 0.10 + 0.01$			
d.	$3.00 + 0.30 + 0.03$			

12. Grandma Nancy has 20 chocolate and 25 red velvet cupcakes. She gives each of her three grandchildren $\frac{1}{3}$ of her cupcakes. Write an expression for how many cupcakes she gave each grandchild.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$3(20 + 25)$			
b.	$(\frac{1}{3})(20 + 25)$			
c.	$(20 + 25)/3$			
d.	$(\frac{1}{3})(20) + (\frac{1}{3})(25)$			

13. Mila bought a sweater for \$25 and a pair of pants for \$40. She had a 25% off coupon for her purchase, write an expression for how much Mila spent.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$0.25(25 + 40)$			
b.	$0.75(25 + 40)$			
c.	$0.75(25) + 0.75(40)$			
d.	$(\frac{3}{4})(25 + 40)$			

14. Paul bought two sandwiches for \$5 each, a drink for \$3.00, and a candy bar for \$1.00. He had a 20% off coupon for the purchase. Write an expression to how much Paul spent for the food.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$0.20(2(5) + 3 + 1)$			
b.	$0.80(2(5) + 3 + 1)$			
c.	$0.80(2(5)) + 3 + 1$			
d.	$(4/5)(2(5) + 3 + 1)$			

For each context: a) write a numeric expression for the context and b) answer the question.

15. Aunt Nancy gave each of her four nieces two dollars, 1 dime, and 3 pennies. How much money did Nancy give away
16. Uncle Aaron gave 8 dimes, 2 nickels, and 20 pennies to each of his two nephews. How much money did he give away?
17. I bought 2 toy cars for \$1.25 each and 3 toy trucks for \$1.70 each. How much did I spend?
18. The football team scored 1 touchdown, 3 field goals, and no extra points. How many points did they score in all? Hint: a touchdown is worth 6 points, a field goal worth 3, and an extra point worth one.
19. I had \$12. Then I spent \$2.15 a day for 5 days in a row. How much money do I have now?
20. I earned \$6. Then I bought 4 candy bars for \$0.75 each. How much money do I have left?
21. Cara bought two candles for \$3 each and three books for \$7 each. She had a 25% off her entire purchase coupon. How much did she spend on her purchases?
22. Mona and Teresa worked together to make \$118 selling phone covers and \$354 fixing computers. If they split the money evenly between the two of them, how much money did they each make?
23. Dora bought three pair of shoes for \$15 each and two pair of shorts for \$20 each. If she had a 15% off coupon for her entire purchase, how much money did she spend?

3.1b Homework: Translating Contexts to Numerical Expressions

For each context: a) write a numeric expression, b) justify an estimate for the answer, and then c) find the answer (a calculator will be helpful for these).

1. Camilla is allowed 1100 text messages a month on her plan. When she got her September bill it showed she had gone over the allowed text messages by 38%. How many text messages did she send in September?
2. Leona made \$62,400 in her first year as an engineer at IM Flash in Lehi. She's up for review and stands to get a 12.5% pay raise if her review is favorable. If she gets the pay raise, what will she be earning?

For each of the five contexts below, there are four numeric expressions offered. Look at each expression offered and determine whether or not it is appropriate for the given context. Explain why the expression "works" or "doesn't work."

3. I bought two toy cars for \$5 each and three toy trucks for \$7 each. Write a numeric expression for how much was spent.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$2(5) + 3(7)$			
b.	$2(3) + 5(7)$			
c.	$(2 + 3)(5 + 7)$			
d.	$(2 + 3) + (5 + 7)$			

4. The football team scored three touchdowns, two field goals, and two extra points. Write a numeric expression for how many points were scored in all. (Hint: a touchdown is 6 points, a field goal is 3 points, and an extra point is 1 point)

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$3(6) + 2(3) + 2(1)$			
b.	$6 + 6 + 6 + 3 + 3 + 1 + 1$			
c.	$(6 + 6 + 6) + (3 + 3) + (1 + 1)$			
d.	$18 + 6 + 2$			

5. I earned \$6. Then I bought 4 candy bars for \$0.50 each. Write a numeric expression for how much money I have left.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$6 - 0.50 - 0.50 - 0.50 - 0.50$			
b.	$6 - 4(0.50)$			
c.	$6 - (0.50 - 0.50 - 0.50 - 0.50)$			
d.	$6 - (0.50 + 0.50 + 0.50 + 0.50)$			

6. I earned \$5. Then I spent \$1 a day for 2 days in a row. Write a numeric expression for how much money I have now.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$5 - 1 + 1$			
b.	$5 - 1 - 1$			
c.	$5 - (1 - 1)$			
d.	$5 - (1 + 1)$			

7. Uncle Aaron bought three books for \$12 each and a candy bar for \$1.35. He had a coupon for 20% off his purchase. Write an expression for how much he spent.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$0.80(3)(12) + 1.35$			
b.	$0.08(3(12) + 1.35)$			
c.	$0.80(3(12) + 1.35)$			
d.	$(4/5)(3(12) + 1.35)$			

8. Leah bought four glue sticks for \$0.50 each and three small cans of paint for \$2.25 each at the craft store. She had a 25% off the entire purchase coupon she applied. Write an expression for how much Leah spent.

	Expression	Evaluate	Does it work?	Why or Why Not?
a.	$4(0.50) + 3(2.25) - 0.25$			
b.	$0.25(4(0.50) + 3(2.25))$			
c.	$0.75(4(0.50) + 3(2.25))$			
d.	$0.75(4(0.50)) + 0.75(3(2.25))$			
e.	$\frac{3}{4}(4(0.50) + 3(2.25))$			

Write an expression of your own for each problem. Then evaluate the expression to solve the problem.

9. Josh made ten 3-pointers and a 2-pointer at his basketball game. How many points did he score?
10. I bought five apples for \$0.30 each and 5 oranges for \$0.30 each. How much money did I spend?
11. I bought two apples for \$0.50 each and four oranges for \$0.25 each. When I got to the cash register, I got a 50% discount. How much money did I spend?
12. At the store I bought three sweaters for \$25 each and two pair of pants for \$30 each. I had a coupon for 25% off the total purchase. How much money did I spend?

Write a context similar to the ones in # 8 – 12 for each numeric expression given.

13. $10 - 3(0.75) - 2(0.50)$

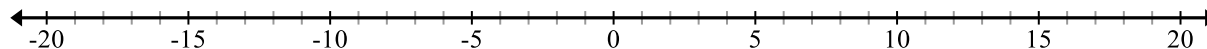
14. $3(2 + 5)$

15. $(\frac{1}{2})(2(60) + 3(10))$

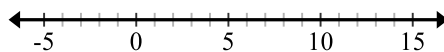
16. $(0.80)(25 + 15)$

Spiral Review

1. Locate each of the following on the line below: **A** = 4 **B** = -4 **C** = -15 **D** = 7 **E** = 18 **F** = -19



2. For the model below, write a numeric expression and then create a context.



3. Simplify: $2\frac{3}{5} - 1\frac{1}{2}$.

4. $18 \div (6 \div 3) =$

5. $3 \div \frac{3}{4} =$

3.1c Class Activity: Algebraic Expressions

Write two *different* numeric expressions for the context below:

Maria bought 5 apples for \$0.35 each.

How would the expression change if she spent \$0.40 on each apple?

What if you didn't know how much each apple cost, how could you write an expression?

For each context, four algebraic expressions are offered. Make a conjecture about the correctness of the expression. Then, evaluate it for the given value, and explain why the expression did or didn't work for the given context.

1. Ryan bought 3 CDs for x dollars each and a DVD for \$15. Write an expression of how much money Ryan spent.

	Expression	Correct expression?	Evaluate $x = 7$	Did it work?	Why or why not?
a.	$3 + x + 15$				
b.	$15x + 3$				
c.	$15 + x + x + x$				
d.	$3x + 15$				

2. I started with 12 jellybeans. Sam ate 3 jellybeans and then Cyle ate y jellybeans. Write an expression for how many jellybeans were left.

	Expression	Correct expression?	Evaluate $y = 6$	Did it work?	Why or why not?
a.	$12 - 3 - y$				
b.	$12 - (3 - y)$				
c.	$12 - (3 + y)$				
d.	$9 - y$				

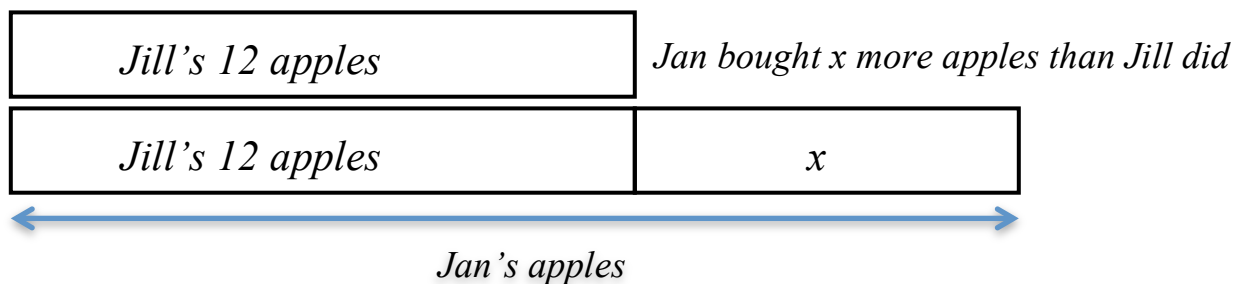
3. Kim bought a binder for \$5 and 4 notebooks for n dollars each. She received a 30% discount on the items. Write an expression for how much she spent.

	Expression	Do you think it will work?	Evaluate (use $n = 3$)	Did it work?	Why or why not?
a.	$0.70(5 + 4 + n)$				
b.	$0.70(4n + 5)$				
c.	$0.70(5n + 4n)$				
d.	$0.30(4n + 5)$				

For each context below, draw a model for the situation, label all parts, and then write an *expression* that answers the question. The first exercise is done for you.

Example: Jill bought 12 apples. Jan bought x more apples than Jill. Write an expression to show how many apples Jan bought.

Jan bought $12 + x$ apples.



4. Josh won 12 tickets. Evan won p tickets fewer than Josh. Write an expression to represent the number of tickets Evan won.

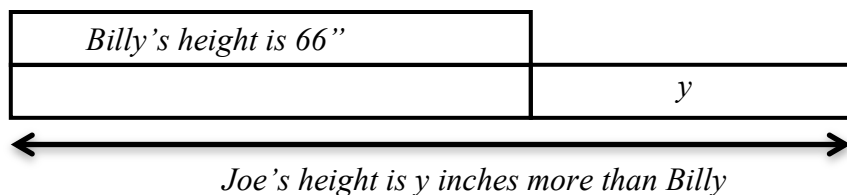
5. Tim is 3 years younger than his brother. If his brother is y years old, write an expression to represent Tim's age.

6. Carol washed 8 windows. Mila washed w windows fewer than Carol. If both Carol and Mila earn \$2 for each window they wash, write an expression to represent how much money Mila earned for washing windows.

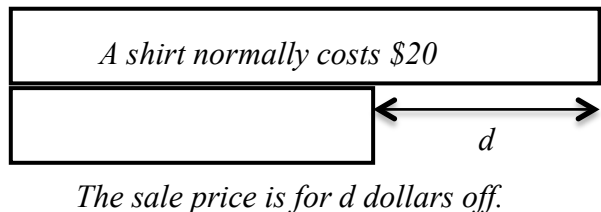
7. Jan bought a more apples than Jill. Jill bought 4 apples. Each apple costs \$0.10. Write an expression to show how much money Jan spent on apples.

For the following models, write a context (with question) and algebraic expression that fits the given model.

8.



9.



Create a context and draw a model for the following algebraic expressions.

10. $r + 15$

11. $1.75(x - 10)$

For each context involving a percent increase or decrease, write an algebraic expression for the questions. Use a model if desired:

12. Tanya had x number of marbles in her bag. She lost 25% of them when they spilled out by accident. Write an expression for the number of marbles she now has.

13. Bruno has an m inch vertical jump. He wants to increase it by 30%. Write an expression for how high he will be able to jump if he's able to increase his vertical jump by 30%.

14. It costs Guillermo d dollars to produce smartphone covers to sell. He wants to sell them for 45% more than it costs him to make them. Write an expression for how much he should sell the smartphone covers.

15. Juliana is training for a race. If she was able to reduce her time t by 17%, write an expression for how much time it will take to her run the race now.

3.1c Homework: Algebraic Expressions

Read each context. Make a conjecture about which expressions will work for the context given. Evaluate the expression for a given value. Explain why the expression did or didn't work for the given context.

1. Bob bought 5 books for x dollars each and a DVD for \$12. Write an expression for how much money Bob spent.

	Expression	Do you think it will work?	Evaluate (use $x = 5$)	Did it work?	Why or why not?
a.	$5 + x + 12$				
b.	$5(x)12$				
c.	$x + x + x + x + x + 12$				
d.	$5x + 12$				

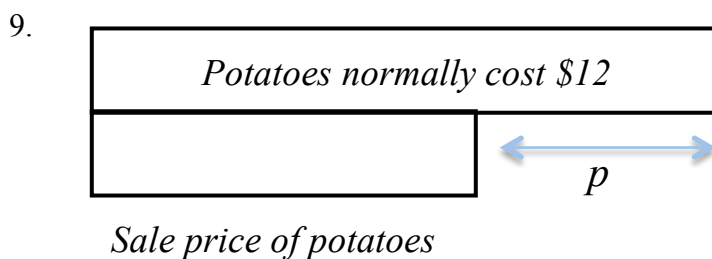
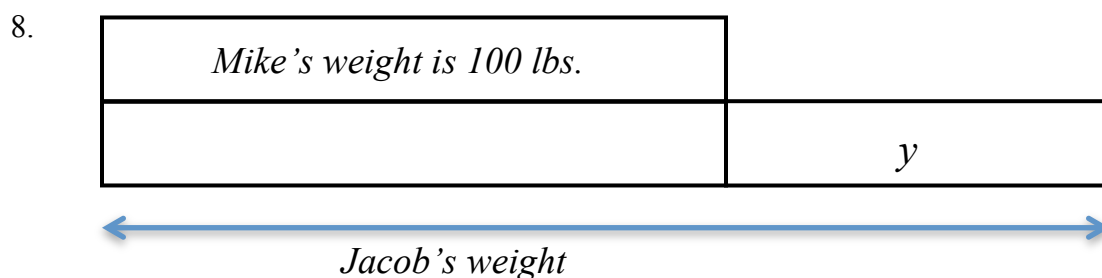
2. Jim won 30 tickets. Evan won y tickets fewer than Jim did. Write an expression for the number of tickets Evan won.

	Expression	Do you think it will work?	Evaluate (use $y = 6$)	Did it work?	Why or why not?
a.	$30 - y$				
b.	$y - 30$				
c.	$y + 30$				
d.	$30 \div y$				

Draw a model and write an expression for each context.

3. I did 4 more problems than Manuel. If I did p problems, write an expression for how many did Manuel did.
4. I bought x pair of shoes for \$25 each and 2 pairs of socks for \$3 each. Write an expression for how much money I spent.
5. I bought m gallons of milk for \$3.59 each and a carton of eggs for \$2.24. Write an expression for how much money I spent.
6. Paul bought s sodas for \$1.25 each and chips for \$1.75. Write an expression for how much money Paul spent.
7. Roberto and Francisco went to the basketball game. Each bought a drink for d dollars and nachos for n dollars. Write an expression for the amount of money they spent all together.

For the following models, come up with a context and an expression that represents the model.



Create a model and write a context for the following expressions.

10. $2(q + 4)$

11. $5 - k$

For each context involving a percent increase or decrease, write an algebraic expression for the questions. Use a model if desired:

12. Marcia ran m miles on Monday. On Tuesday, she ran 18% farther than she ran on Monday. Write an expression for how far Marcia ran on Tuesday.
13. Jorge needs to reduce his expenses by 35%. If he currently spends q dollars a month, write an expression for how much he will be spending once he reduces his expenses.
14. Ruth normally orders p pounds of flour for her bakery every week. She's decided she needs to increase her order by 22%. Write an expression for how many pounds of flours she will now be ordering.

Spiral Review

1. Which number is larger? Justify your answer. -4.03 or -4.3
2. Use long division to show how you can convert this fraction to a decimal and then a percent $\frac{2}{9}$
3. Which number is greater: -5.101 or -5.11 ? Justify.

3.1c Additional Practice

For each problem, draw a model, write out what the unknown stands for, and then write an expression modeling the situation.

1. Marina has \$12 more than Brandon. Represent how much money Marina has.
2. Conner is three times as old as Jackson. Represent Conner's age.
3. Diane earned \$23 less than Chris. Represent how much Diane earned.
4. Juan worked 8 hours for a certain amount of money per hour. Represent how much Juan earned.
5. Martin spent $\frac{2}{3}$ of the money in his savings account on a new car. Represent the amount of money Martin spent on a new car.
6. Brianne had \$47 dollars. She spent \$15 on a new necklace and some money on a bracelet. Represent the amount of money Brianne has now.
7. For 5 days Lydia studied math for a certain amount of time and read for 15 minutes each day. Represent the total amount of time Lydia studied and read over the 5-day period.

8. Carlos spent \$8 on lunch, some money on a drink and \$4 on ice cream. Represent how much money Carlos spent.
9. Nalini has \$26 dollars less than Hugo. Represent the amount of money Nalini has.
10. Bruno ran four times as far as Milo. Represent the distance Bruno ran.
11. Christina earned \$420. She spent some of her earnings on her phone bill and spent \$100 on new clothes. Represent the amount of money Christina now has.
12. Camille has 4 bags of candy. Each bag has 3 snicker bars and some hard candy. Represent the amount of candy Camille has.
13. Heather spent $\frac{1}{4}$ of the money in her savings account on a new cell phone. Represent the amount of money Heather spent on the new cell phone.
14. Miguel is 8 years older than Cristo. Represent Miguel's age.

3.1d Class Activity: Simplifying Algebraic Expressions with Models

In using Algebra tiles, every variable is represented by a rectangle, positive or negative and every integer is represented by a square, positive or negative.

Key for Tiles:

$$\boxed{1} = 1$$

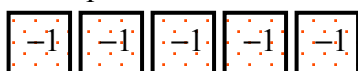
$$\boxed{x} = x$$

$$\boxed{-1} = -1$$

$$\boxed{-x} = -x$$

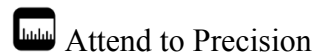
Write an expression for what you see and then write the expression in simplest form.

Example:

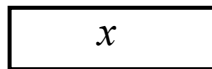
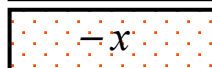
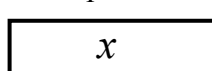


$$-5 + 4$$

$$-1$$



Example

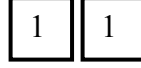
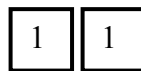


$$2x - x + 4$$

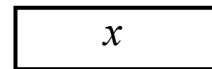
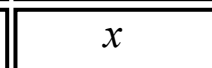
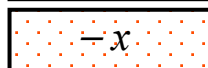
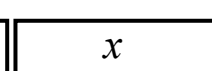
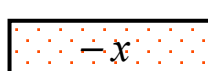
$$2x + -x + 4$$

$$x + -x + x + 4$$

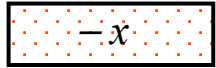
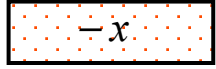
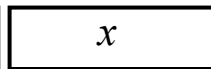
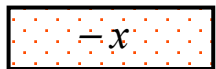
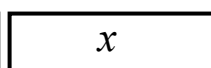
$$x + 4$$



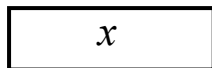
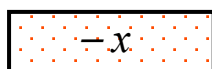
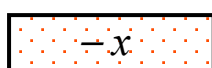
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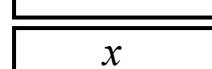
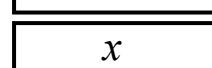
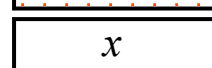
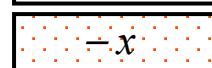
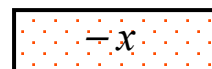
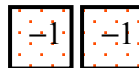
2.



3.



4.



Using algebra tiles, draw two different models or write two different expressions that can be simplified to the following expressions.

5. $x + 6$	6. $-3x + -2$	7. $4x - 1$
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How many ways are there to model each of the expressions above? Justify.


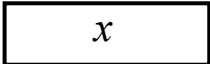

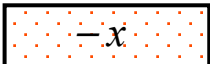
Simplify each expression.

<p>8. $2x + 1 + x$</p> <div style="border: 1px solid black; padding: 2px; margin-bottom: 2px; width: fit-content;">x</div> <div style="border: 1px solid black; padding: 2px; margin-bottom: 2px; width: fit-content;">x</div> <div style="border: 1px solid black; padding: 2px; margin-bottom: 2px; width: fit-content;">1</div> <div style="border: 1px solid black; padding: 2px; margin-bottom: 2px; width: fit-content;">x</div>	<p>9. $3x + 4 + (-2)$</p>	<p>10. $2x + x - 2 + 3$</p>
<p>11. $-3x + 1 + -x$</p>	<p>12. $2x + -3 + -2x$</p>	<p>13. $-x + 3 + 4x$</p>

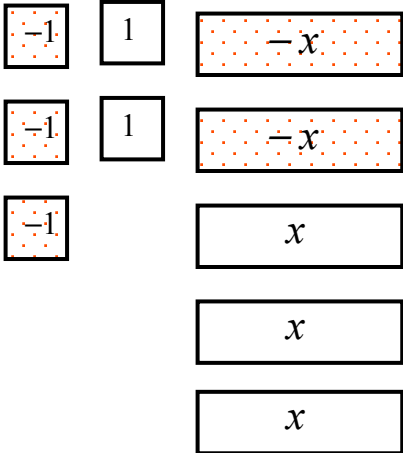
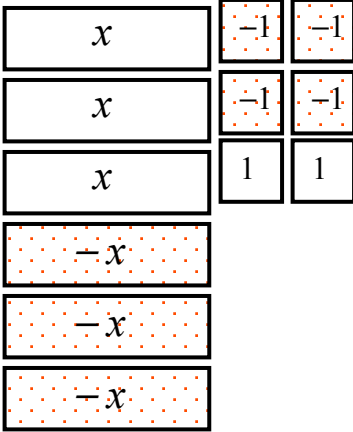
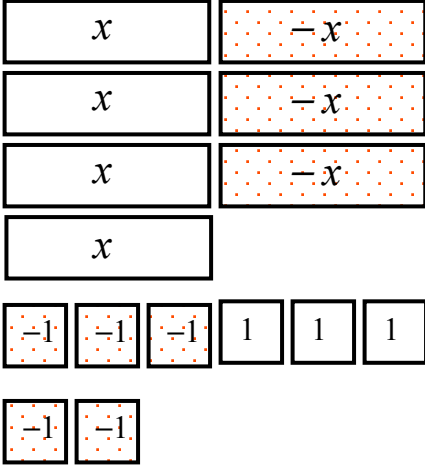
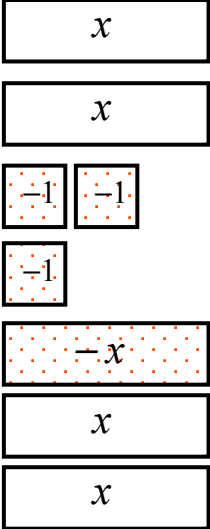
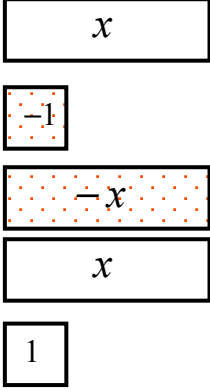
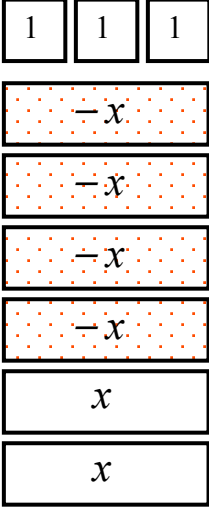
14. $-2x + 4 + x - 7$	15. $4x - 3 + 2 - 2x$	16. $-4x - 1 + 3x + 2 - x$
17. $x + x + x + x$	18. $x - x - x - x$	19. $-x - x - x - x$
20. $3 - 2x + x - 5$	21. $4 - 2 - 4x - 2$	22. $2x - x - 3 + 5$

3.1d Homework: Simplifying Algebraic Expressions with Models

Use the key below to interpret or draw the algebraic expressions in your homework.

<i>Key for Tiles:</i>	
 = 1	 = x
 = -1	 = $-x$

Write an expression for what you see, and then simplify if possible. The first one is done for you.

<p>1.</p>  <p>$-3 + 2 + -2x + 3x$, simplifies to $-1 + x$ or $x + -1$ or $x - 1$</p>	<p>2.</p> 	<p>3.</p> 
<p>4.</p> 	<p>5.</p> 	<p>6.</p> 

Simplify the following expressions. If needed, sketch a model in the space provided.

7. $2x + 4 - x$	8. $x - 5 + 3x$	9. $2x - 3 + 5$
10. $3x + 2 - 2x$	11. $2x + 1 + 3 - 5$	12. $-2x + 4 - 3$
13. $-2x + 3 + 5x - 2$	14. $5x - 3 - 4 + x$	15. $-3x + 1 + 2x - 3$
16. $x + 4 + -3x - 7$	17. $-x - 3 + 2x - 2$	18. $4x - 3 - 7x + 4$

Spiral Review

1. Mark has \$16 more than Becca. Represent how much money Mark has.

2. What property is shown?

$$8 + 7 + 2 \text{ and } 8 + 2 + 7 \underline{\hspace{2cm}}$$

$$18 + 0 \text{ and } 0 + 18 \underline{\hspace{2cm}}$$

3. Find: $\frac{3}{5} + \frac{2}{3} =$

4. Find: $-4 + -7 =$

5. Find: $-3 - 8 =$

3.1e Class Activity: More Simplifying Algebraic Expressions

Activity 1:

Miguel saw the following two expressions:

$$17 + 4 + 3 + 16$$

$$43 - 8 - 3 + 28$$

He immediately knew the sum of the first group is 40 and the sum of the second set is 60. How do you think he quickly simplified the expressions in his head?

Activity 2:

Abby has 3 turtles, 4 fish, a dog and 2 cats. Nia has 2 turtles, 2 fish, and 3 dogs. Mr. Garcia asks them, “how many pets do you two have all together?” How might Abby and Nia respond?

Simplify the following expression with the Key provided below.

$$x + y + 3x - 4y + 2$$

Justify your work:

Key for Tiles:

$$\boxed{1} = 1$$

$$\boxed{x} = x$$

$$\boxed{y} = y$$

$$\boxed{-1} = -1$$

$$\boxed{-x} = -x$$

$$\boxed{-y} = -y$$

Explain how you would simplify:

$$5x - 4 - 3x + 4y - z + 3z$$

For #1-16, model each expression (draw Algebra Tiles representations of the variables and numbers). Then simplify each expression.

1. $8x + 2x + 2y + 7y$

2. $3z + 2x + x$

3. $5x - 9x + 2w - w$

4. $-6 + 4x + 9 - 2x + 6t - 2t$

5. $-3 + 3x + 11 - 5x - 2k - k$

6. $9x - 12 + 12 - 9x$

7. $1 + x + 5x - 2 + 4k - 7k$

8. $-4x - 5x + 12m - 2m + 6q - 10q$

Your friend is struggling to understand what it means when the directions say “simplify the expression.” What can you tell your friend to help him?

Your friend is also having trouble with expressions like problems #5 and #8 above. He’s unsure what to do about the “-.” What might you say to help him?

Simplify each.

9. $\frac{1}{2}x + \frac{1}{2}x + \frac{1}{3} + \frac{1}{2}$

10. $\frac{2}{3}x + x + \frac{3}{4} - \frac{1}{4}$

11. $\frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z$

12. $\frac{5}{3}x + \frac{4}{5} - \frac{2}{3}x - \frac{3}{5}$

13. $\frac{1}{3}y - \frac{5}{3}y + \frac{1}{3}x + \frac{5}{3}x$

14. $3m + \frac{1}{4}m + 2y + \frac{3}{4}y$

15. $5x - 2y + 4 - 3x + y + x - 2$

16. $-5 + \frac{1}{2}x - \frac{1}{5}y - \frac{2}{10}y + \frac{3}{4}x - 7$

3.1e Homework: More Simplifying Algebraic Expressions

For #1-16 model each expression, using Algebra Tiles if necessary. Then simplify each expression, combining like terms.

1. $5x + 3x + 5y$

2. $1 + 3x + x$

3. $3x - 5x + 4y + 3y$

4. $7m - 2x - 9m + 4x$

5. $-4x - 5 + 2x$

6. $5y - 4x + 5x + 10y$

7. $-4x - 5m + x + 7x - 12m$

8. $x - 6x + 14y + 8y - 2x$

9. $2x + x + 2y + x + 1$

10. $x + 3y + x + 2 + y + 1$

11. $\frac{2}{3}t + \frac{2}{3}t + \frac{1}{6} + \frac{2}{3}$

12. $\frac{5}{6}x + \frac{1}{6}x - \frac{2}{3}x + \frac{1}{2}y + \frac{1}{3}y$

13. $-2x + y - 3y + y + x$

14. $\frac{3}{2}y + \frac{5}{2}y - \frac{1}{2}x + \frac{1}{2}$


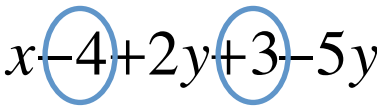

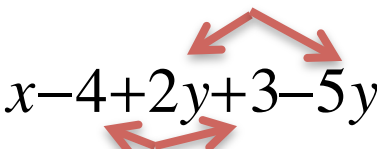
15. $5x - y + 2 - 4x + 2y + x + 2$

16. $\frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}y + \frac{1}{6}y + \frac{1}{5}z - \frac{2}{5}z$

3.1f Class Activity: Vocabulary for Simplifying Expressions

In groups of 2 or 3 students, consider the following expressions: a) $2x + 5 + 3y$, b) $2x + 5 + 3x$, and c) $2x + 5x + 3x$. How are these expressions similar? How are they different?

Parts of an Algebraic Expression: Use the diagrams to create definitions for the following vocabulary words. Be prepared to discuss your definitions with the class.

Terms	Constants
 $x - 4 + 2y + 3 - 5y$	 $x - 4 + 2y + 3 - 5y$
<p>There are five terms in this expression. The terms are x, -4, $2y$, 3, and $-5y$.</p>	<p>The constants are -4 and 3. (Recall that subtracting is like adding a negative number.)</p>
Coefficients	Like Terms
 $x - 4 + 2y + 3 - 5y$	 $x - 4 + 2y + 3 - 5y$
<p>The coefficients are 1, 2, and -5.</p>	<p>$2y$ and $-5y$ are like terms. -4 and 3 are also like terms.</p>

Terms:

Constant:

Coefficient:

Like Terms:

2. Identify the terms, constants, and like terms in each algebraic expression.

Expression	Terms	Constants	Coefficients	Like Terms
$4x - x + 2y - 3$	$4x, -x, 2y, -3$	-3	$4, -1, 2$	$4x, -x$
$3z + 2z + 4z - 1$				
$2 + 3b - 5a - b$				
$a + b - c + d$				

3. Identify the terms, constants, coefficients, and like terms in each algebraic expression.

Expression	Terms	Constants	Coefficients	Like Terms
a. $2y + 5y - 6x + 2$				
b. $-3x + 2x - (2/3)$				
c. $\frac{4}{5}p + \frac{1}{5} - 3h + j$				
d. $0.3x - 1.7 + 1.2 + 4.4y + 3.6y$				

Simplify the above expressions. Remember to show your work.

a.

b.

c.

d.

3.1f Homework: Solidifying Expressions

Matching: Write the letter of the equivalent expression on the line.

- | | |
|--------------------------------------|--------------|
| 1. ____ $3x - 5x$ | a) $-8a$ |
| 2. ____ $4a - 12a$ | b) $2x - 6$ |
| 3. ____ $3x + 5x$ | c) $-x + y$ |
| 4. ____ $16a + a$ | d) $9x - 4y$ |
| 5. ____ $2x - 2y + y$ | e) $-2x$ |
| 6. ____ $2x - 2 - 4$ | f) $2x - y$ |
| 7. ____ $x - y + 2x$ | g) $x + y$ |
| 8. ____ $-y + 2y - x$ | h) $3x - y$ |
| 9. ____ $5x + 4y - 3x - x + 3y - 6y$ | i) $17a$ |
| 10. ____ $4x + 3y + 5x - 7y$ | j) $8x$ |

Simplify each expression. For exercises #11 – 16, **also** list all the coefficients.

- | | |
|---|-------------------------------------|
| 11. $13b - 9b$ | 20. $14w + 9 + 15 - 16w$ |
| 12. $22x + 19x$ | 21. $4m + 6 + 2m - 5 - 19$ |
| 13. $44y - 12y$ | 22. $6z - 16z - 9z + 18z + 2$ |
| 14. $6a + 4a - 2b$ | 23. $16y - 12y - 20$ |
| 15. $16b - 4b + 2$ | 24. $2x - 2 + 3 + 8x$ |
| 16. $\frac{2}{3}a - \frac{3}{4}a$ | 25. $1\frac{2}{3}x + 3\frac{3}{4}x$ |
| 17. $3.4 - 21.4x - 3.4y + 2.2$ | 26. $11 - u - 14u + 13u$ |
| 18. $\frac{12}{20}b - \frac{1}{4}b + \frac{3}{4}$ | 27. $88.5z - 22.4y + 4.04z + 26.3$ |
| 19. $2x + 0.5x - 3y + 4.75y + 9.8$ | 28. $4b - 3 + 2$ |

3.1f Additional Practice

Simplify each expression.

1. $2x - y + 5x$

2. $5y + x + y$

3. $2y + 8x + 5y - 1$

4. $10b + 2 - 2b$

5. $8y + x - 5x - y$

6. $9x + 2 - 2$

7. $-2x - 6 + 3y + 2x - 3y$

8. $6m + 2n + 10m$

9. $7b - 5 + 2b - 3$

10. $2a - 3 + 5a + 2$

11. $8x + 5 - 7y + 2x$

12. $4y + 3 - 5y - 7$

13. $6x + 4 - 7x$

14. $2x + 3y - 3x - 9y + 2$

15. $-2b + a + 3b$

16. $m - 5 + 2 - 3n$

17. $4 + 2r + q$

18. $5h - 3 + 2k - h + k$

19. $5t - 3 - t + 2$

20. $c + 2d - 10c + 4$

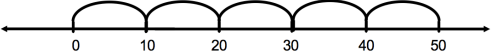
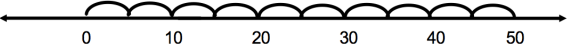
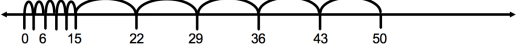
3.1g Class Activity: Multiplying Groups/Distributive Property

Review:

Show as many different ways to write 50 as you can.

Examine the different ways to represent a product of 50 below. Explain how each is an accurate representation of 50.

Review:

NUMBER LINE MODELS for MULTIPLICATION (1 dimensional representation of multiplication)	
 <p>5×10</p>	 <p>10×5</p>
 <p>$5(3 + 7)$</p>	<p>Notice that in the number line to the left, first there are five groups of 3 then there are five groups of 7. This can also be represented as five groups of a length of $(3 + 7)$ or 10.</p>

Activity: On the number line below, show a length of 1 unit. Then make the length of 3 units.



On the number line below, show a length of x units. Then make the length of $3x$ units.



AREA MODELS for MULTIPLICATION (2 dimensional representation of multiplication)

	25
2	50

2×25

<table><tr><td></td><td>10</td></tr><tr><td>5</td><td>50</td></tr></table> <p>5×10</p>		10	5	50	<table><tr><td></td><td>3</td><td>7</td></tr><tr><td>5</td><td>15</td><td>35</td></tr></table> <p>$5(3 + 7)$</p>		3	7	5	15	35
	10										
5	50										
	3	7									
5	15	35									
<table><tr><td></td><td>5</td><td>5</td></tr><tr><td>5</td><td>25</td><td>25</td></tr></table> <p>$5(5 + 5)$ or $5^2 + 5^2$</p>		5	5	5	25	25	Create your own representation of 50:				
	5	5									
5	25	25									

Can you draw an area model representation of $3x$?

1. Model $5(x + 1)$ and then simplify. Justify your steps.

x	1
x	1
x	1
x	1
x	1



2. Model $4(x - 2)$ and simplify.

Write $4(x - 2)$ in three different ways.

What does the number in front of the parentheses tell you about the grouping?

3. a. Explain what $3(2x - 5)$ means, then simplify.

b. Write $3(2x - 5)$ in three different ways.

How can you use what we've learned about integers and what we know about writing expressions with parentheses to rewrite expressions that have " $-$ " in the groupings?

Write the expression below in an equivalent form. If necessary, draw a model.

4. $3(x + 2)$	5. $2(3x + 5)$	6. $3(x + 1)$
7. $4(3x - 1)$	8. $2(3 + x)$	9. $3(3x - 2)$
10. $\frac{1}{2}(4x + 6)$	11. $\frac{3}{4}(2x - 5)$	12. $0.2(7x - 9)$

In sixth grade, you talked about order of operations, what is the order of operations and how is it related to what you've been doing above?

3.1g Homework: Multiplying Groups/Distributive Property

Simplify each of the following. Draw a model or explain the meaning of the expression to justify your answer.

1. $5(x + 1)$	2. $2(3x + 2)$	3. $4(x + 3)$
4. $2(3x - 1)$	5. $3(2x - 3)$	6. $5(x - 1)$
7. $\frac{1}{3}(3x + 12)$	8. $\frac{3}{4}(5x - 8)$	9. $-\frac{2}{5}(4x - 7)$
10. $0.25(4x - 8)$	11. $0.6(-2x + 5)$	12. $-2.4(1.5x - 0.9)$

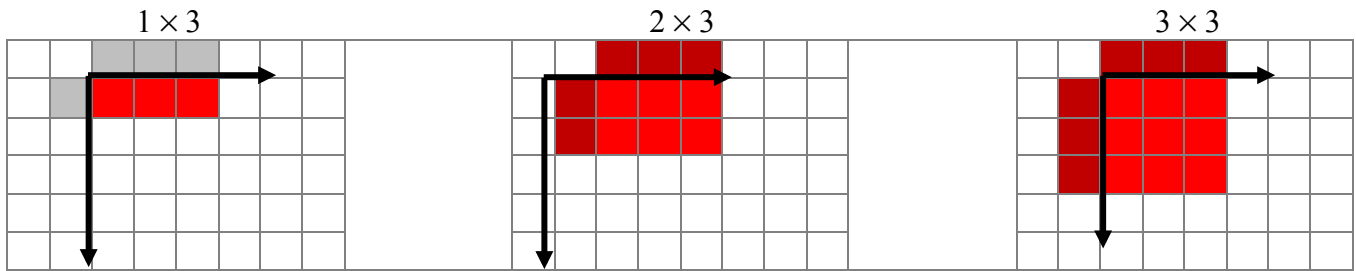
The expressions $2(5x - 3)$ can be written and $10x - 6$ OR $10x + (-6)$.
Write the following expressions in two different ways as the example shows:

15. $4(3x - 5)$

16. $2(7x - 3)$

3.1h Classwork: Modeling Backwards Distribution (Factoring)

Review: below is a review of modeling multiplication with an array.



Factors: 1, 3

Factors:

Factors:

There is one group of 3


Product/Area: 3

Product/Area:

Product/Area:

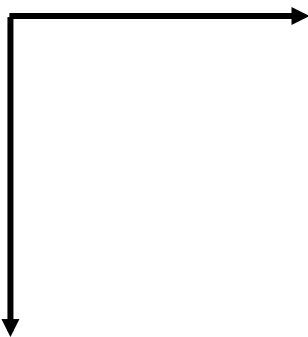
Use the Key below to practice using a multiplication model.

1 

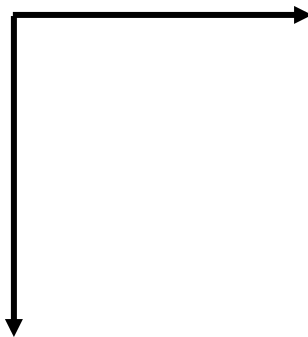
x^2 

x 

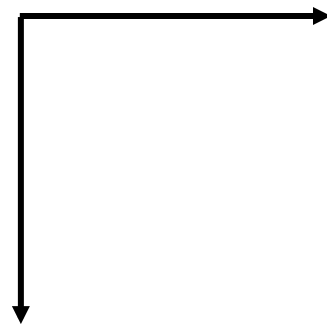
2×2



4×4



$x \cdot x$



Factors:

Factors:

Factors:

Product/Area:

Product/Area:

Product/Area:

Look at the three models above. What do you think 2^2 might be called?
What might 4^2 be called?

What do you think x^2 might be called?

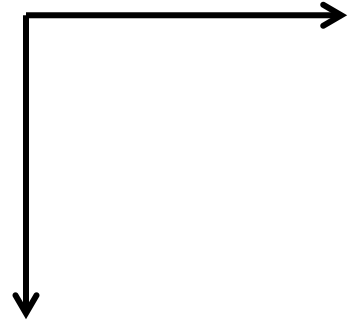
Why?
Why

1. Build the factors for $3(x + 2)$ on your desk. Then build the area model. Draw and label each block below.

What are the factors of the multiplication problem?

What is the area ?

What is the product of the multiplication problem?



2. Build the factors for $3(2x + 1)$ on your desk. Then build the area model. Draw and label each block below.

What are the factors of the multiplication problem?

What is the area?

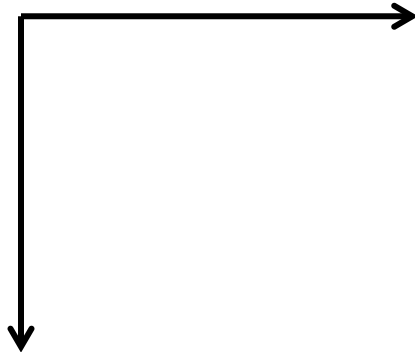
What is the product of the multiplication problem?



3. Build the factors for $2(x + 4)$. Build the area and draw.

What is the area or product?

What are the factors? and



4. Build the factors for $x(x + 3)$. Build the area and draw.

What is the area or product?

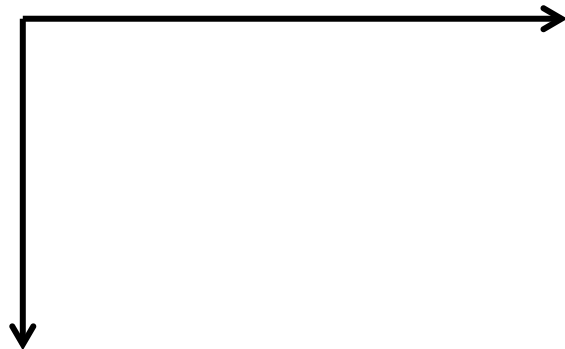
What are the factors? and



5. Build the factors for $x(2x + 5)$. Build the area and draw.

What is the area or product?

What are dimensions or factors? and



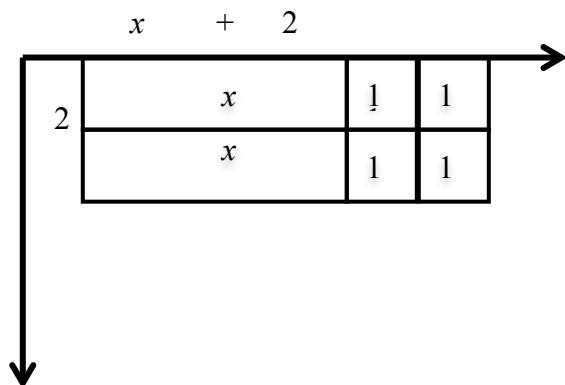
Example: Factor $2x + 4$

We create a rectangle with the terms:
two “ x ” blocks and four “1” blocks.

Notice that the dimensions of the rectangle are
2 by $(x + 2)$

$$2(x + 2) = 2x + 4$$

$2(x + 2)$ is the factored form of $2x + 4$



6. Factor $6x + 3$ by creating a rectangle to see its dimensions.

What are the dimensions (factors)

Width: Height:

What is the area (product) of the rectangle?

Write $6x + 3$ in factored form:

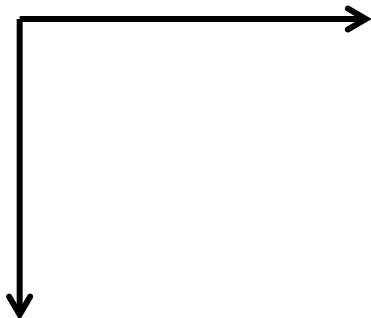


Draw a model to factor each of the following.

7. $10x + 5$

8. $3x + 12$

9. $5x + 10$

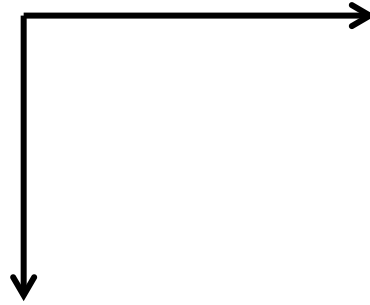


<p>10. $10x + 5$</p> <p>Dimensions of Rectangle:</p> <p>or</p> <p>Greatest Common Factor:</p> <p>$10x + 5$ in factored form:</p>	<p>11. $3x + 12$</p> <p>Dimensions of Rectangle:</p> <p>or</p> <p>Greatest Common Factor:</p> <p>$3x + 12$ in factored form:</p>	<p>12. $5x + 10$</p> <p>Dimensions of Rectangle:</p> <p>or</p> <p>Greatest Common Factor:</p> <p>$5x + 10$ in factored form:</p>
--	--	--

13. $6x + 2$

14. $x + 4$

15. $x^2 + 3x$



<p>$6x + 2$</p> <p>Dimensions of Rectangle:</p> <p>or</p> <p>Greatest Common Factor:</p> <p>$6x + 2$ in factored form:</p>	<p>$x + 4$</p> <p>Dimensions of Rectangle:</p> <p>or</p> <p>Greatest Common Factor:</p> <p>$x + 4$ in factored form:</p>	<p>$x^2 + 3x$</p> <p>Dimensions of Rectangle:</p> <p>or</p> <p>Greatest Common Factor:</p> <p>$x^2 + 3x$ in factored form:</p>
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Practice: Write each in factored form. Be ready to justify your answer.

16. $30x + 6$	17. $4b + 28$	18. $3m - 15$
19. $4n - 2$	20. $25b - 5$	21. $4x - 8$

Look at problems #16, 17, and 21. How else might these expressions be factored?

Write the following in factored form. The first one is done for you.

$\frac{2}{3}x + \frac{5}{3}$ Notice $\frac{1}{3}$ is a factor of both the coefficient and the constant. $\frac{2}{3} = \frac{1}{3} \cdot 2$ and $\frac{5}{3} = \frac{1}{3} \cdot 5$ so $\frac{2}{3}x + \frac{5}{3}$ can be written as $\frac{1}{3} \cdot 2x + \frac{1}{3} \cdot 5$ and factored as $\frac{1}{3}(2x + 5)$	22. $\frac{3}{2}x + \frac{7}{2}$	23. $\frac{20}{3}x - \frac{4}{3} = \frac{20}{3}x + \left(-\frac{4}{3}\right)$
24. $\frac{3}{5}x - 6$	25. $2.3x + 6.9$	26. $1.6x - 2.4$

3.1h Homework: Modeling Backwards Distribution

Write each in factored form. Use a model to justify your answer on problems #1 - 6.

1. $2x + 4$	2. $3x + 12$	3. $2x + 10$
4. $3x + 18$	5. $x^2 + 2x$	6. $x^2 + 5x$
7. $\frac{4}{7}x + \frac{6}{7}$	8. $\frac{2}{3}x - 4$	9. $0.6x + 1.5$

Simplify each expression. Justify your answer with words or a model.

10. $2x + 3x$	11. $(2x)(3x)$
12. $5x - 2x$	13. $(5x)(-2x)$

3.1i Class Activity: More Simplifying

Review:

What is the *opposite* of “forward 3 steps”?

What is the *opposite* of “turn right”?

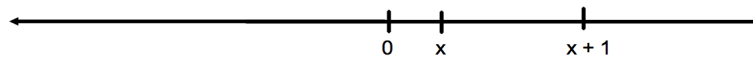
What is the *opposite* of “forward three steps then turn right”?

In math, what symbol do we use that means *opposite*?

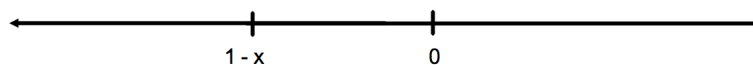
Using the logic above, what do you think each of the following means?

$-x$	$-(x)$
$-(x + 1)$	$-(1 - x)$

- a. Consider the number line below. The location of x and $(x + 1)$ are identified on the line. Locate and label $-x$ and $-(x + 1)$.



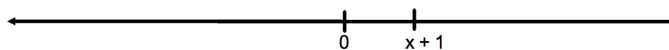
- b. Consider the number line below. The location of $(1 - x)$ is identified on the line. Locate and label $-(1 - x)$.



What does “ $-$ ” in front of a set of parentheses tell us?

On the line below the location of the quantity $x + 1$ is indicated.

- b. Locate $3(x + 1)$
- c. Locate $-3(x + 1)$



Activity:

Suppose Juanita has \$1200 in her savings account and once a week for 5 weeks she withdraws x dollars for bills and \$10 for entertainment. Write an expression for the amount of money she now has in her savings account.

In groups of 2 or 3 students, simplify each of the following. Be ready to justify your answer.

1. $4(x + 1)$	2. $-(x + 2)$	3. $-(3x + 2)$
4. $-2(x + 3)$	5. $-3(x - 2)$	6. $2(3 - x)$
7. $-2(5 - 3x)$	8. $-(4 - 3x)$	9. $-4(2x + 3)$

In groups of 2 or 3 students, simplify the following exercises. Justify your answer.

10. $3x + 5 - x + 3(x + 2)$	11. $3x + 5 - x - 3(x + 2)$
-----------------------------	-----------------------------

12. $2(x - 1) + 4x - 6 + 2x$	13. $-2(x - 1) + 4x - 6 + 2x$
14. $5 + 2(x - 3)$	15. $7x - 2(3x + 1)$
16. $6x - 3 + 2x - 2(3x + 5)$	17. $-9x + 3(2x - 5) + 10$
18. $(5 - 3x) - 7x + 4$	19. $-(4x - 3) - 5x + 2$
20. $9 - 8x - (x + 2)$	21. $15 - 2x - (7 - x)$

3.1i Homework: More Simplifying

Simplify the following expressions:

1. $3(2x + 1)$	2. $-3(2x + 1)$	3. $-3(2x - 1)$
4. $-(x + 4)$	5. $-(x - 4)$	6. $-(4 - x)$
7. $-2(4x - 3)$	8. $-5(3x + 2)$	9. $-2.7(2x - 5)$
10. $5x + 2(x + 3)$	11. $5x + 2(x - 3)$	12. $5x - 2(x + 3)$
13. $5x - 2(x - 3)$	14. $3x + 2 - 4x + 2.1(3x + 1)$	15. $-7x + 4 + 2x - \frac{3}{2}(x + 2)$
16. $10x - 4 - \frac{7}{2}x - \frac{3}{4}(2x - 3)$	17. $4.3x - 5(2.1x - 1.5) - 3x + 4$	18. $x - 7.8 - 2(3.4x - 3.7) + 6.1x$

3.1i Additional Practice: Simplifying

Matching: Write the letter of the equivalent expression on the line

- | | |
|-----------------------|-------------|
| 1. _____ $3(x+6)$ | a) $-3x-3$ |
| 2. _____ $3(x-6)$ | b) $3x+18$ |
| 3. _____ $3(x-1)$ | c) $2x-6$ |
| 4. _____ $-3(x+1)$ | d) $-2x-6$ |
| 5. _____ $6(x-3)$ | e) $-6x+18$ |
| 6. _____ $-(6x+18)$ | f) $-x+6$ |
| 7. _____ $-(6x-18)$ | g) $6x-18$ |
| 8. _____ $(x-6)(-1)$ | h) $-6x-18$ |
| 9. _____ $(x-3)2$ | i) $3x-18$ |
| 10. _____ $(x+3)(-2)$ | j) $3x-3$ |

Practice: Simplify each expression.

- | | | |
|-------------------------|--------------------------|------------------------------------|
| 11. $7(x+3)$ | 20. $8(x-8)$ | 29. $-(6x-6)$ |
| 12. $3(y-3)$ | 21. $4(1-6n)$ | 30. $-0.8(0.5-0.7x)$ |
| 13. $3(k+4)$ | 22. $9.8(p+5.2)$ | 31. $-4(6-6p)$ |
| 14. $\frac{2}{3}(3b-6)$ | 23. $-6(1+2n)$ | 32. $0.25(n-7)$ |
| 15. $7(y-z)$ | 24. $-\frac{1}{2}(2t+6)$ | 33. $9(4-7m)$ |
| 16. $r(s+t)$ | 25. $-1(4+k)$ | 34. $-5\left(m+\frac{1}{3}\right)$ |
| 17. $2(3x+2)$ | 26. $-(3x-6)$ | 35. $-4(k-1)$ |
| 18. $1.6(2v-2)$ | 27. $3(5x+2)$ | 36. $-7(-3n+2)$ |
| 19. $(c-3)(-1)$ | 28. $14(j-12)$ | 37. $(6-t)(-4)$ |

3.1j Extra Practice: Simplifying Expressions

Simplify:

1. $-2x + 5y + 2x - 4y - 2z$

2. $7w - 3q - 5 + 8q - 6 - 10w$

3. $3p - 2q + 4p + 4q - 6 - 4$

4. $17.3v - 2.6 + 12.8v - 15.5$

5. $-3c + 6c - 5c + 2d - 4d - 3d$

6. $-\frac{10}{3}y - \frac{10}{4}y - 3$

7. $31y + 5x - 4 + 12 - 13x - 23y$

8. $-3.7y + 2.9x - 5.6y + 5x - 4.8x$

9. $\frac{4}{3}(2x - 5)$

10. $-6.4(5.1 - 2.5y)$

11. $(-8 - 6v)\left(-\frac{2}{5}\right)$

12. $(3 - 5h)(-3)$

13. $\frac{1}{2}(6a - 8)$

14. $-1.8(x - 2y)$

15. $-(3x + 6)$

16. $-(-3k - 5)$

17. $-(7h + 2k)$

18. $\frac{7}{3}(-x + 5q)$

19. $3 + 4(2x - 5)$

20. $12 - 6(4 - 2y)$

21. $5 + (-8 - 6v)$

22. $4 + (3 - 5h)$

23. $2(6h - 8) + 10$

24. $5y - 1(x - 2y) + 6$

25. $5x - (3x + 6) + 6$

26. $5k - (-3k - 5) + 8$

27. $4h - (7h + 2k) - 5$

28. $7x - 7(-x + 5q)$

3.1k Class Activity: Modeling Context with Algebraic Expressions

Look back at the anchor problem at the beginning of the chapter. In particular, look back at your work for step 10 and step x . Your task was to express the number of units at each step of the pattern.

Look at your model and work for step 10. Use it to help you determine how many units the 100th step would contain.

Step 100

Draw a model if necessary for step 100:

How many units would the 100th step contain? Use your methods for the 10th step to help you determine the units in the 100th step.

Method 1:

Method 2:

Method 3:

Method 4:

Step x

Draw a model if necessary for step x :

How could you adapt the methods you used before to find the number of units for *any* pattern.

Method 1:

Method 2:

Method 3:

Method 4:

In the context above you wrote several expressions for each situation; often there is more than one equivalent way to algebraically model a context. Below are contexts; write two equivalent expressions for each situation. It may be helpful to draw a model.

1. Marty and Mac went to the hockey game. Each boy bought a program for 3 dollars and nachos for n dollars. Write two different expressions that could be used to represent how much money the boys spent altogether.
2. Members of the cooking club would like to learn how to make peach ice cream. There are 14 people in the club. Each member will need to buy 3 peaches and 1 pint of cream to make the ice cream. Peaches cost x cents each, and a pint of cream costs 45 cents. Write two different expressions that could be used to represent the total cost of ingredients for all 14 members of the club. Simplify each expression.
3. Martin's grandma gave him a \$100 gift certificate to his favorite restaurant. Everyday for four days Martin went to the restaurant and bought a drink for \$1.19 and fries for f dollars. Write two different expressions for how much money Martin has left on his gift certificate.
4. Juab Junior High School wants to meet its goal of reading 10,000 books in three months. If 7th grade reads x number of books every week, 8th grade reads y number of books every week, and 9th grade reads z number of books every week, write two different expressions for how many books the school will still need to read after 3 weeks?

For #5 – 10 sketch a model to help you answer the questions.

5. Leo and Kyle are training for a marathon. Kyle runs 10 fewer miles per week than Leo. Draw a model to help clarify contexts.
 - a. Write an expression to represent the distance Kyle runs in terms of Leo's miles, L .
 - b. Write an expression to represent the distance Leo runs in terms of Kyle's miles, K .
 - c. Write an expression to represent the distance Kyle runs in 12 weeks in terms of Leo's miles.
 - d. Write an expression to represent the distance Leo runs in 12 weeks in terms of Kyle's miles.

6. Harry is five years younger than Sue. Bridger is half as old as Harry. Write **two** different expressions that could be used to represent Bridger's age in terms of Sue's age. Simplify each expression. (Hint: use a variable to represent Sue's age.)
7. Maria earned some money babysitting. She spent $\frac{1}{2}$ of her babysitting money on a gift for her mother and then put $\frac{2}{3}$ of the rest into her savings account. Write at least one expression for how much money Maria put into her savings account. (A model is provided below.)
8. Inez ran uphill for two-thirds of the marathon. Half of the rest was downhill and the other half was flat. Write at least one expression to represent the portion of the marathon Inez ran downhill.
9. Cleon went to the bookstore where everything was 50% off. The store manager made an announcement that an additional 50% would be taken off the sale price at the cash register. Write an expression for the price of a book with both deductions taken.
10. Rico went to the store to buy a pair of pants because there was a 30% off all merchandise sale. When he got to the store he learned that purchases made before noon would receive an additional discount of 20% off the sale price. If the pants he picked out cost p dollars and he purchased them before noon, write an expression for the final cost of the pants.

3.1k Homework: Modeling Context with Algebraic Expressions

Write an algebraic expression for each context. In some cases you will be asked to write more than one expression. Be prepared to justify your expressions.

1. Marie would like to buy lunch for her three nieces. She would like each lunch to include a sandwich, a piece of fruit, and a cookie. A sandwich costs \$3, a piece of fruit costs \$0.50, and a cookie costs \$1.
 - a. Write two different expressions that could be used to represent the total price of all three lunches. Then simplify each expression that you wrote to find out how much total money Marie spent on lunch.
 - b. Write two different expressions that could be used to represent the total price of all three lunches if a sandwich costs x dollars, a piece of fruit costs y dollars, and a cookie costs z dollars.
2. Boris is setting up an exercise schedule. For five days each week, he would like to play a sport for 30 minutes, stretch for 5 minutes, and lift weights for 10 minutes.
 - a. Write two different expressions that could be used to represent the total number of minutes he will exercise in five days. Then simplify each expression that you wrote.
 - b. Write two different expressions that could be used to represent the total number of minutes he will exercise in five days if he plays a sport for x minutes, stretches for y minutes, and lifts weights for half the time he plays a sport.
3. James had \$550 in his checking count. Once a week, he spends d dollars on music downloads and \$5 on food. Write two expressions for how much money James has in his account after five weeks.
4. Five girls on the tennis team want to wear matching uniforms. They know skirts will cost \$24 but are not sure about the price of the top. Write two different expressions that could be used to represent the total cost of all five skirts and five tops if x represents the price of one top. Simplify each expression.

5. Drake, Mike, and Vinnie are making plans to go to a concert. They have a total of \$200 between the three of them. The tickets cost \$30 each, and each boy plans to buy a t-shirt for t dollars. Write two different expressions to represent the amount of money the three boys will have left over. Simplify each expression.
6. Miguel has run three marathons. The first time he ran a marathon, it took him m minutes. The next time he ran he cut 10% off his time. The third time he ran a marathon, he was able to cut an additional 5% off his fastest time (the time he got on the second marathon). Write an expression for the amount of time it took Miguel to run the marathon the third time.
7. Ana earned x dollars at her summer job. She put 50% of her money into savings and spent 40% of the rest on a new outfit. Write an expression for the amount of money she has left.
8. Marcela earns money working in her dad's office over the summer. If she spent 40% of her money on a new phone and 50% of the rest on new clothes for school, write an expression for how much of her money she spent.

3.11 Self-Assessment: Section 3.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Recognize and explain the meaning of a given expression and its component parts. [1, 2, 3, 4]	I struggle to understand the meaning of parts of a given expression.	I can recognize the different parts of a given expression and match them with their meaning.	I can recognize the different parts of a given expression and can explain their meaning in my own words.	I can explain the meaning of a given expression and its parts in my own words. I can also write a meaningful expression given a context.
2. Recognize that different forms of an expression may reveal different attributes of the context. [5, 6]	I can't tell when two expressions are really different forms of the same expression.	I can recognize that two expressions are different forms of the same expression, but I struggle to see why I would write it in a different form.	I can recognize that two expressions are different forms of the same expression, and I can explain how the different forms reveal different attributes of the context.	I can write two expressions that are different forms of the same expression, and I can explain how the different forms reveal different attributes of the context.
3. Combine like terms with rational coefficients. [7 (rational c & d)]	I struggle to combine like terms.	I can combine like terms with a model.	I can combine like terms with integer coefficients without a model.	I can combine like terms with rational coefficients.
4. Use the Distributive Property to expand and factor linear expressions with rational numbers. [8 c,d]	I know what the Distributive Property is, but I struggle to use it.	I can use the Distributive Property to expand linear expressions with integers such as Problem 8a.	I can use the Distributive Property to expand linear expressions with rational numbers such as Problems 8a and 8b.	I can use the Distributive Property to expand and factor linear expressions.
5. Recognize properties of arithmetic and use them in justifying work when manipulating expressions. [10]	I know there are properties, but I have a hard time recognizing them in work.	I can name properties when I see them, but struggle to use them to justify manipulations.	I can use properties to justify manipulations with expressions.	I can use properties to justify manipulations with expressions and can use them to critique the arguments of others.
6. Write numeric and algebraic expressions to represent contexts.	I struggle to translate contexts to numeric and algebraic expressions.	Most of the time I can translate contexts to numeric and algebraic expressions.	I can translate contexts to numeric and algebraic expressions.	I have no trouble translating contexts to a variety of numeric and algebraic expressions.

Sample Problems for Section 3.1

1. Given the following situation, match the parts of the expression to the parts of the context.

Luis went to a soccer game with some friends. He bought two sodas for \$1.50 each and four giant candy bars for \$2.25 each. How much did he spend?

$$2(1.50) + 4(2.25)$$

2. Explain the different parts of the following expression:

Matthew likes to buy movies from a DVD club. He pays \$5 per month plus \$2 per movie. If he buys six movies in June, how much did he pay for the month of June?

$$6(2) + 5$$

3. Write an expression to match the following situation. Explain the different parts of the expression.

Nastas plays on the school basketball team. He scores five 3-pointers and 3 lay-ups (worth 2 points each) in one game. How many total points did he score in that game?

4. Write an expression to match the following situation. Explain the different parts of the expression.

Millie bought two sweaters for \$25 and three pair of pants for \$30. She had a 25% off coupon for her entire purchase. Write an expression for the amount of money Millie spent.

5. Identify which expressions match the following situation. Explain how each different correct expression reveals different aspects of the situation.

Ooljee goes to the movies with her two friends. They each pay for their own movie ticket at \$8.50 each and each buys a soda for d dollars. How much money do the girls spend at the theater?

$$3(8.50) + 3d \quad 3(8.50) + d \quad 3(8.50 + d) \quad 3(8.50)d \quad 8.50 + d + 8.50 + d + 8.50 + d$$

6. Write at least two different expressions for the following situation. Explain how each different expression reveals different aspects of the situation.

Phoebe babysits every weekend. She puts 30% of each check into savings and spends 15% of the remainder on new books. How much does she have left?

7. Simplify the following expressions. Use a model if needed.

a. $9m - 3 + 4m$

b. $71b - 4a + 4b - 4a$

c. $50q + 0.1t - 0.3t + 14q$

d. $\frac{9}{10}m - \frac{3}{5} + \frac{4}{5}m + \frac{1}{2}m$

8. Simplify the following expressions.

a. $8(-1 - 7w)$ $-8(7w + (-6))$ $(10 + 7w)(-5)$

b. $-6\left(\frac{1}{5}a - \frac{1}{6}\right)$ $\frac{1}{4}(-6a + 8b)$ $\frac{1}{3}\left(-7 - \frac{1}{6}a\right)$

9. Factor the following expressions.

$3x - 6$ $15 - 20y$ $\frac{5}{3}x + \frac{2}{3}$ $4.9t - 2.8$

10. Identify the arithmetic property used in simplifying each expression.

a. $3x + 2 + x + -1$
 $3x + x + 2 + -1$ Property:

b. $4x + (-4x)$
 0 Property:

c. $(3x + 2y) + y$
 $3x + (2y + y)$ Property:

Section 3.2 Solving Algebraic Equations ($ax + b = c$)

Section Overview:

This section begins by reviewing and modeling one- and two-step equations with integers. Students then learn to apply these skills of modeling and solving to equations that involve the distributive property and combining like terms. Students learn to extend these skills to solve equations with rational numbers and look for errors in their own and other's work

Concepts and Skills to be Mastered (from standards)

By the end of this section, students should be able to:

- 1 Solve multi-step equations fluently including ones involving calculations with positive and negative rational numbers in a variety of forms.
- 2 Connect arithmetic solution processes that do not use variables to algebraic solution processes that use equations.
- 3 Use properties of arithmetic to create an argument or critique the reasoning of others in solving algebraic equations.

3.2 Class Discuss: Using Properties to Justify Steps for Simplify Expressions

At the beginning of 3.1, you named properties you've been using since elementary school. Without looking back or ahead, can you list the properties you've named to this point?

There is one last property to add to the list: Distributive Property of Multiplication over Addition.

The distributive property links addition and multiplication. Like the properties discussed at the beginning of 3.1, you've been using the distributive property informally since elementary school.

Examples:

Without a calculator, find the product 23×7 . Explain your procedure.

Without a calculator, find the product: 4×3.1 . Explain your procedure.

Show two ways one might simplify: $2(3 + 4)$

Use the distributive property to rewrite $a(b + c)$

Review: Generalizing Properties

Properties of Mathematics:

Name Property	Algebraic Statement	Meaning	Examples
Identity Property of Addition			
Identity Property of Multiplication			
Multiplicative Property of Zero			
Commutative Property of Addition			
Commutative Property of Multiplication			
Associative Property of Addition			
Associative Property of Multiplication			
Distributive Property of Multiplication over Addition			
Additive Inverse			
Multiplicative Inverse			

Use properties to justify steps:

Example: Jane wants to find the sum: $3 + 12 + 17 + 28$. She uses the following logic, “3 plus 17 is 20, and 12 plus 28 is 40. The sum of 20 and 40 is 60.” Why is this okay? The table below shows how to justify her thinking using properties for each step.

Statement	Step	Justification
$3 + 12 + 17 + 28$	<i>No change, this is where she started.</i>	<i>This expression was given.</i>
$3 + 17 + 12 + 28$	<i>The 17 and the 12 traded places.</i>	<i>Commutative Property of Addition</i>
$(3 + 17) + (12 + 28)$	<i>Jane chose to add the numbers in pairs first, which is like inserting parentheses.</i>	<i>Associative Property of Addition</i>
$20 + 40$	<i>Jane found the sums in the parentheses.</i>	<i>Jane is now following the Order of Operations.</i>
60	Add, so . . . $3 + 12 + 17 + 28 = 60$	

1. The expression $3(x - 4) + 12$ has been written in five different ways. State the property that allows each change.

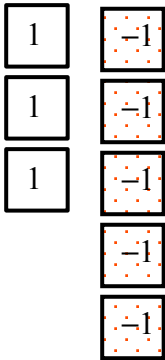
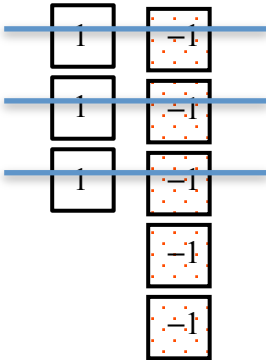
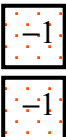
Expression	Step	Justification / Property Used
$3(x + (-4)) + 12$	No change	This expression was given, but rewritten using the idea that $a - b = a + (-b)$
$3x + 3(-4) + 12$		
$3x + (-12) + 12$		
$3x + ((-12) + 12)$		
$3x + 0$		
$3x$		

2. The expression $2(3x + 1) + -6x + -2$ has been written in four different ways. State the property that allows each change.

Expression	Step	Justification
$2(3x + 1) + -6x + -2$	No change	
$6x + 2 + -6x + -2$		
$6x + -6x + 2 + -2$		
$(6x + -6x) + (2 + -2)$		
$0 + 0$		

Review: Look back at Chapter 2 and review addition/subtraction with the chip/tile model.

3. Model $3 + (-5)$ to find the sum

<div><p>$3 + (-5)$</p><p>Step 1</p><p>Step 2</p><p>Step 3</p></div>	<p>Justify each step:</p> <p>Step 1:</p> <p>Step 2:</p> <p>Step 3:</p>
---	--

4. In chapter 2 you learned that a negative times a negative produces a positive product. We used models to discover why this is true. Below is a more formal proof. We start with the the Multiplicative Property of Zero (anything times zero is 0.)

Statement	Step	Justification
$-1(0) = 0$	Given	Multiplicative Property of Zero
$-1(-1 + 1) = 0$	0 was replaced with $(-1 + 1)$	Additive Inverse Property
$-1(-1) + -1(1) = 0$		
$-1(-1) + 1(-1) = 0$		
$-1(-1) + -1 = 0$		
$-1(-1)$ must equal 1 because if we get 0 when we add it to -1 , it must be the additive inverse of -1		

3.2a Classroom Activity: Model and Solve Equations

Use any method you'd like to solve each of the following. Justify your answer with a model or words:

<p>1. $m + 3 = 7$</p>	<p>2. $8 = k - 2$</p>	<p>3. $3n = 18$</p>
<p>4. $17 = 2j + 1$</p>	<p>5. $j/2 = 6$</p>	<p>6. $y + 3 = -5$</p>

It may have been easy to solve some (or all) of the above in your head, that's good; that means you're making sense of the problem. In this section, we are going to focus on the structure of equations and how properties of arithmetic allow us to manipulate equations. Even though the "answer" is important, more important right now is that you understand the underpinnings of algebraic manipulations.

Evaluate the expression $2x + 1$ for each of the given values:

7. Evaluate $2x + 1$ for $x = 3$	8. Evaluate $2x + 1$ for $x = -2$	9. Evaluate $2x + 1$ for $x = -3$
----------------------------------	-----------------------------------	-----------------------------------

Solve each equation in any way you want. Be able to justify your answer with a model or words:

10. $2x + 1 = 5$	11. $2x + 1 = 9$	12. $2x + 1 = -9$
------------------	------------------	-------------------

What do the terms “evaluate” and “solve” mean?

What is the difference between an equation and an expression?

For this activity use the following Key to represent variables and integers. Note: “ x ” or “ $-x$ ” can be any variable.

<div style="border: 1px solid black; padding: 5px; display: inline-block;">x</div>	<div style="border: 1px solid black; padding: 5px; display: inline-block; background-color: #cccccc;">$-x$</div>
<div style="border: 1px solid black; padding: 5px; display: inline-block;">1</div>	<div style="border: 1px solid black; padding: 5px; display: inline-block; background-color: #cccccc;">-1</div>

11. Consider the equation: $x - 1 = 6$. What value of x makes this equation true? Justify:

Draw a model of $x - 1 = 6$ below, then use it to illustrate the algebraic steps need

<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-bottom: 5px;">x</div> <div style="border: 1px solid black; padding: 5px; display: inline-block; background-color: #cccccc; margin-left: 20px;">-1</div>	=	<div style="display: inline-block; vertical-align: middle;"> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-bottom: 5px;">1</div> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-bottom: 5px;">1</div> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-bottom: 5px;">1</div> </div> <div style="display: inline-block; vertical-align: middle;"> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-bottom: 5px;">1</div> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-bottom: 5px;">1</div> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-bottom: 5px;">1</div> </div>	$x - 1 = 6$

12. Consider the equation: $x - 3 = 5$. What value of x makes this equation true? Justify:

Draw a model of $x - 3 = 5$ below, then use it to illustrate the algebraic steps needed to solve.

<div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; padding: 5px; margin-right: 10px;">x</div> <div style="display: flex; flex-direction: column; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px; background-color: #cccccc;">-1</div> <div style="border: 1px solid black; padding: 2px 5px; background-color: #cccccc;">-1</div> <div style="border: 1px solid black; padding: 2px 5px; background-color: #cccccc;">-1</div> </div> <div style="margin: 0 10px;">=</div> <div style="display: flex; flex-direction: column; align-items: center;"> <div style="border: 1px solid black; padding: 2px 5px;">1</div> <div style="border: 1px solid black; padding: 2px 5px;">1</div> <div style="border: 1px solid black; padding: 2px 5px;">1</div> </div> </div>	$x - 3 = 5$

13. Use a model to solve $8 = 7 + m$. Write the algebraic procedure you followed to solve.

	=		$8 = 7 + m$
	=		

14. Consider the equation: $6 = 3x$. What value of x makes this equation true? Justify:

Draw a model of $6 = 3x$ below, then use it to illustrate the algebraic steps need

<div><div>1</div><div>1</div><div>1</div><div>1</div><div>1</div><div>1</div></div>	<div><div></div><div></div><div></div></div>	<div><div><div><div>x</div></div><div><div>x</div></div><div><div>x</div></div></div></div>	<div>$6 = 3x$</div>
	<div><div></div><div></div><div></div></div>		

15. $8 = -2m$. Write the algebraic procedure you followed to solve.

	$=$		$8 = -2m$
	$=$		

16. Consider the equation $-5 + 3n = 7$. What value of n makes the equation true? Justify:

Draw a model of the equation and use it to solve the equation. Write the algebraic procedure you followed to solve.

	$=$		$-5 + 3n = 7$
	$=$		
	$=$		

17. Consider the equation $-4 = -3m + 8$. What value of m makes the equation true? Justify.

Draw a model of the equation and use it to solve the equation. Write the algebraic procedure you followed to solve.

	$=$		$-4 = -3m + 8$
	$=$		
	$=$		

Additional Practice

18. $-7 = m - 9$

23. $40 = -5y$

19. $12 = w + (-4)$

24. $5 = -2x + 9$

20. $-4k = -20$

25. $4x + 7 = -5$

21. $-18 = 2n$

26. $-8 = -3m + 10$

22. $3x = -15$

27. $7z + 1 = 15$

3.2a Homework: Model and Solve Equations

Model and solve each equation below. Use the key below to model your equations.

<div><div>x</div></div>	<div><div>-x</div></div>
<div><div>1</div></div>	<div><div>-1</div></div>

1. $x - 6 = -9$

<div><div>x</div><div><div>-1</div><div>-1</div><div>-1</div><div>-1</div><div>-1</div></div></div>	<div><div>=</div></div>	<div><div><div>-1</div><div>-1</div><div>-1</div><div>-1</div><div>-1</div><div>-1</div><div>-1</div><div>-1</div></div></div>	<div><div>$x - 6 = -9$</div><div>$x + -6 = -9$</div><div>$+ 6 \quad +6$</div><div>Additive</div><div>Inverse</div></div>
<div><div>x</div></div>	<div><div>=</div></div>	<div><div><div>-1</div><div>-1</div><div>-1</div></div></div>	<div><div>$x = -3$</div></div>

2. $-15 = x - 14$

	<div><div>=</div></div>		<div><div>$-15 = x - 14$</div></div>
	<div><div>=</div></div>		

3. $m + 2 = -11$

	$=$		$m + 2 = -11$

4. $4n = -12$

	$=$		
	$=$		

5. $-15 = -3m$

	$=$		
	$=$		

6. $3t + 5 = 2$

	$=$		
	$=$		
	$=$		

7. $8 = 2p - 4$

	$=$		
	$=$		
	$=$		

3.2b Class Activity: More Model and Solve One- and Two-Step Equations

Draw a model, justify your steps and then check your answer. The first one is done for you.

Model/Draw the Equation	What are the solving actions? Record the steps using Algebra	Check solution in the equation.
<div>1. $x + 5 = 8$</div> <div><div><div><div><div></div><div></div><div></div></div><div><div></div><div></div></div></div><div><div><div><div>x</div></div></div></div></div><div>=</div><div><div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div></div></div></div> <div>2. $5 = x + 8$</div> <div><div><div><div></div><div></div><div></div></div><div><div></div><div></div></div></div><div>=</div><div><div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div></div></div></div>	<div>Add -5 to both sides. -5 is the additive inverse of 5. This step isolates the x.</div> <div>$\begin{array}{r} x + 5 = 8 \\ -5 \quad -5 \\ \hline x = 3 \end{array}$</div>	<div>$x + 5 = 8$ $(3) + 5 = 8$ True, so the solution is correct.</div>

Look at # 1 and # 2. Why are the answers different?

3. $3x = -6$				
	=			

Explain the logic above in #3.

How might you use related logic to model $x/3 = -6$?

4. $x/3 = -6$				
	=			
5. $(1/2)x = 3$				
	=			

In problems # 3 and # 4, what happened to the terms on both sides of the equation?

<p>6. $-9 = 2x - 5$</p> <table border="1"> <tr> <td></td> <td>=</td> <td></td> </tr> </table>		=			
	=				
<p>7. $7 = 3x - 2$</p> <table border="1"> <tr> <td></td> <td>=</td> <td></td> </tr> </table>		=			
	=				
<p>8. $-5 = -3 + 2x$</p> <table border="1"> <tr> <td></td> <td>=</td> <td></td> </tr> </table>		=			
	=				
<p>9. $7 + x/2 = -3$</p> <table border="1"> <tr> <td></td> <td>=</td> <td></td> </tr> </table>		=			
	=				
<p>10. $-3 = x/2 + 2$</p> <table border="1"> <tr> <td></td> <td>=</td> <td></td> </tr> </table>		=			
	=				

3.2b Homework: More Model and Solve One- and Two-Step Equations

Model/Draw the Equation	What are the solving actions? Record the steps using Algebra	Check solution in the equation.
1. $2 = x + 5$ <div style="border: 2px solid black; padding: 10px; margin: 10px 0;"> <div style="border: 1px solid black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; left: 50%; top: 50%; transform: translate(-50%, -50%);">=</div> </div> </div>		
2. $-12 = 3x$ <div style="border: 2px solid black; padding: 10px; margin: 10px 0;"> <div style="border: 1px solid black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; left: 50%; top: 50%; transform: translate(-50%, -50%);">=</div> </div> </div>		
3. $-(x/4) = -8$ <div style="border: 2px solid black; padding: 10px; margin: 10px 0;"> <div style="border: 1px solid black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; left: 50%; top: 50%; transform: translate(-50%, -50%);">=</div> </div> </div>		
4. $-2 = (1/3)x$ <div style="border: 2px solid black; padding: 10px; margin: 10px 0;"> <div style="border: 1px solid black; width: 100%; height: 100%; position: relative;"> <div style="position: absolute; left: 50%; top: 50%; transform: translate(-50%, -50%);">=</div> </div> </div>		

5. $-9 = x/2 - 5$

	=	
--	---	--

6. $-3x + 2 = -13$

	=	
--	---	--

7. $-11 = -4x - 3$

	=	
--	---	--

8. $-2x - 5 = 3$

	=	
--	---	--

9.

$$x/3 - 5 = -2$$

	=	
--	---	--

10. $2 + 5x = -8$

	=	
--	---	--

11.

$$(1/2)x - 5 = -3$$

	=	
--	---	--

3.2c Class Activity: Model and Solve Equations, Practice and Extend to Distributive Property

Practice: Solve each.

1. $-16 = 6a - 4$

	=	
--	---	--

$a =$ —

2. $7 = 6 - n/7$

	=	
--	---	--

$n =$

3. $-10 = -10 - 3x$

	=	
--	---	--

4. Review: Simplify each expression. Use words or a model to justify your answer.

a. $3(2x + 1)$

b. $-2(3x + 2)$

c. $-4(2x - 3)$

Solve each equation. In the space to the right, write a justification for your algebraic steps.

5. $2(x + 1) = -8$

	=	
--	---	--

6. $6 = -3(x - 4)$

	=	
--	---	--

7. $-12 = -3(5x - 1)$

	=	
--	---	--

8. $-4(3 - 2m) = -12$

=

9. $-(1/2)(4x + 2) = -5$

=

10. $3(2x - 4) + 6 = 12$

=

3.2c Homework: Model and Solve Equations, Practice and Extend to Distributive Property

Solve each equation. Justify your algebraic manipulations on the right.

1. $9 = 15 + 2p$

	=	
--	---	--

2. $-7 = 2h - 3$

	=	
--	---	--

3. $-5x - 12 = 13$

	=	
--	---	--

4. $6 = 1 - 2n + 5$

	=	
--	---	--

5. $8x - 2 - 7x = -9$

	=	
--	---	--

6. $2(n - 5) = -4$

	=	
--	---	--

7. $-3(g - 3) = 6$

=

8. $-12 = 3(4c + 5)$

=

3.2d Class Activity: Error Analysis

Students in Mrs. Jones' class were making frequent errors in solving equations. Help analyze their errors. Examine the problems below. When you find the mistake, circle it, explain the mistake and solve the equation correctly. Be prepared to present your thinking.

Student Work	Explanation of Mistake	Correct Solution Process
1. $-6t = 30$ $\overline{6 \quad 6}$ $t = 5$		
2. $\frac{3}{4}x = 12$ $\frac{3}{4} \cdot \frac{3}{4}x = 12 \cdot \frac{3}{4}$ $x = 9$		
3. $8 - 5c = -37$ $\begin{array}{r} -8 \quad -8 \\ \hline 5c = -45 \\ \frac{5}{5} \quad \frac{5}{5} \\ c = -9 \end{array}$		
4. $\frac{x+1}{3} = 2$ $\frac{x+1-1}{3} = 2-1$ $3 \cdot \frac{x}{3} = 1 \cdot 3$ $x = 3$		

<p>5. $4x - 3 = 17$</p> $ \begin{array}{r} 4x - 3 = 17 \\ +3 \quad +3 \\ \hline 4x = 20 \\ -4 \quad -4 \\ \hline x = 16 \end{array} $		
<p>6. $3(2x - 4) = 8$</p> $ \begin{array}{r} 6x - 4 = 8 \\ +4 \quad +4 \\ \hline 6x = 12 \\ \frac{6}{6} \quad \frac{6}{6} \\ x = 2 \end{array} $		
<p>7. $3x + 2x - 6 = 24$</p> $ \begin{array}{r} 3x + 2x - 6 = 24 \\ -2x \quad -2x \\ \hline x - 6 = 24 \\ +6 \quad +6 \\ \hline x = 30 \end{array} $		
<p>8. $5x + 1 - (-2x) = -8$</p> $ \begin{array}{r} 5x + 1 - (-2x) = -8 \\ \swarrow \quad \searrow \\ 3x + 1 = -8 \\ -1 \quad -1 \\ \hline 3x = -9 \\ \frac{3}{3} \quad \frac{3}{3} \\ x = -3 \end{array} $		
<p>9. $-2(x - 2) = 14$</p> $ \begin{array}{r} -2(x - 2) = 14 \\ -2x + 4 = 14 \\ +4 \quad +4 \\ \hline -2x = 18 \\ \frac{-2}{-2} \quad \frac{-2}{-2} \\ x = -9 \end{array} $		

10. $3(2x+1)+4=10$

$$6x+3+4=10$$

$$6x+4=10$$

$$\begin{array}{r} 6x+4=10 \\ -4 \quad -4 \\ \hline \end{array}$$

$$6x=6$$

$$\frac{6x}{6} = \frac{6}{6}$$

$$x = \frac{6}{6} = 1$$

3.2d Homework: Practice Solving Equations (select homework problems)

Solve each equation.

1. $8 - t = -25$

2. $2n - 5 = 21$

3. $3 - y = 13$

4. $12 = 5k - 8$

5. $-5 - b = 8$

6. $5 = -6a + 5$

7. $8 = \frac{n}{-7} - 5$

8. $8 = \frac{x}{7} + 5$

9. $\frac{y}{3} + 2 = 10$

10. $\frac{t}{3} + 4 = 2$

11. $9 = \frac{n}{-8} - 6$

12. $\frac{y}{5} + 4 = -12$

13. $8 + 6 = -p + 8$

14. $-7 + 8x - 4x = 9$

15. $8x - 6 - 8 - 2x = 4$

16. $6(m - 2) = 12$

17. $5(2c + 7) = 80$

18. $5(2d + 4) = 35$

19. $3(x + 1) = 21$

20. $7(2c - 5) = 7$

21. $6(3d + 5) = 75$

22. $4 - 14 = 8m + 2m$

23. $-1 = 5p + 3p - 8 - p$

24. $5p - 8p = 4 + 14$

25. $2p - 4 + 3p = -9$

26. $-8 = -x + 5 - 1$

27. $12 = 20x - 3 + 4x$

3.2e Class Activity: Solve One- and Two-Step Equations with Rational Numbers (use algebra to find solutions)

Before we begin...

<p>...how can we find the solution for this problem?</p> $\frac{3}{5}x = 6$	<p>...do you expect the value for x to be larger or smaller than 4 for these problems? Explain.</p> $\frac{2}{3}x = 4$ $0.25x = 8$ <p>...how can you figure out the solutions in your head?</p>
---	--

Solve the equations for the variable. Show all solving steps. Check the solution in the equation (example #1 check: $-13(3) = -39$, true.). Be prepared to explain your work.

1. $-13m = -39$	2. $-2 = \frac{m}{16}$	3. $y - 25 = 34$	4. $-2y = 24$
Check:			
5. $-3x = \frac{3}{4}$	6. $-13 = -25 + y$	7. $\frac{1}{2}x = 6$	8. $\frac{3}{4}x = 6$
Check:			

9. $\frac{2}{3}x = -5$	10. $\frac{a}{1.23} = 0.2$	11. $-3.25d = 5.278$	12. $8x = -\frac{1}{4}$
Check:			
13. $\frac{3}{4}q = 2$	14. $\frac{3}{4}r = \frac{2}{3}$	15. $-5.36a = \frac{67}{5}$	
Check:			
16. $9.2r + 5.514 = 158.234$	17. $0.25x - 2 = 8$	18. $-8.38v + 10.71 = 131.382$	
Check:			
19. $\frac{n}{1.4} - 2.9 = -5.11$	20. $2x - \frac{3}{4} = 3.25$	21. $\frac{1}{4}x - 2 = 3$	
Check:			

3.2e Homework: Solve One- and Two- Step Equations (practice with rational numbers)

Solve the equations for the variable in the following problems. Use models if desired. **Show all solving steps.**
Check the solution in the equation.

1. $22 = -11k$	2. $\frac{x}{7} = -7$	3. $x + 15 = -21$
Check:		
4. $-3y = -36$	5. $\frac{2}{5}x = -2$	6. $-54 = 16 + y$
Check:		
7. $\frac{1}{4}x = 3$	8. $0.25x = 3$	9. $\frac{3}{4}x = -6$
Check:		
10. $\frac{m}{-3.68} = -26.9$	11. $23.45j = -469$	12. $-2x = \frac{4}{7}$
Check:		

13. $5b = 0.2$	14. $\frac{2}{3}r = \frac{2}{5}$	15. $2.5b = 1\frac{4}{5}$
Check:		
16. $-3.8 - 13.4p = -460.606$	17. $0.4x + 3.9 = 5.78$	18. $\frac{m}{2.8} - 4.9 = -7.11$
Check:		
19. $0.4x - 2 = 6$	20. $3x - \frac{2}{3} = 5\frac{1}{3}$	21. $\frac{1}{5}x - 3 = 2$
Check:		

3.2f Extra Practice: Equations with Fractions and Decimals

1. $x + \frac{1}{2} = 5$	2. $v - \frac{3}{8} = \frac{1}{8}$	3. $\frac{2}{3}n = \frac{4}{9}$
Check:		
4. $k + \frac{2}{3} = \frac{4}{5}$	5. $\frac{1}{3} + n = \frac{7}{12}$	6. $\frac{5}{9} = u - \frac{2}{9}$
Check:		
7. $\frac{4}{5}n = 0.625$	8. $n + \frac{5}{7} = \frac{1}{2}$	9. $x + 0.5 = 4$
Check:		

10. $m - \frac{3}{4} = \frac{5}{6}$	11. $\frac{1}{4}x = 1\frac{1}{2}$	12. $a + \frac{3}{4} = 5$
Check:		
13. $\frac{q}{3.1} + 5 = 10$	14. $-9.2 + \frac{k}{6} = 4$	15. $-5 = \frac{h}{3} + 7$
Check:		
16. $20 = \frac{w}{4} - 10$	17. $\frac{2}{5}p = \frac{5}{8}$	18. $\frac{2}{3}x = \frac{5}{8}$
Check:		

3.2g Extra Practice: Solve Equation Review

1. $-6.2d = 124$	2. $k + 12\frac{1}{2} = -20$	3. $\frac{a}{5} = 20$
Check:		
4. $g - 12.23 = 10.6$	5. $-\frac{1}{5}h = 3$	6. $-\frac{2}{3}h = 5$
Check:		
7. $\frac{w}{-1.26} = -2.36$	8. $3d = \frac{2}{3}$	9. $\frac{c}{3} + 1 = 10$
Check:		

10. $28 = 8x - 4$	11. $-0.3x + 3 = 4.2$	12. $\frac{s-4}{11} = 2$
Check:		
13. $-6(x+8) = -54$	14. $5(w-20) - 10w = 5$	15. $\frac{2}{3} = 5(y-0.2)$
Check:		
16. $\frac{1}{2}(4d+2) = \frac{2}{3}$	17. $4\left(v + \frac{1}{4}\right) = 37$	18. $3(x-1) - 2(x+3) = 0$
Check:		

19. $2\left(5a - \frac{1}{3}\right) = \frac{7}{3}$	20. $-2.4 + 0.4v = 16$	21. $7(w + 2) + 0.5w = 5$
Check:		

Solve Multi-Step Equations (distribution, rational numbers)

22. $4(x + 2x) = -24$	23. $-5(5 + x) = -65$	24. $-6(-4 + 6x) = 24$
Check:		
25. $-3(-2x + 3) = -57$	26. $2(4x - 1) = 42$	27. $-7(-2x + 7) = 105$
Check:		

28. $3(7x+8)=150$	29. $5(7x+5)=305$	30. $5(1+7x)=320$
Check:		
31. $3(5x+6)=78$	32. $5(2x+3)=96$	33. $\frac{12.1+a}{4.9}=7.071$
Check:		
34. $-6\left(-2x-\frac{1}{2}\right)=123$	35. $5\left(-\frac{2}{7}x+2\right)=\frac{80}{7}$	36. $\frac{1}{2}(6x+2)=-29$
Check:		

37. $\frac{-10.5+m}{11.57} = -2.748$	38. $-13.9 + \frac{b}{12.8} = -13.306$	39. $\frac{n-12.9}{6.1} = -0.377$
Check:		

40. $2(4x+8) = -32$	41. $7(5x+8) = 91$	42. $-2(-4x+2) = 76$
Check:		
43. $-7(3x+7) = 175$	44. $4(-9+x) = -12$	45. $5(-7+6x) = 175$
Check:		

46. $2(1x + 4) = 18$	47. $3(7 + 4x) = 33$	48. $3(10 + 6x) = 84$
Check:		
49. $5(1x + 7) = 40$	50. $-8 = -3(5 - 2x) + 1$	51. $\frac{r - 8.7}{3.6} = 3.722$
Check:		
52. $2\left(\frac{2}{3}x + 2\right) = 8$	53. $5\left(-4x + \frac{3}{10}\right) = -10$	54. $-\frac{1}{2}(4 + 5x) = -7$
Check:		
55. $\frac{k - 2.6}{5.2} = -0.418$	56. $-13.3 + \frac{k}{11.796} = -0.296$	57. $\frac{-7.3 + r}{9.2} = -0.739$
Check:		

3.2h Self-Assessment: Section 3.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Solve multi-step equations fluently including ones involving calculations with positive and negative rational numbers in a variety of forms. [1]	I can solve one and two-step equations, but I struggle to solve multi-step equations.	I can solve multi-step equations involving calculations with integers with a model.	I can solve multi-step equations involving calculations with integers without a model.	I can solve multi-step equations involving calculations with rational numbers.
2. Connect arithmetic solution processes that do not use variables to algebraic solution processes that use equations.	I do not understand the connection between arithmetic solution processes that do not use variables to algebraic solution processes that use equations.	I can identify the connection between arithmetic solution processes that do not use variables to algebraic solution processes that use equations.	I can explain the connection between arithmetic solution processes that do not use variables to algebraic solution processes that use equations.	I can explain the connection between arithmetic solution processes that do not use variables to algebraic solution processes that use equations using the vocabulary of mathematical properties as appropriate.
3. Use the properties of arithmetic to make an argument and/or critique the reasoning of others when solving algebraic equations. [3]	I struggle to understand the reasoning of algebraic equations.	I can tell if my reasoning or another's reasoning to a solution is correct or not, but I struggle to explain why.	I can tell if my reasoning or another's reasoning to a solution is correct or not, and I can use properties to explain why.	I can tell if my reasoning or another's reasoning to a solution is correct or not, and I can use properties to explain why. If a solution is incorrect, I can also explain with properties how it needs to be changed to show a correct reasoning and solution.

Sample Problems for Section 3.2

1. Solve each of the following equations with or without a model.

a.

$-1 = x + 7$	$-2x + 3 = 7$	$30 = \frac{x}{7} - 20$
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b.

$-3(x - 6) = 9$	$-3 = \frac{2x + 4}{2}$	$-3x + 5x + 4 = 4$
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c.

$0.8x + (-8) = 0$	$\frac{1}{2}(-2x - 7) = \frac{3}{4}$	$\frac{-1}{4} = 2.4x + 10 - 0.35x$
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2. Solve the following equations. Explain why you used fractions or decimals to get your final answer.

$-\frac{1}{7} = x - 4.7$	$-\frac{1}{2} = 2x + 0.8$
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3. Francisca is asked to solve the following equation. Francisca's work is shown below.

$$3.5 = \frac{1}{3}(x - 1.5)$$

$$3.5 = \frac{1}{3}x - 1.5$$

$$+1.5 \qquad +1.5$$

$$5 = \frac{1}{3}x$$

$$\cdot 3 \quad \cdot 3$$

$$15 = x$$

What mistake did Francisca make?

Help Francisca answer the question correctly.

Section 3.3: Solve Multi-Step Real-World Problems Involving Equations and Percentages

Section Overview:

In this section students learn how to solve percent problems using equations. They begin by drawing bar models to represent relationships in word problems. Then they translate their models into equations which they then solve. Students then use similar reasoning to move to problems of percent of increase and percent of decrease. Finally, students put all of their knowledge together to solve percent multi-step problems with various types of rational numbers.

Concepts and Skills to be Mastered (from standards)

By the end of this section, students should be able to:

1. Use variables to create equations that model word problems.
2. Solve word problems leading to linear equations.
3. Solve multi-step real-life percent problems involving calculations with positive and negative rational numbers in a variety of forms.
4. Convert between forms of a rational number to simplify calculations or communicate solutions meaningfully.

3.3a Class Activity: Create Equations for Word Problems and Solve

For each each context, draw a model to represent the situation, write an equation that represents your model, solve the equation, and then answer the question in a full sentence. Check you answers in original problem.

1. Today is Rosa's 12th birthday. She has a savings account with \$515 in it, but her goal is to save \$10,000 by the time she turns 18. How much money should she add to her savings account each month to reach her goal of \$10,000 between now and her 18th birthday?
2. Eisenhower Junior High School made 1250 hotdogs to sell at the county fair as a fundraiser for new computers. They're only going to sell them for two days. On the first day, they sold 436 hot dogs. If they plan on selling for 8 hours on the second day, how many hot dogs per hour will they have to sell in order to sell the rest of their hotdogs?
3. Calisa's grandpa won't tell her how old he is. Instead he told her, "I'm twice your mom's age." Calisa knows her mom had her when she was 24 and Calisa is now 12. How old is Calisa's grandpa?

4. Calisa's grandpa was really impressed that she was able to figure out his age, so he decided to give her another riddle. He said, "The sum of your great grandparents' age is 183. If great grandpa is 5 years older than great grandma, how old is each of them?"

5. Peter tends a total of 87 sheep and goats. He had 15 more sheep than goats. How many sheep and goats does he have?

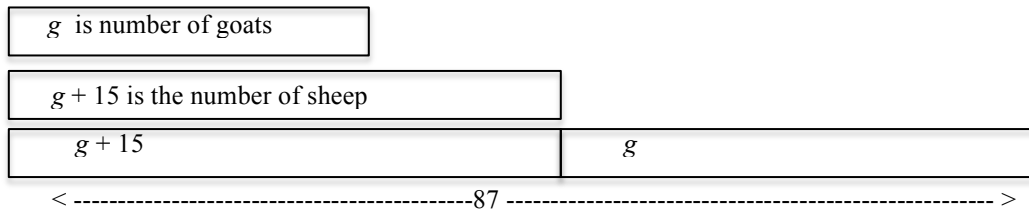
6. Peter's neighbor raises chickens; both hens and roosters. Their family sells the eggs, so they like to have four times as many hens as roosters. If they currently have 45 birds, how many are hens?

7. James is five inches taller than twice his height when he was three years old. If he's now 5' 10" tall, how tall was he when he was three?

3.3a Homework: Create Equations for Word Problems and Solve

Example 1:

Peter tends a total of 87 sheep and goats. He had 15 more sheep than goats. How many sheep and goats does he have?



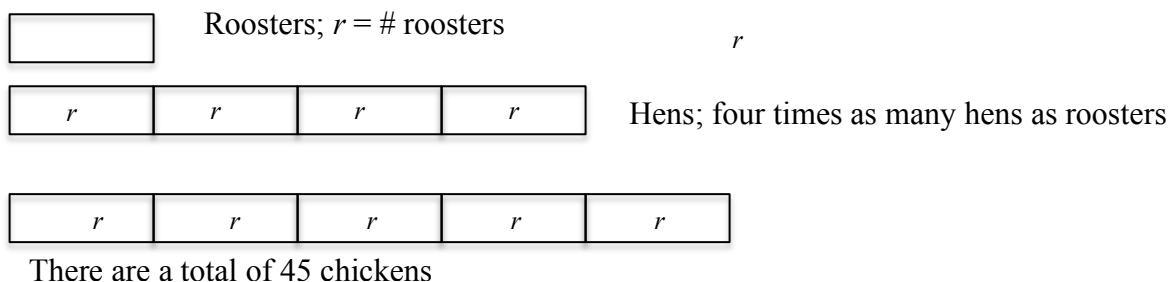
We know that there are more sheep than goat. We know that there are exactly 15 more sheep than goat, so the smaller rectangle represents g goat, and the larger is $g + 15$ sheep. There are total of 87 sheep and goat, so we can write the equation:

$$g + (g + 15) = 87$$

$g = 36$; thus we know that there are 36 goat and 51 sheep.

Example 2:

Peter's neighbor raises chickens; both hens and roosters. Their family sells the eggs, so they like to have four times as many hens as roosters. If they currently have 45 birds, how many are hens?



We know that there are more hens than roosters. Further, we know that there are four times as many hens as roosters. So if r is the numner of roosters, then $4r$ is the number of hen. All together there are 45 chickens, so we can write:

$$r + 4r = 45$$

$$r = 9$$

So there are 9 roosters and 36 hens.

For each context, draw a model, write an equation, and then write a complete sentence to answer the question in the context.

1. The blue jar has 27 more coins than the red jar. If there are a total of 193 coins, how many coins are in each jar?	2. Leah's age is twice her cousin's age. If they add their ages together, they get 36. How old are Leah and her cousin?
3. There are a total of 127 cars and trucks on a lot . If there are four more than twice the number of trucks than cars, how many cars and trucks are on the lot?	4. Art's long jump was 3 inches longer than Bill's. Together they jumped 20 feet. How far did they each jump?
5. Juliana and Maria logged a total of 90 hours working for their dad. Juliana worked 2 hours more than three times as much as Maria. How many hours did Juliana work?	6. There are 17 more boys than girls at Juab Junior High School. If there are a total of 981 students there, how many students are boys?
7. A total of 81 points were scored at the basketball game between White Horse High School and Bear River High School. If White Horse scored 7 points more than Bear River, how many points did White Horse score?	8. Pedro spent 57 minutes doing his math and language arts homework. His language arts homework took him twice as long as his math homework. How much time did he spend on his language arts homework?

3.3b Class Activity: Writing Equations to Solve Percent Problems

Do the indicated for each context.

1. Last week, Dirk jumped y inches in the long jump. This week, he increased the length of his jump by 10%. If this week he jumped 143 inches, how far did he jump last week?

a. Draw a model:

b. Write an equation to answer the question:

c. Solve for the unknown:

d. Write a sentence stating the answer to the question:

2. Consider these two contexts, write an equation for each and then solve:

Luis wants to buy a new smart phone. Because he's had another phone with the carrier, he will get an 18.25% discount. If the phone he wants is normally \$325.50, how much will he pay?

Mica bought a new smart phone for \$258.13. That was the price after his 18.25% discount. What was the original price of the phone?

How are the equations similar? How are they different?

3. Consider the three equations below. Write a context for each and then state the answer to your context question.

a. $x - 0.28x = 252$

b. $0.72x = 252$

c. $0.28x = 252$

4. Drake wants to buy a new skateboard with original price of s dollars. The skateboard is on sale for 20% off the regular price; the sale price is \$109.60.
- Draw a model, write an equation, and solve it to find the original price of the skateboard.
 - Drake decided to buy the skateboard for the sale price. When he got to the counter, he got an additional 10% off, but also had to pay 8.75% sales tax. What was the total amount Drake paid for the skateboard?
5. Hallie has been training for a marathon for about a year. The first time she ran a marathon she had a really slow time. The next time she ran a full marathon, she cut about 25% off her original time. The third time she recorded her time in a marathon, she reduced her time a second time by approximately 15%. If her newest time is 3 hours and 15 minutes, what was her original time?
- Draw a model to represent this situation.
 - Write an equation to answer the question:
 - Solve the equation:
 - Answer the question in a sentence:

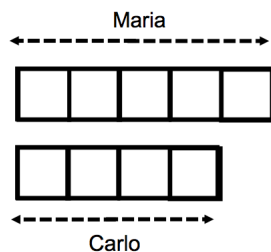
6. Alaya sells cupcakes. She currently sells 200 cupcakes a week, but now she's advertising on several local restaurant blogs and expects business to increase by 10% each week. If her business grows 10% from the previous week each week for two weeks in a row, how many cupcakes will she be selling in 2 weeks?

a. Draw a model and write an expression to represent this situation:

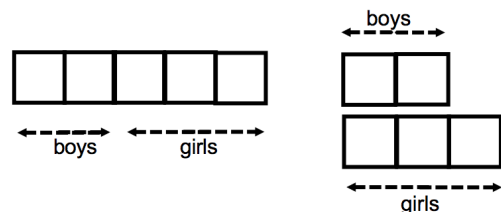
b. Find the answer to the question.

Determine whether each statement below is true or false. Use words, models or an expression/equation to justify your answer. If the statement is false, write a correct statement with the given context.

7. If Maria has 25% more money than Carlo, then Carlo has 25% less money than Maria.



8. If 40% of Ms. Eischeid's class are boys and 60% are girls, then there are 20% more girls than boys in Ms. Eischeid's class.



3.3b Homework: Writing Equations to Solve Percent Problems

For each context, write an equation for the context, solve the equation and then answer the question. Draw a model to help you understand the context where necessary.

1. There were 850 students at Fort Herriman Middle School last year. The student population is expected to increase by 20% next year. What will the new population be?

Draw a model to represent this situation.	Write an equation to represent the new population.
What will the new population be?	

2. A refrigerator at Canyon View Appliances costs \$2200. This price is a 25% mark up from the wholesale price. What was the wholesale price?

Draw a model to represent this situation.	Write an equation to represent the whole sale price
What was the whole sale price?	

3. Carlos goes to Miller's Ski Shop to buy a snowboard, because there's a 30% off sale. When he gets to the store, he gets a coupon for an additional 20% off the sale price. If he paid \$252 for the snowboard, what was the original cost?

Draw a model to represent this situation.	Write an equation to represent the problem situation.
What was the original cost of the snowboard?	

For each context in #4 – 6, write an equation, solve it, and write and a complete sentence to answer the question. Draw a model to assist you in understanding the context.

- Philip took a test and missed 35% of the questions. If he missed 28 questions, how many questions were on the test?
- Dean took his friend to lunch last week. The service was really good, so he left a tip of 20% of the total bill, the meal and tax. If he spent a total of \$28.50, how much was the meal and tax?
- The size of Mrs. Garcia's class increased 20% from the beginning of the year. If there are 36 students in her class now, how many students were in her class at the beginning of the year?

7. Write a context for each equation:

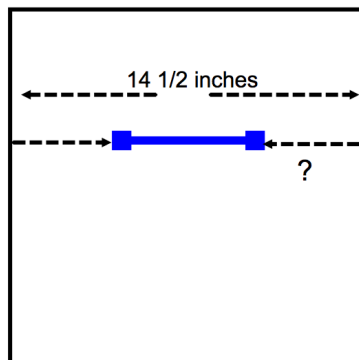
$0.40x = 450$	$0.60x = 450$	$x - 0.40x = 450$
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8. Kaylee is training for a marathon. Her training regiment is to run 12 miles on Monday, increase that distance by 25% on Wednesday, and then on Saturday increase the Wednesday distance by 25%. How far will she run on Saturday?
9. Christina would like to give her brother new gloves for his birthday. She has a lot of coupons but is not sure which one to use. Her first coupon is for 50% off of the original price of one item. Normally, she would use this coupon. However, there is a promotion this week and gloves are selling for 35% off the original price. She has a coupon for an additional 20% off the sale price of any item. Which coupon will get her the lower price? She is not allowed to combine the 50% off coupon with the 20% off coupon.
- a. Draw a model to show the two different options.
- 50% off coupon: 35% off sale with additional 20% off coupon:
- b. Let x represent the original price of the picture frame. Write two different expressions for each option.
- 50% off coupon: 35% off sale with additional 20% off coupon:
- c. Which coupon will get her the lowest price? Explain how you know your answer is correct.

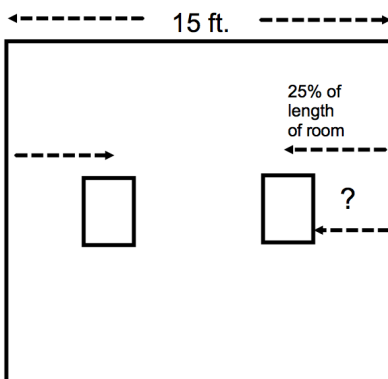
3.3c Class Activity and Homework: Word Problems with Various Forms of Rational Numbers

Use any strategy you want to answer each question. Be able to justify your answers.

1. Randy needs to hang a towel rack on a wall that's $50\frac{3}{4}$ inches wide. The towel rack is $14\frac{1}{2}$ inches wide. How far in from the edge of the wall should Randy place the ends of the towel rack if he centers it?



2. Marcela has two $16\frac{3}{4}$ inch wide paintings to hang on her 15 foot wide wall. She wants the paintings to be centered 25% of the way in from the left and right edge of the wall. How many inches from the edge will (a) the hook need to be placed and (b) will the edge of the frame be from the edge of the wall?



3. Bruno earns \$43,250 a year. Of this amount, he pays 17.8% to taxes. Of the remainder, $\frac{1}{3}$ is for living expenses, $\frac{2}{5}$ for food and entertainment, and $\frac{1}{4}$ for other insurance and car expenses. What percent of the \$43,250 does Bruno have left over for miscellaneous expenses? How much money is left over?

4. There are 200 girls and 300 boys in 7th grade at Alpine Junior High. Cory learns that on Monday one-fifth of the girls and 30% of the boys in 7th grade bought school lunch.
- How many boys and girls bought school lunch on Monday?
 - Did 25% of the 7th graders buy school lunch on Monday? Explain.
 - What fraction of the 7th graders bought school lunch?
5. Nicholas and Martin both have \$2000. They both invest in two different mutual funds. Nicholas earns a 25% return on his investment the first year but then loses 25% of the new amount the next year. Martin loses 25% on his investment the first year, but then earns 25% on his new amount in the next year. At the end of two years, how much will each have?

Would they have different ending amounts if they both started with \$5000 or both started with \$35?

Suppose Martin lost 25% on his investment the first year. What percent increase of his new amount would Martin have to earn in order to be back to his original \$2000 investment?

6. Mario is taking College Algebra at the University of Utah. This final grade is weighted as follows:
- 14% Homework
 - 18% Midterm 1
 - 18% Midterm 2
 - 18% Midterm 3
 - 32% Final

If Mario earned the following grades so far:

- 92% Homework
- 83% Midterm 1
- 79% Midterm 2
- 91% Midterm 3

What is the lowest score Mario needs to earn on the final to earn an A– in the course if the cutoff for an A– is 90%? (Note, all scores are rounded to the nearest whole number.)

7. Wayne and Tino are both selling lemonade. Wayne's lemonade is 70% water while Tino's is 80% water.
- If Wayne has one gallon of lemonade and Tino has two gallons of lemonade and they mix it all together, what percent of the new mixture will be water?
 - How much lemonade would Wayne need to add to Tino's two gallons if they wanted the lemonade to contain 75% water?
8. A man owned 19 cars. After his death, his three children wanted to follow the instructions of his will which said: The oldest child will receive $\frac{1}{2}$ my cars, the second child will receive $\frac{1}{4}$ of my cars and the youngest will receive $\frac{1}{5}$ of my cars. The three children didn't know what to do because there was no way to follow the will without cutting up cars. Not wanting to destroy any of the cars, they decided to ask their aunt (their father's sister) for help. She said she'd loan them one of her cars, that way they'd have 20 cars; 10 would go to the oldest, 5 to the second child and 4 to the youngest. Once they distributed the 19 cars, they could then return the car she lent them. Is this a good solution to the problem?
The above was adapted from http://mathafou.free.fr/pba_en/pb003.html "Aegyptian fractions"
9. Kevin is trying to understand percents, but often gets confused. He knows that in his school 55% of the students are girls and 45% are boys. It seems to him that there are 10% more girls than boys in his school. Is he correct? Explain.
10. Ana Maria says $\frac{3}{4}$ of the students in her class like rap music. Marco says that is $\frac{3}{4}\%$ of the class. Is Marco correct?

3.3d Self-Assessment: Section 3.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Use variables to create equations that model word problems. [1]	I struggle to begin writing an equation that models a word problem.	I can draw a model that represents a word problem. I struggle to use that model to create an equation.	I can write an equation that represents a word problem if I draw a model first.	I can write an equation that models a word problem.
2. Solve word problems leading to linear equations. [1]	I struggle to solve word problems leading to linear equations.	I can usually write an equation to solve a word problem leading to a linear equation, but I struggle using that equation to get a solution.	I can solve word problems leading to linear equations.	I can solve word problems leading to linear equations. I can explain the solution in context.
3. Recognize and explain the meaning of a given equation and its component parts when using percents. [2]	I struggle to understand the meaning of parts of a given equation when using percents.	I can recognize the different parts of given equation and match them with their meaning when using percents.	I can recognize the different parts of a given equation when using percents and can explain their meaning in my own words.	I can explain the meaning of a given equation and its parts in my own words. I can also write an equation given a context.
4. Solve multi-step real-life percent problems involving calculations with positive and negative rational numbers in a variety of forms. [3]	I struggle to solve real-life percent problems involving positive and negative rational numbers.	I can usually write an equation to solve a real-life percent problem, but I struggle using that equation to get a solution.	I can solve real-life percent problems involving positive and negative rational numbers.	I can solve multi-step real-life percent problems involving positive and negative rational numbers. I can explain the solution in context.
5. Convert between forms of a rational number to simplify calculations or communicate solutions meaningfully. [3c]	I struggle to convert between all forms of a rational number.	I can convert between forms of a rational number, but I struggle to know how to use that to solve equations or simplify calculations.	I can convert between forms of a rational number to simplify calculations, solve equations, and communicate solutions meaningfully.	I can convert between forms of a rational number to simplify calculations, solve equations, and communicate solutions meaningfully. I can explain why a particular form communicates the solution meaningfully.

Sample Problems for Section 3.3

1. Write an equation to represent each of the following word problems. Solve each problem. Express answers as fractions or decimals when appropriate.
 - a. Chloe has twice as many cats as her sister has dogs. Her brother has two turtles. Together, they have five pets. How many of each pet do they have?
 - b. Brian is three times older than Sydney. The sum of the ages of Brian and Sydney is eight. How old is Brian?
 - c. Samantha is 8 and two-third years older than Jason. The sum of their ages is 23 years. How old is Jason?
2. Explain the meaning of the following equation that matches the given situation:

I go to a department store with a coupon for 30% off any one item. The pants that I want are on sale for 40% off. What was the original price if I pay \$21?

$$0.70(x - 0.40x) = 21$$
3. Write an equation to represent each of the following real-life percent problems. Solve each problem. Express answers as fractions or decimals when appropriate.
 - a. Victor is saving for retirement. He invested \$500 plus the money his company invested for him. He earned 4% interest per year and had \$780 at the end of one year. How much did his company invest for him?
 - b. William and his family eat at a fancy restaurant that automatically charges 18% gratuity (tip). If his bill total is \$53.02, how much was the bill before gratuity was included?
 - c. Felipe saved some money in a CD with a rate of 1.5% per year. Ellie saved the same amount of money in a different CD with a rate of 0.75% interest per year. David saved $\frac{2}{5}$ the same amount of money in a box under his bed (no interest under there). If they had \$969 total after one year, how much did Felipe save to start?

First published in 2013 by Math Materials Access Improvement for Seventh and Eighth Grades (MMAISE), Salt Lake City, USA.

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Chapter 4 Proportional Relationships, Solving Problems (6 Weeks)

UTAH CORE Standard(s)

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.* 7.RP.1
2. Recognize and represent proportional relationships between quantities. 7.RP.2
 - a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. 7.RP.2a
 - b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. 7.RP.2b
 - c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.* 7.RP.2c
 - d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate. 7.RP.2d
3. Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. 7.RP.3

CHAPTER OVERVIEW:

This chapter focuses on extending understanding of ratio to the development of an understanding of proportionality in order to solve one-step and multi-step problems. The chapter begins by reviewing ideas from 6th grade as well as 7th grade chapters 1 – 3 and transitioning students to algebraic representations. Students will rely on understandings developed in previous chapters and grades to finding unit rates, proportional constants, comparing rates and situations in multiple forms, writing expressions and equations, and analyzing tables and graphs. The goal is that students develop a flexible understanding of different representations of ratio and proportion to solve a variety of problems.

One important thing for teachers to note (and help students understand) is that a ratio can be written in many different ways, including part:part and part:whole. Fractions represent a part:whole relationship. Because of the overlap in expressing part:whole relationships, we often have to rely on the context to determine if a given value a/b is a fraction or a ratio.

Many ratio and proportion concepts are interrelated. Therefore, at the beginning of each section, we list the key points from the 7th grade ratio and proportion standards and, below them, list the concepts and skills that will be the primary focus for the section.

VOCABULARY:

Bar model, constant of proportionality, equation, part-to-part ratio, part-to-whole ratio, percent change, proportion, proportional constant, rate table, and ratio.

CONNECTIONS TO CONTENT:

Prior Knowledge







Student should be able to draw models of part-to-part and part-to-whole relationships. From these models, students should be able to operate fluently with fractions and decimals, especially reducing fractions, representing division as a fraction, converting between mixed numbers and improper fractions, and multiplication and division with fractions. Many of these concepts were reviewed in Chapter 1 and will be reviewed briefly in this chapter. Students worked extensively on ratio in 6th grade where they learned to write ratios as fractions, using a colon, or using words.



At the beginning of this chapter, it is assumed that students can use models to solve percent, fraction and ratio problems with models. Students will connect ratio and proportional thinking to a variety of multi-step problems.

Future Knowledge

A strong foundation in proportion is key to success throughout traditional middle and high school mathematics. Further, it is essential in Trigonometry and Calculus. In the next chapter, students will use proportions as a basis for understanding scaling. In 8th grade, proportions form the basis for understanding the concept of constant rate of change (slope) and the basic measures of statistics. Also in 8th grade students will finalize their understanding of linear relationships and functions; proportional relationships studied in this chapter are a subset of these relationships. Later, in secondary math, students will solve rational equations (such as $\frac{x}{x+2} = \frac{3}{x^2-4}$), apply ratio and proportion to similarity and then to trigonometric relationships, and learn about change that is not linear.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

	Make sense of problems and persevere in solving them.	By building from simpler contexts of ratio and proportion, students develop strategies for working with multi-step problems without a clear path to a solution or containing inconvenient numbers. Additionally, students should think flexibly about the relationship between fractions, ratio and proportion and their multiple representations to solve problems. After solving a problem students should be encouraged to think about the reasonableness of the answer.
	Reason abstractly and quantitatively.	Students will reason abstractly throughout the chapter particularly when they begin to write proportion equations and solve them algebraically. Students should be encouraged to think about the relationship between equations, tables and graphs and the contexts they represent. Students should also be encouraged to reason quantitatively throughout the chapter with the emphasis on units. When an answer is obtained, students should be able to state what the number represents. When students state a unit rate they should be able to give the units represented therein. Further, students should be able to draw models of the quantities and connect various representations to tables, graphs and equations.
	Construct viable arguments and critique the reasoning of others.	Students should construct arguments and critique those of others' throughout. With both, students should refer to representations of ideas and/or arithmetic properties in the construction of their arguments. Arguments should be made orally or in writing.
	Model with mathematics.	As students study proportional relationships, they begin to see examples in their lives: traveling on the freeway at a constant rate, swimming laps, scaling a recipe, shopping, etc. Most proportional relationship from the real world can be modeled using the representations students learn in this chapter. Appropriate models include bar/tape models (either part-part-whole or comparison) or other similar representations, tables or graphs. Models selected should help to reveal relationships and structure. Further, students should be able to connect models to their algebraic representation.
	Attend to precision.	Students should be encouraged throughout the chapter to express unit rates exactly with appropriate units of measure by using fractions and mixed numbers rather than rounded decimals. When graphing or creating tables, labeling is crucial as it is when making an argument. Vocabulary should also be used appropriately in communicating ideas.
	Look for and make use of structure.	Students should make use of structure as they identify the unit rate from tables by looking at the relationship of values both vertically and horizontally. Additionally, students will discover that graphs of proportional linear relationships cross through the origin. When comparing proportional relationships, students should note the structure of equations and how it relates to its table and the steepness of its lines. Lastly, students should solidify their understanding of the difference between ratio and fraction in this chapter. This is an important distinction and one that will allow them to think more flexibly with either.

	<p>Use appropriate tools strategically.</p>	<p>Students continue their transition in this chapter to more abstract representations of ideas. As such, students will be working towards using symbolic representation of ideas rather than models exclusively. This transition will occur at different rates for different students. Students should be encouraged to use mental math strategically throughout. For example, if students are given a rate of 3 pounds per \$2, there is no need for a calculator to determine that the unit rate is $\frac{3}{2}$ pounds per \$1 or $\\$ \frac{2}{3}$ per pound.</p>
	<p>Look for and express regularity in repeated reasoning.</p>	<p>Students should note repeated reasoning in rate tables and graphs throughout the chapter. They will also use repeated reasoning to uncover the unit rate associated with percentages of increase and decrease and in the relationship of ratios and fractions.</p>

4.0 Anchor Problem: Purchasing Tunes-Town Music Company—What’s the Better Deal?

Problem # 1:

Discounts and Markups

(Adapted from illustrative mathematics.org)

In June, Tunes-Town wasn’t making enough money to stay in business, so in July, they reduced all prices by 20% to attract more business.

They found that even at July’s prices they weren’t making enough money, so they decided they had to sell their store to Beat-Street. In August, Beat-Street took over the store and the manager immediately increased all the current (July) prices by 20%, thinking this would go back to the original prices.

1. Track the price of a \$100 car stereo throughout the story.
2. By what percentage should Beat-Street have raised prices to make August’s prices revert back to the original June prices? Use any strategy to solve.

Problem # 2:

What’s the Better Deal?

(From illustrative mathematics.org)

Beat-Street, Tunes-Town, and Music-Mind are music companies. Beat-Street offers to buy 1.5 million shares of Tunes-Town for \$561 million. At the same time, Music-Mind offers to buy 1.5 million shares of Tunes-Town at \$373 per share.

1. Who would get the better deal, Beat-Street or Music-Mind? Explain.
2. What is the total price difference?

Section 4.1: Understand and Apply Unit Rates

Section Overview:

The purpose of this section is to solidify the concept of unit rates. This includes finding unit rates from contexts, tables, graphs and/or proportional relationships written as equations. This section will begin the work of helping students to move fluidly among these representations to find missing quantities. Students will visualize ratios using models—both part-part-whole and comparison tape models. Particular attention should be paid to models of rates with unlike units (for example miles and hours). Students will identify the unit rate for *both* units (i.e. miles per hour *and* hours per mile). Students should then be able to move away from using models and be able to recognize the reciprocal relationship of the two unit rates to solve problems.

Along the way, this section will practice operations with rational numbers, with particular attention paid to precision with “inconvenient” division problems that come up from ratios. Students should be encouraged to find exact unit rates; i.e. they should be encouraged to use fractions to divide numbers, rather than using long division or a calculator and getting decimals approximations. Students should understand that using a rounded unit rate will give inaccurate results when finding missing quantities.

Up until this chapter, students worked primarily with part:whole relationships (percent, fractional portions, and probability). In this section, students will begin to connect how part:whole relationships give information about part:part relationships and vice versa. In 4.4 this idea will be solidified.

Rate tables will be introduced but will be further explored and connected to graphs in the next section.

Key Ratio and Proportion Concepts from Utah Core Standards

RP Standard 1:

1. Extend the concept of a unit rate to include ratios of fractions.
2. Compute a unit rate, involving quantities measured in like or different units.

RP Standard 2:

3. Determine if two quantities expressed in a table or in a graph are in a proportional relationship.
4. Determine a unit rate from a table, graph, equation, diagram or verbal description and relate it to the constant of proportionality.
5. Write an equation for a proportional relationship in the form $y = kx$.
6. Explain the meaning of the point (x, y) in the context of a proportional relationship.
7. Explain the significance of $(0,0)$ and $(1,r)$ in a graph of a proportional relationship, where r is the unit rate.

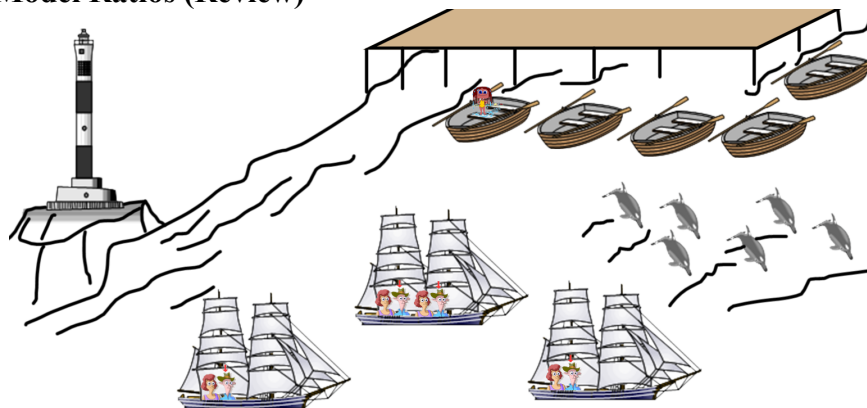
RP Standard 3:

8. Solve multistep problems involving percent using proportional reasoning
9. Find the percent of a number and extend the concept to solving real life percent applications.
10. Calculate percent, percent increase, decrease, and error.

Primary Concepts and Skills to be Mastered in This Section

1. Compute unit rate from a context.
2. Compute a unit rate from a table of values.
3. Compare two rates to determine equivalence or to contrast differences.
4. Find the unit rate for BOTH units (i.e. miles per hour and hours per mile).
5. Use unit rate to find a missing quantity.

4.1a Class Activity: Model Ratios (Review)

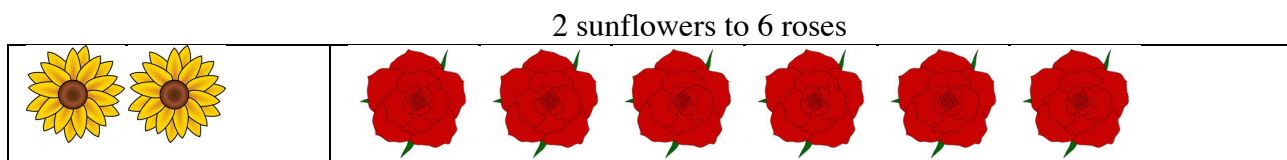


What is a ratio?

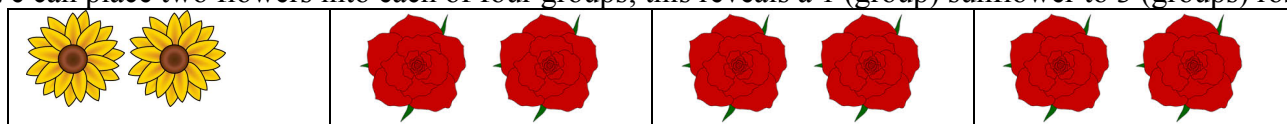
In the space below, write as many ratios as you can find in the picture shown above. Be sure to label what you are comparing.

Review Reducing Ratios and Relating Ratios to Fractions and Percents

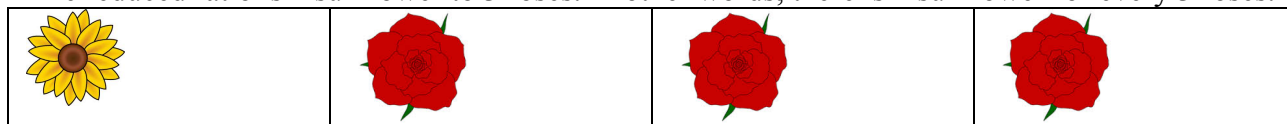
Example 1: Use a model to find ratios that are equivalent to the ratio 2 sunflowers to 6 roses.



We can place two flowers into each of four groups; this reveals a 1 (group) sunflower to 3 (groups) roses

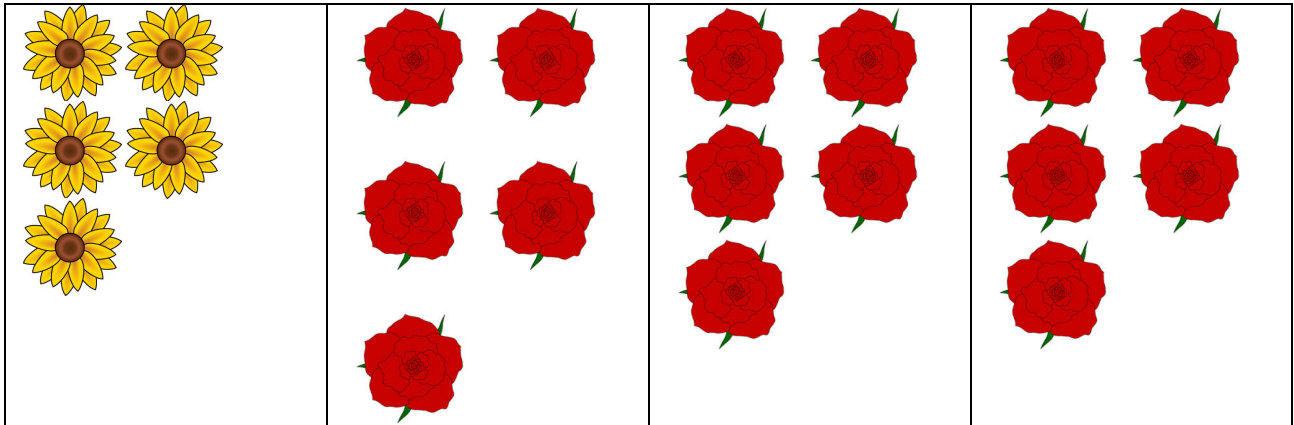


The reduced ratio is 1 sunflower to 3 roses. In other words, there is 1 sunflower for every 3 roses.

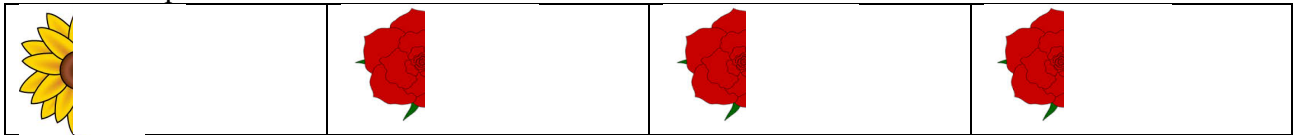


We can write other equivalent ratios by increasing the number of flowers in each group as long as all the groups have the same number of flowers.

For example, 5 sunflowers to 15 roses, notice that all the groupings have five flowers.



The ratio is also equivalent to $\frac{1}{2}$ sunflower to $1\frac{1}{2}$ roses.



Example 2: What percent of the flowers in Example 1 are sunflowers?

Because there are a total of 4 parts (1 part sunflowers and 3 parts roses) and sunflowers are 1 part of *all* the flowers, we know that 25% of the flowers are sunflowers. We could also say $\frac{1}{4}$ of all the flowers are sunflowers and then convert $\frac{1}{4}$ to 25%.

Working with a partner use a model to answer each question.

1. The high school has a new ski club this year. In the club there are 4 girls and 16 boys.
 - a. Use a model to find the ratio of girls to boys in the skiing club.

- b. What percent of the students in the ski club are girls?



2. A cat has given birth to a bunch of kittens. There are 3 black cats and 6 striped cats.
 - a. Use a model to find the ratio of black cats to striped cats.
 - b. What fraction of the kittens are striped?
3. There are 16 girls and 12 boys in Ms. Garcia's class.
 - a. What is the ratio of girls to boys in Ms. Garcia's class?
 - b. What is the ratio of girls to students in her class?
 - c. Approximately, what percent of the students in Ms. Garcia's class are boys?
 - d. What fractional portion of the students in Ms. Garcia's are boys?
 - e. Suppose Ms. Garcia had a huge class of 63 students at the same ratio as above. How many students would you expect to be boys and girls?

Example 3: The ratio of girls to boys in Mrs. Jimenez's class is 3 to 2. Create a model to represent this context.

Girl	Girl	Girl	Boy	Boy
------	------	------	-----	-----

We can model this situation with a part-part-whole tape model as above: three parts girls and two parts boys. In this way we can see that there are a total of five equal parts in the whole.

We might also model the situation as a comparison, where one unit of measure is 3 girls to 2 boys:

Girl	Girl	Girl
Boy	Boy	

Example 4: Use a model to find ratios that are equivalent to the ratio 3 girls to 2 boys.

12 girls to 8 boys is equivalent to 3 girls to 2 boys, as seen in the part-part-whole tape model below.

Girl	Girl	Girl	Boy	Boy
Girl	Girl	Girl	Boy	Boy
Girl	Girl	Girl	Boy	Boy
Girl	Girl	Girl	Boy	Boy

The equivalent ratio 12 Girls to 8 boys can also be shown using a comparison model:

Girl	Girl	Girl
Boy	Boy	
Girl	Girl	Girl
Boy	Boy	
Girl	Girl	Girl
Boy	Boy	
Girl	Girl	Girl
Boy	Boy	

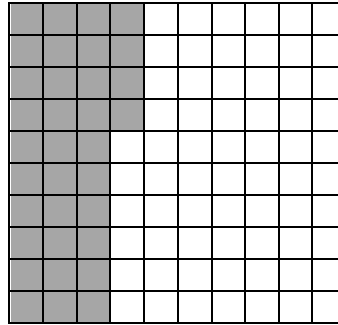
Other equivalent ratios include:

6 girls to 4 boys 30 girls to 20 boys 18 girls to 12 boys

4. What percent of Mrs. Jimenez’s class are girls?
5. What fractional portion of Mrs. Jimenez’s class are boys?

Spiral Review

1. Model $-18 + 6 =$
2. Simplify the following expression. Use a model if needed. $9m - 3 + 4m$
3. Which number is larger? Justify your answer. **-6 or -4**
4. Write this model in fraction form then simplify.



4.1a Homework: Model Ratios (Review)

1. In the closet, there are 10 shirts and 15 skirts.
 - a. Use a model to find the simplified ratio of shirts to skirts.
 - b. Use the same model to find the percent of the items in the closet that are shirts.

2. In my pocket, I have 8 quarters and 6 dimes.
 - a. Use a model to find the simplified ratio of quarters to dimes?
 - b. What fractional portion of the coins in my pocket are dimes?

3. On the bookshelf, there are 16 paperback and 20 hardback books.
 - a. Use a model to find the simplified ratio of paperback to hardback books.
 - b. To the nearest whole number, find the percent of paperback books on the shelf.

4. During gym class, there are 7 footballs and 49 tennis balls.
 - a. Use a model to find the simplified ratio of footballs to tennis balls.
 - b. If the ratio of footballs to tennis balls stays the same and there are 21 footballs, how many tennis balls would you expect to have?

5. In the toolbox, there are 4 nails and 30 screws.
 - a. Use a model to find the simplified ratio of nails to screws.
 - b. What fractional portion of the items are nails?

6. In a classroom, there are 12 girls and 8 boys.
- What fraction of the students in the class are girls?
 - What percent of the class are girls?
7. In the fridge, there are $2\frac{1}{2}$ bananas and 10 apples.
- Use a model to find the simplified ratio of apples to bananas.
 - What percent of the fruit is bananas?
8. A package of Skittles has 2 red skittles, 4 green skittles, and 4 purple skittles.
- Use a model to find the simplified ratio of red : green : purple candies.
 - What percent of the skittles are purple?
 - If the ratio of red : green : purple is always the same and there are 120 skittles in a package, how many of each color do you expect to have?
9. The ratio of rabbits to birds at the pet store is 5:2.
- Find three equivalent ratios and
 - What fraction of the rabbits and birds at this pet store are birds?
10. In a gummy bear package the ratio of red to yellow to clear gummy bears is 3:2:5.
- What percent of the gummy bears are yellow?
 - What fraction of the gummy bears are clear?

4.1b Class Activity: Equivalent Ratios and Rate Tables

Activity 1: It takes you 3 hours to read 82 pages in your novel.

- At this rate, how many pages per hour do you read?
- If you maintain the same rate, how many pages can you read in 9 hours?
- How many pages can you read in 2 hours?

In the previous section we used models to calculate ratios with similar units. You can also use models when working with rates where two attributes of the same situation are stated as a rate with *unlike units*.

Example 1: If you worked 5 hours and make a total of \$42.30, use a model to determine how much money per hour you make.

5 hours		5 hours				
\$42.30		1 hour	1 hour	1 hour	1 hour	1 hour
		\$8.46	\$8.46	\$8.46	\$8.46	\$8.46
		\$42.30				

You can use the same idea to scale up or scale down, as seen in the next example.

Example 2: How much will you make if you work 10 hours?

5 hours					5 hours				
1 hour	1 hour	1 hour	1 hour	1 hour	1 hour	1 hour	1 hour	1 hour	1 hour
\$8.46	\$8.46	\$8.46	\$8.46	\$8.46	\$8.46	\$8.46	\$8.46	\$8.46	\$8.46
\$42.30					\$42.30				

Thus, in 10 hours you will make \$84.60.

Use the model to determine how much you would make in 3 hours.

Activity 2: For your family reunion, you are in charge of planning a day at the lake. You want to choose a kayak rental shop near the lake, but you're not sure how many kayaks you'll need or how long you'll need them. You call around to get some prices and find two shops (Paddle Heaven and Floating Oasis) close to the lake. Both shops will charge you for the exact amount of time you have the kayaks, using a constant rate. Below are the rates each shop quoted you:

Paddle Heaven		Floating Oasis
\$92.50 every 5 hours		\$38 every 2 hours

Draw a model for each kayak shop's rate. Use the model to help you fill out the following tables. Assume that the hours and dollars keep a constant ratio.

Paddle Heaven:

Hours	Dollars
5	92.50
10	
20	
100	

Floating Oasis:

Hours	Dollars
2	38
1	
10	
50	

- Which company is cheaper? Justify your answer.
- You decide to use Floating Oasis because it is closer to your family's campground. How much would it cost if you rented one kayak for 33 hours?
- Challenge: If you only had \$1, approximately how long would you be able to kayak from either shop?
- Challenge: If you budgeted \$150 for kayaks, approximately how long would you be able to kayak from either shop?

Activity 3: A hiker backpacking through the mountains walks at a constant rate. Below is a rate table with how many miles she walked and the hours it took. From the table: a) write as many equivalent ratios as you can and b) write the rate of miles per hour in simplest form.

Miles	Hours
24	8
6	2
1	$\frac{1}{3}$
3	1

What patterns do you see in the table? Justify.

For each table below one ratio is given; fill in the missing values. Use a model to confirm your method.

4.

Servings	Ounces
12	21
4	
16	
8	

5.

Children	Cars
6	2
	1
	5
30	

6.

Soldiers	Battalions
1500	3
	1
	4
	2

Spiral Review

- Order the following rational numbers from least to greatest. $\frac{12}{3}$, 4.5, $\frac{14}{3}$, .94,
- What is the opposite of 7?
- What is 35% of 120? Use a bar model.
- Without using a calculator, determine which fraction is bigger in each pair. Justify your answer with a picture *and* words.
 - $\frac{5}{7}$ or $\frac{3}{5}$
 - $\frac{6}{12}$ or $\frac{8}{14}$

4.1b Homework: Equivalent Ratios and Rate Tables

For each table below one ratio is given; fill in the missing values. Draw a model to confirm your calculation.

1.

Rose bushes	Square feet of garden
60	40
30	
	100
	1

2.

Scoops of corn	Number of deer fed
1.5	1
	5
	10
	30

3.

Hours	Distance travelled (km)
$\frac{1}{2}$	15
1	
$\frac{1}{3}$	
	100

4.

Hours worked	Lawns mowed
6	8
	1
	5
	7

5.

Minutes	Gallons in the pool
90	54
10	
	3
	1

6.

Acres	Houses
$\frac{1}{4}$	1
1	
	100
100	

4.1c Class Activity: Model and Understand Unit Rates

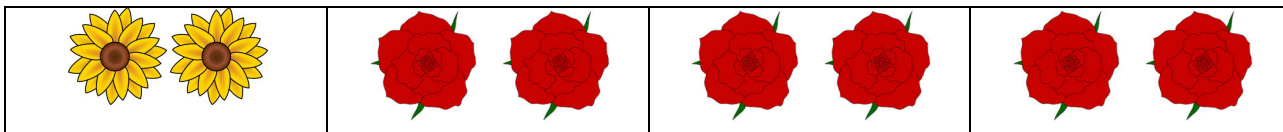
Activity 1: Fill in the missing information for each. Then compare the rate tables:

Miles	Hours
60	48
	7

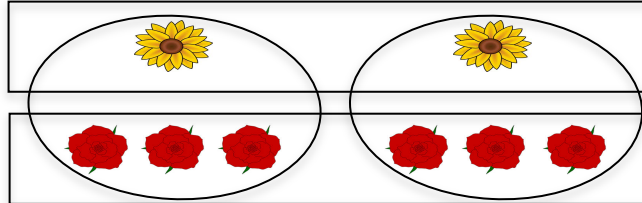
Miles	Hours
$1\frac{1}{4}$	1
	7

What do you notice?

Recall: Example 1 in 4.1a, the ratio of sunflowers to roses.

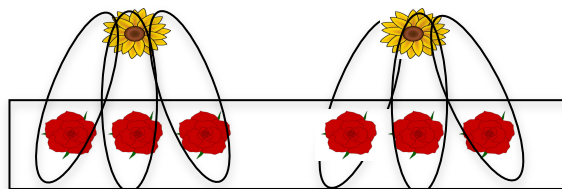


In that example, we found that the reduced ratio of sunflowers to roses is 1 sunflower to 3 roses. Another way to model that reduced ratio is:



What if we wanted to know the ratio of sunflowers to 1 rose?

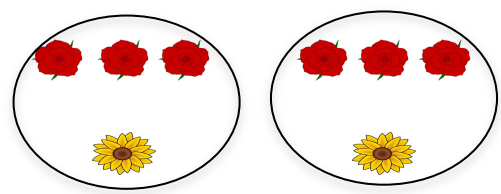
$$\frac{2 \text{ Sunflowers}}{6 \text{ Roses}} = \frac{1 \text{ Sunflower}}{3 \text{ Roses}} = \frac{\frac{1}{3} \text{ Sunflower}}{1 \text{ Rose}}$$



Thus, the unit rate (the ratio of the first quantity per one of the second quantity) of sunflowers to roses is $\frac{1}{3}$ of a sunflower per 1 rose.

We can also find the unit rate for roses to sunflowers.

$$\frac{6 \text{ Roses}}{2 \text{ Sunflowers}} = \frac{3 \text{ Roses}}{1 \text{ Sunflowers}}$$



Here, the unit rate is 3 roses per 1 sunflower.

Activity 2: John walks at a constant rate of 8 miles every 3 hours. Answer the following:

- 1. What is the **rate** written as miles per hours? _____miles : _____hours
- 2. Use a bar model to find the *unit rate* of miles per hour. _____miles : 1 hour
- 3. How far can John walk in 5 hours?
- 4. Fill in the following table

Miles (mi)										
Hours (h)	0	1	2	3	4	5	6	10	20	h

5. Where on the table can you find the unit rate for miles per hour?
6. Use a model to find how long would it take John to walk 1 mile? Write this as a unit rate:

7. Fill in the following table:

Hours (h)													
Miles (m)	0	1	2	3	4	5	6	7	8	9	10	20	m

8. Where on the table can you find the unit rate for hours per mile?
9. Complete each sentence:

The number of miles is equal to _____ the number of hours.

The number of hours is equal to _____

What is the relationship between miles per hour and hour per mile? How might this help you?

Practice: for #1 – 4, use models to write two possible **unit rates** for each given situation. Be sure to include units of measure in your unit rate.

1. The sandwich shop uses 24 slices of meat for every six sandwiches.
2. It took $\frac{3}{4}$ of a cake to feed $\frac{1}{3}$ of the class.
3. Each $\frac{1}{2}$ hour the temperature of the water raises $\frac{1}{5}$ degree.
4. The ant crawls $\frac{2}{3}$ meters every $\frac{1}{4}$ minute.

Spiral Review

1. Mark has \$16 more than Becca. Represent how much money Mark has.
2. What property is shown?
 $8 + 7 + 2$ and $8 + 2 + 7$ _____
 $18 + 0$ and $0 + 18$ _____
3. Add $-\frac{3}{5} + \frac{2}{3} =$
4. Add $-4 + -7 =$

4.1c Homework: Model and Understand Unit Rates

Use models to help you write the two possible unit rates for each given situation. Be sure to label the units in the unit rate.

1. Tony Parker scored 6 points in every quarter of a game.



2. A car traveled 18 miles on $\frac{3}{4}$ of a gallon of gas.



3. A computer company can inspect 12 laptops in 8 hours.



4. Samantha can type 14 sentences in $3\frac{1}{2}$ minutes.



5. John can mow $\frac{2}{5}$ of a lawn in $\frac{2}{3}$ of an hour.

4.1d Classwork: More with Unit Rates

Find each unit rate for the situations. Be prepared to justify your answer with a model.

Activity 1: Mauricio sometimes swims laps at his local recreation center for exercise. He wants to check whether he is swimming laps faster over time. When he first starts swimming, he can swim a 100 meter lap in 90 seconds. Fill in his unit rate for laps per second (which could be a fraction) and his unit rate for seconds per lap (which could be a mixed number).

Number of laps	1 lap	8 laps	5 laps	10 laps
Time spent swimming	90 seconds	600 seconds	390 seconds	720 seconds
Laps per second				
Seconds per lap				

Which set of laps had the fastest pace? Explain your answer.

- You can buy 8 apples for \$2.00.
 - Find the unit rate for 1 apple.
 - Find the unit rate for \$1.
- The flight from Oakland to Salt Lake City was 720 miles and took 180 minutes.
 - Find the unit rate for 1 minute.
 - Find the unit rate for 1 mile.
- At the store, you can get 15 cans of peaches for \$3.
 - Find the unit rate for \$1.
 - Find the unit rate for 1 can.
- Traveling between countries means exchanging currencies (money). Suppose the exchange rate is 3 dollars to 2 Euros.
 - How many Euros would you receive in exchange for 1 dollar (the unit rate for \$1)?
 - How many dollars would a person receive in exchange for 1 Euro (the unit rate for 1 Euro)?
- Emina can swim $\frac{1}{2}$ mile in $\frac{1}{4}$ hour.
 - What is her average rate per hour?
 - What is her average time per mile?
 -

6. In $\frac{1}{10}$ of an hour, Jane can clean $\frac{5}{8}$ of a window.
- What is her average rate per hour?
 - What is her average time per window?
7. Henry eats $2\frac{3}{4}$ slices of pizza in 20 minutes.
- What is his average time per slice?
 - What is his average number of slices per minute?
8. In 9 minutes, Guillermo reads $7\frac{1}{2}$ pages of the novel.
- What is Guillermo's average reading rate in pages per minute?
 - What is Guillermo's average reading rate in minutes per page?

Write unit rates in two ways. Pick the most useful unit rate for each situation (choose out of the two possibilities) and explain. For example choose between “pounds in one bag of flour” or “bags of flour in one pound.”

	Unit rate a	Unit rate b	Pick the more useful unit rate, if you think one is more useful than the other. Explain your choice.
9. The bakery has 8 bags of flour weighing a total of 40 pounds.			
10. In four tennis ball cans, you got 12 tennis balls.			
11. Your dad bought 4 gallons of gas for \$13.			

12. In $\frac{1}{2}$ hour, you can walk 2 miles.			
13. Marco went 300 miles in 5 hours on his road trip.			

Spiral Review

1. You can buy a 12 cans for \$2.40.
Find the unit rate for 1 can.
Find the unit rate for \$1.
2. Use long division to show how you can convert this fraction to a decimal and then a percent
 $\frac{2}{7}$
3. Solve: $\frac{3}{4} - \frac{7}{8} =$
4. Simplify: $x + 3x - 7x + 2x^2$

4.1d Homework: More with Unit Rates

Find each unit rate. (Don't forget to label)

1. You can buy 5 oranges for \$2.00.
 - a. What is the unit rate for 1 orange?
 - b. What is the unit rate for \$1?

2. For three people, there are 5 candy bars.
 - a. What is the unit rate for the number of candy bars for 1 person?
 - b. What is the unit rate for the number of people for 1 candy bar?

3. In $12\frac{1}{2}$ minutes, Dulce read 50 pages.
 - a. On average, how many pages did she read per minute?
 - b. On average, how many minutes does it take to read one page?

4. Yazmin's heart rate was measured at 19 beats in a $\frac{1}{4}$ minute.
 - a. How many beats per minute?
 - b. How many minutes per beat?

5. Traveling between countries means exchanging currencies (money). Look up the current exchange rate for Europe (or replace Euros with the currency of a country of your choice).
 - a. How many Euros would you receive for 1 dollar? How many Euros will you receive for \$10?
 - b. How many dollars would a European receive for 1 Euro? How many dollars would they receive for 10 Euros?

Write unit rates in two ways. Pick the most useful unit rate (you choose) and explain.

	Unit rate a	Unit rate b	Pick the more useful unit rate, if you think one is more useful than the other. Explain your choice.
6. In 8 hours, Mayra can mow 5 lawns.			
7. George can eat 30 hotdogs in 2 hours.			
8. A tree grows 15.5 feet in 10 years.			
9. Over five years, Luz has collected 25 buttons.			
10. After a $2\frac{1}{2}$ hour rainstorm, there are 5 inches of water in a bucket.			
11. Estefania can run $\frac{3}{4}$ the race in $\frac{2}{3}$ of an hour			

4.1e Extra Practice: Complex Fraction Unit Rates

1. Diego can swim $\frac{1}{2}$ of a mile in $\frac{2}{3}$ of an hour.

a) What is his rate in miles per hour?

Use a picture or model:

b) What is his rate in hours per mile?

Use a picture or model:

2. In 20 minutes, I ate $1\frac{1}{2}$ sub sandwiches. What is my rate in sandwiches per hour?

Use a picture or model:

3. Bob ate $3\frac{3}{4}$ sandwiches in 25 minutes. What is his rate in sandwiches per hour?

Use a picture or model:

Use two different methods to solve each of the following problems. Show all your work!

4. In $2\frac{1}{2}$ days Raul can paint $6\frac{1}{2}$ offices. How many offices per day is he painting? Show your work!

5. Napoleon earns $4\frac{1}{2}$ dollars in 5 hours. How much does he make per hour?

6. At the 4th of July Fun Run, I ran $3\frac{1}{2}$ miles in $\frac{3}{4}$ of an hour. On average, how fast is that?

7. In 30 days I lost $8\frac{1}{2}$ pounds. How many pounds per week is that, on average?

8. Oscar put $2\frac{3}{4}$ puzzles together in 6 nights.

a. How many puzzles per night?

b. How many nights per puzzle?

9. In 10 hours, Henry harvested $3\frac{1}{2}$ fields of corn.

a. How many fields per hour is that?

b. If a working day is 8 hours, how many fields can he harvest in a working day?

10. Francisco can read 7.5 books in 5 weeks.

a. What is his rate in books per week?

b. How can you use the rate of books per week to calculate the number of books read in $8\frac{1}{7}$ weeks?

4.1f Class Activity: Compare Unit Rates

I. Find Unit Rates Practice (round to nearest hundredth, if needed)

Ratio	Rate (Ratio as a fraction)	Unit Rate
1. 7 hours to drive 228 miles.	$\frac{228 \text{ miles}}{7 \text{ hours}}$	$\approx 32.57 \text{ miles per hour}$ or $\approx \frac{32.57 \text{ miles}}{1 \text{ hour}}$
2. 375 students for 12 teachers		
3. \$47.95 for 7 hours work		
4. 1 dozen bagels for \$5.40 (price per bagel)		
5. \$19.87 for 5 pounds of turkey. (price per pound)		
6. 5 ounces of chicken for \$2.59 (price per ounce)		
7. $5\frac{1}{2}$ blankets in 11 hours		
8. 2.2 Liters of soda for \$2.19 (price per liter)		

II. Compare Unit Rates: Justify your comparisons.

9. Jean sells M&Ms to friends at 4 for 5 cents. The machine at the store sells 9 for 25 cents. Which is the better deal?	10. Frosted Flakes has 11 grams sugar per ounce and Raisin Bran 13 grams per 1.4 ounces. Which cereal has more sugar per ounce?	11. Tom sells baseball cards at 10 for 35 cents. Is that a better deal than 12 for 40 cents?
12. The hardware store sells 4 th of July sparklers for 19 cents each. The fireworks stand charges 85 cents for four. Where would you purchase sparklers?	13. Ribbon: 5 m for \$6.45 or 240 cm for \$3.19. Which ribbon is cheaper per meter?	14. An 11 ounce can of condensed soup (add 1 can of water), costs \$1.45. A 20 ounce can of ready to serve soup costs \$1.29. Which can of soup has the best price per ounce?

Spiral Review

1. Luis went to a soccer game with some friends. He bought two sodas for \$1.50 each and four giant candy bars for \$2.25 each. How much did he spend?
2. Juliana bought 3 bags of chips and 3 sodas for herself and two friends. The chips were \$0.85 a bag; if she spent a total of \$6, how much was each can of soda?
3. Simplify the following expression. Use a model if needed.

$$71b - 4a + 4b - 4a$$

4. Write a context for the following expression: $4(x - 3)$

4.1f Homework: Compare Unit Rates

I. Find unit rates practice (round if needed)

Ratio	Rate (Ratio as a fraction)	Unit Rate
1. 63 sit-ups in $3\frac{1}{2}$ minutes		
2. 8 minutes to read 500 words		
3. 216 miles on 17 gallons of gas		
4. 36 commercials in 2 hours of TV		
5. 64 ounces of orange juice for \$2.79		
6. \$28.45 for 5 pounds of roast beef		
7. 16 ounces of spaghetti for \$0.50		
8. 24 cans of cat food for \$7.60		

II. Compare Unit Rates: Justify your comparisons. (Example: Which is the better buy? Why?)

9. Applesauce: 24 oz for \$1.25 or 16 oz for \$0.89	10. Grape juice: $\frac{1}{2}$ gal for \$2.80 or 20 oz. for \$0.75. (Note: there are 128 fluid oz. in a gallon).	11. Olive oil: 3 L for \$16.99 or 500 ml for \$3.09
12. Ground beef: 4.3 kg for \$21.70 or 1.6 kg for \$7.85	13. Greeting cards: 3 boxes of 12 for \$30 or 4 boxes of 15 for \$49	14. Stan typed 90 pages in 7.5 hours. Jan typed 110 pages in 9.4 hours.

4.1g Classroom Activity: Using Unit Rates

Use the tables to identify the proportional constants (unit rates). Then use the unit rate to answer additional information. Models may be useful.

1.

Cups of flour	2	4	6	8	10
Number of Cookies	12	24	36	48	60

What is the unit rate of cups of flour per cookie?

How many cups of flour would be used for 20 cookies?

What is the unit rate of cookies per cups of flour?

How many cookies could be made with 5 cups of flour?

How many cookies could be made with 0 cups of flour?

2.

Inches of snowfall	1	2	3	4	5
Hours	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$

What is the unit rate of inches per hour?

At this rate, how many inches of snow would have fallen in $2\frac{1}{3}$ hours?

What is the unit rate of hours per inch?

At this rate, if there is $7\frac{1}{2}$ inches of snow, how long has it been snowing?

3.

Dollars	3	4.50	6	7.50
Pesos	1	1.5	2	2.5

What is the unit rate of pesos per dollars?

At this rate, how many pesos could you get for 8 dollars?

What is the unit rate of dollars per pesos?

At this rate, how many dollars could you get for 18.60 pesos?

If you have 0 pesos, how many dollars is that worth?

4.

Homeruns	1	2	3	4	5
Swings	7	14	21	28	35

What is the unit rate of homeruns per swing?

At this rate, if the batter swings 30 times, estimate the number of homeruns he would have gotten.

What is the unit rate of swings per homerun?

At this rate, if the batter has gotten 9 home runs, how many times did he swing?

5.

Hours	0	1	2	3	4
Plumbing Cost	\$45	\$70	\$95	\$120	\$145

What is different about this table?

Is there a unit rate? Justify your answer.

Do you think this data is continually proportional? Why or why not?

4.1h Unit Rate Projects:

I. Your Reading Rates

What is your reading rate? How long will it take you to read a book (you pick the book)? Use your group plan and be prepared to show your steps and your work.

1. Each member of your group will pick a different book.
2. Brainstorm with your group to come up with strategies for estimating how long it will take you to read your specific book.
3. Create a group plan (steps to follow at home). Be certain to include the following:
 - a. What unit rates will you need to use to help you make this estimate?
 - b. How will time be measured?
 - c. What steps will be recorded?
 - d. What work will need to be shown?
4. Make certain each member of the group has a copy of the plan.
5. Tomorrow, each member of the group will explain his/her steps and reading rate.

II. Grocery Store Comparisons

For this project, you will collect data about three products in your local grocery store. If you are unable to go to the store, use the ads that come in the newspaper or mail or check online for your local grocery store ad. Ads used must be attached to this worksheet. Choose from the following two kinds of comparisons.

1. Choose a product that comes in two sizes. Record the price and measurement (amount) of each size. Compute and compare the unit price for each product. Then tell which product is the best buy.
2. Choose two different brands of a product that comes in the same size. Record the price and measurement (amount) of each. Compute and compare the unit price of each product. Then tell which product is the best buy, explaining your reasoning.
3. Pick something you'd like to buy for yourself. Compare brands or sizes. Compute and compare the unit price for each product. Then tell which product you would choose and why.

4.1i Unit Rate Review

Answer the questions below using unit rates. Models may be helpful in answer each. Justify your answer in words, models and/or numeric expressions.

1. Arturo can run $1\frac{3}{4}$ miles in $\frac{1}{4}$ hour. What is his speed in miles per hour?
2. In 2.5 hours, Daniel's brother can ride 40 miles on his bike. How fast is he going in miles per hour?
3. There were 7 inches of rain in 24 hours. What is the average amount of rain per hour?
4. The table below shows the number of parts produced by a worker on an assembly line.

# of Parts	24	36	42	54
Hours	4	6	7	9

What is the unit rate between parts and hours?

5. Allyson measures her plant every year. It's now 16 years old and it measures 12 inches. What is the unit rate for the plant growth per year?
6. The times for three runners in three different races are given in the table.

Runner	Distance	Time
A	5 kilometers	34 minutes
B	10 kilometers	62 minutes
C	21 kilometers	155 minutes

Which runner finished the race at the fastest average rate?

7. CD Express offers 4 CDs for \$60. Music Place charges \$75 for 6 CDs. Which store offers the better buy?
8. Tia is painting her house. She paints $34\frac{1}{2}$ square feet in $\frac{3}{4}$ hour. At this rate, how many square feet can she paint each hour?
9. In $1\frac{1}{2}$ hours, Sebastian can walk $4\frac{1}{2}$ miles. Find his average speed in miles per hour.

10. Mrs. Rossi needs to buy dish soap. There are four different sized containers.

Dish Soap Prices	
Brand	Price
Lots of Suds	\$0.98 for 8 ounces
Bright Wash	\$1.29 for 12 ounces
Spotless Soap	\$3.14 for 30 ounces
Lemon Bright	\$3.45 for 32 ounces

Which brand costs the least per ounce?

Which brand gives you the most ounces for one dollar?

11. Paola is making pillows for her Life Skills class. She bought $2\frac{1}{2}$ yards of green fabric and $1\frac{1}{4}$ yards of purple fabric. Her total cost was \$15 for the green fabric and \$8.50 for the purple fabric. Which color fabric is cheaper? Justify your answer.

12. The Jimenez family took a four-day road trip. They traveled 300 miles in 5 hours on Sunday, 200 miles in 3 hours on Monday, 150 miles in 2.5 hours on Tuesday, and 250 miles in 6 hours on Wednesday. On which day did they average the greatest speed (miles per hour)?

4.1j Self-Assessment: Section 4.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Compute unit rate from a context. [1]	I struggle to understand how to compute a unit rate from a context.	I can compute a unit rate from a context that uses whole numbers.	I can compute a unit rate from a context that uses whole numbers and decimals.	I can compute a unit rate from a context that uses rational numbers in any form.
2. Compute a unit rate from a table of values. [2]	I struggle to understand how to compute a unit rate from a table of values.	I can compute a unit rate from a table of values that uses whole numbers and/or fractions as long as the values aren't out of order.	I can compute a unit rate from a table of values that uses whole numbers and decimals regardless of the order of the values in the table.	I can compute a unit rate from a table of values that uses rational numbers in any form and values are in any order. Further, I can explain what the rate means for the context.
3. Compare two rates to determine equivalence or to contrast differences. [3]	I can find rates, but I struggle to know how they compare.	I can find rates to determine equivalence or to determine which is greater.	I can change rates to a form that allows me to compare the two to determine equivalence or to determine which is greater. I can also use the different rates to make predications.	I can change rates to a form that allows me to compare the two to determine equivalence or to contrast differences. I can express in my own words similarities and differences between the two and use the information to draw conclusions.
4. Find the unit rate for BOTH units (i.e. miles per hour and hours per mile). [4]	I struggle to understand how to compute a unit rate.	I can find the unit rate for one unit but often not the other.	I can always find a unit rate for BOTH units.	I can find a unit rate for BOTH units. I can determine which unit rate would be more helpful in a given situation.
5. Use unit rate to find a missing quantity. [5]	I struggle to know how to use unit rate to find a missing quantity.	I can use unit rate to find a missing quantity involving whole numbers.	I can use unit rate to find a missing quantity involving whole numbers and decimals.	I can use unit rate to find a missing quantity involving whole numbers, decimals, and fractions and I can explain the process I used.

Sample Problems for Section 4.1

1. Given the following situations, find the indicated unit rate for each:

- a. Corrin has twin boys. She buys a box of 10 toy cars to share evenly between the boys.

_____ cars per boy

- b. Belle works at a donut shop. They sell a box of donut holes for \$1.80. There are 20 donut holes in the box.

_____ cost per donut hole

- c. Abby runs $14\frac{1}{4}$ laps in $42\frac{3}{4}$ minutes.

_____ minutes per lap

2. Given the following tables, find the indicated unit rate for each:

- a. _____ push-ups per day

Days	Total Push-ups
2	30
4	60
29	435
58	870

- b. _____ gallons per minute

Minutes	Gallons of Water in Swimming Pool
2	12.4
3	18.6
17	105.4
27	167.4

- c. _____ hours per kilometer

Kilometers	Hours
5	$\frac{1}{3}$
15	1
7.5	$\frac{1}{2}$
$\frac{1}{4}$	$\frac{1}{60}$

3. The following situations each describe two rates. Compare and contrast the two rates. State equivalence or contrast the differences in context of the situation.
- In a two minute typing test, Stacey types 100 words. In a five minute typing test, Sarah types 250 words.
 - A 20 oz. bottle of ketchup is 99¢. A 38 oz. bottle of ketchup is \$1.99.
4. For each of the following situations, find the unit rate for BOTH units.
- Ivonne drove 357 miles on 10 gallons of gasoline.
 - In the World Cup, Chile scored 8 goals in 4 games.
5. For each of the following situations, use the unit rate to answer the question.
- Katherine is visiting patients in a hospital. She visits 12 patients in 6 hours. At the same rate, how many patients will she visit in 9 hours?
 - If 4 gallons of gas cost \$14.60, how much does 10 gallons of gas cost?
 - Joaquin runs $2\frac{1}{2}$ kilometers in $\frac{1}{6}$ hours. If he continues running at the same pace, how long will it take him to run 11 kilometers?

Section 4.2: Identify and Communicate Unit Rates in Tables and Graphs.

Section Overview:

In this section, students will connect the concept of proportionality to unit rates, tables, and graphs. Students will first use unit rates to check for proportionality. They will then move into graphs and tables to determine whether paired quantities are proportional. They will learn that proportional quantities always make a straight line emanating from the origin when graphed and that tables showing proportional quantities will give a constant unit rate upon division of every pair of quantities. They will begin to write equations to find missing quantities, using the unit rate as the constant of proportionality, but equations will be further developed in section 4.3. They will also continue to explore the relationship between part:part and part:whole relationships (4.2b) with ideas solidifying in section 4.4.

Key Ratio and Proportion Concepts from Utah Core Standards

RP Standard 1:

1. Extend the concept of a unit rate to include ratios of fractions.
2. Compute a unit rate, involving quantities measured in like or different units.

RP Standard 2:

3. Determine if two quantities expressed in a table or in a graph are in a proportional relationship.
4. Determine a unit rate from a table, graph, equation, diagram or verbal description and relate it to the constant of proportionality.
5. Write an equation for a proportional relationship in the form $y = kx$.
6. Explain the meaning of the point (x, y) in the context of a proportional relationship.
7. Explain the significance of $(0,0)$ and $(1, r)$ in a graph of a proportional relationship, where r is the unit rate.

RP Standard 3:

8. Solve multistep problems involving percent using proportional reasoning
9. Find the percent of a number and extend the concept to solving real life percent applications.
10. Calculate percent, percent increase, decrease, and error.

Primary Concepts and Skills to be Mastered in This Section

1. Determine if two rates written as ratios are proportional.
2. Graph a relationship from a table of values.
3. Determine if a relationship is proportional from a graph.
4. Find a unit rate from a table.
5. Find a unit rate from a graph.
6. Explain the significance of $(1, r)$ in a proportional relationship.
7. Compare rates from tables.

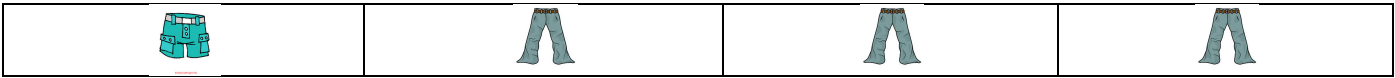
4.2a Classroom Activity: Determine Proportionality (Review)

Ratios are considered proportional when the ratios are equivalent. You can use bar models to help you determine proportionality.

Example: In class A the ratio of students wearing shorts to students wearing pants is 1 to 3. In class B, the ratio is proportional

Draw a model of each situation and use bar models to find the simplified ratios.

Class A:



Class B:



Notice how each ratio could be depicted using the same number of boxes for shorts and for pants for each class with the same number of items in each box. Since the two ratios are equivalent, we can say they are proportional.

- 1. Draw bar models to determine if the two ratios are proportional.
 - a. In one box of cookies, the ratio of chocolate chip cookies to oatmeal cookies is 3 to 8. A bigger box of cookies has a ratio of 12 to 32 chocolate chip cookies to oatmeal cookies.
 - b. The ratio of cats to total pets in the first pet store is 3 to 9. In the second pet store the ratio of cats to total pets is 6 to 18.

The information in both 1a and 1b has been transfer to the tables below. How do the tables help you determine if the ratios are proportional?

Chocolate chip cookies	Oatmeal Cookies	Cats	Total Pets	
3	8	3	9	
12	32	6	18	

2. The ratio of pink flowers to purple flowers in a vase is 6:8. Make a table to determine if 8 pink flowers to 10 purple flowers is proportional.

Now reduce the fractions in #1 and #2. What do you observe?

From #1a: $\frac{3}{8}$ $\frac{12}{32}$

From #1b: $\frac{3}{9}$ $\frac{6}{18}$

From #2: $\frac{6}{8}$ $\frac{8}{10}$

3. Identify which of the four tables below is **not** proportional. For each table that is proportional, find the proportional constant (unit rate.)

proportional constant (unit rate.)																																																					
a.	<table><tr><th>x</th><th>y</th></tr><tr><td>3</td><td>6</td></tr><tr><td>5</td><td>10</td></tr><tr><td>10</td><td>20</td></tr><tr><td>12</td><td>24</td></tr><tr><td>15</td><td>30</td></tr></table>	x	y	3	6	5	10	10	20	12	24	15	30	b.	<table><tr><th>x</th><th>y</th></tr><tr><td>2</td><td>1</td></tr><tr><td>4</td><td>2</td></tr><tr><td>6</td><td>3</td></tr><tr><td>8</td><td>4</td></tr><tr><td>10</td><td>5</td></tr></table>	x	y	2	1	4	2	6	3	8	4	10	5	c.	<table><tr><th>x</th><th>y</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>6</td></tr><tr><td>3</td><td>10</td></tr><tr><td>4</td><td>14</td></tr></table>	x	y	1	2	2	6	3	10	4	14	d.	<table><tr><th>x</th><th>y</th></tr><tr><td>-2</td><td>-6</td></tr><tr><td>-0.5</td><td>-1.5</td></tr><tr><td>2</td><td>6</td></tr><tr><td>3</td><td>9</td></tr><tr><td>4</td><td>12</td></tr></table>	x	y	-2	-6	-0.5	-1.5	2	6	3	9	4	12
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Justify why or why not.		Justify why or why not.		Justify why or why not.		Justify why or why not.																																															
If the relationship above is proportional, you can write an equation that relates y to x : $y = \underline{\hspace{2cm}}$. If NOT proportional write "NOT" and then explain why not.																																																					

4. Determine if each set of ratios below is proportional by reducing them to simplest form.

Note: ratios can be expressed in multiple ways (part:part, part:whole, etc.).

a. $\frac{7}{9}$ $\frac{21}{27}$	d. $\frac{10}{12}$ $\frac{15}{30}$	g. 1:2 3:13
b. $\frac{2}{3}$ $\frac{3}{8}$	e. $\frac{9}{12}$ $\frac{49}{50}$	h. 7:3 28:12
c. $\frac{15}{35}$ $\frac{24}{52}$	f. $\frac{4}{16}$ $\frac{2}{8}$	i. 6:9 8:4

Challenge: Can you think of other ways to determine proportionality?

Spiral Review

1. Use a model to solve:

a. $-14 = 3x - 2$

b. $-8 = -3m + 10$

3. Evaluate $-2(x + 1) - x$ for $x = -3$

4. Write $\frac{17}{20}$ as a percent and decimal (without a calculator.)

5. Malory got an 75% on her math test. To earn that score, she got 27 questions correct. How many questions were on the test?

4.2a Homework: Determine Proportionality (Review)

1. Are the **relationships in the tables** proportional? If the relationship is proportional, state the proportional constant. If not proportional, explain why not.

a.	<table><tr><td>x</td><td>y</td></tr><tr><td>1</td><td>3</td></tr><tr><td>2</td><td>6</td></tr><tr><td>3</td><td>9</td></tr><tr><td>4</td><td>12</td></tr><tr><td>5</td><td>15</td></tr></table>	x	y	1	3	2	6	3	9	4	12	5	15	b.	<table><tr><td>x</td><td>y</td></tr><tr><td>-4</td><td>-10</td></tr><tr><td>-2</td><td>-4</td></tr><tr><td>0</td><td>0</td></tr><tr><td>2</td><td>4</td></tr><tr><td>4</td><td>8</td></tr></table>	x	y	-4	-10	-2	-4	0	0	2	4	4	8	c.	<table><tr><td>x</td><td>y</td></tr><tr><td>3</td><td>1</td></tr><tr><td>6</td><td>2</td></tr><tr><td>9</td><td>3</td></tr><tr><td>12</td><td>4</td></tr><tr><td>15</td><td>5</td></tr></table>	x	y	3	1	6	2	9	3	12	4	15	5	d.	<table><tr><td>x</td><td>y</td></tr><tr><td>-1</td><td>-1.5</td></tr><tr><td>1</td><td>1.5</td></tr><tr><td>3</td><td>4.5</td></tr><tr><td>5</td><td>7.5</td></tr><tr><td>7</td><td>10.5</td></tr></table>	x	y	-1	-1.5	1	1.5	3	4.5	5	7.5	7	10.5
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If proportional, write an equation: $y = \underline{\hspace{2cm}}$. If NOT proportion write "NOT" and then explain why not.																																																							

2. The ratio of pink flowers to purple flowers prepared for a wedding will be 2 to 3.

- a. Complete the table for varying amounts of pink and purple flowers.

Pink Flowers	2	8		20	
Purple Flowers			15		45

- b. Explain how you can see the proportional relationship in the table.

4.2b Classroom Activity: Mixing Lemonade—Expressing Proportionality

Review: Recall the sunflower and rose example from 4.1a. There the ratio of sunflowers to roses was 1:3. We also noted that 25% of the flowers in the example were sunflowers because 1 in 4 of the parts were sunflowers.

You want to sell lemonade by the park. Different brands of lemonade have different formulas. You want to sell the lemonade that tastes the most “lemony.”

- Use a model to show the portion of mixture that is lemon concentrate and water.
- What fraction of the mix is concentrate? What fraction is water?
- What percent of the mix is concentrate? What percent is water?
- Create a unit rate $\frac{\text{water}}{\text{concentrate}}$ for each mixture.
- Answer the question.

<p>1. 2 cups concentrate, 3 cups water</p> <p>a.</p> <table><tr><td>1 c con</td><td>1 c con</td><td>1 c water</td><td>1 c water</td><td>1 c water</td></tr></table> <p>or</p> <table><tr><td>1 c conc</td><td>1 c conc</td></tr><tr><td>1 c water</td><td>1 c water</td><td>1 c water</td></tr></table> <p>b. $w = 3/5, c=2/5$</p> <p>c. $w = 60\%, c = 40\%$</p> <p>d. $3/2$ cups of water to 1 cup concentrate</p> <p>e. If you have 12 cups of concentrate, how much water will you need? 18 cups of water because the ratio of concentrate to water is 1 to $3/2$</p>	1 c con	1 c con	1 c water	1 c water	1 c water	1 c conc	1 c conc	1 c water	1 c water	1 c water	<p>2. 4 cups water mixes with 1 cup concentrate</p> <p>a.</p> <p>b.</p> <p>c.</p> <p>d.</p> <p>f. If you have 20 cups of concentrate, how much water will you need?</p> <p>g.</p>
1 c con	1 c con	1 c water	1 c water	1 c water							
1 c conc	1 c conc										
1 c water	1 c water	1 c water									
<p>3. 4 cups concentrate, 6 cups water</p> <p>a.</p> <p>b.</p> <p>c.</p> <p>d.</p> <p>e. How much concentrate do you need if you're going to use 18 cups of water?</p>	<p>4. 3 cups concentrate, 5 cups water</p> <p>a.</p> <p>b.</p> <p>c.</p> <p>d.</p> <p>e. How much concentrate do you need if you're going to use 18 cups of water?</p>										

<p>5. $\frac{1}{2}$ cup concentrate, 2 cups water</p> <p>a.</p> <p>b.</p> <p>c.</p> <p>d.</p> <p>e. How much concentrate do you need if you're going to use 10 water?</p>	<p>6. 8 cups water for 4 cups of concentrate</p> <p>a.</p> <p>b.</p> <p>c.</p> <p>d.</p> <p>f. How much water do you need if you're going to use 16 water?</p>
<p>7. 1 cup water for every $\frac{2}{3}$ cup concentrate</p> <p>a.</p> <p>b.</p> <p>c.</p> <p>d.</p> <p>e. How much water do you need if you're going to use 9 cups of concentrate?</p>	<p>8. $\frac{1}{2}$ cup concentrate, 1 cup water</p> <p>a.</p> <p>b.</p> <p>c.</p> <p>d.</p> <p>e. How much water do you need if you're going to use 7 cups of concentrate?</p>

9. Are any of the mixes (#1-8) proportional to others in the set? In other words, do any sets have the same unit rates? Explain how you know.

10. For the mixes that are proportional, how does the unit rate relate to the fraction or percent?

11. Which mix will make the strongest lemonade (most lemon flavor)? The weakest? Explain how you know.

12. Order the lemonade from weakest to strongest lemonade flavor. Explain your order.

Spiral Review

1. Order the following fractions from least to greatest

$$-\frac{9}{10}, -\frac{3}{7}, -\frac{3}{4}, -\frac{6}{5}, -\frac{4}{3}$$

2. I go to a department store with a coupon for 20% off any one item. The shoes that I want are on sale for 40% off. What was the original price if I paid \$48?

3. Use the distributive property to rewrite the following expression $-4(5x - 1)$

4. Find each sum without a model.

a. $-27.2 + \frac{4}{5}$

b. $98.1 + (-1.35)$

4.2b Homework: Expressing Proportionality

Winner Lollipops

Winner Candy Company has had three different promotions for selling more lollipops. Each promotion involves randomly wrapping coupons in lollipops for free lollipops. You are the new president of the company and your job is to find the best strategy for this promotion for your company. In other words, you want to find the strategy that gives the least number of free lollipops.

a.
2 coupons per 10 lollipop
wrappers

b.
3 coupons per 9 lollipop
wrappers

c.
4 coupons per
15 lollipop wrappers

Explain the reasoning using unit rates, models, percentages, and explanations

Which would be better for the customer (more chance of a free lollipop)?

Mixing Smoothies

Use any method you want to solve the “smoothie” proportion question below.

Smoothie recipes

a.
 $\frac{1}{4}$ cup bananas (mashed)
 $\frac{1}{2}$ cup Greek Yogurt
 $\frac{2}{3}$ cup strawberries (mashed)

b.
2 cup bananas (mashed)
1.5 cup Greek yogurt
3 cup strawberries (mashed)

c.
1 cup bananas (mashed)
1 cup Greek yogurt
1 cup strawberries (mashed)

If all the smoothies are served in 12 oz. portions, which would have the most...

1. Banana
2. Strawberry
3. Greek yogurt

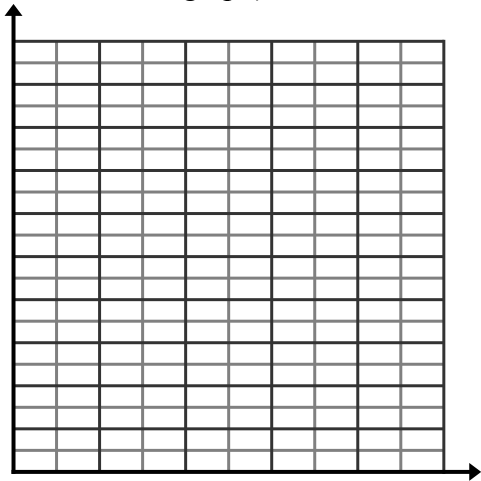
Explain your answer using unit rates, models, percentages, etc.:

4.2c Classwork: Proportions from Tables and Graphs

1. The following table shows the relationship between the amount of cherries (in pounds) and cost.

Cost (\$)	1.50	3.00	4.50	6.00	7.50
Amount (lbs.)	1	2	3	4	5

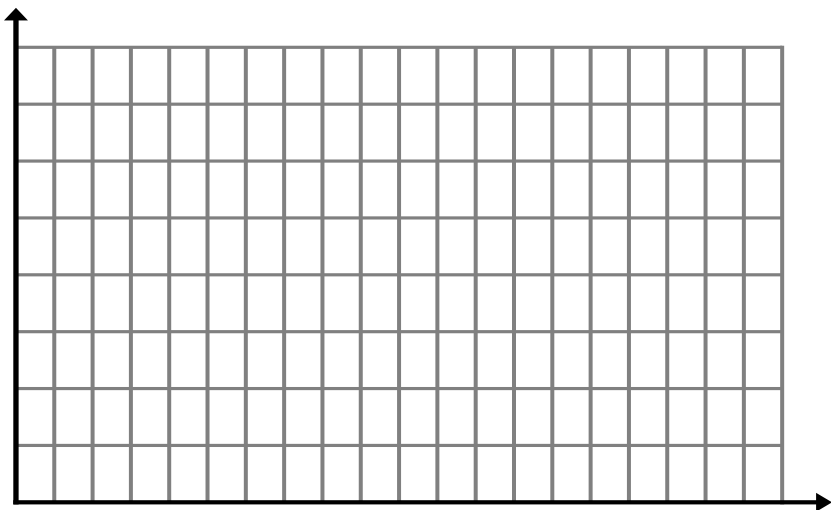
- a. Is this a proportional relationship? How do you know?
- b. Put the information from the table on the graph below, with amount of cherries on the x axis and cost on the y axis (put these labels on the graph).



2. The following table shows the relationship between the number of ounces in soup cans and cost.

Cost (\$)	1.00	1.50	2.00	2.50	3.00
Amount (oz)	8	16	24	30	36

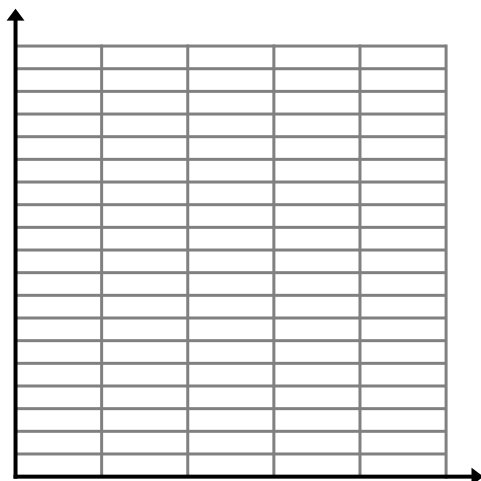
- a. Is this a proportional relationship? How do you know?
- b. Graph the information in the table, using and labeling the x -axis for amount (oz) and y -axis for the cost.



3. The following table shows the pounds of meat on a sandwich and the calories in the sandwich.

Calories	560	650	740	830	920
Meat (lbs)	0.5	1	1.5	2	2.5

- Is this a proportional relationship? How do you know?
- Graph the information from the table. Label the axes, using meat (lbs) on the x -axis.



Do you have a conjecture about how to determine if a **table** of values represents a proportional relationship?

Do you have a conjecture about how to determine if a **graph** of values represents a proportional relationship?

4. For the two scenarios below, complete the table and determine if it shows a proportional relationship. If it is proportional, state the proportional constant.

- A flower that is 3 inches tall when planted grows $\frac{1}{2}$ an inch every 4 weeks.

Weeks	0	4	8	12	16
Height (inches)	3	Type equa		$4\frac{1}{2}$	

- The sum of two numbers is 0 (use x and y to represent the two numbers).

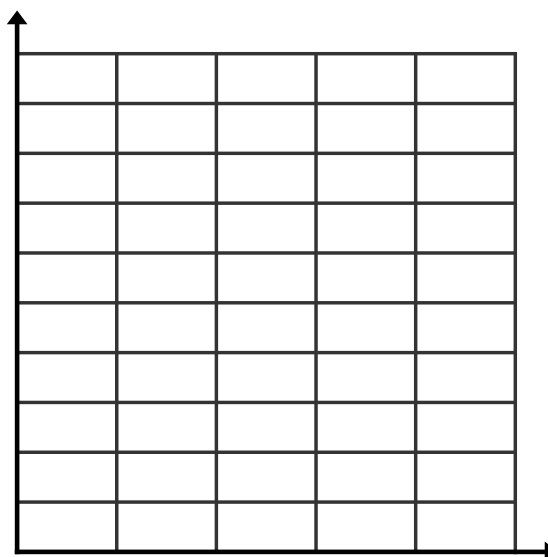
x	-2	-1	0	1	2
y					

5. Peter is renting wave runners for a company party. Each wave runner holds 3 people.

- a. Complete the table below to show the relationship between the number of wave runners rented and the number of people that can ride.

<i># of Wave Runners</i>	<i># of Riders</i>
0	
1	
	6
3	
	15
	24

- b. Graph the relationship number of wave runners to the number of riders with wave runners on the x -axis and number of riders on the y -axis.



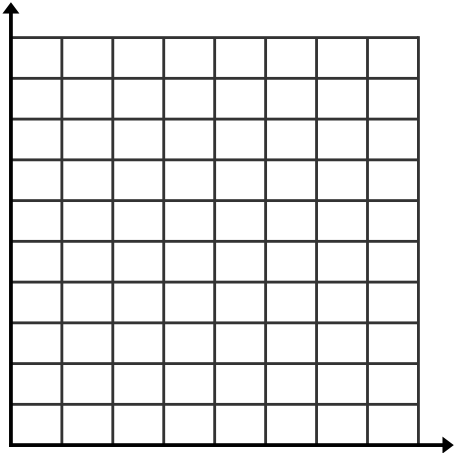
- c. Find the ratio of riders to number of wave runners for several different ordered pairs in the table.

$$\frac{\text{number of riders}}{\text{number of wave runners}} =$$

- d. Where do you see the unit rate in the table?
- e. What is the significance of the point (0, 0)?
- f. What is the relationship between the number of riders, r , and the number of wave runners, w ?
- g. Use the relationship and graph to determine how many wave runners Peter needs to rent if there will be 39 people at his company party that all want to ride a wave runner.
- h. Write an equation expressing the number of riders, r , in terms of the number of wave runners, w . Where do you see the unit rate (proportional constant) in this equation?

6. Bubba’s Body Shop charges \$25 per hour to fix a car.
Complete the table and graph to represent this situation. Be sure to label the axes of your graph.

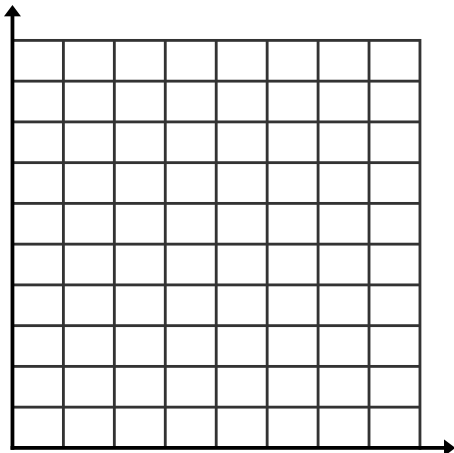
<i>t (time in hours)</i>	<i>C (cost in dollars)</i>
0	
2	
	75
	125
8	



- What is the cost for Bubba to spend 1 hour to fix your car?
- What is the significance of the number found in part a.?
- Write an equation to show the relationship between the cost C Bubba charges to fix a car for t hours?
- Are cost and time proportionally related in this situation? Justify your answer using information from your table, graph, and equation.

7. Sunset Splash and Spa charges an entrance fee of \$5.00 and then \$2.00 per hour.
- Complete the table and graph to represent this situation. Be sure to label the axes of your graph.

<i>t (time in hours)</i>	<i>C (cost in dollars)</i>
0	
1	
2	
3	
4	
5	



- Are cost and time proportionally related in this situation? Justify your answer using information from your table, graph, and equation.

Spiral Review

1. Zach invested \$1500. If he earned 23.2% on his investment, a) write an expression for the amount of money he now has and b) state how much money he now has.
2. Lara has \$1,425 in her bank account. How much money did she start with if that amount reflects a 14% increase on her original amount?
3. There are 36 red and 44 blue marbles in a bag. What is the probability of randomly drawing a red marble?
4. Express each percent as a fraction in simplest form.

44%

17.5%

4.2c Homework: Proportions (Unit Rates) from Tables and Graphs

Label the axes and graph the information from the table. Use the table to determine if the relationship represented is proportional throughout the table. If it is proportional, state the proportional constant (unit rate) as a fraction or as a decimal. If it is not proportional, write N/A (Not Applicable) for unit rate questions.

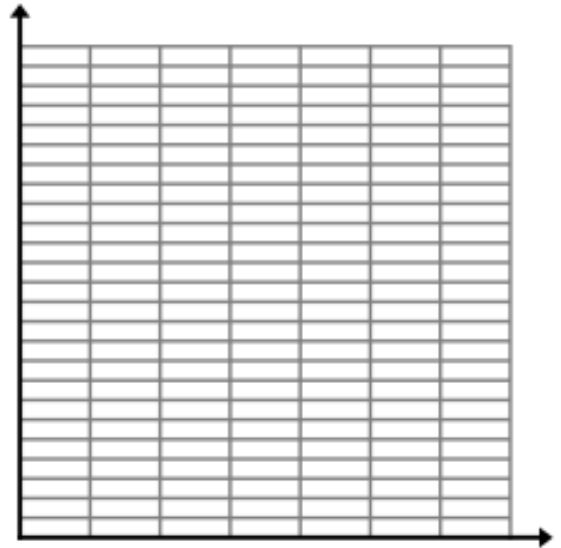
1.

Bananas (lbs)	0	1	2	3	4	5	6
Price (\$)	0	0.40	0.80	1.20	1.60	2.00	2.40

Proportional? Yes or No? Explain your answer.

If yes, what is the unit rate? (pick the most useful unit rate)

If proportional, where do you see the unit rate in the table and in the graph?



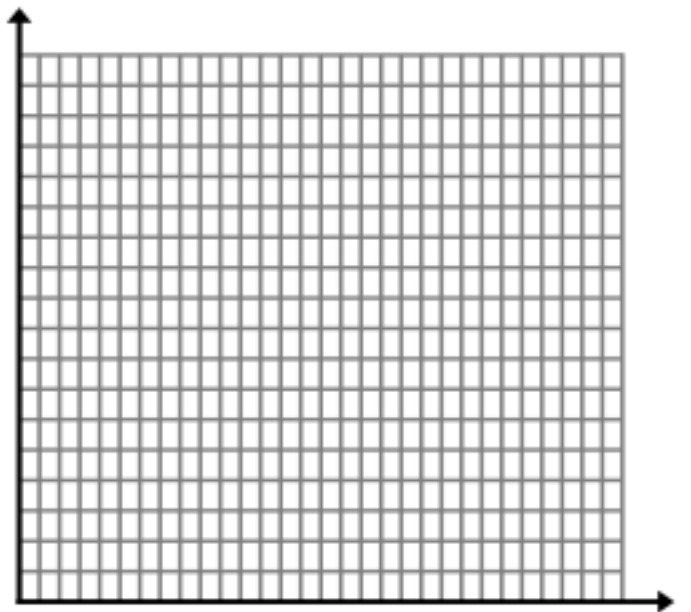
2.

# Bagels	1	2	6	13	19	26
Price (\$)	0.75	1.50	4.50	9.00	13.50	18.00

Proportional? Yes or No? Explain your answer.

If yes, what is the unit rate? (pick the most useful unit rate)

If proportional, where do you see the unit rate in the table and in the graph?



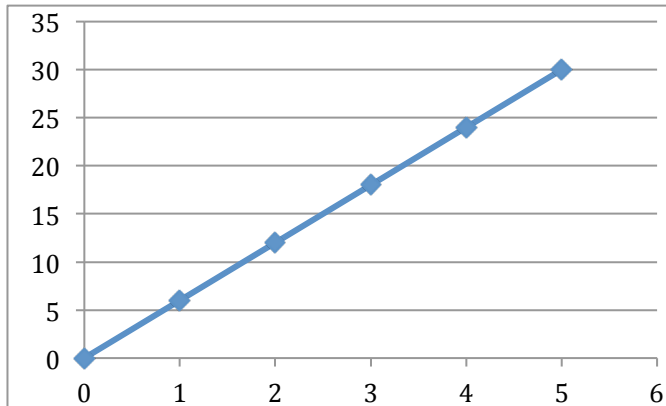
3.	Eggs	1	2	3	4	5	6
	Cups of milk	$\frac{1}{2}$	1	$\frac{13}{10}$	$\frac{9}{5}$	2	3
<p>Proportional? Yes or No? Explain your answer.</p> <p>If yes, what is the unit rate? (pick the most useful unit rate)</p> <p>If proportional, where do you see the unit rate in the table and in the graph?</p>							

4.	# of MP3s	2	3	4	5	6	10
	Price (\$)	1.00	1.50	1.75	1.90	2.00	5.00
<p>Proportional? Yes or No? Explain your answer.</p> <p>If yes, what is the unit rate? (pick the most useful unit rate)</p> <p>If proportional, where do you see the unit rate in the table and in the graph?</p>							

5.	Minutes	0	500	1000	1500	2000	2500
	Cell Phone Monthly Charge	\$10	\$20	\$30	\$40	\$50	\$60
<p>Proportional? Yes or No? Explain your answer.</p> <p>If yes, what is the unit rate? (pick the most useful unit rate)</p> <p>If proportional, where do you see the unit rate in the table and in the graph?</p>							

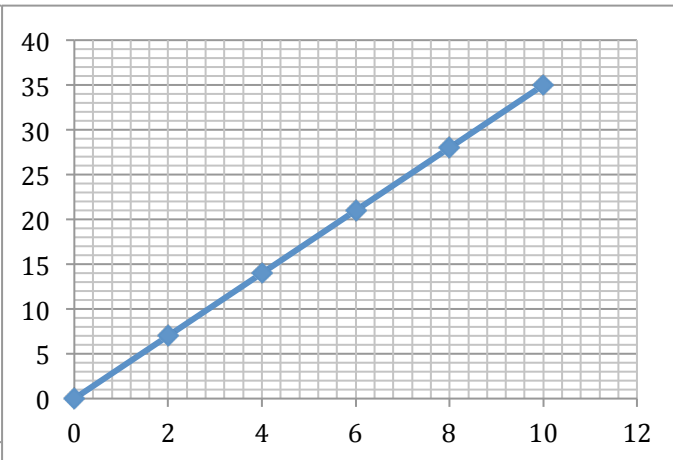
Determine whether or not each graph represents a proportional relationship.

6.



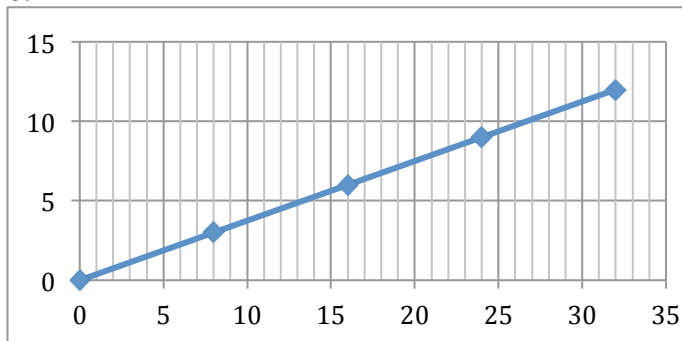
Proportional? Yes or No
If proportional, find the unit rate.

7.



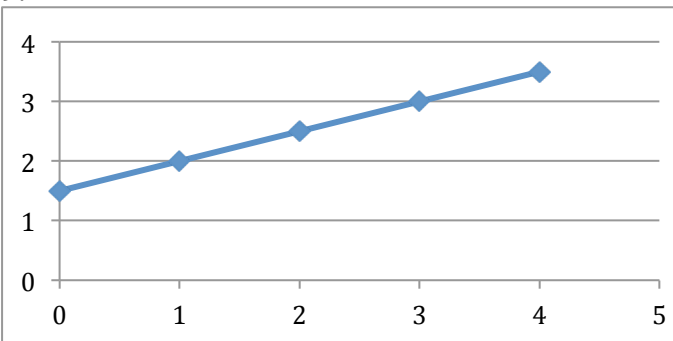
Proportional? Yes or No
If proportional, find the unit rate.

8.



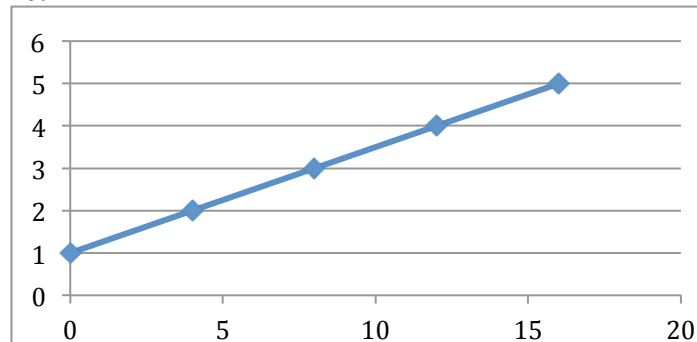
Proportional? Yes or No
If proportional, find the unit rate.

9.



Proportional? Yes or No
If proportional, find the unit rate.

10.

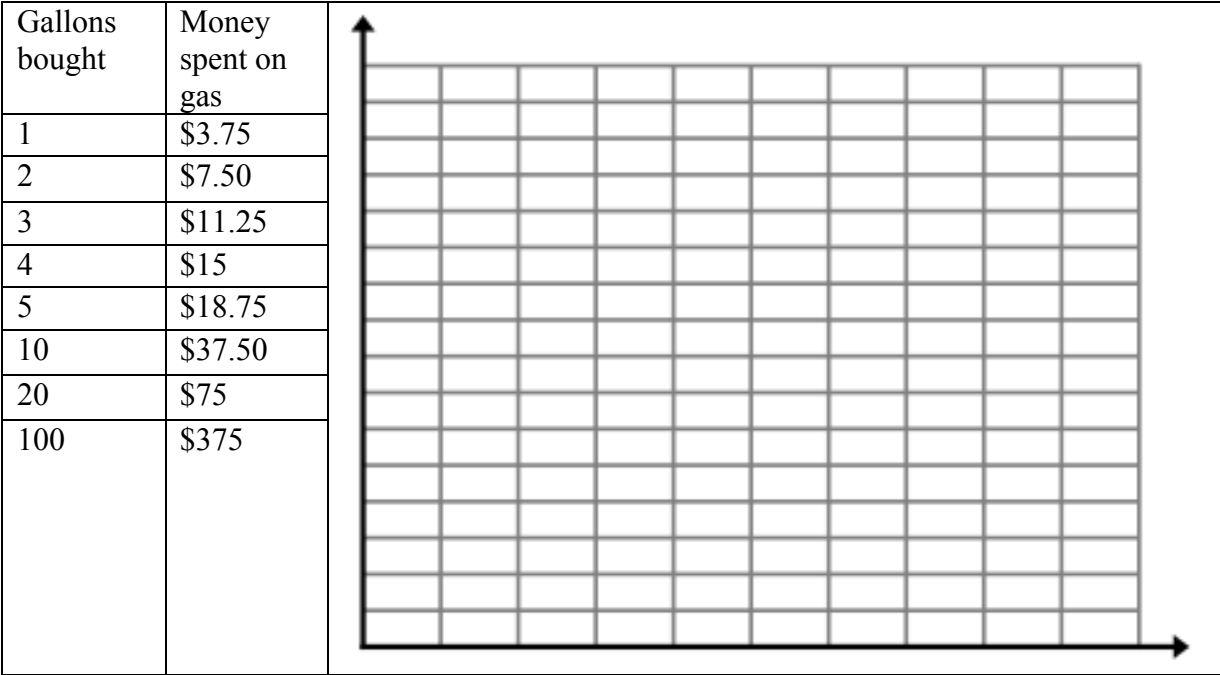


Proportional? Yes or No
If proportional, find the unit rate.

11. Which graphs above show proportional relationships? How do you know?

4.2d Classwork: Review Proportional Tables, Stories, and Graphs and Compare Rates

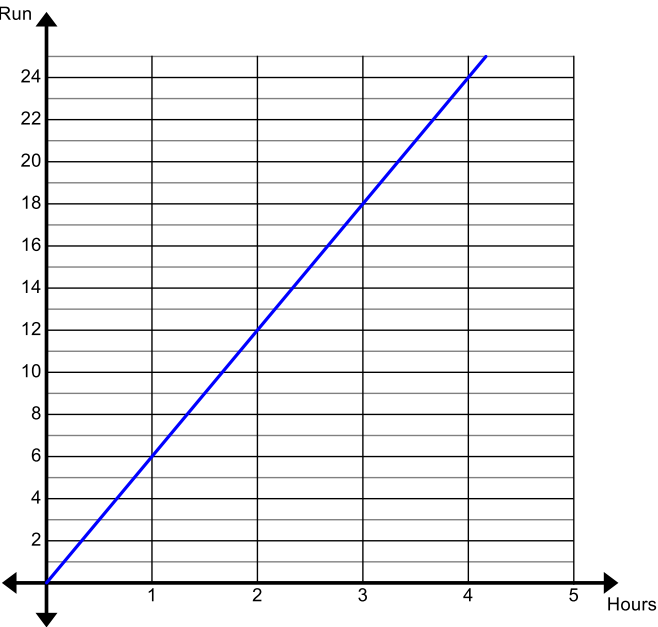
1. For the table below, a) create a context, b) graph the data, making sure to label your axes, c) determine if the relationship is proportional, and c) state the relationship between gallons bought and money spent:



2. Toni ran in a marathon. The graph given below represents the distance she has traveled since the beginning of the race.

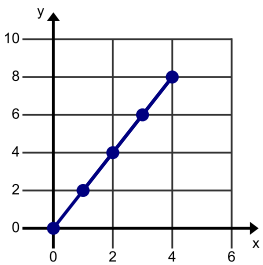
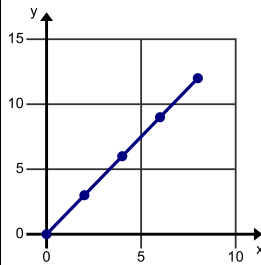
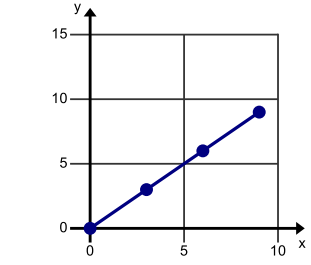
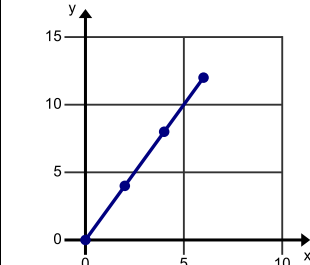
- a. Create a table with at least three entries showing her distance at different times; be sure to label your columns.

- b. Is the relationship between time and distance proportional? Explain.



- c. If proportional, where do you see the unit rate in the table and on the graph?
- d. Suppose another runner started at the same time as Toni and in 3.5 hours ran a distance of 20 miles. Which runner was going faster?

3. Examine the graphs below: a) fill in a table with at least two values and then b) determine if the graphs show a proportional relationship. If the graph is a proportional relationship state the proportional constant.

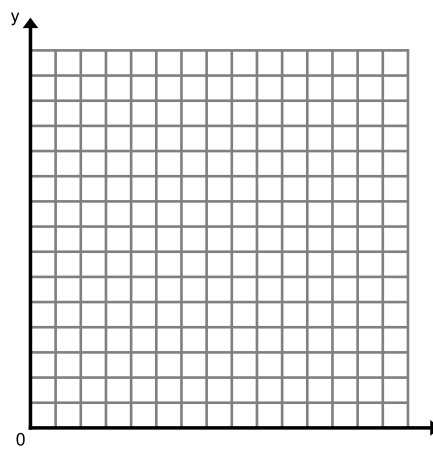
<p>a.</p>  <table border="1" style="margin-left: auto; margin-right: auto; text-align: center;"> <thead> <tr> <th style="padding: 5px;">x</th> <th style="padding: 5px;">y</th> </tr> </thead> <tbody> <tr><td style="height: 30px;"></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> </tbody> </table>	x	y							<p>b.</p>  <table border="1" style="margin-left: auto; margin-right: auto; text-align: center;"> <thead> <tr> <th style="padding: 5px;">x</th> <th style="padding: 5px;">y</th> </tr> </thead> <tbody> <tr><td style="height: 30px;"></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> </tbody> </table>	x	y						
x	y																
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<p>Proportional? (Yes or No) Proportional constant:</p>	<p>Proportional? (Yes or No) Proportional constant:</p>																
<p>c.</p>  <table border="1" style="margin-left: auto; margin-right: auto; text-align: center;"> <thead> <tr> <th style="padding: 5px;">x</th> <th style="padding: 5px;">y</th> </tr> </thead> <tbody> <tr><td style="height: 30px;"></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> </tbody> </table>	x	y							<p>d.</p>  <table border="1" style="margin-left: auto; margin-right: auto; text-align: center;"> <thead> <tr> <th style="padding: 5px;">x</th> <th style="padding: 5px;">y</th> </tr> </thead> <tbody> <tr><td style="height: 30px;"></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> </tbody> </table>	x	y						
x	y																
x	y																
<p>Proportional? (Yes or No) Proportional constant:</p>	<p>Proportional? (Yes or No) Proportional constant:</p>																

Graph each proportional relationship from above on the grid below.

For each x value in the relationship above, state the corresponding y value:

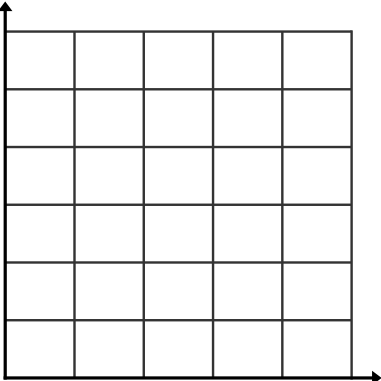
- a) if $x = 1$, $y = \underline{\hspace{2cm}}$
- b) if $x = 1$, $y = \underline{\hspace{2cm}}$
- c) if $x = 1$, $y = \underline{\hspace{2cm}}$
- d) if $x = 1$, $y = \underline{\hspace{2cm}}$

What do you notice about the relationship between the graphs and the proportional constant?



Use the story to complete the following. Milo and Sera each bought chocolate cinnamon bears from different candy stores. Milo paid \$3.00 for 2 pounds and Sera paid \$5.25 for 3 pounds.

4. Fill in each box for Milo’s purchase:

a. Create a table for Milo’s story.	b. Sketch a graph. LABEL axes.	c. Is this a proportional relationship? Explain.														
<table border="1"><tr><td>x</td><td>y</td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table>	x	y														d. If proportional, what is the unit rate? Highlight the corresponding point on graph.
x	y															
		e. State the relationship between pounds and cost:														

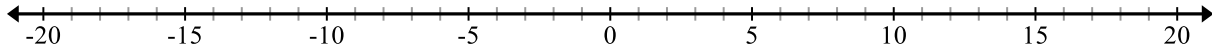
5. Fill in each box for Sera’s purchase:

a. Create a table for Sera’s story.	b. Sketch a graph. LABEL axes.	c. Is this a proportional relationship? Explain.														
<table border="1"><tr><td>x</td><td>y</td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table>	x	y														d. If proportional, what is the unit rate? Highlight the corresponding point on graph.
x	y															
		e. State the relationship between pounds and cost:														

6. Who got the best deal on chocolate cinnamon bears? Explain?

Spiral Review

1. There are a total of 127 cars and trucks on a lot . If there are four more than twice the number of trucks than cars, how many cars and trucks are on the lot?
2. Use the number line below to show why $(-1)(-1) = 1$



3. If the ratio of girls to boys in a class is 3 to 4 and there are 35 students in the class. How many students are girls?
4. Fill in the equivalent fraction and percent for this decimal:

Fraction	Decimal	Percent
	.95	

4.2d Homework: Compare Unit Rates Using Tables and Graphs

1. Mario will pay you 360 gold coins for 2 hours of racing Go-Karts. Luigi will pay you 426 gold coins for 3 hours of racing Go-Karts. Find the best offer using three different methods (table, rate and graph.)

a. Complete the table.				b. Write each offer as rate fraction.			
Mario's Offer		Luigi's Offer		Mario's: Luigi's:			
Hours	Payment	Hours	Payment	Find the unit rate for each offer, including labels.			
1		1		Mario's:			
2		2		Luigi's:			
3		3					
4		4					
5		5					

c. Graph each offer in a different color on the same coordinate plane. Label your axes.

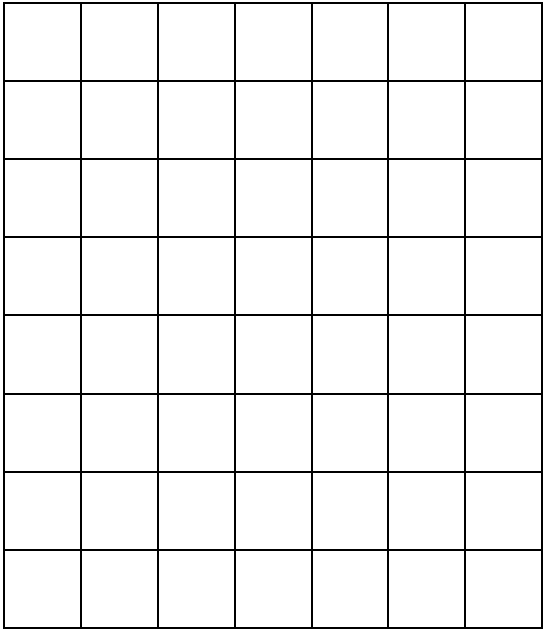
d. Which is the best offer? Why? (In your explanation, explain how the table, unit rate and graph help you to see the best offer.)

2. The Jones family drives 200 miles in 5 hours. The Grant family drives 360 miles in 6 hours. Compare the car trips. Assume they both traveled at a constant rate.

a. Complete the table for each family.

Jones Family	
Hours	Miles
Unit rate:	

Grant Family	
Hours	Miles
Unit rate:	



- b. Graph each family’s rate in a different color on the same coordinate plane. Label axes.
- c. Compare the driving trips of the two families. In your comparison, explain how the table, unit rate and graph help you to see the difference in the two trips.

3. The tortoise can walk $\frac{1}{2}$ a mile in $\frac{1}{4}$ of an hour. The hare can run $1\frac{1}{2}$ miles in $\frac{1}{2}$ hour. Compare their speed.

a. Complete the table for each animal.

Tortoise	
Hours	Miles
Unit Rate:	

Hare	
Hours	Miles
Unit Rate:	

b. Graph each animal’s rate in a different color on the same coordinate plane.

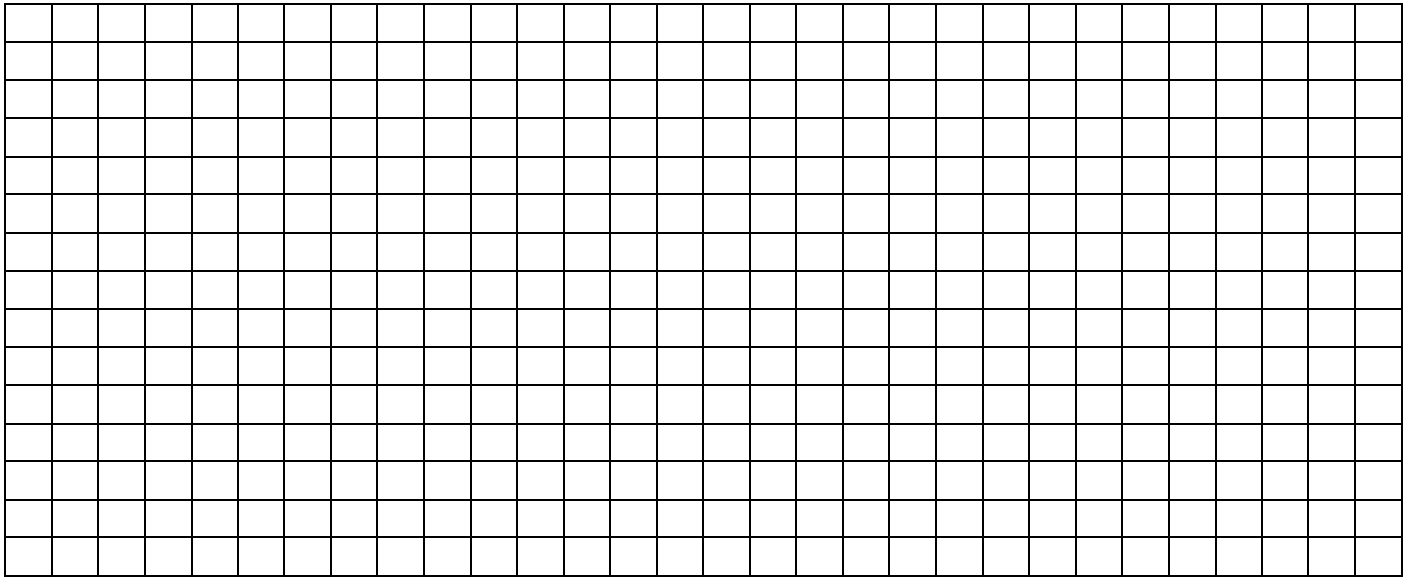
- c. Compare the speeds. In your comparison, explain how the table, unit rate and graph help you to see the difference in the two trips.

4.2e Classroom Activity: Jon’s Marathon—Find Unit Rate (Proportional Constant) in Tables and Graphs

Jon is running a marathon. His average speed is 15 miles in 3 hours. Pretend his average speed was constant for the entire marathon. Fill in the table below.

Hours	0	1	2	3	4	7	10	24	h
Miles									

- What is the unit rate (proportional constant) and how does this unit rate help you figure out the number of miles traveled at any given hour?
- If m represents the number of miles and h represents the number of hours, state the relationship between m and h .
- Use your rule to predict the number of miles that can be run in 2.6 hours and the number of hours it would take to run 55 miles.
- Create a graph showing Jon’s progress in the marathon. Label the x -axis (horizontal) as “hours” and the y -axis (vertical) as “miles.” Think carefully about the scale before you start graphing- you won’t see much information if you make each unit on the graph represent 1 mile.



- Where does the unit rate show up in the graph?
- Use the graph to find the number of miles run in 12 hours.
- Use the graph to find the number of hours it took to run 30 miles.
- How does the mathematical rule (from question 3 above) relate to the graph?

Spiral Review

1. Luke has 90 baseball cards. He sells $\frac{1}{3}$ of the cards. He stores 20% of the rest in a safty deposit box and the rest in his dresser drawer. How many baseball cards are in his dresser drawer?
2. Find the product without a calculator: $15.5(-8)$
3. Suppose you were to roll a fair 6-sided number cube once, then flip a coin. List all the possible outcomes.
4. Using the information in question 3, what is the probability of getting a heads and an even number?

4.2e Homework Activity: Revisit Making Lemonade

1. Find the “mix-matches” in the lemonade formulas below (that is the ones with the same proportional constant [or unit rate]).

1. $\frac{1}{2}$ cup concentrate, 1 cup water	2. $\frac{2}{3}$ cup concentrate, 1 cup water	3. $\frac{1}{2}$ cup, 2 cups water	4. 2 cups concentrate, 3 cups water
5. 1 cup concentrate, 4 cups water	6. 4 cups concentrate, 8 cups water	7. 4 cups concentrate, 6 cups water	8. 3 cups concentrate, 5 cups water

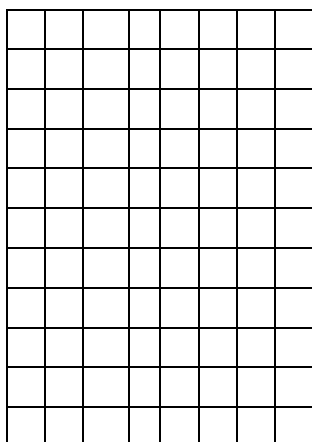
2. Fill in the information for the “mix-matches” in the tables below.
- a. Record the “mix-matches” in the top of the table (from least “lemony” to most “lemony”).
 - b. Complete a table for each “mix-match” (use numbers from the chart).
 - c. Circle the data in each table which shows the unit rate for water to concentrate.
 - d. Write the unit rate (of water to concentrate) as a fraction with labels.
 - e. State the proportional relationship between concentrate and water.

Least Lemony						Most Lemony	
A. Mixes		B. Mixes		C. Mixes		D. Mixes	
Conc. (x)	Water (y)	Conc. (x)	Water (y)	Conc. (x)	Water (y)	Conc. (x)	Water (y)
Unit rate:							
State the relationship:							

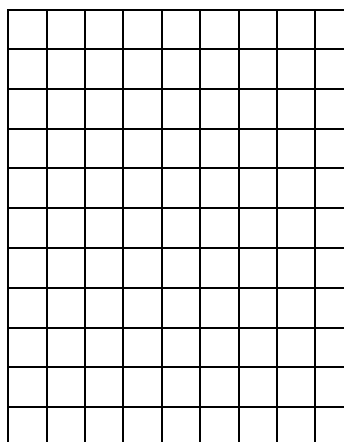
3. Predict what you think the graphs will show about lemonade mixes A, B, C and D.

4. Graph the data from the tables for Mixes A, B, C, and D. Use the same scale on each graph.

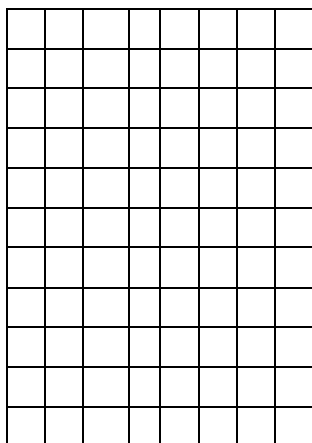
Mix A



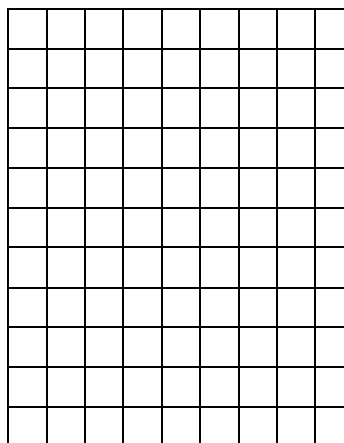
Mix B



Mix C



Mix D



5. Were your predictions about the graphs correct? What do the graphs show?
- How do they show the proportional constant or unit rate?
 - How do they show “lemony-ness?”
6. Compare the lemonade tables of data, the equations and the graphs.
- What do they have in common?
 - Which do you like best to tell the story of lemonade mixes? Why?

4.2f Review: Representing a Proportional Relationship with a Table, Graph, and Equation

Follow these steps for each situation given below.

- a. Create a table
- b. Determine the unit rate (proportional constant)
- c. Create a graph (use the “unit” label across the bottom of the graph).
- d. Write a simple mathematical formula for the situation.

1. Rhonda was paid \$24 for 4 hours of babysitting.

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2. Carlos made 6 goals in 3 soccer games. (assume this is his regular habit)

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3. Itzel drove 30 miles in a half hour.

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4. Carmen bought 3 pieces of candy for $\frac{1}{4}$ of a dollar.

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5. Alejandra bought $\frac{1}{4}$ of a pound of gummy bears for 2 dollars.

[illegible]

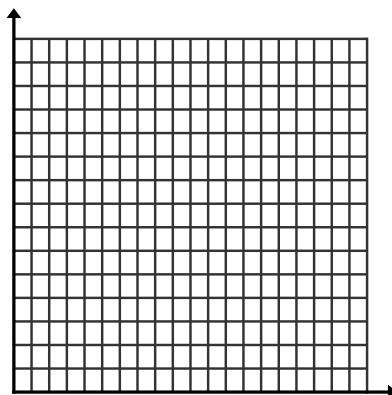
6. Charlie swam part of a mini-triathlon $\frac{1}{2}$ mile in $\frac{1}{3}$ hour.
 - a. What was his average speed? How do you know?
 - b. Is the unit rate a proportional constant? Explain why or why not.
 - c. What might the graph look like?(sketch if desired)

7. Carmen is making rice crispy treats for an upcoming bake sale. The number of marshmallows to the number of cups of crispy rice is proportionally related. If Carmen uses 36 jumbo marshmallows, she will need to use 6 cups of crispy rice.
- a. Complete the table below to show the relationship between number of marshmallows and number of cups of crispy rice needed to make rice crispy treats.

Number of Marshmallows (x)	Cups of Crispy Rice (y)
0	
6	
12	
	5
	8
60	
1	

- b. What is the proportional constant for this relationship?
- c. Write an equation that shows the relationship between the number of marshmallows x and the number of cups of crispy rice y needed to make rice crispy treats.

8. Jane took 16 pounds of aluminum cans to the recycling center and received \$12. She knows that the amount she earns is proportional to the number of pounds of cans she turns in.
- a. What rate did the center pay for cans?
- b. Write an equation that relates the pounds to the amount of money received.
- c. Graph this relationship on the grid below and highlight the unit rate/slope.



4.2g Self-Assessment: Section 4.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Determine if two rates written as ratios are proportional. [1]	I don't understand how to determine if two rates written as ratios are proportional.	I can determine if two rates written as ratios are proportional with a model.	I can determine if two rates written as ratios are proportional without a model, using unit rate or simplifying.	I can determine if two rates written as ratios are proportional by using either unit rate or simplifying. I can explain which method is most appropriate in a given situation.
2. Graph a relationship from a table of values. [2]	I struggle to graph a relationship from a table of values.	I can graph a relationship from a table of values, but I don't accurately label and scale both axes.	I can graph a relationship from a table of values. I can appropriately and accurately label and scale both axes.	I can graph a relationship from a table of values. I can appropriately and accurately label and scale both axes. I can also justify why I chose a particular variable for each axis.
3. Determine if a relationship is proportional from a graph. [3]	I don't understand how to determine if a relationship is proportional from a graph.	I can usually determine if a relationship is proportional from a graph.	I can determine if a relationship is proportional from a graph. I can show on the graph why it is or isn't proportional.	I can determine if a relationship is proportional from a graph. I can explain why or why not in my own words.
4. Find a unit rate from a table. [4]	I struggle to find unit rate from a table.	I can usually determine the unit rate from a table but sometimes I miscalculate.	I can find unit rate from a table.	I can find unit rate from a table. I can explain in my own words the meaning of the unit rate.
5. Find a unit rate from a graph. [3]	I struggle to find unit rate from a graph.	I can find unit rate from a graph if I convert it to a table first.	I can find unit rate from a graph.	I can find unit rate from a graph in two different ways.
6. Explain the significance of $(1, r)$ in a proportional relationship. [4]	I struggle to understand the meaning of the point $(1, r)$ in a proportional relationship.	I can usually explain the meaning of the point $(1, r)$ in a proportional relationship.	I can explain the contextual meaning of the point $(1, r)$ in a proportional relationship.	I can explain in my own words the contextual meaning of the point $(1, r)$ in a proportional relationship.
7. Compare rates from tables. [5]	I struggle to determine rates from tables.	I can find rates from tables. I struggle to compare those rates in a meaningful way.	I can find rates from tables. I can compare those rates in context.	I can find rates from tables in BOTH ways. I can compare those rates in context using whichever is most appropriate.

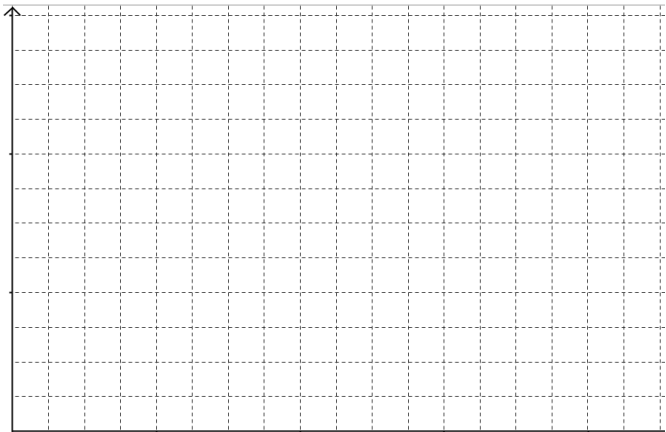
Sample Problems for Section 4.2

1. Determine if each of the following pairs of rates are proportional.

$\frac{3}{2}$ and $\frac{2}{3}$	$\frac{5}{15}$ and $\frac{10}{30}$	14 : 18 and 4 : 8
---------------------------------	------------------------------------	-------------------

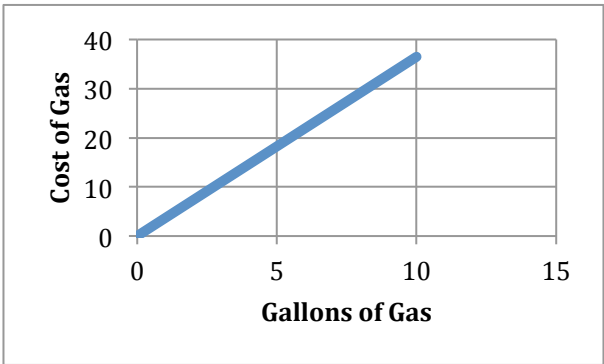
2. Last year, Sheryl got a Labrador puppy. The table below shows the age and weight of the puppy over several months. Graph the following relationship. Write a sentence justifying your placement of each variable.

Weight of dog (in pounds)	Age of dog (in months)
23	3
30	4
45	6
55	9
60	12

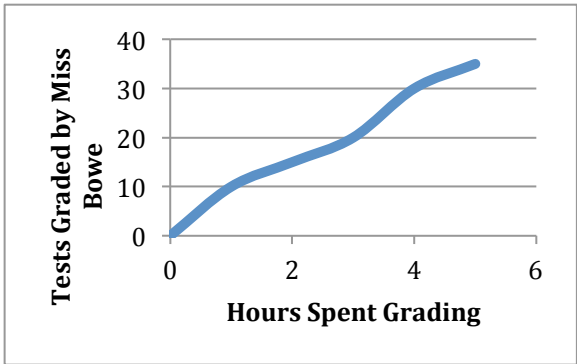


3. Given the following graphs, state whether each graph is proportional or not. Show or explain how you know. If it is proportional, find the unit rate.

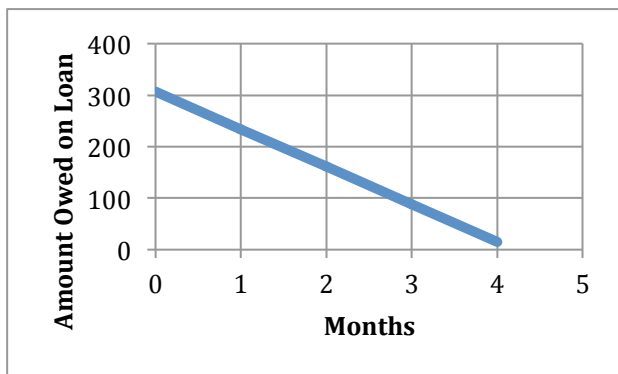
a.



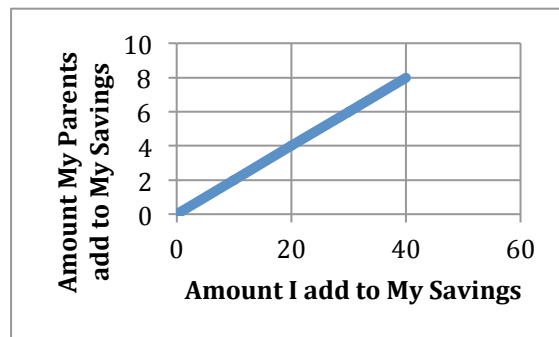
b.



c.



d.



4. For each table below, find the unit rate. Then explain the meaning of the point $(1, r)$.

a.

Kelsey's run	
Miles	Hours
$\frac{1}{2}$	$\frac{1}{8}$
2	$\frac{1}{2}$
3.5	$\frac{7}{8}$
12	3

b.

Brand X Peanut Butter	
Cost	Ounces
13.92	48
11.60	40

5. Find the unit rate for each table below. Then compare to the corresponding context in # 4.

a.

Romina's run	
Hours	Miles
$\frac{1}{2}$	2.5
2	10
4	20
5	25

b.

Brand Y Peanut Butter	
Cost	Ounces
20.16	96
6.30	30
3.15	15

Section 4.3: Analyze Proportional Relationships Using Unit Rates, Tables, Graphs and Equations.

Section Overview:

In this section, students will extend fluency with graphs and tables and solidify writing equations for a proportional relationship. Throughout the section, there will be problems with relationships that are not proportional to be used as contrast to those that are. Problems in this section will have more steps and/or will be more advanced than those in section 4.2. By the end of the section, students should be able to go from any one representation to any other representation (graph to equation, equation to table, graph to table, etc.) and should be able to state what points on the graphs, solutions to the equation, or entries on the table mean in the context of the problem. For problems finding missing quantities, the emphasis should be on using the unit rate (scaling), equation, table, or graph to solve. Formal proportions and cross multiplication should not be used (these ideas will be solidified in chapter 5).

Key Ratio and Proportion Concepts from Utah Core Standards

RP Standard 1:

1. Extend the concept of a unit rate to include ratios of fractions.
2. Compute a unit rate, involving quantities measured in like or different units.

RP Standard 2:

3. Determine if two quantities expressed in a table or in a graph are in a proportional relationship.
4. Determine a unit rate from a table, graph, equation, diagram or verbal description and relate it to the constant of proportionality.
5. Write an equation for a proportional relationship in the form $y = kx$.
6. Explain the meaning of the point (x, y) in the context of a proportional relationship.
7. Explain the significance of $(0,0)$ and $(1, r)$ in a graph of a proportional relationship, where r is the unit rate.

RP Standard 3:

8. Solve multistep problems involving percent using proportional reasoning
9. Find the percent of a number and extend the concept to solving real life percent applications.
10. Calculate percent, percent increase, decrease, and error.

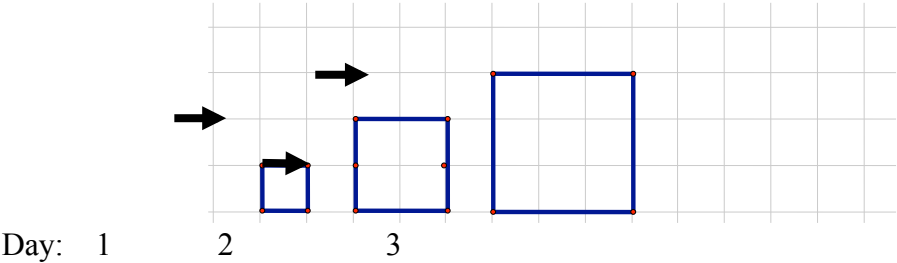
Primary Concepts and Skills to be Mastered in This Section

1. Write an equation for a proportional relationship from a table of values.
2. Write an equation for a proportional relationship from a graph.
3. Use rate to find a missing value in a table or context.
4. Use ratios to convert between units.
5. Explain why a relationship is or is not a proportional relationship.

4.3a Classroom Activity: Writing Equations from Patterns

Activity 1: Roberto is recovering from an injury. He increases the number of blocks he walks each day in the following pattern. (The arrow represents one street block.)

a. Draw the figure for Day 4 and write the number of blocks Roberto will walk.



b. Complete the table.

Day (<i>d</i>)	1	2	3	4	5	8	14		
Blocks (<i>b</i>)	4	8	12					64	92

c. How did you figure out the number of blocks for 8 days and 14 days? What would you do to find the number of blocks for any given number of days?

d. How did you find the number of days when you knew the number of streets?

e. What is the unit rate $\frac{\text{blocks}}{\text{day}}$? What is the unit rate $\frac{\text{days}}{\text{block}}$?

- f. Write equations to find
- days for *b* number of blocks
 - blocks for *d* number of days

g. How can you identify the unit rate in an equation of a proportional relationship?

Activity 2: For each sequence below: fill in the missing numbers, find the 50th term in the sequence and write equation(s) for the relationships found in the tables. Note: Write two equations for each example below (one for each possible unit rate). For numbers that are not integers, represent them in an exact form (i.e. without using a repeating decimal.)

a.	Hours (h)	1	2	3	4	5	6	.	50 th	Equations:
	Miles (m)	6.8	13.6	20.4	27.2			.		
b.	Days (d)	1	2	3	4	5	6	.	50 th	Equations:
	Fees (f)	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{6}{7}$	$1\frac{1}{7}$.		
c.	Days (d)	1	2	3	4	5	6	.	50 th	Equations:
	Earnings (e)	20	40	60	80			.		
d.	Boxes (b)	1	2	3	4	5	6	.	50 th	Equations:
	Pounds (p)	$\frac{1}{4}$	$\frac{1}{2}$				$\frac{3}{2}$.		
e. Challenge	Paper Tears (p)	1	2	3	4	5	6	.	50 th	Equation for size:
	Size of each paper piece (s)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$.		

Practice:

- Complete the table below. Write an equation to show the relationship between the output (y value) to the input (x value). Then answer the questions.

Number of Tickets (x)	1	2	3	4	5	6	7	Equation: $y =$
Total cost (y)	\$3	\$6	\$9	\$12	\$15			

- Is the relationship in this table proportional?
- What connections do you see between the unit rate, the table, and the equation?
- This equation gives you a rule when you input a number of tickets (x) to get an output of cost (y). Write the equation that does the opposite job: you input cost (y) and get an output of number of tickets (x).
- What is the mathematical relationship between the constant of proportionality in the two equations?

2. Complete the table below. Write an equation relating the output (y value) to the input (x value). Then answer the questions.

# of Rides (x)	Cost (y)
0	\$4
1	\$6
2	\$8
3	\$10
4	\$12
5	
8	

Equation:
 $y =$

- a. Is the relationship in the table proportional? Explain.
- b. What connections do you see between the unit rate, the table, and the equation?
- c. This equation gives you a rule when you input a number of rides (x) to get an output of cost (y). Write the equation that does the opposite job: you input a cost (y) and get an output of number of rides (x).

Spiral Review

- Maria is setting up a game for a school fundraiser carnival. The game will be a fishing pond with 300 rubber ducks. On the bottom of each rubber duck will be either a red, blue or white sticker to indicate the type of prize the player will win. A red sticker will earn the player the top prize. Maria wants the probability of getting a red sticker to be 5%. On how many of the ducks should Maria affix a red sticker?
- Factor the following expressions.

$3x - 6$
 $15 - 20y$
 $\frac{5}{3}x + \frac{2}{3}$
 $4.9t - 2.8$
- Use the distributive property to rewrite $a(b + c)$
- Use a model to solve $7 = 3x - 2$
- Chloe has twice as many cats as her sister has dogs. Her brother has 3 turtles. Together, they have six pets. How many of each pet do they have?

4.3a Homework: Writing Equations from Patterns

1. Complete the table below. Write an equation relating the output (y value) to the input (x value). Then answer the questions.

Hours (x)	1	2	3	4	5	6	7	Equation: $y =$
Miles Driven (y)	25	50	75	100	125			

- Is the relationship in the table proportional? Explain your answer.
 - What connections do you see between the unit rate, the table, and the equation?
 - This equation gives you a rule when you input a number of hours (x) to get an output of miles (y). Write the equation that does the opposite job: you input a miles (y) and get an output of hours (x).
 - What is the mathematical relationship between the constant of proportionality in the two equations?
2. Complete the table below. Write an equation relating the output (y value) to the input (x value). Then answer the questions.

Feet (x)	Yards (y)
1	$\frac{1}{3}$
2	$\frac{2}{3}$
3	1
4	$1\frac{1}{3}$
5	$1\frac{2}{3}$
6	
10	
Equation: $y =$	

- Is the relationship in the table proportional? Explain your answer.
- What connections do you see between the unit rate, the table, and the equation?
- This equation gives you a rule when you input a number of feet (x) to get an output of number of yards (y). Write the equation that does the opposite job: you input a number of yards (y) and get an output of number of feet (x).
- What is the mathematical relationship between the constant of proportionality in the two equations?

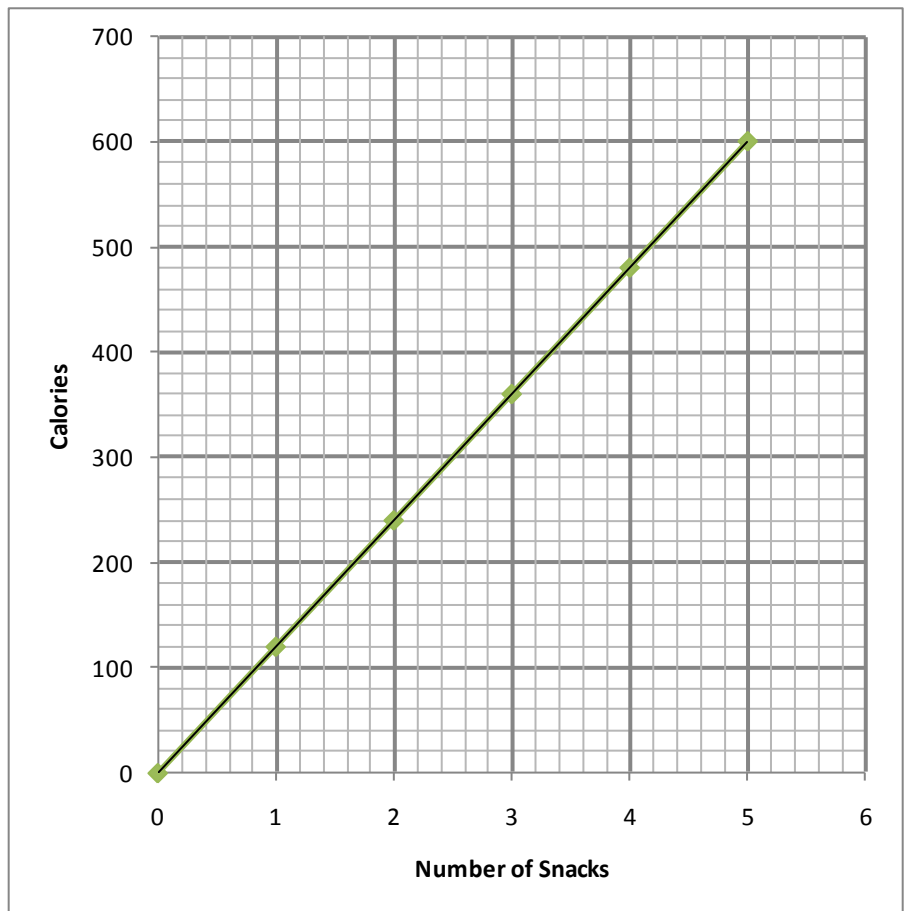
4.3b Classwork: Writing Equations from Graphs

1. Use Graph A below to fill in the table relating calories to snacks.

Number of Snacks	Calories	Ordered Pair	Write a complete sentence describing the meaning of this point on the graph.
0	0	(0, 0)	Zero snacks have zero calories.
1			
2			
3			
4			

Graph A

- Is this a proportional graph? Why or why not?
- What is the unit rate (include labels)? How do you know?
- Write the equation for calories related to number of snacks.
- Where does the unit rate show up on the graph? In the equation?

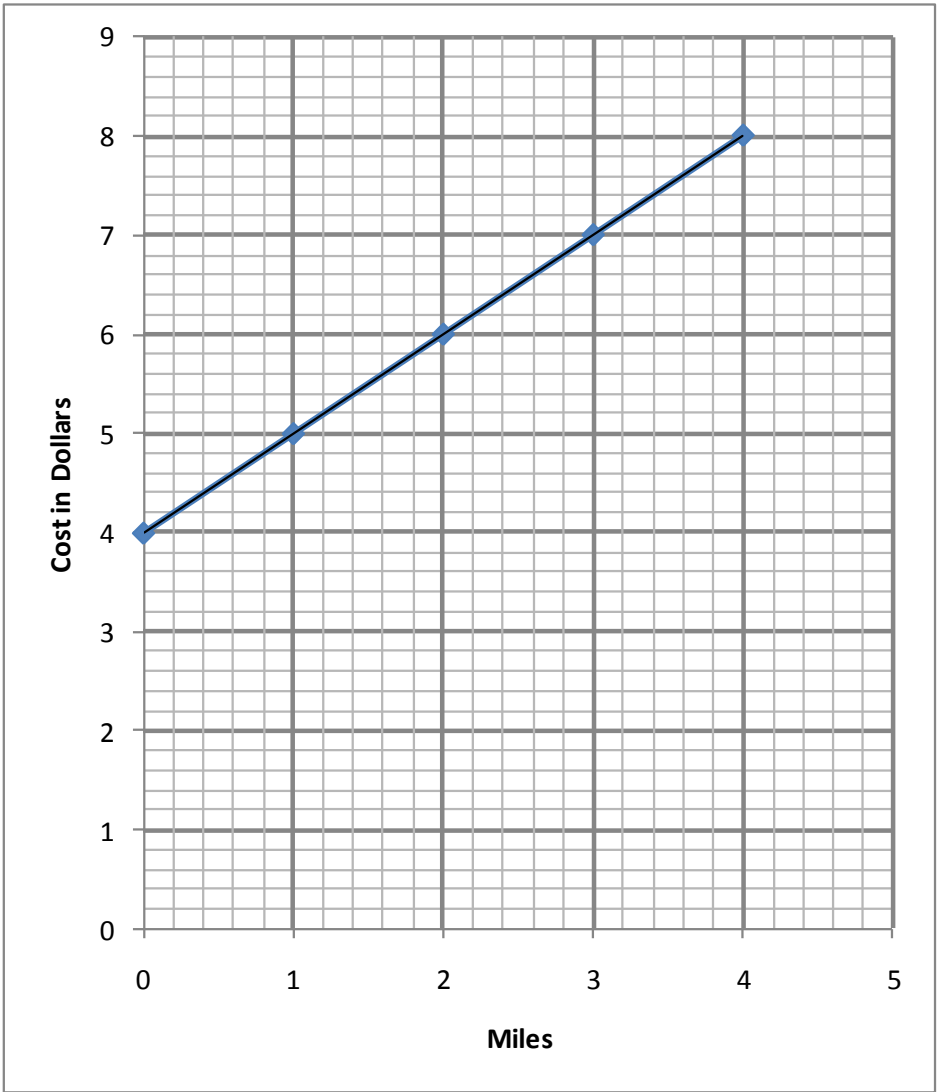


2. Use Graph B to fill in the table relating cost to cab-ride distance.

Miles	Cost	Ordered Pair	Write a complete sentence describing the meaning of this point on the graph.

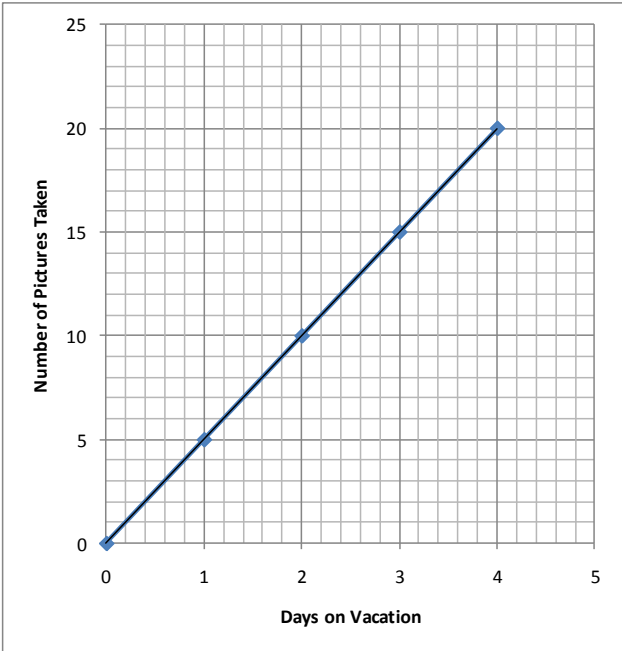
Graph B

- a. Is this a proportional graph? Why or why not?
- b. Looking at the graph, describe what’s happening to the cost as miles increase.
- c. Is there a unit rate? If so, what is the unit rate? Is the rate consistent for the entire graph? Explain.
- d. Challenge: Write the equation relating cost to cab-ride distance. Explain the equation in words.



4.3b Homework: Writing Equations from Graphs

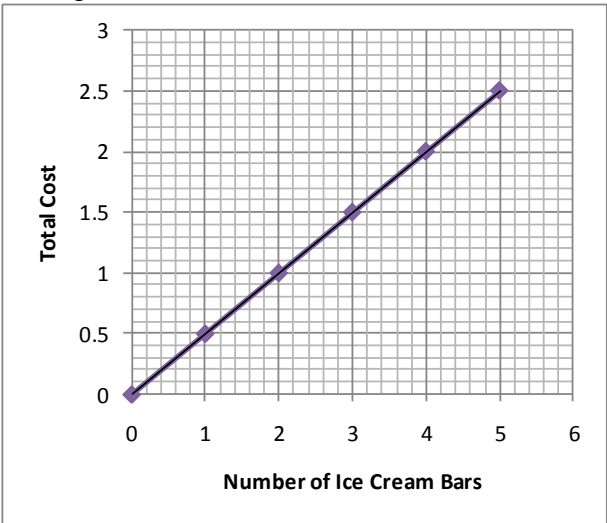
1. Graph C:



- a. Is this a proportional graph? Why or why not?
- b. Fill in the table above for the graph

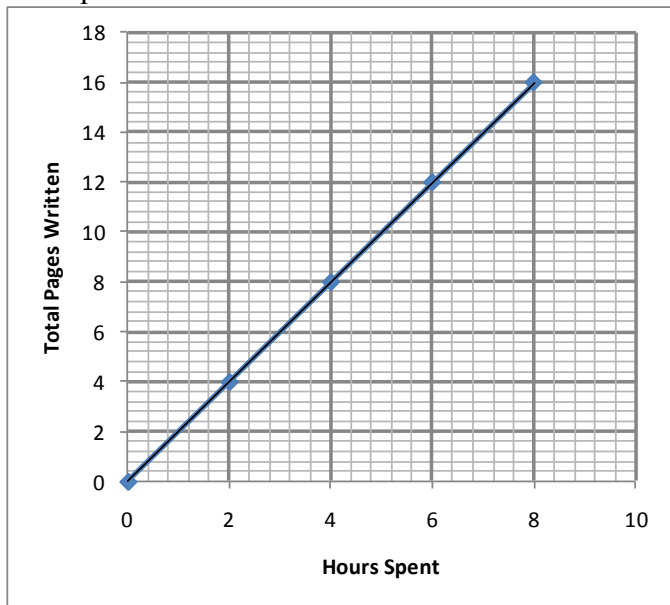
- c. What is the unit rate (include labels)?
- d. Write the equation relating number of pictures taken to days on vacation.

2. Graph D:



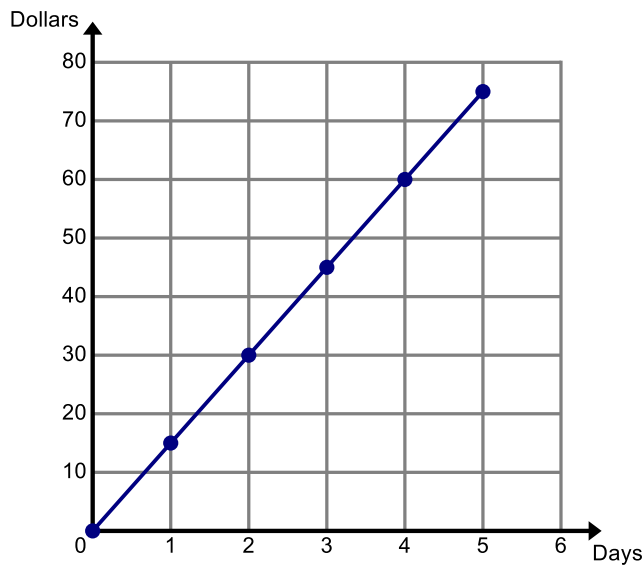
- a. Is this a proportional graph? Why or why not?
- b. Fill in the table for the graph
- c. What is the unit rate (include labels)?
- d. Write the equation relating cost to number of ice cream bars purchased.

3. Graph E:



- Is this a proportional graph? Why or why not?
- Fill in the table for the graph.
- What is the unit rate (include labels)?
- Write the equation relating pages written to hours spent.

4. Graph F:

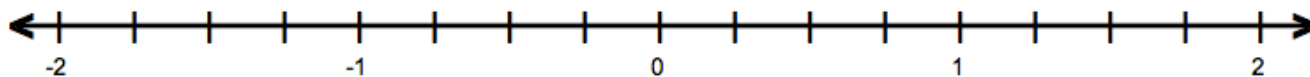


- Is this a proportional graph? Why or why not?
- Fill in the table for the graph:
- What is the unit rate (include labels)?
- Write the equation relating dollars (spent or earned) to days.

Spiral Review

1. Place the fractions on the number line below.

$$-\frac{3}{4}, \quad \frac{5}{12}, \quad -1\frac{1}{4}, -\frac{4}{3}$$



2. Write $\frac{1}{5}$ as a percent and decimal.

3. Solve $-3x + 5.5x + 4 = 7.2$

4. Simplify the following expression. Use a model if needed. $2(m - 1) + 3m - 4 + m$

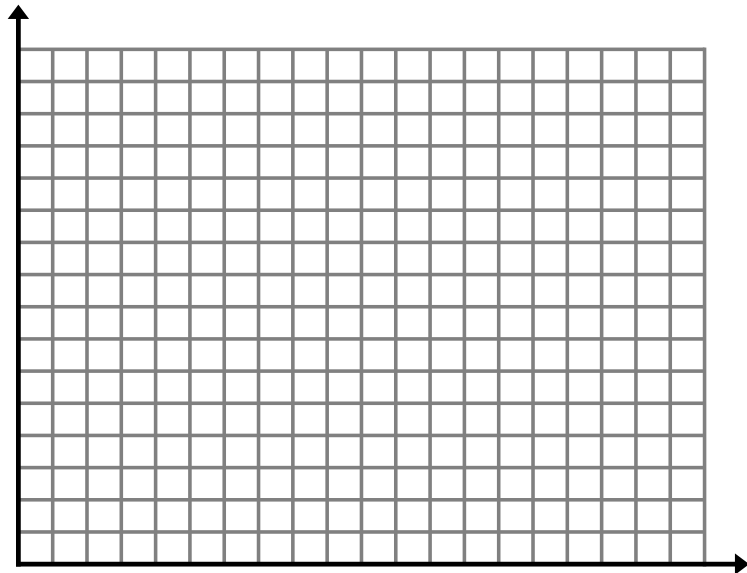
4.3c Classroom Activity: Convert Units—Proportion in Tables and Graphs

A. Quarts related to Gallons:

Fill in the missing amounts of quarts and gallons. Then find the ratio for the number of quarts (y) to the number of gallons (x).

Gal (x)	1	2	4	5	10		25	30		100	x
Quarts (y)		8				80			180		
Ratio $\frac{y}{x}$											

1. What does the $\frac{y}{x}$ ratio tell us?
2. Write an equation to show how to find the number of quarts in any number of gallons.
3. Make a graph of the relationship between quarts and gallons (put gallons on the horizontal axis).



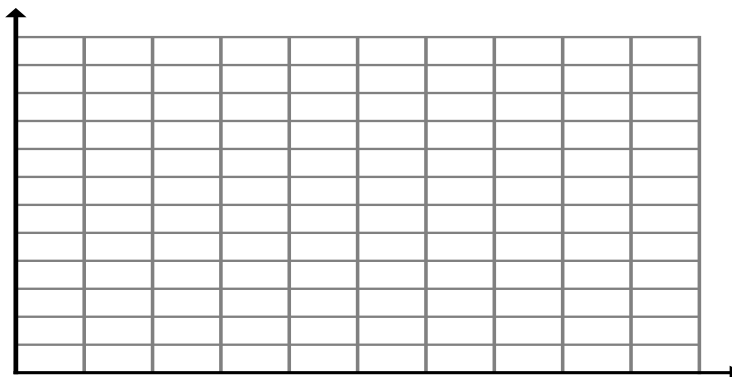
4. What does the graph tell us about the relationship of quarts to gallons?
5. On the graph, find the following coordinate points. What is the missing y value?
Point A (0,) Point B (1,) Point C (3,) Point D (4,)
6. Draw the points and label them A, B, C, and D on the graph.
7. Describe how you “move” on the grid from one point to the next (you must stay on the grid lines to move from A to B, B to C, C to D). Use words like up, down, right, left, vertical, horizontal.
8. How does this “moving” relate to rate?
9. Would the ordered pair (6, 26) lie on the graph? Why or Why not?

B. Feet related to yards:

Fill in the missing data.

Yards (x)	1	2	4	5	10		25	30		100	x
Feet (y)						48			144		
Ratio $\frac{y}{x}$											

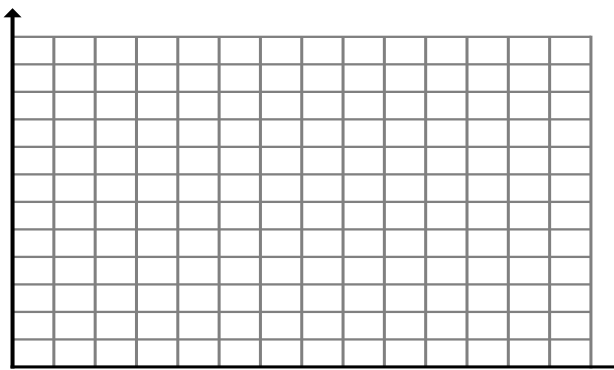
10. What is the unit rate? How do you know?
11. What did you do to find the number of feet in a given number of yards?
12. Write an equation to find the number of feet in a given number of yards.
13. Predict the movement between points on a graph. Explain--use words like up, down, right, left, vertical, horizontal.
14. Sketch the graph, including labeling the axes.

**C. Centimeters related to inches:**

Fill in the missing data.

Inches (x)	1	2	4	5	10		25	30		100	x
Centimeters (y)			10.16			38.1			127		
Ratio $\frac{y}{x}$											

15. What is the unit rate? How did you find it?
16. What did you do to find the number of centimeters in a given number of inches?
Write an equation to find the number of centimeters in a given number of inches.
17. Predict the movement between points on a graph. Explain--use words like up, down, right, left, vertical, horizontal.
18. Sketch the graph, including labels for the axes.



Spiral Review

1. Add $\frac{2}{7} + \frac{2}{3}$

2. Simplify the following expression. Use a model if needed. $71b - 4a + 4b - 4a$

3. Find the sum or product of each:

- a. $-3 + -7$
- b. $(-3)(-7)$
- c. $-3 - (-7)$
- d. $3(-7)$
- e. $-3 + (7)$
- f. $-3 - 7$

4.3c Homework: Convert Units—Proportion in Tables and Graphs

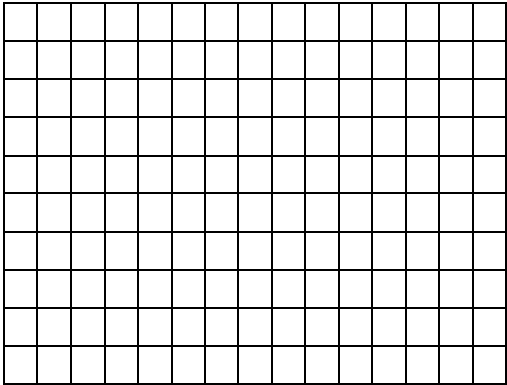
A. Minutes related to Hours

Fill in the missing data.

Hours (x)	1	2	3	5	10		15		x
Minutes (y)				300		720		1200	
Ratio $\frac{y}{x}$									

- 1. What is the unit rate? How do you know?
- 2. What did you do to find the number of minutes in a given number of hours?
- 3. Write an equation to find the number of minutes in a given number of hours.
- 4. Predict the movement between points on a graph. Explain--use words like up, down, right, left, vertical, horizontal.

Sketch the graph, including labeling the axes.



B. Hours related to Minutes: (use the table in #A above --the ratio would be reversed)

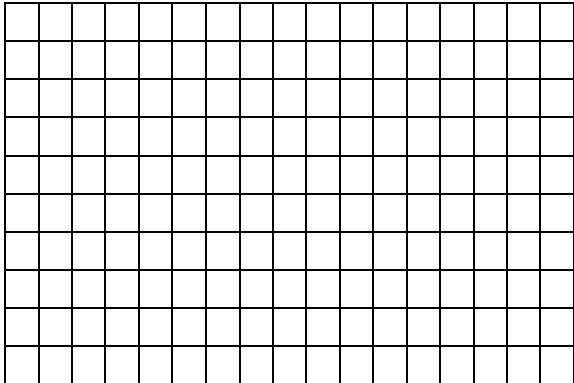
- 6. What is the unit rate? How do you know?
- 7. What did you do to find the number of hours in a given number of minutes?
Write an equation to find the number of hours in a given number of minutes.
- 8. Predict the movement between points on a graph. Explain--use words like up, down, right, left, vertical, horizontal.
- 9. How will the graph be different than the graph in #A above?

C. Kilometers related to miles: (1 mile ≈ 1.6 kilometers)

Fill in the missing data.

Miles (x)	1	2		4	5	10		25		100	x
Kilometers (y)			4.8		8		19.2		80		
Ratio $\frac{y}{x}$											

10. What is the unit rate? How do you know?
11. How could you find the unit rate if the kilometers to miles equation wasn't given?
12. What did you do to find the number of kilometers in a given number of miles?
Write an equation to find the number of kilometers in a given number of miles.
13. Predict the movement between points on a graph. Explain--use words like up, down, right, left, vertical, horizontal.
14. Sketch the graph, including labels for the axes.



D. Miles related to Kilometers: (1 kilometer ≈ 0.62 mile) (Use the data in #C above—the ratio would be reversed)

12. What is the unit rate? How do you know?
13. How could you find the unit rate if the miles to kilometers equation wasn't given?
14. What would you do to find the number of miles in a given number of kilometers?
Write an equation to find the number of miles in a given number of kilometers.
15. Predict the movement between points on a graph. Explain--use words like up, down, right, left, vertical, horizontal.
16. How will this graph be different from the graph in #C above?

4.3d Extra Assignment: Revisit Unit Rate Situations, Write Equations

Answer the questions for each situation below.

1. It costs \$0.80 to buy 4 apples. You want to buy 7 apples. How much will that cost?

a. Write the two possible unit rates.	c. Create a table (at right) to show the unit rate increasing with each additional apple.	<table border="1"><thead><tr><th>Apples</th><th>Cost</th></tr></thead><tbody><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></tbody></table>	Apples	Cost								
Apples	Cost											
b. Which unit rate will help you solve the problem?	d. Describe the number pattern you see in the table.											
e. Since you know the unit price, write a number sentence for finding the cost of 7 apples.												
f. Write an equation to find the cost of any number (n) of apples.												

2. At a bank in England, Ms. Vega wanted to exchange 20 U.S. dollars for Euros. The exchange rate was 3 U.S. dollars to 2 Euros. How many Euros did Ms. Vega receive?

a. Write the two possible unit rates.	c. Create a table (at right) to show the unit rate increasing with each additional dollar.	<table border="1"><thead><tr><th>Dollars</th><th>Euros</th></tr></thead><tbody><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></tbody></table>	Dollars	Euros								
Dollars	Euros											
b. Which unit rate will help you solve the problem?	d. Describe the number pattern you see in the table?											
e. Since you know the unit price, write a number sentence for finding how many Euros Ms. Vega will receive in exchange for 20 U.S. dollars.												
f. Write an equation to find the amount of Euros received (3 dollars to 2 Euros) for any amount of U.S. dollars (d).												

3. Your dad can buy 4 tires for \$360. He needs 6 new tires for the two family cars. How much will that cost?

a. Write the two possible unit rates.	c. Create a table (at right) to show the unit rate increasing with each additional tire.	<table border="1"> <thead> <tr> <th>Tires</th> <th>Cost</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table>	Tires	Cost								
Tires	Cost											
b. Which unit rate will help you solve the problem?	d. Describe the number pattern you see in the table.											
<p>e. Since you know the unit price, write a number sentence for finding the cost of 6 new tires.</p> <p>f. Write an equation to find the cost of any number of tires (t).</p>												

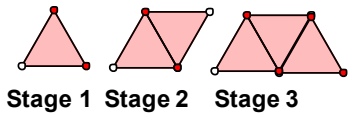
4. At a bank in Chile, Mr. Manycattle wanted to exchange 5000 pesos for U.S. dollars. The exchange rate was 1 U.S. dollars to 500 pesos. How many pesos did Mr. Manycattle receive?

a. Write the two possible unit rates.	c. Create a table (at right) to show the unit rate increasing with each additional dollar.	<table border="1"> <thead> <tr> <th>Dollars</th> <th>Pesos</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table>	Dollars	Pesos								
Dollars	Pesos											
b. Which unit rate will help you solve the problem?	d. Describe the number pattern you see in the table?											
<p>e. Since you know the unit price, write a number sentence for finding how many dollars Mr. Manycattle will receive in exchange for 5000 pesos.</p> <p>f. Write an equation to find the amount of dollars received (1 dollars to 500 pesos) for any amount of pesos (p).</p>												

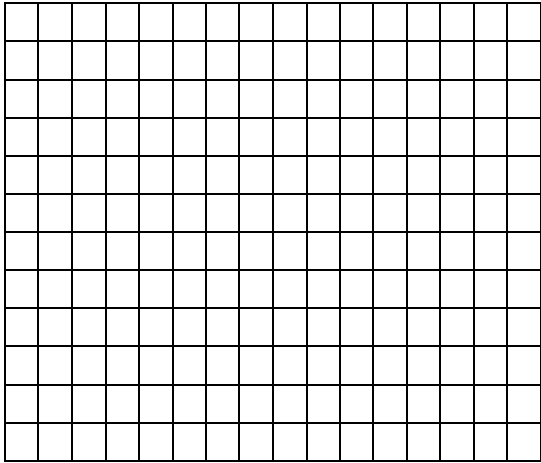
4.3e Classroom Activity: Compare Proportional, Non-Proportional Patterns and Equations

Activity 1: Triangle Train

- a. Fill in the table relating area to perimeter of the triangle trains at right.
- b. Create the graph of that relationship (be sure to mark and label the axes).
- c. Describe the growth of the perimeter as the stage areas increase.



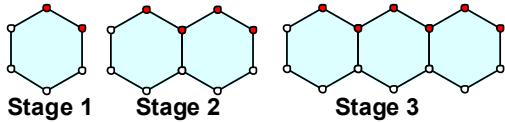
Stage # or area	Perimeter
1	3
2	
3	
4	
5	
10	
100	
x	



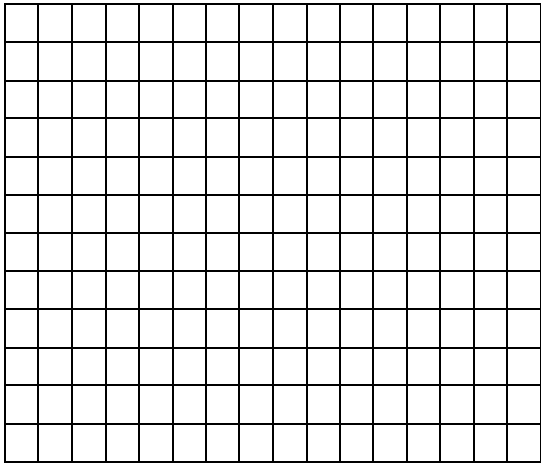
Create the equation. Perimeter (triangle train)=
Is the equation proportional? Why or why not?

Activity 2: Hexagon Train:

- a. Fill in the table relating area to perimeter of the hexagon trains at right.
- b. Create the graph of that relationship (be sure to mark and label the axes).
- c. Describe the growth of the perimeter as the stage areas increase.



Stage # or area	Perimeter
1	6
2	
3	
4	
5	
10	
100	
x	

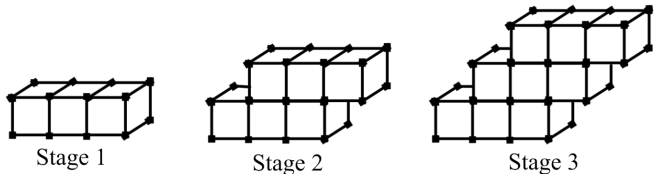


- d. Create the equation. Perimeter (hexagon train) =
Is the equation proportional? Why or why not?

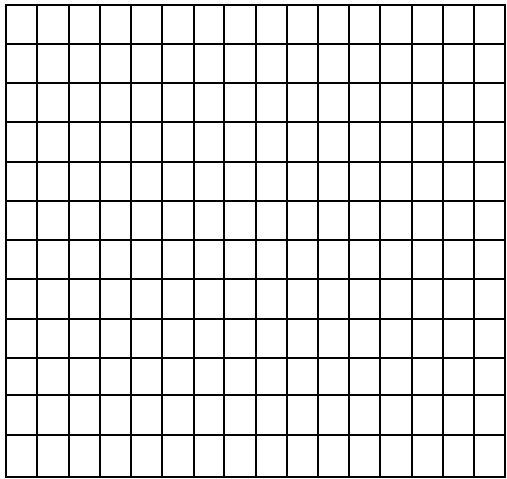
Activity 3: Staircase Volume and Surface Area

Build a 3 unit staircase using linking cubes.

- a. Fill in the table relating surface area to staircase volume.
- b. Create the graph of that relationship (be sure to mark and label the axes).
- c. Describe the growth of the volume as related to the stage.
- d. Describe the growth of the surface area as related to the stage.



Stage #	Volume	Surface Area
1		
2		
3		
4		
5		
10		
25		
x		

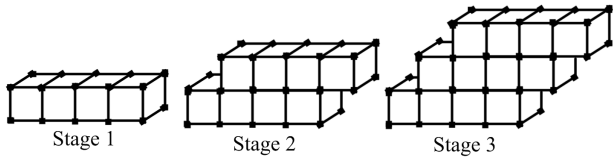


- e. Create the equations. Volume (staircases) =
- f. Are the equations proportional? Why or why not?

Surface Area (Staircases) =

Activity 4: Build a 4 unit staircase. Follow the same steps.

- a. Describe the growth of the volume as related to the stage.
- b. Describe the growth of the surface area as related to the stage.



Stage #	Volume	Surface Area
1		
2		
3		
4		
5		
10		
25		
x		

- c. Create the equations.
Volume (4-unit staircases) =
Surface Area (4-unit staircases) =
Are the equations proportional? Why or why not?

Activity 5. Build a 5 unit staircase like the ones above (Activity 3 was a 3 unit staircase, Activity 4 was a 4 unit staircase). Follow the same steps.

- Describe the growth of the volume as related to the stage.
- Describe the growth of the surface area as related to the stage.

Stage #	Volume	Surface Area
1		
2		
3		
4		
5		
10		
25		
x		

- Create the equations. Volume (5- unit staircases) =
Surface Area (5 unit staircases) =
Are the equations proportional? Why or why not?

Activity 6: Using the patterns found in the equations for all of the previous “staircase patterns,” predict the equations for a **six unit staircase** without building the steps.

- Volume (6-unit staircases) = Surface Area (6 unit staircases) =
Are the equations proportional? Why or why not?
- Use your equations to fill in the blanks in the table.

Stage #	Volume	Surface Area
1		
2		
3		
4		
5		
10		
25		
x		

Spiral Review

1. Write 0.612 as a percent and fraction.
2. If 4 gallons of gas cost \$14.60, how much does 10 gallons of gas cost?
3. Find each product or quotient.
 1. $-6\left(\frac{1}{6}\right)$
 - b. $-10 \div \frac{1}{2}$
4. A mouse can travel 1.5 miles in $\frac{3}{4}$ of an hour. At that pace,
 - a. how far can it travel in 1 hour?
 - b. how long does it take it to travel one mile?

4.3e Homework: Write Equations from Tables, Patterns, Graphs

For each problem #1-5,

- Find the unit rate in the form $\frac{y}{x}$.
- Write a unit rate statement (___ miles per hour)
- Write the equation.

1.

Hours	1	2	3	4	5
Miles	20	40	60	80	100

Unit rate:

Unit rate statement:

Equation:

2.

Hours	3	4	5	6	7
Pay	18	24	30	36	42

Unit rate:

Unit rate statement:

Equation:

3.

Shirts	5	6	7	8	9
Price	28.75	34.50	40.25	46.00	51.75

Unit rate:

Unit rate statement:

Equation:

4.

Minutes	2	4	6	8	10
Problems	3	6	9	12	15

Unit rate:

Unit rate statement:

Equation:

5.

Hour	5	12	21	23	29
Parts	10	24	42	46	58

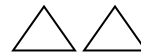
Unit rate:

Unit rate statement:

Equation:

Fill in the table. Find the equation.

6.



Stage 1

Stage 2

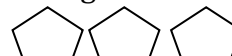
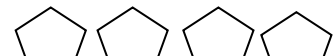
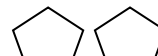
Stage 3

Stage (s)	1	2	3	4
Perimeter units (p)	3			

Equation:

Proportional (yes or no)?

7.



Stage 1

Stage 2

Stage 3

Stage (s)	1	2	3	4
Perimeter units (p)	10			

Equation:

Proportional (yes or no)?

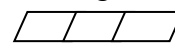
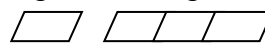
Note: Perimeter involves outside edges. In # 8–9 the inside lines are not parts of the perimeter.

8. DRAW STAGE THREE. Then fill in the table and write the equation for the pattern.

Stage 1

Stage 2

Stage 3

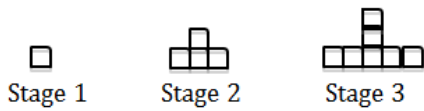


Stage (s)	1	2	3	4
Perimeter (p)				

Equation:

Proportional (yes or no)?

9.

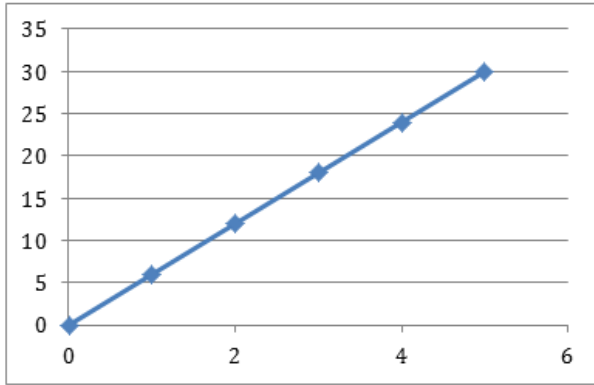


Equation:

Stage (s)	1	2	3	4
Perimeter units (p)	4	10		

Proportional (yes or no)?

10.



Proportional? (yes or no)

Unit rate ($\frac{y}{x}$):

11. Correct the error: The number of pages read and the time spent reading is proportional. Josie fills out the rate table below.

- What mistake did Josie make?
- Use another representation to show that her table doesn't show a proportional relationship.
- Correct the entries in her table.

Pages	Time (minutes)
5	3
10	<i>8</i>
15	<i>13</i>
20	<i>18</i>

} Josie's work

12. Correct the error: Marcus is filling out a rate table below, which shows a proportional relationship between the amount of snow accumulated and the hours it has been snowing at a constant rate.

- What mistake did Marcus make?
- Use another representation to show that his table doesn't show a proportional relationship.
- Correct the entries in his table.

Inches of snow	Hours
2	4
3	<i>9</i>
4	<i>16</i>
5	<i>25</i>

4.3f Classroom Activity and Homework: Time Trials (Rates of Speed)

In this activity, you will compare the rate of speed of two battery operated toy vehicles.

- Create a runway in the classroom. Test out the cars on the runway. You might use the width of the classroom or a certain distance in the hallway.
- Determine what your units of measurement will be. For example, tiles per second (if your classroom has tiles), centimeters per second, inches per second, feet per second, etc.
- Practice timing, (for example counting the seconds on a watch with a second hand or starting and stopping a stop watch).
- Do some trial runs to practice.
 - You might mark time and distance by having one person drop a cube or marker behind the car as the second person counts off each second.
 - You will measure the distance between markers and record in the table below.
- You will probably want to do one car (or toy) at a time.

1. Begin the trial. Collect the data for time and distance. After the data is collected, complete the table.

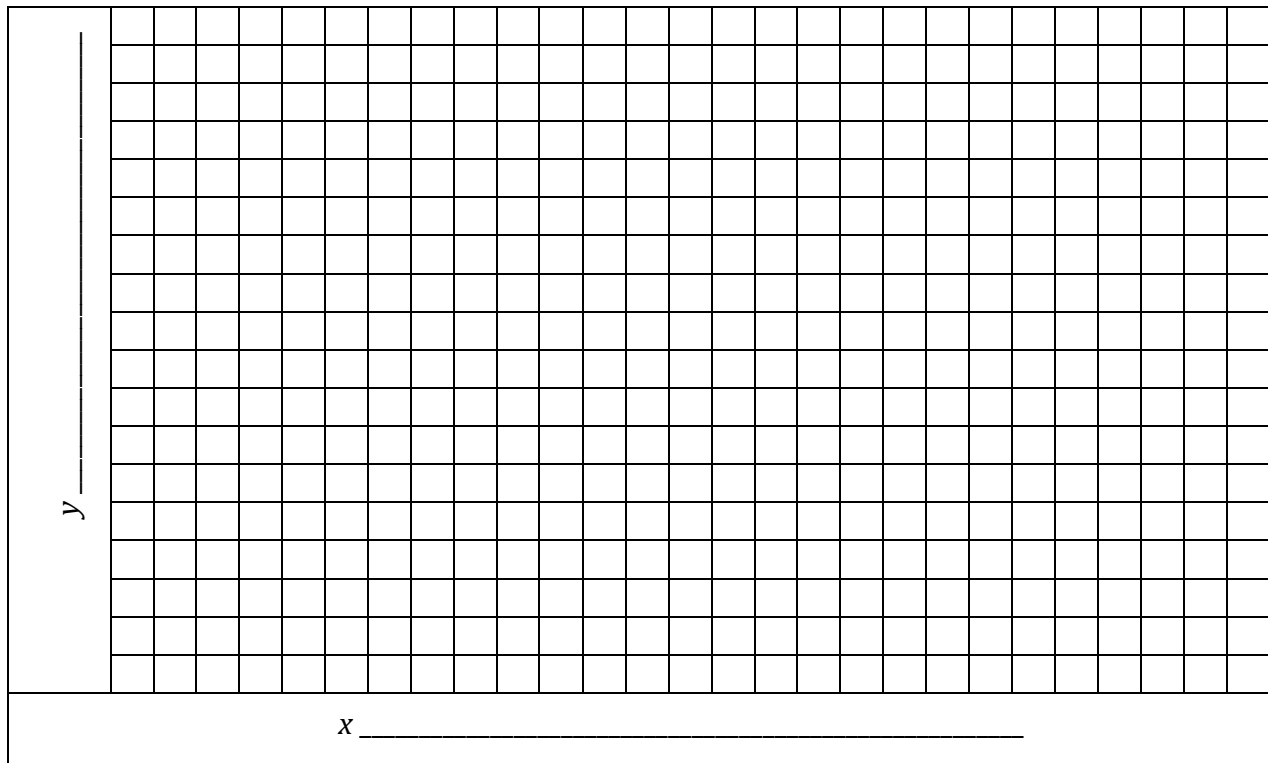
Toy # 1

Time (in sec)	Distance (in ____)	Ratio $\frac{dist}{time}$	Ratio (as a decimal)

Toy # 2

Time (in sec)	Distance (in ____)	Ratio $\frac{dist}{time}$	Ratio (as a decimal)

2. Analyze the data.
 - a. What does the ratio of distance and time tell you?
 - b. Why is it helpful to change the $\frac{dist}{time}$ ratio to a decimal?
 - c. What is the unit rate for the two vehicles? How did you figure it out?
3. Create the graph to chart the data for both vehicles (toys). Label the axes and the graph. Label the graphed lines as 1st and 2nd Vehicle or with the names.



4. Compare the speeds of the two cars. Write your comparison below. Use the words, the math and the graphs to help you. This comparison might involve...
 - a. Explaining the data in the table.
 - b. Explaining the unit rate
 - c. Explaining how you arrived at the unit rate (considering that your measurement tools weren't exactly accurate).
 - d. Describing the graph.

4.3g Class Activity and Homework: Jet Ski Rentals—Proportional vs. Non-proportional Equations

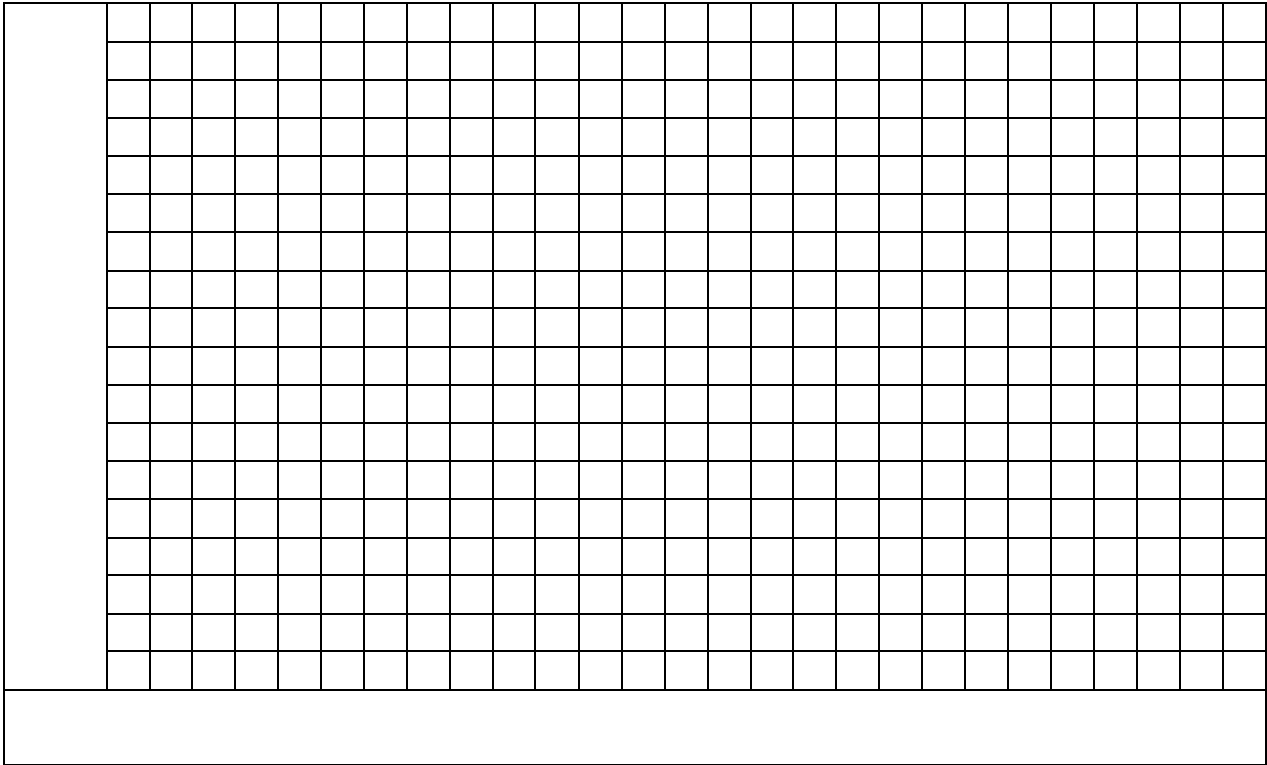
Activity 1: Your dad has researched renting personal water-crafts (jet skis) for your family vacation. He found the following two cost fee plans:

- a. Rental plan A: \$30.00 initial one time rental fee plus \$45.00 per hour.
- b. Rental plan B: \$50.00 per hour with no initial one time rental fee.

Create data tables to show how much it will cost to rent jet skis from each company.

Rental Plan A				Rental Plan B			
	Hours	Expression for Total Cost	Total Cost (in dollars), simplified		Hours	Expression for Total Cost	Total Cost (in dollars), simplified
	1	$\$30(1) + \$45(1)$	\$75		1	$\$50 (1)$	\$50
	2						
	3						
	4						
	5						
	x						

Create graphs for each situation on the same grid. Label the axes. Put a title on the graph. Label graphed lines by plan A or B.



Answer the questions related to the two plans shown on the tables and graphs above.

- How long can you rent a jet ski if your family has budgeted \$350 for the rental fee?
- Look at the tables. Compare the total cost to the number of hours for each row of the table. Is there a set unit rate? Does the table show a proportional relationship for this plan?
- Write a sentence and an equation that relates the total cost, y , and the number of hours, x .
- Use your graph to find the cost to rent one jet ski for 10 hours.
- Explain why and when you would you choose to use each plan.

Plan A	Plan B
a.	a.
b.	b.
c.	c.
d.	d.
e.	e.

4. Explain how you can determine whether or not a relationship is proportional or non-proportional...

a. ...looking at tables.

b. ...looking at graphs.

c. ...looking at equations.

Spiral Review

1. Estimate by rounding to the nearest integer. $3\frac{1}{3} \div \frac{7}{9} \approx \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} \approx \underline{\hspace{1cm}}$

Is your answer an over estimate or under estimate, explain?

2. David is in a submarine at 200 feet below sea level. Casey is above him in a helicopter at 5,900 feet altitude. How far apart are David and Casey?

3. Find each difference without a model.

a. $16 - 29$

b. $-2 - (-8)$

c. $-90 - 87$

d. $5 - (-3)$

4.3h Self-Assessment: Section 4.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Write an equation for a proportional relationship from a table of values. [1 a & b]	I struggle to begin writing an equation for a proportional relationship from a table of values.	I know what an equation for a proportional relationship from a table of values should look like, but I often mix up the parts.	I can write an equation for a proportional relationship from a table of values.	I can write an equation for a proportional relationship from a table of values. I can explain how that equation matches the table of values.
2. Write an equation for a proportional relationship from a graph. [1 c & d]	I struggle to begin writing an equation for a proportional relationship from a graph.	I know what an equation for a proportional relationship from a graph should look like, but I often mix up the parts.	I can write an equation for a proportional relationship from a graph.	I can write an equation for a proportional relationship from a graph. I can explain how that equation matches the graph.
3. Use rate to find a missing value in a table or context. [2]	I struggle to know how to use rate to find a missing value in a table or context.	I can use rate to find a missing value in a table or context involving whole numbers.	I can use rate to find a missing value in a table or context involving whole numbers and decimals.	I can use rate to find a missing value in a table or context. I can explain what it means in my own words.
4. Use ratios to convert between units. [3]	I struggle to use ratios to convert between units.	I can use ratios to convert between units in one of the systems (English or metric).	I can use ratios to convert between units in both English and metric systems.	I can use ratios to convert between units in any direction in both systems.
5. Explain why a relationship is or is not a proportional relationship. [4]	I struggle to recognize if a relationship is or is not a proportional relationship.	I can recognize if a relationship is or is not a proportional relationship, but I struggle to know why or why not.	I can recognize if a relationship is or is not a proportional relationship. I can show how I know with pictures or words.	I can recognize if a relationship is or is not a proportional relationship. I can explain why or why not in my own words.

Sample Problems for Section 4.3

2. Write an equation for each of the following proportional relationships from either a table or a graph.

a.

Months	Books Read
1	2
3	6
5	10
7	14

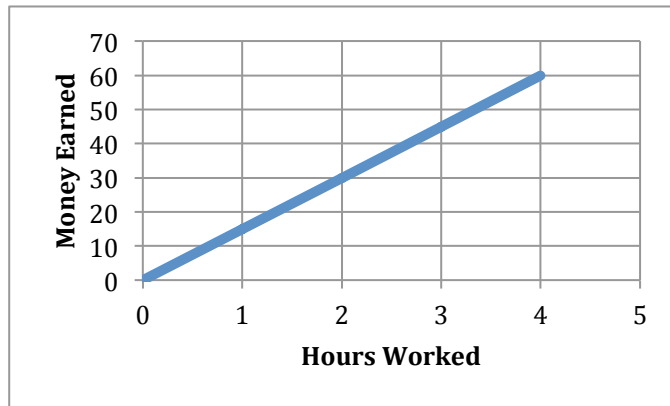
Equation:

b.

Guests at Party	Cost
2	\$5.60
6	\$16.80
10	\$28
14	\$39.20

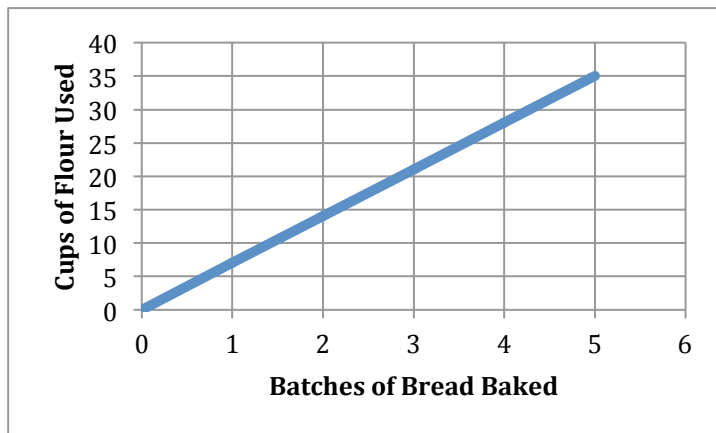
Equation:

c.



Equation:

d.



Equation:

3. Use the rate to find the missing value(s) in each problem:

a. Roxana bikes to and from work every day. She bikes 28 miles in two days. She bikes 42 miles in three days. How many miles does she bike in five days?

b. Chocolate Milkshake Recipe

Chocolate Syrup	Vanilla Ice Cream
?	1 cup
10 Tbsp	?
20 Tbsp	10 cups

<p>c. Filling Buckets of Sand</p> <table border="1"> <thead> <tr> <th>Minutes</th><th>Buckets of Sand</th></tr> </thead> <tbody> <tr> <td>10</td><td>12</td></tr> <tr> <td>?</td><td>3.9</td></tr> <tr> <td>1</td><td>?</td></tr> </tbody> </table>	Minutes	Buckets of Sand	10	12	?	3.9	1	?	<p>d. Tommy works 3 hours and gets paid \$21.90. How much would he get paid for working for 4.5 hours?</p>
Minutes	Buckets of Sand								
10	12								
?	3.9								
1	?								
<p>e. Chocolate Chip Cookies Recipe</p> <table border="1"> <thead> <tr> <th>Cups Sugar</th><th>Cups Flour</th></tr> </thead> <tbody> <tr> <td>$\frac{3}{4}$ cups</td><td>$2\frac{1}{4}$ cups</td></tr> <tr> <td>?</td><td>1 cup</td></tr> <tr> <td>$1\frac{1}{2}$ cups</td><td>?</td></tr> </tbody> </table>	Cups Sugar	Cups Flour	$\frac{3}{4}$ cups	$2\frac{1}{4}$ cups	?	1 cup	$1\frac{1}{2}$ cups	?	<p>f. Sebastian walks $\frac{1}{2}$ mile in $\frac{1}{6}$ of an hour. How far does he walk in a half hour?</p>
Cups Sugar	Cups Flour								
$\frac{3}{4}$ cups	$2\frac{1}{4}$ cups								
?	1 cup								
$1\frac{1}{2}$ cups	?								

4. Convert the following units using the ratios given:

a. 20 mL = _____ L (1 L = 1000 mL)

b. _____ feet = 37 inches (1 foot = 12 inches)

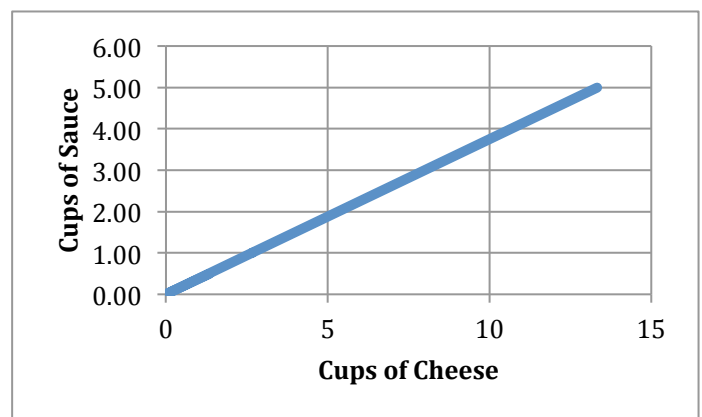
c. $\frac{3}{4}$ tons = _____ pounds (1 ton = 2000 pounds)

5. Look at each of the following situations. Explain why it is or isn't proportional.

a. Recipe for Pizza

Pizza Diameter	Cheese
6 in	78 g
12 in	312 g
18 in	700 g
2 ft	1244 g

b.



Section 4.4: Analyze and Use Proportional Relationships and Models to Solve Real-World and Mathematical Problems.

Section Overview:

The concepts studied in sections 4.1 - 4.3 will be applied throughout this section. The formal definition of a proportion will be introduced, and students will set up and solve proportions for real-world problems, including problems with percentages of increase and decrease. It's likely, especially for advanced classes, that before the topic is formally introduced students will set up proportions and solve them (a) using properties of equality, (b) by finding a common denominator on both sides, or even (c) by cross multiplying, if they have seen this in previous years or at home. Any method that can be justified using proportional reasoning can lead to a meaningful discussion. Students should be encouraged to justify their answer with other representations (graph, bar model, table, or unit rates) to bring out the possible misconception that proportions can be solved with an additive rather than a multiplicative strategy (e.g. $\frac{1}{2} = \frac{3}{6}$ so $\frac{1 \cdot 3}{2} = \frac{3 \cdot 3}{6}$ but $\frac{1+3}{2} \neq \frac{3+3}{6}$).

In addition to solving proportions algebraically, students will examine part-to-whole ratios and compare them to the part-to-part ratios they have been using in the first three sections of the chapter. Odds and probability will be examined together to give students practice in fluently switching between part-to-part and part-to-whole relationships and solving for missing quantities using either type of ratio.

Key Ratio and Proportion Concepts from Utah Core Standards

RP Standard 1:

1. Extend the concept of a unit rate to include ratios of fractions.
2. Compute a unit rate, involving quantities measured in like or different units.

RP Standard 2:

3. Determine if two quantities expressed in a table or in a graph are in a proportional relationship.
4. Determine a unit rate from a table, graph, equation, diagram or verbal description and relate it to the constant of proportionality.
5. Write an equation for a proportional relationship in the form $y = kx$.
6. Explain the meaning of the point (x, y) in the context of a proportional relationship.
7. Explain the significance of $(0,0)$ and $(1, r)$ in a graph of a proportional relationship, where r is the unit rate.

RP Standard 3:

8. Solve multistep problems involving percent using proportional reasoning
9. Find the percent of a number and extend the concept to solving real life percent applications.
10. Calculate percent, percent increase, decrease, and error.

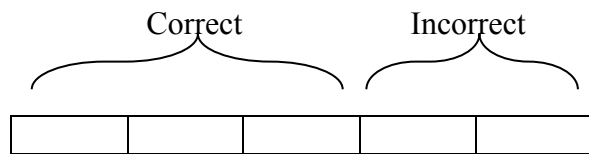
Primary Concepts and Skills to be Mastered in This Section

1. Rewrite a part:part ratio as a part:whole ratio and vice versa.
2. Find any missing value in ratio relationships with a model.
3. Write a proportion to find missing values.
4. Solve a proportion equation.
5. Explain the difference between probability and odds.
6. Solve one- and multi-step problems involving percent using proportional reasoning.

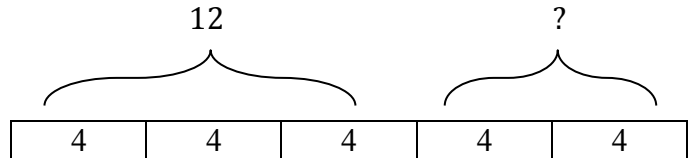
4.4a Classroom Activity: Use Models to Solve Proportional Problems

Review: On a quiz, the ratio of Fernanda’s correct to incorrect answers is 3:2. If she got 12 correct, how many were incorrect and what was the total number of problems?

Model Step 1: Draw the model to match the ratio and label.



Model Step 2: Match the numbers to the model and solve. Since there are 3 parts for 12 correct answers, then each part is worth 4.

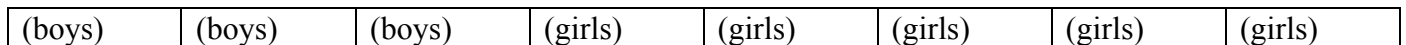


The incorrect number is 8 and the total number of problems is 20.

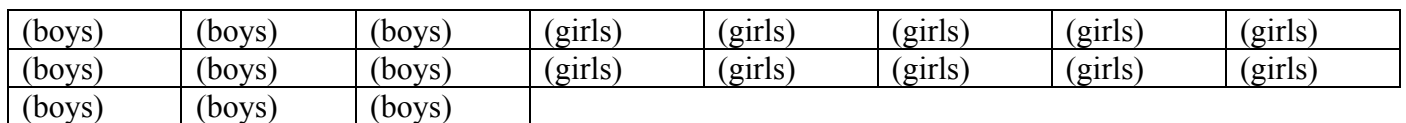
Example 1:

The ratio of boys to girls in math club last year was 3 boys to 5 girls. This year, the amount of boys tripled, while the amount of girls doubled. Now there are 133 kids in math club. How many kids were in the club last year?

This time you have a ratio of 3:5, but you don’t know how many “items” are in each group:



The above diagram represents the club last year. Next modify the diagram to represent the club this year:



Notice there are three times as many sections for the boys, and twice as many for the girls. The entire diagram represents 133 kids. Remember that each section has the same number of items (people) in it.

How many people must be in each section? Then how many kids were in the club last year?

Practice: Solve each problem, using a model or equation as needed.

1. You uncle wants the ratio of flowers and vegetables planted his garden to be 3 flowers : 4 vegetables. He has space for 91 plants. How many flowers should he plant?
2. A recipe for sauce calls for a ratio of 1 part butter to 3 parts chocolate. You want to make a total of 2 pounds of total sauce. How much butter and how much chocolate should you use?
3. The lengths of two ribbons are in the ratio 2:5. If the length of the longer ribbon is 20 inches, find the length of the shorter ribbon.
4. The lengths of three ribbons are in the ration 3:5:7. If the length of the longest ribbon is 42 centimeters, find the lengths of the other ribbons.
5. The number of male teachers is $\frac{2}{3}$ of the number of female teachers in a school. If there are 75 teachers altogether, how many more female teachers than male teachers are there?
6. The ratio of the number of cars to the number of motorcycles is 6:2. If there are 30 cars, how many motorcycles are there?

7. The ratio of the number of cars to the number of vans is 2:1. The ratio of the number of vans to the number of motorcycles is 3:5.
- Draw a model representing cars to vans to motorcycles.
 - If there are 25 motorcycles, how many cars are there? Explain.
 - Find the simplified ratio of the number of cars to the number of vans to the number of motorcycles. Explain.
8. The ratio of the number of Rocio's postcards to the number of Troy's postcards is 2:7. Together, they have 72 postcards. How many postcards does Rocio have?
9. Members of the book club voted for their favorite snack. Four out of every 7 votes were for ice cream. The other votes were for fruit. If ice cream received 32 votes, how many votes did fruit receive?
10. The ratio of time spent on homework to time spent babysitting your little sister was 4:9 yesterday. Today, you spent double the amount of time on your homework, but you spent the same amount of time taking care of your sister. It took you a total of 204 minutes today for homework and babysitting. How many minutes did you spend on homework today?
11. Both Joe and Conner have the same amount of money. If Joe gives $\frac{1}{3}$ of his money to Conner, what will be the ratio of Joe's money to Conner's?
12. Half of Sergio's money is the same amount as one-fourth of Henry's money. If Henry has \$60 more than Sergio, how much do they each have?

4.4a Homework: Use Models to Solve Proportional Problems

Solve each problem, using a model or equation as needed.

1. Zoey made some trail mix by combining raisins and chocolate in a 3:4 ratio. Zoey used 18 ounces of raisins. How many ounces of chocolate were used?
2. For every 2 miles that I run, Rubio runs 5 miles. If I ran 12 miles last week, how many miles did Rubio run?
3. For every 8 sit-ups that Clara can do, Angela can do 3. If Clara did 96 sit-ups, how many more sit-ups did Clara do than Angela?
4. The ratio of the number of rock songs to the number of country songs on my MP3 player is 6:1. If there are 42 rock songs on my MP3 player, how many country songs are there?
5. Rafael, Victor, and Gabriel raised money for the Housing Project in the ratio 6:3:2. Rafael raised \$120. How much money was raised altogether?
6. At first, the ratio of John's money to Peter's money was 4:7. After John spent half of his money and Peter spent \$60, Peter had twice the amount of money as John. How much money did John have at first?
7. Alice, Ben and Carol shared a sum of money they received. Alice received $\frac{1}{5}$ of the money. The rest of the money was divided between Ben and Carol in the ratio 1:3. If Carol received \$6 more than Alice, how much money did Ben receive?

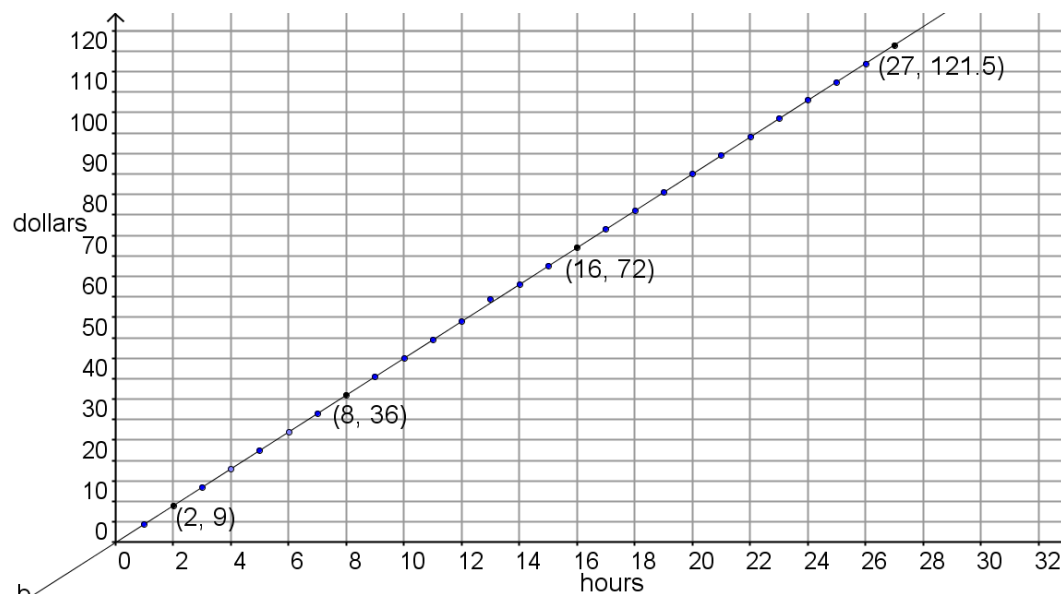
4.4b Classroom Activity: Writing Proportions

Review: Jennifer received \$9.00 for 2 hours of babysitting. At this rate, if she baby-sat for 27 hours last month, how much did she make?

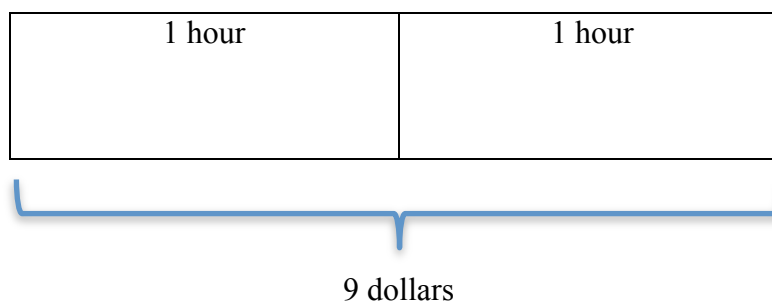
- We have solved similar problems from tables. Solve this problem using the table, and **explain your reasoning in the space next to the table.**

Hours (x)	2	1	27
Dollars (y)	9		

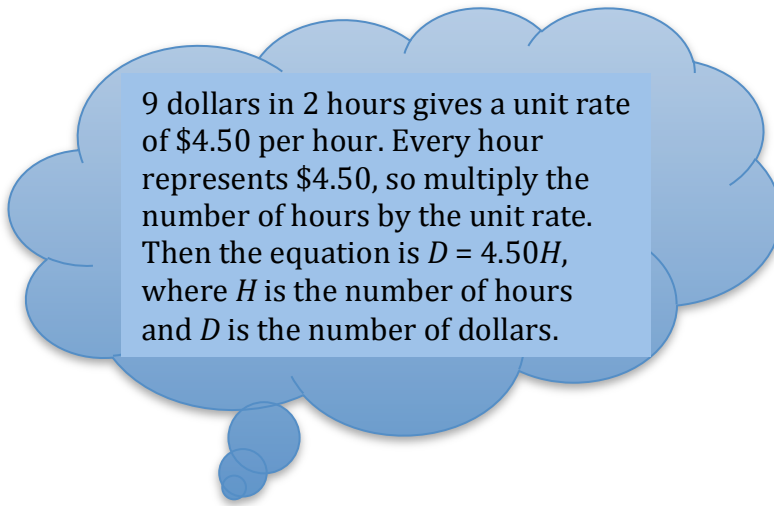
- We have also solved similar problems using graphs. Solve this problem using the graph below, and **explain the reasoning next to the graph.**



- We have also solved similar problems using bar models. Solve this problem using the bar model below, and **explain your reasoning in the space next to the bar model.**



- We have used equations to solve problems like this also. By reading the thought bubble, **explain how the equation can be used to solve the problem.**



- Another way to solve this problem is to set up an equation of two equal ratios, and then solve that equation. An equation showing that two ratios are equal is called a proportion, because it applies to proportional relationships.

$$\frac{a}{b} = \frac{c}{d}$$

This proportion means “ a is to b as c is to d ”

Be sure you are consistent in the units of the quantities you are comparing. In the proportion below, both quantities are comparing dollars to hours:

Proportion showing $\frac{\text{dollars}}{\text{hours}} : \frac{9}{2} = \frac{D}{27}$

“9 dollars is to 2 hours as how many (D) dollars is to 27 hours?”

Is this the only possible proportion for the question? Explain.

What strategies can you use to solve the proportion $\frac{9}{2} = \frac{D}{27}$?

Example 1: A 5 inch frog can leap 50 inches in one jump! If jumping distance were proportional to body length, how far should a 60 inch human be able to jump? Set up two possible proportions.

Solution:

We will first compare body length to jumping distance:

$$\frac{\text{body}}{\text{jumping}} : \quad \frac{5}{50} = \frac{60}{J}$$

Then we will compare jumping distance to body length:

$$\frac{\text{jumping}}{\text{body}} : \quad \frac{50}{5} = \frac{J}{60}$$

Set up two proportions for each problem. Be sure to write the units of your comparisons. You don't need to solve for the missing quantity yet. You will come back and solve these problems as part of 4.4c homework.

1. Pablo won 2 out of 3 chess games one day. At this rate, if he played 33 games, how many did he win?
2. Nalini and Hugo won first and second place in the spelling contest. They had to share a cash prize in the ratio 8:5. Nalini received 72 dollars. How much did Hugo receive?
3. Bargain Betty's sells sneakers and scooters in the ratio 3:8. The store sold 21 sneakers yesterday. How many scooters were sold?
4. In 1.5 hours, a sprint-race sled dog can run 30 miles. How long would it take the dog to run 12 miles?
5. Bliss chocolates have 220 calories for 6 pieces. How many pieces should you eat if you want 150 calories of chocolate?

4.4b Homework: Writing Proportions

For each problem, write the two proportions that represent the problem. You do not need to solve them now. You will come back and solve them as part of the homework for the next section.

1. 12-packs of soda are on sale for 5 for \$12. At this rate, what is the cost for three 12-packs?

2. You are making a cinnamon sugar topping for your Snicker Doodles.
 - a. The ratio of cinnamon to sugar is 1:4. You use 3 tablespoons of cinnamon. How much sugar?

 - b. A different recipe has a ratio of cinnamon to sugar that is 2:3. If you need a total of 20 tablespoons of topping, how much cinnamon and sugar will you need?

3. For your batch of cookies, you'll need 2 cups of chocolate chips for every 5 cups of cookie dough mix.
 - a. You have 30 cups of cookie dough mix, how many cups of chocolate chips?

 - b. You have 1 cup of chocolate chips, how much cookie dough mix do you need?

4. Students are collecting cans of food for the needy. They count cans every 3 days and get an average of 145 cans when they count. At this rate, how many cans will they have at the end of the month (20 school days)?

4.4c Class Activity: Solving Proportions

The following proportion compares two quantities:

$$\frac{3}{x} = \frac{5}{9} \text{ means "3 is to what number, as 5 is to 9?"}$$

What is another comparison that could be made between the two quantities?

Now consider the equation $\frac{x}{3} = \frac{9}{5}$. Whether the numbers came from a ratio with like units, a rate with unlike units, or fractions, we can solve this equation for x using the multiplication property of equality. Example #1 shows a path for the solution. Write a justification of why each step shows an equation with the same solution as the previous step.

Example #1: Write a justification for each step, using algebraic properties.

Equation	Justification
$\frac{x}{3} = \frac{9}{5}$	
$(3)\frac{x}{3} = \frac{9}{5}(3)$	
$\frac{3x}{3} = \frac{27}{5}$	
$x = \frac{27}{5}$	

For the proportions below, **solve using properties of equality**. You may reverse the order of the ratio comparison.

1. $\frac{3}{2} = \frac{12}{x}$

4. $\frac{3}{2} = \frac{y}{1}$

2. $\frac{5}{2} = \frac{8}{x}$

5. $\frac{x}{27} = \frac{4}{6}$

3. $\frac{5}{x} = \frac{2}{3}$

6. $\frac{9}{x} = \frac{6}{5}$

4.4c Homework: Solving Proportions

Solve each proportion using properties of equality.

1. $\frac{5}{3} = \frac{15}{x}$

4. $\frac{5}{3} = \frac{y}{1}$

2. $\frac{2}{4} = \frac{x}{15}$

5. $\frac{5}{x} = \frac{9}{6}$

3. $\frac{7}{3.5} = \frac{9}{x}$

6. $\frac{9}{x} = \frac{12}{8}$

After solving problems # 1 – 6, go back and solve the problems from section 4.4b (where you wrote proportion equations but did not yet solve).

4.4d Class Activity: PART-TO-PART and PART-TO-WHOLE Proportion Problems

Use models and equations to solve the problems below.

- a. Ginger and her brother Cal shared some stamps in the ratio 3:1. If Cal had 5 stamps, how many stamps did Ginger have?

- b. Ginger and Cal have some stamps. Three out of four of the stamps belong to Ginger. How many stamps do they each own if there are 20 total stamps?

1. How are the problems different?

2. Write proportion equations for finding the missing number. Use a model if necessary.
- a. In a group of 72 people, 2 out of every 9 people are left-handed. How many people are right-handed?

- b. Five out of 8 students voted for Jose. The other students voted for Carlos. If 39 students voted for Carlos, how many students voted in all?

3. Rewrite the problems a. and b. from #2 above to be part-to-part. Draw models to solve. Write proportion equations to find the missing number.
- a.
- b.
4. The ratio of rock tunes to jazz tunes on Abby's iPod is 3:2. If Abby has 12 jazz tunes, how many rock tunes does she have? Use a model to solve. Then write a proportion equation for finding the missing number.
5. Rewrite the problem from #4 above to be part-to-whole. Draw a model to solve. Write a proportion equation to find the missing number.

Label the ratio in the problem as part-to-part (pp) or part-to-whole (pw). Draw models if desired.

Write proportion equations to solve. (Be careful! To answer the question, you might have to do two steps.)

- Divide # 6-21 below evenly in your groups of four (count off every fourth problem).
 - All students will do #22.
 - Work in pairs to complete one each of the individually assigned problems.
 - Pair check the other three individual problems.
 - Present problems in groups of four. As students present their problems, other group members complete the problems and critique if needed.
6. Two numbers are in the ratio 9:5. If the smaller number is 20, what is the sum of the two numbers?
7. Clarence the Clown has 20 purple and pink balloons to give out to children at the circus. 1 out of 4 balloons is pink. How many more balloons are purple?
8. One out of every 4 children attending the art show is a girl. There are 21 boys. What is the total number of children at the art show?
9. Tina and Alma shared a cash prize in the ratio 6:7. If Alma received 35 dollars, how much money did Tina receive?
10. Maria downloaded pop tunes and hip hop tunes to her computer in the ratio 3:2. She has 10 hip-hop tunes. How many more pop tunes than hip-hop tunes does Maria have?
11. Jorge built a gaming website. The website had 40 visitors on Monday. Three out of every 8 visitors played Planet Zak. The other visitors played Cosmic Blobs. How many visitors played Cosmic Blobs?
12. For every 4 candy bars that Ella sells, Ben sells three. Ella sold 24 candy bars last month. How many candy bars did Ben sell?
13. The ratio of the weight of Christian's hamster to the weight of Javier's hamster is 1:3. Christian's hamster weighs 7 ounces. How much do the two hamsters weigh together?

14. Gustavo spends 4 out of every 5 dollars he earns on software. He uses the rest of the money to buy snacks. Last month, Gustavo spent 20 dollars on software. How much less money did he spend on snacks?
15. Kevin spends 3 out of every 4 dollars he earns on games. He uses the rest of the money to buy comic books. Last month, Kevin spent 18 dollars on games. How much less money did he spend on comic books?
16. The ratio of the weight of Rachel's radio to the weight of Belle's radio is 5:7. Rachel's radio weighs 35 ounces. How much less does Rachel's radio weigh than Belle's radio?
17. Mrs. Ruiz has 20 students in her class. One out of 4 students stayed after school yesterday for soccer practice. The other students stayed for band. How many students stayed for band practice?
18. For every 7 push-ups Dulce can do, Sara can do 6. If Dulce did 28 push-ups during gym class. How many push-ups did they do altogether?
19. The ratio of the number of comic books in Yazmin's collection to the number of comic books in Itzel's collection is 4:1. Itzel has 6 comic books. How many comic books do they have altogether?
20. A group of fourth grade students voted for their favorite sport. One out of 9 students voted for baseball. The other students voted for basketball. There were a total of 36 votes. How many students voted for basketball?
21. Angela and Diana share some lollipops in the ratio 4:7. Diana has 28 lollipops. How many lollipops do they have altogether?
22. (All students do this problem.) A recipe calls for a ratio of flour and water to be $\frac{1}{2} : \frac{1}{3}$. Sara has 2 cups of flour. How many cups of water does she need?

4.4d Homework: PART-TO-PART and PART-TO-WHOLE Proportion Problems

1. Write proportion equations for finding the missing number and then solve, using a model if necessary.
 - a. Edgar and Raul participated in a team bike-athon. Edgar rode 6 out of every 7 miles in the course. Raul covered the rest of the mileage. If Edgar rode 48 miles, how many miles did Raul ride?
 - b. The school basketball team won 1 out of every 5 games it played this season. The team lost 36 games. Assuming there were no ties, how many games did the team play this season?
2. Rewrite the problems from #1 above to be part-to-part. Draw models to solve. Write proportion equations to find the missing number.
 - a.
 - b.

3. Yesterday, Brainy Games sold robots and chess sets in the ratio 2:3. The store sold 16 robots. How many toys were sold in all? Use a model to solve. Then write proportion equations for finding the missing number.
4. Rewrite the problem from #3 above to be part-to-whole. Draw a model to solve. Write a proportion equation to find the missing number.

Label the ratio in each problem as part-to-part (pp) or part-to-whole (pw). Draw models if desired. Write proportion equations to solve. (Be careful—to answer the question, you might have to do two steps.)

5. For every 3 hats that Lexi sells, Maddie sells two. Lexi sold 12 hats last month. How many hats did they sell in all?
6. For every 7 games the Cherico Cheetahs played, the team won 6. If the Cherico Cheetahs won 30 more games than they lost, how many games did they win?
7. A website conducted a survey about musical tastes. According to the survey, 1 out of 8 people prefers hip hop music. The others prefer rock music. If 56 people took part in the survey, how many more people prefer rock music?
8. On Friday morning, Harmon's had 35 boxes of Captain Crush on the shelf. 1 out of every 5 boxes contained a prize. How many boxes did not contain a prize?
9. Marco and Justin shared a cash prize in the ratio 3:4. If Justin received 16 dollars, how much money did Marco receive?

10. The ratio of girls to boys at the track meet was 4:1. Eric counted 9 boys. How many children were at the track meet altogether?
11. One out of every 4 children attending the soccer game is a girl. There are 27 boys. What is the total number of children at the soccer game?
12. Clarence the Clown has 36 pink and white balloons to give out to children at the circus. 1 out of 4 balloons is white. How many more balloons are pink?
13. The ratio of the length of Jake's wire to the length of Maddie's wire is 1:4. Maddie's wire measures 28 inches. How long is Jake's wire?
14. For every 7 sit-ups Naomi can do, Morgan can do 6. If Naomi did 49 sit-ups, how many more sit-ups did Naomi do than Morgan?
15. On Wednesday morning, Smith's had 99 boxes of Sweet Smacks on the shelf. 4 out of every 11 boxes contained a prize. How many boxes did not contain a prize?
16. Kyle and Megan participated in a team bike-athon. Kyle rode 5 out of every 8 miles in the course. Megan covered the rest of the mileage. If Kyle rode 45 miles, how many fewer miles did Megan ride than Kyle?
17. For every 3 books that Corey sells, Lexi sells one. Corey sold 21 books last week. How many fewer books did Lexi sell than Corey?
18. Will and Abby shared some marshmallows in the ratio 7:3. Abby had 27 marshmallows. How many fewer marshmallows did Abby have than Will?

4.4e Extra Assignment: Use Models or Proportion Equations

Use a model or proportion equations to solve for the missing value.

1. The ratio of cheerleaders to football players at the game is 3:4. There are 21 cheerleaders. How many football players are there?
2. The ratio of purple jellybeans to green jellybeans is 4:9. There are 45 green jellybeans. How many purple jellybeans are there?
3. The ratio of Phineas to Ferb toys at the toy store is 5:11. If there are 35 Phineas toys, how many Ferb toys are there?
4. The ratio of blue birds to gerbils at the pet store is 4:16. At the store there are 10 blue birds. How many gerbils are there?
5. The ratio of triangles to rectangles is 3:7. There are 21 triangles. How many rectangles are there?
6. The ratio of roses to sunflowers is 2:12. There are 4 roses. How many sunflowers are there?
7. The ratio of trumpet players to flute players is 16:23. There are 46 flute players. How many trumpet players are there?
8. In a class, there is a ratio of 3 boys to 2 girls. There are 10 girls. How many boys are there?
9. In a recipe, the ratio of oil to sugar is 3: 1.25. You have 3.75 cups of sugar. How much oil will you need?
10. To make the right paint color, you have a ratio of 4 parts blue: 2 parts yellow: 3 parts brown. You have 16 parts blue. How much yellow and brown do you need to use all the blue paint?
11. To make paste, you need the ratio of $\frac{3}{5}$ cups water to $\frac{3}{4}$ cups flour. If you have 3 cups of water how many cups of flour do you need?

4.4f Classroom Activity: Odds and Probability: Chance Proportions

Activity 1: A school basketball team wins 3 games for every 1 game it losses. So, *the odds* that the school basketball team will lose a game is 1:3. Is this a part-to-part or part-to whole ratio? Explain any parts or wholes.

Rework the statement above to state the probability of the basketball team winning. Would this be part-to-part or part-to whole? Explain any parts or wholes.

What is the difference between odds and probability?

Draw models if desired. Set up proportion equations to find needed information to answer the questions.

1. The chance of getting “heads” when you flip a coin is $\frac{1}{2}$. What are the odds of flipping “heads”? If you flip a coin 80 times, how many times (theoretically) would you expect to get “heads”?
2. The odds of randomly guessing the answer right on a multiple choice test are 1:4. What is the probability of guessing the correct answer? If the test has 20 questions, how many of them would I expect to guess correctly?
3. The chance of rain during April is $\frac{5}{6}$. How many days can you expect rain during the month of April?
4. What is the probability of rolling a number less than 3 on a six-sided number cube? What are the odds? If you roll the cube 27 times, how many rolls (theoretically) would have a number less than 3?
5. The probability of spinning an even number on a given spinner is $\frac{2}{9}$. If you spin the spinner 91 times, approximately how many of them would you expect to be even?
6. A certain student is late for school 3 out of 8 times. What are the odds for him/her to be late? If there are 180 days of school, how many days would he/she likely be late?
7. Cami has a record for winning Tic-Tac-Toe 3 out of 7 times. What are the odds for Cami to win? What is the probability of her winning? How many games would she need to play in order to be likely to win 12 games?

4.4f Homework: Odds and Probability: Chance Proportions

Set up proportion equations to help find needed information to answer the questions. Draw models if desired.

1. The chance of snowfall on any day in February is $\frac{6}{7}$. What are the odds of snow on a day in February? If there are 28 days in February, how many of those days would you expect there to be snow fall?
2. Approximately 12 out of every 100 males are left-handed. What are the odds that a randomly chosen boy is left-handed? Out of 75 boys how many of them would you expect to be left-handed?
3. Over time, you have found that your probability of bowling at least one strike in a game is $\frac{3}{10}$. What are the odds of getting at least one strike in a game? If you bowled 24 games, about how many games would you expect to bowl at least one strike?
4. The odds of finding a four-leaf clover in a clover patch are 1:34. What is the probability that a certain clover has four leaves? If I pick 245 clovers, how many of them would I expect to have four leaves?
5. The odds of getting an even number on a given spinner are 4:3. What is the probability of getting an even number on this spinner? How many times would you expect to spin the spinner if you wanted to land on an even number 32 times?
6. The probability of randomly selecting a pair of blue socks from a given sock drawer is $\frac{5}{9}$. If I randomly draw a pair of socks 45 times, how many of those draws (theoretically) will result in a pair of blue socks?
7. The odds of selecting a green marble from a given bag are 5 : 18. How many times (theoretically) would I need to randomly select marbles in order to pull out a green marble 75 times?

4.4g Classroom Activity: Percent Proportions

Review: Percentage involves part-to-whole relationships. If we use percent in a proportion equation, we write the percent as a fraction out of 100, for example 72% is $\frac{72}{100}$. In a percent proportion equation, we would always be looking to find one of three possible missing pieces of information, the “percent” out of 100, the “part”, or the “whole”. $\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}$.

1. A survey reveals 252 out of 350 students in 7 th grade like to read. What percent like to read? Write a percent proportion equation to solve this question.	2. 72% of the students in 8 th grade like to read and there are 350 students in 8 th grade. How many students in 8 th grade like to read? Write the percent proportion equation to solve this question.	3. Let’s say 72% of the students in 6 th grade like to read and we know that 252 students like to read, but we don’t know how many students are in 6 th grade. Write the percent proportion equation to answer this question.
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Write proportion equations to find the answer to the questions posed below. Then solve the problem.

4. You are taking a math test and you get 32 problems correct on the test. Your grade is 80%. How many problems were on the test?
8. Serena got the same CD as Tony for \$10.50 at another store. What percent did she get off?
5. Your best friend takes a math test with 40 problems. She solves 75% of the problems correctly. How many problems did your friend get correct?
9. Eddie bought a book of sports stories. It was 25% off the original price of \$15. How much did he pay?
6. Another friend gets 36 out of 40 problems correct. What percent did your friend solve correctly?
10. There are 75 students in the eighth grade. 45 of those go to music class. The rest of the students have art. What percent have art? music?
7. Tony bought a CD on sale for \$3 off the original price of \$15. What percent was the CD marked down?
11. Of all the 8th grade students, 60 walk to and from school. If 80% of 8th graders walk to and from school, how many students are in 8th grade?

4.4g Homework: Write and Solve Three Percent Problems:

- Brainstorm as a class to make a list of situations in which you might find percent problems.
- In your groups of four, decide on four different contexts for writing percentage problems.
- Assign one to each member of the group.
- Each member of the group writes three word problems.
 - One in which the percent is not known but the “whole” and “part” are known.
 - One in which the percent and the “whole” are known, but the “part” is not known.
 - One in which the percent and the “part” are known, but the “whole” is not known.
- Each member of the group solves the three word problems he/she wrote.
- Group members present and critique problems and solutions.
- Twelve completed problems are turned in as homework for the group.

4.4h Classroom Activity: Unit Rates and Proportions in Markups and Markdowns

Review:

Activity 1: A sales manager at a clothing store is decreasing prices on six items by 20%.

- a. Draw a bar model to show a decrease of 20% off the original cost of an item.
- b. Suppose one of the items is a scarf that originally cost \$14.50. What would the sale price for the scarf be?
- c. Explain in your own words this relationship: $\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}$
- d. Use the relationship in c. to write a proportional relationship between percentage and the cost of the scarf.
- e. Using your model, proportions, or another method, find the sale price for each item (sale price of each item is 20% off original price):

Original price (x)	\$14.50	\$18.00	\$25.25	\$40.10	\$39.95	\$5.00
Sale price (y)						
- f. What is the unit rate?
- g. Convert your unit rate into a percentage and use your model to explain why this makes sense.
- h. Write an equation for y , the sale price. Verify that your equation is true for two columns in the table.
- i. If the sale price is \$29.80 for a pair of shoes at this store, how much was the original price?
- j. Use a calculator or graph paper to graph the ordered pairs for (original price, sale price). What do you notice?

Activity 2: A used car dealership marks up the cost of the cars they buy at auctions by 75%.

- Draw a bar model for increasing by 75%, where the original bar represents the original cost of the car at auction.
- If 100% is the original whole, what part of the original is the new amount? Justify your answer.

- Use a proportion in the form $\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}$ to find the selling price (part) for a car whose original auction price (whole) was \$1500.

- Using your model, proportions, or another method, find the selling price if there is always a 75% mark-up of the auction price:

Auction price (x)	\$1500	\$1900	\$1750	\$6400	\$9350	\$6044
Selling price (y)						

- What is the unit rate for $\frac{\text{selling price}}{\text{auction price}}$?
- Write an equation for y, the selling price. Verify that your equation is true for two columns in the table.
- If you go to the car dealership and see a selling price of \$7700, how much did the car cost at auction? Justify your answer.
- Using a graphing calculator or on graph paper, graph the ordered pairs (auction price, selling price). What do you notice?

Activity 3: Find the missing price for each. Use a unit rate, proportional constant, proportions or models to solve, showing your work.

a. Original price: \$16.20
Increase by 40%
Final price:

f. Original price: \$55.00
Increase by 150%
Final price:

b. Original price: \$1050.00
Increase by 0.5%
Final price:

g. Original price: \$100.00
Decrease by 20%
THEN increase by 20%
Final price:
How does this compare with the original price?

c. Original price: \$19.80
Decrease by 5%
Final price:

h. Original price: \$80
Increase by 15%
THEN decrease by 15%
Final price:
How does this compare with the original price?

d. Original price:
Decrease by 85%
Final price: \$12.99

i. Original price: \$40.00
Decrease by 25% (product is on sale)
THEN increase by 6% (pay sales tax)
Final price:

e. Original price:
Increase by 180%
Final price: \$81.20

j. Original price: \$150.00
Decrease by 30% (item is on sale)
THEN decrease by 10% (apply store coupon)
Final price:

Activity 4. Find the unit rate for each of problems a – f above in activity 3.

4.4h Homework activity: Unit Rates in Markups and Markdowns

1. All the dimensions of a drawing must be decreased by 15%.
- a. Draw a bar model for decreasing by 15%, where the original bar represents the original length of an object in the drawing.

- a. Using your model or another method, find the length if it is decreased by 15%:

Original length (x)	18 cm	9.6 cm	12 cm	1 cm	22.8 cm	32 cm
Final length (y)						

- b. What is the unit rate of $\frac{\text{length in final picture}}{\text{length in original picture}}$?
- c. Convert your unit rate into a percentage and use your model to explain why this makes sense.
- d. Write an equation for y , the final length. Verify that your equation is true for two columns in the table.
- e. If the roof of a barn in the final picture is 20.4 cm long, how long was the roof of the barn in the original picture?

2. At the bake sale fundraiser, George marks up all items 40% to make a profit for the math club.
- a. Draw a bar model for increasing by 40%.

- b. Using your model or another method, find the new price if the original is increased by 40%:

Original price (x)	\$0.50	\$1.00	\$0.75	\$1.05	\$3.00	\$2.50
Sale price (y)						

- c. What is the unit rate of $\frac{\text{sale price}}{\text{original price}}$?
- d. Convert your unit rate into a percentage and use your model to explain why this makes sense.
- e. Write an equation for y , the sale price. Verify that your equation is true for two columns in the table.
- f. If you go to the bake sale and purchase a brownie for \$3.15, how much did the brownie originally cost? Justify your answer.

3. Find the missing price for each. Use a unit rate, bar model, or equation to solve. Show your work.

a. Original price: \$22.00

Increase by 40%

Final price:

f. Original price: \$35.00

Increase by 250%

Final price:

b. Original price: \$9000.00

Decrease by 0.5%

Final price:

g. Original price: \$60.00

Decrease by 25%

THEN increase by 25%

Final price:

Compare the final price to original price and explain any differences.

c. Original price: \$39.80

Decrease by 10%

Final price:

h. Original price: \$100.00

Decrease by 30%

THEN increase by 10%.

Final price:

d. Original price:

Decrease by 85%

Final price: \$25.98

i. Original price: \$72.00

Increase by 12.5%

THEN increase by 10%

Final price:

e. Original price:

Increase by 120%

Final price: \$66.00

j. Original price: \$900.00

Decrease by 40%

THEN decrease by 5%

Final price:

4.4i Anchor Problem Revisited: Purchasing Tunes-Town Music Company

I. Considering all you've learned in chapter 4, revisit this problem. How might you enhance your original response to this problem (which we considered at the beginning of chapter 4)? Prepare a response to present to the class.

What's the Better Deal

(From illustrative mathematics.org)

Beat-Street, Tunes-Town, and Music-Mind are music companies. Beat-Street offers to buy 1.5 million shares of Tunes-Town for \$561 million. At the same time, Music-Mind offers to buy 1.5 million shares of Tunes-Town at \$373 per share.

1. Who would get the better deal, Beat-Street or Music-Mind? Why?
2. What is the total price difference?

II. Now consider a different situation for Purchasing Tunes-Town Music Company. Prepare a response to present to the class.

Price Per Share (variation 2)

(from illustrative mathematics.org)

Beat-Street, Tunes-Town, and Music-Mind are music companies. Beat-Street and Music-Mind are teaming up together to make an offer to acquire 1.5 million shares of Tunes-Town worth \$374 per share. They will offer Tunes-Town 20 million shares of Beat-Street worth \$25 per share. To make the swap even, they will offer another 2 million shares of Music-Mind.

What price per share (in dollars) must each of the shares of Music-Mind be worth?

4.4j Self-Assessment: Section 4.4

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Corresponding sample problems, referenced in brackets, can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Rewrite a part:part ratio as a part:whole ratio and vice versa. [1, 2]	I struggle to begin rewriting a part:part ratio as a part:whole ratio and the other way around.	I can rewrite the ratio in the other form only if I use a model.	I can rewrite the ratio in the other form without drawing a model.	I can rewrite the ratio in the other form without a model. I can justify how I arrived at my answer.
2. Find any missing value in ratio relationships with a model. [3]	I struggle to know how to find a missing value in ratio relationships with a model.	I can draw a model of a ratio relationship, but I can't always use that model to find a missing value in ratio relationships.	I can find a missing value in ratio relationships with a model.	I can find a missing value in ratio relationships with a model. I can connect my model to other methods of finding missing values.
3. Write a proportion to find missing values. [4]	I struggle to know how to use a proportion to find a missing value.	I can find a missing value when given a proportion, but I struggle to write my own proportions.	I can write and use a proportion to find a missing value.	I can write and use a proportion to find a missing value. I can explain how the proportion relates to the context.
4. Solve a proportion equation. [5]	I don't know what a proportion equation is.	I can solve a proportion equation with whole numbers.	I can solve any proportion equation.	I can solve any proportion equation. I can explain how a proportion works to accurately find the answer.
5. Explain the difference between probability and odds. [6]	I struggle to know the difference between odds and probability.	I can recognize the difference between odds and probability.	I know the difference between odds and probability. I can solve problems with either odds or probability.	I can explain the difference between odds and probability. I can solve problems with either odds or probability.
6. Solve one- and multi-step problems involving percent increase/decrease using proportional reasoning. [7]	I struggle to solve percent increase / decrease problems using proportional reasoning.	I can solve percent increase / decrease problems using proportional reasoning if they only have one step.	I can solve multi-step percent increase / decrease problems using proportional reasoning.	I can solve multi-step percent increase / decrease problems using proportional reasoning. I can explain the reasoning behind each step.

Sample Problems for Section 4.4

1. Rewrite the following part:part ratios as part:whole ratios. Include labels of units.
 - a. The ratio of boys to girls in Gabrielle's family is 3:8.
 - b. On a recent quiz, Felipe's ratio of correct to incorrect problems is 5:1.
 - c. A slushie recipe calls for 2 parts frozen fruit to 1 part juice.
2.
 - a. Rewrite the following part:whole ratio as a part:part ratio.
Eida likes to travel. The ratio of states Eida has visited to the total states is 19:25.
 - b. Rewrite the following part:part ratio as a part:whole ratio.
3. Find the missing information given the ratios in each problem using a model. (Extra challenge: connect your model to another method of finding the missing information.)
 - a. Another slushie recipe calls for a 3:1 ratio of water to juice. If you use one cup water, how much juice would you use?
 - b. The ratio of two numbers is 8:9. If the larger number is 18, find the sum of the two numbers.
 - c. Daniel got 4 out of every 5 questions correct on a recent multiple choice test. If he got 64 questions correct, how many did he miss?
4. Write a proportion equation and use it to find the missing information in each problem.
 - a. At the local city park, the ratio of ducks to geese is 25:9. If there are 18 geese, how many ducks are there?
 - b. Scientists tag fish in a pond. When they check back a few months later, the ratio of tagged to untagged fish is 1:6. If they captured 21 fish, how many fish were tagged?
 - c. In the above fish pond, they originally tagged 20 fish. How many fish would the scientists predict are in the pond?

5. Solve the following proportion equations:

a. $\frac{3}{4} = \frac{15}{x}$	b. $\frac{9}{x} = \frac{15}{25}$	c. $\frac{x}{6} = \frac{1}{4}$	d. $\frac{4}{27} = \frac{x}{3}$
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6. Alex and Ben love to play with toy cars. One day, Ben brings his 12 cars over to play with Alex and his 15 cars. Alex's baby sister comes and grabs one of the cars.

a. What is the probability that she picks one of Alex's cars?

b. What are the odds that she picks one of Alex's cars?

c. Explain how the answers to part a. and part b. are similar. Explain how they are different.

7. Use proportional reasoning (unit rate, proportional constant, and/or proportion equations) to solve for the following missing price:

a. Original price: \$65.00

Decrease by 30%

Final price:

d. Original price:

Increase by 120%

Final price: \$127.60

b. Original price: \$36.50

Increase by 45%

Final price:

e. Original price: \$125.00

Decrease by 20% (product is already on sale)

THEN decrease by 15% (extra coupon is applied)

Final price:

c. Original price:

Decrease by 20%

Final price: \$67.20

f. Original price: \$59.00 (wholesale price)

Increase by 50% (store mark up)

THEN decrease by 25% (product is on sale)

Final price:

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Chapter 5: Geometric Figures and Scale Drawings (3-4 weeks)

UTAH CORE Standard(s)

Draw construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. 7.G.1
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. 7.G.2
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. 7.G.A3

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. 7.G.4
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. 7.G.5
6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. 7.G.6

CHAPTER 5 OVERVIEW:

Chapter 5 builds on proportional relationships studied in chapter 4 and extends those ideas to scaling and the affects of scaling. The chapter starts by expiring conditions under which one can construct triangles and when conditions establish unique triangles. From there, students move in section 2 to scaling objects and exploring how scaling a side of an object affects the perimeter and area measures. Section 3 helps students understand that all circles are scaled versions of one another and how that fact allows us to connect the diameter of a circle to its circumference and area. Finally in section 4 students use angle relationships to find missing angles.






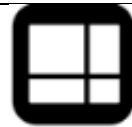


VOCABULARY: acute triangle, angle, congruent, corresponding parts, equilateral triangle, included angle, included side, isosceles triangle, obtuse triangle, right triangle, scalene triangle, triangle inequality theorem.

CONNECTIONS TO CONTENT:

Prior Knowledge: In elementary school students found the area of rectangles and triangles. They measured and classified angles and drew angles with a given measure. Though students learned about circles informally, haven't learned how to find circumference and area.

Future Knowledge: The term “similarity” is formally defined in the 8th grade. In 7th grade we will say “same shape”. In 8th grade students will justify that the angles in a triangle add to 180° and will extend that knowledge to exterior angles and interior angles of other polygons. In 8th grade students will extend their understanding of circles to surface area and volumes of 3-D figures with circular faces. In 9th grade students will formalize the triangle congruence theorems (SSS, SAS, AAS, ASA) and use them to prove facts about other polygons. In 8th grade students will expand upon “same shape” (scaling) and extend that idea to dilation of right triangles and then to the slopes of lines. In 10th grade students will formalize dilation with a given scale factor from a given point as a non-rigid transformation (this will be when the term “similarity” will be defined) and will solve problems with similar figures. The understanding of how the parts of triangles come together to form its shape will be deepened in 8th grade when they learn the Pythagorean Theorem, and in 11th grade when they learn the Law of Sines and Law of Cosines.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

	Make sense of problems and persevere in solving them.	Students will analyze pairs of images to determine if they are exactly the same, entirely different or if they are the same shape but different sizes. With this information they will persevere in solving problems.
	Reason abstractly and quantitatively.	Students will find scale factors between objects and use them to find missing sides. They will also note that proportionality exists between two sides of the same object. Students should move fluidly from $a:b = c:d \rightarrow a:c = b:d$, etc. and understand why all these proportions are equivalent. Further, students should fluidly transition between proportionality and scale factor. E.G. If $\triangle ABC: \triangle DEF$ is 2:3 then the scale factor that takes $\triangle ABC$ to $\triangle DEF$ is $3/2$.
	Construct viable arguments and critique the reasoning of others.	Students should be able to construct a viable argument for why two objects are scale versions of each other AND how to construct scale versions of a given object. Further, students should be able to explain why, for example, if the scale between two objects is 5:3 then a length of 20 on the first object becomes 12 on the new object by using pictures, words and abstract representations.
	Model with mathematics.	Students should be able to create a model (table of values, bar model, number line, etc.) to justify finding proportional values. Additionally, students should be able to start with a model for a proportional relationship and then write and solve a mathematical statement to find missing values.
	Attend to precision	Students should attend to units throughout. For example, if a scale drawing is $1\text{ mm} = 3\text{ miles}$, students should attend to units when converting from 4mm to 12 miles. Students should also carefully attend to parallel relationships: for example for two triangles with the smaller triangle having sides a, b, c and a larger triangle that is the same shape but different size with corresponding sides d, e, f , the proportion $a:d = b:e$ is equivalent to $a:b = d:e$ but sets up relationships in a different manner.
	Look for and make use of structure	Students will link concepts of concrete representations of proportionality (bar models, graphs, table of values, etc.) to abstract representations. For example, if a length 20 is to be scaled down by a factor of 5:3 one can think of it as something times $(5/3)$ is 20 OR 20 divided by 5 taken 3 times.
	Use appropriate tools strategically.	By this point, students should set up proportions using numeric expressions and equations, though some may still prefer to use bar models. Calculators may be used as a tool to divide or multiply, but students should be encouraged to use mental math strategies wherever possible. Scaling with graph paper is also a good tool at this stage.
	Look for and express regularity in repeated reasoning	Students should connect scale to repeated reasoning. For example if the scale is 1:3 then each length of the shorter object will be multiplied by 3 to find the length of the larger scaled object; then to reverse the process, one would divide by three.



from: <http://disneyscreencaps.com/wreck-it-ralph-2012/5/#/>

Cartoon characters are supposed to be illustrated versions of human beings. In a way, we could think about a cartoon character as a scale drawing of a human.

What if Wreck-it Ralph was a scale drawing of you!? If you were Wreck-it Ralph but still at your current height, how tall would your head be?

How big would your hands be?

How long would your legs be?

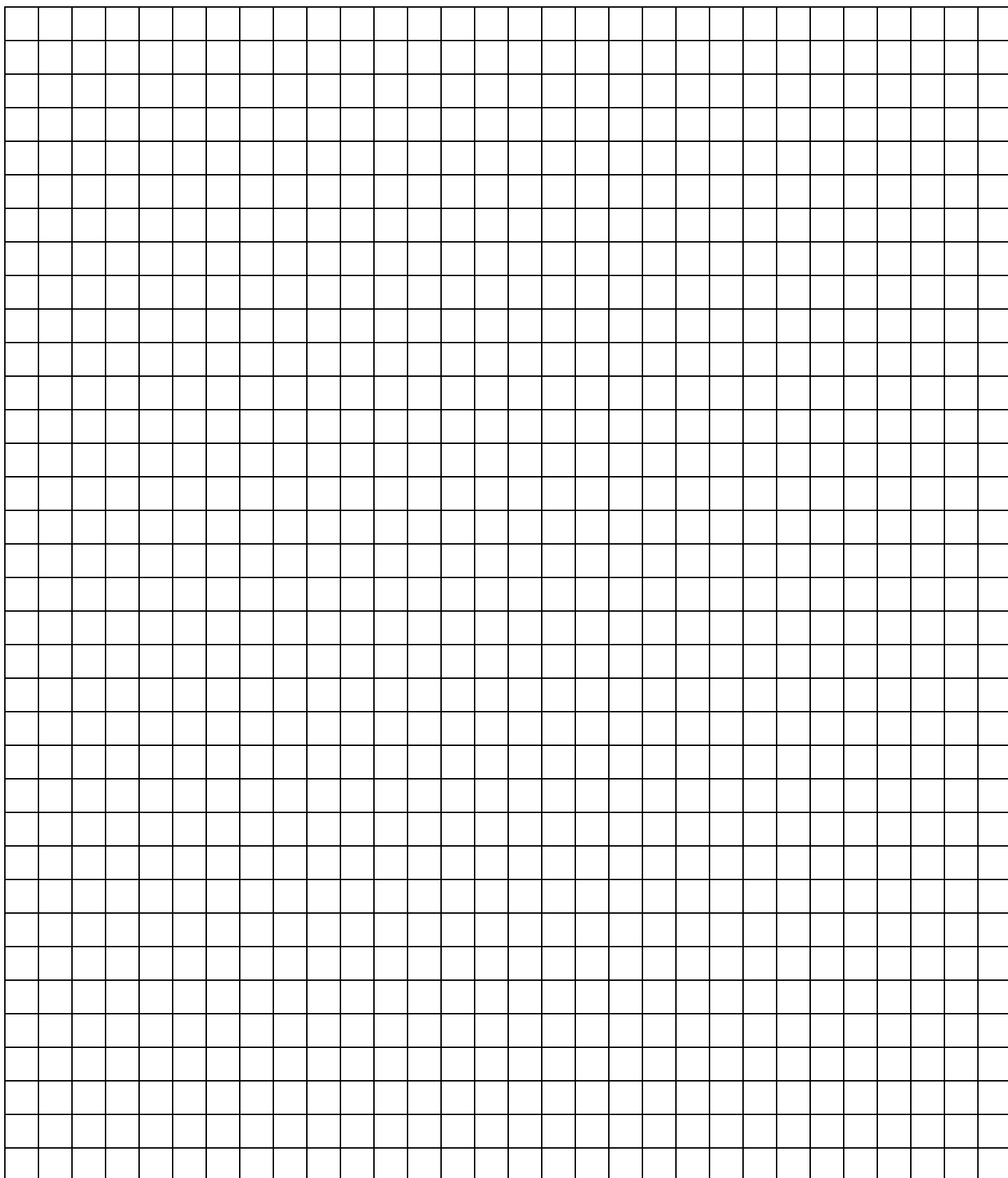
5.0a Chapter Project: Constructing Scale Drawings 1

Task: Making a scale drawing of the top of your desk

- Use measuring tools and scissors to cut a piece of letter-sized plain paper or graph paper to be a scale model of the top of your desk.
- Write the scale on the corner of the paper.
- Place *at least* 3 items on your desk. For example, a pencil, eraser, book, soda can, cell phone, etc.
- Fill in the table of values below with measurements or calculations. You can choose what to measure for the five blank rows. The blanks in the column headings are to write what units you measured in (cm, inches, mm, units of graph paper, etc.)

Item measured	Real measurement is	Measurement in scale drawing is
	_____	_____
A) Short side of top of desk		
B) Long side of top of desk		
C)		
D)		
E)		
F)		
G)		

- Use measuring tools to draw your three items, to scale and in the same position as they are on your desk, on your scale drawing.
- Graph the information from your table on the grid on the next page, labeling each point with the letter A-G from the row of the table. Be sure to label the axes.

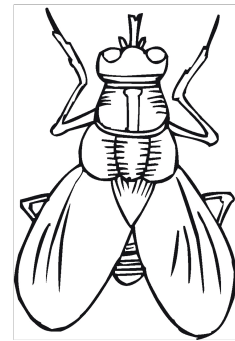
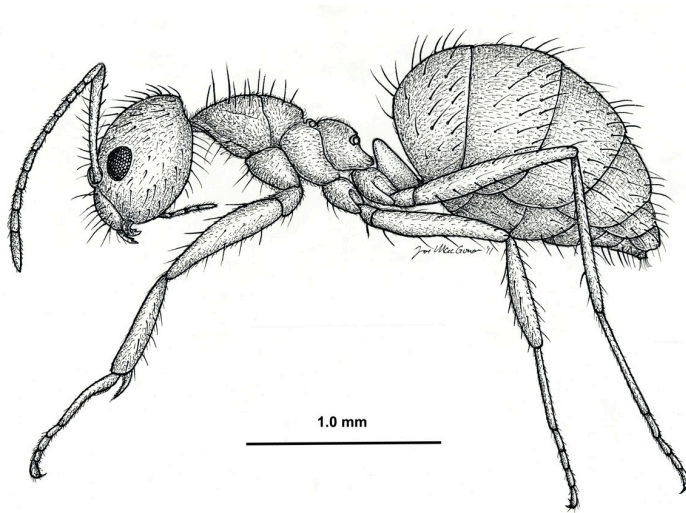


5.0a Chapter Project: Constructing Scale Drawings 2

Task: Making a scale drawing as a class. Choose *one* of the images below and make a scale drawing.

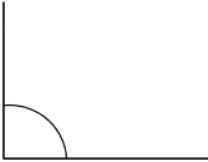


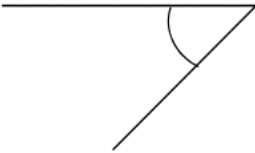
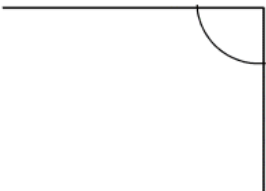



Below is an image of an ant (Nylanderia Pubens Worker ant) and a common house fly (Diptera).

- Use measuring tools to make a scale drawing of either the ant or fly picture below.
- Determine your scale.



5.0b Review: Angle Classification and Using a Protractor

Review Angles: For each angle below, a) use a protractor to find the measurement in degrees and b) write its classification (acute, obtuse, right or straight.)

Angle	Degrees	Classification	Angle	Degrees	Classification
1. 	_____	—	2. 	_____	
3. 	_____		4. 	_____	
5. 	_____		6. 	_____	
7. 	_____		8. 	_____	

Section 5.1 Constructing Triangles from Given Conditions

Section Overview:

In this section, students discover the conditions that must be met to construct a triangle. Reflecting the core, the approach is inductive. By constructing triangles students will note that the sum of the two shorter lengths of a triangle must always be greater than the longest side of the triangle and that the sum of the angles of a triangle is always 180 degrees (*see the mathematical foundation for a discussion of these ideas.*) They then explore the conditions for creating a unique triangle: three side lengths, two sides lengths and the included angle, and two angles and a side length—whether or not the side is included. This approach of explore, draw conclusions, and then seek the logical structure of those conclusions is integral to the new core. It is also the way science is done. In later grades students will more formally understand concepts developed here.

In 7th grade, students are to learn about “scaling.” The concepts in this section are foundational to scaling and then lead to proportional relations of objects that have the same “shape” in section 5.2. In 8th grade, students will extend the idea of scaling to dilation and then in Secondary I to similarity. The word “similar” may naturally come up in these discussions; however, it is best to stay with an intuitive understanding. A definition based on dilations will be floated in eighth grade, but will not be fully studied and exploited until 10th grade. In this section emphasis should be made on conditions necessary to create triangles and that conditions are related to knowing sides and angles.

Concepts and Skills to be Mastered (from standards)

Geometry Standard 2: Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. 7.G.2

Geometry Standard 6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. 7.G.6

1. Given specific criteria, construct a triangle and know if it's a unique triangle
2. Understand and explain the side length requirements of a triangle.
3. Understand and explain the angle and side requirements for constructing a unique triangle.
4. Know the angle measure requirements of a triangle. (Triangle Angle Sum Theorem)

5.1a Class Activity: Triangles and Labels—What’s Possible and Why?

Review Triangle: Write a description and sketch at least one example for the following terms:

a. Acute triangle:

b. Obtuse triangle:

c. Right triangle:

d. Equilateral triangle:

e. Isosceles triangle:

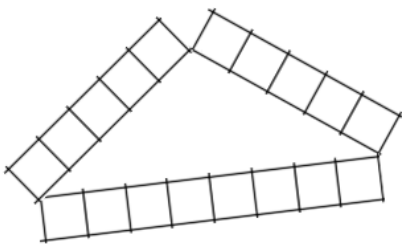
f. Scalene triangle:

Activity: The Engineer’s Triangle: How many different triangles can an engineer make out of an 18 unit beam?

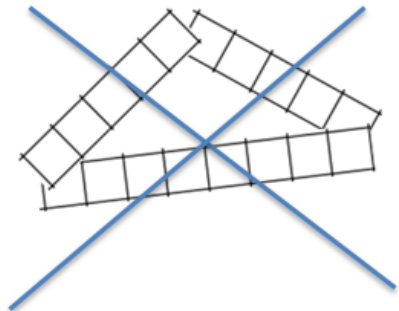
You’re an engineer and you need to build triangles out of 18 foot beams. How many different triangles can you make with an 18 foot beam? What do you notice about the sides of different triangles? What do you notice about the angles of different triangles?

In groups of 2-4, cut out several “18 foot beams.” Your group’s task is to make different triangles with your “beams.” For each triangle you construct, use the table below to classify it by angles and sides.

Be careful to line up the **exact corners** of the strips, like this:



NOT at the center, like this:



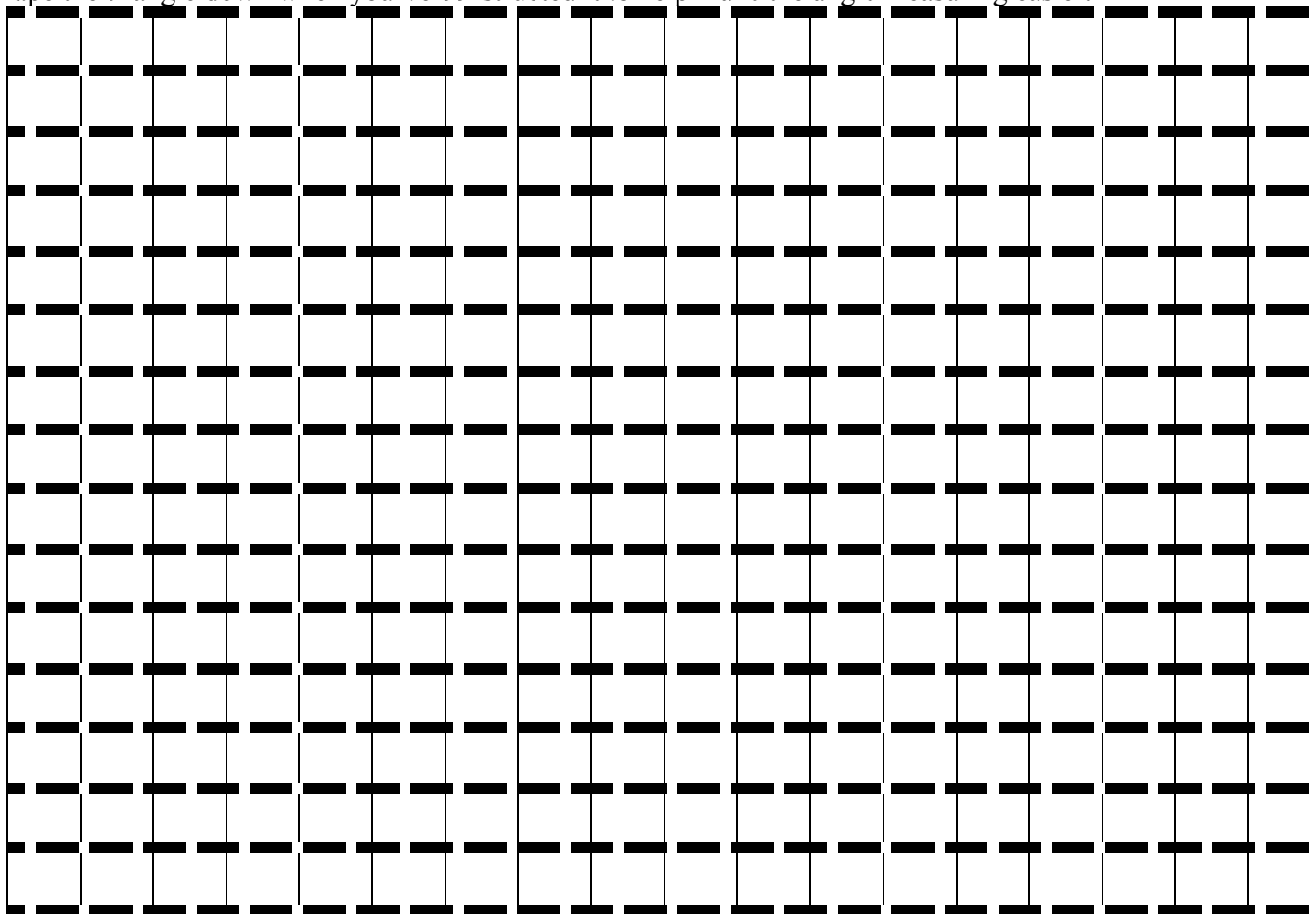
GROUP RECORDING SHEET

For each triangle you construct: 1) record the length of each side, 2) the measure of each angle, 3) classify by angle and 4) classify by side. Pay attention to patterns. If you discover a pattern, write it down your conjecture.

The lengths of each side. Be sure to state units.	The measure of each angle, to the nearest 5° .	Classify each triangle by side: Scalene, Isosceles, or Equilateral	Classify each triangle by angle: Right, Acute, or Obtuse
A.			
B.			
C.			
D.			
E.			
F.			
G.			
H.			
I.			
J.			
K.			

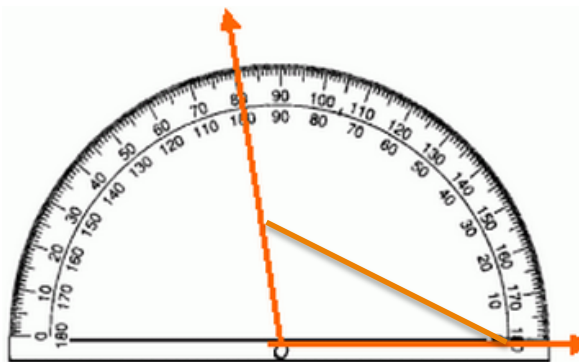
Write the three numbers for the lengths of the side dimensions.	Write the three numbers for the angle measures, to the nearest 5° .	Write whether the triangle was Right, Acute, or Obtuse	Write whether the triangle was Scalene, Isosceles, or Equilateral
L.			
M.			
N.			
O.			
P.			
Q.			
R.			
S.			
T.			
U.			

Instructions: Cut along the dotted lines to get strips 18 units long. Each unit represents one foot. For each trial, one member of your group will cut the strip into three pieces for the three side lengths of a possible triangle. Tape the triangle down when you've constructed it to help make the angle measuring easier.



Review: Using a protractor to find the measure of the angle.

Use the *inside* row,
since this an obtuse
angle. Thus, the angle
is 100° , not 80° .



1. Is there more than one way to put together a triangle with three specific lengths?

2. What pattern do you see involving the sides of the triangle?



3. What pattern do you see involving the angles of the triangle?

4. For each group of three side lengths in inches, determine whether a triangle is possible. Write yes or no, and justify your answer.

a. $14, 15\frac{1}{2}, 2$

b. $1, 1, 1$

c. $7, 7, 17$

d. $3, 9, 4$

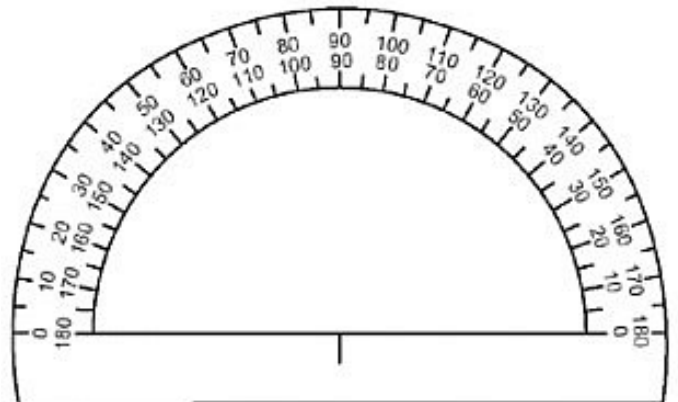
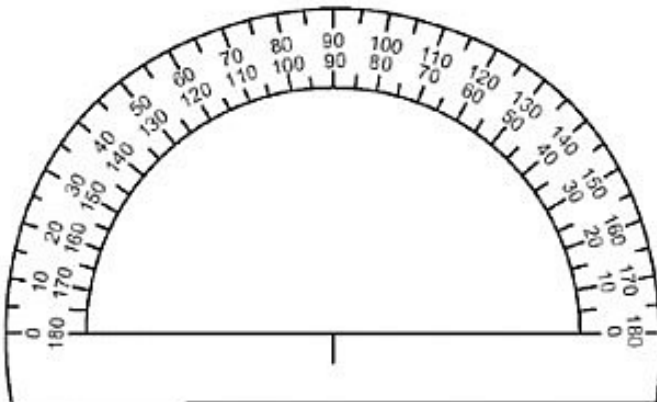
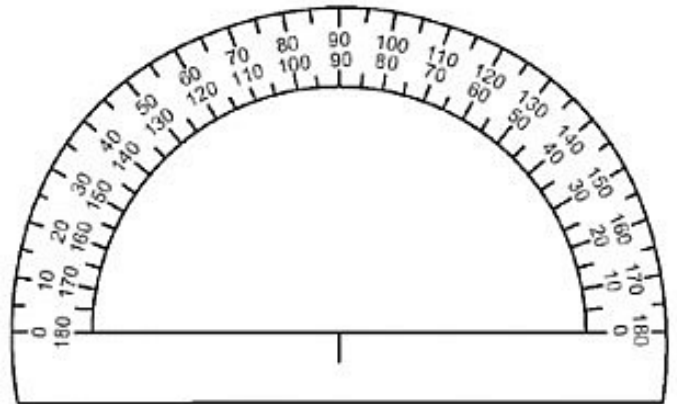
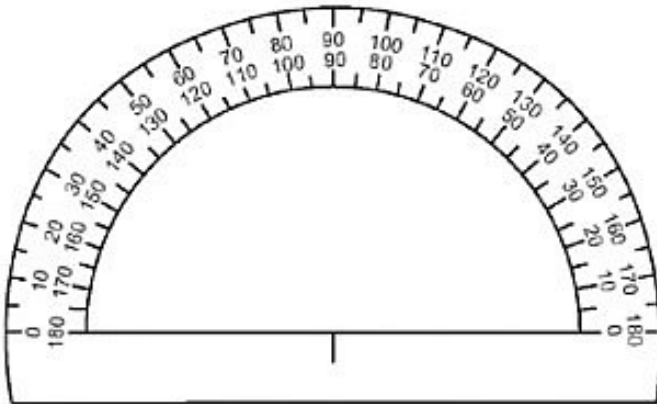
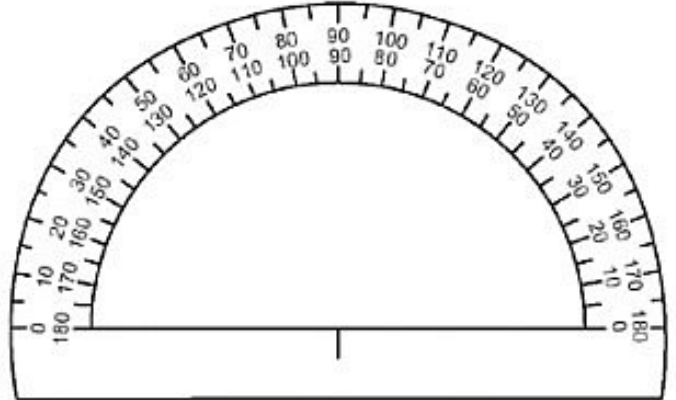
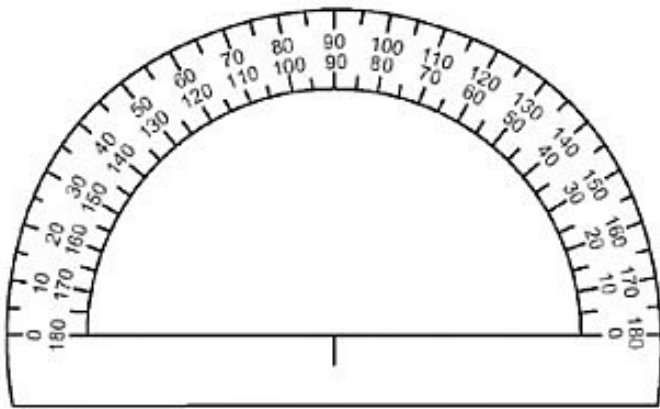
e. $6\frac{1}{3}, 5, 4$

f. $3, 2, 1$

5. If two side lengths of a triangle are 5 cm and 7 cm, what is the smallest possible **integer** length of the third side?



6. If two side lengths of a triangle are 5 cm and 7 cm, what is the largest possible **integer** length of the third side?



Spiral Review

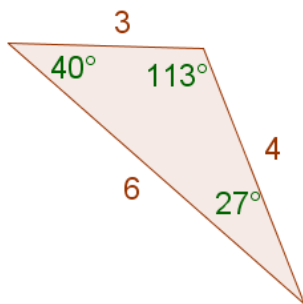
1. Simplify $-27.3 + 6.2 =$
2. Find 45% of 320 without a calculator
3. Evaluate $-3x + 1.4$ for $x = -3$
4. Kim, Laurel, and Maddy are playing golf. Kim ends with a score of -8 . Laurel's score is -4 . Maddy scores $+5$. What is the difference between the scores of Maddy and Kim?
5. The temperature increased 2° per hour for seven hours. How many degrees did the temperature raise after seven hours?

5.1a Homework: Triangle Practice

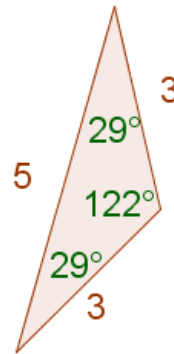
1. Explain what you know about the lengths of the sides of a triangle (the triangle inequality theorem.)

For #2-7, (a) Carefully copy the triangle using a ruler and protractor, technology, or other means. Label the side lengths and angle measures for your new triangle. For side length units, use either centimeters or inches. (b) Write an inequality that shows that the triangle inequality holds. (b) Classify the triangle as equilateral, isosceles, or scalene by examining the side lengths. (d) Classify the triangle as right, obtuse, or acute by examining the angle measures.

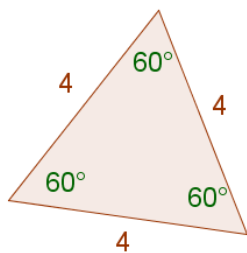
2.



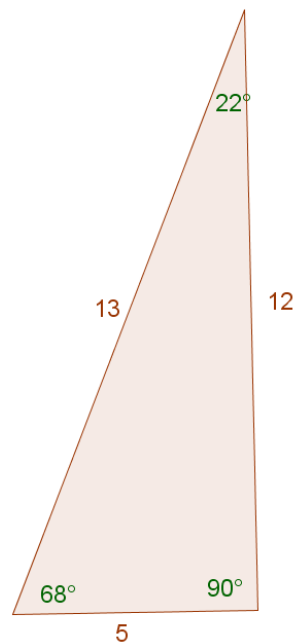
3.



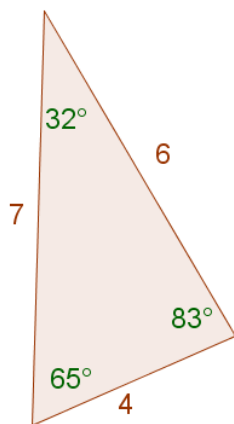
4.



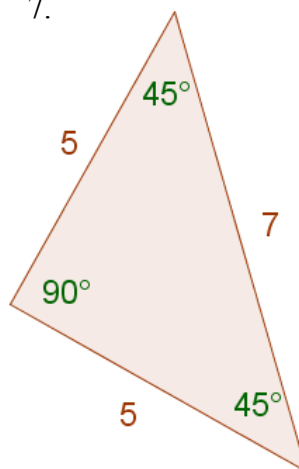
5.



6.



7.



True or false. Explain your reasoning—if false, give a counter example.

8. An acute triangle has three sides that are all different lengths.

9. A scalene triangle can be an acute triangle as well.

10. An isosceles triangle can also be a right triangle.

11. If two angles in a triangle are 40° and 35° , the triangle must be acute.

12. An obtuse triangle can have multiple obtuse angles.

13. A scalene triangle has three angles less than 90 degrees.

14. A triangle with a 100° angle must be an obtuse triangle.

15. The angles of an equilateral triangle are also equal in measure.

16. Write whether the three given side lengths can form a triangle. If not, draw a sketch showing why it doesn't work.

a. 8, 1, 8

b. 3, 3, 3

c. 1, 4, 10

d. $1\frac{1}{2}$, 5, $3\frac{1}{2}$

e. 15, 8, 9

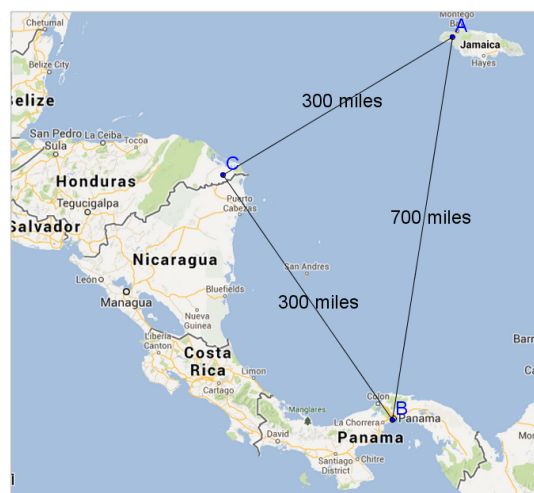
f. 2, 2, 15

g. 4, 3, 6.9

h. $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$

17. Anna used the scale on a map to calculate the distances “as the crow flies” (meaning the perfectly straight distance) from three points in Central America and the Caribbean islands, marked on the map to the right.

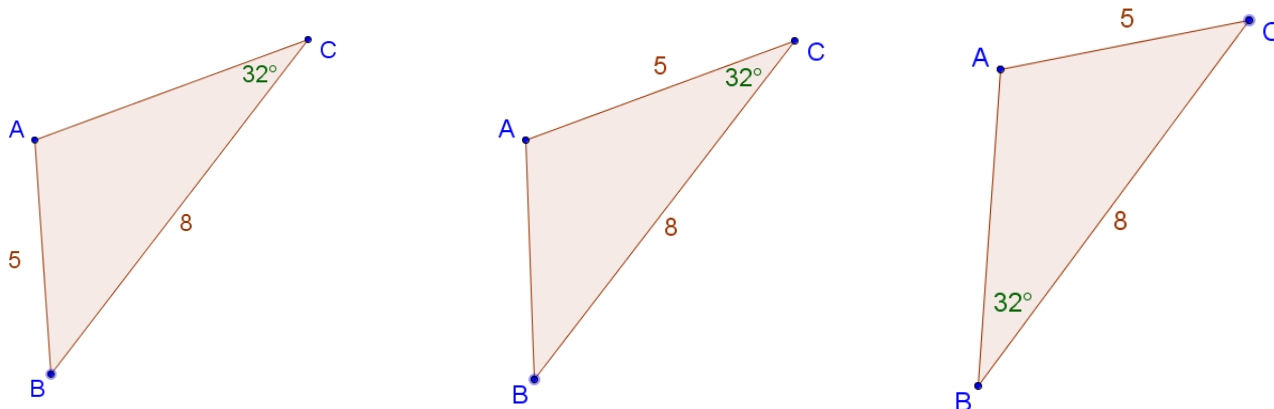
- According to Anna, how far is it from Jamaica to Panama if you don't go through Honduras?
- According to Anna, how far is it from Jamaica to Panama if you do go through Honduras?
- Another way of stating the triangle inequality theorem is “the shortest distance between two points is a straight line.” Explain why Anna must have made a mistake in her calculations.



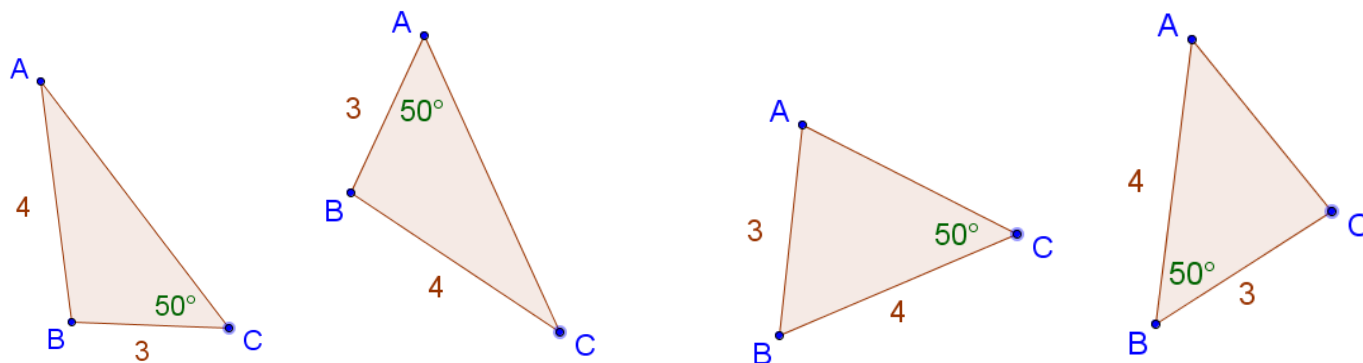
5.1b Class Activity: Building Triangles Given Three Measurements

Preparation for task: included angles.

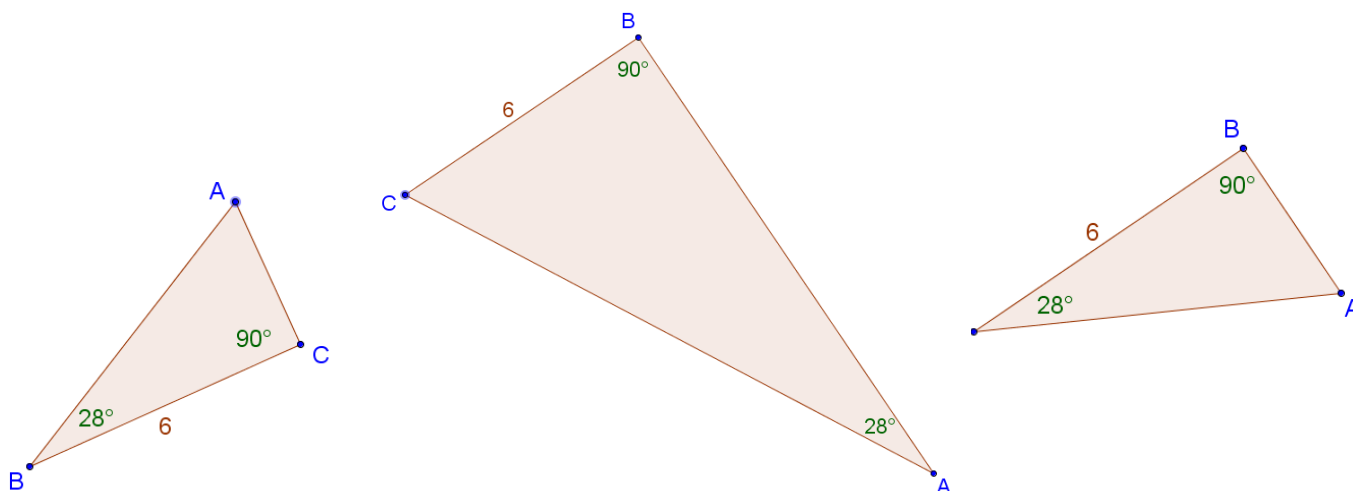
1. Circle all the triangles with side lengths 8 and 5 and an *included angle* of 32° .



2. Circle all the triangles with side lengths 4 and 3, with a *non-included angle* of 50° adjacent to the side with length 3.



3. Circle all the triangles with two angles 90° and 28° with an *included side* of 6 units.



Discussion questions: What if you only have three pieces of information about a triangle, like two angle measurements and one side length? Is it possible to create more than one unique triangle with that information?

Materials: GeoGebra and 5.2 GeoGebra files

Instructions: Open the file. Read the criteria that your triangle must satisfy. Drag the pieces to carefully form a triangle with the information required. If no triangle is possible, explain why. If only one triangle is possible, draw it and measure the remaining sides and angles using the tools in the program. If there is more than one way to make the triangle, draw both triangles.

Trial #1: SSS How many different triangles?	Draw the triangle(s), or write why no triangle is possible.
Trial #2: SAS How many different triangles?	Draw the triangle(s), or write why no triangle is possible.
Trial #3: ASA How many different triangles?	Draw the triangle(s), or write why no triangle is possible.
Trial #4: SSA How many different triangles?	Draw the triangle(s), or write why no triangle is possible.
Trial #5: AAS How many different triangles?	Draw the triangle(s), or write why no triangle is possible.

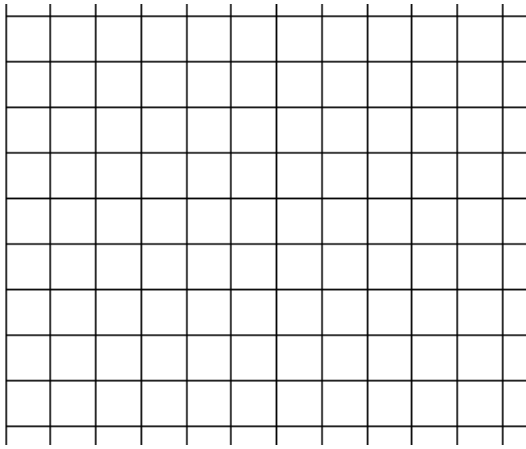
Discussion notes:

Spiral Review

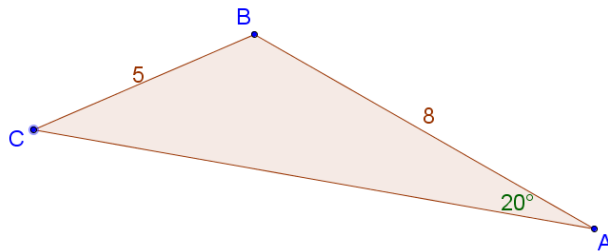
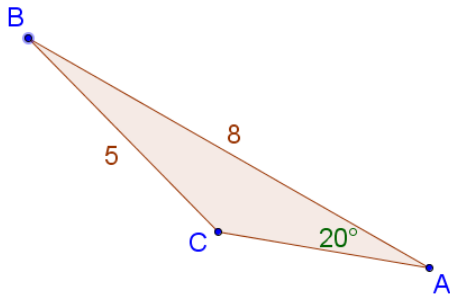
1. $-8 = -3m + 10$
2. $-12 = 3x$
3. Write $\frac{3}{5}$ as a percent and decimal.
4. Show how to simplify the following expression with a number line $-6 + (-3)$
5. Kurt earned \$550 over the summer. If he put 70% of his earnings into his savings, how much money did he have left over?

5.1b Homework: Building Triangles Given Three Measurements

1. Use the grid below to draw and label $\triangle ABC$ with the length of \overline{AB} 5 units, the length of \overline{BC} 7 units, and the measure of $\angle ABC = 90^\circ$.



2. Is $\angle ABC$ the *included* angle to the given sides? Explain.
3. What is the area of the triangle in #2? Justify your answer.
4. Will all your classmates who draw a triangle like #2 get a triangle with the same area? Explain.
5. Which triangle(s) has two sides 5 and 8 units, and a non-included angle of 20° adjacent to the side of length 8?



6. Are the two triangles in #5 exactly the same size and shape?

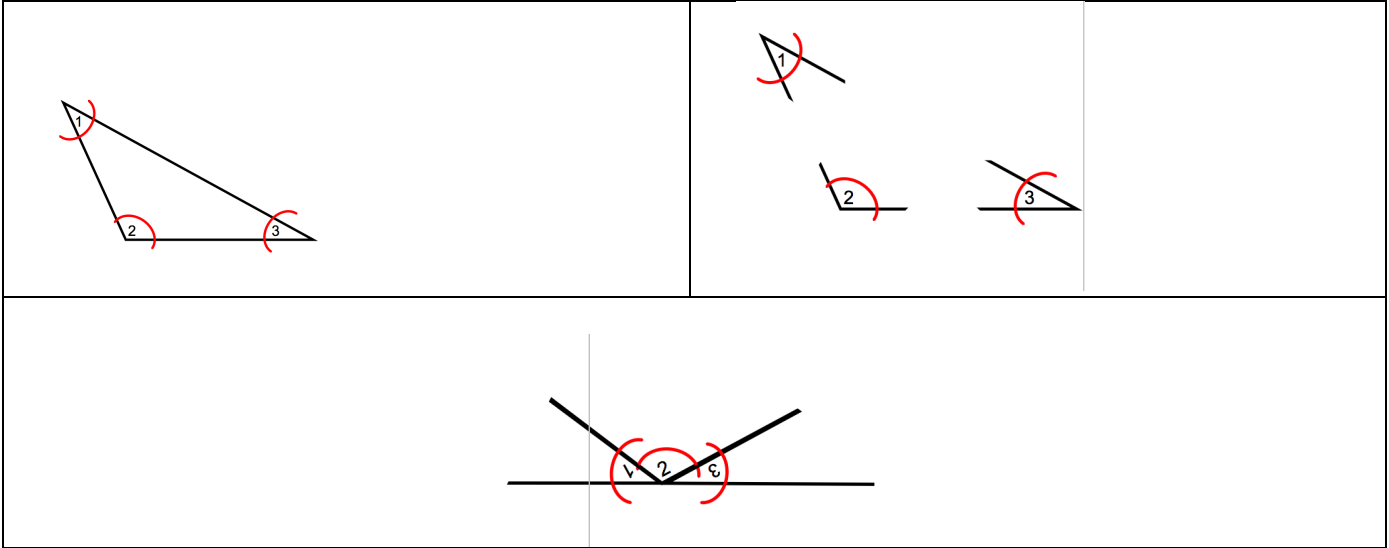
For #7-13, decide whether there are 0, 1, or more than one possible triangles with the given conditions. Use either a ruler and protractor, graph paper strips and protractor, or GeoGebra to draw the triangles.

<p>7. A triangle with sides that measure 5, 12, and 13 cm.</p> <p>How many possible triangles?</p> <p>If possible, what kind of triangle? Sketch, label.</p>	<p>8. A triangle with sides that measure 4 and 6, and an included angle of 120°.</p> <p>How many possible triangles?</p> <p>If possible, what kind of triangle? Sketch, label.</p>
<p>9. A triangle with angles 100° and 20°, and an included side of 2.</p> <p>How many triangles?</p> <p>If possible, what kind of triangle? Sketch, label.</p>	<p>10. A triangle with two angles of 40° and 30°, and an included side of 6.</p> <p>How many triangles?</p> <p>If possible, what kind of triangle? Sketch, label.</p>

<p>11. A triangle with two sides of 5 and 7, and an included angle of 45°.</p> <p>How many triangles?</p> <p>If possible, what kind of triangle? Sketch, label.</p>	<p>12. A triangle with angles of 30° and 60°, and a non-included side of 5 units adjacent to the 60° angle.</p> <p>How many triangles?</p> <p>If possible, what kind of triangle? Sketch, label.</p>
<p>13. A triangle HIJ in which $m\angle HIJ = 60^\circ$, $m\angle JHI = 90^\circ$, and $m\angle IJH = 55^\circ$.</p> <p>How many possible triangles?</p> <p>If possible, what kind of triangle? Sketch, label.</p>	<p>14. A triangle ABC in which $AB = 3$ cm., $m\angle ABC = 60^\circ$, and $m\angle BCA = 60^\circ$.</p> <p>How many possible triangles?</p> <p>If possible, what kind of triangle? Sketch, label.</p>
<p>15. Paul lives 2 miles from Rita. Rita lives 3 miles from the shopping mall. What are the shortest and longest distances Paul could live from the mall?</p> <p>Draw and explain.</p>	

5.1c Class Activity: Sum of the Angles of a Triangle Exploration and 5.1 Review

Activity: Cut out 5 *different* triangles. Mark the angles and then tape them together as shown below. Use a protractor to find the measure of the sum of the angles and fill in the table.



Type of Triangle	Sum of Angles

5.1c Honors Extension: Sum of the Angles of a Polygon Exploration

Activity: Cut out 5 *different* triangles, quadrilaterals, pentagons, and hexagons (all convex). Mark the angles and then tape them together. Use a protractor to find the measure of the sum of the angles and fill in the table.

POLYGON	Number of Angles	Sum of Angles
Triangle	3	180°

1. What pattern do you notice?
2. What do you expect the sum of the angles of an OCTAGON will be?
3. Cut out a convex octagon and test your conjecture.

4. Cut out a triangle and quadrilateral. Trace out the triangle on to a piece of grid paper. After you have traced it, rip the corners off and try to construct as many new triangles as you can with the original angles. Do the same with the quadrilateral. What do you notice?
5. Predict the sum of the angles of a 10-agon.

Spiral Review

1. $5 \times (-11)$

2. Without a calculator, what percent of 80 is 60?

3. Find the following with or without a model:

$$26 + (-26) =$$

$$\frac{1}{6} + \frac{3}{7} =$$

4. Order the numbers from least to greatest.

$$-2.15, \frac{17}{7}, 2.7, -2.105$$

5. Given the following table, find the indicated unit rate:

Days	Total Push-ups
2	30
4	60
29	435

_____ push-ups per day

5.1c Homework: Sum of the Angles of a Triangle Exploration and 5.1 Review

1. Determine whether the three given side lengths can form a triangle. If not, explain why it doesn't work.
 - a) 5.1 inches, 3.24 inches, 8.25 inches
 - b) $4\frac{1}{2}$ cm, $3\frac{3}{4}$ cm, 8 cm
 - c) 6.01 cm, $5\frac{1}{4}$ cm, 11.22 cm
 - d) $4\frac{1}{2}$ m, $3\frac{1}{4}$ m, 8 m
2. Your friend is having a hard time understanding how angle measures of 30° , 60° , 90° might create more than one triangle. Draw two different triangles that have those angle measures and explain why the two triangles are different.
3. Your friend is having a hard time understanding why knowing the lengths of two sides of a triangle and the measure of an angle not between the two sides may not be adequate information to construct a unique triangle. Draw an example where two sides and a non-included angle give two different triangles.

5.1d Self-Assessment: Section 5.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Given specific criteria, construct a triangle and know if it's a unique triangle	I don't know what information I need to make a unique triangle. Nor do I know how to work with information given to me.	I can usually construct a triangle with information given, but I don't always know if the triangle is unique or if I'll even get a triangle by just looking at the information.	I can tell if the information given will make a triangle and I can construct it. I also know if the triangle I constructed is the only one I can construct from the information	I can tell if the information given will make a triangle and I can construct it. I also can explain why the information gives a triangle and why it is or is not unique.
2. Understand and explain the side length requirements of a triangle.	I struggle to understand how and when length will or will not make a triangle.	I can determine if lengths given will make a triangle.	I can determine if lengths given will make a triangle and can apply that knowledge to contexts.	I can determine if lengths given will make a triangle and can apply that knowledge to contexts. I can also explain in my own words why lengths will or will not form a triangle.
3. Understand and explain the angle and side requirements for constructing a unique triangle.	I struggle to understand the angle and side requirements for constructing a unique triangle.	I can determine if the sides and angles given will make a unique triangle.	I understand the angle and side requirements for constructing a unique triangle and can use it to explain whether or not given sides and angles will make a unique triangle.	I understand and can explain the angle and side requirements for constructing a unique triangle. I can explain in my own words why some will not necessarily make a unique triangle.
4. Know the angle measure requirements of a triangle. (Triangle Angle Sum Theorem)	I don't know what the sum of the angles of a triangle is.	Given three angles, I know if they will make a triangle OR given two angles of a triangle, I can figure out the third.	Given three angles, I know if they will make a triangle OR given two angles of a triangle, I can figure out the third. I can use this knowledge in context as well.	Given three angles, I know if they will make a triangle OR given two angles of a triangle, I can figure out the third. I can use this knowledge in context as well. I can explain in my own words what this all works.

Sample Problems for Section 5.1

1. Given the following criteria, construct a triangle:
 - a. Draw and label $\triangle ABC$ with the length of \overline{AB} 5 units, the length of \overline{BC} 7 units, and the length of \overline{CA} 10 units.
 - b. Draw and label $\triangle ABC$ with the measure of $\angle CAB = 40^\circ$, the measure of $\angle BCA = 50^\circ$, and the measure of $\angle ABC = 90^\circ$.
 - c. Draw and label $\triangle ABC$ with the measure of $\angle CAB = 60^\circ$, the length of \overline{CA} 4 units, and the length of \overline{BA} 6 units.
2. Determine if the following side lengths will make a triangle. Explain why or why not.
 - a. 2, 5, 7
 - b. 33, 93.2, 70
 - c. 1, 2.7, 7
3. Determine if the given information will make a unique triangle. Explain why or why not.
 - a. Side lengths 3 and 5 and an included angle of 67°
 - b. Angles 73° , 7° , 100°
 - c. Angles 80° and 25° and included side of 12
4. Determine if the given angles will make a triangle. Explain why or why not.
 - a. Angles 25° , 70° , 95°
 - b. Angles 30° , 40° , 20°

Section 5.2: Scale Drawings

Section Overview: The central idea of this section is scale and its relationship to ratio and proportion. Students will use ideas about ratio, proportion, and scale to: a) change the size of an image and b) determine if two images are scaled versions of each other.

By the end of this section, each student should understand that we could change the scale of an object to suit our needs. For example, we can make a map where 1 inch equals one mile; lie out a floor plan where 2 feet equals 1.4 cm from a diagram; or draw a large version of an ant where 3 centimeters equals 1 mm. In each of these situations, the “shape characteristics” of the object remain the same, what has changed is size. Objects can be scaled up or scale down. Through explorations of scaling exercises, students will see that all lengths of the given object are changed by the same factor in the scaled representation and this factor is called the *scale factor*.

The term “similar” will not be defined in 7th grade; here students continue to develop an intuitive understanding of “the same shape,” so that the concept of similarity (introduced in 8th grade) is natural. Throughout this section students should clearly distinguish between two objects that are of the same shape and dimension and objects that are scaled versions of each other. In particular students will come to understand that two polygonal figures that are scaled versions of each other have equal angles and corresponding sides in a ratio of $a:b$ where $a \neq b$. Students should also distinguish between saying the *ratio* of object A to object B is $a:b$ while the *scale factor* from A to B is b/a . This idea links ratio and proportional thinking to scaling.

Students will learn to find the scale factor from one object to the other from diagrams, values and/or proportion information. Students should be able to fluidly go from a smaller object to a larger scaled version of the object or from a larger object to a smaller scaled version giving either or both the proportional constant and/or the scale factor.

In this section and through the chapter, abstract procedures will be emphasized for finding scale factors and/or missing lengths, however some students may prefer to continue to use bar models. Bar model strategies will become increasingly more cumbersome, so it will be important to help students transition to more efficient procedures. This can be done by helping students connect concrete models (i.e. bar/tape models) to the algorithmic procedure as was done in Chapter 4.

Concepts and Skills to be Mastered (from standards)

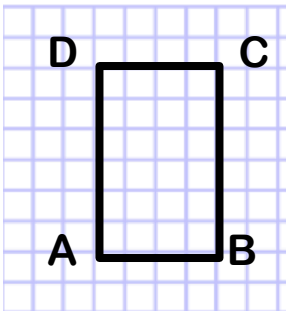
Geometry Standard 1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. 7.G.1

Geometry Standard 6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. 7.G.6

1. Draw a scaled version of a triangle, other polygon, or other object given lengths.
2. Find a measure of a scaled object given the scale factor and measure from the original.
3. Find the scale factor between two objects that are the same shape but different sizes/proportional.
4. Use proportional reasoning in explaining and finding missing sides of objects that are the same shape but different sizes.
5. Find the scale factors for perimeter or area for proportional objects.

5.2a Classwork: Comparing the Perimeter and Area of Rectangles

Activity 1: Below is an image of a rectangle, which are 4 units by 6 units. On the first grid provided, draw a rectangle with sides lengths TWICE as long as this image (label the new vertices E, F, G, H) and on the other grid draw a rectangle with side lengths HALF as long as this image (label the new vertices I, J, K, L.)



- a) Describe the method you used to double the length of each side.
- b) Describe the method you used to make each side have a length half the length of the original.
- c) Do you think the area of the new rectangles changes at the same rate as the sides? Explain.

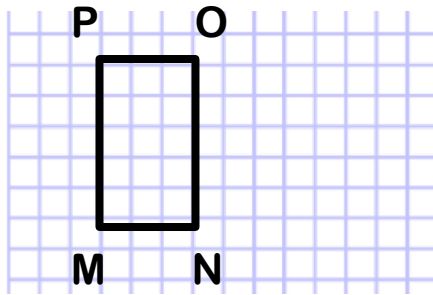
Sides that are twice as long as the image above.

Sides that are half as long as the image above.

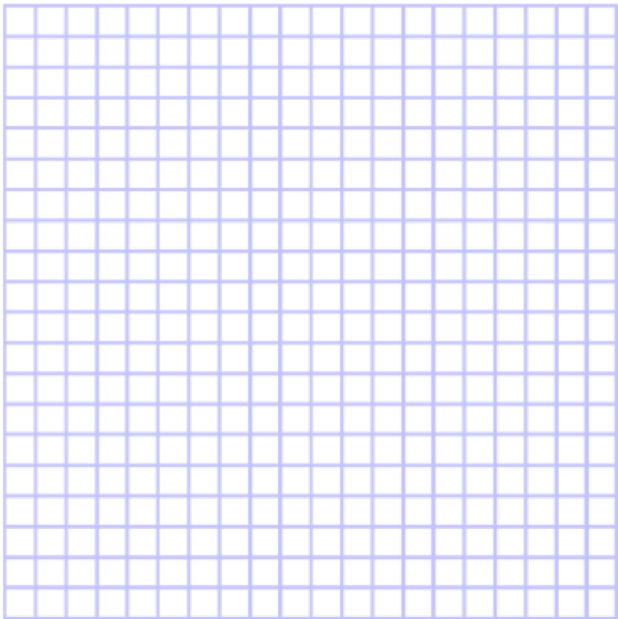
Fill out the missing blanks in the chart below:

Rectangle	Dimensions	Perimeter	Area	Change from original rectangle ABCD
ABDC	4×6	20	24	Same
EFGH				
IJKL				

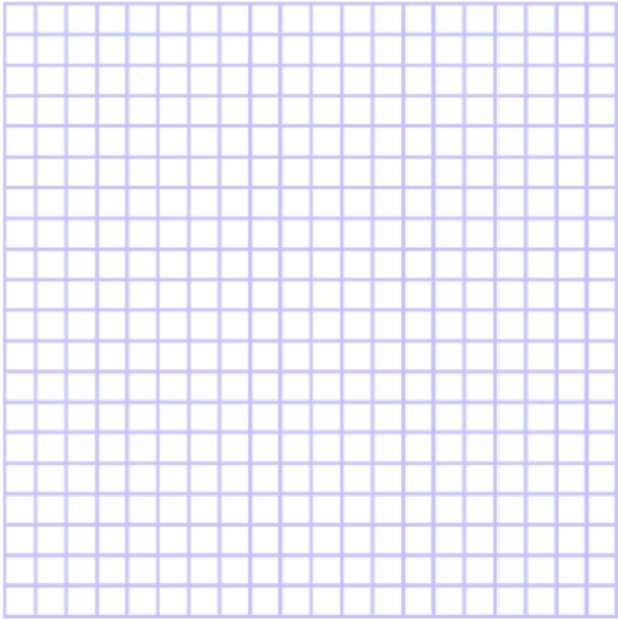
Activity 2: Below is an image of a rectangle which *is 3 units by 5 units*. On the first grid provided, draw a rectangle with sides lengths TWICE as long as this image (label the new vertices Q, R, S, T) and on the other grid draw a rectangle with side lengths HALF as long as this image (label the new vertices U, V, W, X).



Sides that are twice as long as the image above.



Sides that are half as long as the image above.



Fill in the missing blanks in the chart below:

Rectangle	Dimensions	Perimeter	Area	Change from original rectangle ABCD
MNOP	3 × 5	16	15	Same
QRST				
UVWX				

Spiral Review

1. What are triangles with all sides the same length called? _____
2. Katherine is visiting patients in a hospital. She visits 18 patients in 6 hours. At that rate, how many patients will she visit in 9 hours?

hours	Patients visited
6	18
9	

3. What is an obtuse angle?
4. The price for two bottles of ketchup are given below:

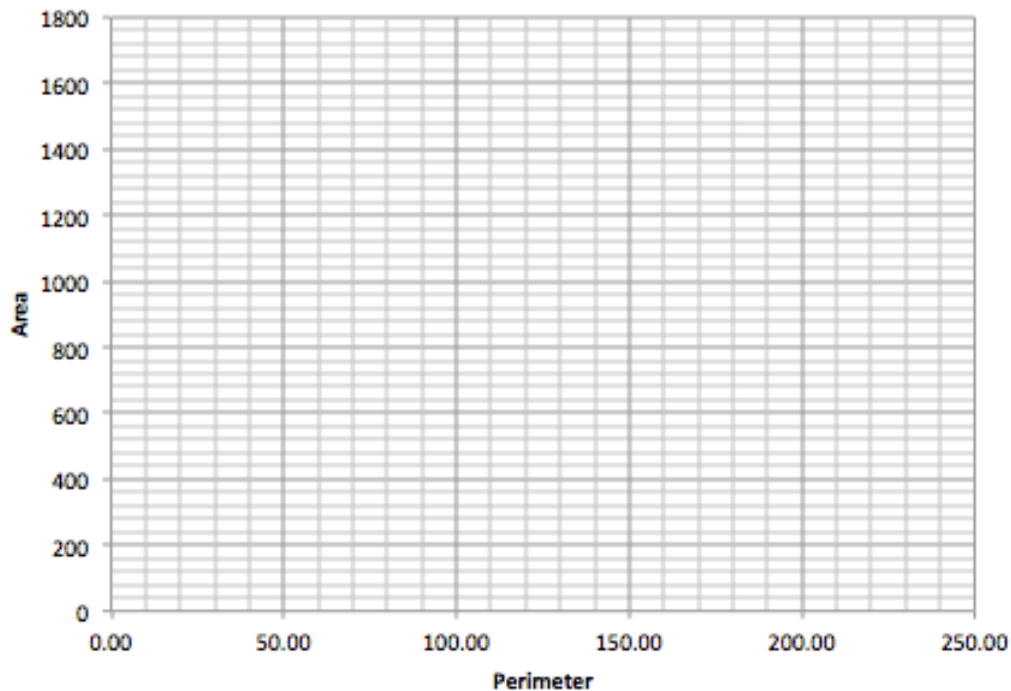
A 20 oz. bottle of DELIGHT ketchup is 98¢ at the grocery store. A 38 oz. bottle of SQUEEZE ketchup is \$1.99 at the same grocery store.

 - a. find the unit rate for each product
DELIGHT is _____ per ounce SQUEEZE is _____ per ounce
 - b. What conclusions can you draw from this information?

5.2a Homework: Comparing the Perimeter and Area of Rectangles

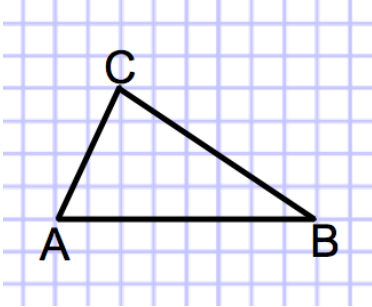
Below is a table that describes the dimensions, perimeter, area and change of side lengths for a rectangle that's scaled in different ways. In the first row you find that the 5x20 rectangle has a perimeter of 50, area of 100; there is no change in side length here because it's the original rectangle. In the next row the dimensions become 10x40 giving it a perimeter of 100 and area of 400; the side length change is twice the original. Examine the other rows to understand what's happening to the rectangle. Then graph the relationship between the perimeter (x axis) and area (y axis) on the graph below. Describe the pattern you notice.

Dimensions	Perimeter	Area	Change of side lengths from original rectangle
5 by 20	50	100	Same
10 by 40	100	400	Twice
5/2 by 10	25	25	Half
15 by 60	150	900	Three times
5/3 by 20/3	16.67	11.11	One third
20 by 80	200	1600	Four times



5.2b Classwork: Scaling Triangles

Activity 1: Below is an image of a triangle. On the first grid provided below, draw a triangle with sides lengths TWICE as long as this image (label the new vertices D, E, F) and on the other grid draw a triangle with side lengths HALF as long as this image (label the new vertices G, H, I.)

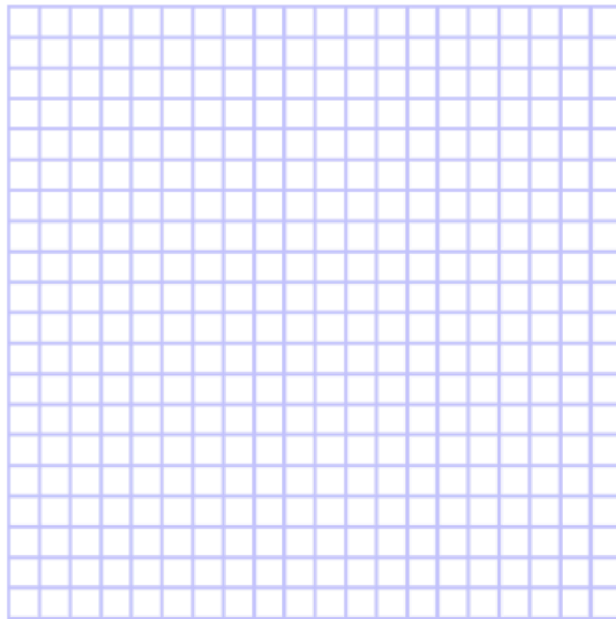


- Describe the method you used to double the length of each side.
- Describe the method you used to make each side have a length half the length of the original.
- Do you think the angles of the new triangles are the same or different than the original triangle? Explain.

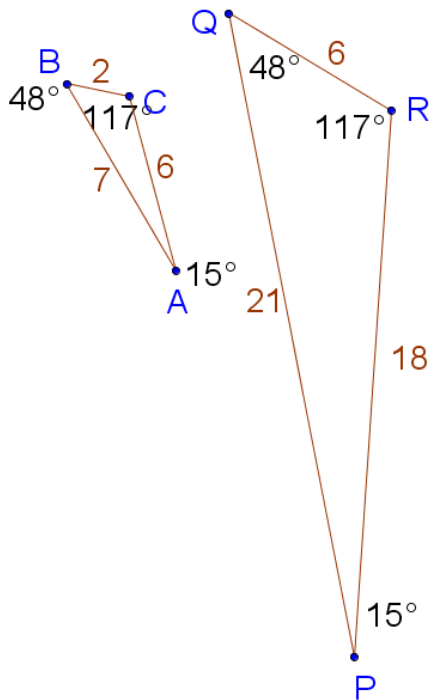
Sides that are twice as long as the image above.



Sides that are half as long as the image above.



Activity 2: Exploring the relationship between triangles that have the same angle measures but side lengths that are different.



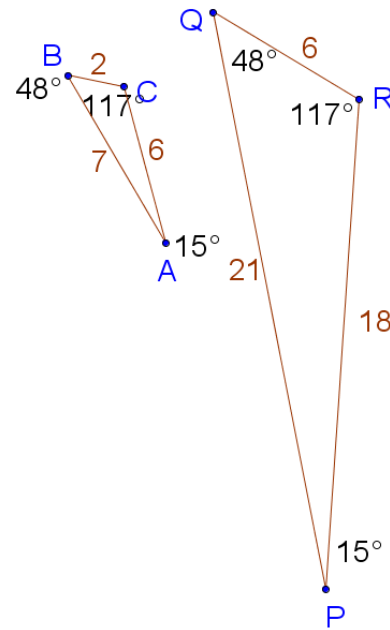
Which sides and angles correspond to each other? List corresponding angles and sides:

- Describe how these triangles are similar and different:
- What do you notice about the lengths of the sides of the two triangles?

Look back at Activity 1 in this section. Write the ratio of the original triangle to the triangle with side lengths that are twice the original and then write the ratio of the original triangle to the triangle with side lengths that are half as long as the original.

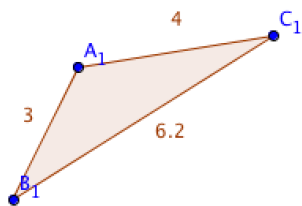
3. Find the ratios for the lengths of the given sides.

a) \overline{BC} to \overline{QR}	b) \overline{AB} to \overline{PQ}
c) \overline{AC} to \overline{PR}	d) Perimeter of $\triangle ABC$ to Perimeter of $\triangle PQR$
e) \overline{BC} to \overline{AB}	f) \overline{QR} to \overline{QP}
g) \overline{AC} to \overline{BC}	h) \overline{RP} to \overline{QR}
i) \overline{AC} to \overline{AB}	j) \overline{RP} to \overline{QP}

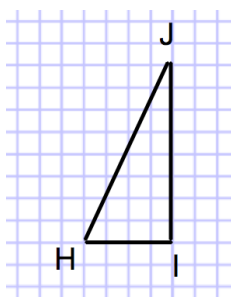


- Some of the ratios above make comparisons of measurements in the **same triangle**. Other ratios make comparisons of measurements of corresponding parts of **two different triangles**. Put a star by the ratios that compare parts of two different triangles. What do you notice?
- Since the corresponding parts have the same ratio, there is a **scale factor** from $\triangle ABC$ to $\triangle PQR$. The scale factor is the number you would multiply a length in $\triangle ABC$ to get the corresponding length in $\triangle PQR$. What is the scale factor from $\triangle ABC$ to $\triangle PQR$? Explain how you arrived at your answer.
- What is the scale factor from $\triangle PQR$ to $\triangle ABC$?
- What is the mathematical relationship between the scale factor of $\triangle PQR$ to $\triangle ABC$ and the scale factor of $\triangle ABC$ to $\triangle PQR$?
- The scale factors from $\triangle ABC$ to $\triangle PQR$ is an *enlarging* scale factor, and the other is a *reducing* scale factor. What would be the scale factor to keep a figure the *same size*?

9. Use a straight edge and protractor to construct $\triangle ABC$ then construct a new triangle, $\triangle DEF$, that is the same shape as $\triangle ABC$, but the scale factor from $\triangle ABC$ to $\triangle DEF$ is 2.

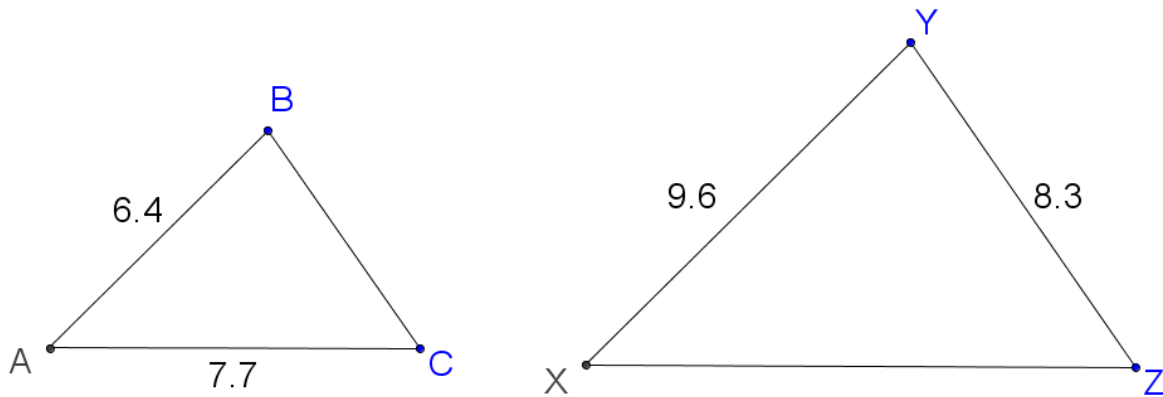


10. If the ratio of $\triangle HIJ$ to $\triangle DEF$ is $\frac{4}{3}$, (see $\triangle HIJ$ below), draw $\triangle DEF$. Show the length of each of the sides.



11. The ratio of $\triangle ABC$ to $\triangle DEF$ is 5:4. If \overline{BC} is 15, what is the length of \overline{EF} ? You may need to draw a diagram.

12. The two triangles below are scaled versions of each other. Use a protractor to find the measure of each of the angles of the two triangles below.



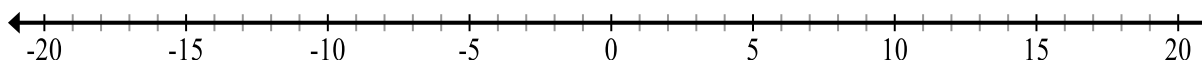
13. What is the ratio of $\triangle ABC$ to $\triangle XYZ$? What is the scale factor that takes $\triangle ABC$ to $\triangle XYZ$?
14. Which of the following proportions would be valid for finding the length of \overline{BC} ? Circle all that apply.
- $\frac{9.6}{6.4} = \frac{8.3}{BC}$
 - $\frac{6.4}{BC} = \frac{8.3}{9.6}$
 - $\frac{6.4}{BC} = \frac{9.6}{8.3}$
 - $\frac{BC}{7.7} = \frac{8.3}{9.6}$
15. Write different proportions that could be used to solve for the length of \overline{BC} . Solve your proportion.
16. Write and solve a proportion to find the length of \overline{XZ} .

Spiral Review

1. Suppose you were to flip a coin 3 times. What is the probability of getting heads all three times?
2. Samantha has 120 bracelets. She sells $\frac{2}{3}$ of the bracelets and then decides to donate 50% of the rest.. How many bracelets does she still have?

3. Place each of the following integers on the number line below. Label each point:

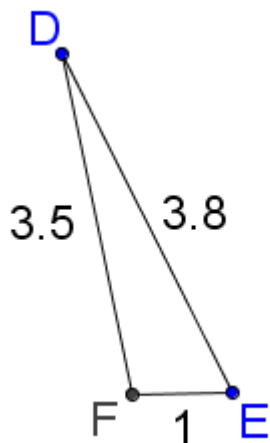
A = 4 B = -4 C = -15 D = 7 E = 18 F = -19



4. Write 0.672 as a percent.

5.2b Homework: Scaling Triangles

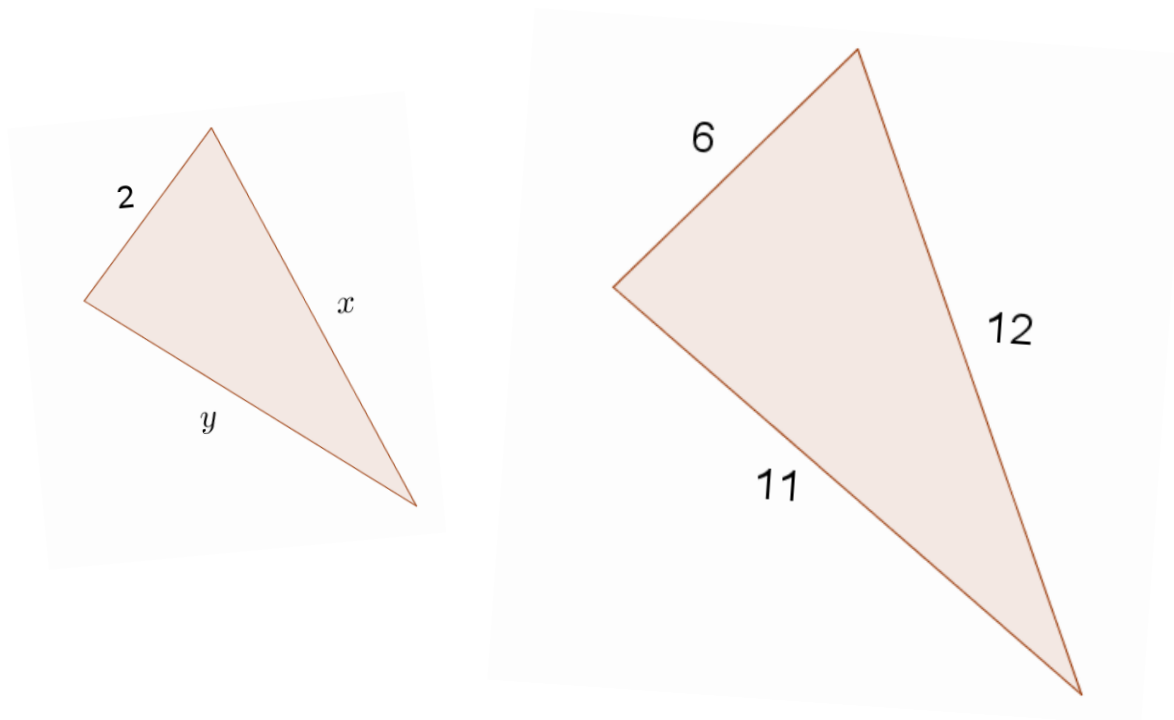
1. The measure of each of the sides of $\triangle DEF$ is given. Draw a $\triangle GHI$ that has side lengths that are three times as long as $\triangle DEF$



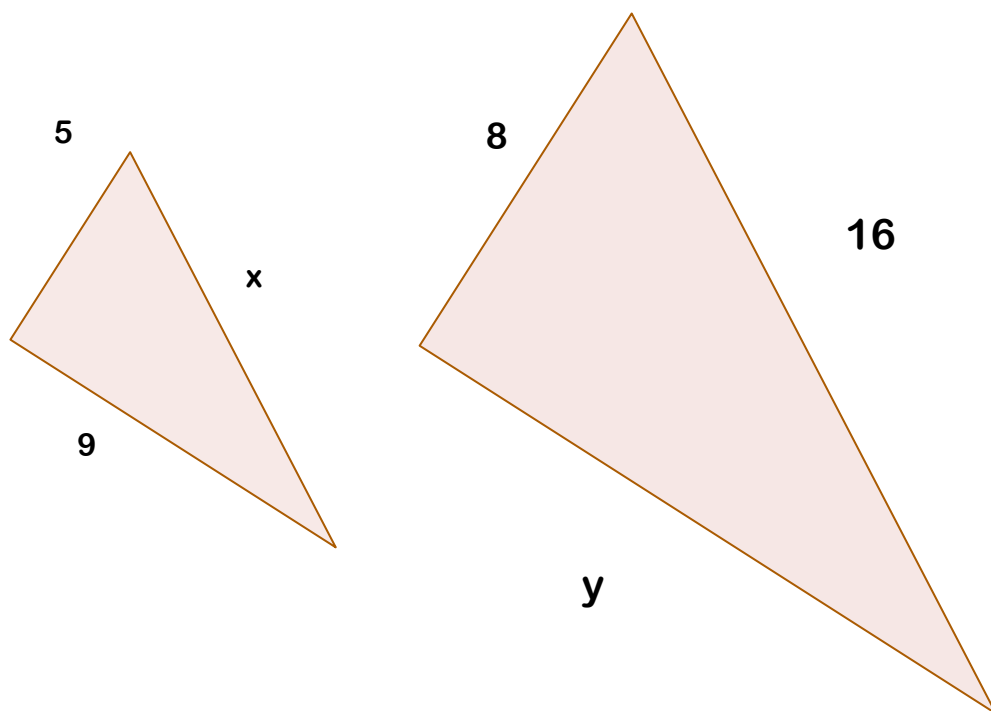
2. If the ratio of $\triangle BCD$ to $\triangle EFG$ is 3:5 and the length of \overline{BC} is 6", what is the length of \overline{EF} ? Justify your answer.

For #3-4, the triangles given in each are proportional. Do the following: a) solve for the unknowns by using proportions and b) state the scale factor between the two triangles. Express all answers exactly. Figures are not necessarily drawn to scale. Show your work.

3.



4.

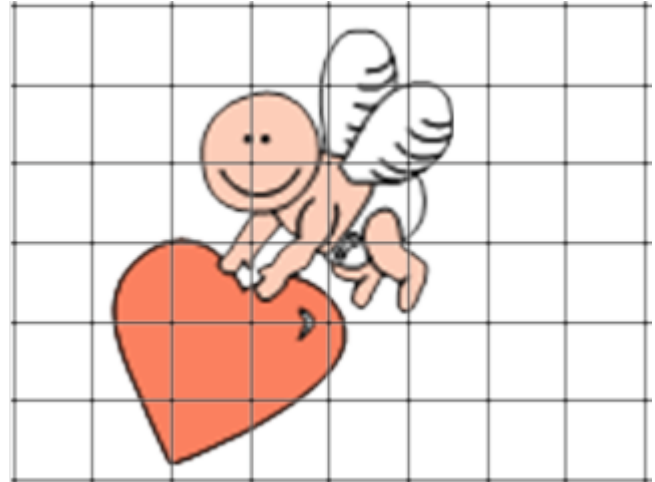


5.2c Class Activity: Solve Scale Drawing Problems, Create a Scale Drawing

1. Your sister wants a large poster version of a small drawing she made. She drew it on centimeter graph paper.

a. What are the dimensions of her original picture as shown to the right?

b. She wants the poster version to have a height of at least 2 feet. What **scale** should she use so that her poster is 2 feet tall?



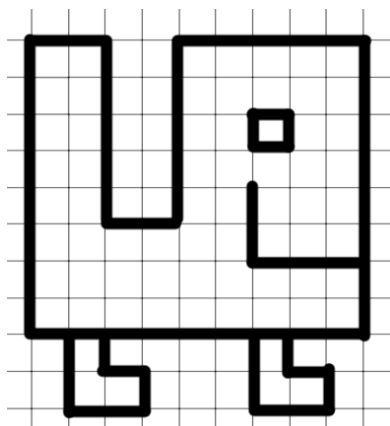
_____ : _____
Height of original Height of large poster

c. The side of a square in the original picture is 1 cm long. How long will the side of a square be in the final poster?

d. What will be the dimensions of the final poster?

2. Hal used the scale 1 inch = 6 feet for his scale model of the new school building. The actual dimensions of the building are 30 feet (height), by 120 feet (width) by 180 feet (length). What are the dimensions of his scale model?

3. On a separate sheet of grid paper, create the creature below so that it is a 1:3 enlargement of the original model. Write your **strategy** for calculating lengths to the right of the picture.



4. Ellie was drawing a map of her hometown using a scale of 1 centimeter to 8 meters.
- The actual distance between the post office and City Hall is 30 meters. What is the exact distance between those two places on Ellie's map?
 - In her drawing, the distance from the post office to the library is 22 centimeters. What is the actual distance?
5. Allen made a scale drawing of his rectangular classroom. He used the scale $\frac{1}{2} \text{ inch} = 4 \text{ feet}$. His actual classroom has dimensions of 32 feet by 28 feet.
- What are the dimensions of his scale drawing of the classroom?
 - The simplified unit ratio of **classroom length : drawing length**, written as a fraction, $\frac{\text{classroom length}}{\text{drawing length}}$, is the scale factor for lengths. What is it?
 - The simplified unit ratio of **classroom area : drawing area**, written as a fraction, $\frac{\text{classroom area}}{\text{drawing area}}$ is the scale factor for areas. What is it?
 - What is the mathematical relationship between the scale factor for lengths and the scale factor for areas?

6. Audrey wants to make a scale drawing of the stamp below. She makes a scale drawing where 1 cm in the drawing represents 3 cm on the stamp. Which of the following are true?

- The drawing will be $\frac{1}{3}$ as wide as the stamp.
- The stamp will be 3 times as tall as the drawing.
- The area of the stamp will be 3 times as large as the drawing.
- The area of the stamp will be 6 times as large as the drawing.
- The area of the stamp will be 9 times as large as the drawing.

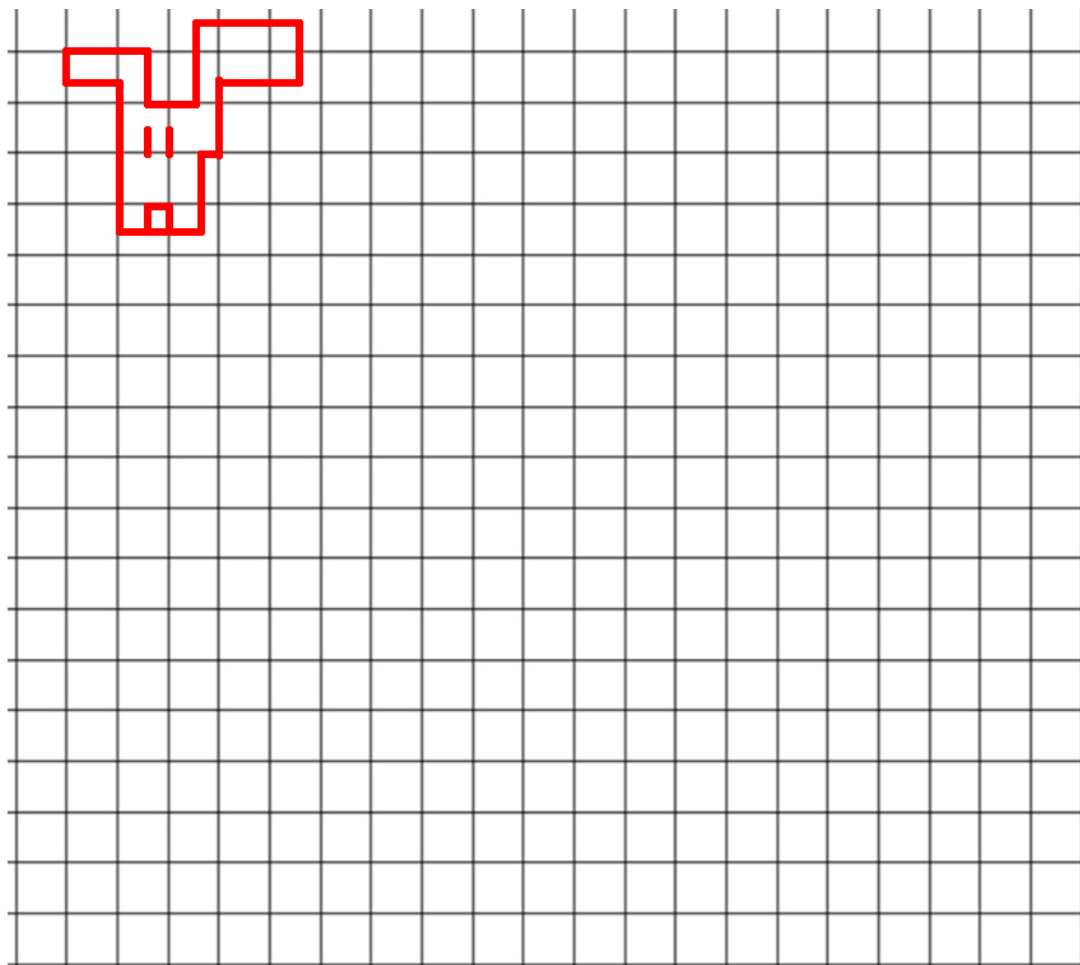
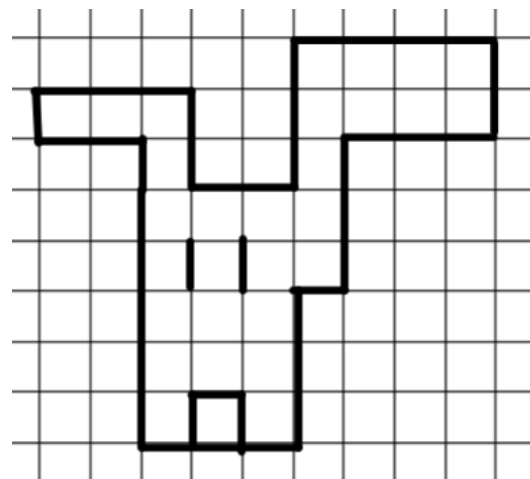


7. The following images are taken from four different maps or scale drawings, each shows a different way of representing scale:

Scale $\frac{1}{4}" = 1 \text{ Foot.}$	Scale 1:10 (1 foot: 10 mm)
	drawing size = 1 inch real size = 1 meter

Which scale represents a drawing that has shrunk the most from the original? Justify your answer.

8. The scale of the drawing on the right is 1 unit = $\frac{1}{4}$ foot. Use the grid below to draw a new scale drawing where 1 unit = $\frac{1}{2}$ foot.

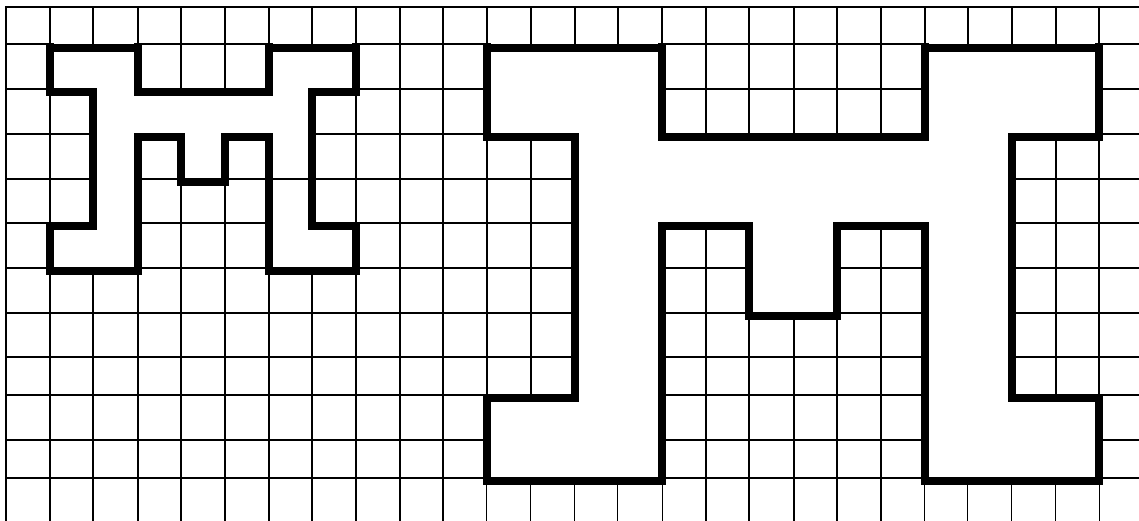


Spiral Review

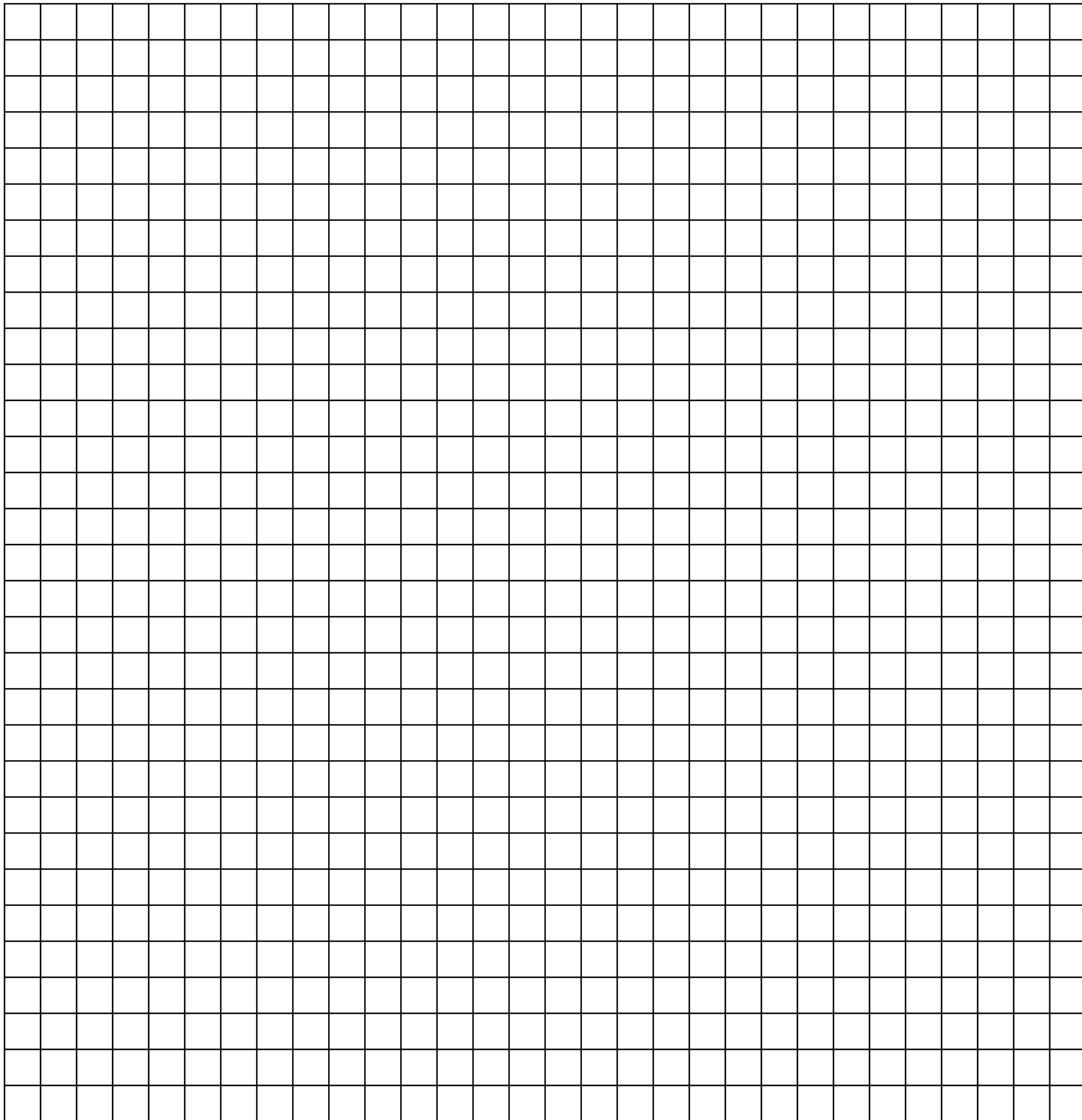
1. Suppose you flip a coin 3 times; what is the probability that you get a heads at exactly two times?
2. For each group of three side lengths in inches, determine whether a triangle is possible. Write yes or no. Justify your answer.
 - a. $14, 15\frac{1}{2}, 2$
 - b. $1, 1, 1$
 - c. $3.2, 7.2, 2.3$
3. Which number is greater: -24.41 or -24.4 ? Explain.
4. Use long division to show how you can convert this fraction to a decimal and then a percent: $\frac{4}{7}$
5. Athena has \$24 less than Bob. Represent how much money Athena has.

5.2c Homework: Solve Scale Drawing Problems, Create a Scale Drawing

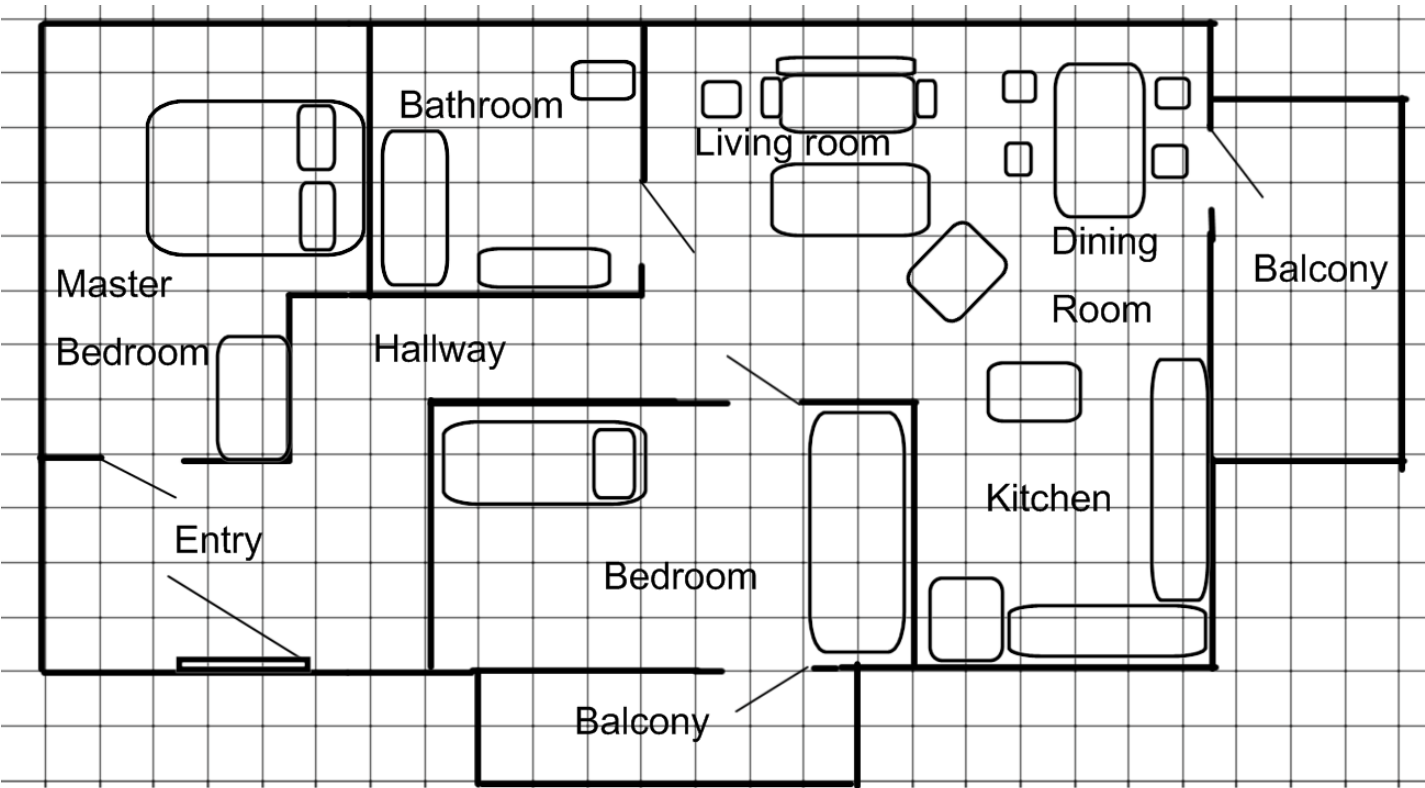
1. On a map, Breanne measured the distance (as the crow flies) between Los Angeles, California and San Francisco, California at 2 inches. The scale on the map is $\frac{1}{4} \text{ inch} = 43 \text{ miles}$. What is the actual straight-line distance between Los Angeles and San Francisco?
2. Janie made a 2.5 inch scale model of one of the tallest buildings in the world: Taipei 101. The scale for the model is $\frac{1}{4} \text{ inch} = 167 \text{ feet}$. Find the actual height of Taipei 101.
3. What scale was used to enlarge the drawing below? How do you know?



4. On the 0.25 inch grid below, create a scale drawing of a living room which is 9 meters by 6.25 meters. In the scale drawing include the following:
- a. Two windows on one of the walls. Each window is 1.25 meters wide.
 - b. An entrance (1.5 meters wide) from the side opposite the windows.
 - c. A sofa which is 2 meters long and 0.75 meters wide.
 - c. Record the drawing scale below the grid.



5.2d Extension Class Activity and Homework: Scale Factors and Area



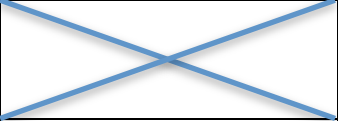
The scale drawing of a house is shown. The scale is 1 unit : 2 feet. Any wall lines that are between units are exactly halfway.

1. The balcony off the bedroom has dimensions in the drawing of 7×2 . Complete the table, including the appropriate units:

	Balcony length	Balcony width	Balcony area
Drawing	7 units	2 units	
Actual house			

2. What is the scale factor to get from units in the drawing to feet in the house?
3. What is the scale factor to get from square units in the drawing to square feet in the house?
4. If the architect includes a bench on the balcony that has dimensions 2.5×1 units, what are the dimensions of the bench in the house?

5. Complete the table for the bathroom, including the appropriate units:

	Bathroom length	Bathroom width	Bathroom area
Drawing			
Actual house			

6. What is the ratio of $\frac{\text{area in house}}{\text{area in drawing}}$?

7. If the backyard in the drawing has an area of 150 square units, how big is the area of the actual backyard of the house?

8. If the walkway to the entryway is 6.5 units long in the drawing, how long is the walkway on the house?

9. Use the drawing to determine how wide each interior door in the house is.

10. Approximately how long is each bed in the house?

11. By counting the number of square units in the master bedroom in the drawing, calculate the area of the master bedroom in the house. Include the area taken up by furniture.

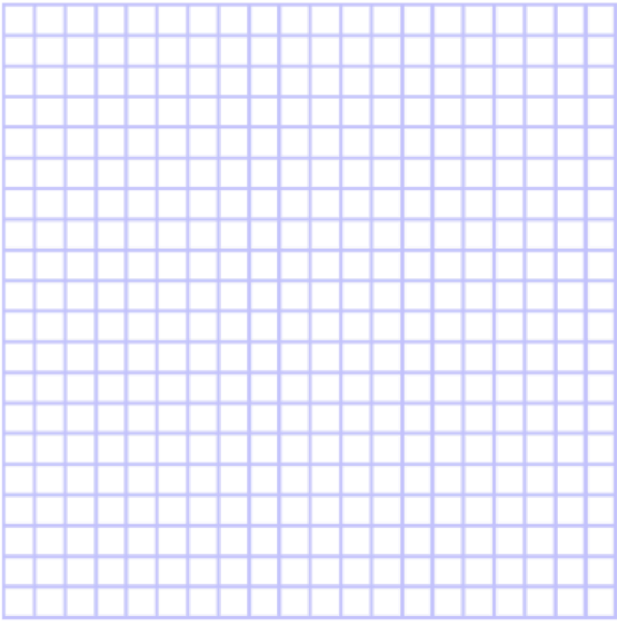
12. Challenge: What is the total square footage of the house, including the balconies? Show your work, labeling the expressions for each step with what they represent in the house.

5.2d Homework: Scale Factors and Area

1. Mouse’s house is very small. His living room measures 2 feet by 4 feet. Draw his living room on the grid. Label the dimensions and the area.

2. Double-Dog’s living room dimensions are double Mouse’s living room. Draw his living room on the grid. Label the dimensions and the area.

3. Triple-Threat-Tiger’s living room is triple the dimensions of Double-Dog’s. Draw Tiger’s living room on the grid. Label the dimensions and the area.



4. Fill in the table below with the dimensions and areas of the living rooms from the sketches above.

Homeowner	Living Room Dimensions (Length and Width)		Living Room Area
Mouse			
Double-Dog			
Triple-Threat-Tiger			

5. Compare measurements for Mouse and Double-Dog’s living rooms. Use scale factor in the comparison.

a. the dimensionsb. the area
6. Compare measurements for Double-Dog and Triple-Threat-Tiger’s living rooms. Use scale factor.

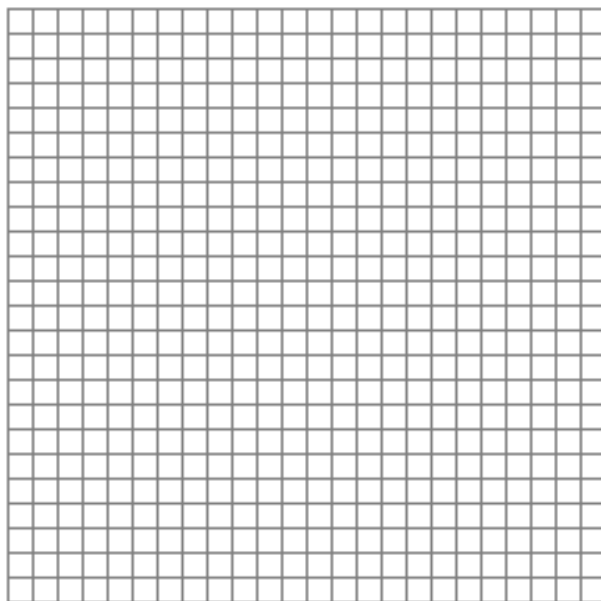
a. the dimensionsb. the area
7. Compare Mouse and Triple-Threat-Tiger’s living rooms. Use scale factor in the comparison.

a. The dimensionsb. the area

8. Generalize a rule related to the scale factors of dimensions and area. You might use “if...., then....”

For example: “If the dimensions of an object are multiplied by _____, the area will be multiplied by _____.”

9. If the area of Mouse’s **kitchen** is 12 square feet, and the dimensions of Double-Dog’s kitchen are twice as big as the dimensions of Mouse’s, what will the area of Double-Dog’s kitchen be?
10. If the dimensions of Triple-Threat-Tiger’s kitchen are three times the dimensions of Double-Dog’s, what will the area of Triple-Threat-Tiger’s kitchen be?
11. If the area of Mouse’s bathroom is 5 square feet, the dimensions of Double-Dog’s bathroom are twice Mouse’s, and Triple-Tiger’s are three times Double-Dog’s, what will the area of Triple-Threat-Tiger’s bathroom be? Is there a shortcut?
12. Ms. Herrera decided to shrink a picture so that it would fit on a page with some text. She went to the copy machine and pushed the 50% button, meaning that the **dimensions** of the paper would be **half** as big as normal.
- a. If the original dimensions of the picture were 8.5 in. X 11 in., what will be the dimensions of the new picture?
- b. Draw the original and new picture on the grid. Label the dimensions.
- c. If you know the area of the original picture, how might you figure out the area of the smaller picture (besides multiplying length and width)?
13. Let’s say that Mouse, Double-Dog and Triple-Threat-Tiger all have swimming pools. What would you predict about the scale factor of the volume as related to double or triple dimensions? Prove or adjust your prediction—say that Mouse has a pool which is 1 foot deep and 2 feet by 3 feet.



5.2e Self-Assessment: Section 5.2

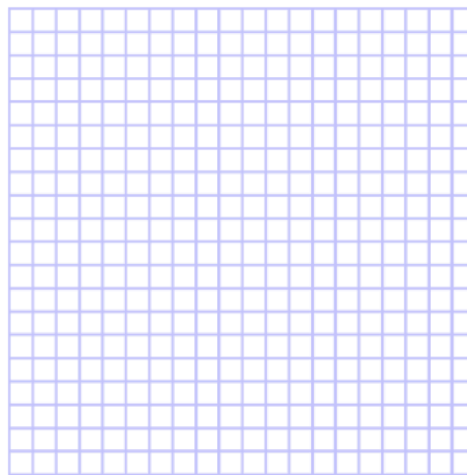
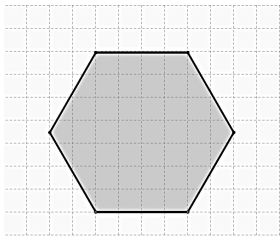
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Draw a scaled version of a triangle, other polygon, or other object given lengths.	I can't draw a scaled version of a triangle, other polygon, or other object.	I can draw a scaled version of a triangle, other polygon, or other object if given some assistance.	I can draw a scaled version of a triangle, other polygon, or other object given the lengths without assistance.	I can draw a scaled version of a triangle, other polygon, or other object given the lengths without assistance. I can explain the method used to draw the scaled version.
2. Find a measure of a scaled object given the scale factor and measure from the original.	I struggle to find a measure of a scaled object given the scale factor and measure from the original.	I can find measures of scaled objects given the scale factor and measure from the original if the scale factor is a whole number.	I can find measures of scaled objects given the scale factor and measure from the original.	I can find measures of scaled objects given the scale factor and measure from the original. I can explain the logic of the method used to find the new measures.
3. Find the scale factor between two objects that are the same shape but different sizes/proportional.	I struggle to find the scale factor between two objects.	I can find the scale factor between two objects if the scale factor is a whole number and/or I sometimes get confused between proportion and scale factor.	I can find the scale factor between two objects and know how it's related to proportionality.	I can find the scale factor between two objects. I can explain the logic of the method used to find the scale factor and how scale factor is related to proportionality.
4. Use proportional reasoning in explaining and finding missing sides of objects that are the same shape but different sizes.	I struggle to know how to find a missing side using proportional reasoning.	I can usually set up a proportion to find missing sides, but I sometimes mess up and/or I confuse proportionality and scale factor sometime.	I can find missing sides using proportional reasoning either from a proportional relationship or from a scale factor.	I can find a missing sides, and I can explain the relationship between two objects that are the same shape, different size using proportional reasoning and extend that explanation to scale factor.
5. Find the scale factors for perimeter or area for proportional objects.	I struggle to understand how changes in length affect the scale factor for perimeter and/or area of two objects of the same shape.	I know how changes in length affect the scale factor for perimeter and area of two objects of the same shape. I can also usually find the scale factors or new perimeters and areas.	I know how changes in length affect the scale factor for perimeter and area of two objects of the same shape. I can also always find the scale factors or new perimeters and areas.	I know how changes in length affect the scale factor for perimeter and area of two objects of the same shape. I can also always find the scale factors or new perimeters and areas and explain why the procedure works.

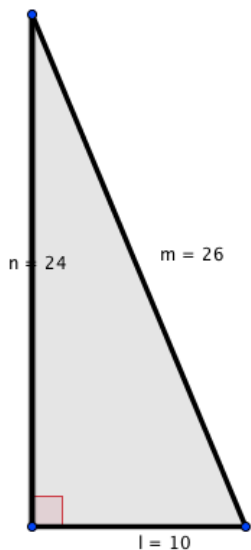
Sample Problems for Section 5.2

1. Draw a scaled version of the following polygons given the scale factor.

a. Scale Factor: 2



b. Scale Factor: $\frac{1}{2}$

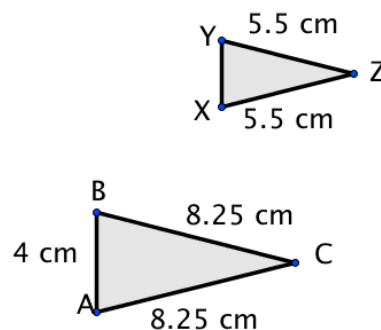
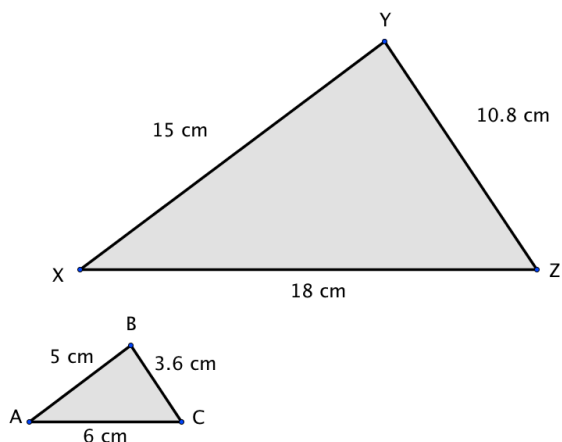


2. Find the missing measurement in each problem.

a. The scale factor $\triangle ABC$ to $\triangle DEF$ is 3. If \overline{BC} is 3, what is the length of \overline{EF} ?

b. The scale factor $\triangle GEL$ to $\triangle HOP$ is $\frac{3}{5}$. If \overline{OP} is 30, what is the length of \overline{EL} ?

3. In each of the following, what is the scale factor that takes $\triangle ABC$ to $\triangle XYZ$?
- a. b.



4. Answer each of the following questions using proportional reasoning.
- a. Becky is drawing a scale model of her neighborhood. She uses a scale of 1 in = 200 feet. If her block is 1000 feet long, what will be the length of the block on her scale model?
- b. Carlos is reading a map of his town. The scale says 1 in = 4 miles. The distance from his house to the school is $\frac{3}{8}$ in. on the map. If Carlos wants to walk to school from his house, how far will he have to walk?
5. The dimensions of Romina's rectangular garden are 2 feet by 3 feet. The dimensions of Santiago's garden are all quadrupled.
- a. Find the perimeter and area of each garden.
- b. Find the scale factor between the perimeters of each garden.
- c. Find the scale factor between the areas of each garden.
- d. In general, how are the scale factors of perimeters and areas of scaled objects related?

Section 5.3: Solving Problems with Circles

Section Overview: In this section circumference and area of a circle will be explored from the perspective of scaling. Students will start by measuring the diameter and circumference of various circles and noting that the ratio of the circumference to the radius is constant (2π). This should lead to discussions about all circles being scaled versions of each other. Next students will “develop” an algorithm for finding the area of a circle using strategies used throughout mathematical history. In these explorations, students should discuss two ideas: 1) cutting up a figure and rearranging the pieces so as to preserve area, and 2) creating a rectangle is a convenient way to find area. Students will connect the formula for finding the area of a circle (πr^2) to finding the area of a rectangle/parallelogram where the base is $\frac{1}{2}$ the circumference of the circle and the height is the radius ($A = Cr/2$.)

Students end the section by applying what they have learned to problem situations. In Chapter 6, students will use ideas of how circumference and area are connected to write equations to solve problems, but in this section, students should solve problems using informal strategies to solidify their understanding.

Concepts and Skills to be Mastered (from standards)

Geometry Standard 4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Geometry Standard 6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. 7.G.6

1. Explain the relationship between diameter of a circle and its circumference and area.
2. Explain the algorithm for finding circumference or area of a circle.
3. Find the circumference or area of any circle given the diameter or radius; or given circumference or area determine the diameter or radius.

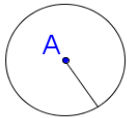
5.3a Class Activity: How Many Diameters Does it Take to Wrap Around a Circle?

1. Create *one* circle using either a) manual construction: a compass, tracing the base of a cylindrical object, or using a string compass, OR b) technology: GeoGebra, etc. (technology will allow for far more accurate measurement). Then, measure the *circumference* and diameter of the circle; collect measurements from five other students to fill in the table below.

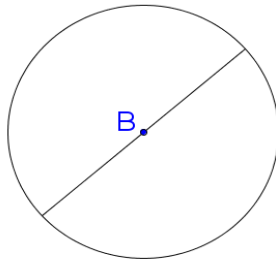
Measurement of the diameter in _____ units	Measurement of the circumference in _____ units (must be the same units as the diameter)	Ratio of circumference : diameter (C/d), as a decimal rounded to the nearest hundredth. Note: C represents <i>circumference</i> and d represents <i>diameter</i> .

2. What do you notice about the values in the third column?
3. If you made a huge circle the size of a city and measured the diameter and circumference, would the ratio of circumference to diameter (C/d) be consistent with the other ratios in the third column of the table? Justify your answer using what you learned from the previous section.
4. If you know the diameter of a circle is 5 inches, what is the approximate measure of the circumference? Justify your answer.

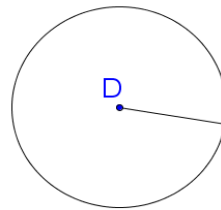
5. Write and justify a formula for circumference, in terms of the diameter.
6. Write and justify a formula for the circumference, in terms of the radius.
7. For each of the three circles below, calculate the circumference of the circle. Express your answer both in terms of π , and also as an approximation to the nearest tenth. *Please note: drawing is not to scale.*



radius = 0.5 cm
circumference = C
Solve for C.



Diameter = 2.5 cm
circumference = C
Solve for C.



radius = 3 units
circumference = C
Solve for C.

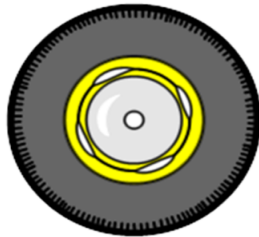
8. If the circumference of a circle is 8π (approximately 25.1) inches, which of the following is true? Rewrite false statements to make them true.
 - a. The ratio of Circumference: Diameter is 8.
 - b. The radius of the circle is $\frac{1}{2}$ the circumference.
 - c. The diameter of the circle is twice the radius.
 - d. The radius of the circle is 8 inches.
 - e. The diameter of the circle is 8 inches.

9. The circumference of 5 objects is given. Calculate the diameter of each object, to the nearest tenth of a unit.



Circumference
of bike wheel

76.9"



Circumference
of car tire

78.1"



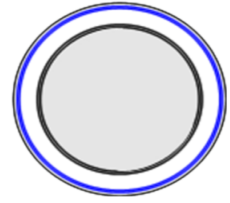
Circumference
of lid

11"



Circumference
of top of
garbage can

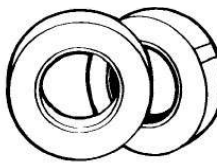
50"



Circumference
of plate

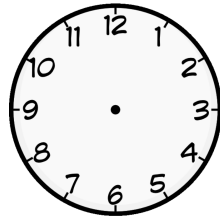
31"

10. The diameter or radius of 5 objects is given. Calculate the circumference of each object, to the nearest tenth of a unit.



Diameter of
masking tape

5"



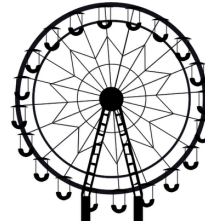
Radius of
clock face

11"



Diameter of
ring

2.5 cm



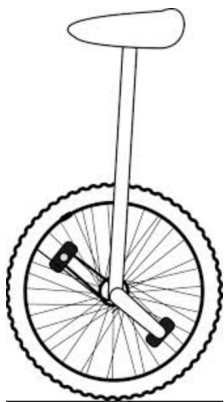
Diameter of
Ferris wheel

50'



Radius of
steering wheel

9"



11. When a unicyclist pedals once, the wheel makes one full revolution, and the unicycle moves forward the same distance along the ground as the distance around the edge of the wheel. If Daniel is riding a unicycle with a diameter of 20 inches, how many times will he have to pedal to cover a distance of 50 feet? Show all your work.

Spiral Review

1. Factor the following expressions.

$$4x - 10$$

$$21x + 35$$

$$4t - 2.7$$

2. Use a model to represent $-7 + 14 =$

3. Find 30% of 240 without a calculator.

4. Without using a calculator, determine which fraction is bigger in each pair. Justify your answer with a picture *and* words.

a. $\frac{21}{25}$ *or* $\frac{3}{5}$

b. $\frac{6}{11}$ *or* $\frac{8}{14}$

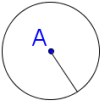
6. Milly bought two sweaters for \$30 and three pair of pants for \$25. She had a 20% off coupon for her entire purchase. Model or write an expression for the amount of money Millie spent.

5.3a Homework: How Many Diameters Does it Take to Wrap Around a Circle?

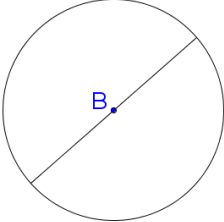
1. Identify 5 circular objects around the house (canned foods, door knobs, cups, etc.). Find the measure of each object’s diameter and then calculate its circumference. Put your results in the table below:

Description of item	Diameter (measured)	Circumference (calculated)	Ratio of $C : d$ (calculated) to the nearest hundredth

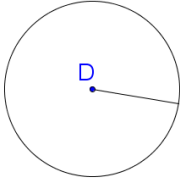
2. What is the exact ratio of the circumference to the diameter of every circle?
3. If the radius of a circle is 18 miles,
- a. What is the measure of the diameter?
 - b. What is the measure of the circumference, exactly in terms of pi?
 - c. What is the approximate measure of the circumference, to the nearest tenth of a mile?
4. For each of the three circles below, calculate the circumference. Express your answer both in terms of pi, and also as an approximation to the nearest tenth.



radius = 1.5 cm
circumference = C
Solve for C.



Diameter = 5 cm
circumference = C
Solve for C.



radius = 9 units
circumference = C
Solve for C.

The decimal for π starts with 3.141592653589... Which fraction is closest to π ? (Note: there is no fraction that is *exactly* equal to π .)

a) $3\frac{1}{4}$	b) $3\frac{1}{5}$	c) $3\frac{1}{7}$	d) $3\frac{1}{6}$	e) $3\frac{1}{8}$
-------------------	-------------------	-------------------	-------------------	-------------------

5. If the circumference of a circle is 20π feet, which of the following statements are true? Rewrite false statements to make them true.
- The circumference of the circle is exactly 62.8 feet.
 - The diameter of the circle is 20 feet.
 - The radius of the circle is 20 feet.
 - The ratio of circumference : diameter of the circle is π .
 - The radius of the circle is twice the diameter.
6. The circumference of 5 objects is given. Calculate the diameter of each object, to the nearest tenth of a unit.



Circumference
of bottom of
cupcake
6.5"



Circumference
of top of mug
13"



Circumference
of wheel
314 cm

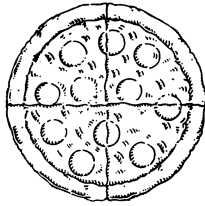


Circumference
of top of water
pail
100 cm



Circumference
of earth
24,901 miles

7. The diameter or radius of 5 objects is given. Calculate the circumference of each object, to the nearest tenth of a unit.



Diameter
of pizza
16"



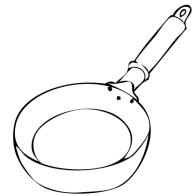
Diameter of
rim of a drum
24"



Radius of
table top
2'



Diameter of
coin
5 cm

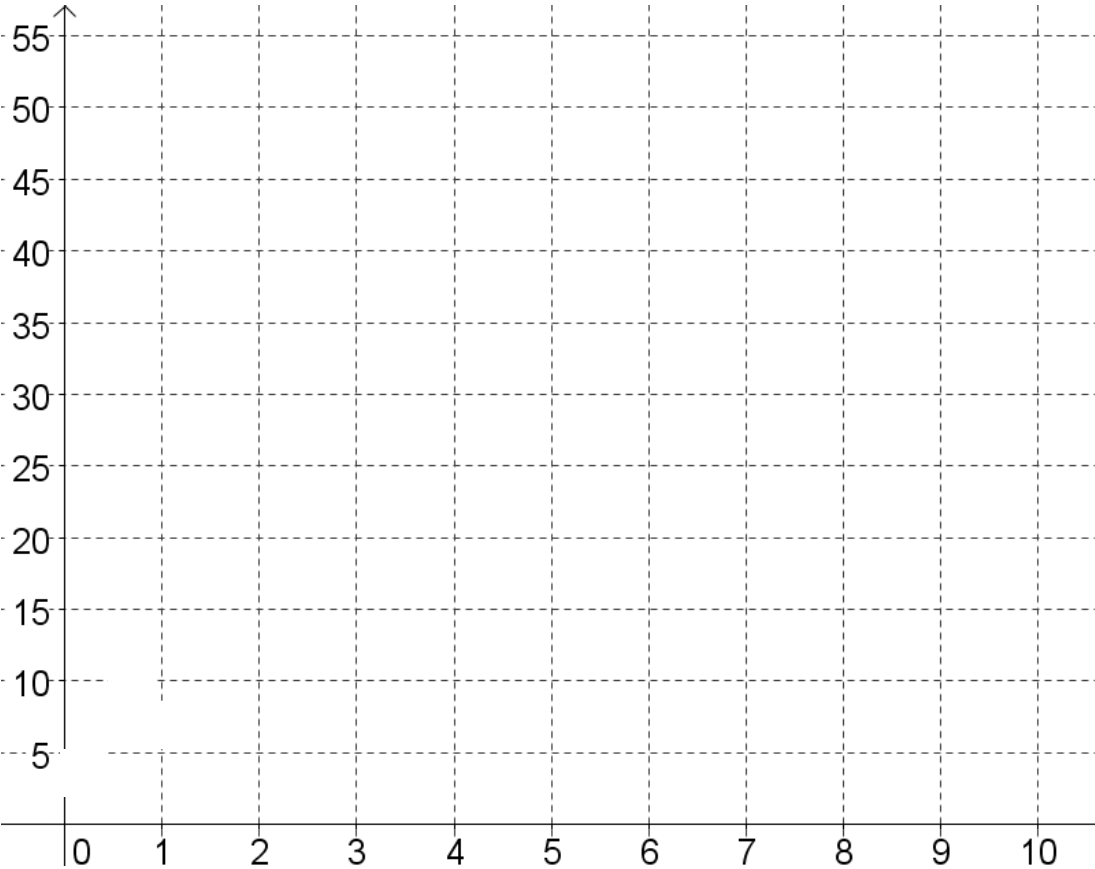


Radius of
frying pan
5.5"

8. Three tennis balls are stacked and then tightly packed into a cylindrical can. Which is greater: the height of the can, or the circumference of the top of the can? Justify your answer.

9. Calculate the radius for each circle whose circumference is given in the table (the first entry is done for you). Then graph the values on a coordinate plane, with the radius on the x axis and the approximate circumference on the y axis.

Radius of circle	4 units							
Circumference of circle	8π un ≈ 25 un	10π ≈ 31 un	2π un ≈ 6 un	16π un ≈ 50 un	6π un ≈ 19 un	18π un ≈ 57 un	4π un ≈ 13 un	12π un ≈ 38 un



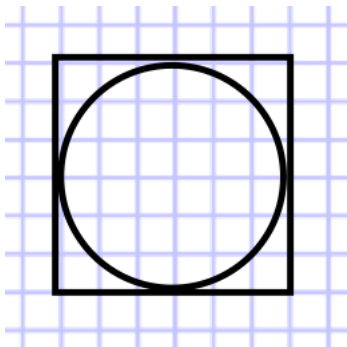
10. Is the radius of a circle proportional to the circumference of the circle? Justify your answer.

5.3b Classwork: Area of a Circle

Activity: Circle Area

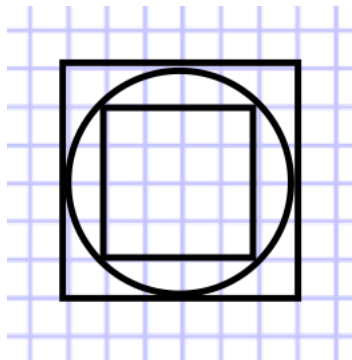
History: Methods for computing the area of simple polygons were known to ancient civilizations like the Egyptians, Babylonians and Hindus from very early times in Mathematics. But computing the area of circular regions posed a challenge. Archimedes (287 BC – 212 BC) wrote about using a method of approximating the area of a circle with polygons. Below, you will try some of the methods he explored for finding the area of a circle of diameter 6 units.

- a. Estimate the area of the circle by counting the number of square units in the circle.



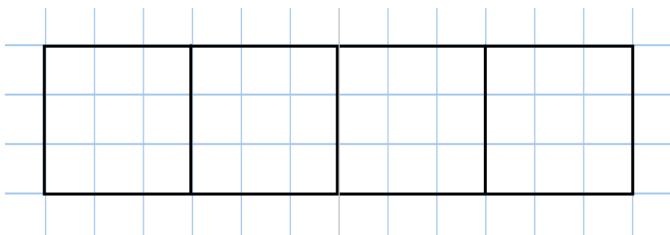
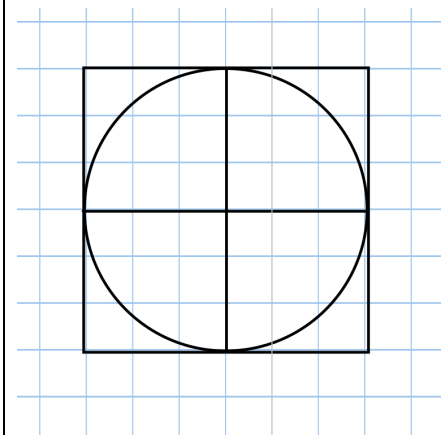
Estimated area = _____

- b. Estimate the area of the circle by averaging the inscribed and circumscribed squares.



Estimated area = _____

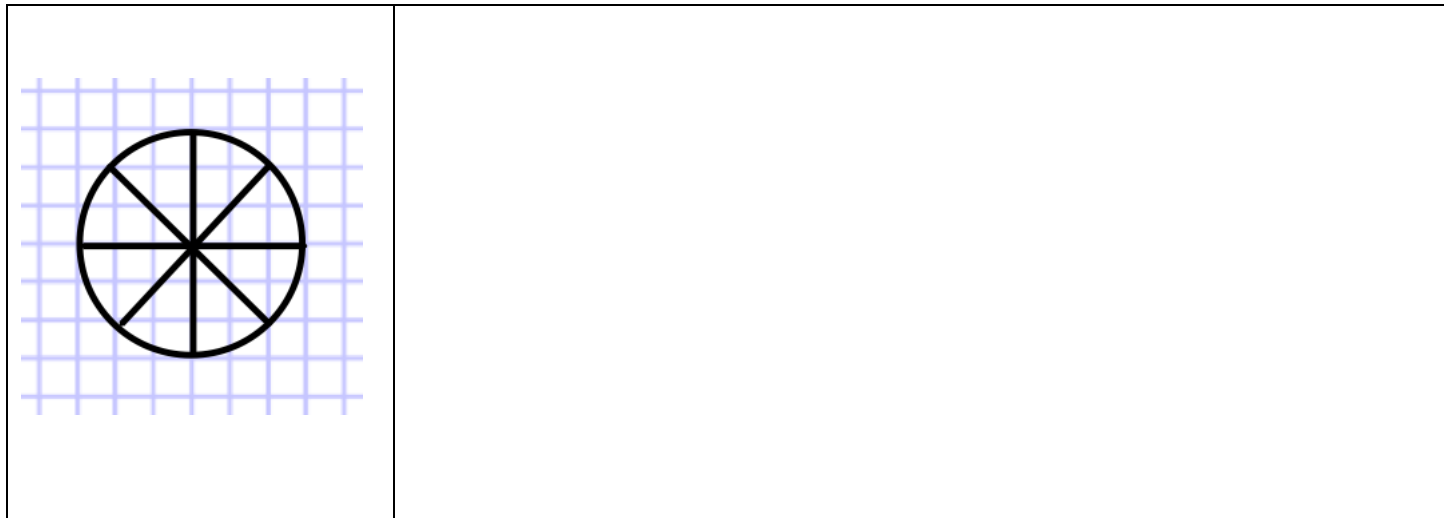
- c. In the figure below on the left, the large square circumscribing the circle is divided into four smaller squares. Let's call the four smaller squares "radius squares." The four radius squares are lined up below on the right. Estimate the number of squares units (grid squares) there are in the circle and then transfer them to the four radius squares below.

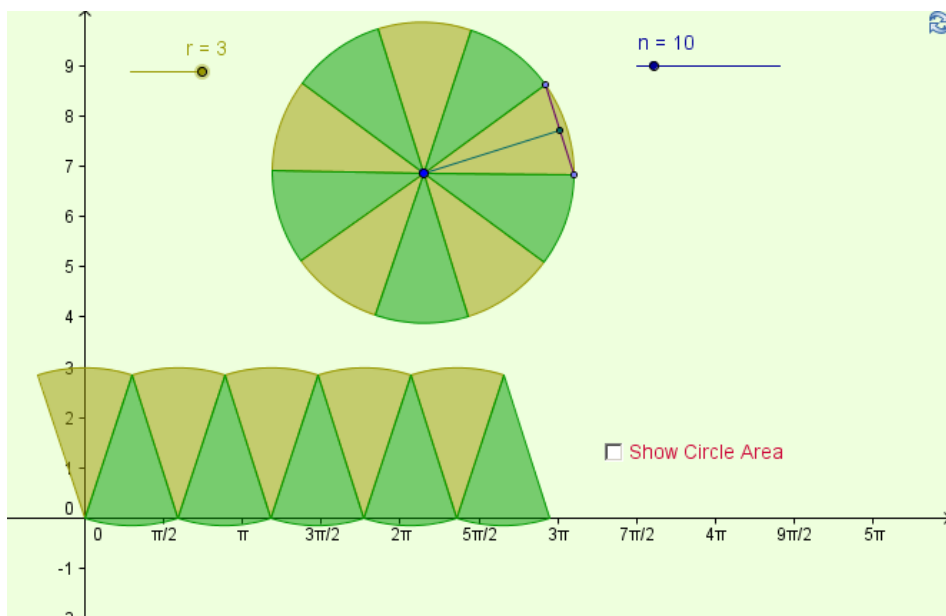


How many radius squares cover the same area as the circle? _____

Johannes Kepler (1571-1630) tried a different approach: he suggested dividing the circle into “isosceles triangles” and then restructuring them into a parallelogram. Refer to the Mathematical Foundation for more information about this approach.

Cut the circle into eighths. Then fit and paste the eighths into a long line (turn the pie pieces opposite ways) to create a “parallelogram.”

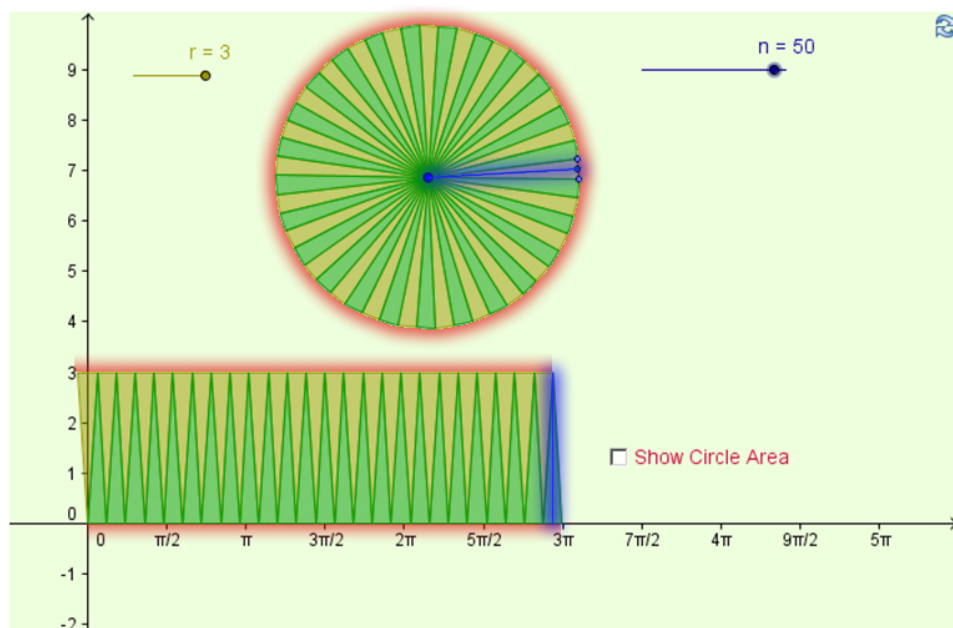




The figure to the left shows the same circle of radius 3 as the previous example, but this time cut into 10 wedges. How will this parallelogram compare to the one created with 8 wedges above?

Will the area created by reorganizing the pieces be the same or different than the original circle? Explain.

In the next diagram, the same circle of radius 3, but this time it's cut into 50 wedges. Again it is packed together into a parallelogram.

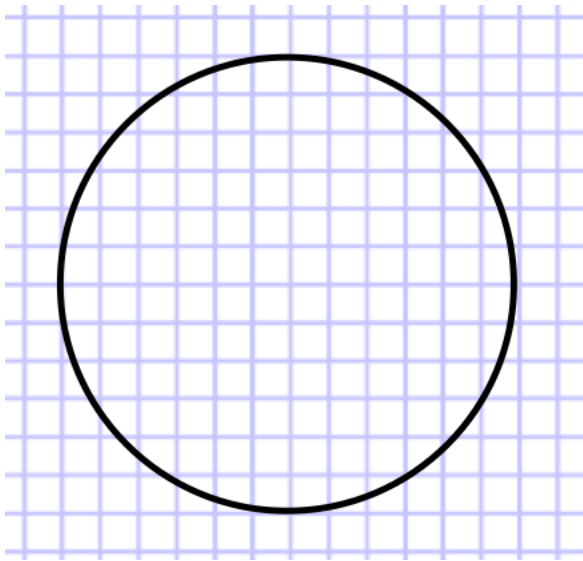


Highlight the circumference of the circle. Then highlight where the circumference is found in the new diagram. Explain why the base of the “parallelogram” is half the circumference of the circle.

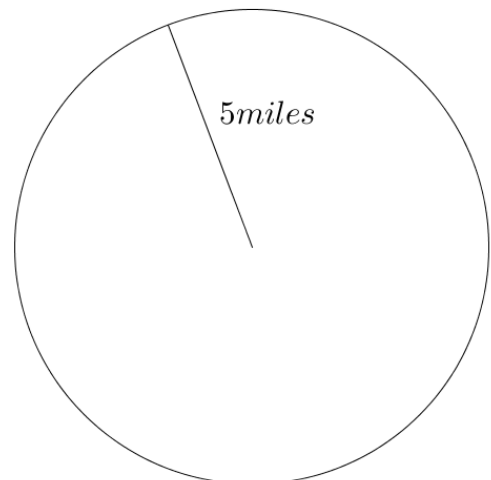
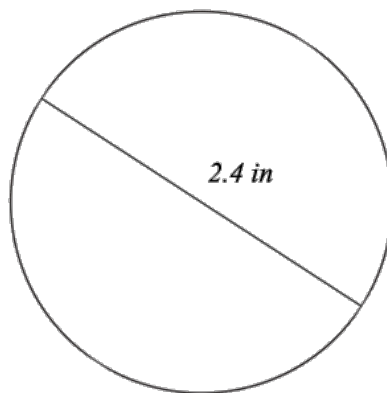
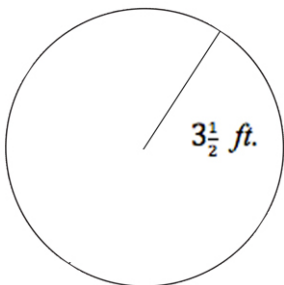
Highlight the radius of the circle in a different color. Then highlight where the radius is found in the new diagram. Explain why the height of the “parallelogram” is the same as the radius.

Use the figure and what you know about the area of a rectangle to write an expression for the area of the circle.

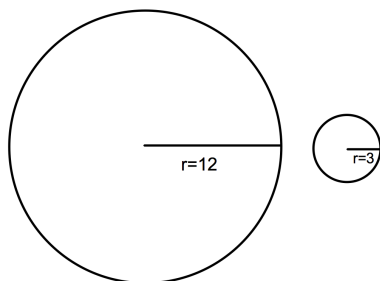
1. Estimate the area of the circle in square units by counting.



2. Use the formula for the area of a circle to calculate the exact area of the circle above, in terms of π .
3. Calculate the area for #2 to the nearest square unit. How accurate was your estimate in #1?
4. Calculate the area of each circle. Express your answer both exactly (in terms of π) and approximately, to the nearest tenth of a unit.

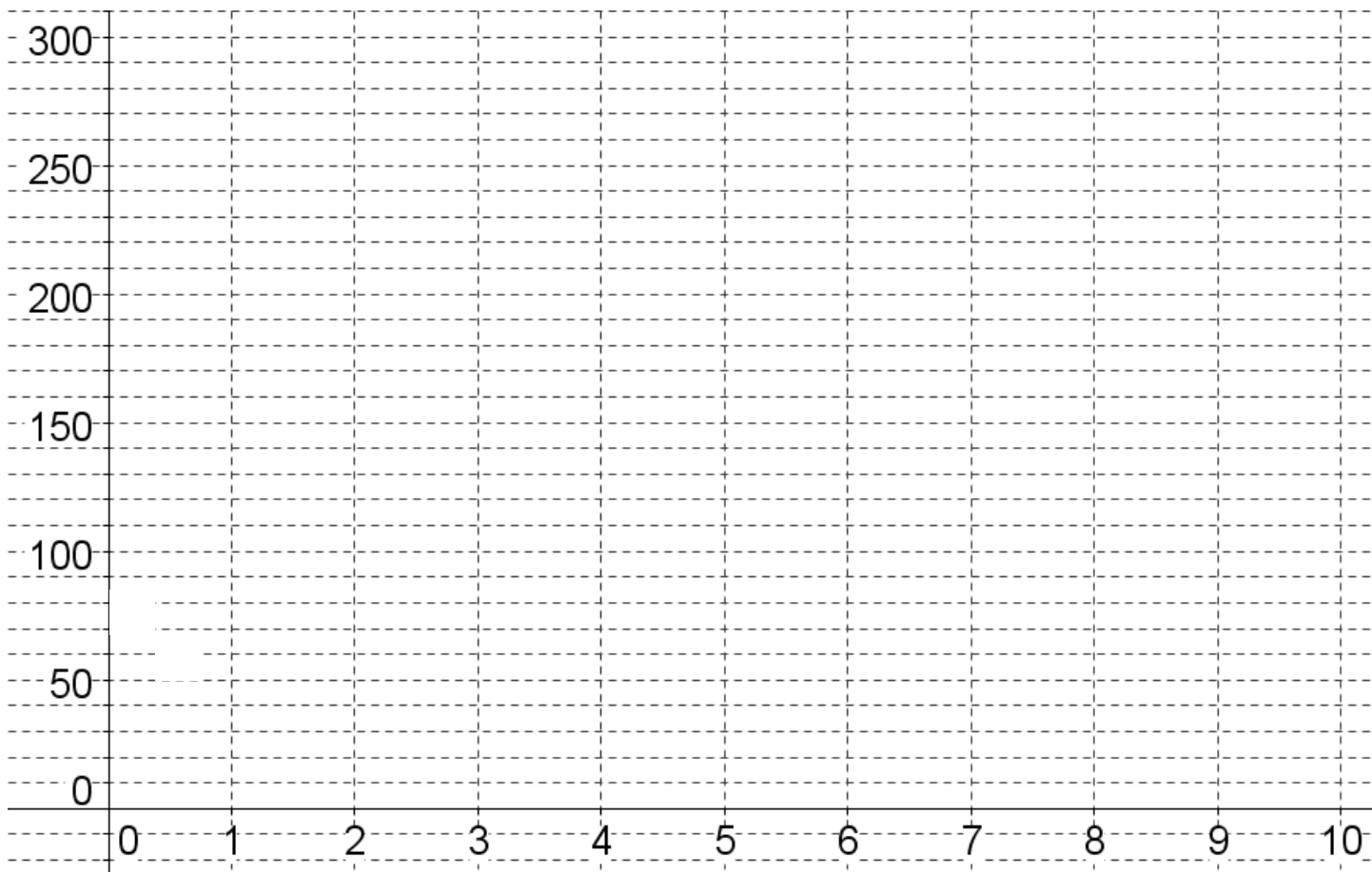


5. A certain earthquake was felt by everyone within 50 kilometers of the epicenter in every direction.
- Draw a diagram of the situation.
 - What is the area that felt the earthquake?
6. There is one circle that has the same numeric value for its circumference and its area (though the units are different.) Use any strategy to find it. Hint: the radius is a whole number.
7. Explain the difference in the units for circumference and area for the circle in #6.
8. Draw a diagram to solve: A circle with radius 3 centimeters is enlarged so its radius is now 6 centimeters.
- By what scale factor did the circumference increase? Show your work or justify your answer.
 - By what scale factor did the area increase? Show your work or justify your answer.
 - Explain why this makes sense, using what you know about scale factor.
9. How many circles of radius 3" can you fit in a circle with radius 12" (if you could cut up the smaller circles to tightly pack them into the larger circle with no gaps)? See the image below. Justify your answer.



10. Calculate the radius for each circle whose area is given in the table (the first entry is done for you). Then graph the values on a coordinate plane, with the radius on the x axis and the approximate area on the y axis.

Radius of circle	6 units							
Area of circle	$36\pi \text{ un}^2$ $\approx 113 \text{ un}^2$	16π $\approx 50 \text{ un}^2$	$81\pi \text{ un}^2$ $\approx 254 \text{ un}^2$	$25\pi \text{ un}^2$ $\approx 79 \text{ un}^2$	$9\pi \text{ un}^2$ $\approx 28 \text{ un}^2$	$64\pi \text{ un}^2$ $\approx 201 \text{ un}^2$	$49\pi \text{ un}^2$ $\approx 154 \text{ un}^2$	$4\pi \text{ un}^2$ $\approx 12 \text{ un}^2$



11. Is the radius of a circle proportional to the area of the circle? Justify your answer.

12. The area of 5 objects is given. Calculate the radius of each object's surface, to the nearest hundredth of a unit.



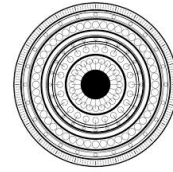
Area of a
smiley face
 3.14 in^2



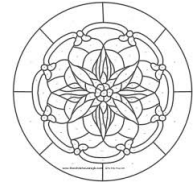
Area of the
base of a
plant pot
 50.24 in^2



Area of a
target
 153.86 in^2



Area of circular
tile pattern
 78.5 ft^2



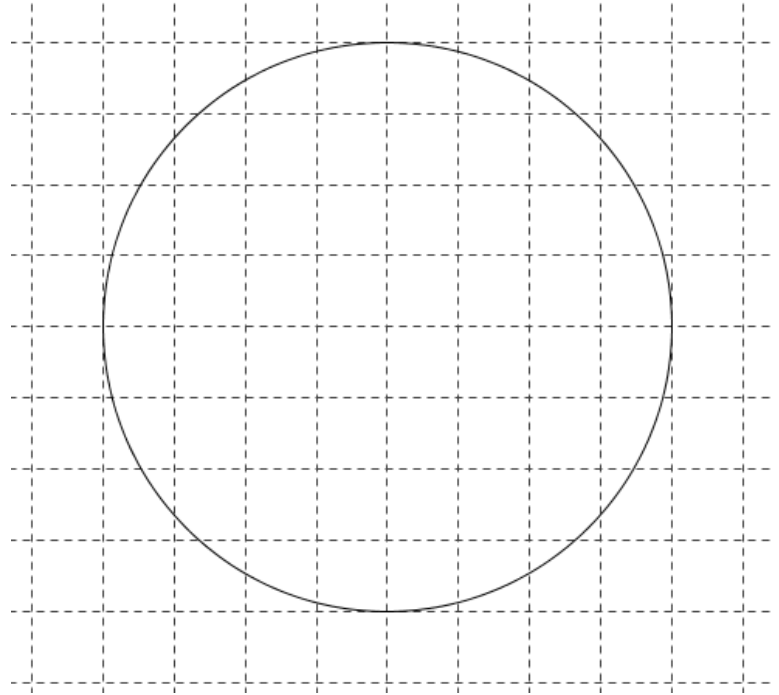
Area of glass in
round window
 12.56 ft^2

Spiral Review

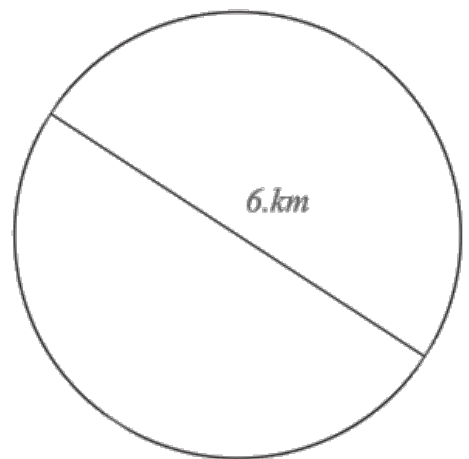
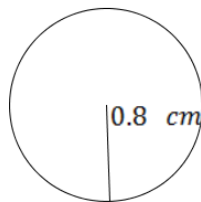
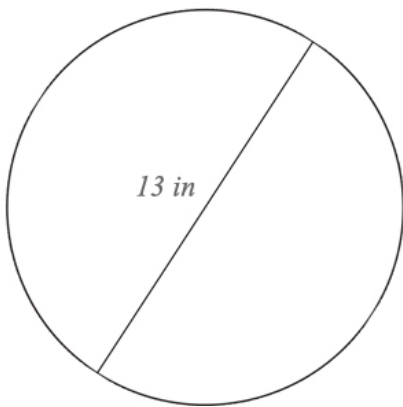
1. $7z + 1 = 15$
2. $-15 = 1.2m + 2.4$
3. Show two ways one might simplify: $5(3 + 4)$
4. There are a total of 214 cars and trucks on a lot. If there are four more than twice the number of trucks than cars, how many cars and trucks are on the lot?
5. $-1 \times -4 \times -7$

5.3b Homework: Area of a Circle

1. Estimate the area of the circle in square units by counting.



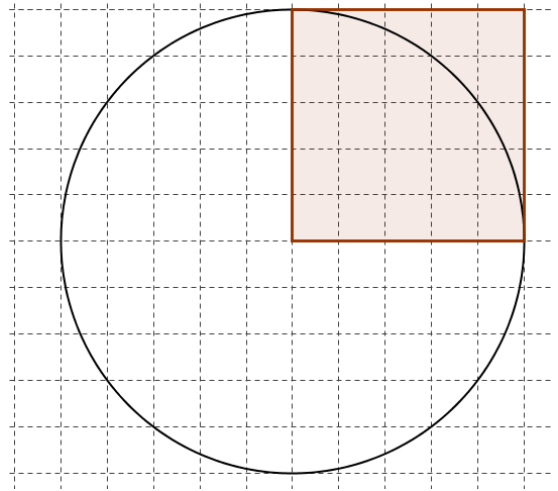
2. Use the formula for the area of a circle to calculate the exact area of the circle above, in terms of π .
3. Calculate an approximation for the area expression from #2, to the nearest square unit. How accurate was your estimate in #1?
4. Calculate the area of each circle. Express your answer both exactly (in terms of π) and approximately, to the nearest tenth of a unit.



5. The strongest winds in Hurricane Katrina extended 30 miles in all directions from the center of the hurricane.
- Draw a diagram of the situation.

b. What is the area that felt the strongest winds?

6. By calculating the areas of the square and the circle in the diagram, determine how many times larger in area the circle is than the square.

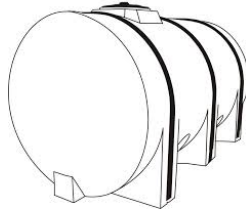


7. Draw a diagram to solve: A circle with radius 8 centimeters is enlarged so its radius is now 24 centimeters.
- By what scale factor did the circumference increase? Show your work or justify your answer.
 - By what scale factor did the area increase? Show your work or justify your answer.
 - Explain why this makes sense, using what you know about scale factor.

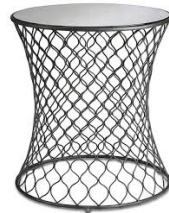
8. How many circles of radius 1" could fit in a circle with radius 5" (if you could rearrange the area of the circles of radius 1 in such a way that you completely fill in the circle of radius 5)? Justify your answer.
9. The area of 5 objects is given. Calculate the radius of each object's surface, to the nearest hundredth of a unit.



Area of a
glass in a
porthole
 3.14 ft^2



Area of side
of a water
tank
 153.86 ft^2



Area of wicker
table top
 28.26 ft^2



Area of base
of trash can
 12.56 ft^2



Area of round
area rug
 153.86 ft^2

5.3c Self-Assessment: Section 5.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Explain the relationship between diameter of a circle and its circumference and area.	I struggle to understand the relationship between diameter of a circle and its circumference or area.	I know there is a relationship between diameter of a circle and its circumference and area, but I have difficulty explaining it.	I can explain the relationship between diameter of a circle and its circumference and area.	I can explain the relationship between diameter of a circle and its circumference and area. Additionally, I can also apply my understanding to a variety of contexts.
2. Explain the algorithm for finding circumference or area of a circle.	I can't explain why the algorithm for circumference or area of a circle works or where it came from.	I can sort of explain why the algorithm for circumference or area of a circle works or where it came from.	I can explain why the algorithm for circumference or area of a circle works or where it came from using pictures and words.	I can explain why the algorithm for circumference or area of a circle works or where it came from using pictures and words. I can also apply my understanding to a variety of contexts.
3. Find the circumference or area of any circle given the diameter or radius; or given circumference or area determine the diameter or radius.	I struggle to find the circumference and/or area of a circle given the diameter or radius AND/OR determine the diameter or radius given the circumference or area.	I can usually find the circumference or area of a circle given the diameter or radius AND/OR determine the diameter or radius given the circumference or area.	I can always find the circumference or area of a circle given the diameter or radius AND/OR determine the diameter or radius given the circumference or area.	I can always find the circumference or area of a circle given the diameter or radius AND/OR determine the diameter or radius given the circumference or area. I can also apply my understanding to a variety of contexts.

Sample Problems for Section 5.3

1. Use pictures and/or words to explain:
 - a. The relationship between the diameter of a circle and its circumference
 - b. The relationship between the diameter of a circle and its area
2. Use pictures and/or words to explain:
 - a. The algorithm for finding the circumference of a circle
 - b. The algorithm for finding the area of a circle
3. Use the given information to find the missing information. Give each answer exactly and rounded to the nearest hundredth unit.
 - a. Radius: 2 m
Circumference: _____
 - b. Diameter: 2 m
Circumference: _____
 - c. Radius: 5.5 in
Area: _____
 - d. Diameter: 40 in
Area: _____
 - e. Circumference: 69.08 cm
Diameter: _____
 - f. Area: 153.86 cm^2
Radius: _____

Section 5.4: Angle Relationships

Section Overview: In this sections students will learn and begin to apply angle relationships for vertical angles, complementary angles and supplementary angles. They will practice the skills learned in this section further in Chapter 6 when they write equations involving angles. Students will also use concepts involving angles to relate scaling of triangles and circles. At the end of this section there is a review activity to help students tie concepts together.

Concepts and Skills to be Mastered (from standards)

Geometry Standard 5: Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

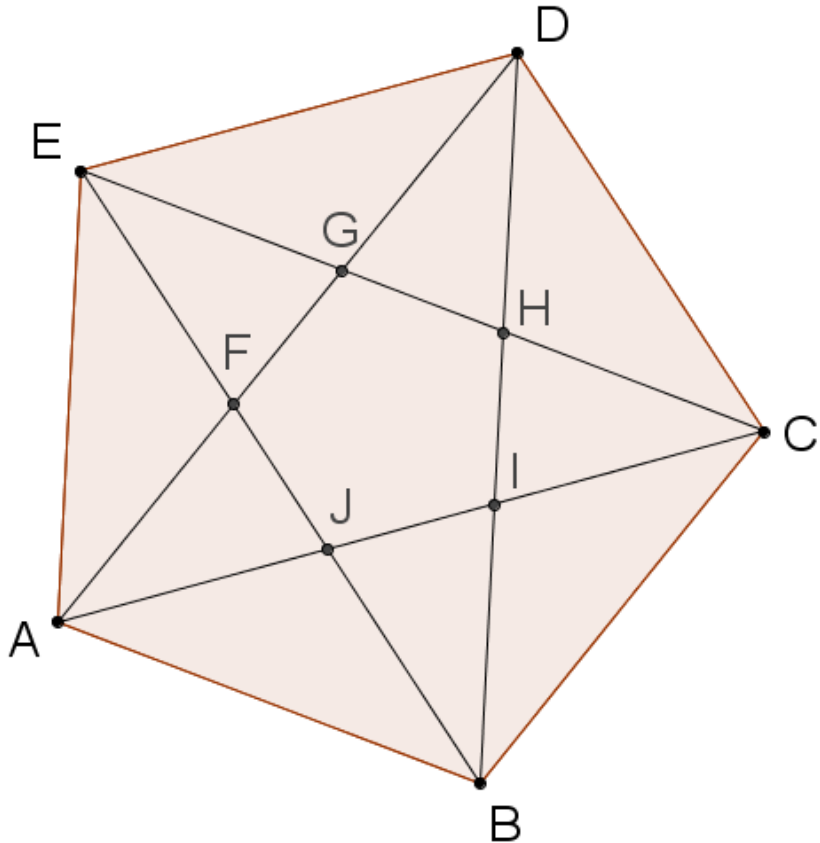
Geometry Standard 6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. 7.G.6

1. Identify vertical, complementary and supplementary angles.
2. Find the measures of angles that are vertical, complementary or supplementary to a known angle.
3. Apply angle relationships to find missing angle measures. Given angle measures, determine the angle relationship.

5.4a Classwork: Special Angle Relationships

This diagram is a regular pentagon with all its diagonals drawn and all points labeled.

1. How many *non-overlapping* angles are in the diagram?
2. There are groups of angles that all have the same measure. For example, $\angle DEG$ and $\angle AEF$ have the same measure. How many *different measures* of angles are there in the diagram? Use a protractor.

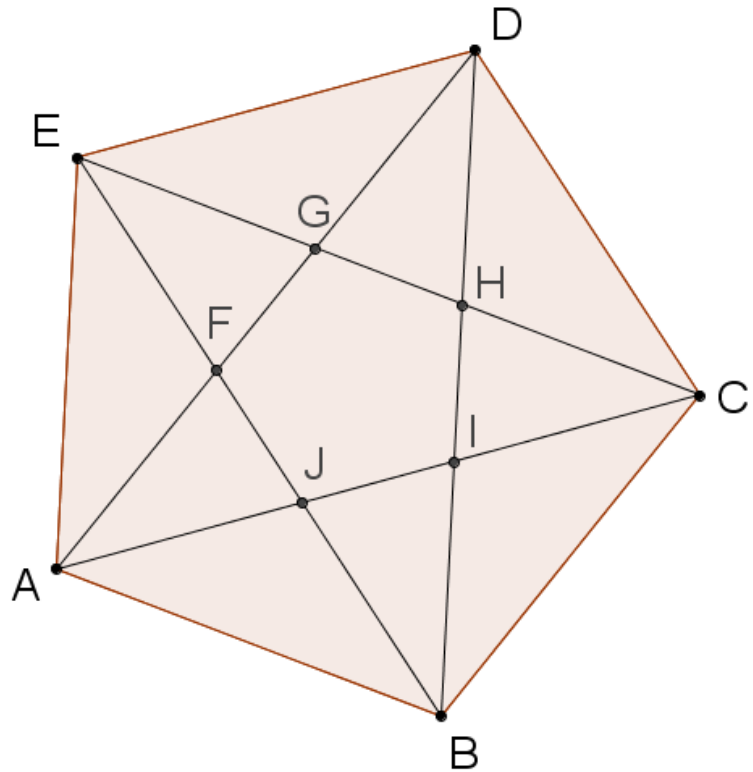


Vertical Angles: Two lines that intersect form vertical angles. Vertical angles are pairs of angles that are always opposite one another (rather than adjacent to each other). For example, $\angle EFG$ and $\angle AFJ$ are vertical angles.

Adjacent Angles: Two angles are adjacent if they have a common ray (side) and vertex. For example, $\angle EFG$ and $\angle EFA$ are adjacent angles.

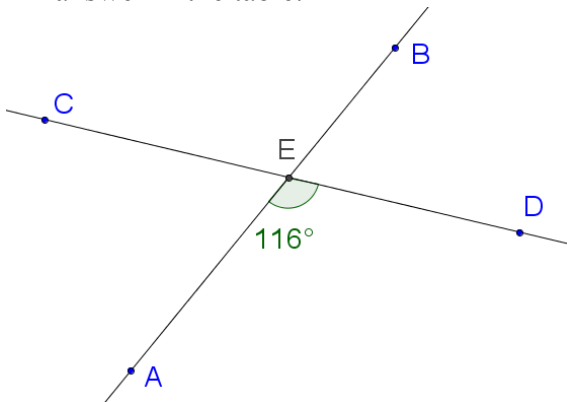
3. Name at least five vertical angle pairs. Use a different colored pencil to mark each pair in the diagram above.
4. Name at least five adjacent angle pairs. Use a different colored
5. What seems to be the relationship between measures of two vertical angles? Draw another pair of vertical angles below by constructing two intersecting lines and measure the two angles in the vertical pair. Does this example support your conjecture?

$\angle DGE$ and $\angle EGF$ are called *supplementary* angles because their measures add to 180° . When supplementary angles are adjacent, you can see that they form a straight line with the two outside rays. Supplementary angles don't always have to be next to each other.

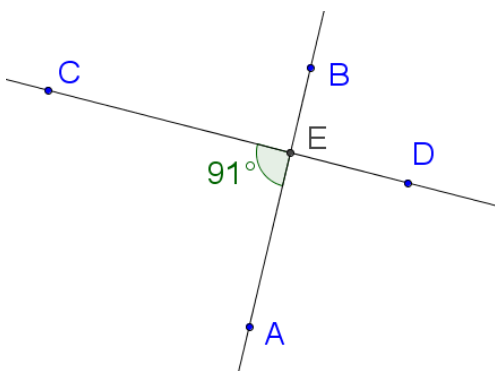


6. Find and name at least 5 pairs of supplementary angles in the diagram. Use a different color to mark each pair in the diagram.

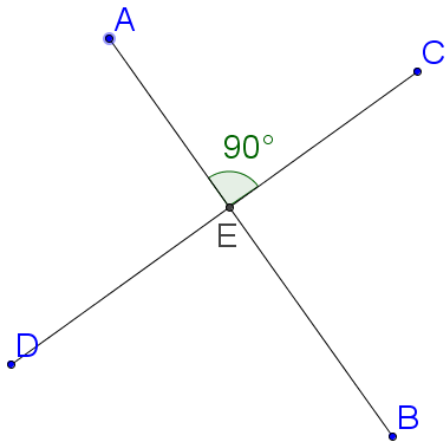
7. For each pair of intersecting lines below, find the three missing measures of angles formed. Justify your answer in the table.



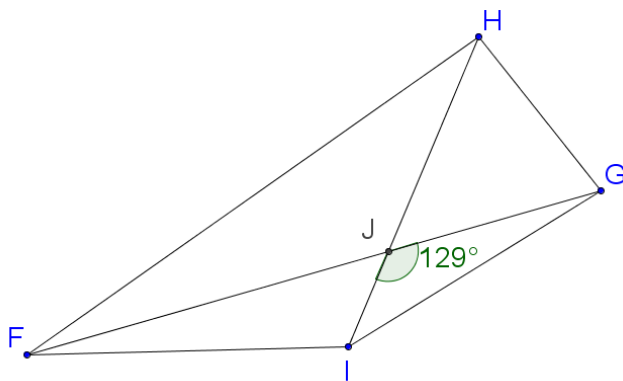
Angle	Measure of angle	Justification
$\angle CEB$		
$\angle DEB$		
$\angle AEC$		



Angle	Measure of angle	Justification
$\angle CEB$		
$\angle DEB$		
$\angle AED$		

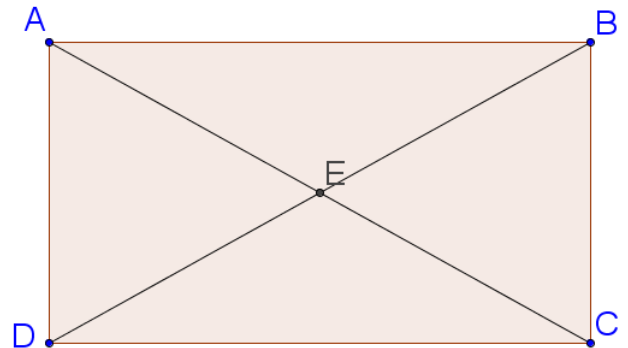


Angle	Measure of angle	Justification
$\angle CEB$		
$\angle DEB$		
$\angle AED$		



Angle	Measure of angle	Justification
$\angle FJI$		
$\angle HJG$		
$\angle FJH$		

8. For this rectangle with diagonals drawn in, there is one place where you can see supplementary and vertical angles. Use a protractor to measure one of the angles, and then calculate the measures of the other three angles that have vertex at E using facts about vertical and supplementary pairs.

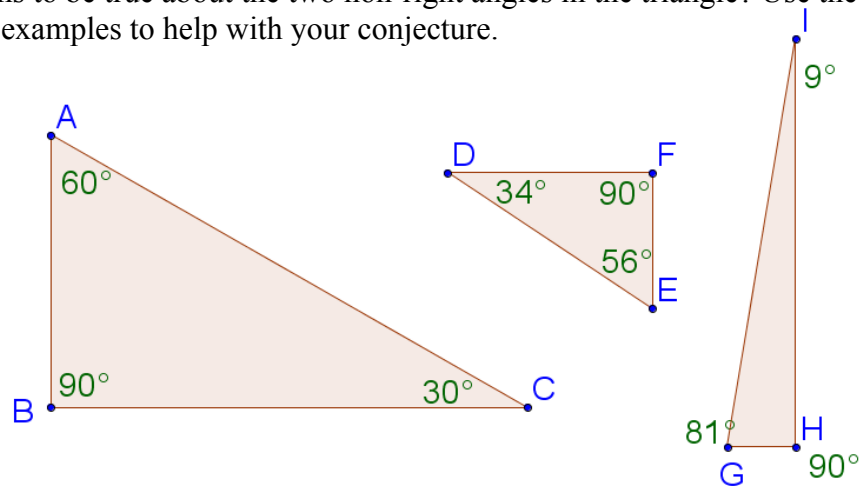


$\angle EAB$ and $\angle DAE$ are *complementary* because their measures add to 90° . When complementary angles are adjacent, you can see the right angle that is formed by the outside rays. However, complementary angles don't need to be adjacent; as long as their measures add to 90 degrees, two angles form a complementary pair.

9. Find and name at least five more pairs of complementary angles in the figure above. Use a different highlighter to mark each pair on the diagram.

10. Review: What is the sum of all the angles in a triangle?

11. Consider a right triangle. What seems to be true about the two non-right angles in the triangle? Use the examples below, or draw your own examples to help with your conjecture.

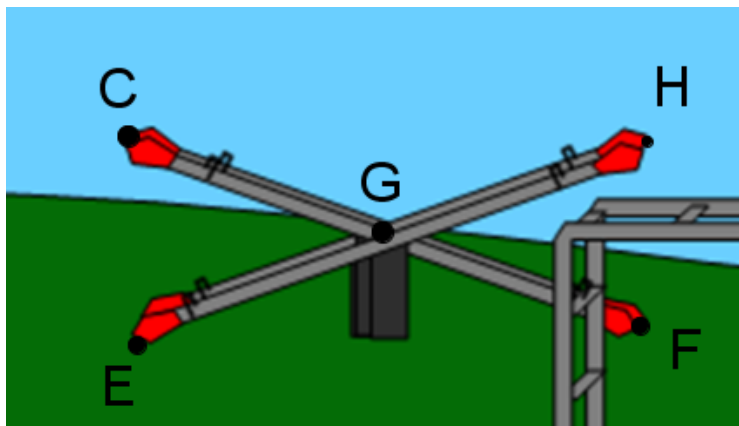


12. Look around the room you're in right now. Find examples of angles in the furniture, tiles, posters, etc. Can you see any complementary angles? Can you see supplementary angles? Can you see vertical angles? Draw sketches for at least three angle pairs you find.

For #13-14, use properties of complementary, supplementary, and vertical angles to find missing measures.

13. Two pair of seesaws sit unused at a playground, as shown. $\angle EGF$ has a measure of 140° .

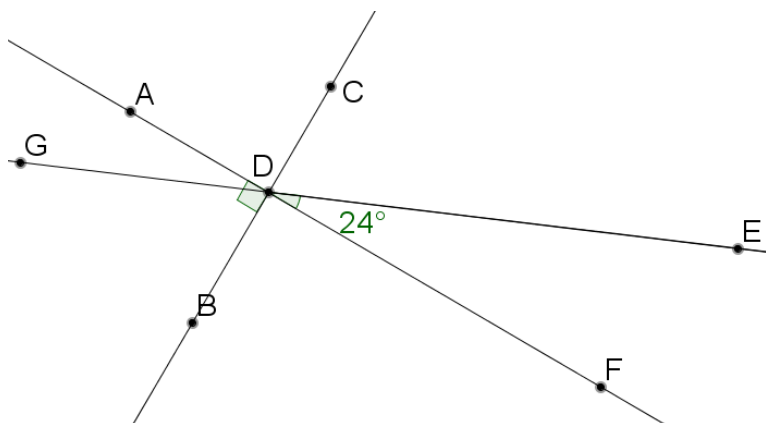
- Which angle is vertical to $\angle EGF$?
- What is the measure of $\angle CGH$?
- Name an angle that is supplementary to $\angle EGF$.
- What is the measure of $\angle CGE$?
- What is the measure of $\angle HGF$?



14. In the diagram below, $\angle ADB$ is a right angle. The figure is formed by 3 intersecting lines.

Fill in the measures and justifications in the table:

Angle	Measure	Justification
$\angle CDA$	90°	Supplementary to $\angle ADB$, which is 90° .
$\angle ADG$		
$\angle GDB$		
$\angle BDF$		
$\angle EDC$		



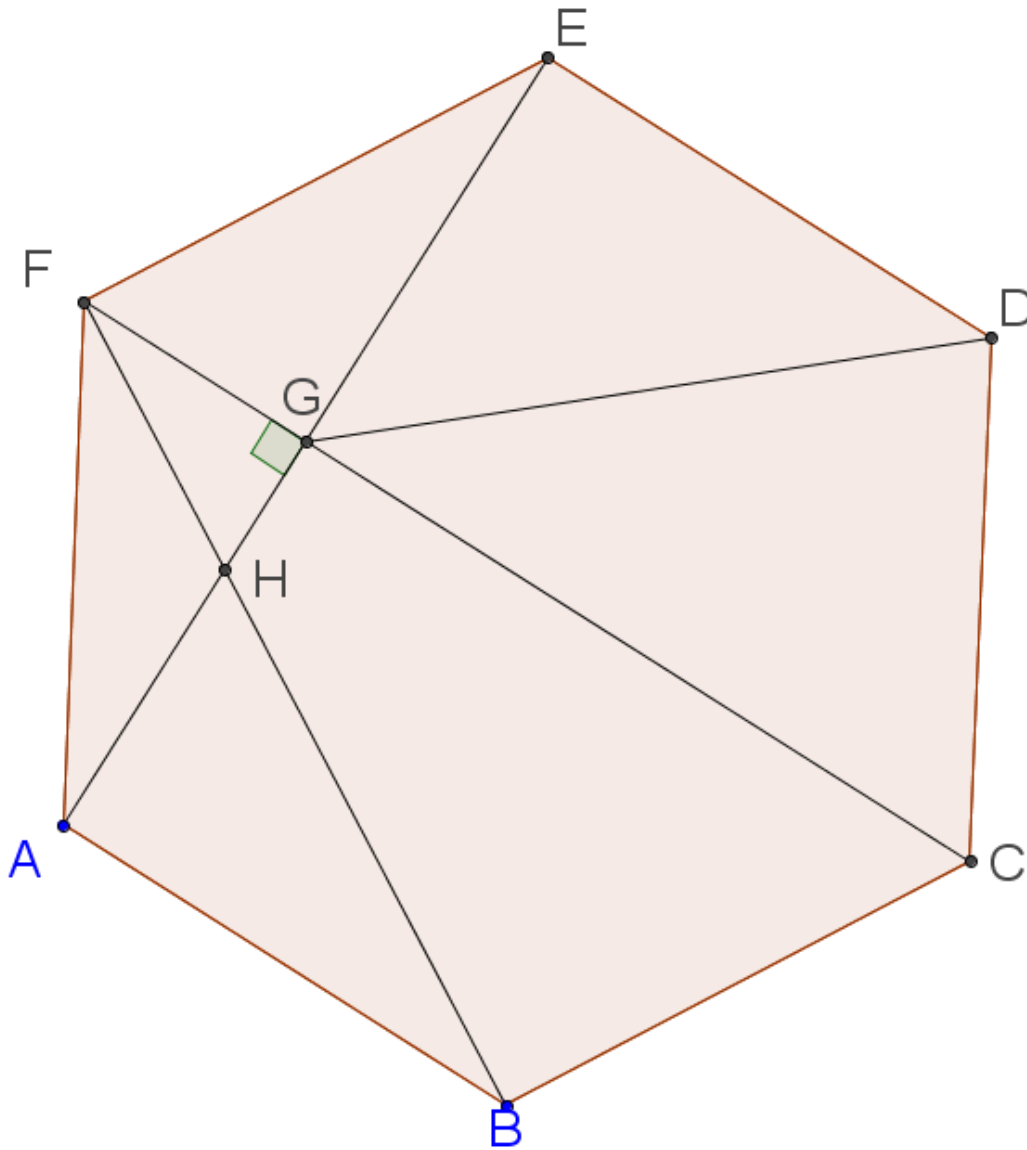
Spiral Review

1. Order the numbers from least to greatest. $-\frac{1}{2}$, $\frac{1}{4}$, $-\frac{1}{4}$, -1.2 , -1.02 , $-.75$
2. Find the quotient: $-\frac{8}{9} \div \left(-\frac{3}{4}\right)$
3. Find the unit rate for BOTH units.
Izzy drove 357 miles on 10 gallons of gasoline.
4. Convert the following units using the ratios given:

_____ feet = 37 inches (1 foot = 12 inches)
5. The temperature at midnight was 8°C . By 8 AM, it had risen 1.5° . By noon, it had risen a additional 2.7° . Then at 6 PM a storm blew in causing it to drop 4.7° . What was the temperature at 6 PM?

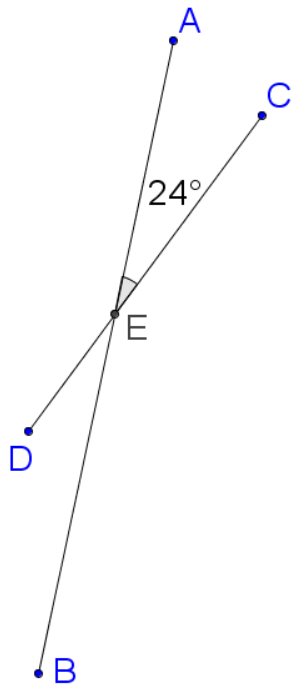
5.4a Homework: Special Angle Relationships

1. Find at least one example of each angle relationship in the diagram. Name the angle pairs below, and highlight the pairs of angles in the diagram, using a different color for each relationship.

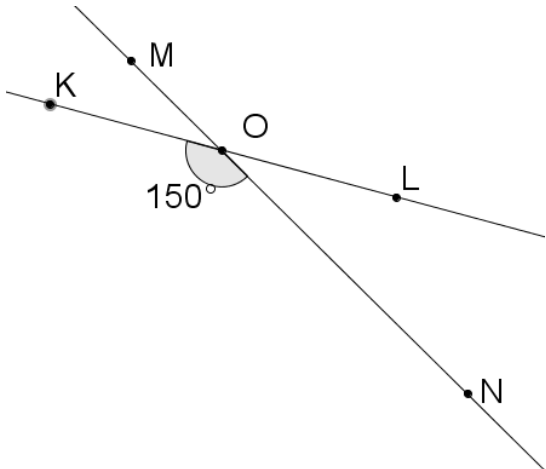


- a. Vertical angles
- b. Supplementary angles
- c. Complementary angles

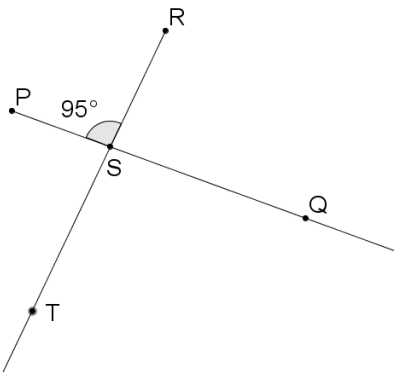
2. For each figure of two intersecting lines, calculate the three missing measures, justifying your answer.



Angle	Measure of angle	Justification
$\angle CEB$		
$\angle DEA$		
$\angle BED$		

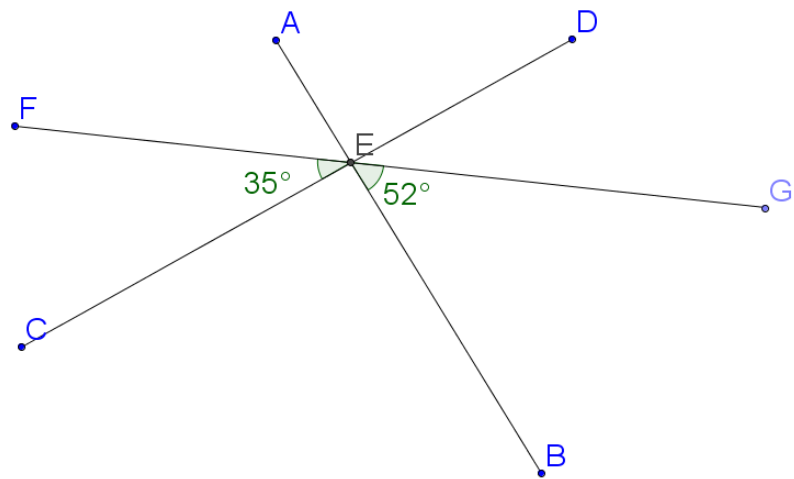


Angle	Measure of angle	Justification
$\angle LOM$		
$\angle MOK$		
$\angle NOL$		



Angle	Measure of angle	Justification
$\angle PST$		
$\angle RSQ$		
$\angle QST$		

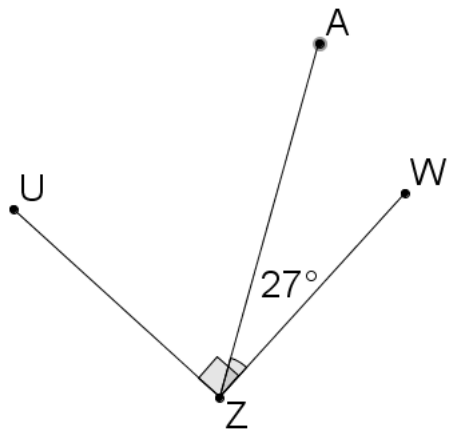
3. For the figure formed by three intersecting lines, calculate the four missing measures, justifying your answer.



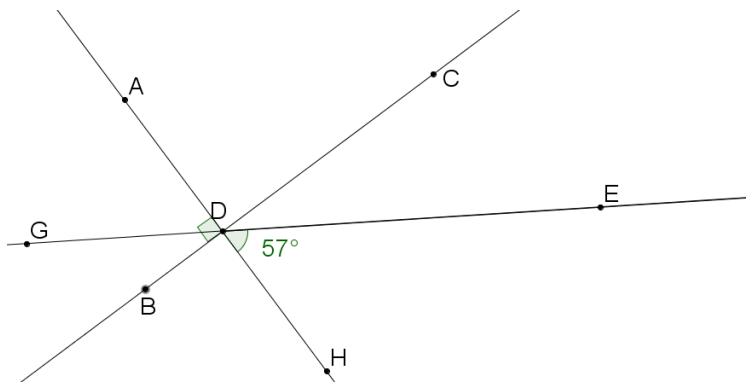
Angle	$\angle AEF$	$\angle AEC$	$\angle AED$	$\angle DEG$	$\angle CEB$	$\angle FEG$
Measure	52°					
Justification	Vertical to $\angle GEB$					

4. Refer to the figure below.

$m\angle AZU =$ because



5. Fill in the missing angle measurements in the table, and give a justification for each measurement.



Angle	$\angle ADG$	$\angle GDB$	$\angle BDH$	$\angle CDH$	$\angle CDE$	$\angle CDA$
Measure	57°					
Justification	Vertical to $\angle EDH$					

For #6-8, draw a diagram to illustrate the situation, and then choose the correct answer.

6. If $\angle G$ is complementary to $\angle H$, and $m\angle H = 20^\circ$, then $\angle G$ must be:

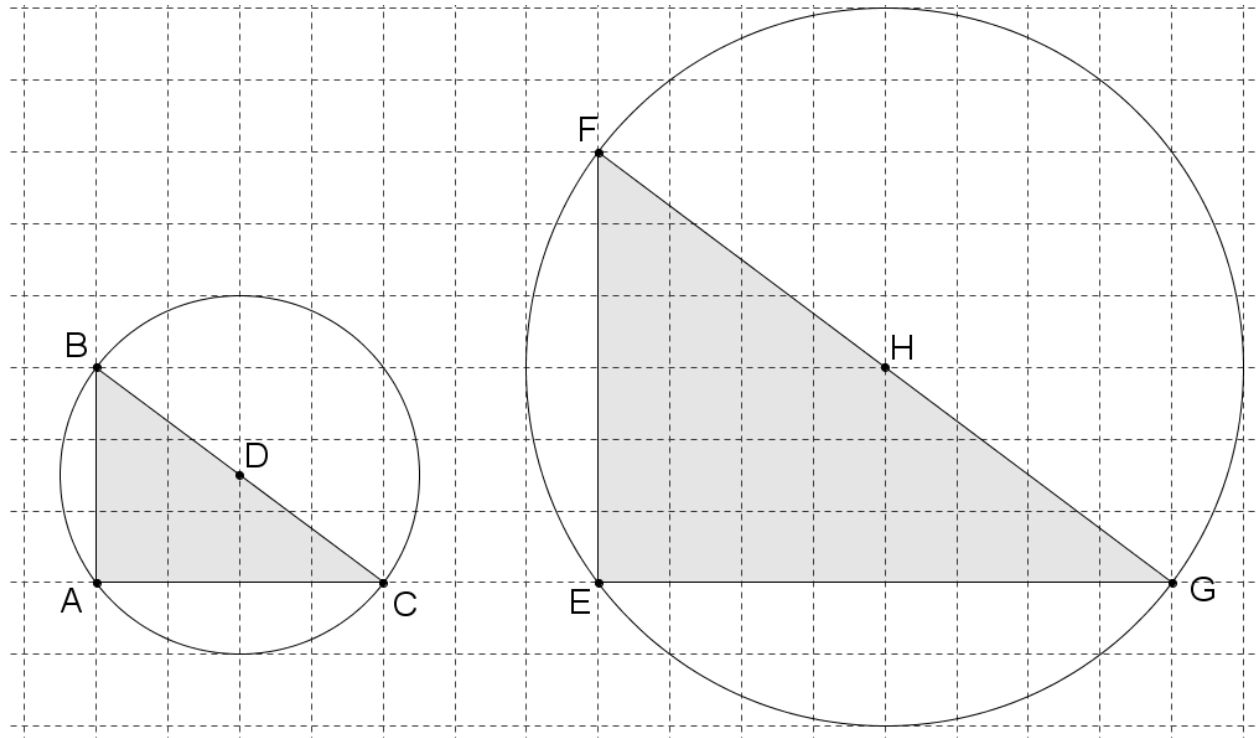
 - a. Obtuse
 - b. Acute
 - c. Right
7. If $\angle B$ is supplementary to $\angle C$, and $m\angle C = 90^\circ$, then $\angle B$ must be:

 - d. Obtuse
 - e. Acute
 - f. Right
8. If $\angle D$ is vertical to $\angle E$, and $m\angle E = 115^\circ$, then $\angle E$ must be:

 - g. Obtuse
 - h. Acute
 - i. Right

5.4b Classwork: Circles, Angles, and Scaling

Examine the figures below. \overline{BC} has a length of 5 units.



1. Write a complete sentence explaining why the figures are scale drawings of each other. You may use a protractor. Use words like *corresponding angles*, *scale factor*, *corresponding sides*, *circle*.
2. Remember that you have been given that \overline{BC} has length 5. What is the radius of each circle? Justify your answer.
3. Calculate the area of each triangle.
4. Classify the triangles by their sides and angles.

5. Name two pairs of complementary angles in the figures.
6. What is the area of each circle?
7. How many of the smaller circle would fit inside the bigger circle, if you could put all the area in without overlapping and with no empty space?
8. What is the circumference of each circle?
9. How many of the circumferences of the smaller circle equal the bigger circumference?
10. Fill in the blanks: When you enlarge a figure with a scale factor of two, the side lengths and circumference _____, the areas _____, and the angles _____.
11. You could construct other figures that are similar to these two figures with a different scale factor. What would be the dimensions of the triangle with scale factor $\frac{1}{2}$ from the figure on the left? What would be the radius of the circle?
12. Follow the steps below to create another figure in the space below:
 - a. Make a dot for the center of a circle, and use a compass to construct a circle around that dot.
 - b. Use a straightedge to draw in a diameter with endpoints labeled A and B .
 - c. Choose any point on the edge of the circle, and label it C .
 - d. Draw in segments AC and BC so you can see a triangle ABC inscribed in the circle.
 - e. Measure the angles in the triangle, and classify the triangle.
 - f. Are there any pairs of complementary angles? If so, name them.

Spiral Review

1. Convert the following units using the ratios given: $\frac{3}{4}$ tons = _____ pounds (1 ton = 2000 pounds)
2. Solve the following proportion equations: $\frac{9}{x} = \frac{15}{25}$
3. Without a calculator, what percent of 90 is 60? Type equation here.
4. Use a model to show $-17 + 5$
5. Alli owes her mom \$124. Alli made four payments of \$20 to her mom. How much does Alli now owe her mother?

5.4b Homework: Review Assignment

Decide if the figures below are possible. Justify your conclusion with a mathematical statement. To construct the triangles use:

- A ruler or strips of centimeter graph paper cut to the given lengths
- A protractor
- Construction technology like GeoGebra

2. A triangle with angles that measure 20° , 70° , and 90° ?

Possible or not? Why or why not?

If so, what kind of triangle? Sketch, label.

3. A triangle with sides 8 and 3 cm. The angle opposite the 3 cm side measures 45° .

Possible or not? Why or why not?

If so, what is the measure of the 3rd side? Sketch, label.

4. A triangle with sides of 8 cm and 3 cm. The angle opposite the 8 cm side measures 45° .

Possible or not? Why or why not?

5. Two students were building a model of a car with an actual length of 12 feet.

a. Andy's scale is $\frac{1}{4}$ inch = 1 foot. What is the length of his model?

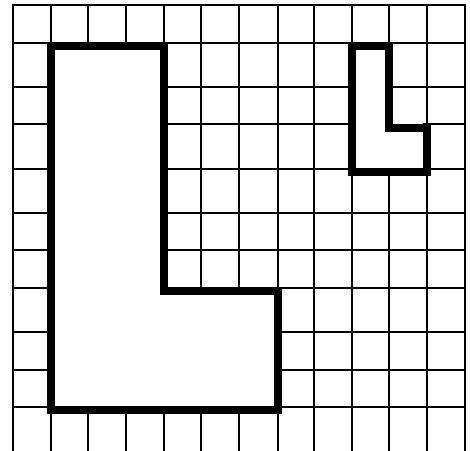
b. Kate's scale is $\frac{1}{2}$ inch = 1 foot. What is the length of her model?

6. At Camp Bright the distance from the Bunk House to the Dining Hall is 112 meters and from the Dining Hall to the Craft Building is 63 meters (in the opposite direction). The scale of the a map for the camp is $0.5\text{cm} = 14\text{meters}$. On the map,...

- a. ...what is the scaled distance between the Bunk House and the Dining Hall?
- b. ...what is the scaled distance between the Dining Hall and the Craft Building?

7. In the similar L figures,

- a. What is the ratio of height of left figure : height of right figure?
- b. What is the reducing scale factor?
- c. What is the ratio of area of left figure: area of right figure?



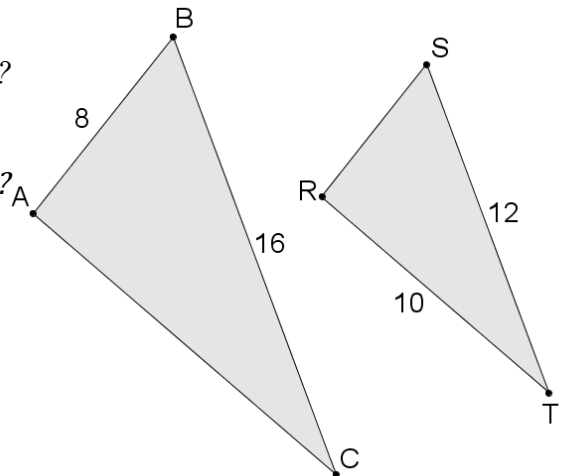
8. Triangles ABC and RST are scale versions of each other.

- d. What is the scale factor from $\triangle ABC$ to $\triangle RST$?

- e. What is the scale factor from $\triangle RST$ to $\triangle ABC$?

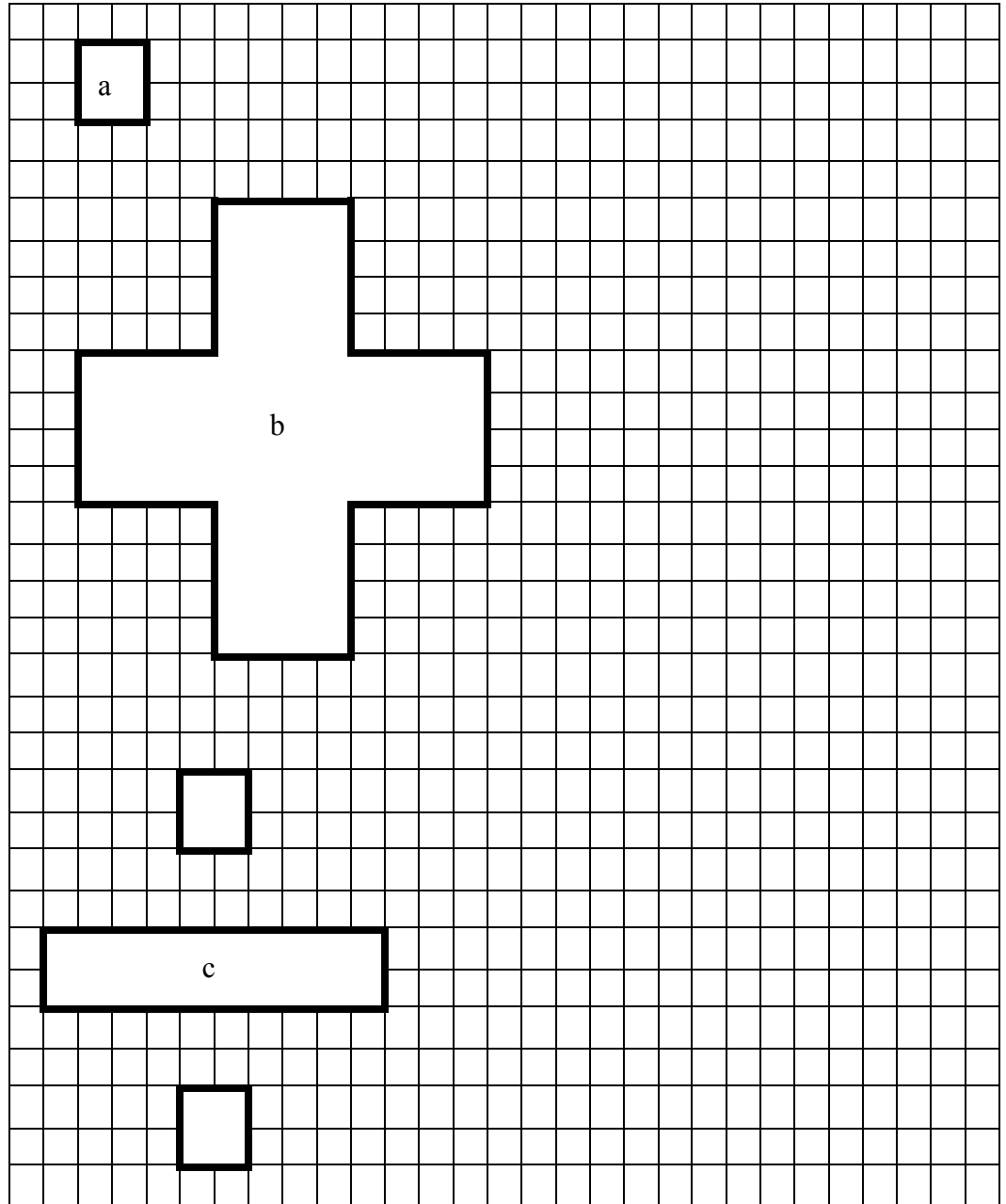
- f. What is the distance between A and C ?

- g. What is the distance between R and S ?



9. Redraw the figures at right using the scale factors below.

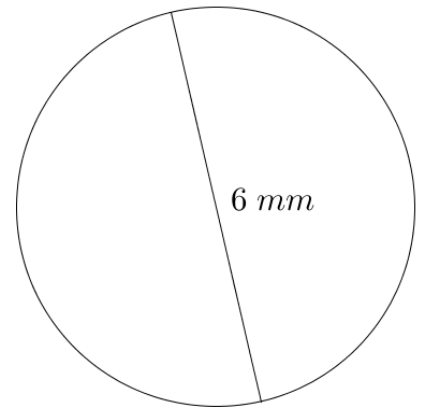
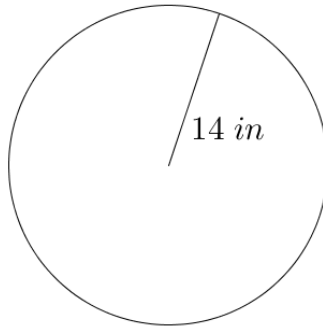
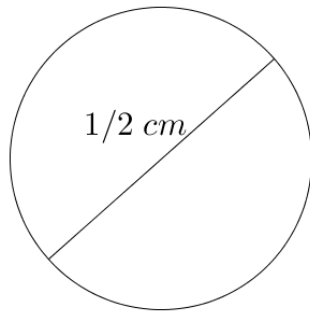
- Use a scale factor of 4 to re-draw the square.
- Use a scale factor of $\frac{1}{4}$ to re-draw the addition sign.
- Use a scale factor of 1.5 to re-draw the division sign.



10. The Washington Monument is 555 feet and $5\frac{1}{8}$ feet tall. Bob wants to create a scale model of it that is no more than 6 feet tall. What scale would you suggest Bob use for his model?

11. Calculate the circumference and area of the circles below. Express each measurements both exactly in terms of pi, and as an approximation to the nearest tenth of a unit.

C =	C =	C =
C \approx	C \approx	C \approx
A =	A =	A =
A \approx	A \approx	A \approx



12. How many times would a circle with radius 4 units fit inside a circle with radius 12 units, if you could pack the area tightly with no overlapping and no leftover space?

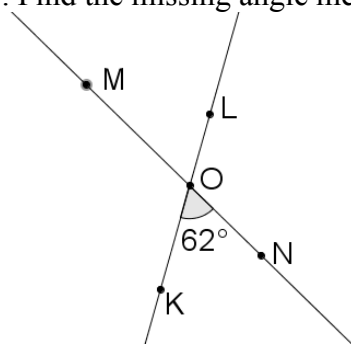
13. Are all circles similar? Justify your answer.

14. Are all squares similar? Justify your answer.

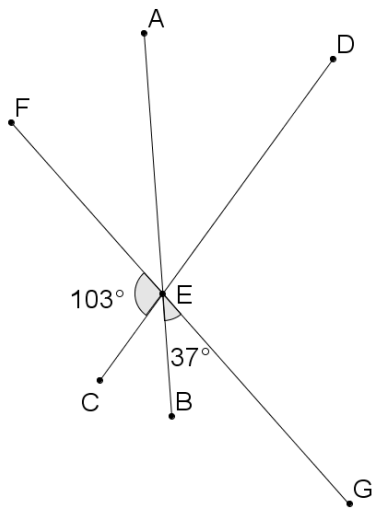
15. Are all rectangles scaled versions of each other? Justify your answer.

16. A circle has an area of $144\pi \text{ mm}^2$. What is its circumference? Show your work.

17. Find the missing angle measures for the figure below. Justify each answer.



18. Find all the missing angle measures for the figure below. Justify each answer.



19. Draw and label two intersecting lines for which $\angle CDE$ and $\angle ADR$ are vertical angles.

20. Draw and label two intersecting lines for which $\angle HOG$ and $\angle GOX$ are supplementary.

21. Draw and label a pair of adjacent complementary angles $\angle ABC$ and $\angle BCD$.

5.4c Self-Assessment: Section 5.4

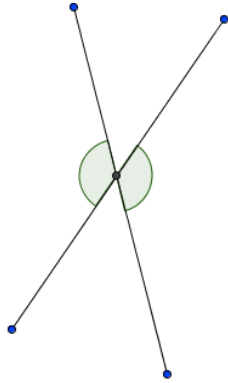
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Identify vertical, complementary and supplementary angles.	I struggle to use the terms vertical, complementary and supplementary angles.	Most of the time I can identify vertical, complementary and supplementary angles.	I can identify vertical, complementary and supplementary angles.	I can identify vertical, complementary and supplementary angles. I can also explain the relationship of the measures of the angles in each angle relationship.
2. Find the measures of angles that are vertical, complementary or supplementary to a known angle.	I struggle to find measures of angles that are vertical, complementary and supplementary angles to a known angle.	I can usually find measures of angles that are vertical, complementary and supplementary angles to a known angle.	I can find the measure of angles that are vertical, complementary or supplementary to a known angle.	I can find the measure of angles that are vertical, complementary or supplementary to a known angle. I can justify how I solved for the missing angle.
3. Apply angle relationships to find missing angle measures. Given angle measures, determine the angle relationship.	I struggle to find measures of angles using angle relationships.	I can usually find measures of angles that are vertical, complementary and supplementary to other angles. I can also usually identify relationships given angle measures.	I can always find measures of angles that are vertical, complementary and supplementary to other angles. I can also usually identify relationships given angle measures.	I can always find measures of angles that are vertical, complementary and supplementary to other angles. I can also usually identify relationships given angle measures. I can also explain how these relationships are similar and different.

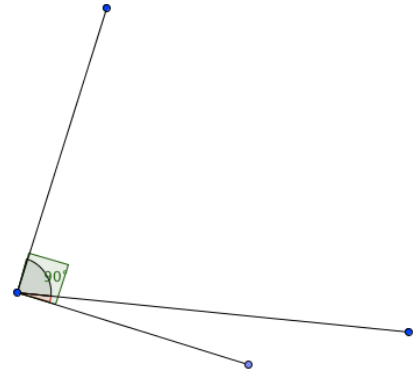
Sample Problems for Section 4.4

1. Identify the shaded angle pairs diagramed below as vertical, supplementary, or complementary.

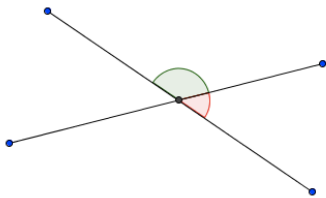
a.



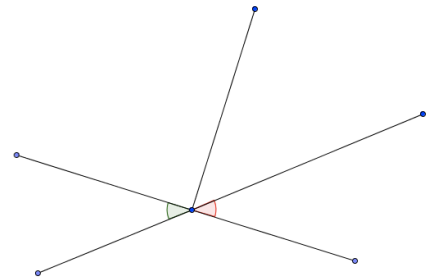
c.



b.



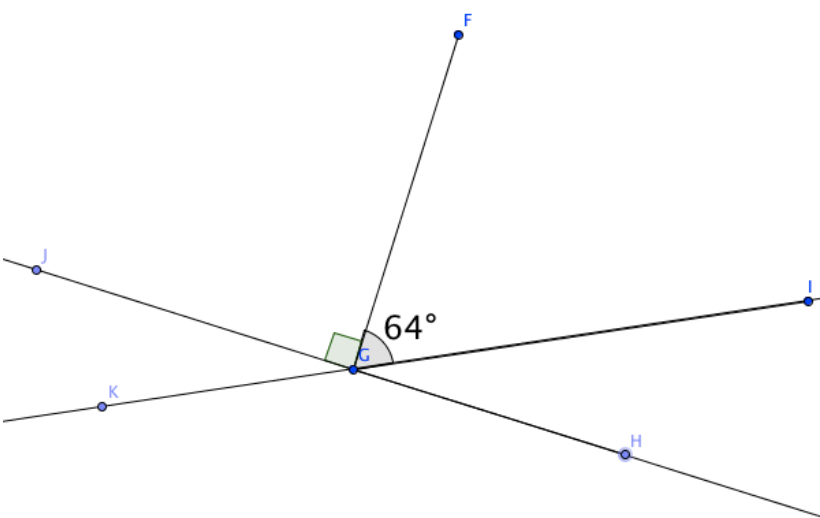
d.



2. Use the relationship given to find the missing angle.

- a. If $\angle G$ is complementary to $\angle H$, and $m\angle H = 52^\circ$, then $\angle G$ must be _____.
- b. If $\angle J$ is supplementary to $\angle K$, and $m\angle J = 168^\circ$, then $\angle K$ must be _____.
- c. If $\angle N$ is vertical to $\angle M$, and $m\angle M = 98^\circ$, then $\angle N$ must be _____.

3. In the diagram below, find all missing angles. Justify with the appropriate angle relationship.



Angle	Measure	Justification
$\angle FGH$		
$\angle IGH$		
$\angle KGH$		
$\angle KGJ$		

4. Given the measures of the following angles, identify the possible angle relationship(s).

a. $m\angle ABC = 10^\circ$ and $m\angle DBC = 80^\circ$

b. $m\angle ABC = 24^\circ$ and $m\angle EBF = 24^\circ$

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Chapter 6: Real World Equations and Inequalities (2-3 Weeks)

UTAH CORE Standard(s)

1. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”* 7.EE.A.2
2. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* 7.EE.B.3
3. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. 7.EE.B.4
 - a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* 7.EE.B.4a
 - b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.* 7.EE.B.4b
4. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. 7.G.5

CHAPTER OVERVIEW:

This chapter brings together several ideas. The theme throughout however is writing equations or inequalities to represent contexts. In the first section students work with ideas in geometry and represent their thinking with equations. Also in that section students solidify their understanding of the relationship between measuring in one-, two-, and three-dimensions. In the second section, students will be writing equations for a variety of real life contexts and then finding solutions. The last section explores inequalities. This is the first time students think about solutions to situations as having a range of answers.

VOCABULARY:

algebraic, inequality, equation, inverse operations, solution, at most, at least, less than, greater than, $<$, $>$, \leq , \geq , supplementary, complementary, vertical angles, adjacent angles, intersecting lines

CONNECTIONS TO CONTENT:

Prior Knowledge






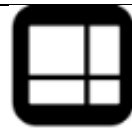


In Chapter 3 students learned how to solve one-step and simple multi-step equations using models. In this chapter students extend that work to more complex contexts. In particular they build on understandings developed in Chapter 5 about geometric figures and their relationships. Work on inequalities in this chapter

builds on 6th grade understandings where students were introduced to inequalities represented on a number line. In this chapter student move to solving simple one-step inequalities. Also by this chapter, students should move to representing ideas symbolically rather than with models.

Future Knowledge

Throughout mathematics, students need to be able to model a variety of contexts with algebraic expressions and equations. Further, algebraic expressions help shed new light on the structure of the context. Thus the work in this chapter helps to move students to thinking about concrete situations in more abstract terms. Lastly, by understanding how an unknown in an expressions/equations can represent a “fixed” quantity, students will be able to move to contexts where the unknown can represent variable amounts (i.e. functions in 8th grade.)

MATHEMATICAL PRACTICE STANDARDS (emphasized):

	Make sense of problems and persevere in solving them.	Students must read, interpret and understand problem situation and transfer that understanding to algebraic equations or inequalities that represent the context. Students should develop flexible strategies for doing this work that will extend to more complicated situations. Additionally, students should make reasonable predictions about what they believe their final answer will be and then use that to both guide their strategy for writing expressions and equations and for checking their answer.
	Reason abstractly and quantitatively.	Students should fluidly connect problem contexts to algebraic representations of them. Each portion of the expression (e.g. variables, operations, groupings etc.) should connect to the context and the abstract representation should shed new light on context.
	Construct viable arguments and critique the reasoning of others.	Students are able to explain and defend the reasonableness of their answer by connecting the context to the abstract representation. Further, students should be able to critique the work of others by connecting the context to the algebraic expression and/or equation.
	Model with Mathematics.	Students will model a variety of contexts with algebraic expressions and equations. Further, students should be able to take an algebraic expression or equation and model it with a context.
	Attend to Precision	Students should use precision in translating between contexts and abstract representations. For example, students should understand when two expressions should be “equal” versus “greater than or equal to” each other OR distinguish when a quantity is being “increased by two” versus “increased by a <i>factor</i> of two.”
	Look for and make use of structure	Students should recognize and interpret structures both within a context and an algebraic expression/equation. Structures either in contexts or in abstract representations should shed light on how to solve a problem and the reasonableness of an answer.
	Use appropriate tools strategically.	Students demonstrate their ability to select and use the most appropriate tool (paper/pencil, manipulatives, pictorial models and calculators) while solving real-life word problems. By this chapter, students should recognize that their ability to reason through computations is often much faster than using a calculator.
	Look for and express regularity in repeated reasoning	Students look for structure and patterns in real-life contexts to help them identify solution strategies. Further, students should begin to recognize how things are <i>changing</i> in a context (operationally.) In 7 th grade the change is fixed and equations help us find one unknown but in 8 th grade students begin to understand change can be continuous (as in functions) and they begin to see how an unknown can represent a variable quantity.

Section 6.1 Write and Solve Equations to Find Unknowns in Geometric Situations.

Section Overview: This section builds upon what students learned about geometric relationships in chapter 5 and in earlier grades. Students start with a review of solving equations. They then move to applying those skills to writing and solving one-step and multi-step equations involving finding missing measures of unknown values in contexts involving various angle relationships with triangles, areas, perimeters, circles and scaling. Students should pay close attention to the relationship between the structure of algebraic equations and expressions and the contexts they represent.

Concepts and Skills to be Mastered (from standards)

By the end of this section, students will be able to:

1. Use properties of supplementary, complementary, vertical, and adjacent angles to solve for unknown angles in figures.
2. Write and solve equations based on a diagram of intersecting lines with some known angle measures.
3. Write and solve equations to find the measure of a missing angle in a triangle.
4. Write and solve equations to find the radius or diameter given the area or circumference of a circle.
5. Write and solve equations using a scale factors.
6. Write and solve equations to find areas, perimeters, or unknown lengths of polygonal figures.

Class Activity: Solving Equations *Review*

In Chapter 3, you learned to solve various equations using models. *If necessary*, use the Key to draw a model to solve the following equations:

Key for Tiles: <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin: 5px;">1</div> = 1 <div style="border: 1px solid black; padding: 5px; margin: 5px;">x</div> = x </div> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin: 5px;">-1</div> = -1 <div style="border: 1px solid black; padding: 5px; margin: 5px;">$-x$</div> = $-x$ </div>		
--	--	--

Model the Equation	What are the solving action? Record the steps using Algebra	Check solution in the equation.
1. $2x + 7 = 9$ <div style="border: 2px solid black; padding: 10px; display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; padding: 5px; margin: 5px;">x</div> <div style="border: 1px solid black; padding: 5px; margin: 5px;">x</div> <div style="border: 1px solid black; padding: 5px; margin: 5px;">1</div> <div style="border: 1px solid black; padding: 5px; margin: 5px;">1</div> <div style="border: 1px solid black; padding: 5px; margin: 5px;">1</div> <div style="border: 1px solid black; padding: 5px; margin: 5px;">1</div> <div style="border: 1px solid black; padding: 5px; margin: 5px;">1</div> <div style="border: 1px solid black; padding: 5px; margin: 5px;">1</div> <div style="border: 1px solid black; padding: 5px; margin: 5px;">1</div> </div>	Add -7 to both sides. Divide by 2 on both sides. $ \begin{array}{r} 2x + 7 \div 9 \\ -7 \div -7 \\ \hline 2x \div 2 \\ 2 \div 2 \\ \hline x \div 1 \end{array} $	
2. $-7 = 3x - 1$ <div style="border: 2px solid black; height: 150px; width: 100%;"></div>		
3. $0 = 3 - 7x + 10$ <div style="border: 2px solid black; height: 150px; width: 100%;"></div>		

4. $(\frac{1}{2})(3x - 4) = -8$ <div style="border: 2px solid black; padding: 10px; display: flex; align-items: center; justify-content: center; height: 180px;"> <div style="border-right: 2px solid black; width: 200px; height: 180px;"></div> <div style="border-right: 2px solid black; width: 30px; height: 180px; display: flex; align-items: center; justify-content: center;"> = </div> <div style="width: 200px; height: 180px;"></div> </div>		
5. $(\frac{1}{3})x - 6 = 2x + 1$ <div style="border: 2px solid black; padding: 10px; display: flex; align-items: center; justify-content: center; height: 180px;"> <div style="border-right: 2px solid black; width: 200px; height: 180px;"></div> <div style="border-right: 2px solid black; width: 30px; height: 180px; display: flex; align-items: center; justify-content: center;"> = </div> <div style="width: 200px; height: 180px;"></div> </div>		

Solve the following either with or without a model.

6) $1 = 5x + 7 - 2x$	7) $-10 = (\frac{1}{2})x - 3$	8) $(\frac{2}{3})x + 4 = -2 + x$
9) $14 = -2(3x + 1)$	10) $-5 = (\frac{1}{2})(3x - 1)$	11) $(-\frac{1}{3})(2x + 5) = 7$

Homework: Solving equations *review*.

Solve the following equations. Draw a model if needed. Show all steps using Algebra. A calculator may be helpful.

1. $5x + 3 = -2$

2. $34 = -2(1 - 9x)$

3. $2(3x - 3) - 8 = 4$

4. $\quad = 5$

5. $7x - 7 - 4 = -25$

6. $6(b - 5) = 30$

7. $2j - 6 - \frac{1}{2}j = -12$

8. $8(v + 1) = 4$

9. $2b - \frac{1}{9} = \frac{1}{3}$

10. $-5 + 75n + 1 = 146$

11. $357 = 3(x - 9)$

12. $48 = 15 + \frac{1}{3}d$

13. $48 - \frac{1}{3}d - \frac{2}{3}d = 50$

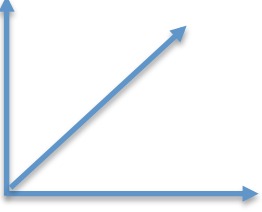
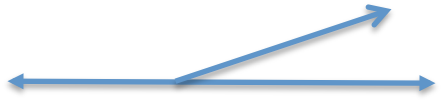
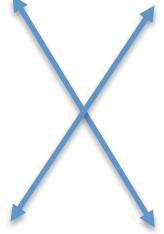

14. $8y + \frac{2}{3} = 9$

15. $0.75n + 1 - 0.25n = -100$


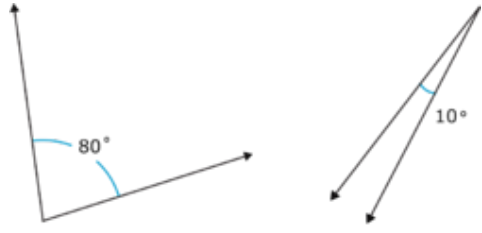
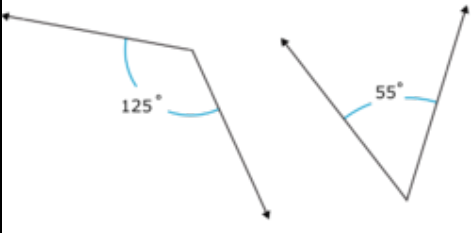
6.1a Class Activity: Complementary, Supplementary, Vertical, Adjacent Angles

Review: Chapter 5 concepts:

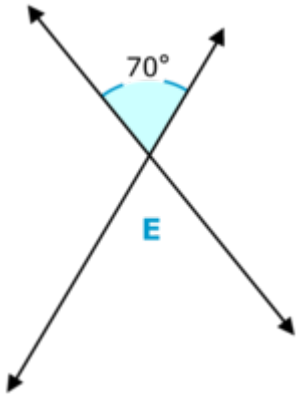
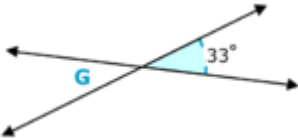
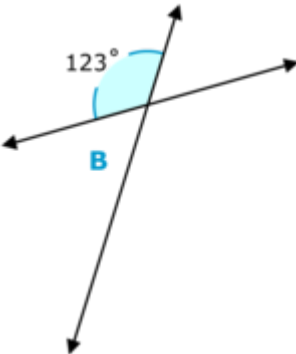
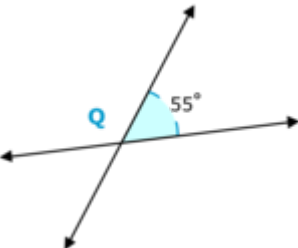
Draw an example for each type of angle pair then explain their relationship.

1. Complementary Angles 	2. Supplementary Angles 
3. Vertical Angles 	4. Adjacent Angles 

Identify whether the example pairs below are complementary, supplementary or neither.

5. 	6. 
7. 	8. Angle A = 37° , Angle B = 53°
	9. Angle C = 110° , Angle B = 70°

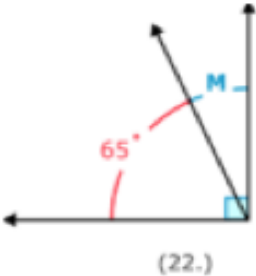
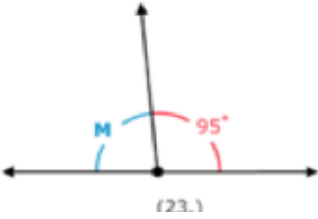
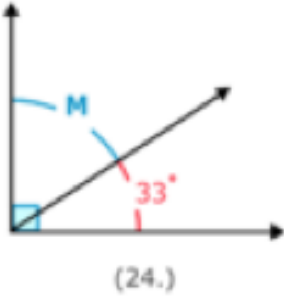
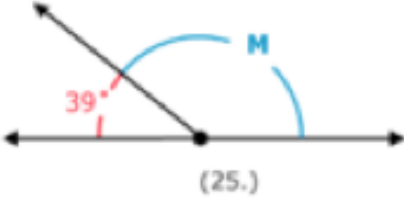
Find the measure of the identified angle:

<p>10.</p> 	<p>11.</p> 
<p>Are the angles in #10 adjacent or vertical? Explain:</p>	<p>Are the angles in #11 adjacent or vertical? Explain:</p>
<p>12.</p> 	<p>13.</p> 
<p>Adjacent or vertical?</p>	<p>Adjacent or vertical?</p>

In 10-13 above the adjacent angle pairs are also examples of supplementary angles. Are adjacent angles always supplementary? Why or why not?

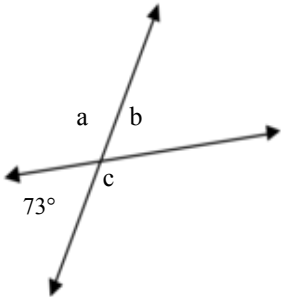


Use angle relationship (complementary, supplementary, vertical) to write a simple equation to find the missing angle (example: $180^\circ = 50^\circ + x$, or $x = 180^\circ - 50^\circ$)

<div>14.</div>  <div>(22.)</div>	<div>15.</div>  <div>(23.)</div>
Equation involving M:	Equation involving M:
<div>16.</div>  <div>(24.)</div>	<div>17.</div>  <div>(25.)</div>
Equation involving M:	Equation involving M:

18.


Write equations for a, b, and c below:



a→

b→

c→



For 19-25 draw a model to help you find the measure of the indicated angle.

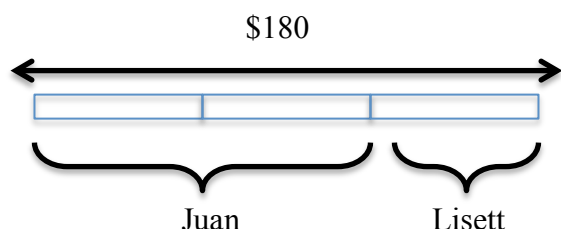
19. Find the measure of an angle vertical to a 56° angle.

20. Find the measure of an angle whose supplement is 89° .

21. Find the measure of an angle whose supplement is 9° .

22. Find the measure of an angle whose complement is 28° .

Review: Juan has twice as much money as Lisett, if they have \$180 all together, how much does Juan have?



Equation $x + 2x = 180$
 $x = 60$; Juan has \$120



23. Two angles are supplementary; one is two times the measure of the other. What are the measures of the two angles?

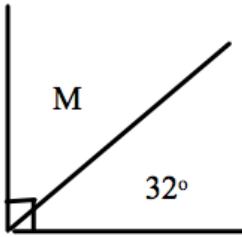
24. Two angles are complementary. One angle is 5 times the measure of the other angle. What are the measures of the two angles?

25. One angle is 25° bigger than another angle. The two angles are supplementary. What is the measure of the two angles?

6.1a Homework: Complementary, Supplementary, Vertical, Adjacent Angles

For numbers 1-6, use angle relationship (complementary, supplementary, vertical) to write a simple equation to find the missing angle. Then find the measure of the missing angle.

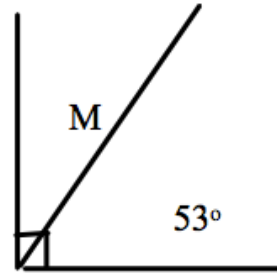
1.



Equation:

$M =$

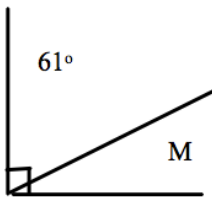
2.



Equation:

$M =$

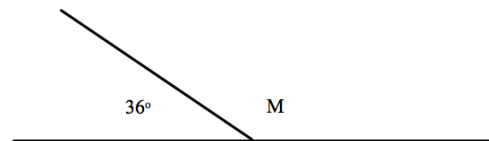
3.



Equation:

$M =$

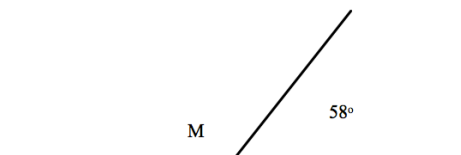
4.



Equation:

$M =$

5.



Equation:

$M =$

$M =$



For each of the following: a) write an equation to find the missing angles and b) find the missing angle.

7. Find the measure of an angle whose complement is 20° .	8. Find the measure of an angle whose supplement is 121° .
9. Two angles are supplementary, and they are equal. What are the measures of the two angles?	10. Two angles are complementary. One angle is 4 times the size of the other angle. What are the measures of the two angles?

Determine if the statement below is *always*, *sometimes* or *never* true. If it is *sometimes* true, give an example when it is true and when it is false. If it is *never* true, give a counter example.

11. Adjacent angles are also supplementary angles.



12. Vertical angles are complementary angles.

13. If one of two adjacent angles is 70° , then the other is also 70° .

14. Vertical angles have the same measure.



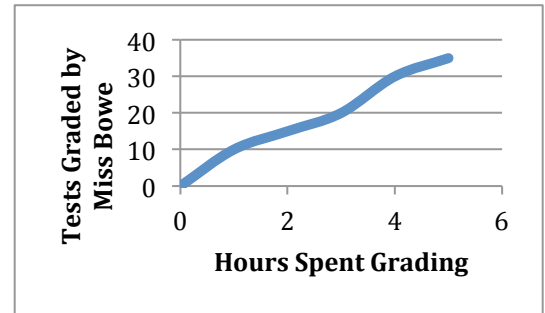
15. Perpendicular lines will never intersect.

Spiral Review

1. Solve $-6x + 4 = -2$
2. Simplify the following expression. Use a model if needed.
 $-18m + 8 - 12m$
4. If $\angle A$ is complementary to $\angle F$ and $m\angle A = 79^\circ$, what is $m\angle F$?
4. You can buy 8 apples for \$2.00.
 - a. Find the unit rate for 1 apple.
 - b. Find the unit rate for \$1.

5. Is this graph proportional? _____

How do you know?



6.1b Class Activity: Angle Pairs and Solving Equations

To begin: Talk in a group about your answers to numbers 11-15 from homework 6.1a. Present your arguments to the class.

Review: Solve each of the equations below:

1. $3x + 1 = x - 5$

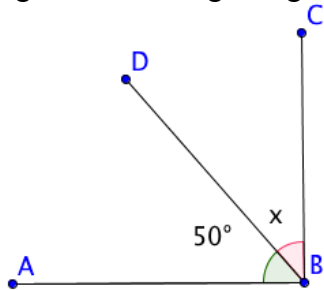
3. $13 = 2(x - 1) + 1$

2. $14 = 5 - 3x$

4. $(2x + 3) + (x - 1) = 17$

For each situation below, write an equation and then find the missing angles.

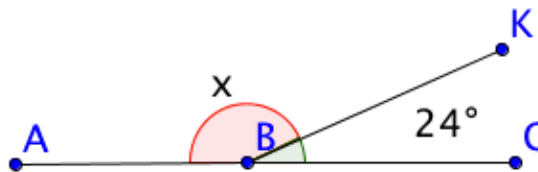
5. Angle ABC is a right angle.



Equation:

$x =$ $\angle DBC =$

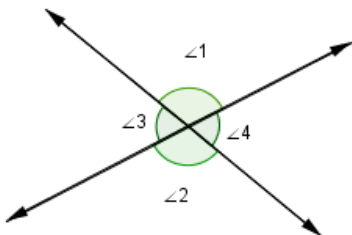
6. Figure ABC is a straight line.



Equation:

$x =$ $\angle ABK =$

7. Given the $m\angle 1 = (3x + 2)$ and the $m\angle 3 = (2x - 7)$.

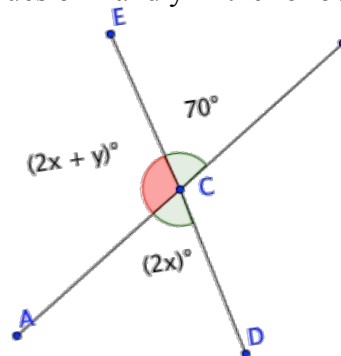


Equation:

$x =$ $\angle 1 =$ $\angle 3 =$ $\angle 2 =$

$\angle 4 =$

8. Find the values of x and y in the following figure.



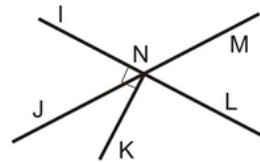
Equation:

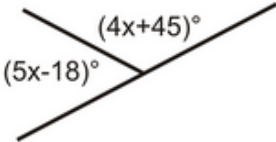
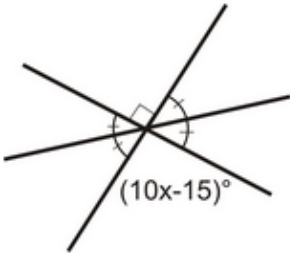
$x =$ $y =$
 $\angle ACD =$ $\angle ACE =$ $\angle DCB =$

Write an equation and find the indicated missing angles. Draw a model to show your thinking.

<p>9. A pair of angles are equal. Their sum is 144°. Find the angle measure.</p>	<p>10. Two adjacent angles (A and B) are in the ratio of 4:5. The sum of the angles is equal to 54°. Find the angle measures.</p>
<p>11. $\angle R$ and $\angle W$ are adjacent. $\angle R$ is 30° larger than $\angle W$. Their sum is 70°. Find the angle measures.</p>	<p>12. $\angle A$ and $\angle B$ are supplementary angles whose ratio is 2:3. Find the angle measures.</p>
<p>13. One supplementary angle is 15 degrees less than twice the other. Find the measure of the two supplementary angles.</p>	<p>14. Angles A and B together create a 90° angle. $\angle A = 4x - 10$ and $\angle B = 2x - 20$. Find the angle measures.</p>

Use the diagram to the right for 15 and 16.



<p>15. Given $\overline{IL} \perp \overline{NK}$.</p> <p>Name two complementary angles:</p> <p>Name two supplementary angles:</p>	<p>16. Given: $m\angle MNL = 70^\circ$. Find the measures of the following angles</p> <p>$m\angle JNK =$</p> <p>$m\angle JNL =$</p> <p>Explain how you arrived at your answer:</p>
<p>17. Find the value of x:</p> 	<p>18. Find the value of x:</p> 

19. Explain how you might check your answer for number 13 and then use the strategy to check your answer

20. Explain how you might check your answer for number 14.

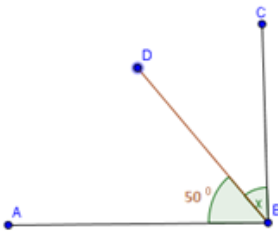
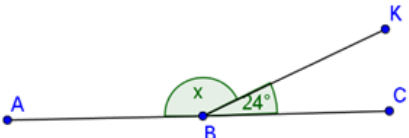
21. Make up a problem like #17 or #18 and solve it. Be prepared to share it with others.

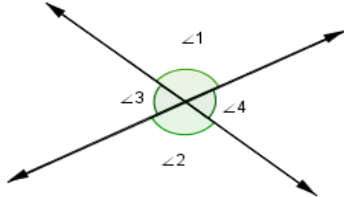
6.1b Homework: Angle Pairs and Solving Equations

Solve each:

1. $4(x - 2) = -24$	2. $\frac{1}{2}(x + 5) = 4$
3. $-3x + 7 - 2x - 1 = 18$	4. $-5 = 3x - 2 - 4x + 1$

Draw a model (where necessary); then write an equation and find the measure of the indicated angle.

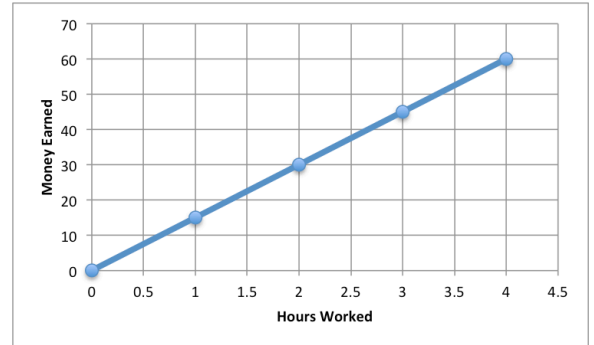
5. A pair of angles are equal. Their sum is 156° . Find the angle measure.	6. Two adjacent angles (A and B) have a ratio of 2:3. The sum of the angles is equal to 80° . Find the angle measures.
7. Angle ABC measures 92° . Find the measure of angle x ? 	8. Figure ABC measures 178° . What is the measure of angle x ? 
9. Angles B and C are adjacent. Angle C is 25° larger than Angle B. Their sum is 80° . Find the angle measures.	10. $\angle A$ and $\angle B$ are supplementary angles whose ratio is 2:7. Find the measures of $\angle A$ and $\angle B$.

<p>11. One supplementary angle is 12 degrees less than twice the other. Find two supplementary angles.</p>	<p>12. Angles A and B together create a 90° angle. $\angle A = 3x - 2$ and $\angle B = x + 12$. Find the angle measures.</p>
<p>13. Given the $m\angle 1 = 2x + 35$ and the $m\angle 2 = 3x + 7$. Find the angle measures.</p> 	<p>14. Angles 3 and 4 are complementary. The $m\angle 3 = 2y$ and the $m\angle 4 = y - 18$. Find the value of y and find the measure of angles 3 and 4.</p>
<p>15. Two angles are complementary. One of the angles is 34°, what's the measure of the other?</p>	<p>16. Find two supplementary angles such that the measure of the first angle is 30° less than five times the measure of the second.</p>
<p>17. Find two complementary angles such that the measure of the first angle is 40° more than four times the measure of the second.</p>	<p>18. Challenge: If two equal angles are supplements to each other, find the measure of each angle in terms of one variable.</p>

Spiral Review

1. Simplify the following expression. $-4x + 4.5 - 0.94 + 14x$

2. The graph to the left shows how many hours Gabriel worked and how much he was paid. What does the point (1, 15) mean in context of the situation? Use the unit rate in your explanation.



3. Use a proportion to find what 35% of 120 is.

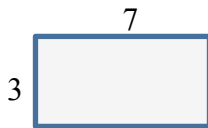
4. If $\angle A$ is vertical to $\angle F$ and $m\angle A = 16^\circ$, what is $m\angle F$

5. Solve $41 = \frac{1}{2}x + (-4)$

6.1c Class Activity: Perimeter and Area with Variables

Review:

1. Find the perimeter and area of the rectangle shown below



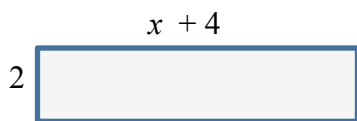
Perimeter:

Area:

2. What is the difference between perimeter and area?

RECALL: if a rectangle's sides are scaled by a factor of 4, then the new rectangle will have a perimeter 3 times the original and an area 16 times the original. Why is this true?

3. Find an expression for the perimeter and area of the rectangle shown below



Perimeter:

Area:

If the perimeter is 20, what is the value of x ?

Equation:

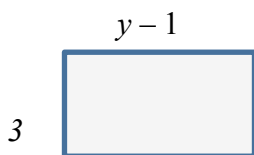
Solution:

If the area is 30, what is the value of x ?

Equation:

Solution:

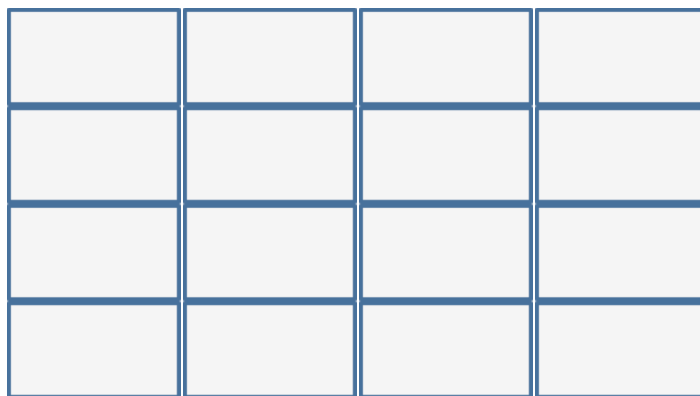
4. Find an expression for the perimeter and area of the rectangle shown below

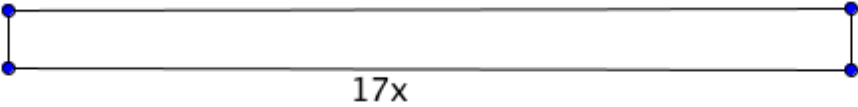


Perimeter:

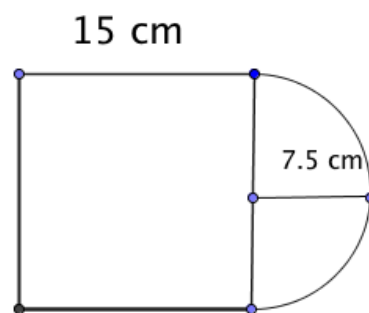
Area:

Scale this rectangle by a factor of 4. Write an expression for the scaled rectangle's perimeter and area.



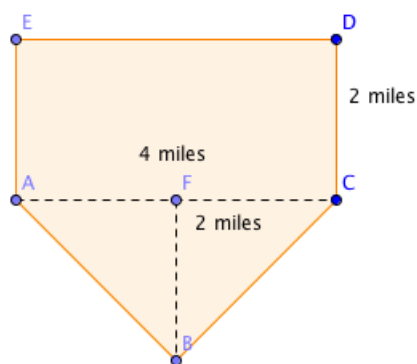
<p>5. The perimeter of a rectangle is 48 in. Its length is twice its width.</p> <p>a. Draw a model of the context.</p> <p>b. Write an equation relating the length and width to the perimeter.</p> <p>c. Solve the equation and state the length and width.</p>	<p>6. The area of a rectangle is 18 in^2. The length is $x + 4$ and the width is 3 in. Draw a model of the context.</p> <p>a. Draw a model of the context.</p> <p>b. Write an equation relating the length and width to the area.</p> <p>c. Solve the equation and state the length and width.</p>
<p>7. A rectangular garden has an area of 48 square feet. One of the sides is currently 6.4 feet while the other is 7.5 feet.</p> <p>a. By how much would the shorter side (the 6.4 foot side) have to be increased to get a garden with an area of 60 square feet? 7</p> <p>b. How much would the longer side (the 7.5 foot side) have to be increase to get the same area (60 square feet)?</p>	<p>8. The area of a rectangular garden is 128 ft^2. On a map of the garden, the scale is $\frac{1}{4} \text{ in.} = 1 \text{ ft}$. What is the area of the garden on the map?</p>
<p>9. The ratio of length to width of a rectangular photograph is 1:2. The short side is 8 units. If it is reduced by a scale factor of $\frac{1}{3}$, what are the dimensions of the reduced photo, and what is the area of the new photograph?</p>	<p>10. Thirty percent of a photograph is in black ink. In this particular photograph, that means 100 cm^2 is black ink. If the dimensions of the photo are enlarged by a factor of 3, how many square centimeters are black, and what is the area of the enlarged photograph?</p>
<p>11. If the base of the rectangle below is $17x$ and its perimeter is $1 + 34x$, a) what is the length of the vertical sides? b) find x if the area is 50.</p> 	

12. What is the perimeter of the figure to the right?

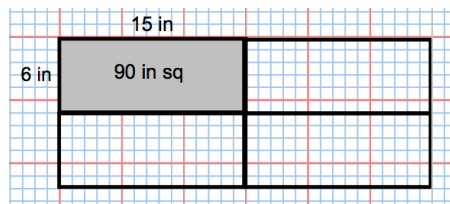


13. What is its area?

14. Find the area of the figure below:


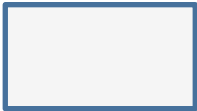


15. The ratio of the length to width of a rectangular photograph is 2:5. The longer side is 15 inches. a) What is the length of the longer side? b) If the area of the photo is quadrupled, what will the new dimensions of the photo be?

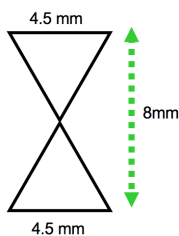
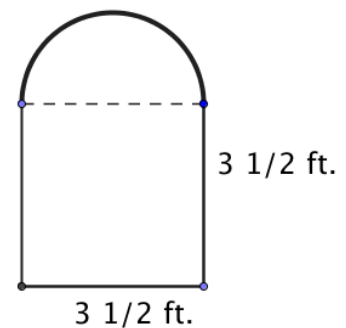


6.1c Homework: Perimeter and Area with Variables

It will be VERY helpful to draw a model of each context.

<p>1. Find an expression for the perimeter and area of the rectangle shown below</p> <p style="text-align: center;">$x - 6$</p> <p>4 </p> <p>Perimeter = Area =</p> <p>If the perimeter is 60, what is the value of x?</p> <p>If the area is 80, what is the value of x?</p>	<p>2. Find an expression for the perimeter and area of the rectangle shown below</p> <p style="text-align: center;">$x + 21$</p> <p>5 </p> <p>Perimeter = Area =</p> <p>If the perimeter is 100, what is the value of x?</p> <p>If the area is 200, what is the value of x?</p>
<p>3. The perimeter of a rectangle is 64 in. Its length is three times its width. Find a) the length and width and b) the area of the rectangle.</p>	<p>4. The area of a rectangle is 28 in^2. The length is $x + 2$ in. and the width is 7 in. Find a) the value of x, b) the length and c) the perimeter of the rectangle.</p>
<p>5. The area of a rectangle is 81 cm^2. The length equals the width. Find a) the length, b) the width, and c) the perimeter.</p>	<p>6. Challenge: What's the biggest area you can enclose with 1000 meters of fencing? Suppose you have a horse and exactly 1000 feet of fencing. You want to create an enclosure for your horse to give it the most area to roam. How would you configure your fence.</p>

7. The area of a rectangular garden is 224 ft^2 . On a map of the garden, the scale is $1 \text{ in.} = \frac{1}{2} \text{ ft.}$ What is the area of the garden on the map?
8. The ratio of length to width of a rectangular photograph is 3:5. The shorter side is 3 units. If the dimensions are enlarged by a scale factor of 6, a) what are the dimensions of the enlarged photo? and b) what is the area of the new photograph?
9. 20% of a photograph is in black ink. In this particular photograph, that means 100 cm^2 is black ink. If the area of the photo is enlarged by a factor of 9, a) how many square centimeters are black, and b) what is the area of the enlarged photograph?
10. The perimeter of a square is $4x + 6$. What is its side length.
11. Find the perimeter of the figure to the *right*:
12. Find its area of the figure to the *right*:
13. Find the area of the figure *below*:



Spiral Review

1. The length of a rectangle is 16 in. longer than the width. Represent the length of the rectangle.
2. What property is shown?
 $8(7 + 2) = 8(2) + 8(7)$ _____
 $18 + 0 = 18$ _____
3. Determine if the given angles will make a triangle. Explain why or why not.
 - a. Angles 25° , 60° , 95°
 - b. Angles 30° , 40° , 20°
4. Add $-4 + -7 =$
5. Solve $6(-1 + 8x) = 18$

6.1d Class Activity: Triangles and Circles

In chapter 5, you worked with angle measures in triangles. Now, you are going to practice writing equations to solve for a missing angle measure. Recall from Chapter 5 that the angles of a triangle sum to 180° .

Example:

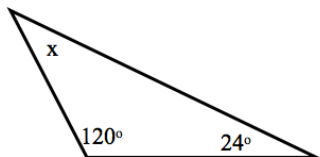
Find the missing angle measure:

Equation: $120 + 24 + x = 180$

Solution: $144 + x = 180$

$$\begin{array}{r} -144 \quad -144 \\ x = 36^\circ \end{array}$$

$$x = 36^\circ$$



Sometimes, x is not the missing angle measure:

Equation: $2x + 42 + 64 = 180$

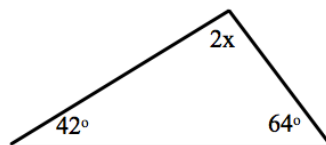
Solution: $2x + 106 = 180$

$$\begin{array}{r} -106 \quad -106 \\ 2x = 74 \end{array}$$

$$\frac{2x}{2} = \frac{74}{2}$$

$$x = 37$$

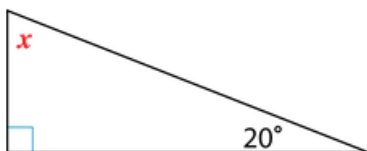
$$x = 37$$



However, x is not the missing angle measure. $2x$ is, so the missing angle measure is $2(37^\circ) = 74^\circ$

Write an equation and solve for x , then find the missing angle measures. *Pictures are not drawn to scale.*

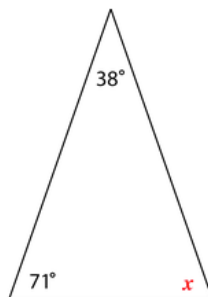
1.



Equation:

Solution:

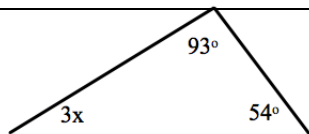
2.



Equation:

Solution:

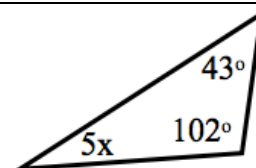
3.



Equation:

Solution:

4.



Equation:

Solution:

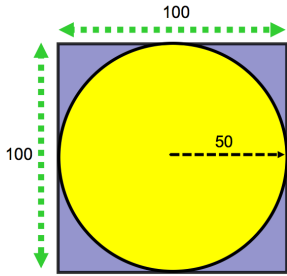
<p>5. One angle of a triangle is 5 times the smallest angle. The other angle is 40°.</p> <p>Equation:</p> <p>Solution:</p>	<p>6. The sum of 2 angles of a triangle is $37\frac{1}{2}^\circ$. What is the measure of the other angle?</p> <p>Equation:</p> <p>Solution:</p>
<p>7. The ratio of angles of a triangle is 3:2:1. What are the angle measurements?</p> <p>Equation:</p> <p>Solution:</p>	<p>8. One of the angles of a triangle is one-fourth the size of the largest angle. The other angle is one-half the size of the largest angle. What are the measures of all the angles?</p> <p>Equation:</p> <p>Solution:</p>
<p>9. The ratio of the angles of a triangle is $5:2\frac{1}{3}:1$. What are the measures of all three angles?</p> <p>Equation:</p> <p>Solution:</p>	<p>10. One angle of a triangle is 63°. The ratio of the other two angles is 5:2. What are the measures of all the angles of the triangle?</p> <p>Equation:</p> <p>Solution:</p>
<p>11. The angle of a triangle is 54°. The ratio of the other two angles is 3:5. What are the measurements of all the triangles.</p> <p>Equation:</p> <p>Solution:</p>	<p>12. Two angles of a right triangle have the ratio 5:7. What are the measures of the angles?</p> <p>Equation:</p> <p>Solution:</p>

Now for circles:

Review: What is the formula for the *Circumference* of a circle?

What is the formula for the *Area* of a circle?

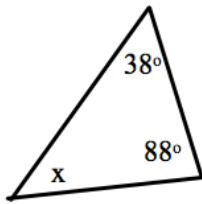
13. What if you were only given the circumference? Could you find the radius or diameter?

<p>14. Michael loves swimming. He swam around the edge of a circular pool and found that it took him 176 strokes to swim one complete time around the pool. About how many strokes will it take him to swim across the pool? (Use 3.14 for pi)</p>	<p>15. The circumference of the center circle of a soccer field is 31.416 yards. What is the radius of the circle?</p> <p>Equation:</p> <p>Solution:</p>
<p>16. Find the radius of a circle with a circumference of 22 feet.</p> <p>Equation:</p> <p>Solution:</p>	<p>17. A farmer has a 100 feet x 100 feet plot of land he needs to water. He has a sprinkler that waters in a circle. The sprinkler has a reach of 50 feet. If he puts the sprinkler in the center of the plot of land, what <i>percent</i> of the plot will be watered? (Hint: Draw a picture first)</p>
<p>18. Pizzas are sold according to diameter. For example, a 6 inch pizza is a pizza with a diameter of 6 inches. At Francesco's pizzeria, there are two pizzas. Pizza A is a 12 inch, and Pizza B has an area of 450 in^2. Which pizza is bigger? What is the percent of increase from the smaller pizza to the larger pizza?</p>	
<p>19. A bike's wheel diameter is 50 cm. If the wheel rotates 45 times a minute, how far has the bike traveled after 30 minutes?</p>	<p>20. Mary has a circular table whose diameter is 7 feet. She would like to put a tablecloth on it, but the packaging only gives the area. The tablecloth she bought says it is 110 ft^2. Will the tablecloth fit?</p>

6.1d Homework: Triangles and Circles

Write an equation to find x , then find the measure of the missing angle. *Pictures are not drawn to scale.*

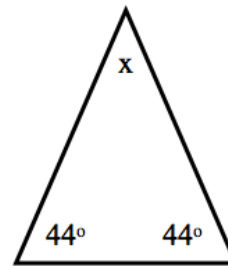
1.



Equation:

Solution:

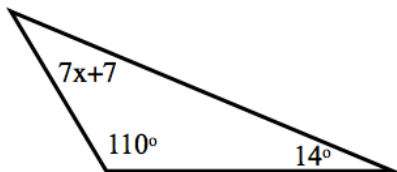
2.



Equation:

Solution:

3.



Equation:

Solution:

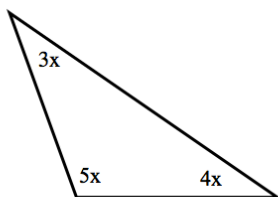
4.



Equation:

Solution:

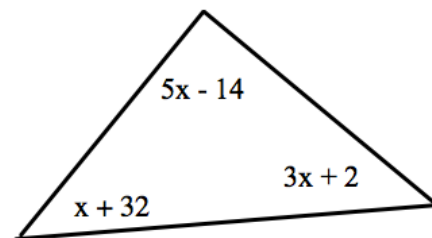
5.



Equation:

Solution:

6.



Equation:

Solution:

<p>7. One angle of a triangle is 3 times the smallest angle. The third angle is 60°.</p> <p>Equation:</p> <p>Solution:</p>	<p>8. The sum of 2 angles of a triangle is $39\frac{1}{4}^\circ$. What is the measure of the other angle?</p> <p>Equation: $180 - 39\frac{1}{4} = x$</p> <p>Solution:</p>
<p>9. One angle of a triangle has a measure of x. Another angle is $3\frac{1}{2}$ times the size of angle x. The third angle is half the size of angle x. What are the measures of all three angles?</p> <p>Equation:</p> <p>Solution:</p>	<p>10. One of the angles of a triangle is three-fourths the size of the largest angle. The other angle is one-half the size of the largest angle. What are the measures of all the angles?</p> <p>Equation:</p> <p>Solution:</p>
<p>11. The ratio of angles of a triangle is 3:1:1. What are the angle measurements?</p> <p>Equation:</p> <p>Solution:</p>	<p>12. One angle of a triangle is 80°. The ratio of the other two angles is 3:2. What are the measures of all the angles of the triangle?</p> <p>Equation:</p> <p>Solution:</p>
<p>13. The angle of a triangle is 52°. The ratio of the other two angles is 3:4. What are the measurements of all the triangles.</p> <p>Equation:</p> <p>Solution:</p>	<p>14. Two angles of a right triangle have the ratio 2:3. What are the measures of the angles?</p> <p>Equation:</p> <p>Solution:</p>

<p>15. Phil has a lamp with a circular base that he would like to fit onto a circular side table in his house. The area of the base of the lamp is 70 in^2. The table has a radius of 5 in. Will the lamp fit? Be sure to show all your work.</p>	<p>16. The circumference of a pizza is 81 in. What is the radius?</p>
<p>17. The circumference of a circular hot tub cover is 200 ft. What is the area of the cover?</p>	<p>18. The circumference of a basketball hoop is 125.6 in. What is the area inside the hoop?</p>
<p>19. A pizza place charges \$12 for a 12 inch pizza, \$14 for a 14 inch pizza, \$16 for a 16 inch pizza, and \$20 for a 20 inch pizza. Which pizza is the best deal? Justify your response.</p>	

Spiral Review

1. Luis went to a soccer game with some friends. He bought two sodas for \$1.50 each and four giant candy bars for \$2.25 each. Write a numeric expression showing how much he spent. Then calculate the total he spent.
2. Use long division to show how you can convert this fraction to a decimal and then a percent
 $\frac{2}{7}$
3. Solve: $\frac{3}{4}x - \frac{7}{8} = \frac{5}{8}$
4. Simplify: $x + 3x - 7x + 2x^2$
5. Write a context for the following expression: $4(x - 3)$

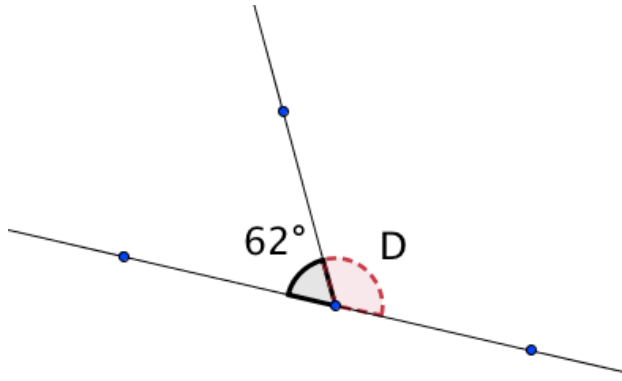
6.1e Self-Assessment: Section 6.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Use properties of supplementary, complementary, vertical, and adjacent angles to solve for unknown angles in figures.	I know what the properties of supplementary, complementary, vertical, and adjacent angles are, but I struggle to use them to solve for unknown angles.	With some help, I can find an unknown using properties of supplementary, complementary, vertical, and adjacent angles.	I can find an unknown angle using properties of supplementary, complementary, vertical, and adjacent angles.	I can find an unknown angle using properties of supplementary, complementary, vertical, and adjacent angles by drawing a diagram or writing an equation. I can justify my procedures and explain how to find other missing angles in the same context.
2. Write and solve equations based on a diagram of intersecting lines with some known angle measures.	I struggle to write an equation based on a diagram of intersecting lines.	With help I can write and solve an equation based on a diagram of intersecting lines.	I can write and solve an equation based on a diagram of intersecting lines with some numeric angle measures OR with angle measures given as algebraic expressions.	I can write and solve an equation based on a diagram of intersecting lines with some numeric angle measures OR with angle measures given as algebraic expressions. I can also justify all my procedures.
3. Write and solve equations based on area and perimeter.	I struggle to write an equation based on area or perimeter.	With help, I can write an equation based on area or perimeter.	I can write and solve an equation based on area or perimeter with a numeric side length OR written as algebraic expressions.	I can write and solve an equation based on area or perimeter with a numeric side length OR written as algebraic expressions. I can also justify my procedures.
4. Find the measure of missing angles in a triangle.	I know what the property of angle measures in a triangle, but I struggle to use it to solve for unknown angles.	With help I can solve for missing angles in a triangle.	I can solve for unknown angles in a triangle.	I can solve for unknown angles in a triangle. I can explain why my procedure works.
5. Find the measure of missing lengths of a circle.	I know algorithms for circumference of a circle, but I struggle to use them to solve for missing lengths of a circle.	I can usually write and solve an equation based on a circle to find missing lengths of a circle.	I can write and solve an equation based on the circumference of a circle.	I can write and solve an equation based on the circumference or the area of a circle. I can also explain the relationship between circumference and area.

Sample Problems for Section 6.1

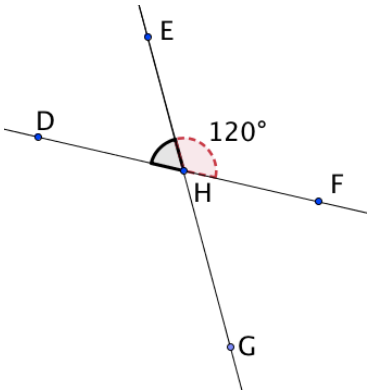
1. Find the measure of the missing angle.

<p>a.</p> 	<p>b. Find the measure of an angle vertical to a 94° angle.</p>
<p>c. Find the measure of an angle whose complement is 48.4°.</p>	<p>d. Two angles are complementary. One angle is 4 times the measure of the other angle. What are the measures of the two angles?</p>

2. Find the measure of the missing angles in each diagram by first writing an equation:

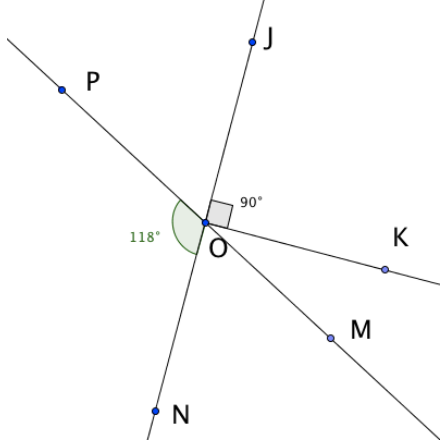
a.

Angle	$\angle EHD$	$\angle GHD$	$\angle FHG$
Measure			



b.

Angle	$\angle JOP$	$\angle MOK$	$\angle MON$
Measure			



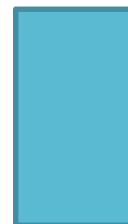
3. Write an equation to solve the following problems.
- a. Terry is building planter boxes for his garden. He wants each box to have an area of 12.5 ft^2 . If each box is 5 feet wide, how long should he make them?

- b. If the perimeter of rectangle to the right is 144, what is the value of t ?

4

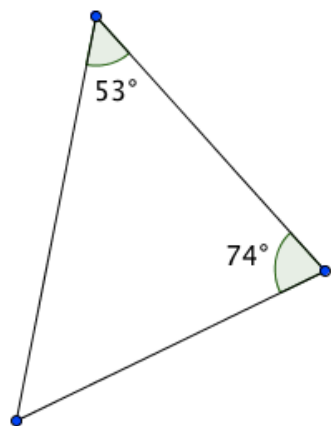
- c. If the area of the rectangle to the right is 52, what is the value of t ?

$t - 1$



4. Find the missing angles in each triangle drawn or described below.

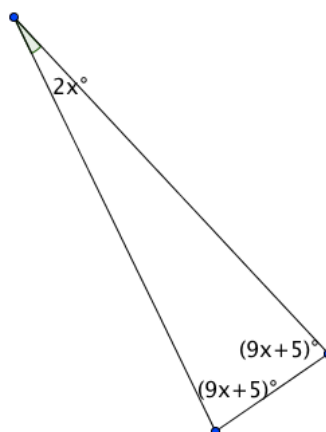
a.



- b. A triangle has the following angles: 37° and 53° . What is the measure of the third angle?

- c. One angle in a triangle is 90° . The ratio of the other two angles is 1:8. What is the measure of each angle?

d.



5. Write an equation to solve the following problems involving circles:
- The circumference of Fabian's watch is 125.6 mm. What is the diameter of his watch?
 - General Sherman is a giant sequoia tree in California. The largest in the world, its circumference is 102.6 feet. What is its diameter?
 - The circumference of a carousel is 21.98 yd. What is the area of ground covered by the carousel?
 - Everlie buys a take-and-bake pizza. The area of the pizza is 154 in^2 . Her pizza pan to bake it on has a 16 in. diameter. Will the pizza fit on her pan?

Section 6.2 Write and Solve Equations from Word Problems

Section Overview:

This section begins with a review of solving multi-step equations, both with and without models, within a context. Students then build on that understanding to work with more complicated situations. Activities in this section have students working in two “different directions”—in some sections students will be given a context and asked to find relationships and solutions, while in other sections, students will be given relationships and asked to write contexts. The goal is to help students understand the structure of context in relationship to algebraic representations.

Concepts and Skills to be Mastered

By the end of this section, students should be able to:

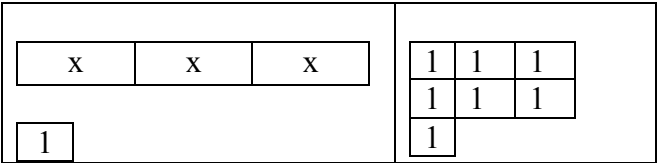
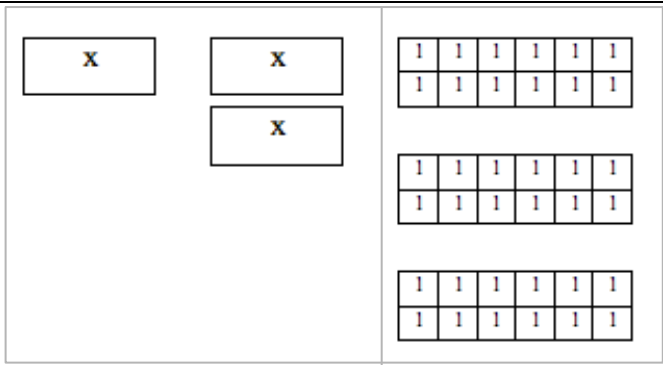
1. Recognize and explain the meaning of a given expression or equation and its component parts.
2. Solve multi-step context problems involving calculations with positive and negative rational numbers in a variety of forms.
3. Use variables to create equations that model word problems.
4. Solve word problems leading to linear equations.

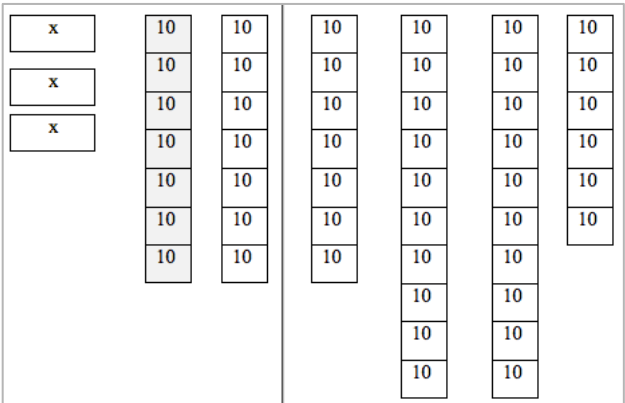
Anchor Problem: Cookies for a Party

I go to the store to buy cookies, milk, napkins, and cups for a party. I need to know how many packages of cookies I can buy and still have money left over. I have \$35.50 in my wallet. I know that I need 2 packages of napkins at \$1.50 each and two packages of cups at \$3.50 each. I need one gallon of milk for every two packages of cookies. Each gallon of milk costs \$2.50. Each package of cookies costs \$3. How many packages of cookies can I buy and still have money left over?

6.2a Class Activity and Homework: Write and Solve Equations for Word Problems I

In the exercises below you are given either a context/word problem OR a model/equation. Provide the missing information. Solve each problem. An example is given.

Word Problem	Model and/or equation	Solution
EXAMPLE Three times some number plus one is seven. What is the number?	 $3x + 1 = 7$	$x = 2$
1.	$4x + 2 = 30$	
2. Eleven less than 5 times a number is 24.		
3. The quotient of a number and -9 is increased by 10 the result is 11.		
4.	 $x + 2x = 36$	

5.	$-3x - 5 = -2$	
6.	$m - 26 = 15$	
For the next set of exercises, write contexts that are more “real life”.		
Example: For two months in a row I made the same amount babysitting. I deposited my babysitting money into my savings account that had \$80, I now have \$350 in my account. How much money did I make each month babysitting?	$2x + 80 = 350$	
7.	 $3x - 70 = 260$	
8.	$x/2 - 5 = 10$	
9.	$(2/3)x - 20 = 280$	

10.	$3x + 2x = 85$	
11.	$x + (x+3) + x + (x+3) = 34$	
12.	<div> <div>x</div> <div> x x x </div> <div> x x x x x </div> <div> 20 20 20 20 20 20 20 20 20 </div> </div> $x + 3x + 5x = 180$	
13.	$(2/5)x + (1/5)x = 360$	
14.	$0.2x + x = 240$	

Spiral Review

1. Write an expression to model the following situation in two different ways: The price of the car was reduced by 15%.
2. Juliana bought 3 bags of chips and 3 sodas for herself and two friends. The chips were \$0.85 a bag. Write an equation to find the price of each can of soda if she spent a total of \$6, then solve.
3. Simplify the following expression.
$$71b - 4a + 4b - 4a$$
4. Solve: $7 = 3x - 2$
5. Write a context for the following expression: $26 + 8x$

6.2b Class Activity and Homework: Write Word Problems for Equations I

Working backwards, writing the word problems for the equations. The first one is done for you.

Given Information & Equation	Write a word problem to go with the information and the equation to the left.	Solve the equation. Then figure out all missing information.
<p>1. Given information:</p> <ul style="list-style-type: none"> a = Ali's age now $2a - 3$ = Mel's age now <p>Equation: $a + 2a - 3 = 39$ years</p>	<p>The sum of Ali and Mel's age is 39. If Mel is 3 years younger than twice Ali's age, how old are Ali and Mel?</p>	<p>$a = 14$ Ali is 14 and Mel is 25</p>
<p>2. Given information:</p> <ul style="list-style-type: none"> s = small angle $4s$ = larger angle <p>Equation: $s + 4s = 70$ degrees</p>		
<p>3. Given information:</p> <ul style="list-style-type: none"> w = width of rectangle $2w + 3$ = length of rectangle <p>Equation: $w + 2w + 3 + w + 2w + 3 = 78$ in.</p>		
<p>4. Given information:</p> <ul style="list-style-type: none"> v = Vicki's money $5v$ = Wally's money <p>Equation: $5v + v = \\$72$</p>		

<p>5. Given information:</p> <ul style="list-style-type: none"> • $2m$ = Number of red marbles • $3m$ = Number of blue marbles <p>Equation: $2m + 3m = 155$ marbles.</p>		
<p>6. Given information:</p> <ul style="list-style-type: none"> • e = Elisabeth's age now • $e - 7$ = Zack's age now • $e + 4$ = Gail's age now • $2e$ = Bob's age now <p>Equation: $e + (e - 7) + (e + 4) + 2e = 82$</p>		
<p>7. Given information:</p> <ul style="list-style-type: none"> • a = Measure of angle A • $a + 20$ = Measure of angle B • $a - 10$ = Measure of angle C <p>Equation: $a + a + 20 + a - 10 = 180^\circ$.</p>		
<p>8. Given information:</p> <ul style="list-style-type: none"> • c = cost of a shirt • $.25c$ = 25% of the cost of the shirt <p>Equation: $c - 0.25c = \\$28.80$</p>		

<p>9. Given information:</p> <ul style="list-style-type: none"> • m = height of a maple tree • $0.15m$ = 15% of the height of the maple tree <p>Equation: $m + 0.15m = 97.75$ feet</p>		
<p>10. Given information:</p> <ul style="list-style-type: none"> • z = length of a side of a triangle • $2z$ = length 2nd side of a triangle • $2z + 3$ = length of 3rd side of triangle <p>Equation: $z + 2z + (2z + 3) = 73$ cm.</p>		
<p>11. Given information:</p> <ul style="list-style-type: none"> • s = the price of a pair of socks • $5s - 1$ = the price of a pair of shoes <p>Equation: $3s + 2(5s - 1) = \\$56.50$</p>		
<p>12. Given information:</p> <ul style="list-style-type: none"> • g = Gordon's allowance • $2g - 1$ = Chris's allowance • $0.5g$ = Drew's allowance <p>Equation: $g + 2g - 1 + 0.5g = \\$51.50$</p>		

<p>13. Given information:</p> <ul style="list-style-type: none"> • x = length of blue ribbon • $2x + 3$ = length of red ribbon • $3x - 1$ = length of green ribbon <p>Equation: $x + 2x + 3 + 3x - 1 = 92$ inches</p>		
<p>14. Given information:</p> <ul style="list-style-type: none"> • x = Amount of money Jaime earned • $(1/5)x$ = one-fifth of the money Jaime made. • $(2/5)x$ = two-fifths of the money Jaime made. <p>Equation: $(1/5)x + (2/5)x = \\$165$</p>		

Spiral Review

1. Solve:

a. $-14 = 3x - 2$

b. $-8 = -3m + 10$

2. Write an expression to represent the following situation. Danielle is having a birthday party and inviting 8 friends. She wants to give each friend a gift bag with a large candy bar and a notebook. Show the total price if each large candy bar costs \$1.40.

3. Use the distributive property to rewrite the following expression $-4(5x - 1)$

4. Express each percent as a fraction in simplest form.

44%

17.5%

5. Malory got a 75% on her math test. To earn that score, she got 27 questions correct. Write an equation to find how many questions were on the test. Then solve.

6.2c Class Activity: Write and Solve Equations for Word Problems II

1. Matt, Rosa, and Kathy are cousins. If you combine their ages, they would be 40 yrs. old. Matt is one-third of Rosa's age. Kathy is five years older than Rosa. How old are they? Show several ways to solve the problem. Be able to explain how you came to your answer.

a. If necessary, use a model to help you write an equations for this context.

Rosa	Matt is $\frac{1}{3}$ Rosa's age so Matt is	Kathy is five years older than Rosa so Kathy is
Write an equation for the sum of the cousin's ages: Find x		

b. How old is Rosa?

c. How old is Matt?

d. How old is Kathy?

e. How can you be sure you have the correct answer?

2. The senior class has 412 students. They are assigned to different homerooms. There are 28 students in the smallest home room and the remaining 12 homerooms have the same number of students. How many students are in each of the remaining 12 homerooms?

a. If necessary, use a model to help you write an equations for this context.

b. How many students are in each of the 12 remaining homerooms?

c. How do you know your answer is correct?

3. Billy is three years older than his sister Anne. Together, the sum of their ages is 25. How old are Billy and Anne?
- a. Write an equation for this context. Use a model to help you if necessary.
 - b. How old is Billy?
 - c. How old is Anne?
 - d. Show that your answer is correct.
4. At the store, you find a pair of jeans and a t-shirt. Together, they'll cost \$80.20. The jeans cost three times the cost of the t-shirt. How much does each cost?
- a. Use a model to help you write an equation for this context.
 - b. How much do the jeans cost?
 - c. How much does the t-shirt cost?
 - d. How do you know your answer is correct?

5. The sum of two numbers is 41. The larger number is 1 more than 5 times the smaller number. What are the two numbers?
- Use a model to help you write an equation for this context.
 - What are the two numbers?
 - How do you know your answer is correct?
6. The sum of three consecutive integers is 21. What are the numbers?
- Use a model to help you write an equation for this context.
 - What are the three numbers?
 - How do you know your answer is correct?

6.2c Homework: Write and Solve Equations for Word Problems II.

Write equations for the sentences below. Then solve. If needed, draw a model.

1. Fifteen more than twice a number is -23. Equation: Solution:	2. The sum of three times a number and -23 is 28. Equation: Solution:	3. The difference between 5 times a number and 4 is 16. Equation: Solution:
4. Nine more than -8 times a number is -7. Equation: Solution:	5. The difference between 12 and ten times a number is -28. Equation: Solution:	6. Seven more than three times a number is 52. Equation: Solution:
7. Eleven less than five times a number is 19. Equation: Solution:	8. Thirteen more than four times a number is -91. Equation: Solution:	9. Seven less than twice a number is 43. Equation: Solution:

Write equations for the word problems below. Then solve. If necessary, draw a model.

10. The total cost of a suit and 4 ties is \$292. The suit cost \$200. The ties are all the same price. What is the cost of a tie?

Equation:

Solution:

11. Mary's sister is 7 years older than Mary. Their combined ages add up to 35. How old is Mary?

Equation:

Solution:

12. If Cassandra had 8 more dollars, she could buy the \$40 pair of hiking boots she wants. How much money does Cassandra have?

Equation:

Solution:

13. Tammy is training for a marathon. She ran the 3 miles from home to a lake, twice around the lake and then home again. She ran 11 miles. How far is it around the lake?

Equation:

Solution:

Spiral Review

1. Solve $4(x+1)+4x=36$
2. Phoebe has \$10. Write an expression showing how much money she will have left after buying 3 candy bars and a pack of pencils.
3. Simplify $-2(x+1)-x+5$
4. Find each sum without a model.
 - a. $-27.2 + \frac{4}{5}$
 - b. $98.1 + (-1.35)$
5. I go to a department store with a coupon for 20% off any one item. The shoes that I want are on sale for 40% off. Write and solve an equation to find the original price if I paid \$48.

6.2d Class Activity: Write and Solve Equations from Word Problems III

Do the following four things for each problem:

- Write the equation as complete as possible to include all the information.
- Solve the equation.
- Answer the question(s) in complete sentences.
- Check. Does your answer(s) make sense? Why?

1. For a field trip, 331 students went to the museum. Most of the students rode on the 6 buses provided by the school, but 7 students traveled in cars. If the same number of students rode each bus, how many students were in each bus?	
a.	c.
b.	d.

2. Ivan was broke (as usual!) Then he got his weekly allowance, but because he has a hard time saving money, he spent half his weekly allowance playing mini-golf. He then earned 4 dollars cleaning out his parents' car. If he now has \$12, how much is his weekly allowance?	
a.	c.
b.	d.

3. The cooking club made some pies to sell to raise money for new math books. The cafeteria contributed four pies to those made by the club. All of the pies were then cut into five pieces each and sold by the piece. There were a total of 60 pieces to sell. How many pies did the club make?	
a.	c.
b.	d.

4. Jan bought 2 shirts (same style and cost but different colors) and 2 pair of pants (same style and cost but different colors). Each shirt was \$3 less than a pair of pants. She spent \$49.80 (before tax). What is the price of a shirt? What is the price of a pair of pants?

a.	c.
b.	
	d.

5. Tom was training for a marathon. During the first week he ran a certain distance. The second week he ran 1.5 times further than the first week. During the third and fourth weeks, he ran 3 miles more than twice what he ran the first week. He ran a total of 136 miles in those four weeks. How many miles did he run each week?

a.	c.
b.	
	d.

6. Cassie and Tom wanted hamburgers for lunch. Cassie ordered a hamburger for \$4 and an order of fries. Tom ordered twice Cassie's order. The total price was \$16.65 (before tax). What is the cost of one order of fries?

a.	c.
b.	
	d.

7. Lupe and Carlos work in an office. Carlos makes \$16,000 less than twice Lupe's salary. The sum of their two salaries is \$104,000. How much are their salaries?

a.	c.
b.	
	d.

8. The neighborhood candy store sold 336 candy items this week. Twice as many M&Ms were sold as Snickers, and three times as many Crunch bars were sold as Snickers bars. How many of each kind of candy were sold this week?

a.

c.

b.

d.

9. Paul and Ringo went to the music store. Paul bought some guitar picks for \$6 and a package of guitar string. Ringo bought some new drumsticks for \$8. Then Paul remembered he had a coupon for \$5 off. The final bill after the coupon was \$15. How much was the package of guitar string?

a.

c.

b.

d.

10. Steve, Tyrel, and Josh spent a total of \$20 at the soccer game. It costs \$3 for each one of them get into the game. Each boy also bought a program. Josh bought a foam hand to wave in the bleachers. The foam hand was \$5. What is the cost of one program?

a.

c.

b.

d.

11. A total of 960 students attend Bosco Middle School. Some students walk to school, some ride the bus, and the rest come by car. The ratio of bus-riders to walkers to car riders is 6:3:1. How many students come to school by each form of transportation?

a.

c.

b.

d.

12. Mr. Manycattle drove to Mexican Hat, Utah from southern New Mexico in 4 days. On Monday and Wednesday, he traveled exactly the same distance. On Tuesday, he traveled 2 times as far as he did on Monday, and on Thursday, he traveled 3 times as far as he did on Monday. If the total trip covered 602 miles, how far did Mr. Manycattle travel each day of his trip? Which fraction describes the part of the trip covered on each day?

a.	c.
b.	
	d.

13. Ana had \$60 to spend at the mall. She bought 2 shirts for \$12.99 each, and 3 pounds of candy for \$2.89 per pound. How much money does she have left?

a.	c.
b.	
	d.

6.2d Homework: Write and Solve Equations from Word Problems III

For each context: a) write an equation, b) solve your equation, c) answer the question in a complete sentence, and d) check your answer. You may need to do your work on a separate sheet of paper.

1. Allie had \$24. After buying seven art pencils and a \$0.35 eraser, she had \$10 left. How much did each pencil cost?
2. Sarah won 40 super bouncy balls playing horseshoes at her school's game night. Later, she gave two each to some of her friends. If she has 8 remaining bouncy balls, to how many friends did she give bouncy balls?
3. At the local clothing store all shirts were on sale for one price and sweaters for a different price. Lonnie purchased three sweaters and two shirts for \$130. If the sale price of a shirt was five dollars less than the sale price of a sweater, how much did each item cost Lonnie?
4. Brock ate 16 Girl Scout cookies in 5 days (he wasn't suppose to eat any cookies because they belonged to his sister.) The second day he ate 3 more than the first (he felt pretty bad about that.) The third day he ate half as much as the 1st day (he was able to get better control of himself.) The fourth and fifth days, he ate twice each day what he ate the first day (he really likes Girl Scout cookies.) How many cookies did he eat each day?

5. Cassie and Tom went to the hamburger stand. Cassie ordered a hamburger for \$4 and an order of fries. Tom was really hungry, so he doubled Cassie's order for himself. The total price was \$18 (before tax). What's the cost of one order of fries?

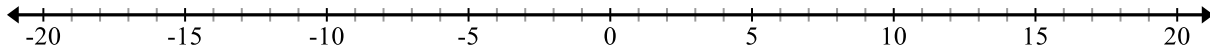
6. A collection of marbles has been divided into 3 different sets. The middle sized set is 2 times the size of the smallest set, and the largest set is 3 times as large as the middle-sized set. What fraction describes each part of the total marble collection?

7. Challenge: Brian buys 1 pack of baseball cards to add to the 2 cards a friend gave him. Then his mother gives him 2 more packs as a special treat. Now he has as many cards as Marcus who owns 1 pack plus 12 loose cards. How many cards are in each pack?

Spiral Review

1. Zach invested \$1500. If he earned 23.2% on his investment, a) write a proportion to find the amount of money he earned and b) state how much money he now has.

2. Use the number line below to show why $(-1)(-1) = 1$



3. If the ratio of girls to boys in a class is 3 to 4 and there are 35 students in the class. How many students are girls?
4. Lin is drawing a scale model of his school. He uses a scale of 1 in = 5 feet. If his classroom is 30 feet by 25 feet, what will be the dimension of the classroom on his scale model?
5. The equation $p = 6h$ shows how many pies *Pi Tree Bakery* sells on a given day. What is the unit rate?

6.2e Extra Practice: Write and Solve Equations

<p>1. You bought a magazine for \$5 and four erasers. You spent a total of \$15. How much did each eraser cost?</p>	<p>2. Old McDonald's 3 hens each lay the same number of eggs one week. This gives Old McDonald's wife enough eggs to make two recipes. One recipe requires 10 eggs and the other recipe requires 2 eggs. How many eggs did each hen lay?</p>
<p>3. Paul owns a set of model cars. His brother gives him 3 more sets for his birthday. Then Paul gives 1 set to a friend who really likes model cars but doesn't have any. Now Paul has 30 model cars left. How many model cars are in each set?</p>	<p>4. Tanner likes to collect comic books. He has 3 sets of the same title comics and 5 other comic books. His friend, Scott, has 1 set (the same as Tanner's) and 19 other comic books. The total number of comic books owned by Tanner and Scott is the same. How many books are in each set?</p>
<p>5. Erin can buy 5 Putt-putt tickets and 2 one-dollar boxes of popcorn for the same price as 3 putt-putt tickets and 12 one-dollar boxes of popcorn. How much does each putt-putt ticket cost?</p>	<p>6. Allison has 2 aquariums. In each aquarium she has 2 families of guppies and 3 tetras. Leigh has 1 aquarium with 10 tetras and 3 families of guppies. Allison and Leigh have the same number of fish and their guppy families each have the same number of members. How many guppies are in each family?</p>

<p>7. A whole object has been broken into 4 pieces, all of different sizes. Each piece is 2 times the size of the next smallest piece. What fractions describe each piece of the whole object?</p>	<p>8. Mrs. Smith rode the bus 720 miles in 3 days. On the first day, she traveled 3 times as far as she did on the second day. On the third day, she traveled 2 times as far as she did on the second day. How far did she travel each day?</p>
<p>9. The neighborhood grocery store sold 1463 bottles of soft drinks last month. Twice as many bottles of root beer were sold than lemon-lime soda, and twice as many bottles of cola were sold than root beer. How many bottles of each type of soft drink were sold?</p>	<p>10. Jill runs 5 miles to get to work. After work, she runs home, to a restaurant, and then back home again. In total, she runs 14 miles that day. How many miles is it from the restaurant to her house?</p>

6.2f Self-Assessment: Section 6.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Create an algebraic expression or equation to model a context.	I struggle to begin writing an algebraic expression or equation that models a context.	I can draw a model that represents a context. I struggle to use that model to create an algebraic equation.	I can write an algebraic equation that represents a context if I draw a model first.	I can write an equation that models a word problem.
2. Create a context that models an algebraic expression or equation.	I struggle to begin writing a context that models an algebraic expression or equation.	I can write a numeric context that models an algebraic expression or equation.	I can create a numeric or real-life context that models an algebraic expression or equation.	I can create a real-life context that models an algebraic expression or equation. I can verify that the answer is reasonable in that context.
3. Solve multi-step real-life problems involving calculations with positive and negative rational numbers in a variety of forms.	I can solve one and two-step real-life problems, but I struggle to solve multi-step real-life problems.	I can solve multi-step real-life problems involving calculations with integers with an equation by first drawing a model.	I can solve multi-step real-life problems involving calculations with integers with an equation without a model.	I can solve multi-step real-life problems involving calculations with rational numbers.
4. Determine the reasonableness of an answer to a contextual problem.	I struggle to determine the reasonableness of an answer to a contextual problem.	I can determine if the answer to a contextual problem is reasonable, but I struggle to explain why.	I can determine if the answer to a contextual problem is reasonable, and I can to explain why.	I can determine if the answer to a contextual problem is reasonable, and I can to explain why. If it is not reasonable, I can rethink that problem to find a reasonable answer.

Sample Problems for Section 6.2

1. Write an equation for each context. Draw a model first if necessary.
 - a. Tom has \$12,484 in his bank account. Six months ago, he had \$1,240. How much does Tom get paid each month if he has only deposited his monthly paycheck into his account?
 - b. Sixty-five pounds of candy was divided into four different boxes. The second box contained twice the amount of the first box. The third box contained two more pounds than the first box. The last box contained one-fourth the amount in the second box. How much candy was in each box?
 - c. Simon is ordering equipment for his tennis team. He orders a racquet and can of three tennis balls for each player. Each can of balls costs \$4. For a team of 5 players, his bill total is \$415. How much does each racquet cost?

2. Write a context that models the following equations. At least one context should be real-life.

$2L + 2(3L) = 990$	$3m + 4(m - 0.50) = 12$	$-46 = 6x + 2$

3. Solve the following real-life problems.
- The sum of two numbers is 52. One number is three less than quadruple the other number. What are the two numbers?
 - Martha divides \$94 amongst her four friends. Leon gets twice as much money as Kokyangwuti. Jill gets five more dollars than Leon. Isaac gets ten less dollars than Kokyangwuti. How much money does each friend get?
 - Elizabeth has \$26 left after shopping at the mall. She bought 2 shirts for \$22.99 each, a drink for \$2.02, and 2 books for \$16 each. How much money did she start with?
4. For each of the contexts in problem 3, answer the following questions:
- Is the answer reasonable or not?
 - Why is or isn't it reasonable?
 - If it isn't reasonable, explain what is wrong with the answer and rethink the problem.

6.3 Solve and Graph Inequalities, Interpret Inequality Solutions

Section Overview: Students begin this section by reviewing from 6th grade how to write inequalities and graph them on a number line. They then move to solving and graphing one-step and multi-step inequalities using their knowledge of solving one-step and multi-step equations. The section ends with students writing and solving one- and multi-step contextual inequality problems.

Throughout this section it is important that students understand the similarities and differences between finding the solution to an equation and finding solution(s) to an inequality. Students should also understand the relationship of each to the real line.

Language is particularly difficult for some students in this section. Phrases like “less than” or “greater than” in the previous section indicated an operation (e.g. subtract or add), in this section they may indicate $<$ or $>$. Help students to look at contexts holistically. Making sense of problem situations is critical with writing equations and/or expressions. Also help students predict the type of answers they will be getting as a way of interpreting how to write the context in algebraic form.

Students will also be reviewing number sense with integers in this section.

Concepts and Skills to be Mastered (from standards)

By the end of this section, students will be able to:

1. Use variables to create inequalities that model word problems.
2. Solve word problems leading to linear inequalities.
3. Use symbols of inequality to express situations in which solutions are greater than or less than a given value.

6.3a Class Activity and Homework: Review of Inequality Statements.

Review from 6th grade: writing and graphing inequalities.

Write inequalities for each statement below. For statements 1 – 4, the variable is identified for you. For statements 5 – 15, you must write what the variable will represent.

Example: The Garcia family car seats seven (with seat-belts) at most. " <i>x</i> " is the number of people that can sit in the Garcia's car.	$x \leq 7$
1. A school bus can seat at most 48 students. " <i>x</i> " is the number of people that can ride the bus.	
2. In many states you must be at least 16 years old to obtain a driver's license. " <i>x</i> " is the age you must be to obtain a drivers' license.	
3. It isn't safe to use a light bulb of more than 100 watts in many light fixtures. " <i>x</i> " is how many watts a light fixture has.	
4. At least 250 parents attended back-to-school night. " <i>x</i> " is the number of parents that attended back-to-school night.	
5. You must be no more than 15 years old to attend the middle school dance.	
6. A plane must travel at least 120 miles per hour to stay in the air. So as not to break the sound barrier, a plane must travel under 760 miles per hour.	
7. Children must be at least 48 inches tall to ride the roller coaster.	
8. You must have less than 3 tardies to get a satisfactory citizenship grade.	
9. Children younger than age 5 can get in free.	
10. To hunt big game in Utah a hunter must be at least 12 years old.	
11. The elevator can hold a maximum of 20 people	
12. To work the track at the community gym, you must be at least 16 years old.	
13. To join the FBI, you must be at least 23, but younger than 37 years old.	

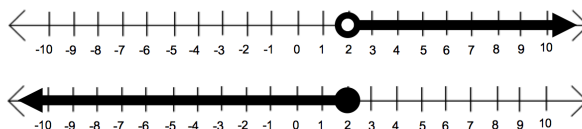
14. To run the class they must have no less than 12 participants registered.	
15. On the seven day family vacation, the Jones family traveled 12 miles on the shortest driving day and 500 miles on the longest driving day.	

Write situations to go with the following inequalities. Make up the situation and inequality for the last one.

16.	$x < 7$
17.	$x \geq 13$
18.	$x \leq -6$

Review: graphing inequalities on a number line:

Examine the inequality graphs below. Discuss the questions below as a class.



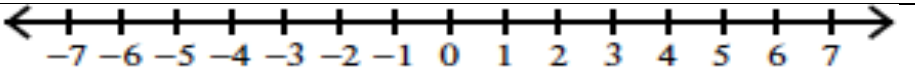
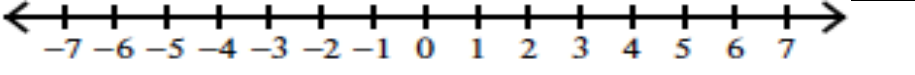
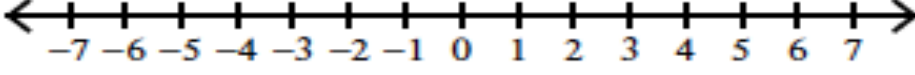
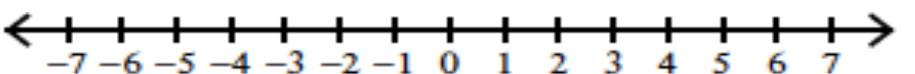
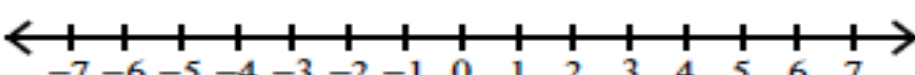
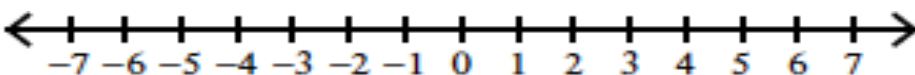
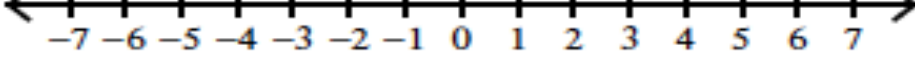
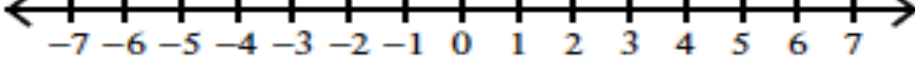
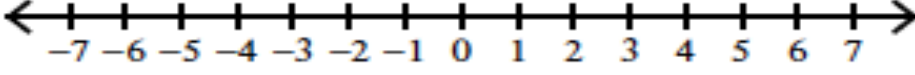
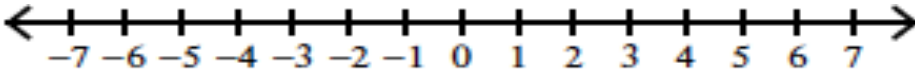
How are the inequalities shown on the number line?

Open circle means:

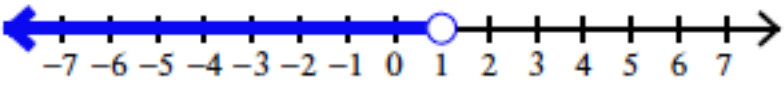
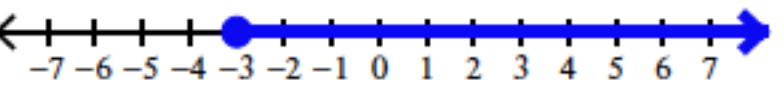
Closed circle means:

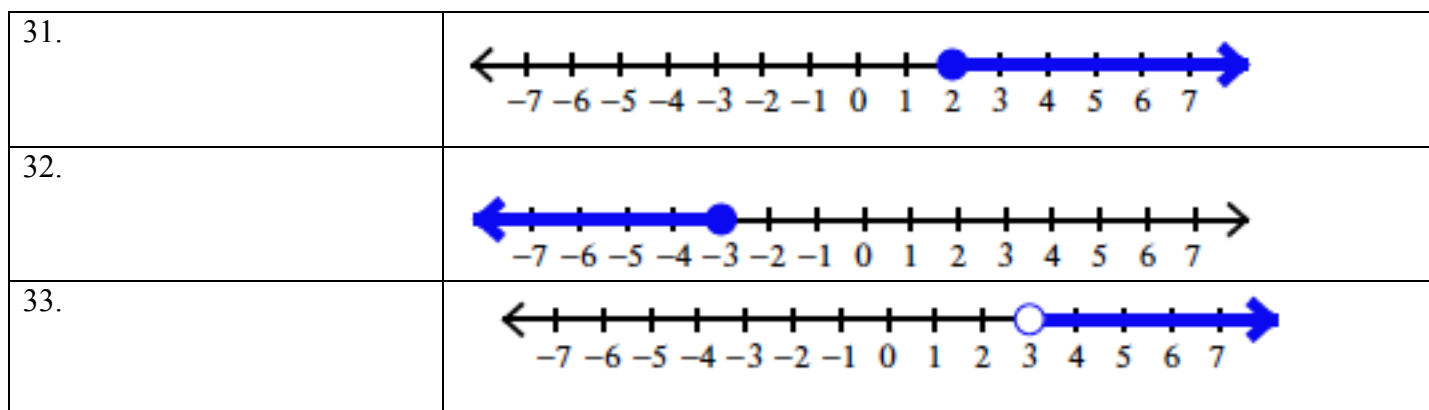
Next to each number line above, write the inequality represented by the number line above.

Practice Graphing Inequalities on a number line.

19. $x > 2$	
20. $a < -1$	
21. $y \leq 2$	
22. $b > -3$	
23. $p \geq 3$	
24. $x < .5$	
25. $y > -1\frac{1}{2}$	
26. $m \leq 3.5$	
27. $c \geq \frac{-15}{3}$	
28. $d \leq 4.25$	

Write an inequality for each graph below.

29.	
30	

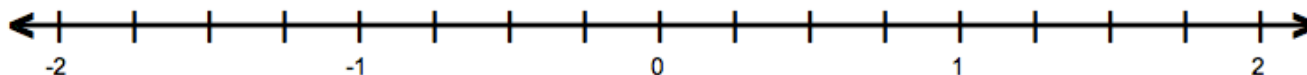


Spiral Review

1. There are a total of 127 cars and trucks on a lot. If there are four more than twice the number of trucks than cars, how many cars and trucks are on the lot?

2. Place the fractions on the number line below.

$$-\frac{3}{4}, \quad \frac{5}{12}, \quad -1\frac{1}{4}, \quad -\frac{4}{3}$$



3. A mouse can travel 1.5 miles in $\frac{3}{4}$ of an hour. Write an equation showing how far it travels.

4. Solve $-3x + 5.5x + 4 = 7.2$

5. Fill in the equivalent fraction and percent for this decimal:

Fraction	Decimal	Percent
	0.2	

6.3b Class Activity: Solve and Graph Inequalities

Activity 1: Every hour 92 people enter an office building and 30 people exit. If the building starts empty, after how many hours will there be more than 350 people in the building.

- Draw a model to illustrate what's happening in this context.
- In this context, what does "more than"?
- Write an inequality for the situation.
- If you double the people entering and exiting, how will it affect your time?

Activity 2: A 150-pound person burns 6 calories per minute when walking at a speed of 3.5 miles per hour. While walking, this person eats a snack that has 40 calories. This snack subtracts from the calories burned while walking.

- How long must the person walk at this speed to burn at least 190 calories?
- Explain what will happen if the person walks for a longer period of time? Shorter period of time?
- Write and graph an inequality that describes the solution to this situation.
- Extension: What if the person wanted to spend less time exercising but burn the same number of calories. How is this possible?

Activity 3: Explore Inequality Statement: Consider the following two inequality statements






$$x \leq 2 \quad \text{and} \quad 2 \leq x$$

In your own words, describe the solution set for each and then draw a graph of the solution set.

Activity 4: Exploring Inequality Statements:

- a. What is the solution set for $x - 1 > 6$. In other words, what value(s) of “x” make this statement true? Write both an inequality statement and graph your solution on a number line.
- b. What is the solution set for $2x < 6$. In other words, what value(s) of “x” make this statement true? Write both an inequality statement and graph your solution on a number line.
- c. What is the solution set for $3x - 7 \geq 13$. In other words, what value(s) of “x” make this statement true? Write both an inequality statement and graph your solution on a number line.
- d. Describe what you did to find the solution set.

Find the solution set for each inequality. Then graph the solution set. Scale the number lines appropriately.

1. $x + 8 \geq 18$	
2. $-2 + x \leq -16$	
3. $-28 < v - 10\frac{1}{2}$	
4. $4y > -8$	
5. $21 \leq 3p$	

6. $-15 > 2x$



7. $4r + 13 < 9$



8. $\frac{r-2}{3} > \frac{1}{3}$



9. $11 \geq 8 + 6n$



10. $5n - 75 \geq -135$



11. $18.\overline{66} + 2k \leq 10$















12. $-4 \geq 4x + 16$



6.3b Homework: Solve and Graph Inequalities

Solve to find the boundary. Then graph the inequalities below. Scale the number lines appropriately.

1. $n - 2 \leq 4$	
2. $-18 + n < -7$	
3. $20 \geq p + 16.5$	
4. $2x < 6$	
5. $-24 \leq 3p$	
6. $3y > 17$	
7. $-4 \geq -2 + 6n$	
8. $3x - 1 \leq 3$	
9. $21n - 63 > -126$	
10. $9.5 + 2n > -5$	
11. $2x - 1 > 6$	
12. $\frac{3}{4} > \frac{-2 + 2x}{4}$	

Spiral Review

1. Solve $-3(x - 6) = 9$
2. Find each product or quotient.
 - a. $-6\left(\frac{1}{6}\right)$
 - b. $-10 \div \frac{1}{2}$
3. Suppose you were to roll a fair 6-sided number cube once, then flip a coin. List all the possible outcomes.
4. Using the information in question 3, what is the probability of getting a heads and an even number?
5. Dawn is researching car rental companies. The following tables show their rates based on the days you rent the car. Which company's rates are proportional? Explain how you know.

Company E	
Days	Cost
1	73
2	76
3	79
4	82

Company F	
Days	Cost
1	21
2	42
3	63
4	84

6.3c Class Activity: Multiplying by a negative when Solving Inequalities

Consider the inequality $4 > 1$.

- Question: If you multiply both sides of an inequality by the same *positive* number do you expect the inequality to remain true? Why or why not?
- Question: If you multiply both sides of an inequality by the same *negative* number do you expect the inequality to remain true? Why or why not?

1. Test your answers from above. Complete the table by filling in the middle column with $<$, $>$, or $=$.

4	$>$	1	True or untrue?	If untrue, what must be done for the inequality to be true?
$4 \bullet 3$	$>$	$1 \bullet 3$		
$4 \bullet 2$	$>$	$1 \bullet 2$		
$4 \bullet 1$	$>$	$1 \bullet 1$		
$4 \bullet 0$	$>$	$1 \bullet 0$		
$4 \bullet -1$	$>$	$1 \bullet -1$		
$4 \bullet -2$	$>$	$1 \bullet -2$		
$4 \bullet -3$	$>$	$1 \bullet -3$		

2. Under what conditions did the inequality become *untrue*? Why does that condition make the inequality untrue?

3. Solve the inequality $\frac{x}{-2} < 2$, explain your procedure. Write and graph the solution.



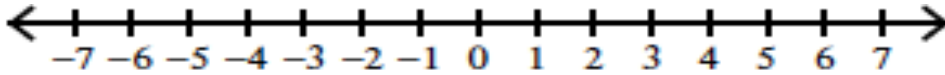
4. Check your solution for varying values for x . Is your graph correct?

In the inequality $\frac{x}{-2} < 2$, if the value for x is...	-8	-6	-3	-2	0	2	4	6	8
Write the inequality.									
...is your solution set true?									

5. Solve and graph the solution to $\frac{x}{-3} \geq -9$.

Check your values in your solution set to make sure your answer is correct.

6. What about dividing by a negative number? What do you expect? Solve and graph the inequality $-2x > 6$, explain your procedure.



7. Test your solution.

In the inequality $-2x > 6$, if the value for x is...	-8	-6	-3	-2	-1	0	2	4	6
Write the inequality.									
...is your solution set true?									

8. Finish this sentence: When you multiply or divide both sides of an inequality by the same negative number...

Solve the following problems by first writing an inequality and then solving the inequality.

9. Kyle and Mika aren't very good at the trivia game. In the game a wrong answer gives your team a negative two-thirds of a point. If at the end of the game, Kyle and Mika have never scored more than a negative 8 points, how many wrong answers do they usually give?

Inequality:






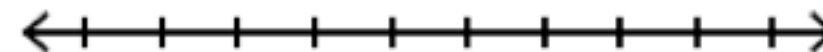

Solution:

10. You have \$10 to spend at the school carnival. If each game costs \$0.25, how many games can you play?

Inequality:








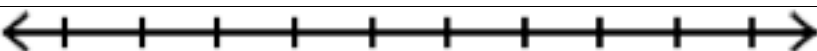
Solution:





Practice solving inequalities involving multiplication or division by negative numbers.

11. $-5t \geq -25$	
12. $\frac{-2y}{3} < -4$	
13. $-\frac{1}{2}p > -4$	
14. $\frac{y-3}{-4} \leq 5$	
15. $-5 \geq -2 - 3n$	
16. $-18 \geq \frac{-x}{2}$	
17. $-2(x+4) \leq 28$	

6.3c Homework: Multiplying by a negative when Solving Inequalities

Solve the following inequalities. Graph your solution on the number line.

1. $-3t \geq 27$	
2. $\frac{-y}{3} < -10$	
3. $-\frac{3}{4}m > 6$	
4. $\frac{5n}{-2} > -1.5$	
5. $-28 \geq -4n - 4$	
6. $7.5 - 2n > -2$	
7. $-3(x - 4) \leq 24$	
8. $-\frac{3}{4}n - 21 > -15$	

9. $-6.5 + 2n > -5$	
10. $2x - \frac{2}{3} > \frac{16}{3}$	
11. $0.75 > \frac{-2x}{4}$	
12. $\frac{x}{7} - \frac{2}{3} > \frac{13}{3}$	


Spiral Review

- Solve for a . $a + \frac{3}{4} = 5$
- Factor the following expressions.
 $3x - 6$ $15 - 20y$ $\frac{5}{3}x + \frac{2}{3}$ $4.9t - 2.8$
- Bubba can make $7\frac{1}{2}$ sandwiches in $\frac{1}{2}$ hour. Find the following unit rates
 - _____ sandwiches per hour
 - _____ hours per sandwich
 - _____ minutes per sandwich
- Solve: $-2x + 1 < -3$
- Chloe has twice as many cats as her sister has dogs. Her brother has 3 turtles. Together, they have six pets. How many of each pet do they have?


6.3d Class Activity: Write and Solve Inequalities for Word Problems

Follow the structure below to solve and graph each inequality. The first problem has been started for you as an example.


1. Andy has \$550 in a savings account at the beginning of the summer. He wants to have at least \$200 in the account by the end of the summer. He withdraws \$25 each week for food, clothes, and movie tickets. How many weeks will his money last?

Known Information <ul style="list-style-type: none"> Starts with \$550 Takes away \$25 each week Wants at least \$200 in the end 	Variable and what it represents w = number of weeks money can last	What's the relationship? The money Andy has needs to be more (bigger) than or equal to \$200	Inequality $550 - 25w \geq 200$
Solution and what it means			


2. On vacation, Katelyn wanted to have her hair braided in multiple braids to cover her head. The beautician charges a flat rate of \$4, plus \$0.75 per braid. She's saved \$29 to get braids. How many braids can she get?

Known Information	Variable and what it represents	What's the relationship?	Inequality
Solution and what it means			


3. Maria is starting a small DVD business online. She makes \$2.25 on each DVD she sells. To start her business though, she had to invest \$750. How many DVDs does she need to sell before he starts to make a profit?

Known Information	Variable and what it represents	What's the relationship?	Inequality
Solution and what it means			


4. The Community Swimming Pool charges a flat rate of \$50 for a birthday party plus \$2.50 for each person. Deborah can't spend more than \$100. How many friends can she invite?

Known Information	Variable and what it represents	What's the relationship?	Inequality
Solution and what it means			


5. David owns a Yellow Cab. The company charges a flat rate of \$2.50 for every cab ride, plus \$0.85 per mile. David figures he needs to average at least \$12 for each cab ride to make a profit. At least how many miles must rides average to make a profit?

Known Information	Variable and what it represents	Inequality Sign	Inequality
Solution and what it means			

6. Jacques has a pre-paid phone plan. He has \$45 to spend and each minute costs \$0.39, what is the most minutes he can buy?

Known Information	Variable and what it represents	Inequality Sign	Inequality
Solution and what it means			

7. Harry wants to download some songs to his mp3 player. If he gets a \$20 gift card for his birthday and each song costs \$0.90, at most how many songs can he download?

Known Information	Variable and what it represents	Inequality Sign	Inequality
Solution and what it means			

6.3d Homework: Write and Solve Inequalities for Word Problems

Write and solve inequalities for each word problem below. Use the structure from Class Activity 6.3d as a frame for solving each.









1. Kimberly took her 6 nieces and nephews to a hockey game. She wants to buy them snacks. How much can she spend on snacks for each child if Kimberly wants to spend less than \$33 in total?	2. The school is having a fundraiser. They are running a carnival. Tickets sell for \$0.50 each. They are planning on buying supplies for the carnival that cost \$50. How many tickets must they sell to raise at least \$200?
3. Billy needs to read 500 minutes this week for his English class. He is going to read 6 days. If he already reads 15 minutes every day, how many additional minutes does he need each day to read at least 500 minutes?	4. Erin is buying cupcakes for her birthday party. Each cupcake costs \$1.50. How many guests can she invite if her budget is \$80 and she has already spent \$16 on paper cups and plates? By the way, Erin thinks that each guest will want two cupcakes to eat.
5. Lauren got \$321 from various relatives on her birthday. If she wants to put 20% of the money into her savings account, how much will she have left over to spend on new clothes?	6. Peter is trying to set a new record for pizza deliveries. His previous record is 20 pizzas in one hour. He has already delivered 2 pizzas in 5 minutes. How many pizzas will he need to average per minute to beat his previous record?



7. Mrs. Brown is ordering pictures of her new baby. There is a \$20 sitting fee and each 5x7 portrait she orders is \$4. She also has a coupon for \$10 off. If she wants to spend less than \$50, how many 5x7 portraits can she order?	8. Stuart's Painting Service charges a \$50 supplies fee plus \$10 per hour painting. Andrew's Awesome Painting charges a \$20 supplies fee plus \$20 per hour painting. For how many hours does Andrew charge less than Stuart?
9. The technology department is having a fundraiser. They want to make at least \$1000 by selling hoodies for \$25. Each hoodie costs them \$15. How many will they need to sell to reach their goal?	

Spiral Review





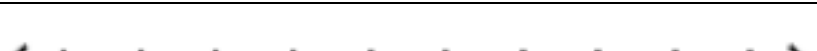
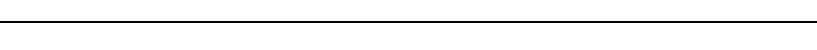


- Solve the following inequality: $25 > 2x + 8 - 66$
- Lara has \$1,425 in her bank account. Write and solve an equation to show how much money she started with if that amount reflects a 14% increase on her original amount.
- There are 36 red and 44 blue marbles in a bag. What is the probability of randomly drawing a red marble?
- Jordyn runs $2\frac{1}{2}$ kilometers in $\frac{1}{3}$ hours. If she continues running at the same pace, how long will it take her to run 40 kilometers? (Hint: First, find the unit rate.)
- Solve the following equation. Use a model if needed. $2(m - 1) + 3m - 4 + m = 18$



6.3e Class Activity: Solve Inequalities Review

1. $-13m + 46 > -45$	
2. $-6h - 16 < -34$	
3. $11a - 19 \geq 9a + 5$	
4. $12y + 8 > 56$	
5. $\frac{x}{7} - 20 \geq -12$	
6. $25 \leq -\frac{5}{6}d$	
7. $-12 - 14n < 20 - 6n$	
8. $-16 \geq -5(y + 0.2)$	

9. $-\frac{1}{2}x - 1 > 6$	
10. $\frac{-3x+1}{2} \leq 8$	
11. Aimee wants to order some DVDs from Amazon. Each DVD costs \$8.49 and shipping for the entire order is \$5. She has only \$70 to spend. How many DVDs can she order?	
12. On vacation, Jocelyn wants to have her hair braided in multiple braids to cover her head. It costs a flat rate of \$3, plus \$0.85 per braid. She had saved \$32. How many braids can she get?	
13. The Community Swimming Pool charges a flat rate of \$60 for a birthday party plus \$2.25 for each person. Juan can't spend more than \$120. How many friends can he invite?	

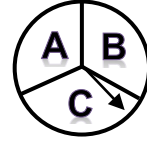
6.3e Homework: Solve Inequalities Review

1. $8e \geq -64$	
2. $d + (-13) \leq 26$	
3. $3 > \frac{-2+2x}{-2}$	
4. $-\frac{4}{3}(y-3) \leq \frac{3}{4}$	
5. $-9(m-6) > 99$	
6. $\frac{-x}{5} - \frac{3}{5} > 4.4$	
7. $15 - 3y + 8y > 7$	
8. $-\frac{2}{5}(6+x) < 2$	

9. $28 \leq 9 + 7y + 5$	
10. $-23 \geq -4(y - 0.25)$	
11. Van has an old cell phone he has to buy minutes for. He has \$55 to spend and each minute costs \$0.49, what is the most minutes he can buy?	
12. The Yellow Cab Taxi charges a flat rate of \$3.50 for every cab ride, plus \$0.95 per mile. Tofi needs a ride from the airport. He only has \$30 cash. How many miles can he go?	
13. Vicki wants to play a video game that charges you \$0.12 per minute. If she has \$15 to spend, how many minutes can she play at most?	

Spiral Review

1. A spinner contains three letters of the alphabet.



- How many outcomes are possible if the spinner is spun three times?
- List all of the outcomes for spinning three times.
- What is the probability of getting exactly one A in three spins?

2. Express each percent as a fraction in simplest form.

35%

22%

3. Simplify the following expression. Use a model if needed. $71b - 4a + 4b - 4a$

4. What is 30% of 150? Use a bar model.

5. Will the following side lengths make a triangle? Why or why not?

- 5 cm, 4 cm, 18 cm
- 3 ft, 3 ft, 2 ft
- 3 in, 6 in, 3 in

6.3f Self-Assessment: Section 6.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Solve a linear inequality and check your solution.	I can solve one and two-step inequalities with whole numbers, but I struggle to solve multi-step equations or ones with rational numbers.	I can solve multi-step inequalities involving calculations with whole numbers.	I can solve multi-step equations involving calculations with integers. I can also check my solution.	I can solve multi-step inequalities involving calculations with rational numbers. I can also check my solution.
2. Create an inequality with variables that models a context.	I struggle to begin writing an inequality that models a context.	I can draw a model that represents a context. I struggle to use that model to create an inequality.	I can write an inequality that represents a context if I draw a model first.	I can write an inequality that models a context.
3. Solve word problems leading to linear inequalities.	I struggle to solve word problems leading to linear inequalities.	I can usually write an inequality to solve a word problem leading to a linear inequality, but I struggle using that inequality to get a solution.	I can solve word problems leading to linear inequalities.	I can solve word problems leading to linear inequalities. I can explain the solution in context.
4. Determine the reasonableness of a solution to a contextual inequality.	I struggle to determine the reasonableness of an answer to a contextual inequality.	I can determine if the answer to a contextual inequality is reasonable, but I struggle to explain why.	I can determine if the answer to a contextual inequality is reasonable, and I can to explain why.	I can determine if the answer to a contextual inequality is reasonable, and I can to explain why. If it is not reasonable, I can rethink that problem to find a reasonable answer.

Sample Problems for Section 6.3

1. Solve each of the following equations with or without a model.

a.

$-1 < x + 7$	$2x + 3 > 7$	$30 \leq \frac{x}{7} - 20$
--------------	--------------	----------------------------

b.

$3(x - 6) \geq 9$	$3 \leq \frac{2x + 4}{2}$	$3x + 5x + 4 > 4$
-------------------	---------------------------	-------------------

c.

$-2(x - 9) \geq -7$	$4 \leq \frac{-6x + 6}{6}$	$-2x + 8x + 4 > -8$
---------------------	----------------------------	---------------------

d.

$0.8x + (-8) < 0$	$\frac{1}{2}(-2x - 7) > \frac{3}{4}$	$-\frac{1}{4} \leq 2.4x + 10 - 3.5x$
-------------------	--------------------------------------	--------------------------------------

2. Write an inequality for each context. Draw a model first if necessary.
- A moving company is shipping my stuff on a move from Sandy to St George. The maximum weight on Utah roads for a truck is 80,000 lbs. If the empty truck and trailer weigh 32,500 lbs and other people have stuff on the truck already weighing 35,000 lbs total, how much can my stuff weigh?
 - In Whatsitville, speeding tickets are fined \$75 plus \$13 for every mile over the speed limit. If Mr. Bo is pulled over and told his ticket will be at least \$400. At least how much over the speed limit was he?
 - Jenna's credit card requires that she pay at least 10% of her balance each month. If she pays at least \$44, what is the most her balance could be?
3. Write an inequality to represent each of the following word problems. Solve each problem. Explain your solution in context.
- Loralie is having a birthday. She wants to bring treats for her friends at school. Her mom gives her \$20. How much can she spend per friend if she wants to bring treats for herself and eight friends?
 - Jeremy is two years older than Rachel. The sum of the ages of Jeremy and Rachel is less than 46. How old could Jeremy be?
 - Kathryn is five years old and so excited to ride all the rides at the local amusement park. Unfortunately, she is 42.5 in tall and one ride requires that she be at least 50 in. You want to tell her in about how many years, she'll be able to ride. A quick google search shows that the average child grows $2\frac{1}{2}$ in per year. How many years will you tell Kathryn it will be until she can ride all the rides?
4. For each of the contexts in problem 3, answer the following questions:
- Is the answer reasonable or not?
 - Why is or isn't it reasonable?
 - If it isn't reasonable, explain what is wrong with the answer and rethink the problem.

First published in 2013 by the University of Utah in association with the Utah State Office of Education.

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Chapter 7: Probability and Statistics (3 weeks)

UTAH CORE Standards

Probability and Statistics:

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **7.SP.1**
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* **7.SP.2**

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* **7.SP.3**
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* **7.SP.4**

Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **7.SP.5**
6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **7.SP.6**
7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. **7.SP.7**
 - a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* **7.SP.7a**
 - b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **7.SP.7b**
8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. **7.SP.8**
 - a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. **7.SP.8a**

- b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. **7.SP.8b**
- c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?* **7.SP.8c**

Chapter 7 Summary:

Throughout this chapter students engage in a variety of activities: gathering data, creating plots, and making comparisons between data sets. Activities are designed to help students move from experiences to general speculations about probability and number.

Section 1 begins with an exploration of basic probability and notation, using objects such as number cube (dice) and cards. Students will develop modeling strategies to make sense of different contexts and then move to generalizations. In order to perform the necessary probability calculations, students work with fraction and decimal equivalents. These exercises should strengthen students’ abilities with rational number operations. Some probabilities aren’t known, but can be estimated by repeating a trial many times, thus estimating the probability from a large number of trials. This is known as the Law of Large Numbers, and will be explored by tossing a Hershey’s Kiss many times and calculating the proportion of times the Kiss lands on its base.

Section 2 investigates the basics of gathering samples randomly in order to learn about characteristics of populations, in other words, the basics of inferential statistics. Typically, population values are not knowable because most populations are too large or difficult to measure. “Inferential statistics” means that samples from the population are collected, and then analyzed in order to make judgments about the population. The key to obtaining samples that represent the population is to select samples randomly. Students will gather samples from real and pretend populations, plot the data, perform calculations on the sample results, and then use the information from the samples to make decisions about characteristics of the population.

Section 3 uses inferential statistics to compare two or more populations. In this section, students use data from existing samples and also gather their own data. They compare plots from the different populations, and then make comparisons of center and spread of the populations, through both calculations and visual comparisons.

Terms and phrases used in this chapter are informally explained below.

VOCABULARY:

random sample – a set of data that is chosen in such a way that each member of the population has an equal probability of being selected

population – the set of possibilities for which data can be selected

independent events – events that are not affected by each other

compound events – an event made up of two or more independent events

expected value – the average value of repeated observations in a replicated experiment

frequency – the number of times that a particular value occurs in an observation

probability – the chance or likelihood that an event will occur, expressed from a scale from 0 (impossible) to 1 (certain)

relative frequency – the ratio of the frequency of an event in an experiment to the total frequency

Law of Large Numbers – the long run relative frequency of an experiment, based on a large number of trials

sample – a subset of a population collected by a defined procedure for the purpose of making inferences from the sample to the population

simulation – an experiment that models a real-life situation

probability model – a mathematical representation of a random phenomenon that includes listing the sample space and the probability of each element in the sample space

uniform probability model – when all of the outcomes of a probability model are equally likely

CONNECTIONS TO CONTENT:

Prior Knowledge

Students should be familiar with the following content from 6th grade:

- Understands that a set of data has a distribution that can be described by its center, spread, and overall shape. **6.SP.2**
- Displays numerical data in plots on a number line, dot plots, histograms, and box plots. **6.SP.4**
- Gives quantitative measures of center (median and/or mean) and variability (IQR and/or mean absolute deviation) **6.SP.5c**
- Describes any overall patterns of data and any striking deviations from the overall pattern. **6.SP.5c**
- Relates the choice of measure of center and variability to the shape of the data distribution and context. **6.SP.5d**









Chapter 7 begins by reviewing standard 7.SP.5, basic probability content that was covered in Chapter 1.

Future Knowledge

This unit introduces the importance of fairness in random sampling, and of using samples to draw inferences about populations. Some of the statistical tools used in 6th grade will be practiced and expanded upon as students continue to work with measures of center and spread to make comparisons between populations. Students will investigate chance processes as they develop, use, and evaluate probability models. Compound events will be explored through simulation, and by multiple representations such as tables, lists, and tree diagrams.

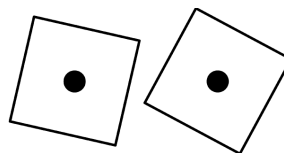
The eighth grade statistical curriculum will focus on scatter plots and bivariate measurement data. Bivariate data is also explored in Secondary Math I, however, Secondary Math I, II, & III statistics standards return to exploration of center and spread, random probability calculations, sampling and inference.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

	Make sense of problems and persevere in solving them.	Students will make sense of probability calculations by connecting rational numbers to probabilities, and creating models to support calculations. Additionally, students will use sense-making skills to compare data sets using measures of center and spread.
	Reason abstractly and quantitatively	Students are able to utilize the mathematics necessary to solve simple probability problems using both ratios and percents, and interpret data using appropriate measures of center and spread.
	Construct viable arguments and critique the reasoning of others	Students are able to assess the reasonableness of their answers and will solve problems in a variety of ways, where they will be able to discuss and validate their own approaches and solutions.
	Model with mathematics	Students will use multiple representations to model probability problems and create appropriate graphical representations for data.
	Attend to precision	Students will identify whether or not their answer makes sense (e.g. probability values less than 0 or greater than 1 are not valid, measures of center and spread should be reasonable for the data).
	Look for and make use of structure	Students are able to recognize the key phrases of compound probability models and use of diagrams or tables to assist with calculations and data analysis.
	Use appropriate tools strategically	Students demonstrate their ability to select and use the most appropriate tool(s), such as diagrams, tables, lists, box plots, dot plots, etc., while solving real-life word problems.
	Look for and express regularity in repeated reasoning	Students look for structure and patterns in real-life word problems, which will help them identify a solution strategy.

7.0 Anchor Problem: The Teacher Always Wins

The Teacher Always Wins Game (to be introduced by the teacher)



Why does the teacher always seem to win? Is it certain that the teacher will win? After the introduction of this activity, your job is to determine the answers to these questions, and see if you can discover the secret to the game.

Section 7.1: Probability Models to Analyze Real Data, Make Predictions

Section Overview:

This section starts with a review of concepts from Chapter 1 section 1 and then extends to a more thorough look at probability models. A complete probability model includes a sample space that lists all possible outcomes, including the probability of each outcome. The sum of the probabilities from the model is always 1. A uniform probability model will have relative frequency probabilities that are equivalent. A probability model of a chance event (which may or may not be uniform) can be approximated through the collection of data and observing the long-run relative frequencies to approximate the theoretical probabilities. Probability models can be used for predictions and determining likely or unlikely events.

There are multiple representations of how probability models can be displayed. These include, but are not limited to: organized lists (including a list that uses set notation), tables, and tree diagrams.

Students will also consider the ramifications of rounding, what it means to have “independent events,” how to create a simulation, and further explore the difference between theoretical probability and real life situations. There will be several exploration activities in the section giving students ample opportunity to discuss ideas.

Concepts and Skills to be Mastered

1. Develop a probability model and use it to find probabilities of events.
2. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
3. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
4. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams.

7.1a Class Activity: The Horse Race Game Revisited—Probability Basics



1. Review from Chapter 1 (1.1.b): in the Horse Race Game you predicted which horse (#2-12) would win the race. The winning horse was determined by tossing two dice and observing the sum of the die. Fill in the table below to find the *possible* sums of two dice.

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Use the table to answer the following questions. Some of the questions may review Chapter 1 content.

2. What is the number of the horse that is most likely to win? Explain how you know.
3. How many times did that horse's number occur in the table?
4. What is the number of the horse (or horses) that is/are the least likely to win?
5. How many times did that horse(s) number occur in the table? _____
6. How many total outcomes are there altogether on the table? _____

RECALL: Probability is written as a part-to-whole ratio of possible outcomes to the number of total outcomes. For example, the chances that horse #3 will win is: two *possible* ways to win out of thirty-six *total* possibilities (or $2/36$.)

7. What is the probability that horse #8 will win? _____

The table shows the probabilities for each horse winning. Recall, this is called the **theoretical probability** of winning.

Mathematical Notation: the mathematical shorthand way of writing a probability looks like this:

$$P(\text{horse \#4 wins}) = 3/36 \text{ OR } P(4) = 3/36$$

8. Fill in the table below with the theoretical probability for each horse to win. Write the values both as a fraction and as a percent. Round the percents to the nearest whole number.

Number	2	3	4	5	6	7	8	9	10	11	12
Probability as a fraction											
Probability as a percent											

9. Add up all the fractions. What is the total?

10. Add up all of the percents in the table. What is the total?

11. If there are 200 races (200 rolls of the dice), how often would you predict horse #7 would win? Show all your work and explain your reasoning.

12. Suppose the horses race...

a. ...500 times, what is your prediction for how many times horse #7 will win? Show your calculations.

b. ...1000 times, what is your prediction for how many times horse #2 will win? ...horse #12 will win?

c. Suppose we watched the horses race 500 times. Which of the following values would be the most likely result for horse #5? 11 wins 50 wins 100 wins 250 wins

Explain the reasoning for your choice.

7.1a Homework: Probability Problem Solving

M&M Probability (refer to the table below for amounts of colors)

1. The color mix in a large bag of M&Ms is shown in the table below. What is the total number of M&M's in the bag?
2. Calculate the probability of drawing each of the colors. Finish the probability model by recording the experimental probability of drawing each color. Show the probabilities as both a fraction and as a percent.

Color and number	RED 60	GREEN 40	BROWN 45	YELLOW 25	ORANGE 20	BLUE 30
Fractions						
Percents						

3. If you drew 50 M&M's, one at a time (returning the M&M to the bag each time), how many of each color would you expect, based on the probabilities in the table above? Put your answers in the table.

Predicted Sampling Estimate for 50 draws					
RED	GREEN	BROWN	YELLOW	ORANGE	BLUE

4. Suppose you went to the store and bought a large bag of M&Ms. From that bag you took a sample of exactly 50 M&Ms and calculated the percent of each color in your sample. Do you think the percents would be the same as in the first table? Why or why not?

Probability model – a mathematical representation of a random phenomenon that includes listing the sample space and the probability of each element in the sample space.

5. The Bag Game

There are three bags of chips: one with 25 red and 5 blue, another with 20 red and 10 blue, and the last with 10 red and 20 blue. You're randomly given one bag. To win the game, you must guess correctly which bag you've been given but you cannot see its contents.

To make your guess you are given three options:

- | | |
|---|---|
| a. Draw 5 chips and guess correctly, win \$100. | d. Draw 20 chips and guess correctly, win \$25. |
| b. Draw 10 chips and guess correctly, win \$75. | e. Draw 25 chips and guess correctly, win \$10. |
| c. Draw 15 chips and guess correctly, win \$50. | |

Note: to play the bag game, you draw one chip at a time, record the color, replace the chip, and then repeat.

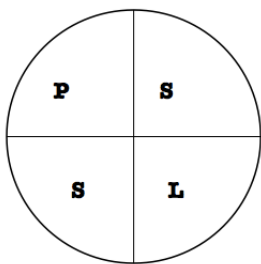
You want to win as much money as possible. Which option do you choose for guessing which bag you've been given? Why? Be certain to explain all of the probabilities.

6. Rolling Doubles

If TWO dice are rolled 36 times, how many doubles would you expect to see? What is the probability of rolling doubles with two fair die?

Spiral Review

- Write 0.612 as a percent and fraction.
- If 4 gallons of gas cost \$14.60, how much does 10 gallons of gas cost?
- If you spin the following spinner once, what is the theoretical probability of spinning an L?



- A mouse can travel 1.5 miles in $\frac{3}{4}$ of an hour. At that pace,
 - how far can it travel in 1 hour?
 - how long does it take it to travel one mile?

Madison is riding her horse around the outside of a circular arena. She knows that 14 laps is $\frac{1}{2}$ mile. What is the diameter of the arena? (Hint: 1 mi = 5280 ft)

7.1b Class Activity: Probability Models

Probability models (like “tree” models) show the outcome of random processes. A probability model includes the following:

- A listing of the sample space (all the possible outcomes.) For example, you might use set notation $S = \{ _, _, _, \dots \}$, a tree diagram, a table, etc.
- Probability for each possible event in the sample space. Remember, probabilities always add up to 1.

1. Suppose you are going to toss a coin and see how it lands.

- List the sample space using set notation. $S =$
- What is the probability for tossing a head? $P(\text{head}) =$
- What is the probability for tossing a tail? $P(\text{tail}) =$



2. Consider the theoretical outcomes for tossing a fair coin 3 times.

- What is the sample space? Use set notation. (Hint: there should be 8 outcomes in the sample space.)
- What is the probability of each of the 8 outcomes?

A probability model for which all outcomes are equally likely (have the same probability) is called a **uniform probability model**.

3. Create a tree diagram to display the sample space for tossing a coin 3 times.

4. Use the list or the tree diagram for 3 coin tosses to fill in the theoretical probability of the following events:

3 heads	3 tails	2 heads and 1 tail (in any order)	2 tails and 1 head (in any order)

5. Use all or part of the tree diagram in #4 to calculate the following probabilities:

Notation: if you see $P(TTH)$, that is the same as writing “the probability of a tail, and a tail, and a head”

$$P(H) =$$

$$P(T) =$$

$$P(HT) =$$

$$P(TH) =$$

$$P(HTH) =$$

$$P(TTT) =$$

Compound Event: an event made up of two or more independent events.
--

6. Fill in the blanks for calculating probabilities for **compound events**, in other words, for two or more events occurring together. Suppose we call the first event A , the second event B , the third event C , etc.

If $P(A) = 0.5$ and $P(B) = 0.5$
then the compound probability $P(A \text{ and } B) = \underline{\hspace{2cm}}$

If $P(A) = 0.5$ and $P(B) = 0.5$ and $P(C) = 0.5$
then the compound probability $P(A \text{ and } B \text{ and } C) = \underline{\hspace{2cm}}$

Using words and symbols, state your conjecture for the general rule for calculating probabilities for independent compound events.

7. Use the rule you found in the prior question to calculate $P(\text{HTHHH})$.

8. Suppose that you have an *unfair* coin where the $P(H) = 0.8$ and the $P(T) = 0.2$. Compute the following probabilities:

$P(H) =$ $P(T) =$

$P(HT) =$ $P(TH) =$

$P(HTH) =$ $P(TTT) =$

9. Compare the calculations that you used for the fair coin and the unfair coin. How are the calculations similar? How are they different?

7.1b Homework: Probability Models

Suppose you rolled a dice and tossed a coin at the same time.

1. Create a probability model, BOTH a tree model and table, for rolling a die once then tossing a coin once.
2. How many total outcomes are represented by either the tree or table model ?
3. What is the sample space for the possible outcomes? List the sample space using set notation.
 $S = \{ _, _, _ \dots \}$
4. What is the probability for each outcome in the sample space? Write the probabilities both as a ratio and as a percent.
5. If you collected experimental data from rolling a die and then tossing a coin, would the calculated probabilities from the experiment match the theoretical probabilities? Why or why not?

Spiral Review

1. Rewrite the following part:part ratio as part:whole ratio.
a. The ratio of boys to girls in Gabrielle's family is 3:8.

2. Solve the following proportion equation: $\frac{x}{5} = \frac{4}{10}$

3. Simplify each.

a) $-6(-5)$

b) $-10 \cdot 31$

4. Kim had a bag with red, green, purple, yellow and orange marbles. The following table shows what color she drew each time.

Draw	1	2	3	4	5	6	7	8	9	10
Result	red	orange	purple	orange	orange	purple	yellow	green	red	green

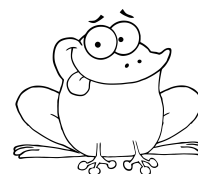
- a) Find the experimental probability of drawing a red marble.

- b) If there are 100 marbles in the bag, how many of them do you think are red? Justify your answer.

5. If $\angle N$ is vertical to $\angle M$, and $m\angle M = 98^\circ$ and $m\angle N = (6x + 2)^\circ$, the x must be _____.

7.1c Class Activity: Rolling Along

Can you roll your tongue? Some people can roll their tongue, some cannot. Approximately 1 out of every 3 people *cannot* roll their tongue.



Consider: Ria is doing a survey on the number of people with different genetic traits. She asks people, one at a time, if they can roll their tongue. Ria was surprised that she asked 5 people before she found someone who wasn't able to roll their tongue. **Does this mean the statement “approximately 1 out of 3 people cannot roll their tongue” must be false? Is it unusual that after surveying 5 people she did not find anyone who could not roll their tongue?**



To answer this question we can do a simulation of Ria's experiment.

Simulation – an experiment that models a real-life situation

Select a method to simulate a 1 out of 3 chance (die, slips of paper, software, etc.). Run the simulation until you get the “1 out of 3” chance you're looking for. For example, there is a 1 out of 3 chance of rolling a 1 or 2 with a six sided die. *One simulation is the number of times it takes to roll a 1 or 2 with a die.* Record a tally mark under the number of times it takes to get the 1 or 2 in the table below. Run the simulation 20 times recording your result each time. Once you've done your 20 simulations, compile your results with two other people so that you have 60 total simulations. Record the data in the table.

Number of attempts	1	2	3	4	5	6	7	8	9	10
Record a tally										
Combined Results (60 simulations)										
% out of 60 simulations										

1. Based on the combined trials, calculate the probability that it would take 5 or more attempts.
2. Were Ria's results unusual? Write a paragraph summarizing your conclusion, based on the simulation.

4. Suppose that 60% of students choose chocolate ice cream, 30% choose vanilla ice cream, and 10% choose strawberry ice cream. Create a simulation for the number of trials it takes to get a student to choose chocolate, vanilla and strawberry ice cream.

Law of Large Numbers – the long run relative frequency of an experiment, based on a large number of trials.

7.1c Homework: Finding Probability

The colors of M&Ms in a large bag are distributed according to the probabilities shown in the table:

Color	Brown	Red	Yellow	Green	Orange	Blue
Probability	0.25	0.25	0.20	0.10	0.10	?

1. Finish the table above by finding $P(\text{blue})$.
2. Suppose you draw an M&M out of the bag and record the color. List the sample space using set notation.
 $S =$
3. What is required in order to have a complete probability model?

Compound Probabilities of M&M Colors

4. Compute the following theoretical probabilities. Use the probabilities from the M&M table given above.

$$P(\text{red and yellow}) =$$

$$P(\text{brown, orange}) =$$

$$P(3 \text{ blues in a row}) =$$

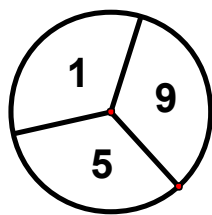
Simulation

5. Your favorite M&M's are red, so you want to create a simulation for modeling the drawing of red M&M's from a bag with the color probabilities as listed above. Describe a simulation. Remember: you are only trying to simulate drawing a red M&M.

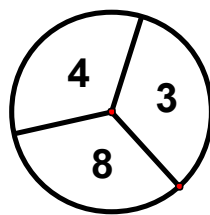
Spiral Review

1. If you flip a penny three times, what is the probability of getting two tails and one head in any order?
2. The scale factor $\triangle GEL$ to $\triangle HOP$ is $\frac{1}{6}$. If \overline{OP} is 30, what is the length of \overline{EL} ?
3. Daniel got 4 out of every 5 questions correct on a recent multiple choice test. If he got 64 questions correct, how many did he miss?
4. Find the sum or difference for each:
 - a. $\frac{5}{3} - \frac{3}{4}$
 - b. $-\frac{2}{3} + \frac{1}{4}$
5. Paul left a \$25 tip for the waiter at a restaurant. If the tip was 25% of the bill, how much was the bill?

7.1d Class Activity: More Models and Probability



Spinner A



Spinner B

Win the Spin!

Determine the probability that Player 1 wins the spin (highest number wins). Player 1 uses spinner A and Player 2 uses spinner B. Assume that the areas on each spinner are equal in size.

1. Create a probability model for the outcomes of the “Win the Spin” game, using a tree diagram.
2. How many possible outcomes are there? How do you know?
3. What is the probability of each of the outcomes? How do you know?
4. What is the probability that Player 1 will win? How do you know?
5. What is the probability that Player 2 will win? How do you know?
6. What is the probability of a draw (tie)? How do you know?
7. Is this a fair game? Why or why not?

8. If you were to play the game, your outcome would not necessarily match the probabilities above. Explain why this is true.

Odd or Even Game

For a different game Player 1 and 2 each spin once (Spinner A, then B) and add the numbers. If the sum is odd, then Player 1 gets a point. If the sum is even, then Player 2 gets a point.

9. Create a probability model for the outcomes of the “Odd or Even” game, using a tree diagram.
10. Use the tree diagram to figure out if the game is fair or not. Explain.
11. Use the rule for calculating compound probability to calculate the probabilities of the different combinations of the spins for Players 1 and 2. Show all your work.

$$P(\text{odd, even}) =$$

$$P(\text{odd, odd}) =$$

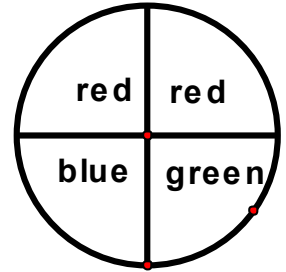
$$P(\text{even, odd}) =$$

$$P(\text{even, even}) =$$

7.1d Homework: Probability Models and Spinner Games

1. The spinner at right is spun twice. List the sample space for the possible outcomes from two spins. Use set notation. (Hint: there are 9 outcomes)

$S =$



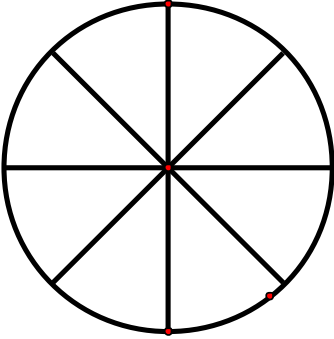
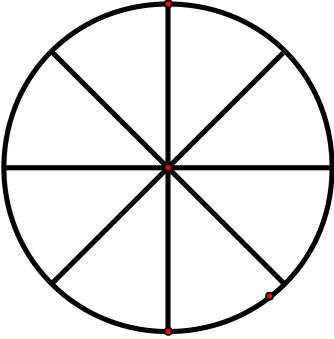
2. Are all the outcomes in the sample space equally likely? Why or why not?
3. How might you figure out the number of outcomes without making a list or diagram? Explain.
4. Create a probability model for the outcomes for spinning the spinner twice (organized list, tree diagram, or table) to show all possible outcomes and probabilities from two spins of the spinner.
5. Fill in the spaces below and make a conjecture about a rule for the number of possible outcomes for compound events.

If there are 3 possible outcomes in the first event, and 2 possible outcomes in the second event, then there will be _____ possible outcomes in the compound event.

If there are 5 possible outcomes in the first event, and 3 possible outcomes in the second event, then there will be _____ possible outcomes in the compound event.

If there are “ a ” possible outcomes in the first event, and “ b ” possible outcomes in the second event, then there will be _____ possible outcomes in the compound event.

6. Examine the model you created above to determine these probabilities.
 - a. $P(\text{red, red})$
 - b. $P(\text{one red and one green, in any order})$
 - c. $P(\text{blue, red})$ Note: blue must come first.
7. Create a probability game for each spinner. Design the spinners to make one game fair and the other unfair. Write the rules to tell how to play each game.

 <p style="margin-top: 10px;">Game 1</p>	 <p style="margin-top: 10px;">Game 2</p>
<p>Game 1 Rules</p> 	<p>Game 2 Rules</p>

8. Explain the probability of winning each game and why the game is fair or unfair.

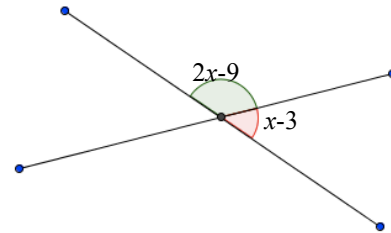
Extra Challenge: Spinners for Math Day

Howard is in charge of the Spinner Game for the Math Fair. There will be about 300 people at the fair and he believes everyone will buy a ticket to play the Spinner Game. The school wants to raise money for some math software. Spinner tickets cost \$1. Winners of the spinner game will be given cash prizes. Hal wants to make \$100 profit from the game.

Design a plan that should net Hal \$100 from the Spinner Game. Be sure to show the spinners you would recommend and the rules you think would work. Explain why your spinners and rules make sense for this context.

Spiral Review

1. Simplify each:
 - a. $-6(-5)$
 - b. $-16(-3)$
 - c. $-10 \cdot 31$
 - d. $-89 + (-6)$
2. Kelsey puts each letter of her name on a piece of paper. What is the probability that she will draw a K and an E in any order?
3. Order the following rational numbers from least to greatest. $\frac{12}{3}, -4.5, -\frac{14}{3}, -0.94,$
4. Matthew wants a bigger cage for his bearded dragon. He wants to length to be 2 inches less than twice the width. If the perimeter of the cage should be 104 in, what dimensions should the cage be?
5. Use the diagram at the left to find the angle measures.



Long run relative frequency: the probability of an outcome obtained after many trials
Variable (the verb, not the noun): not consistent or having a fixed pattern; liable to change
Experimental Probability: the ratio of the number favorable outcomes to the total number of trials, from an actual sequence of experiments
Theoretical Probability: the probability that a certain outcome will occur, as determined through reasoning or calculation

7.1e Class Activity: Probability of a Kiss

When you toss a coin, it will either land heads or tails. That isn't very interesting. But suppose you toss a Hershey's Kiss in the air, and then observe how it lands. That is much more interesting.



The sample space for tossing a Kiss has two possible outcomes: $S = \{\text{base, side}\}$. "Base" means the Kiss landed on the flat base, and "side" means the Kiss landed on its side.

What is the probability for each outcome? We don't know the answer. First we must do an experiment, and then calculate the experimental probability that the Kiss will land on its base, $P(B)$.

1. Define "long run relative frequency".
2. Make a guess for the probability that a Kiss tossed in the air will land on its base:
 $P(B) =$
3. Record the results of your experiment in the table after each toss and calculate the experimental probability after each trial.
4. Make a plot of your results by plotting the trial number against the experimental probability. Connect the points from trial 1, to trial 2, to trial 3, ... and end with trial 30.

Trial Number	Outcome (B or S)	# of times on Base (running total)	Experimental Probability
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
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30			

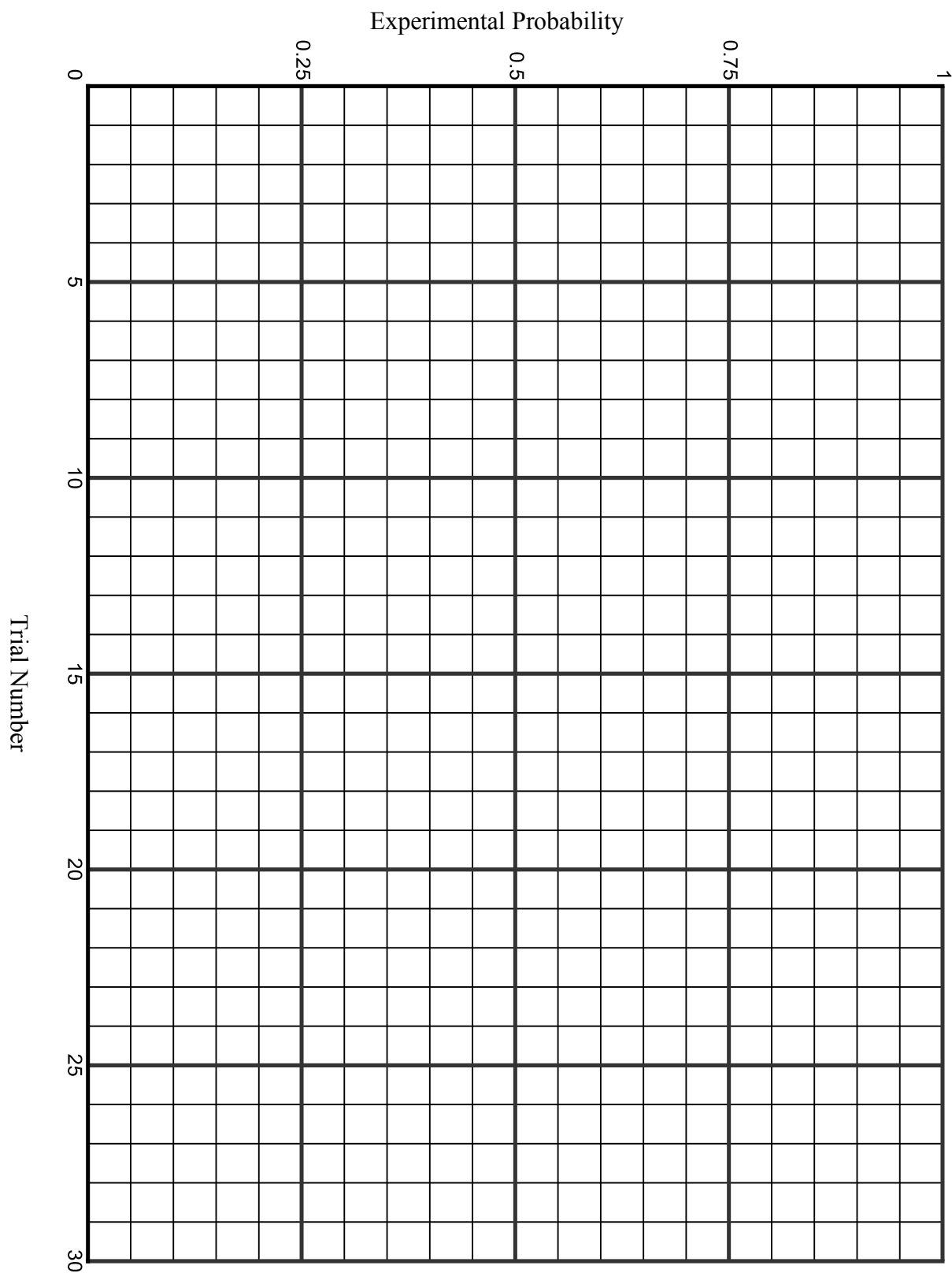
5. What is the experimental probability of a Kiss landing on its base after 30 trials for your experiment?
 $P(B) =$

6. Compare the value for the experimental probability at Trial 2, compared to Trial 20. Which value was closer to the final experimental value you found at Trial 30?

7. Examine the appearance of the plot. Why is the plot so variable at the beginning compared to at the end?

8. Why is it important to perform many trials in an experiment, and not just a few?

Probability of a Kiss: Graph your results below. Draw lines between the points when you are finished.



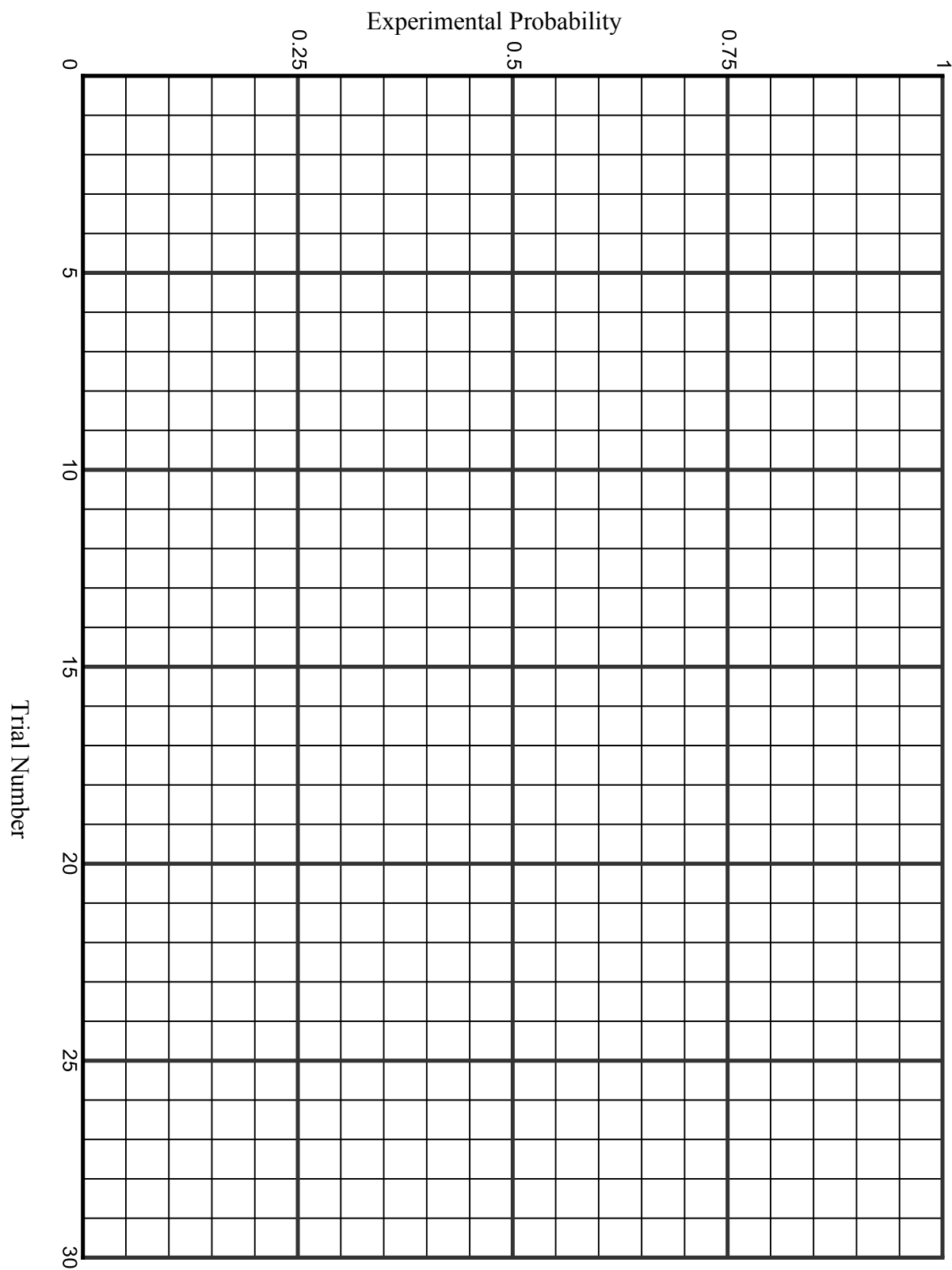
7.1e Homework: Experimental Probabilities

1. Choose an object that has two outcomes when tossed, such as a spoon (face up or face down) or a marshmallow (circular base or side.) Use the techniques from the class activity to find the experimental probability that the object will land on one of the sides.

Plot the data, using the same graph as you used for the class activity. However, plot the outcomes for the homework using a different color, and then label it.

Trial Number	Outcome (B or S)	# of times on Base (running total)	Experimental Probability
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			

Graph your results of # 1 below. Draw lines between the points when you are finished.



Spiral Review

1. Estimate by rounding to the nearest integer. $-3\frac{1}{3} \div -\frac{3}{4} \approx \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} \approx \underline{\hspace{1cm}}$
Is your answer an over estimate or under estimate, explain?
2. David is in a submarine at 200 feet below sea level. Casey is above him in a helicopter at 5,900 feet altitude. How far apart are David and Casey?
3. Find each difference without a model.
 - a. $16 - 29$
 - b. $-2 - (-8)$
 - c. $5 - (-3)$
 - d. $-90 - 87$
4. Given the measures of the following angles, identify the possible angle relationship(s).
 - a. $m\angle UTS = 9^\circ$ and $m\angle DCB = 9^\circ$
 - b. $m\angle BCD = 71^\circ$ and $m\angle PSV = 109^\circ$
 - c. $m\angle PSV = 60^\circ$ and $m\angle GFE = 30^\circ$
5. A baby toy has rings with a radius of 3 inches. What is the circumference of the rings?

7.1f Optional Class Activity: Free Throws or Monty Hall

Activity 1: Free Throws—Will We Win?

You are the coach in the final state basketball championship game; your team is losing by one point. The other team has the ball. You have one of your players foul the person with the ball from the other team. The player from the other team will now shoot two free throws. After the free throws, there will only be enough time to quickly get off a three point shot. The player at the foul line has a free throw percentage of 60%. Your best three point shooter is only a 25% shooter at any three point range.

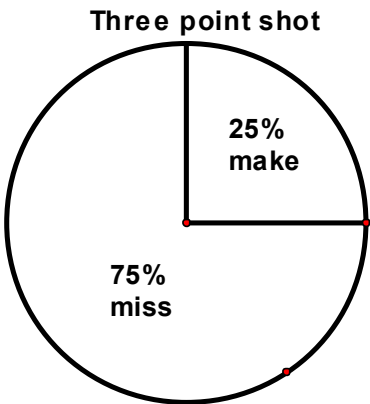
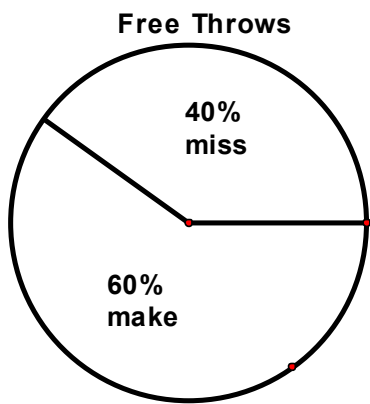
Your task is to run a simulation to better understand the probability of winning the game.

Using spinners, simulate the situation (spinner can be made out of paper or use internet based spinners, e.g. <http://www.mathsisfun.com/data/spinner.php>).

- Spin and record the result of the spin for each of the two free throws.
- Spin and record the result of the one three point shot.
- Record if you would win, lose, or tie.
- Repeat this process 10 times.

1. Based on your simulation, what are your chances for winning?
2. What is the theoretical probability for a win, loss and tie?

Free Throws	3-point Shots	Win, Lose, or Tie



Activity 2: The Monty Hall Question

In the game show Let's Make a Deal, Monty Hall would sometimes show three doors to a contestant. He then informs the contestant that a valuable prize is hidden behind one of the three doors. The contestant would be asked to pick a door. After the contestant chooses a door (without opening), Monty then removes from play one of the doors where the prize is NOT HIDDEN. There are now 2 doors remaining, one of which has the prize. The contestant has already chosen one of these two doors. At this point, Monty gives the contestant the option of switching doors or remaining with his/her original choice.

Which statement below do you think is true:

- The probability of getting the prize is greater by switching doors.
- The probability of getting the prize is greater by not switching doors.
- It doesn't matter.

a. What is your conjecture? Explain.

<http://betterexplained.com/articles/understanding-the-monty-hall-problem/>

- b. Run a simulation to see if there is a difference in the probabilities between staying with the same door or switching. Keep track of the results.
- “Monty” rolls a die out of the contestant’s view. If the die reads 1 or 2, then the prize (you can use a paper clip or a quarter to represent the prize) is placed behind Door 1. If the die reads 3 or 4, place the prize behind Door 2. If the die reads 5 or 6, place the prize behind door 3.
 - The contestant rolls a die. The roll of the die will decide which door the contestant chooses. Use the same numbers as above.
 - “Monty” removes from play one of the doors where the prize is NOT HIDDEN. The contestant is asked to remain with the original choice or switch.
 - For the sake of consistency, have the contestant REMAIN with the original choice (no switch) for a set number of times. Later, the contestant ALWAYS SWITCHES for an equal number of times.
 - “Monty” reveals where the prize is, and the recorder writes down the results in the appropriate column.
- c. What are your conclusions?
- d. Combine your results with those of the other groups assigned to this problem. What are your conclusions?
- e. Explain the results of the game simulation.

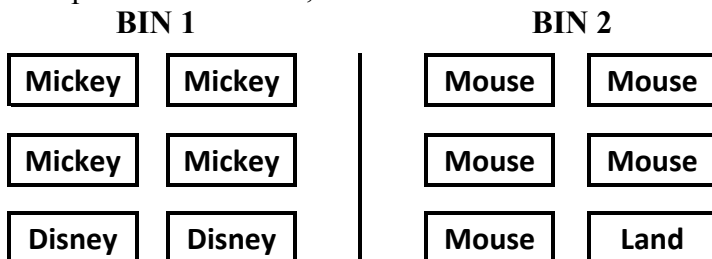
7.1f Optional Homework Project: Mickey Match

At the school fundraiser there are two games with prizes. For Game 1 the prize is a Disney decal. For Game 2 there are two different prizes, a t-shirt or a day pass for four to Disneyland. Game 1 costs \$1 to play while game 2 costs \$5 to play.

The game is played by picking cards without seeing what is written on the card. The cards are:



The cards are placed in two bins, as shown below:



Game 1: To win a Disneyland decal, you pick a card from the left bin. If you pick “Disney” you win a decal. What is the probability of winning a decal?

Game 2: To play game 2 you must draw one card from Bin 1 and one from Bin 2. Prize options:

Option 1: If you draw Mickey + Mouse, you win the t-shirt.

Option 2: If you draw Disney + Land, you win a day pass for four to Disney Land.

Option 3: If you draw Disney + Mouse or Mickey + Land, you go home with no prize.

Multiple Representations: What is the sample space? Create an organized list, a tree diagram, or a table to see all the possible outcomes.

3. What is the probability for winning a day pass for four to Disney Land?

7.1g Self-Assessment: Section 7.1

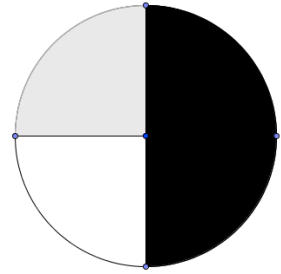
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Develop a probability model and use it to find probabilities of events.	I struggle to find probabilities of events.	Given a probability model, I can use it to find probabilities of events.	I can develop a probability model and use it to find probabilities of events.	I can develop a probability model and use it to find probabilities of events. I can show how my model represents the event.
2. Compare probabilities from a model to observed frequencies.	I don't understand the relationship between probabilities from a model and observed frequencies.	I know that probabilities from a model and observed frequencies may be different, but I struggle to explain why.	I can compare probabilities from a model to observed frequencies.	I can compare probabilities from a model to observed frequencies. I can explain possible sources of any discrepancies if applicable.
3. Represent sample spaces for compound events using various methods.	I struggle to represent sample spaces for compound events.	I can represent sample spaces for compound events using one of the following: organized lists, tables or tree diagrams.	I can represent sample spaces for compound events using organized lists, tables and tree diagrams.	I can represent sample spaces for compound events using organized lists, tables and tree diagrams. I can explain which choice would be best in a given situation.
4. Understand that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.	I don't understand how probability is the fraction of desired outcomes in the sample space.	I understand the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs, but I sometimes have trouble applying what I know.	I understand the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs, which can be written as a fraction, decimal, or percent and can use that knowledge in contextual problems.	I understand the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. I can compare this to probability of simple events.

Sample Problems for Section 7.1

1. For each of the following situations, create a probability model, showing possible outcomes. Then find the probability of the given event.
- a. Sylvia has a collection of books. She has 30 reference books, 18 nonfiction books, and 64 fiction books. Find $P(\text{fiction book})$.

- b. The spinner illustrated to the right is spun twice. Find $P(\text{white, black})$.



2. Don rolled a two on a fair twenty-sided die seven times out of 80 rolls. Would you expect this result? Why or why not?
3. Represent the sample space for each of the following events. If possible, use various methods for representing the same space.
- a. Sum from rolling a six-sided die twice
- b. Flipping a quarter four times
- c. Choosing an outfit out of a plaid, striped or solid shirt and jeans, khakis or shorts
4. Crysta puts each letter of her name on a piece of paper. What is the probability that she will draw a C and an A in any order?

Section 7.2: Use Random Sampling to Draw Inferences about a Population

Section Overview: In this section students will be looking at data from samples and then making inferences from the samples to populations. Students will utilize graphs of data along with measures of center and spread to make comparisons between samples and to make an informal judgment about the variability of the samples. After examining the samples, students will actually make conclusions about the population.

It is important that students think about the randomness of a sample as well as how variations may be distributed within populations. These ideas are quite sophisticated. Activities within this section are designed to surface various ideas about sampling. Teachers, students and parents are strongly encourages (as always) to review the mathematical foundation for a more in-depth examination of the topics within this section.

Concepts and Skills to be Mastered

1. Use random sampling to obtain a sample from a population.
2. Understand that random sampling procedures produce samples that can represent population values.
3. Create appropriate plots of collected data to provide a visual representation of the samples.
4. Compare samples of the same size from a population in order to gauge the variation in the samples. Use this variation to form an estimate of range of where a population value might lie.
5. Make predictions about a population, based on the samples.

7.2a Class Activity: Getting Your Opinion

Your teacher will describe an amazing proposal and then ask which one you'd prefer.

Survey 1: Do you think that survey #1 represents the opinions of the class? Why or why not?

Survey 2: Do you think survey #2 represents the opinions of the class? Why or why not?

Survey 3: Do you think survey #3 represents the opinions of the class? Why or why not?

Survey 4: Do you think survey #4 represents the opinions of the class? Why or why not?

Survey 5: Do you think survey #5 represents the opinions of the class? Why or why not?

Which of the five surveys is likely to be most representative of the class opinion? Explain your reasoning.

Vocabulary that should be discussed during this lesson: population, sample, random sample.

7.2a Homework: Getting Your Opinion

1. You want to determine the most popular brand of shoe among students in your school. Which of the following samples would provide a good representative sample? Explain your choice, and why you didn't choose each of the others.
 - a. Ask every tenth student who comes into the school.
 - b. Ask ten of the girls on the basketball team.
 - c. Ask all the students in your class.
 - d. Ask ten of your friends.

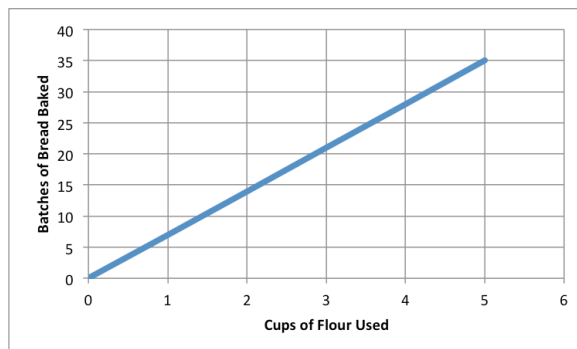
2. You are trying to find out who might come to an evening school play performance. Which of the following samples would provide a good representative sample of the community around the school? Explain your choice, and why you didn't choose the others.
 - a. Ask fifty people at the local grocery store.
 - b. Ask five adults from several randomly selected streets around the school area.
 - c. Call random names from the school telephone directory.
 - d. Place questionnaires at local stores with a sign asking people to fill them out and drop in a box.

3. **Inquiring Students Want to Know!** What are you and other students thinking about? Make a list of topics of interest to you and students in your school. For example: What college do students want to attend? Would students prefer starting school early in the morning and getting out early or starting school later in the morning and then staying later in the afternoon? Etc. Choose a question and design a sampling method for collecting data from 10 or more *randomly selected* students. Then collect the data. Write a paragraph describing the results, and why your method is or isn't a representative sample from the population.

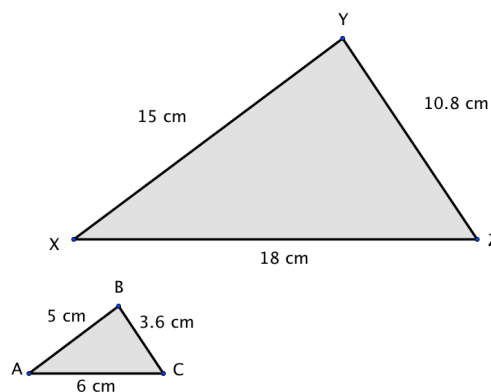
Spiral Review

1. Lisa owes her mom \$78. Lisa made four payments of \$8 to her mom. How much does Lisa now owe her mother?

2. Kaylee's Bakin' Kitchen sells fresh bread. The graph to the left shows batches of bread she can make and how much flour it takes. Is it a proportional relationship? If it is, estimate the unit rate?



3. What is the scale factor that takes $\triangle XYZ$ to $\triangle ABC$?

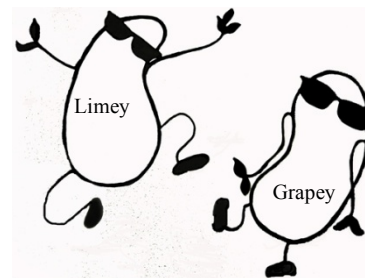


4. Dave is thinking of his favorite number. He tells you that it is one more than three times Emma's favorite number. The sum of their two numbers is 17. What are Dave's and Emma's favorite numbers?
5. Harry's football team loses 13 yards on one play. On the next play, the quarterback throws to a receiver for a gain of 13 yards. What was the change in their position?

7.2b Class Activity: Cool Jelly Beans!

7.2b Class Activity: Cool Beans!

The big election in Jelly Town is coming in November! The Jelly Beans living in Jelly Town (the bag) will cast a vote, either for Limey or for Grapey Bean. Up until now, Limey Bean and Grapey Bean have been tied in the polls. Limey Bean decided that if she wanted to win the election, she needed to do something drastic! So in October she came up with a new campaign slogan “I promise free sunglasses for every Jelly Bean!” Will her new slogan change the way the beans vote? She hired teams of experts to survey the population and answer this question.

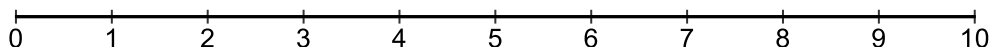


The student groups in class are the experts hired by Limey Bean.

# of green in each sample	0	1	2	3	4	5	6	7	8	9	10
Tally Marks											
Total											

Binomial calculator can be found at <http://stattrek.com/online-calculator/binomial.aspx>

Create a dot plot with your group's data.



Now that Limey Bean is using the new campaign slogan, there are three possibilities for election results:

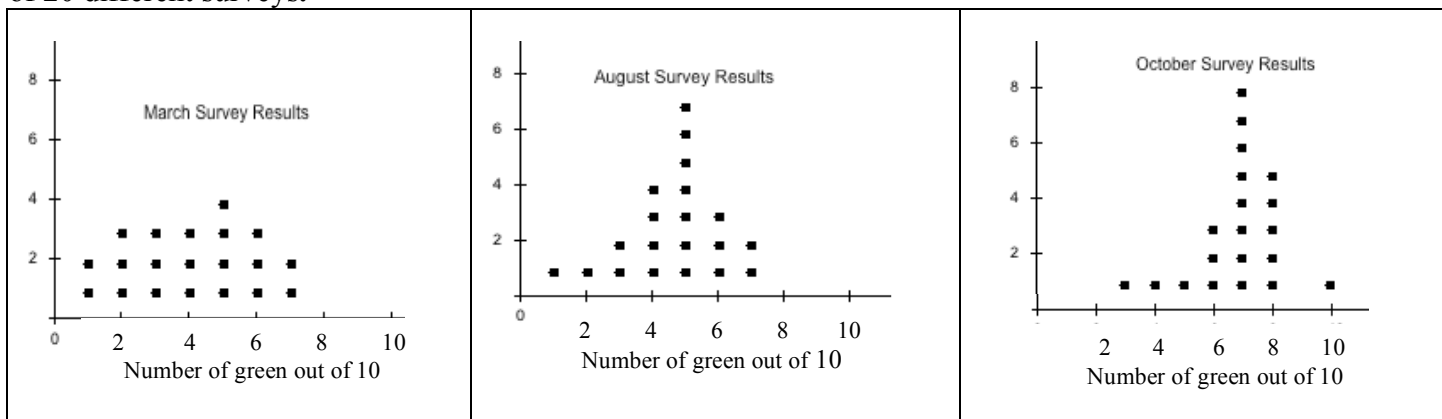
- The new slogan may have made no difference, Limey Bean and Grapey Bean could still be tied.
- The new slogan may have backfired, so that voters now prefer Grapey Bean.
- The new slogan may have worked as Limey Bean hoped, so that voters now prefer her over Grapey Bean.

- Which of the three possibilities (a, b, or c) do your samples support? Justify your answer using your team's survey results.

2. Consider your team's survey results. How many of the samples had more votes for Limey Bean? How many samples showed a tie? How many showed more votes for Grapey Bean?
3. Based on your samples, find the percent of surveys where Limey Bean had the most votes. Do you think you have enough evidence to declare the winner? Explain why or why not.
4. How variable were the results of your samples? In other words, what was the highest number of green beans recorded from any survey, and what was the lowest number of green beans recorded from any survey?
5. Based on your answer above, is it possible that Limey Bean will lose the election? Is it probable
6. Summarize who you think will win the November election, and why.

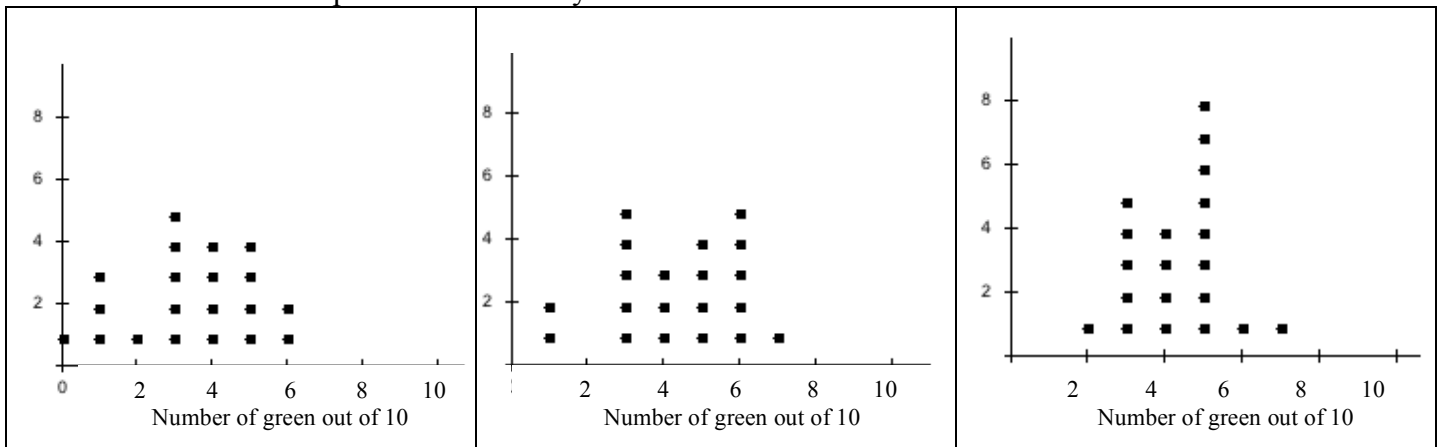
7.2b “Cool Beans!” Homework

The campaign in Jelly Bean Town actually began in March with the election in November. The plots below represent surveys taken during the election process in March, August, and October. Each plot shows the results of 20 different surveys.



1. In the March surveys, circle the dots on the graph where Grapey Bean and Limey Bean were tied. Based on the graph, who is ahead in the campaign in March? Explain your answer.
2. In the August results, circle the dots on the graph where Grapey Bean and Limey Bean were tied. Based on the graph, who is ahead in the campaign in August? Explain your answer.
3. In the October results, circle the dots on the graph where Grapey Bean and Limey Bean were tied. Based on the graph, who is ahead in the campaign in October? Explain your answer.
4. Based on the plots, is it possible that Grapey Bean could be ahead in the campaign in October? Explain.
5. Grapey Bean isn't going to let Limey Bean win the election based on a catchy slogan! In the week before the election Grapey decides to fight back by promising “More Coolness, Less Darkness!” Grapey Bean quickly recruited several expert survey teams to sample the Jelly Bean Town population, in hopes that the new slogan will turn the tide back in Grapey's favor.

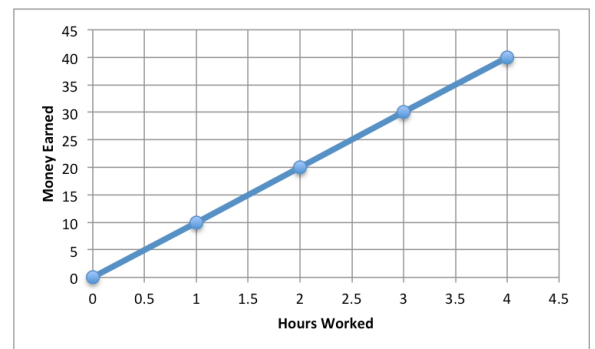
After advertising Grapey’s new campaign slogan, the three different survey teams gathered data and plotted their results. There is one plot for each survey team.

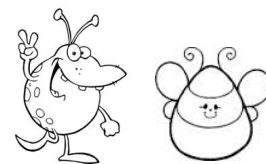


Was Grapey Bean’s slogan successful? Will Grapey win the election now? Use the combined results from the 3 teams’ surveys to justify your answer

Spiral Review

- $-1 \times -3 \times -6$
- Show two ways one might simplify: $2(3 + 4)$
- Convert 0.37 to a percent.
- Art’s long jump was 7 feet shorter than Bill’s. Together they jumped 41 feet. Write and solve an equation to find how far they each jumped?
- Examine the graph to the right showing how many hours worked versus how much money was made. Explain what the point (2, 20) means in context of the situation and the unit rate.





Critter World

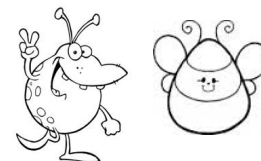
1	2	3
4	5	6
7	8	9
10	11	12



1	2	3
4	5	6
7	8	9
10	11	12

7.2c Class Activity (Optional): Critter Sampling

Your space ship has been orbiting a new flat planet full of life. You are a member of a group of scientists who has been sent to study the flat planet. Your job as an alien biologist is to gather data about the critters, analyze the data, make plots and provide a summary about the area you will study.



The big question is: how is the critter population in your world similar or different from other worlds?

Materials:

Quart sized bags of critters

Sheets of paper, rulers, pencils, tape, scissors, pieces of butcher paper to display graphs

Follow the instructor's directions for setting up the critter worlds. The bags of mixed food items represent different populations of critters. Give your world a name.

1. Open the bag of critters and sprinkle them evenly into the world. Don't use your hands to arrange the critters, just sprinkle them about. Spacing between critters does not have to be exactly equal. If needed, shake the world a bit or stir it with the eraser end of a pencil so that there are critters in every rectangle. Once you are done, hands off! Be careful to not shake or knock the world. (Don't eat the critters until the activity is over!)
2. Random Sampling: Each student in the group will randomly select a rectangle to study. To perform a random selection, each group should write the numbers 1-12 on similar sized pieces of paper and then place the numbered slips of paper into a container, mix well, and have each student draw a slip without looking. Some areas of the "world" will not be selected.
3. How should you count critters that are partway between two rectangles? Make a group decision.
4. Count the critters in your rectangle. Record the data in the table.

Critter Description or sketch	Critter 1:	Critter 2:	Critter 3:	Critter 4:	Critter 5:	
How many?						Total =
Percent of total?						Total = 100%

- Using the table data, create a graph showing the frequencies of each of the critter types in your sample. (Think: you will be comparing your graphs to others in the group. Why might it be better to use percents rather than counts to make the graph?)
- Scientists use data from samples in order to make conclusions about the world. Compare your graph to the graphs made by the other members of your group. Using the data from your samples, come to an agreement on an *estimate* for the total number of each type of critter in your world, and for the percent of each type of critter.

World Population Estimate (count)	Critter 1:	Critter 2:	Critter 3:	Critter 4:	Critter 5:	Total critter estimate =
World Population Estimate (percent)						Does your percent total = 100 %?

- Describe the method your group used to find the estimates of the world population.
- Within your group, decide how well your samples represented the world population. Explain, using complete sentences, any problems that you might have observed with how the samples may have misrepresented the population.

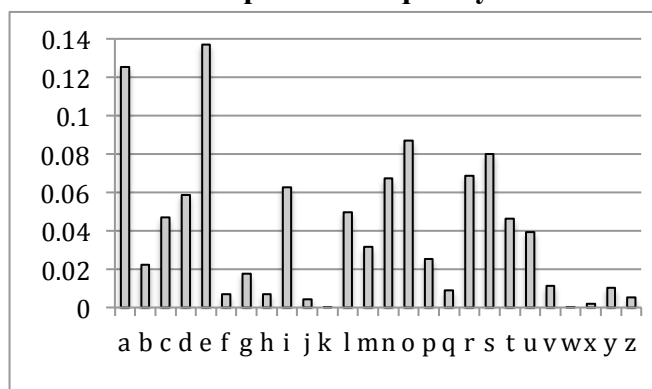
9. Create a graph of the estimate for the frequencies of the critters in your world. Post this graph in the room. You can eat your critters while all the other groups are posting their graphs.

10. Every world (bag) studied today has at least one other world with a similar population. Look at all the graphs posted by the different groups. Use the graphs to see if you can find the matching sets of worlds. Verify with your instructor to see if you were right!

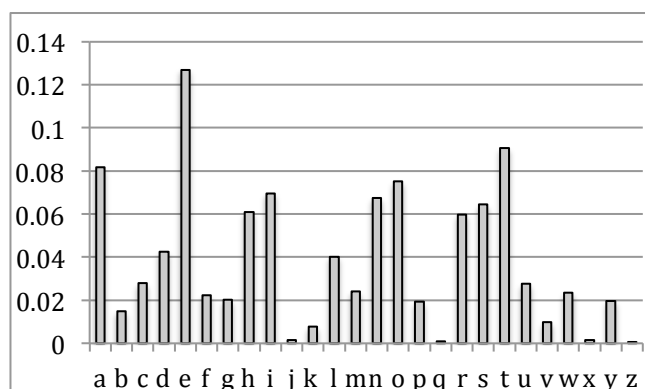
11. There is variability in between all the samples taken by students. What is “variability” and why did it make matching the worlds challenging?

12. Why is it important to use random sampling and not just *choose* a rectangle to use as your sample? Explain why this would create a problem.

7.2c Homework: Alphabet Frequency



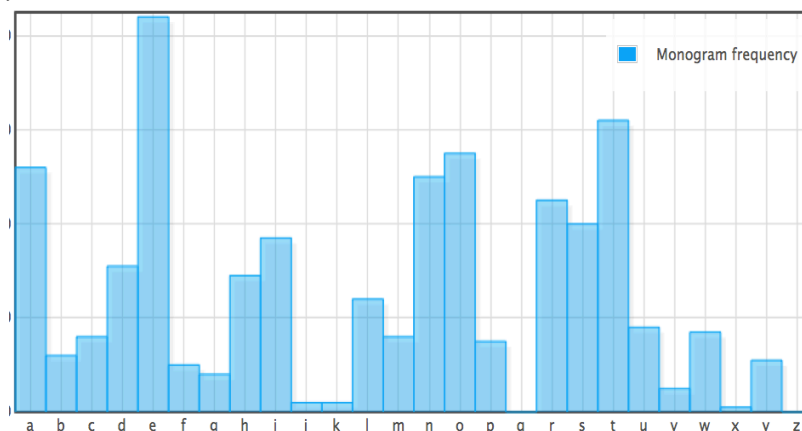
Graph A



Graph B

The two bar graphs above represent the frequency that letters occur in two languages, one graph represents the English language, the other represents the Spanish language.

1. Which 3 vowels and 3 consonants occurred most frequently in the language represented by Graph A? Write the letters in order from most frequent to least frequent.
2. Which 3 vowels and 3 consonants occurred most frequently in the language represented by Graph B? Write the letters in order from most frequent to least frequent.
3. Use the graph, estimate the frequency of “a” in each language. Find the difference between the two.
4. The most frequent word in the English language is the word “the”. Based on that hint, which graph, A or B, represents the frequency of the letters of the English language? Explain your choice.
5. The bar graph below is a sample of the frequency of letters used in the first two paragraphs of the book “*Artemis Fowl: The Lost Colony*”, by Eoin Colfer, written in English. Since the graph is from a sample, the frequencies will vary a bit from the overall letter frequencies of the English language. Compare the book sample to Graphs A and B. Which graph is the book sample most similar to, A or B? Explain your choice.
The frequency plot was created on the weblink “Practical Cryptography” at: <http://practicalcryptography.com/cryptanalysis/text-characterisation/monogram-bigram-and-trigram-frequency-counts/>



7.2c Homework Extension: Cryptograms

Cryptograms are puzzles where a symbol or letter is substituted for the actual letter. Each of the Artemis Fowl books has a cryptogram at the bottom of the pages of the book, where symbols are substituted for letters, and readers are challenged to solve the hidden message in the cryptogram.

One way people find clues in cryptograms is by looking at letter frequencies. The three most common symbols (in order) from the book “Artemis Fowl: The Lost Colony” are shown below. Which letters do they likely represent?



Use letter substitution to try to solve this famous quote:

YJMIR SKCN MRKCDRMU; MRXS LXIKVX YKNPU.

YJMIR SKCN YKNPU; MRXS LXIKVX JIMZKQU.

YJMIR SKCN JIMZKQU; MRXS LXIKVX RJLZMU.

YJMIR SKCN RJLZMU; MRXS LXIKVX IRJNJIMXN.

YJMIR SKCN IRJNJIMXN; ZM LXIKVXU SKCN PXUMZQS.
--WJK-MHX

Hints:

I = c

R = h

S = y

L = b

Y = w

L = b

H = z

J = a

K = o

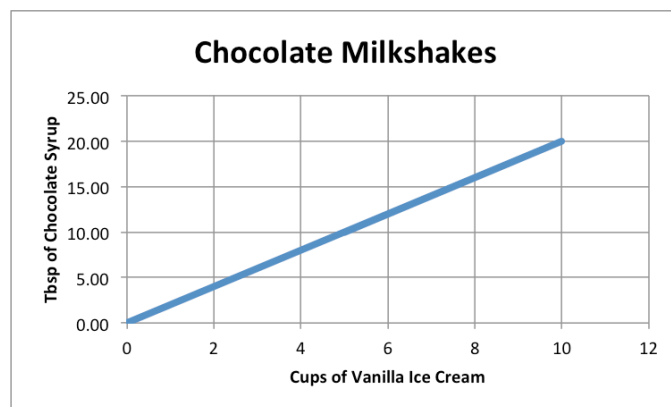
Spiral Review

1. Determine if the given information will make a unique triangle. Explain why or why not.
- a. Side lengths 10, 10, 19

b. Angles 51° , 9° , 120°

2. Solve: $3\frac{3}{4} + (-2\frac{1}{2})$.

3. Examine the graph to the right showing ice cream and chocolate syrup needed to make chocolate milkshakes. Is the relationship proportional? If so, write an equation to represent the relationship.



4. Marta is planting a garden as designed to the right. The width of the rectangle is 2 feet. A semicircle is attached to the width of the rectangle. How long should the length of the rectangle be if the total area is 36.56 feet?



5. Two angles are complementary. One is 48 degrees more than twice the other angle. What are the two angles?

7.2d Self-Assessment: Section 7.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Use random sampling to obtain a sample from a population.	I know what a random sampling is, but I don't know how to use random sampling to obtain a sample from a population.	I can choose which procedure would produce random sampling from a population.	I can use random sampling to obtain a sample from a population. I can explain my procedure for obtaining a random sample.	I can use random sampling to obtain a sample from a population. I can explain why the procedure I used obtains a random sample of a population.
2. Understand that random sampling procedures produce samples that can represent population values.	I don't understand what random sampling is.	I can use random sampling, but I don't understand how that represents the population.	I understand that random sampling procedures produce samples that can represent population values.	I understand and can explain how random sampling procedures produce samples that can represent population values.
3. Create appropriate plots of collected data to provide a visual representation of the samples.	I can't create a plot of collected data.	I can create a plot of collected data, but it doesn't seem to provide a good visual representation of the samples.	I can create a plot of collected data. It is a good visual representation of the samples.	I can create a plot of collected data. I can explain why it is an appropriate plot that provides a visual representation of the samples.
4. Compare samples of the same size from a population in order to gauge the variation in the samples. Use this variation to form an estimate of range of where a population value might lie.	I struggle to compare samples in order to gauge the variation in the samples.	I can compare samples of the same size from a population in order to gauge the variation in the samples, but I struggle to use this variation.	I can compare samples of the same size from a population in order to gauge the variation in the samples, and I can use this variation to form an estimate of range of where a population value might lie.	I can compare samples of the same size from a population in order to gauge the variation in the samples, and I can use this variation to form an estimate of range of where a population value might lie.
5. Make predictions about a population, based on the samples.	I struggle to make predictions about a population, based on the samples.	I can make predictions about a population, based on the samples, but I'm very unsure of my predictions.	I can make predictions about a population, based on the samples.	I can make predictions about a population, based on the samples. I can write a justification for my prediction.

Sample Problems for Section 7.2

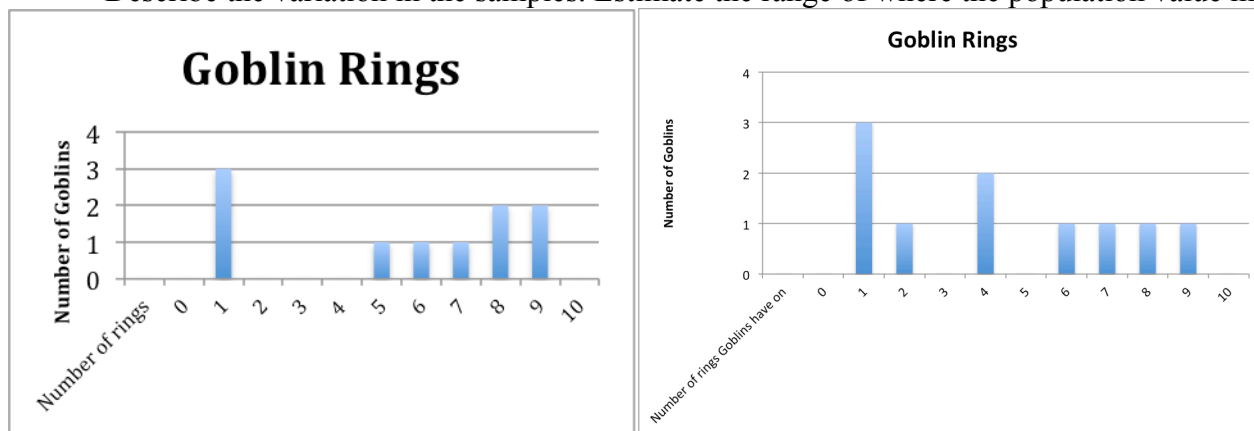
1.
 - a. Choose the procedure that would produce a random sampling in the following situation:

A car insurance company wants to know how many miles people drive each year.
 - Ask the teachers at your school how much they drive each year.
 - Call every 100th name in the phone book and ask how much they drive each year.
 - Ask truck drivers how much they drive each year.
 - b. Describe a procedure that would produce a random sampling in the following situation. Explain why your procedure will produce a random sampling.

Your school is choosing a new mascot. The principal wants the students' opinions.
2. Explain how your random samples in question 1 will represent the population.
3. Belle surveyed her classmates on how many donuts they eat in a month. The following table shows their responses. Make a visual representation of the data.

6	6	6	5	15	19	0	0	0
31	2	3	4	6	6	2	8	8
4	4	4	4	5	5	5	15	15
5	9	2	45	1	1	1	0	5
20	21	25	20	7	7	20	6	7

4. Compare the following two visual representations of how many rings some goblins are wearing. Describe the variation in the samples. Estimate the range of where the population value might lie.



5. Chloe is having a sale on rings in her store. Using the data from the charts in question 4. How many rings should Chloe sell in a set? Explain your reasoning.

Section 7.3: Draw Informal Comparative Inferences about Two Populations

Section Overview: In this section students calculate measures of center and spread from data sets, and then use those measures to make comparisons between populations and conclusions about differences between the populations.

Concepts and Skills to be Mastered

1. Make comparisons of data distributions by estimating the center and spread from a visual inspection of data plots.
2. Compare two populations by calculating and comparing numerical measures of center and spread.
3. Calculate the mean absolute deviation (MAD) as a measure of spread of a population. Measure the distance between the centers of two populations of similar variability using the MAD as the unit of measure.

7.3a Class Activity: Viva la Diferencia! (Celebrate the differences!)



How do female and male populations compare? With a partner, choose a question below to compare female and male responses. Choose a question that you believe you will find a difference between populations.

- a. How many letters are in your first, middle, and last name (total)?
- b. How many states can you list in 30 seconds?
- c. How many pens or pencils did you bring to class?
- d. How many buttons do you have on the clothes you are wearing right now? Include your pants buttons.
- e. How many words can you write in 30 seconds that start with the letter “g”?
- f. How many minutes does it take you to travel to school in the morning?
- g. How many pets do you have?
- h. What is the length of your shoe (in centimeters)?
- i. How many hours of television do you watch per week?
- j. How tall are you (in centimeters)?
- k. How long is your hair (in inches)?

Our Question:

Quietly go around the room and record the responses to your question in the table below. When a student asks you their question, you should also ask them your question. Continue until you have at least 10 female responses and 10 male responses.

When you have finished gathering your data, go to your teacher to have the die determine how you will display it.



Boys	Girls

- If the die roll is even, construct a **histogram** to show male and female results.
- If the die roll is odd, construct a **dot plot** to show male and female results.

1. Does the male data or female data have a larger measure of spread? Explain your reasoning.
2. Find the centers of the data for males and females. Which data has the higher center, male or female?
3. Are there any data points that you would consider to be outliers?
4. What conclusions do you draw from the comparison of males and females for your question? Write three to four sentences about your conclusions.

Review your work. Prepare to present your data to the class.

Create a poster that has the following:

1. Your question
2. A table of your data
3. Histograms or dot plots of the data (determined from the all-knowing die)

7.3a Homework: Review Measures of Center

REVIEW FROM 5th and 6th GRADE:

MODE: The data that occurs with the greatest frequency, or “the most”. The mode is an indicator of the shape of a distribution; it is not a measure of center.

MEAN: The mean is a measure of center. To find the mean of a set of data, add all the values together, then divide by the number of values in the data set.

The mean of 18, 6, 0, 22, 5, 19, 7 is calculated by: $\frac{18+6+0+22+19+7}{6} = \frac{72}{6} = 12$

MEDIAN: The median is a measure of center. To find the median of a set of data, arrange the data in order from least to greatest. If there is an odd number of values, the median will be the middle value. If there are an even number of values, the median will be the midpoint between the values in the middle.

Example 1: Find the median of 33, 35, 10, 19, 7, 0, 0, 6, 7. Arrange in order: 0, 0, 6, 7, 7, 10, 19, 33, 35
There are 9 data points. The middle (median) value is the 5th one, which is 10.

Example 2: Find the median of 14, 6, 8, 42, 6, 11. Arrange in order: 6, 6, 8, 11, 14, 42
There are 6 data points. The median is the midpoint between 8 and 11, so the median = 9.5.

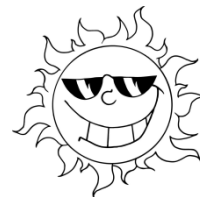
The midpoint between two data points can be found by finding the mean of the two points. $(8 + 11)/2 = 9.5$

Find the mean for the following data sets:

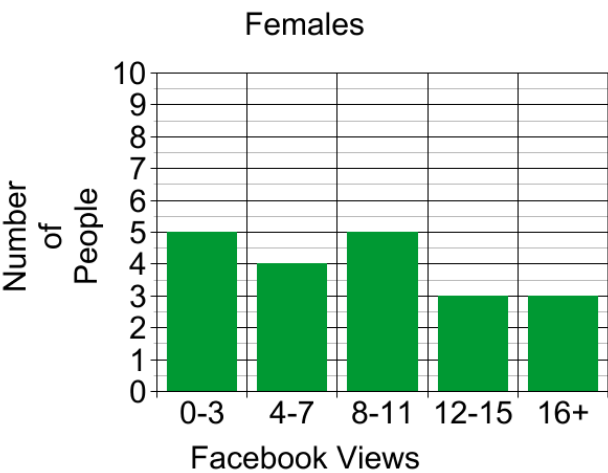
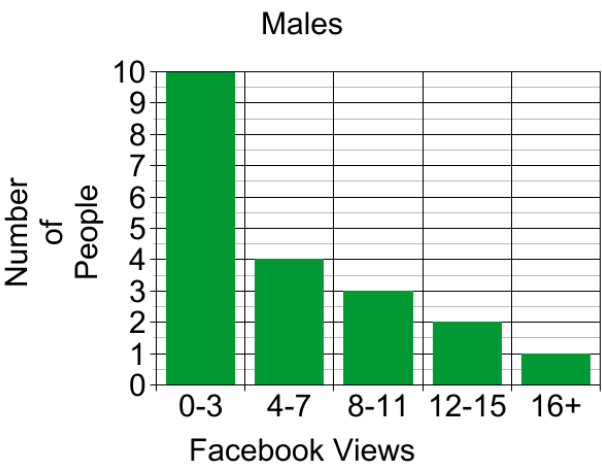
1. 2, 6, 1, 8, 10, 2, 3, 6
2. 24, 14, 8, 9, 6, 5, 18, 10, 16, 22

Find the median for the following data sets:

3. 12, 8, 7, 6, 9, 5, 1, 2, 3
4. 5, 1.6, 3, 8, 7, 11, 15.5, 18, 20, 11
5. A survey was conducted where respondents gave their favorite summer temperature (in degrees Fahrenheit). The results are as follows: 65, 76, 64, 78, 72, 68, 73, 72, 71, 68, 64, 85, 80, 90. Find the mean temperature from the survey. Round to the nearest degree.



20 males and 20 females were asked to approximate the number of times that they viewed Facebook each day. Histograms for the data are shown below.



6. Based on those that were surveyed, which group had a greater median, the boys or the girls? Explain your answer.
7. Why would mode not be a good measure of center for the female data distribution?
8. Create your own dot plots below that follow these rules:

Rule #1: Dot Plot #1 must have a larger spread

Rule #2: Dot Plot #2 must have a greater measure of center

DOT PLOT #1

DOT PLOT #2



Spiral Review

1. Simplify the following expressions.

$$-6\left(\frac{1}{5}a - \frac{1}{6}\right)$$

$$\frac{1}{3}\left(-7 - \frac{1}{6}a\right)$$

2. Find each sum, difference, product, or quotient:

a. $-4 + -7 =$

b. $3 - 10 =$

c. $-9(9) =$

d. $\frac{-32}{-8} =$

3. Solve and graph the following inequalities:

a. $6x - 1 < 17$



b. $9 \geq -4x - 7$



4. There were 850 students at Vista Heights Middle School last year. The student population is expected to increase by 20% next year. Draw a model to find what the new population will be.

5. Shawn surveyed his coworkers on how many times they eat out in a month. The following table shows their responses. Make a visual representation of the data.

36	13	19	24	0	12	35	0	30
31	26	3	16	7	27	9	34	27
25	1	35	16	18	17	27	19	6
27	19	11	27	19	26	25	20	23
30	18	26	16	0	0	20	20	18

7.3b Class Activity: The Glorious Mean and Median

Michael Jordan was a professional basketball player in the NBA for 15 years. He is frequently mentioned as the greatest basketball player of all time. He played for the Chicago Bulls team for most of his basketball career. He retired from the NBA in 2003.

The 1997-98 season is one of the years that the Chicago Bulls won the NBA championship. Below is a list of points scored by Chicago Bulls players, from team members who played over 40 games in the season.

	Chicago Bulls	1997/98
1	Michael Jordan	2357
2	Toni Kukoc	984
3	Scottie Pippen	841
4	Ron Harper	764
5	Luc Longley	663
6	Scott Burrell	416
7	Steve Kerr	376
8	Dennis Rodman	375
9	Randy Brown	288
10	Jud Buechler	198
11	Bill Wennington	167
	TOTAL:	7429

The Toronto Raptors basketball team came in last in their division in the 1997-98 season. Below is a list of points scored by team members.

	Toronto Raptors	1997/98
1	Kevin Willis	1305
2	Doug Christie	1287
3	John Wallace	1147
4	Chauncey Billups	893
5	Charles Oakley	711
6	Dee Brown	658
7	Gary Trent	630
8	Reggie Slater	625
9	Tracy McGrady	451
10	Oliver Miller	401
11	Alvin Williams	324
12	John Thomas	151
	TOTAL:	8583

1. Compare the data in the tables without doing any calculations (only using estimates). What interesting features do you see within each data set and between the two data sets?
2. Without calculating the actual values, which team do you think has a higher *points per player* mean? Why do you think so?
3. Calculate the mean and median number of points for each team.
 Bulls' mean = _____ Bulls' median = _____
 Raptors' mean = _____ Raptors' median = _____
4. Is the mean or the median a more accurate measure of center for the number of points scored by the Bulls? Explain your choice.
5. What would happen if you replaced Michael Jordan's 2357 points with 10,000 points? Find the new mean and median for the Bulls' points per player. Did either value change by much? Explain.
 Bulls' Mean = _____ Bulls' Median = _____

7.3b Homework: The Glorious Mean and Median

1. Ms. Parrish gave her students a math test and recorded their scores. The following is data for all 16 students in her class: 84, 91, 78, 94, 79, 82, 0, 98, 75, 0, 86, 91, 98, 77, 85, 90. Find the following values:

a. Mean _____ Median _____ Mode _____

- b. The two scores that are listed as zeros are from students who were absent. Re-calculate the measures of center without the zeros.

Mean _____ Median _____ Mode _____

- c. Explain the effect that the zeros had on the mean, and which values provide the better indication of the center with respect to students' scores.

2. Students tried out for the school play by memorizing a part. The students were rated on how well they performed and how much they were able to memorize. Their ratings were scored on a scale from 0-100. The scores for the 20 students are shown below.

14	79	68	88	84
96	74	94	98	89
97	88	80	94	67
100	98	88	74	88

- a. Sort the data from smallest to largest.

- b. Find the following values:

Mean _____ Median _____ Mode _____

- c. The first student on the list got a sick stomach during the tryouts and couldn't finish, so only scored a 14. The student was allowed to try again later that day, and now scores a 99.

What is the new mean score? _____

How much does the mean score change?

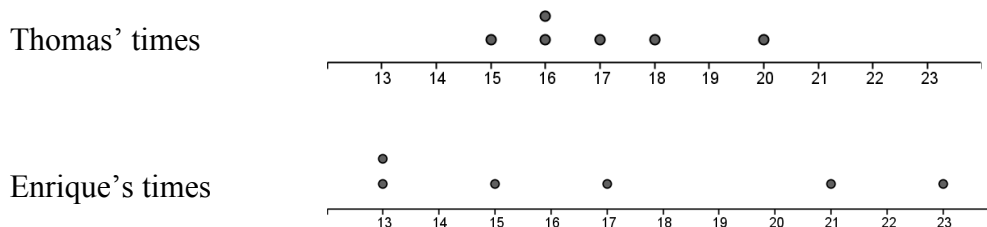
- d. Hamlet is calculating the new mean. Instead of replacing the re-do score of 99, Hamlet adds the re-do score to the end of the list, and then divides the sum by 20. What is result of Hamlet's calculation?
- e. Explain why Hamlet's calculation isn't really an average. What should Hamlet do to fix the calculation?

3. Ten members of the Ceramics Club meet after school to make pottery. A survey was taken to see how far (in city blocks) each member of the club had to travel to get home carrying their heavy pots. The results of the survey are the following distances:

12, 8, 14, 4, 16, 7, 4, 128, 11, 9

- a. Mean _____ Median _____ Mode _____
- b. Which would be the best measure of center for the data: mean, median, or mode? Explain your answer.
- c. Remove the outlier and find the mean of the remaining nine data values. New Mean _____

4. Thomas and Enrique run 2 miles every week and record their times (in minutes). Their data is recorded in the table below:



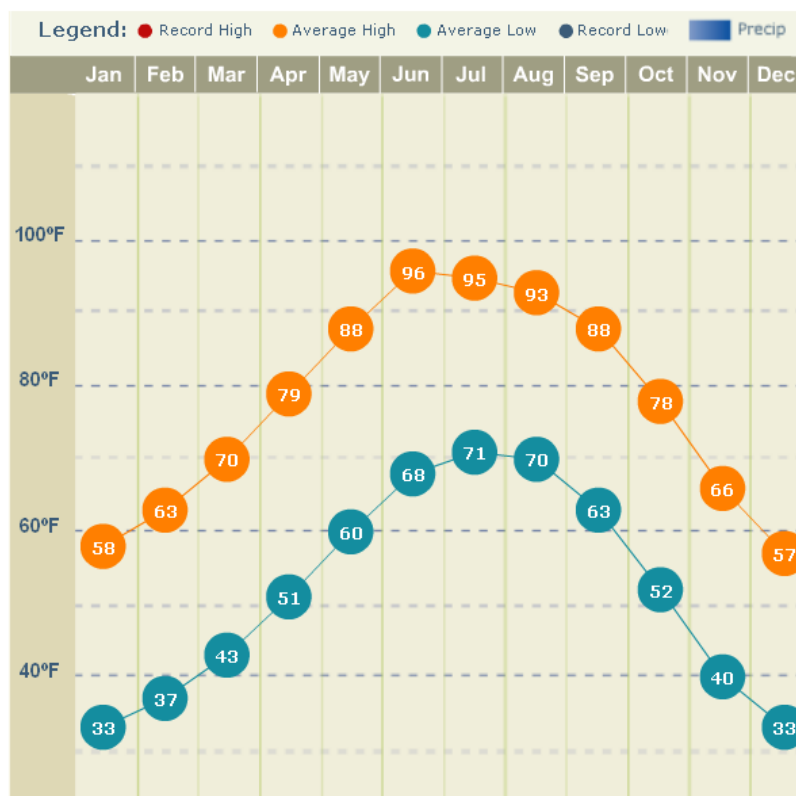
- a. Which runner has data showing the greatest spread? Explain using the plots and comparing data points.
- b. Which runner has the fastest mean time? The fastest median?
- c. If you wanted to select one of these runners to represent your class in a running competition, which one would you chose, and why?

5. Caitlin recently took a tour through Mt. Timpanogos Cave. The cave is at an elevation of 6,730 feet and the temperature inside the cave stays a steady 45°F throughout the entire year. Caitlin finds it interesting that the temperature in the cave stays the same year-round. She wonders if the average annual temperature of the air outside of the cave is same or different than the average temperature inside the cave. Caitlin collected data from a nearby community at a similar elevation and found the following typical monthly temperatures for January through December.

23.6 °, 27°, 34.2°, 42°, 50.3°, 59.1°, 66.4°, 65.6°, 56.6°, 46°, 33°, 24.5°

- a. Determine if the annual average temperature of the nearby community is the same as the temperature inside the cave. Explain your answer.
- b. The Carlsbad Cave system is in the northern Chihuahuan Desert in New Mexico. Steven's family was going on vacation to see the caves. Caitlin told Steven about how cave temperatures seem to be the same as the mean outside temperature. Steven thought he would use Caitlin's information to find out if he would need a coat while he is inside the Carlsbad Cave system. Steven looked up the mean daily temperatures of the nearest large city, which was El Paso, Texas. Although he wasn't able to find the averages he wanted, he found the chart below.

Use the chart of the average high and low temperatures of El Paso to find an estimate mean daily temperature of the outside air and help Steven decide if he will need a coat while he is in the caves.



Spiral Review

1. What property is shown?
 - a. $19 + 0$ and $0 + 19$ _____
 - b. $9 + 7 + 3$ and $9 + 3 + 7$ _____
2. Thomas flipped two quarters 80 times. He tails on both quarters 8 times. Would you expect this result? Why or why not?
3. Willy, Abby, and Maddy are playing golf. Willy ends with a score of -9. Abby's score is -10. Maddy scores +6. What is the difference between the scores of Maddy and Abby?
4. Beth's golf ball has a circumference of 4.71 in. What is the radius of her golf ball?

Bidziil is examining a scale drawing of the national park near his home. He wants to hike from the park entrance to a hot spring. On the map, the entrance and hot spring are 2.5 inches apart. There is a scale on the map: 1 in = 2.5 mi. How far will he have to hike?

7.3c Class Activity: Got the Point?

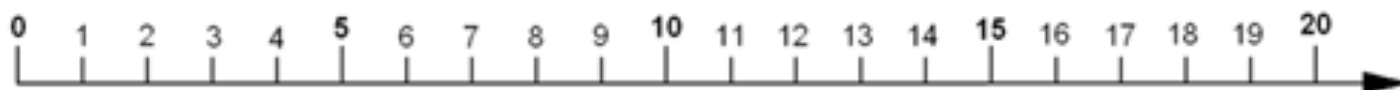
1. With a group of 4-5 students, record the number of pens and pencils that each of you have. Write down each of the numbers in the table provided below.
2. Find the mean of your data. What does the mean represent?

Mean Absolute Deviation (Review from 6th Grade): The mean absolute deviation (MAD) is a measure of variation in a set of numerical data. It is computed by adding the distances between each data value and the mean, then dividing by the number of data values.

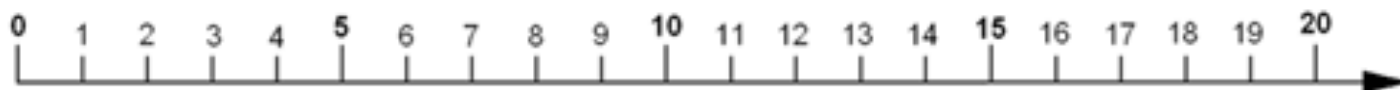
3. Find the mean absolute deviation for the data you collected.

	Number of pens/pencils	Mean All values in this column will be the same	number – mean All values should be positive.
Student 1			
Student 2			
Student 3			
Student 4			
Student 5			
			TOTAL:
			AVERAGE (MAD):

4. In problem #3 above, you found the mean absolute deviation for your group's data. On the number line below, mark the position of the mean. Put large bracket symbols [] above and below the mean at a distance of one mean absolute deviation.



5. Write down the mean and MAD for another group. Like you did above, mark the position of the mean on the number line below. Put large bracket symbols [] above and below the mean at a distance of one mean absolute deviation.



6. Is the MAD for your group higher or lower than the other group? What does it mean if a group has a higher MAD?

7.3c Homework: Analyzing Data Using MAD

EXAMPLE for Mean Absolute Deviation (MAD):

The MAD is a measure of the spread of data. The higher the MAD, the more the data is spread out. The table below shows the 6 fastest birds in the world and their maximum recorded speeds.

Animal	Maximum Recorded Speed (in mph)
Peregrine Falcon	242
White-throated Needletail	105
Eurasian Hobby	100
Frigatebird	95
Anna's Hummingbird	61
Ostrich	60



To find the MAD, first find the mean (average) speed of the birds by adding all of the data and dividing the sum by the number of values.

$$\frac{242 + 105 + 100 + 95 + 61 + 60}{6} = \frac{663}{6} = 110.5$$

Next, find the deviation (distance) from the mean for each bird. Recall that absolute value means you're looking for a "distance" between values and distance is always positive.

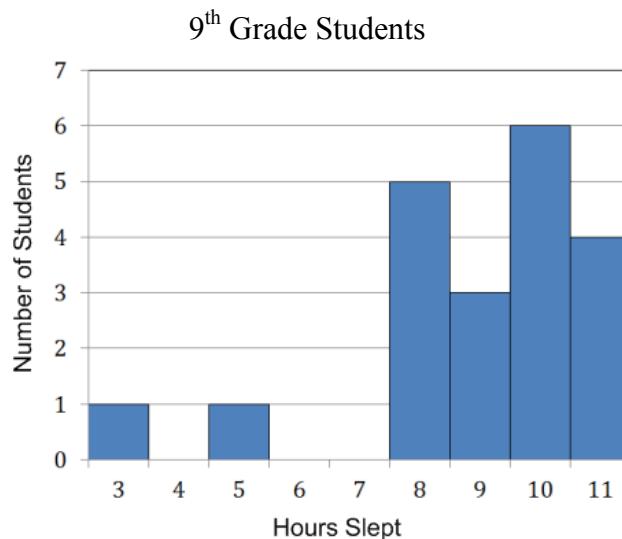
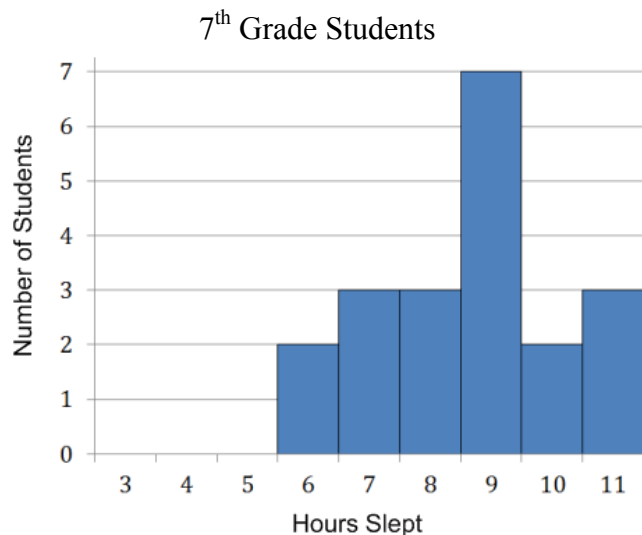
Finally, calculate the average of the deviations from the mean, known as the mean absolute deviation, or MAD.

	speed for each bird – mean	Deviation from the mean
Peregrine Falcon	242 – 110.5	131.5
White-throated Needletail	105 – 110.5	5.5
Eurasian Hobby	100 – 110.5	10.5
Frigatebird	95 – 110.5	15.5
Anna's Hummingbird	61 – 110.5	49.5
Ostrich	60 – 110.5	50.5
	Mean Absolute Deviation =	43.83

What does the MAD indicate? For this data, the MAD shows that average difference between each bird's speed and the mean is 43.83 mph.

Notice that the Peregrine Falcon's speed is the farthest from the mean. If you use the MAD as a unit of measure, anything that is 3 MAD from the mean is very unusual. The Peregrine Falcon is three MAD's from the mean. $(3 \cdot \text{MAD}) = (3 \cdot 43.83) = 131.5$. The Peregrine Falcon is unusually fast, even compared to the other 5 fastest birds in the world!

- Students in 7th and 9th grade were asked the number of hours they slept on non-school nights. Data from 20 students in each grade were randomly selected and the histograms for the data are shown below.



- Find the median of the sleep data for both the 7th grade students and the 9th grade students.

7th grade median: _____ 9th grade median: _____

- Without computing, would sleep data for the 7th grade students or the 9th grade students have a larger MAD? Explain your answer.



- Compare the two graphs. Using the graphs and your calculations, write a few sentences about the conclusions that can be made about the amount of sleep that these twenty 7th grade students get compared to the twenty 9th grade students.
- One statistic used in baseball is how many bases that players steal. This table shows the number of bases stolen by Ken Griffey Jr. and Rickey Henderson each year from 1990 – 2000.

Ken Griffey, Jr. aka “The Kid”

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
Stolen Bases	16	18	10	17	11	4	16	15	20	24	6

Rickey Henderson aka “The Man of Steal”

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
Stolen Bases	65	58	48	53	22	32	37	45	66	37	36

- Create dot plots or histograms to provide a visual comparison between the two sets of data.
- Which player had the highest measure of center for the number of stolen bases? Calculate, and explain your answer.
- Which player had the greatest spread for the number of stolen bases? Explain your answer by calculating the mean absolute deviation (MAD) for each player.
- Based on the data, which player would you say has the greater number of stolen bases for their entire career?

Spiral Review

1. Shawn surveyed his coworkers on how many times they eat out in a month. The following table shows their responses. Find the mean, median, and mode of the data.

Mean: _____

Median: _____

Mode: _____

2. Find the following quotients:

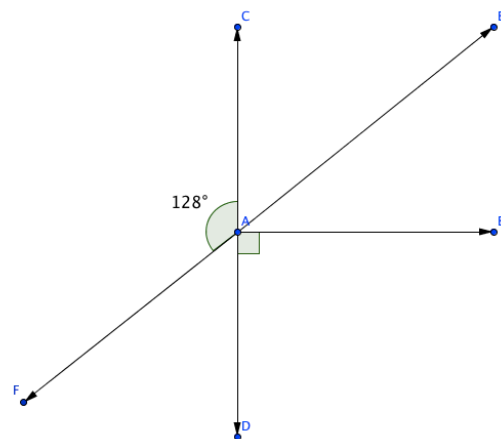
a. $\frac{3}{5} \div \left(-\frac{1}{8}\right)$

b. $\frac{-0.78}{-0.02}$

c. $\frac{-10}{4}$

3. In the diagram to the left, find the missing angles' measures:

angle	measure
$\angle BAE$	
$\angle CAE$	
$\angle DAF$	



4. Ms. Stanford lives in Alaska. When she leaves for work one wintry morning, the temperature is -7°F . By the time she comes home, the temperature has increased 12° . What is the temperature when she comes home?

5. Nellie's bedroom is triangular. She measures the walls as having the following lengths: 10 feet, 10 feet, and 20 feet. How can you tell that she didn't measure correctly?

7.3d Class Activity: NBA Heights

In this activity we will use the MAD to compare the spread of two populations.

Just how much taller are NBA basketball players than students?

You will compare the heights of 25 professional basketball players to the heights of members of your math class.



Your Height (centimeters) _____

Record your height, and the height of your classmates in the table below.

Basketball Player Heights (centimeters)	
181	204
184	205
186	205
190	206
192	208
194	210
196	210
198	211
199	213
200	214
201	215
202	221
203	

Student Heights (centimeters)		
1] _____	13] _____	25] _____
2] _____	14] _____	26] _____
3] _____	15] _____	27] _____
4] _____	16] _____	28] _____
5] _____	17] _____	29] _____
6] _____	18] _____	30] _____
7] _____	19] _____	31] _____
8] _____	20] _____	32] _____
9] _____	21] _____	33] _____
10] _____	22] _____	34] _____
11] _____	23] _____	35] _____
12] _____	24] _____	36] _____

2. Compare the typical height of students and basketball players:
 - a. Calculate the mean of each population. Show all calculations. Round to the nearest centimeter.

Basketball player's mean height = _____

Students' mean height = _____

- b. How far apart are the mean heights of basketball players and the students, measured in centimeters?

3. Calculate the spread of the student heights :

- a. In the table below, write down the heights of each member in your group. Use the class mean to calculate how much each student in your group varied from the mean.

Student Number	Height (cm)	Deviation From Mean height – mean

- b. With the direction of the teacher, record all the other groups responses in the table below.

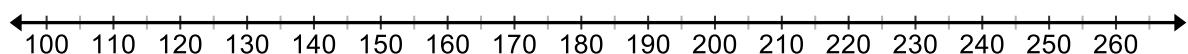
Height Deviations from the Class Mean height – mean		
1] _____	13] _____	25] _____
2] _____	14] _____	26] _____
3] _____	15] _____	27] _____
4] _____	16] _____	28] _____
5] _____	17] _____	29] _____
6] _____	18] _____	30] _____
7] _____	19] _____	31] _____
8] _____	20] _____	32] _____
9] _____	21] _____	33] _____
10] _____	22] _____	34] _____
11] _____	23] _____	35] _____
12] _____	24] _____	36] _____

- c. Calculate the mean absolute deviation (MAD) for the class. Round to the nearest centimeter.
4. The MAD for the heights of the basketball players is 8 cm. Use measure of center, spread and MAD to compare and contrast your class's height to that of the NBA team. Discuss your findings below:

7.3d Homework: NBA Heights

Someone who is more than 3 MAD's shorter (or taller) than the mean height is considered unusual. Use the basketball player height data to answer the following questions about some unusual basketball players.

1. Mark the mean height for the basketball players on the number line (you calculated the mean in the class activity, #2). Measure 1 MAD (8 cm) above and below the mean, and mark each with a "1". Then measure 2 MAD above and below the mean and mark that distance with a "2". Repeat for 3 MAD above and below the mean.



2. Tyrone "Muggsy" Bogues was a professional basketball player from 1987-2001. He was only 5 ft. 3 in tall, which is 160 centimeters. Place a mark on the number line for Muggsy's height, and estimate how many MAD's his height is from the mean.
3. Yao Ming played professional basketball from 2002-2010. He was 7 ft 6 in tall, which is 229 cm. Place a mark on the number line for Yao Ming's height, and estimate how many MAD's his height is from the mean.
4. Whose height was more unusual, compared to the basketball players in this data set, Muggsy's or Yao Ming's?

Spiral Review

1. Find the following quotients:

a. $-\frac{3}{7} \div \left(-\frac{1}{4}\right)$

b. $\frac{-1.7}{0.1}$

c. $\frac{93}{-5}$

2. Choose the procedure that would produce a random sampling in the following situation:

In compiling a brochure about Mathville, the city council wants to know how long people have lived in the city.

- Ask everyone in one neighborhood how long he or she has lived there.
- Call every 10th name in the phone book and ask how long he or she has lived there.
- Ask students at the university how long he or she has lived there.

3. Amie is making cookies for a math party. She has a triangular cookie cutter that is 3 in. on the base and 3.5 in. tall. She rolls her cookie dough into a square with a length of 15 in. About how many cookies will Amie be able to make?

4. Wayne buys a new tie. The tie is 20% off and then he has a coupon for an additional \$2 off. If Wayne pays \$46, how much was the tie originally?

5. The following list is the names of students in Ms. Jones' kindergarten class. Find the mean, median, and mode for the lengths of their names.

a. Mean: _____

b. Median: _____

c. Mode: _____

Jillian	Justina	Chris	Jodi	Casey
Carl	Nick	Bart	Diego	Doug
Amber	Pat	Kristie	Kaylee	Louise

7.3e Class Activity: MAD about M&M's

Below is data collected for the number of M&Ms in 30 small and 30 large bags of M&Ms. As you can see a small bag contains 1.69 oz. while a large bag contains 3.14, however the actual number of candies varies. The mean and MAD for both small and large bags are also provided. On the next page you will display these data.

M&M Data for Students

SAMPLE #	Small Bag (1.69 oz)	Large Bag (3.14 oz)
1	52	103
2	54	107
3	56	99
4	53	103
5	52	104
6	54	99
7	57	100
8	55	107
9	54	107
10	54	104
11	52	103
12	53	102
13	55	104
14	56	103
15	59	103

SAMPLE #	Small Bag (1.69 oz)	Large Bag (3.14 oz)
16	55	104
17	53	104
18	56	102
19	52	107
20	56	103
21	54	104
22	51	103
23	55	101
24	56	102
25	54	103
26	52	104
27	54	102
28	50	102
29	52	103
30	54	100

	Small Bag	Large Bag
Mean	54	103
MAD	1.47	1.48

In the space below, create a dot plot for the number of M&M's in the small bags of M&M's and create a dot plot for the number of M&M's in the large bags of M&M's.

1. How does the spread for the number of M&M's in a small bag compare to the number of M&M's in a large bag? Explain your answer.
2. On your dot plot below, circle the mean for each data set. What is the difference between the mean for a small bag of M&M's and the mean for a large bag of M&M's?

What is the difference between the centers as a multiple of the MAD value?

3. The mean absolute deviation for both data sets is approximately 1.5 M&M's. Approximate the number of MAD's between 54 and 103.

Estimate the following:

1. $35/4$
 2. $14/3$
 3. $35/6$
 4. $63/5$
 5. $120/8$
 6. $1850 / 15$
 7. $7/0.5$
 8. $654 / 4$
 9. $18 / 2.5$
 10. $12/1.5$
4. Calculate the number of MAD's between 54 and 103 by dividing the distance between the means by the MAD.

Distance between the means = _____ mean absolute deviations



5. Suppose that a 2.17 oz. bag of Skittles has a MAD of approximately 1.5. There are an average number of 57 Skittles in a bag.
- a. Measure the distance between the means of the 2.17 oz bag of Skittles and the 1.69 oz. bag of M&M's.
 - b. Rewrite your answer in part (a) using MAD as the unit of measure.

_____ pieces is the same as _____ MAD units
(Difference in means)

6. Suppose that a 14 oz. bag of Skittles also had a MAD of approximately 1.5 with a mean of 360 Skittles.
- a. Measure the distance between the means of the 14 oz. bag of Skittles and the 1.69 oz. bag of M&M's.
 - b. Rewrite your answer in part (a) using MAD as the unit of measure.

7.3e Homework: MAD About Precipitation

<http://www.currentresults.com/Weather/US/average-annual-state-precipitation.php>

1. Utah has an average precipitation of 12.2 inches per year, with an MAD estimated of 4.5 inches. Utah is ranked 49th for precipitation out of all the states. (Nevada is 50th.)
 - a. What does mean absolute deviation (MAD) measure, in terms of precipitation in Utah?
 - b. One of the driest cities in Utah is Wendover, getting only 4.1 inches of precipitation per year. How many MAD away from the mean is the precipitation amount for Wendover?
 - c. One of the wettest places in Utah is Alta Ski Resort, getting about 54 inches of precipitation per year. How many MAD away from the mean is the precipitation amount for Alta?
2. The state of Hawaii has an average precipitation of 63.7 inches per year, with an MAD estimated of 14 inches. Hawaii is ranked in 1st place for precipitation out of all the states.
 - a. One of the driest places in the state of Hawaii is Makena Beach, on the island of Maui. It gets about 17 inches of precipitation per year. How many MAD away from the mean is the precipitation amount for Makena Beach?
 - b. One of the wettest places in Hawaii is Hilo, on the island of Hawaii. It gets about 127 inches of rain per year. How many MAD away from the mean is the precipitation amount for Hilo?
3. How much larger is the MAD for precipitation in the state of Hawaii than the MAD for precipitation in Utah?
4. Recall that the MAD for precipitation in Utah is estimated at 4.5 inches, and for the state of Hawaii it is estimated at 14 inches. What does that tell you about the range of precipitation values for Utah compared to the range for the state of Hawaii?

Spiral Review

1. While living in Mexico City as a foreign exchange student, Ricky kept track of the temperature at noon every day in February. Find the mean, median, and mode temperature in February.

Mean: _____

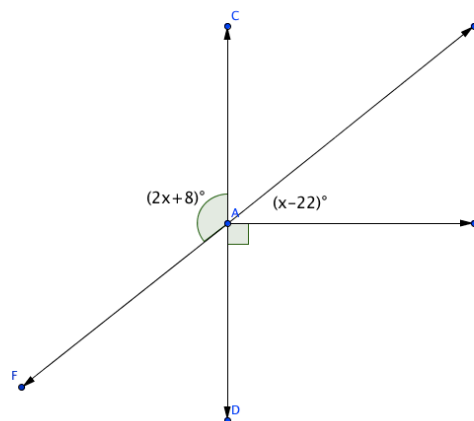
Median: _____

Mode: _____

55	57	58	52
54	55	57	58
56	59	63	62
65	64	65	64
60	61	62	57
58	54	58	60
59	64	62	63

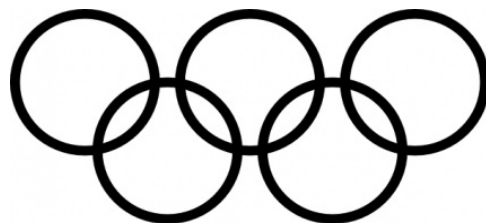
2. Every morning, Myles picks a random shirt and random pants from his closet. If he has blue, red, brown, and orange shirts and jeans or khakis for pants, what is the probability the Myles will be wearing a brown shirt and jeans?

3. Find the value of x in the diagram to the right:



4. Eden is planting a garden. Her garden plot is $14\frac{1}{2}$ feet long. Strawberry plants should be planted about $1\frac{1}{2}$ feet apart. How many strawberry plants can she fit in one row if she has a $\frac{1}{2}$ foot empty space on each side?
6. Ines is standing on a dock 3 feet above the surface of the lake. She dives down 10 feet below the dock. Then she comes up 7 feet. Where is she now? Write a number sentence showing her movement.

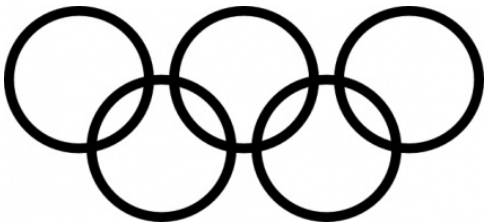
7.3e MAD Olympic Games!



<http://www.alcula.com/calculators/statistics/mean/>

<http://www.alcula.com/calculators/statistics/mean-absolute-deviation/>

7.3f Classwork: MAD Olympic Games!



Instructions: As a class, you will be competing in two MAD Olympic events: Penny Races and Blind Balance. You will be working with a partner as you compete.

Penny Races:



- With your partner, decide who will be competing first. The goal for this event is to roll a penny as far as you can across the floor.
- Roll the penny across the tile floor.
 - Count the number of full tiles the penny traveled from the starting line. Partial tiles don't count!
 - Record the data in the table at the bottom of the page by the star for the first student and by the diamond for the second student...



Blind Balance:

The partner who went second in Penny Races gets to go first in Blind Balance. The goal for this event is to see how long you can stand in the crane position: standing on one leg, with your arms outstretched, and your eyes closed. One partner will balance while the other records time using a timer. Timing ends when your foot touches the ground, you tip over, or if you open your eyes.

- Student #2: Stand on one foot, with your arms outstretched, and your eyes closed
- Student #1: Use a timer to measure how long your partner can stay in the crane position. Record their time in the bottom of the page by the heart ♥ . Round to the nearest second.

Switch roles and repeat. Record Student #1's time by the smiley face. ☺

Once you have finished recording the data, tear out and turn in. Only turn in one for both you and your partner.

PENNY RACES:	NUMBER OF FULL TILES:
Student #1:	★
Student #2:	◆

BLIND BALANCE	TIME:
Student #1:	☺
Student #2:	♥

MAD Olympic Games: Day II

Gold Medals in the MAD Olympic Games:

Write down the top records in the two event:

Penny Races: _____ tiles

Blind Balance: _____ seconds



“Best of the Best” Title Winner:

1. The MAD Olympic Officials want to give a “Best of the Best” Title. Which winner do you think did the best compared to the rest of the class, the Penny Races winner or the Blind Balance winner?
2. MAD Olympic Officials insist that the “Best of the Best” Title must be given to the player that performed the best in their event compared to the other competitors. The officials have included the mean and mean absolute deviations for each event. Record these values below:

PENNY RACES	BLIND BALANCE
Mean:	Mean:
MAD:	MAD:

3. Ms. Needa Winna suggested that the winner of the title should be the person that has the record that is the farthest from the mean in that event.
 - a) Calculate the absolute difference between the Penny Races record and the class mean for the Penny Races Event
 $|\text{Penny Races Record} - \text{Mean}| =$
 - b) Calculate the absolute difference between the Blind Balance record and the class mean for the Blind Balance Event.
 $|\text{Blind Balance Record} - \text{Mean}| =$
 - c) Is this a good method in determining a winner? Why or why not?

4. Mr. Hooda Champ suggests that the winner should be whichever event winner has the record that is the greatest number of MAD units away from the mean.
- a) Calculate the number of MAD units away the Penny Race record is from the mean by dividing the absolute deviation (#3A) by the Penny Races MAD.
 - b) Calculate the number of MAD units away the Blind Balance record is from the mean by dividing the absolute deviation (#3A) by the Blind Balance MAD.
 - c) Is this a good method in determining a winner? Why or why not?

5. Who should receive the “Best of the Best” Title? Explain your answer.

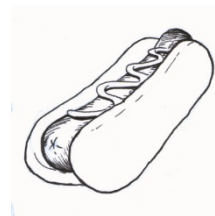


7.3f Homework : MAD Olympic Games

1. Martin participated in a hot dog eating contest. He ate 30 hot dogs in 10 minutes. The average number of hot dogs eaten by the contestants was 12 hot dogs with a MAD of 6 hot dogs.

a. Martin ate _____ more hot dogs than the average contestant.

b. Find the number of MAD units Martin was from the mean.



2. According to Pew Internet (2012), teenagers send an average of 60 texts per day. Suppose that the mean absolute deviation is 15 texts. Lily sends about 35 texts per day.

a. Lily sends _____ fewer texts per day than the average teenager.

b. Find the number of MAD units Lily is from the mean.

Anya recently got an 85% on her Geography test and a 90% on her Spanish test. She knows that she got a higher grade on her Spanish test, but wonders which test she did better on compared to the class.

3. How many MAD units away was her Geography test score from the class average? The Geography test had a mean of 70% with a MAD of 5%.

4. How many MAD units away was her Spanish test score from the class average? The Spanish test had a mean of 80% with a MAD of 5%.

5. Which test did Anya do better on compared to the rest of the class?

Who's more unique??

Joyti Amge – Height: 25 inches
Mean Height: 65 inches
MAD: 3.5 inches

Sultan Kösen – Hand Span: 28 cm
Mean Hand Span: 21 cm
MAD: 2.2 cm

6. Use the data in the table to determine who is more unique, Joyti Amge, the shortest woman in the world, or Sultan Kösen, the man with the largest hand span in the world?

Spiral Review

1. Find the missing information about the following circle:

Diameter: _____

Circumference: 15.7 cm

Area: _____

2. Write and solve an inequality for the following problem: The five times the sum of a number and 11 is at most 175.
3. Represent the sample space for each of the following events. If possible, use various methods for representing the same space.
- Sum from rolling a four-side die twice
 - Flipping a quarter twice
 - Choosing an ice cream sundae from vanilla, chocolate or strawberry ice cream and sprinkles, hot fudge, whip cream or caramel topping
4. Write and solve an equation for the following problem: Emanuela is arranging her living room. The living room is 10.4 feet wide. Her couch is 7 feet long. How much space should be on each side of the couch for it to be centered along the wall?
5. The following table shows the amount of dry cereal and water to make hot wheat cereal. Is the relationship of cereal to water proportional? Why or why not?

Dry Cereal	Water
$\frac{3}{16}$ c	1 c
$\frac{1}{3}$ c	$1\frac{2}{3}$ c
$\frac{2}{3}$ c	$3\frac{1}{4}$ c

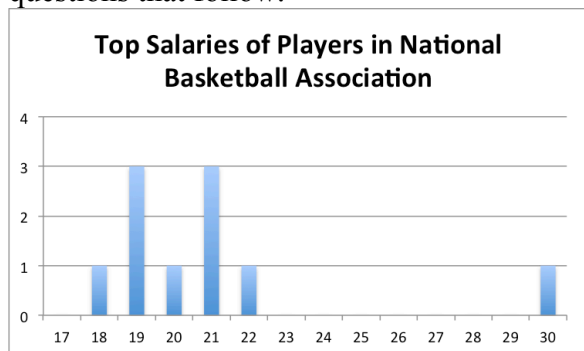
7.3g Self-Assessment: Section 7.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Make comparisons of data distributions by estimating the center and spread from a visual inspection of data plots.	I can make an appropriate plot of data, but I struggle to visually compare two data distributions.	I can compare two data distributions by visually inspecting the data plots.	I can compare two data distributions by estimating the center and spread from a visual inspection of data plots.	I can compare two data distributions by estimating the center and spread from a visual inspection of data plots. I can write an explanation of how they compare.
2. Compare two populations by calculating and comparing numerical measures of center and spread.	I struggle to calculate the center of a population.	I can calculate the center (mean, median, mode) and spread (MAD), but I struggle to use those measures to compare two populations.	I can compare two populations by calculating and comparing the center and spread.	I can compare two populations by calculating and comparing the center and spread. I can write an explanation of how they compare using those measures.
3. Calculate the mean absolute deviation (MAD) as a measure of spread of a population. Measure the distance between the centers of two populations of similar variability using the MAD as the unit of measure.	I struggle to calculate the mean absolute deviation.	I can calculate the mean absolute deviation.	I can calculate the mean absolute deviation. I can measure the distance between the centers of two populations of similar variability using the MAD as the unit of measure.	I can calculate and explain the meaning of the MAD of a population. I can measure the distance between the centers of two populations of similar variability using the MAD as the unit of measure.

Sample Problems for Section 7.3

1. The top ten salaries for sports players in the NBA and NFL are shown in histograms below. Answer the questions that follow.



- Which set of data has a higher center?
 - Which set of data has a larger spread?
 - How do the two data sets compare in similarities and differences?
2. For each data set listed below, calculate the center and spread. Then write a comparison of the two data sets. (data from <http://www.math.hope.edu/swanson/data/cellphone.txt>)

Length of Last Phone Call for Males (in seconds)	Length of Last Phone Call for Females (in seconds)
292	653
360	73
840	10800
60	202
60	58
900	7
60	74
328	75
217	58
1565	168
16	354
58	600
22	1560
98	2220
73	2100
537	56
51	900
49	481
1210	60
15	139
59	80
328	72
8	2820
1	17
3	119

3. Ms. Christensen gave two of her history classes a test. The following table shows the scores from her classes. Find the mean absolute deviation of each. If the MAD is similar for both, find the distance between the centers of the two classes.

5 th Period	6 th Period
90	72
88	80
100	91
98	90
71	55
83	52
92	76
94	67
87	75
78	80
91	78
99	83
60	78
90	75

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Chapter 8: Geometry Part 2: Measurement in 2- and 3-Dimensions, Plane Sections of Solids (2 weeks)

UTAH CORE Standard(s)

Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. 7.G. 3

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

2. Know the formulas for the area and circumference of a circle and use them to solve problems. 7.G.4
3. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. 7.G.6

Chapter Summary:

Throughout this chapter students develop and explore ideas in geometry around measures in one-, two-, and three-dimensions. Additionally, students will tie concepts learned previously in 6th and 7th grade to content in this chapter. Section 1 begins with a brief review and practice with perimeter and area of polygonal figures. Students then use their knowledge of area and perimeter to solve real-world problems, find the area of non-standard shapes and review ideas around percent increase/decrease and scale factor, and connect their understanding of one- and two-dimensional measures to adding and multiplying algebraic expressions by finding the perimeter and area of figures with variable expression side lengths.

Section 2 starts with students exploring plane sections of 3-D solids. Exercises in this section are designed to help students develop an intuitive understanding of dimension (building up and cutting down). Students will notice that plane sections of 3-D objects are affected by: a) the type of solid with which one starts, e.g. right prism versus a pyramid and b) the angle of the cut to the base and/or other faces. This observation should help students understand when and why parallel (and/or perpendicular) cuts to a specific face are needed to create uniform cross sections, and then by extension, uniform cubed units for finding volume. In other words, the study of plane sections here is to help students develop an understanding of the structure of a solid and procedures for finding volumes. Throughout the exercises, students should develop a stronger understanding of units of measure for one-, two-, and three-dimensions. Towards the end of this section students review the use of nets (a concept from 6th grade) to find surface area of prisms and cylinders and then to differentiate this measure from volume, which they will also find.



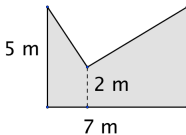




VOCABULARY: area, axis of symmetry, circle, circumference, cross-section, cube, cylinder, edges, equidistant, face, plane section, perimeter, polygon, polyhedron, prism, pyramid, quadrilateral, rectangle, square, surface area, symmetry, trapezoid, triangle, volume, vertices.


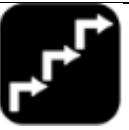
CONNECTIONS TO CONTENT:

Prior Knowledge: In 6th grade students found areas of special quadrilaterals and triangles. They also found areas of other objects by decomposing them into rectangles and triangles. In Chapter 5 of this text, students extended that understanding to finding the area of a circle. Also in 6th grade, students found volume of rectangular prisms and their surface areas by using nets. During their study of geometry in 6th grade, students should have learned that the height and base of an object are always perpendicular to each other. They will build on this understanding as they apply their knowledge of area and volume to real life contexts and as they explore cross sections and plane sections.

Future Knowledge: In 8th grade, students will continue working with volume, formalizing algorithms for volume of cylinders and adding methods for finding the volume of cones and spheres. Students explore cross sections of objects in 7th grade to understand how dimensions are related to each other and the algorithms for surface area and volume. Further, ideas developed through cross section activities are foundational in the study of calculus. Lastly, ideas about planar sections will be extended in secondary mathematics when students explore Cavalieri's Principle.

MATHEMATICAL PRACTICE STANDARDS (emphasized):

	<p>Make sense of problems and persevere in solving them.</p>	<p>Students will be given a variety of contextual problems with which they will need to make sense and persevere. For example, 8.0 Anchor Problem: The city is looking to build a new swimming pool in city park. In their city council meeting, they have determined that they want the pool to hold no more than 2500 m^3 of water, or it will cost too much to keep it filled.</p> <p>Help the city council to choose a design for the swimming pool. Design 3 different swimming pools that will each hold at least 2000 m^3 but no more than 2500 m^3 of water.</p>
	<p>Reason abstractly and quantitatively.</p>	<p>Students will use their understanding of area and volume to reason in a variety of contexts such as the 8.1e Class Activity: Write an expression to find the area. If possible, find the exact area.</p> <div data-bbox="954 583 1138 716" data-label="Figure">  </div>
	<p>Construct viable arguments and critique the reasoning of others.</p>	<p>Students will apply their understanding of perimeter and area to construct and critique arguments. 8.1g Class Activity: <i>Mike, Juliana and Joe were working together to make a garden larger. Mike said, “We have to buy more fencing because if we increase the area of the garden we will need more fencing to go around.”</i></p> <p><i>Juliana had a different opinion. “That’s not true,” she said. “We can use the same amount of fencing and move it to make the area of the garden larger.”</i></p> <p><i>Joe disagreed with both Mike and Juliana. He said, “I know a way that we can make the garden larger and use less fencing.</i></p> <p><i>Who is right?</i></p>
	<p>Model with Mathematics.</p>	<p>Students will use models to explore concepts in geometry such as using play-dough and string to create cross sections of prisms. Throughout the chapter, they will connect models to algorithms.</p>
	<p>Attend to Precision.</p>	<p>Careful attention should be paid to explanations and units throughout this chapter. Students will be expected to attend to several ideas at the same time. Students should attend to precision as they explain ideas throughout this chapter. For example, when discussing cross sections, students should use <u>precise language in describing the angle of cuts to the base, faces and/or edges.</u></p>
	<p>Look for and make use of structure.</p>	<p>Students will connect ideas of one-, two-, and three-dimensional measures to simplifying numeric and algebraic expressions, including this example from the 8.2 d Homework: Determine which expression(s) will give the surface area or volume for a 3-D object.</p> <div style="display: flex; flex-wrap: wrap;"> <div style="width: 50%;"> <p>a. $2(6 + 2 + 3)$</p> <p>b. $3 \cdot 2 \cdot 6$</p> </div> <div style="width: 50%;"> <p>c. $(2 \cdot 6 + 2 \cdot 3 + 3 \cdot 6)2$</p> <p>d. $2 \cdot 3 \cdot 2 + 2 \cdot 3 \cdot 6 + 2 \cdot 6 \cdot 3$</p> </div> </div>

	Use appropriate tools strategically.	Students will use a variety of tools in this chapter including play dough, rulers, graph paper, and calculators. Encourage students to make sense of ideas with tools.
	Look for and express regularity in repeated reasoning.	Students will note in this chapter that length is one-dimensional, area is two-dimensional and requires 2 length (2 one-dimensional measures) that are perpendicular, and volume is three dimensional and requires 3 lengths that are each perpendicular to each other.

8.0 Anchor Problem; Designing a Swimming Pool:

The city is looking to build a new swimming pool at the city park. In their city council meeting, they determined that they want the pool to hold no more than 2500 m^3 of water in order to conserve water.

- A. Help the city council to choose a design for the swimming pool. Design three different swimming pools that each hold at least 2000 m^3 but no more than 2500 m^3 of water.
- B. Choose your favorite of the three designs from part A. Find new measures for the pool so that: a) it holds twice as much water as your original design and b) it remains the same overall shape (but not size) as the original.



Section 8.1: Measurement in Two Dimensions

Section Overview:

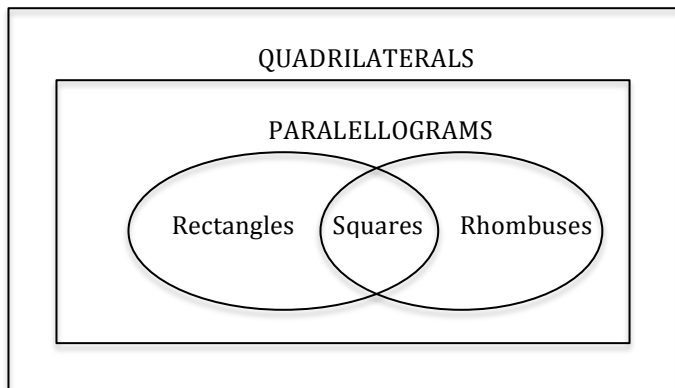
This section involves a review and extension of previously learned skills (from 6th grade) involving perimeter and area of plane figures and volumes of right solids. The section begins with a review of perimeter and area and then moves to finding area of irregular figures, first with numeric side lengths and then with variable expression side length. Additionally, students continue work on skills developed earlier in the year including percent increase/decrease, scale factor for perimeter and area, and simplifying algebraic expressions. The end of the section focuses on all these skill in various real-world and mathematical contexts.

Concepts and Skills to be Mastered (from standards)

1. Know the formula for the area and circumference of a circle and use them to solve problems.
2. Give an informal derivation of the relationship between area and circumference of a circle.
3. Solve real-world and mathematical problems involving area, volume, and surface are of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

8.1a Class Activity: Differentiate Area and Perimeter

REVIEW:



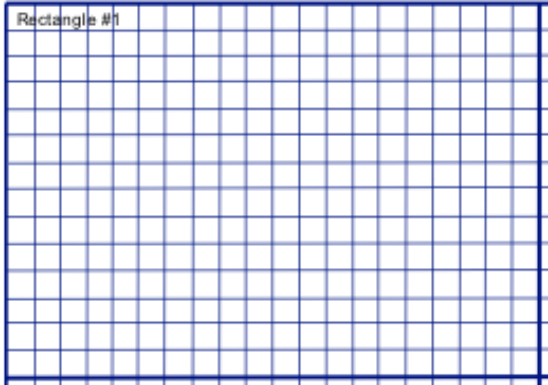
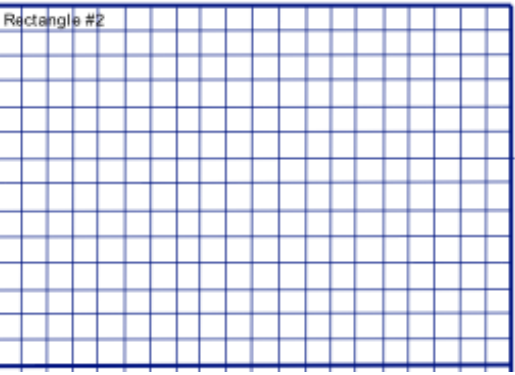
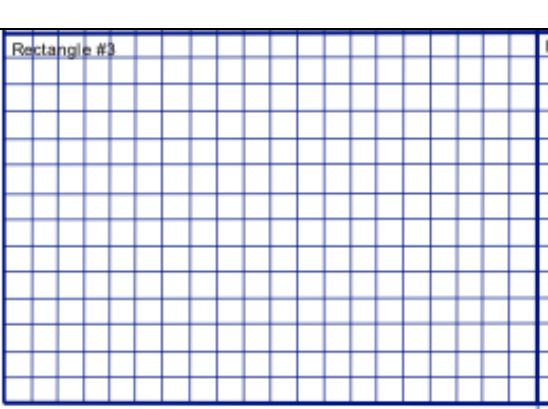
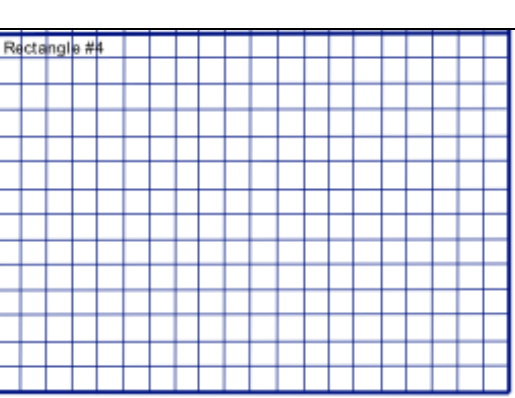
- Rectangle Riddle #1: Can you figure out the dimensions for each rectangle? Use the clues below to draw the four different rectangles described.
 - Rectangle A has a perimeter equal to that of Rectangle B.
 - Rectangle A is a square.
 - Rectangle B has an area of 24 square units. It is as close to a square as possible for that area if the side lengths are **whole numbers**.
 - Rectangle C has a perimeter equal to the measure of the area of Rectangle B. Rectangles C's length is 3 times the width.
 - Rectangle D has two different odd integers as length and width; each is 1 unit greater than the length and width of Rectangle B.

A		B	
C		D	

Use words and models to explain the difference between perimeter and area.

2. Rectangle Riddle #2: Can you figure out the dimensions for each rectangle? Use the clues below to draw the four different rectangles described.

- The numeric measure of the area of Rectangle #1 is twice the number representing the perimeter. Rectangle #1 is 8 units wide.
- Rectangle #2 has the same number representing the area and the perimeter; the perimeter is $\frac{1}{2}$ the perimeter of Rectangle #1.
- Rectangle #3 has the same length as Rectangle #1. The area of Rectangle #3 is equal to the difference between the areas of Rectangles #1 and #2.
- Rectangle #4 has an area 6 square units less than the area of Rectangle #3 and a width 2 units more than the width of Rectangle #2.

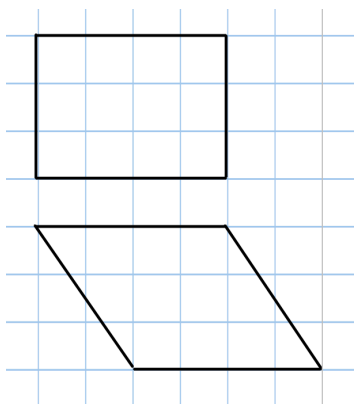
A	 <p>Rectangle #1</p>	B	 <p>Rectangle #2</p>
C	 <p>Rectangle #3</p>	D	 <p>Rectangle #4</p>

3. What helped you most to accomplish “Rectangle Riddle #2”?

For questions #4-6 use a piece of graph paper to answer each:

4. List as many rectangles as you can that have an area of 36?

5. DRAW two different parallelograms that have the same area. State their base and height.



6. DRAW two different triangles that have an area of 12. State their base and height.

7. Suppose you start with a rectangle that has a base length of 6 and height of 4. If you triple the length of the base, what do you have to do to the height to have a new rectangle of the same area?



Activity:

Mike, Juliana and Joe were working together on a garden area and perimeter problem their teacher gave them. The problem is to make the area of a garden larger without having to buy more fencing (increasing the perimeter.) Their task is to figure out if that's possible.

Mike said, "We have to buy more fencing because if we increase the area of the garden we will need more fencing to go around."

Juliana had a different opinion. "That's not true," she said, "We can use the same amount of fencing and move it to make the area of the garden larger."

Joe disagreed with both Mike and Juliana. He said, "I know a way that we can make the garden larger and use less fencing."

Who is right?

- A. Group response: Use graph paper and string to help you think about the problem.
 - a. Decide who you think is right (Mike, Juliana or Joe.)
 - b. Come up with a possible answer with your group.
 - c. Prepare your group explanation and presentation.
 - d. Be certain to justify your conclusion with reasoning and calculation.

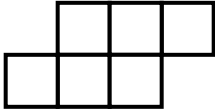
- B. Challenge: Examine the Garden Problem using a spreadsheet. Consider all possible rectangles with perimeters of 36 units. How does the area change as you change the configuration of the perimeter?



8.1a Homework: Perimeter and Area Problem Solving

Answer each of the questions below.

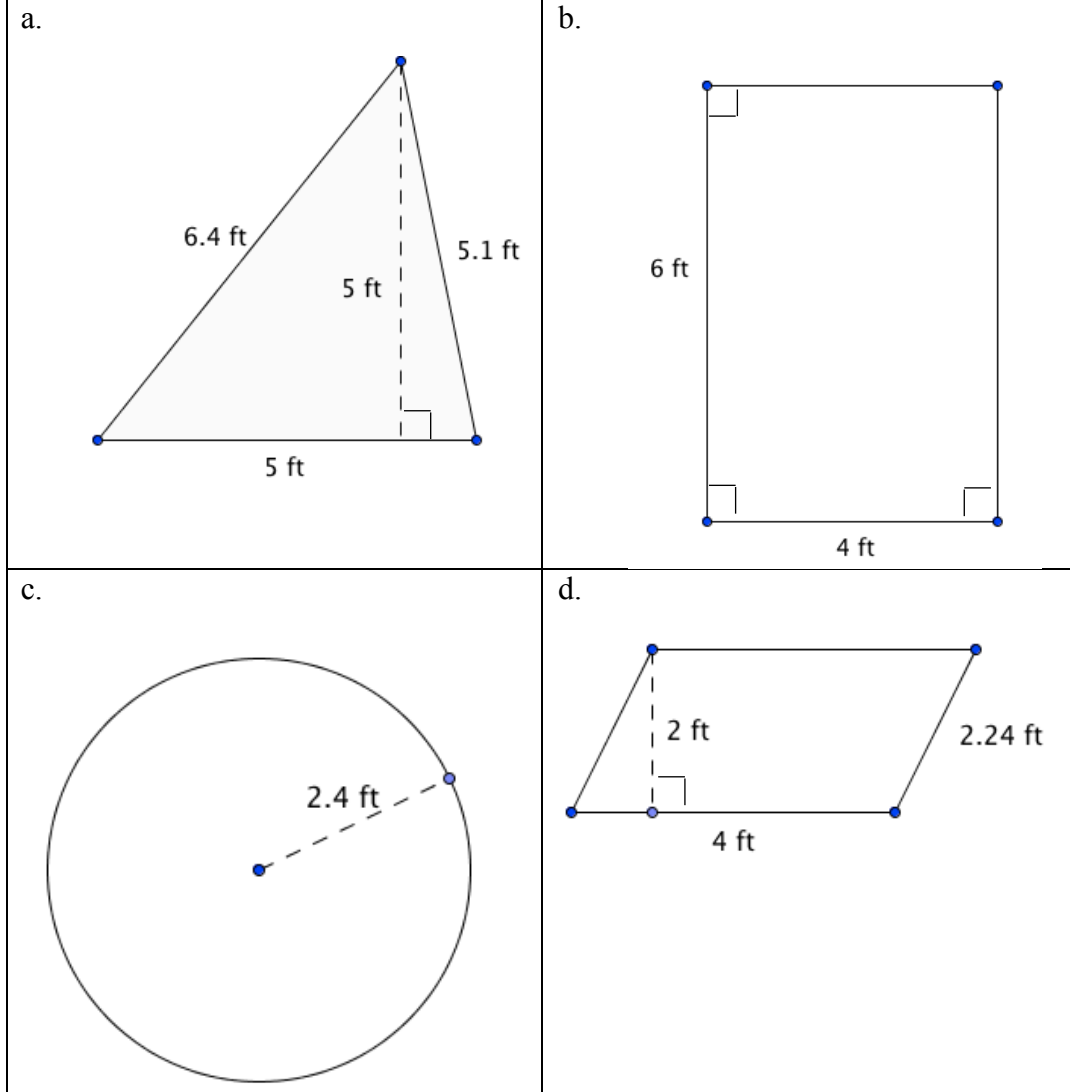
1. What is the least number of tiles you can add to the figure below to create a shape with a perimeter of 16?
Note: When adding a tile, the new tile must share at least one side with the original shape; each tile is 1 unit by 1 unit.



2. Use the original figure given above to answer a-d. Draw your answers. Note, an answer of just “yes” or “no” is not sufficient. Use pictures or words to justify your answer.
 - a. Can you add a tile to this figure to increase the perimeter by 1? If so, how?
 - b. Can you add a tile to this figure to increase the perimeter by 2? If so, how?
 - c. Can you add a tile to this figure to increase the perimeter by 3? If so, how?
 - d. Can you add a tile to this figure so that the perimeter doesn't change? If so, how?
3. Can you make more than one shape with the same perimeter, but different areas? Show your ideas with grid paper.
4. Can you make more than one shape with the same area, but different perimeters? Show your ideas with grid paper.
5. If you pick any whole number between 12 and 24, use that many unit tiles, can you make a shape where the area and perimeter are equal? Show your ideas.

Spiral Review

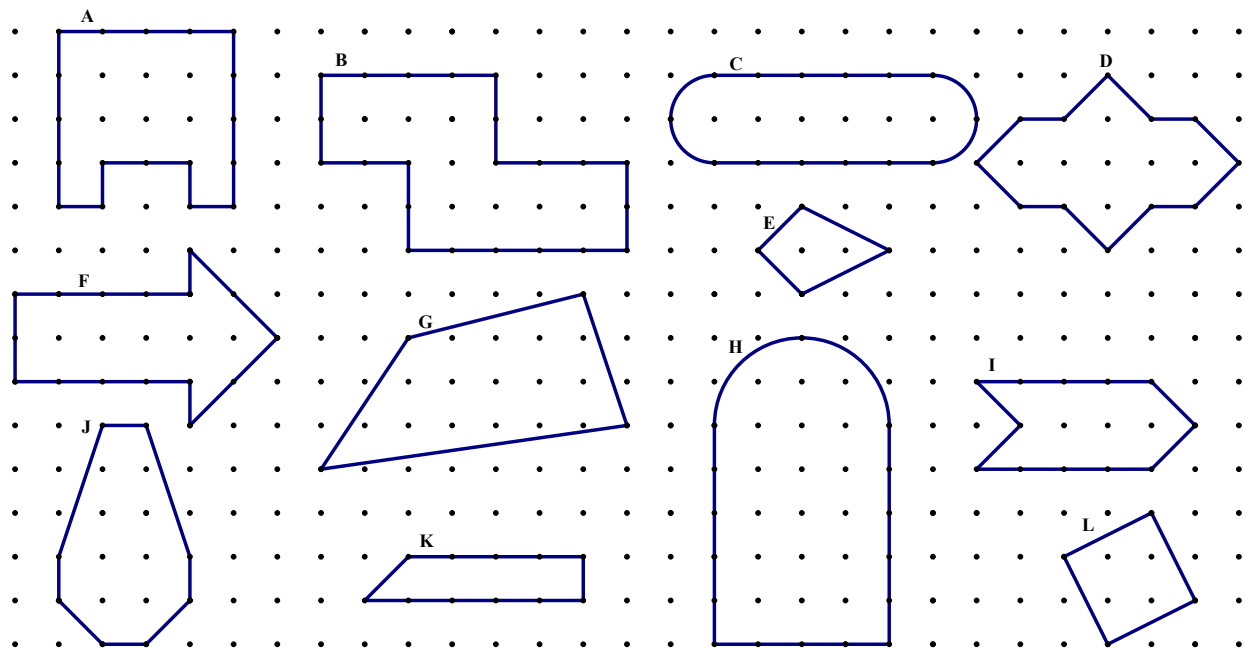
1. Find the area of each of the following shapes:



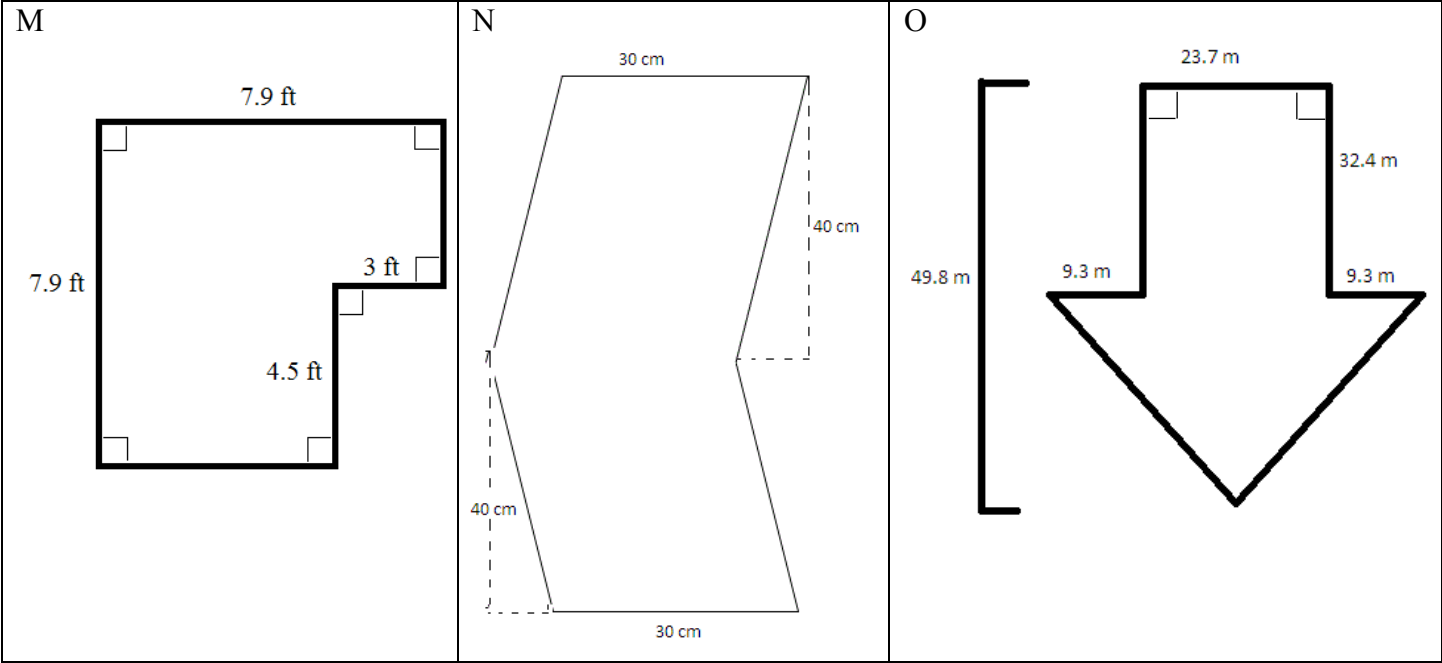
2. Solve: $-8 < -3m + 10$
3. Solve: $-12 > 3x$
4. A website says that the odds of Mr. and Mrs. Durrand having a baby with blue or hazel eyes is 50:50. Describe a simulation that models the color of eyes their baby will have.
5. Write $\frac{3}{5}$ as a percent and decimal.

8.1b Class Activity: Areas of Irregular Shapes

The following shapes have been drawn on square dot paper. The distance between each dot represents one unit. Use what you have learned about area to find the area of each shape (A-L).



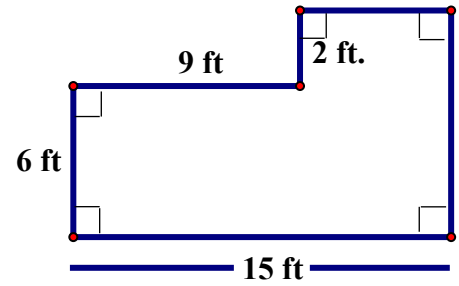
Use your knowledge of area and the problems that you completed above to find the area of the following irregular shapes.



1 a. Johona is building a deck off the back of her house. To the right is a sketch of it. She will need to have a full concrete foundation below the deck. Find the surface area of the concrete foundation.

b. Suppose Johona wants to build her deck onto a concrete foundation that is 1.5 ft. thick and has the same surface area as the deck. How many cubic feet of concrete will she need?

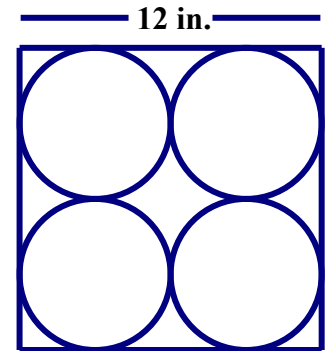
c. How many cubic yards will she need?



2. a. Hugo is making the tile pattern shown. The tile is a square with four circles of the same size inside. He will paint the circles blue and the remaining part of the tile yellow. Find the area of the portion of the tile that will be yellow.

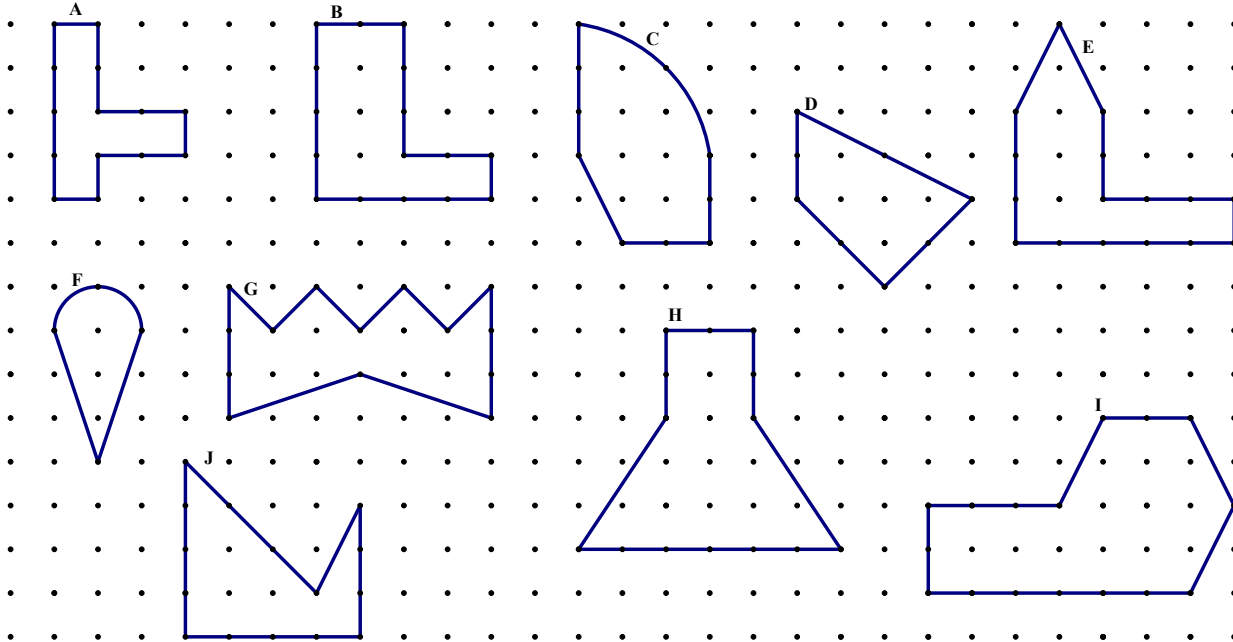
b. Find the area of the diamond-shaped piece in the middle of the tile.

c. What portion (percent) of the tile will be yellow?



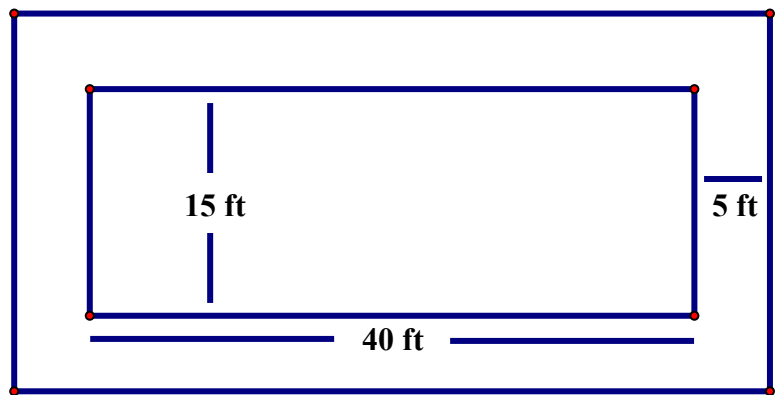
8.1b Homework: Areas of Irregular Shapes

1. The following shapes have been drawn on square dot paper. The distance between each dot represents one unit. Use what you have learned about area to find the area of each shape (A-J).



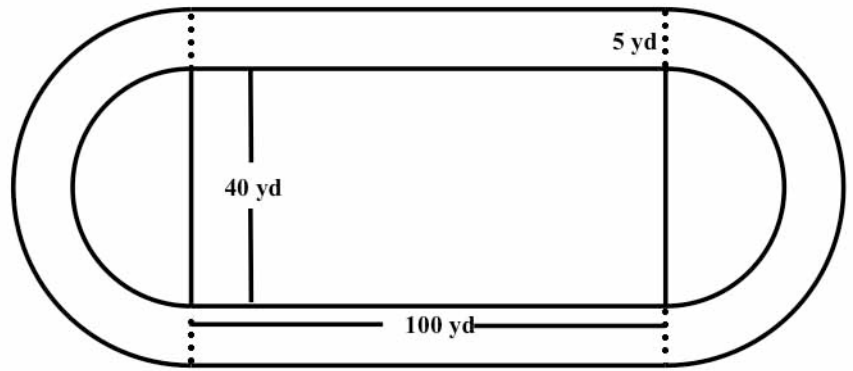
Solve the following area problems.

2 a. A rectangular lap pool with a length of 40 ft. and a width of 15 ft. is surrounded by a 5-ft. wide deck. Find the area of the deck.

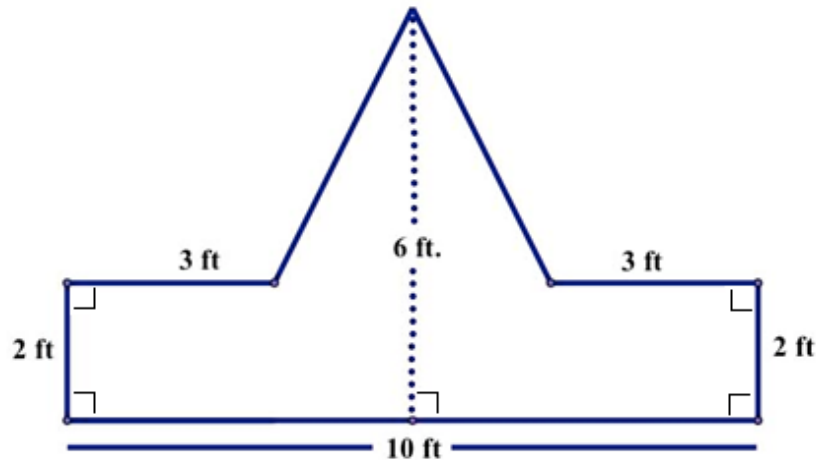


2 b. Draw a picture of the deck if the deck is extended 3 feet in every direction, then find the area of the new deck.

3. A rectangular field with two semi-circles at each of the shorter ends of the field measures 100 yards long and 40 yards wide. It is surrounded by a track that is 5 yards wide. Find the area of the field which includes the two semi-circles on each of the shorter ends of the rectangle. Find the area of the track.



4. Laura is painting a sign for the new Post Office. She will paint the triangular portion blue and the lower rectangular portion red. Find the area of sign that she will paint blue. What percent of the sign will be in blue?



Spiral Review

1. Simplify the following:

$$2.6 + (-2.6) =$$

$$\frac{1}{6} + \frac{3}{7} =$$

2. Simplify the following expressions:

$$-60 + 5x - 1 - 17x$$

$$9(5 - x) + 3x - 8$$

3. Anya and Bartholomew clean windows. Anya charges 50 cents per window plus \$10 per job. Bartholomew charges 90 cents per window plus \$6 per job. If on one job, they make the same amount of money, how many windows did they each clean?

4. Alfonso is tossing his baby brother's cylinder toy. The following table shows the results of the tosses. Based on the observations, what is the probability the cylinder will land on its side?

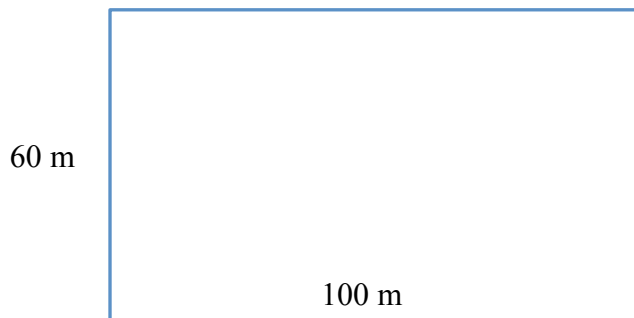
Land on side	Land on top or bottom
9	11

8.1c Class Activity: Areas of Irregular Shapes and Expressions



Use your knowledge of area to find the area of the following shapes, if possible. Write an expression to show how you arrived at your answer.

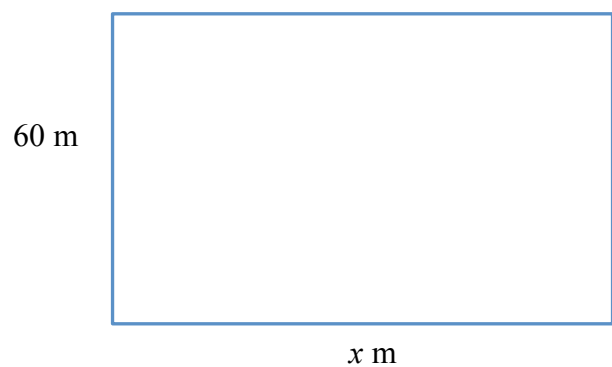
1 a.



1 b.

Suppose you had a 60×100 meter plot of ground on which you were going to plant a garden. In the corner of your plot you want to build 5×8 storage shed. How much area will be left for your garden? Does it matter which way the storage shed is oriented?

2 a



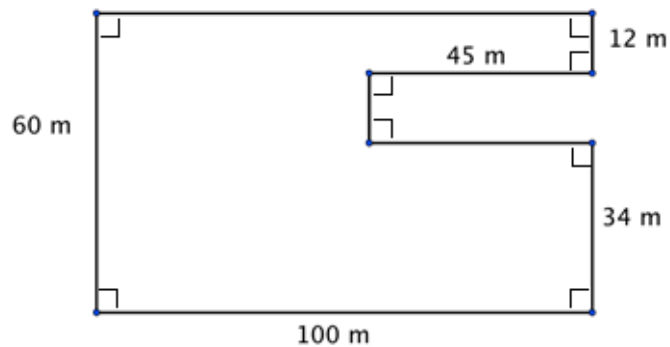
2 b.

Suppose you have a 60 by 200 meter plot of ground on which you are going to plant a garden. You don't want to plant the whole thing, just an area of 8000 m^2 . If one side is 60 meters, how long will the other side (the 200 meter side) have to be to have an area of 8000 m^2 ? What percent of the other side would you be using?

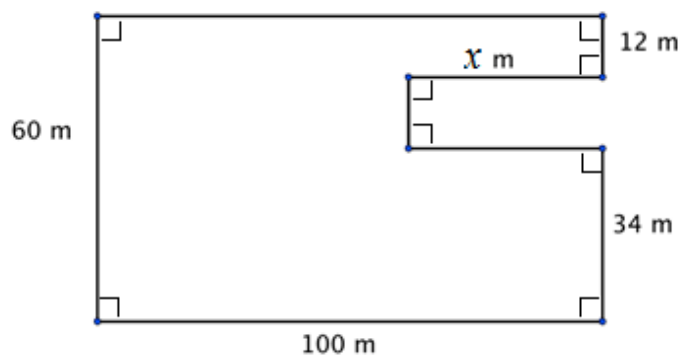
2 c.

What percent of your 60×200 meter plot of ground will be planted?

3. a

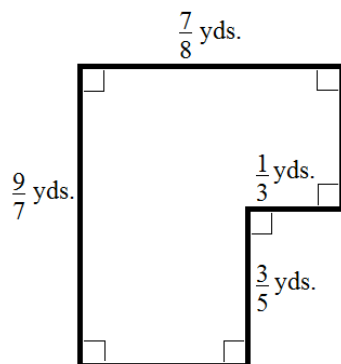


3 b.

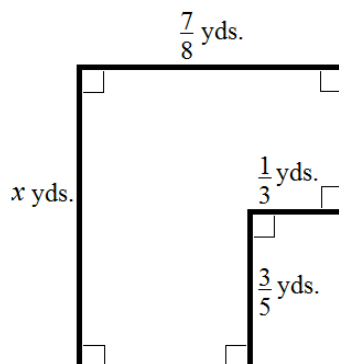


3 c. Suppose you're building a rectangular storage enclosure. The base of the enclosure is to be 60 by 100 meters. You need to construct a ramp into the enclosure as illustrated in 3 b. How long should the ramp be if you want the remaining area to be 5250 m^2 ?

4a.

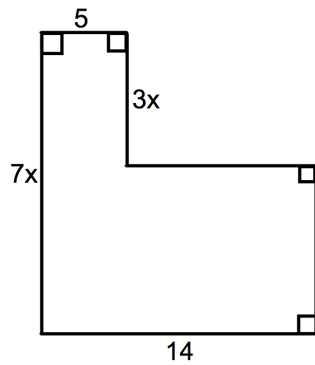


4b.

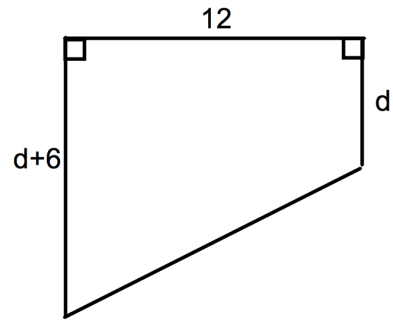


Write an expression to represent the indicated measure for each of the following irregular shapes.

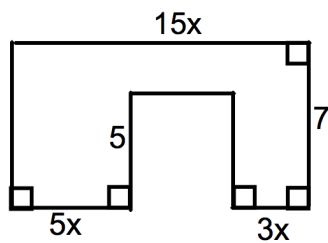
5. Find the area and perimeter for the figure below.



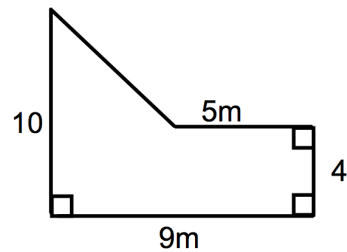
6. Find the area of the figure below.



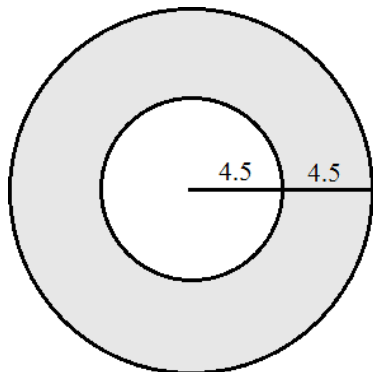
7. Find the area and perimeter of the figure below:



8. Find the area of the figure below:



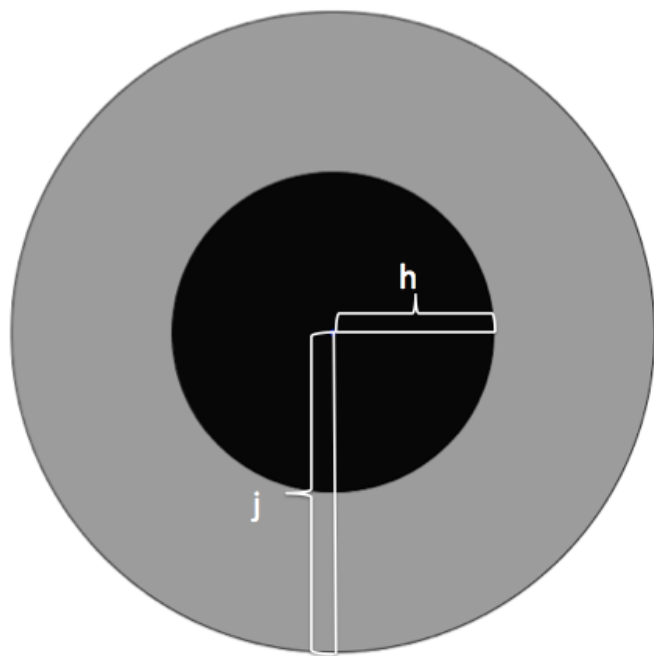
9. A bull's eye is made of two concentric circles as shown below. The radius of the smaller circle is 4.5 inches. The larger circle has a radius of 9 inches.
- Use 3.14 to approximate π in calculating the area of:
- The smaller circle
 - The larger circle
 - The space between the smaller circle and larger circle (the outer ring)



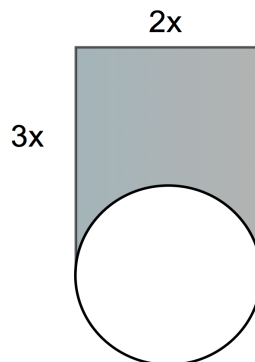
Assuming the dart hits the target somewhere, what is the probability of hitting the smaller circle's area?

- What is the probability of hitting the outer ring?

10. Find the area of the grey part of the figure below.



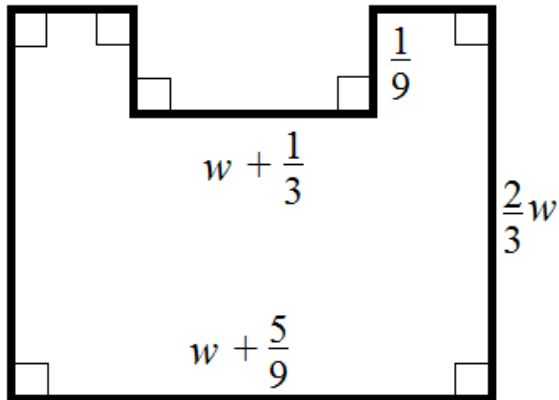
11. The figure below shows a circle and a rectangle. The circle's diameter is equal to the rectangle's base. Find area of the shaded region and its perimeter. (use 3.14 for π)



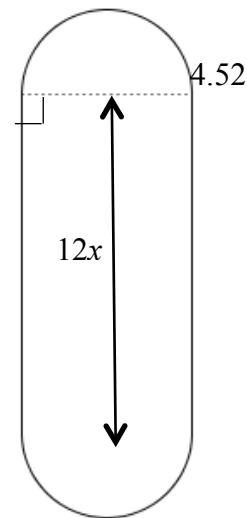
8.1c Homework: Areas of Irregular Shapes and Expressions

Solve the following area problems. Write an expression showing how you got the area.

1. Find the area of the region.

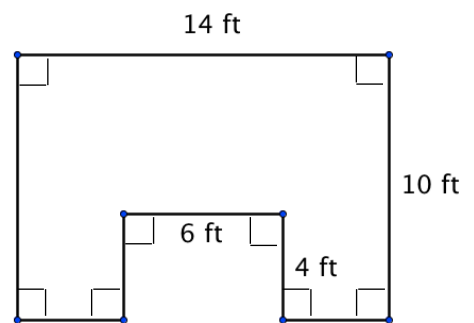


2. Find the perimeter and area of the region.
(use 3.14 for pi)



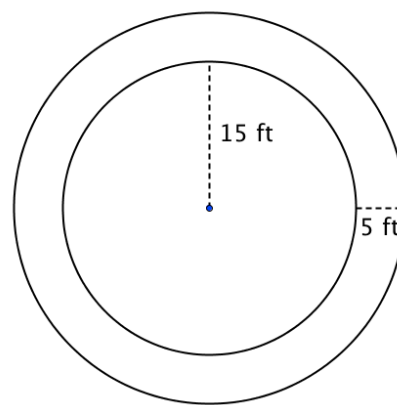
3. Jeremy needs to buy soil for the garden spot in his backyard. A sketch of the plot is to the right.

- Find the area of the garden.
- How many cubic feet of soil will he need to buy if he covers the area in 6 inches of soil?
- How many cubic feet of soil will he need to buy if he covers the area in one and a half feet of soil?
- Howmany feet of fencing will he need to enclose the garden if he fences the exact shape of the garden?



4. Nico is building a deck around the circular pool in his back yard. The pool has a radius of 15 ft. The deck will be 5 ft wide.

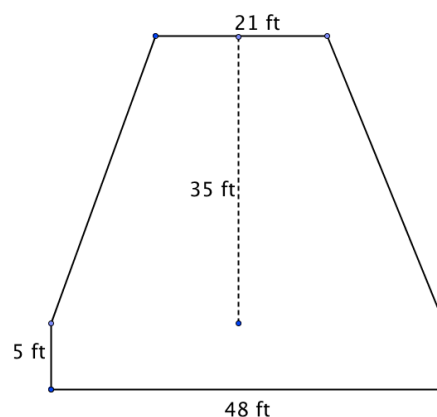
a. Find the area of the deck.



5. a. A stage with a trapezoidal area upstage and a rectangular area downstage is illustrated in the figure to the right. Find the area of the stage.

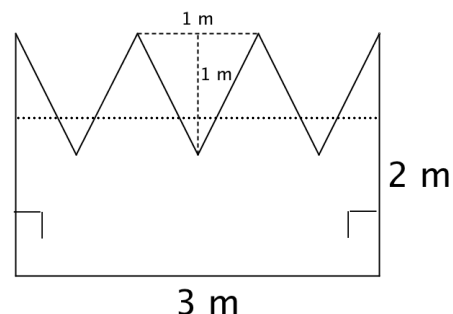
b. What is the area of just the rectangular portion of the stage?

c. What portion of the stage is the rectangular portion?



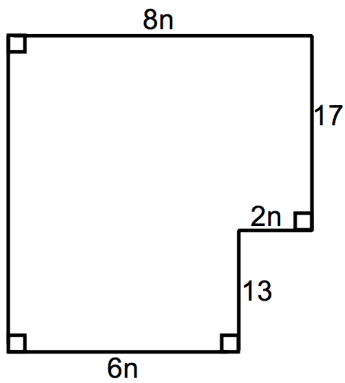
6. Rachel is painting a sign for the new Health Center. Find the area of sign that she will need to paint red if she paints the entire area red.

Suppose Rachel decides to paint the lower portion (the 3×1 bottom portion, shown below the dotted line) of the sign blue. What percent of the sign would be blue?



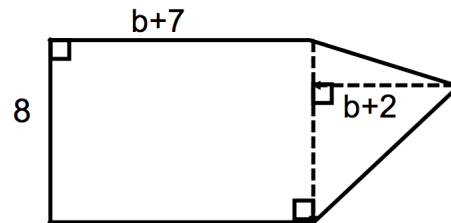
Write an expression to represent the **white area** of the following irregular shapes.

7.

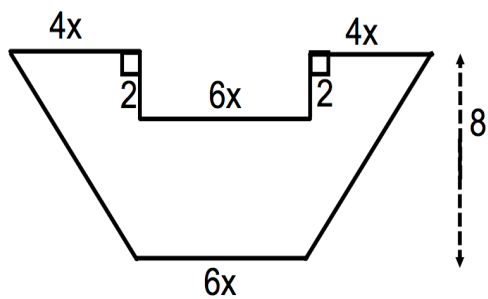


Find the perimeter of this region:

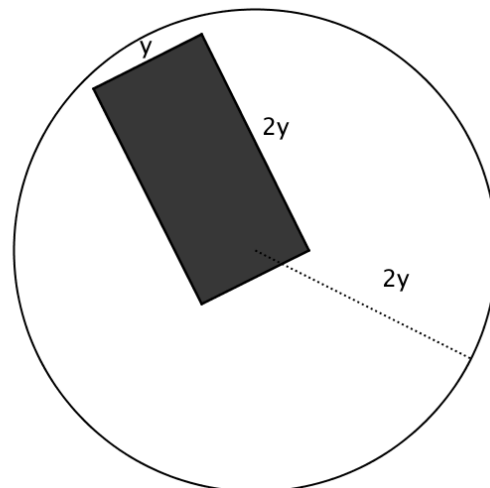
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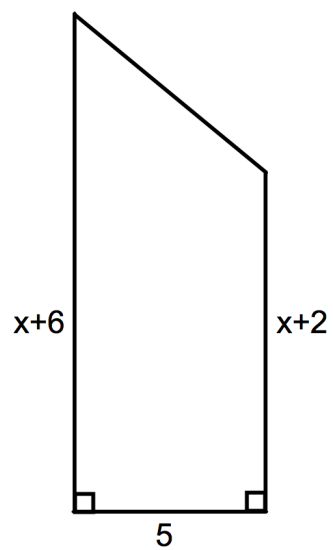
9.



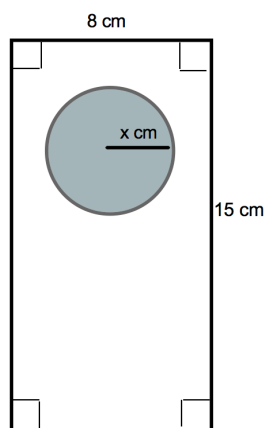
10. The circle has a radius of $2y$.



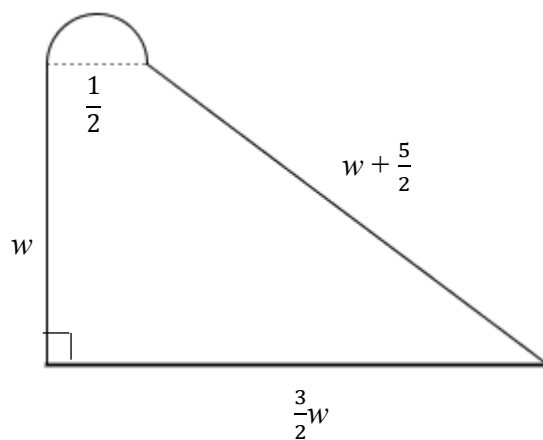
11.



12.



13.



Spiral Review

1. Write and solve a proportion to find what percent of 27 is 3?
2. A local college football team is known for its awesome offense. The table below shows a season's rushing yards for 9 players. Find the mean absolute deviation of their rushing yards.

Rushing Yards
460
399
180
158
110
95
55
15
1

3. Two angles are supplementary. One angle is 49 degrees less than the other. What are the measures of each angle?
4. Write 0.672 as a percent.
5. Sam has lots of bracelets. She gets 3 more bracelets. Then she sells $\frac{1}{3}$ of the bracelets. Write two different equivalent expressions showing how many bracelets Sam has left.

8.1d Class Activity: Review Areas of Triangles, Parallelograms, Trapezoids; Circle Area and Circumference

Activity: In pairs, answer each of the following. Use a model to justify your answer.

1. Look back at question #3 from your homework (8.1c). How much more soil will Jeremy need to buy if he decided to cover his garden in 1.5 feet of soil rather than six inches of soil? Give your answer as a scale factor (twice as much? Three times as much? 1.5 times as much?) Explain:

2. Kara and Sharice are in a quilting competition. Both are stitching rectangular-shaped quilts. So far Kara's has an area of 2,278 square inches with a height of 44 inches. Sharice's quilt has an area of 2,276 square inches with a height of 47 inches. Whose quilt is wider? By how many inches is it wider?

3. Rufina bought two 12-foot pieces of lumber and two 8-foot pieces of lumber to create a border for her garden in her yard. She wants to use all the wood to enclose her garden. How should she use the four pieces to create a rectangular garden of largest possible area? What is the largest area she can get with her four pieces of wood?

4. A triangle has an area of 90 square centimeters.
 - a. If its height is 15 cm, what is the length of the base?
 - b. Draw a triangle with an area of 90 square centimeters, height of 15 cm and base you found in part a. Is there more than one triangle you can draw with those dimensions?
 - c. Draw a triangle that has an area that is 150% of the original 90 cm² triangle. What are the dimensions of your larger triangle?

5. Julius drew a trapezoid that had bases of 15 and 11 inches and a height of 4 inches.
 - a. What is the area of the trapezoid Julius drew?
 - b. Can you draw a trapezoid that has the same area, but different dimensions?
 - c. Draw a trapezoid with the same height but with bases that are a $\frac{5}{2}$ scale factor of the original trapezoid.
 - d. What is the area of the new trapezoid?

Review:

Draw a model and explain how to find the circumference of a circle:

Draw a model and explain how to find the area of a circle:

6. You're making a 12 inch diameter pizza. You want the sauce to cover the pizza with a 1.5 inch ring left around the outside without sauce. (Use 3.14 as an approximation for pi.)
 - a. What is the area that the sauce will cover?
 - b. What percent of the dough will be covered by sauce?
 - c. If one 8 oz can of tomato sauce covers about 125 sq. inches of pizza dough, how many cans of sauce will you need to buy?

7. A circular swimming pool with a diameter of 32 feet is located exactly in the middle of a $40 \text{ ft} \times 40 \text{ ft}$ square lot. For safety reasons the lot needs to have an 8 ft fence on the perimeter of the entire lot.
- How long will the fence need to be?
 - If the fence was around only the circular pool, how long would the fence be?
 - Explain how much longer a fence around the whole yard is than a fence only around the pool using percent increase.
 - What percent of the yard area does the pool take up?
 - What is the area of the yard NOT taken up by the pool?
8. Look back at question #4 from the homework (8.1c). Nico's pool currently takes up approximately 706.5 ft^2 of space in his yard (using 3.14 as an approximation for pi). If Nico adds the five foot deck around the pool, what percent increase of space will this be?

8.1d Homework: Review Areas of Triangles, Parallelograms, Trapezoids; Circle Area and Circumference

1. Wallpaper comes in rolls that are 60 feet long and 2 feet wide. How many rolls of wallpaper will it take to cover 700 square feet?

2. A rectangular garden has an area of 45 square feet. One of the sides is 6 feet.
 - a. What is the other side?
 - b. You want to put a fence around it. How long will the fence need to be?
 - c. You decide you want to increase the length of each side by a scale factor of 3.2. What are the new dimensions of your garden?
 - d. What is the area of your new garden?

3. Mrs. Garcia has a table shaped like an isosceles trapezoid in her third grade classroom. The two parallel sides have lengths of 6 feet and 8 feet. The distance between them is 4 feet.
 - a. What is the area of the top of Mrs. Garcia's table?
 - b. Suppose Mrs. Garcia has a 1.75 ft by 0.8 ft puzzle on the table. How much surface area is now available on her table?

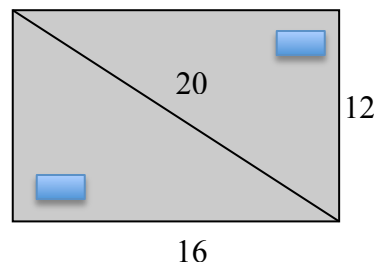
4. The diameter of the earth is about 7926 miles.
 - a. Find the distance around the earth at the equator.

 - b. If there are 5280 feet in every mile, what is the distance around the Earth in feet?

 - c. Suppose you can jog at a rate of 2 miles every 15 minutes. At this rate, how long would it take you to walk around the Earth?

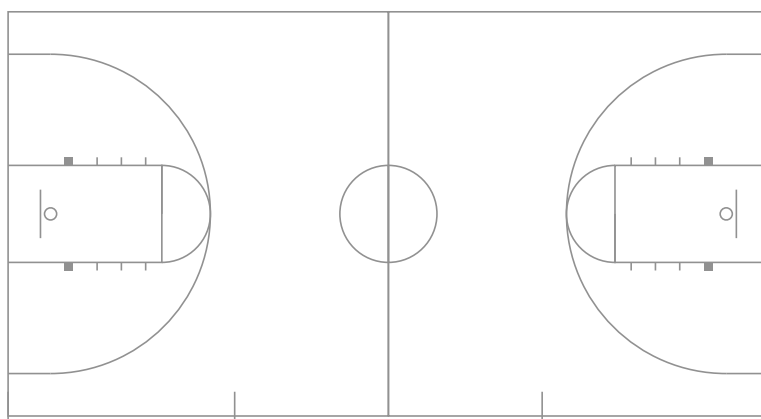
5. A 12 foot by 16 foot rectangular office is being sectioned off into two triangular areas so that desks can be placed in opposite corners. The diagonal of the office (from corner to corner) is 20 feet. The manager needs a dividing curtain to hang from the ceiling around one of the triangles.

a. How long does the curtain need to be?

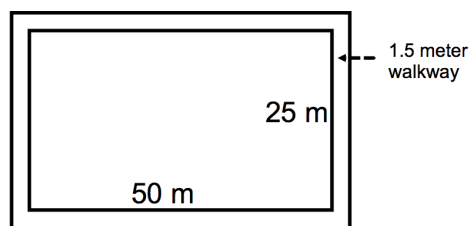


b. The carpet in each section will be a different color. How many square feet of carpet will be needed to cover each triangular section?

6. The three-point line in basketball is approximately a semi-circle with a radius of 19 feet and 9 inches. The entire court is a rectangle 50 feet wide by 94 feet long. What is the approximate area of the court that results in 3 points for a team?

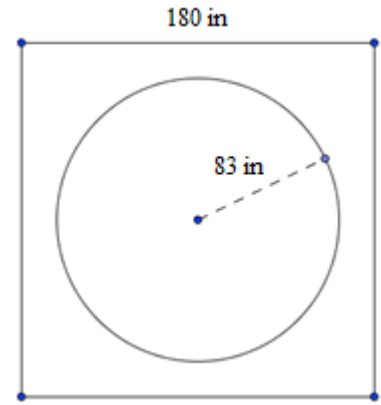


7. Your neighbor's backyard lawn is shaped like a rectangle. The back fence is 38.2 feet long and the side fence is 32.6 feet long. He will pay you \$0.04 per square foot for mowing and \$0.11 per foot for trimming all the edges. How much will you get paid total for mowing and trimming? Remember to show your work.
8. You're the manager of a county recreation center that has a 50 by 25 meter rectangular pool. Currently, there is an 1.5 meter cement walkway around the pool (see diagram). The community is concerned about the safety of the walkway and would like to cover it with a non-slip rubber substance that costs \$78 a square meter to be installed. The county has budgeted \$15,000 for the project. Is that enough money to cover the walkway? Explain your answer.



Spiral Review

1. A hot tub is surrounded by a square deck as pictured to the right. What is the area of the deck?



2. Cristian is building a rectangular garden in his backyard. The width of the garden is set at 29 inches. He wants the fence to be 5 inches longer than the garden on each side. If he wants the area enclosed by the fence to be 2028 square inches, how long should the garden be?
3. Eugene's math class has 20 boys and 10 girls. If the teacher draws a student's name at random for a candy bar, what is the probability Eugene will be chosen? What is the probability that a girl will be chosen?
4. There are a total of 214 cars and trucks on a lot. If the number of cars is four more than twice the number of trucks, how many cars and trucks are on the lot?
5. $-1(-4)(-7)$

8.1f Self-Assessment: Section 8.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Solve problems involving area and circumference of a circle with numeric measures.	I can solve problems finding circumference or area of any circle as long as I'm given the radius, formulas and a picture of the situation.	I can solve problems finding circumference or area of any circle as long as I'm given a picture of the situation.	I can solve problems finding circumference or area of any circle when information is given pictorially or in context.	I can solve problems finding circumference or area of any circle regardless of how the information is given. I can also apply information to new contexts.
2. Solve real world and mathematical problems involving area and perimeter of two-dimensional objects composed of triangles and quadrilaterals.	I struggle to solve problems involving area and perimeter of triangles and quadrilaterals.	I can solve problems involving area and perimeter of triangles and quadrilaterals if the needed information is given.	I can solve problems involving area and perimeter of triangles and quadrilaterals even if I have to find missing information to solve it.	I can solve problems involving area and perimeter of triangles and quadrilaterals even if I have to find missing information to solve it. I can also explain why my answer is correct.
3. Understand and explain the difference between perimeter and area.	I struggle to understand the difference between perimeter and area.	I can calculate perimeter and area of various two-dimensional objects.	I understand and can show visually the difference between perimeter and area.	I understand and can explain in my own words the difference between perimeter and area.
4. Find area or perimeter of an object with algebraically measured lengths.	I can find the area or perimeter of an object with numeric measured lengths, but struggle if there are variables involved.	I can usually find perimeter or area of an object with algebraic measured lengths.	I can always find perimeter or area of an object with algebraically measured lengths.	I can solve area and perimeter problems involving missing sides with algebraically measured lengths.
5. Solve problems involving scale or percent increase/decrease and area and/or perimeter.	I can find the perimeter or area of objects, but I struggle to solve problems involving scale or percent increase/decrease.	I can find the perimeter and areas of two scaled objects. I can usually find the scale factor between those measurements.	I can solve problems involving scale or percent increase/decrease and area and/or perimeter.	I can solve problems involving scale or percent increase/decrease and perimeter or area. I can also explain the relationship between perimeters or areas of those objects.

Sample Problems for Section 8.1

1. Use the given information to find the missing information. Round each answer to the nearest hundredth unit.

a. Diameter: 18 km
Circumference: _____

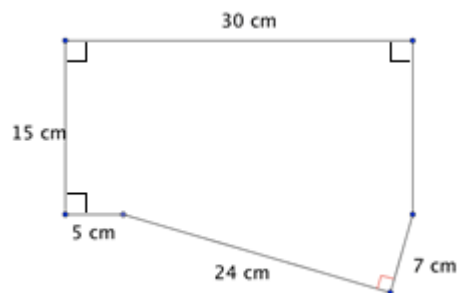
c. Circumference: 50.24 in
Radius: _____

b. Radius: 0.1 m
Area: _____

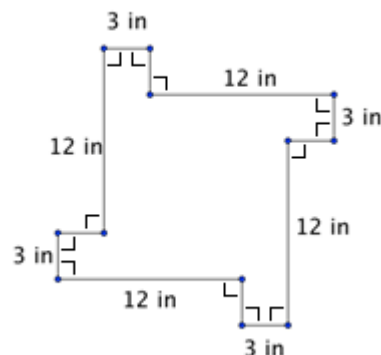
d. Area: 105.63 in^2
Radius: _____

2. Answer the following questions using the object pictured.

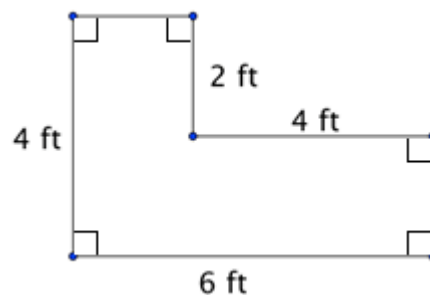
a. Find the perimeter and area of the object to the left.



b. What percent of the total area is the area of one of the outer squares in the object to the left?



c. Phoebe is buying an L-shaped desk. How much area will the desk pictured take up in her apartment?



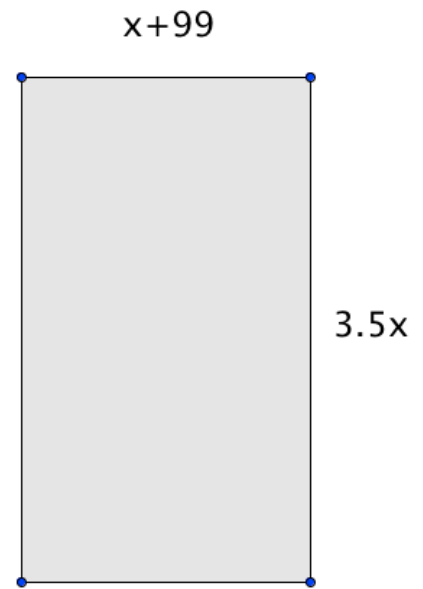
3. Explain the difference between perimeter and area in words and/or pictures. Use the figure in 2c to help you in your explanation.

4. Use the rectangle to the right to answer the following questions:

a. What is the perimeter?

b. What is the area?

c. If the perimeter is 1098 units, what is the value of x ?



5. The dimensions of Gabrielle's rectangular garden are 12 feet by 16 feet. Daniel is building a garden. He is going to decrease the dimensions by 25%.

a. Find the perimeter and area of each garden.

b. Find the percent of decrease between the perimeters of each garden.

c. Find the percent of decrease the areas of each garden.

d. In general, how are the percents of change of perimeters and areas of scaled objects related?

Section 8.2: 2D Plane Sections from 3D Figures and 3D Measurement

Section Overview:

The goal of this section is to help students better understand: a) attributes of various prisms, b) how those attributes affect finding the surface area and volume of different prisms, c) the relationship between measurements in 1-, 2-, and 3-dimensional figures and d) connect ideas of scale factor/percent change to ideas of surface area and volume. Students begin this section by examining three-dimensional figures. They should observe which faces for an object are parallel and/or perpendicular and which are the same size and shape. Students quickly move to taking cross-sections of various figures to notice what two dimensional shapes are generated by different types of cuts. Attention will be paid to when parallel plane sections generate surfaces that are the same and when they are different. The exercises in this section are designed to solidify students' understanding of the algorithms for finding perimeter, area, and volume. Additionally, exercises should help students better understand units of measure for perimeter, area, and volume. Next, students examine the nets of 3D figures. Nets were introduced in 6th grade; in 7th grade, students extend their understanding by differentiating surface area from volume. Students will use their understanding of surface area and volume to solve various problems. Specifically, attention will be paid to scale factor and percent change in problems involving volume and surface area.

Concepts and Skills to be Mastered (from standards)

1. Describe the two-dimensional figures that result from slicing three dimensional figures.
2. Solve real-world and mathematical problems involving volume and surface area of three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

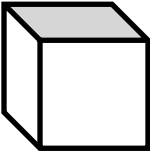
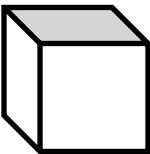
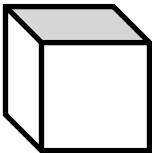
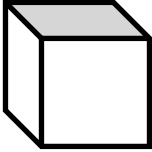
Background knowledge for making and describing 2D cross-sections:

- Right regular polyhedron prisms are three dimensional figures with at minimum two parallel polygonal faces of the same size and shape (called bases); remaining lateral faces are rectangles not necessarily the same as the bases (note that the base may be any polygon but the lateral faces are always rectangles.)
- Right cylinders have parallel circular bases of the same size (radii are equal) whose centers are aligned directly above each other (centers lie on a line that is perpendicular to both bases).
- There are several ways to take plane sections of a prism or cylinder (for the descriptions below, we assume the prism or cylinder is standing on one of its bases):
 - Parallel to the base
 - Perpendicular to the base
 - Slice at an angle (in a tilted direction) to the base
- A cross-section is a special type of plane section cut perpendicular to a face or base. Most cuts in this section will be cross-sections.
- “Equidistant” means an equal distance. “Vertices” are the corners of the figure.

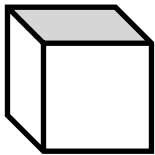
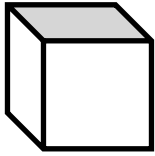
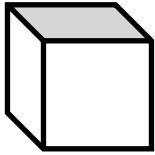
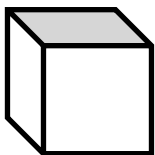
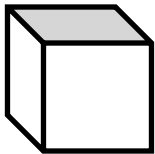
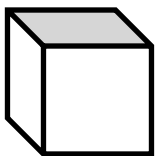
This section is primarily exploratory in nature. Attention to precision will be very important as the teacher and students explain their thinking.

8.2a Class Activity: 2D Plane Sections of Cubes and Prisms (play dough & dental floss)

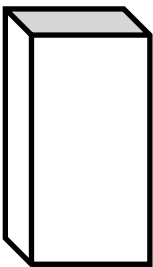
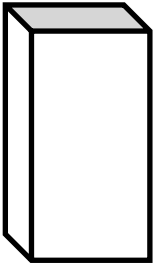
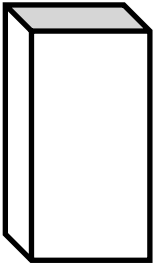
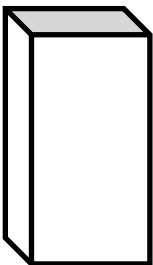
1. Mold a **CUBE** from play-dough.

Perform the following cuts.	Sketch where you cut	Sketch the exposed surface(s).	NOTES
a. Cut <i>parallel</i> to the base.			
b. Cut parallel to the base again, but at a different distance from the base.			How does the new exposed surface compare to the previous surface?
<p>What is true about all cuts of a cube parallel to the base?</p> <p>What is true about other cuts of a cube NOT parallel to the base but through at least one lateral face?</p>			
c. Cut perpendicular to the base and parallel to a face.			
d. Cut perpendicular to the base and parallel to a face again, but at a different location on the cube.			How does the new exposed surface compare to the previous surface?
<p>What is true about all cuts of a cube perpendicular to the base and parallel to a face?</p> <p>What is true about cuts of a cube perpendicular to the base but not parallel to a face?</p>			

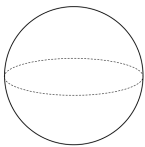
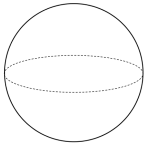
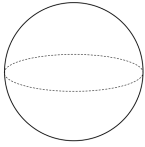
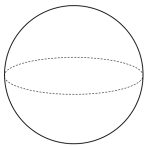
2. Mold a **CUBE** again. Perform a single cut to create the following:

Create...	Sketch where you cut	NOTES
a. Triangle		
b. Square		
c. Rectangle		
d. Pentagon		
e. Hexagon		
f. Circle or Trapezoid		

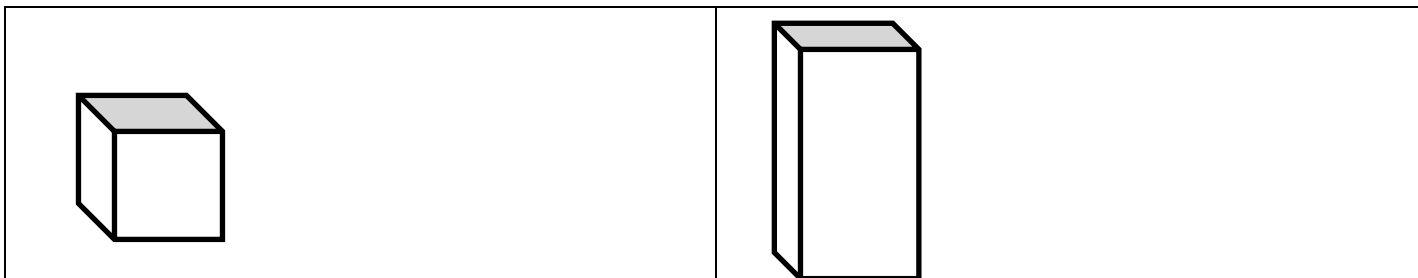
3. Mold a Right **RECTANGULAR PRISM** (that is not a cube.)

Perform the following cuts.	Sketch where you cut	NOTES: describe the plane section and what might happen with other similar cuts.
a. Cut parallel to the base.		
b. Cut parallel to the base again, but at a different distance from the base.		
<p>What is true about all cuts of a rectangular prism parallel to the base?</p> <p>What is true about cuts of a rectangular prism NOT parallel to the base?</p>		
c. Cut perpendicular to the base and parallel to a face.		
d. Cut perpendicular to the base but not parallel to a face.		
<p>What is true about all plane sections of a rectangular prism perpendicular to the base and parallel to a face?</p> <p>What is true about any plane section of a rectangular prism perpendicular to the base but not parallel to a face?</p>		

4. Mold a **SPHERE**.

Perform the following cuts.	Sketch where you cut	NOTES: describe the plane section and what might happen with other similar cuts.
a. Cut parallel to the table at different distances from the table.		
b. Make cuts that are not parallel to the table.		
<p>What is true about all cuts of a sphere parallel to the table?</p> <p>What is true about all cuts of a sphere NOT parallel to the table?</p>		
c. Cut perpendicular to the table.		
d. Cut perpendicular to the table again, but at a different location on the sphere.		
<p>What is true about all cuts of a sphere?</p>		

5. Make a cube and a rectangular prism like in exercises #1 and #3 again. Orient them as in the figure below:

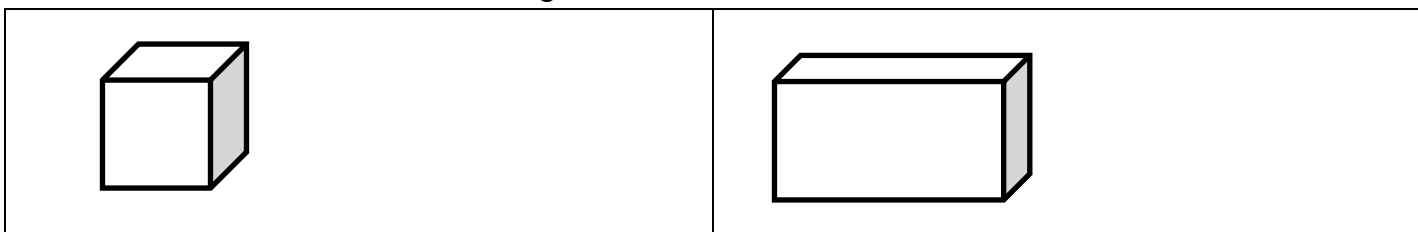


What is the shape of the base of the cube and the rectangular prism?

Will all cuts parallel to the base result in the same planar figure for the cube?

Will all cuts parallel to the base result in the same planar figure for the rectangular prism?

Now rotate each 90° as shown in the figure below:



What is the shape of the base of the cube and the rectangular prism now?

Will all cuts parallel to the base result in the same planar figure for the cube?

Will all cuts parallel to the base result in the same planar figure for the rectangular prism?

6. Compare and contrast plane sections of rectangular prisms, cubes and spheres.

8.2a Homework: 3D Objects

Use your knowledge of each three-dimensional object to answer the following questions.

Cubes

1. How many faces does a cube have?
2. What do you know about each face of a cube?
3. How many edges does a cube have?
4. How many vertices does a cube have?

Rectangular Prisms

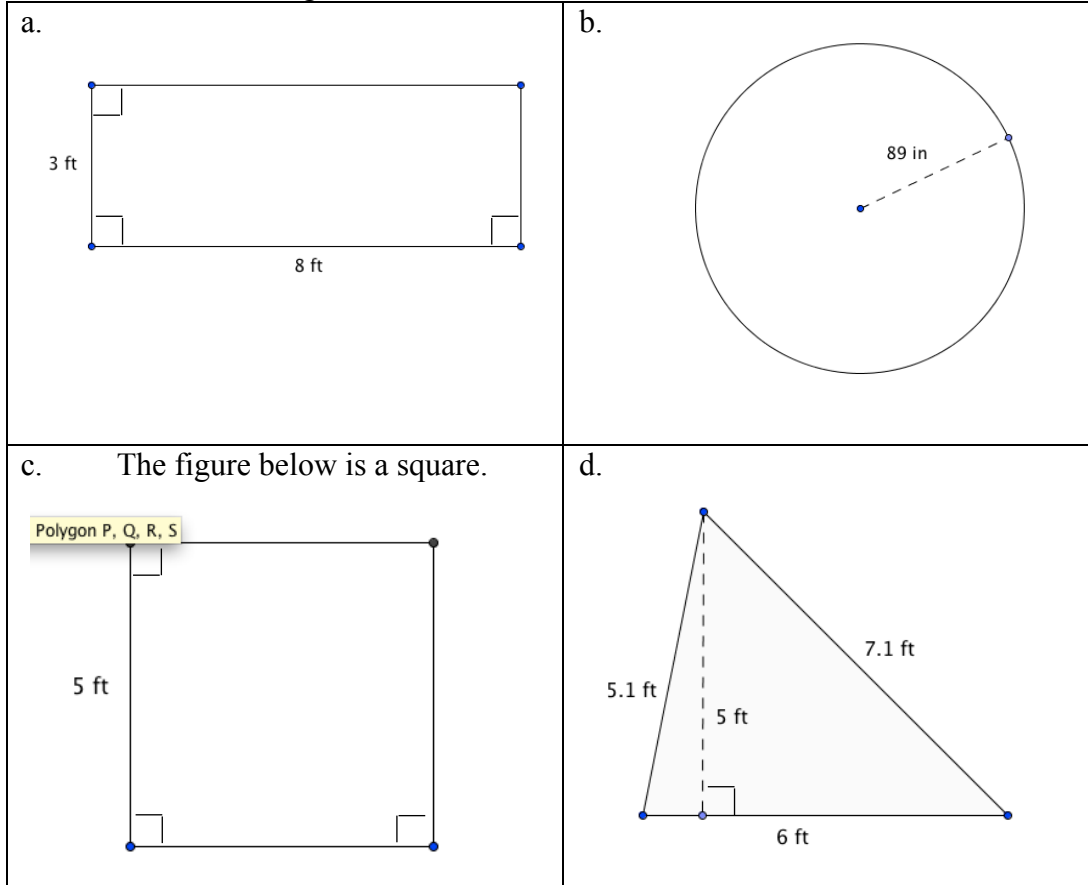
5. How many faces does a rectangular prism have?
6. How many edges and vertices does a rectangular prism have?
7. How are a rectangular prism and a cube similar and different?

Sphere

8. Does a sphere have any edges or vertices?
9. What makes a sphere different from all the other 3D objects named above?

Spiral Review

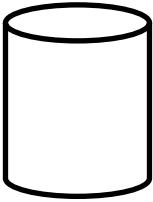
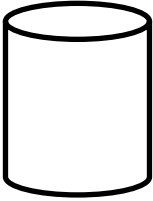
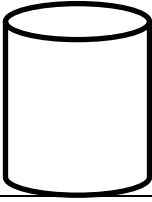
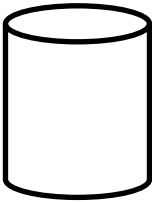
1. Find the area of each figure below:



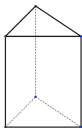
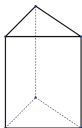
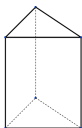
2. Simone rolls a die 64 times. Approximately, how many times will she roll a 6?
3. Find the unit rate for BOTH units.
Izzy drove 357 miles on 10 gallons of gasoline.
4. Convert the following units using the ratios given:
_____ feet = 37 inches (1 foot = 12 inches)
5. The temperature at midnight was 8°C . By 8 am, it had risen 1.5° . By noon, it had risen another 2.7° . Then a storm blew in, causing it to drop 2.7° by 6 pm. What was the temperature at 6 pm?

8.2b Class Activity: 2D Plane Sections on Cylinders and More

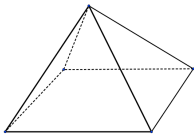
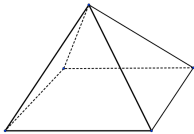
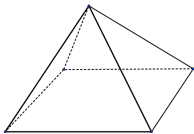
1. Mold a right **CYLINDER**, put the circular base on the table.

Perform the following cuts.	Sketch where you cut	NOTES: describe the plane section and what might happen with other similar cuts.
a. Cut parallel to the base.		
b. Make cuts not parallel to the base but through the lateral face of the cylinder.		
<p>What is true about all cuts of a cylinder parallel to the base?</p> <p>What is true about all cuts of a cylinder NOT parallel to the base?</p>		
c. Cut perpendicular to the base.		
d. Make cuts that go through at least one base of the cylinder but are not perpendicular to the base.		
<p>What is true about all cuts of a cylinder parallel to the to the base?</p> <p>What is true about cuts of a cylinder perpendicular to the base?</p> <p>What is true about any other cut (NOT parallel to the base or perpendicular to the base)?</p>		

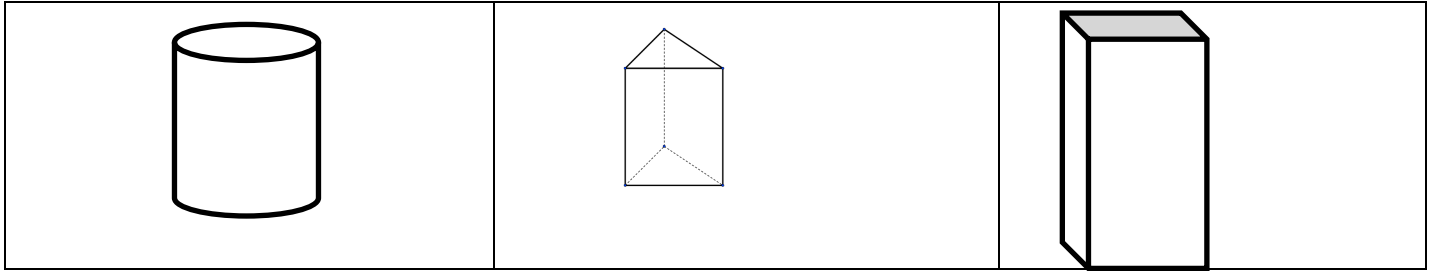
2. Make a **TRIANGULAR-BASED RIGHT PRISM**. Put the triangle base on the table.

Perform the following cuts.	Sketch where you cut	NOTES: describe the plane section and what might happen with other similar cuts.
a. Cut parallel to the base.		
b. Make cuts that are not parallel to the base but go through at least one face of the prism.		
What is true about all cuts of a triangular prism parallel to the base?		
c. Cut perpendicular to the base.		
What is true about all cuts of a triangular prism perpendicular to the base?		

3. Make a **SQUARE-BASED RIGHT PYRAMID**. Put the square base on the table.

Perform the following cuts.	Sketch where you cut	NOTES: describe the plane section and what might happen with other similar cuts.
a. Make cuts parallel to the base.		
What do you notice about cuts of a square-based pyramid parallel to the base?		
b. Cut perpendicular to the base and parallel to one of the base's edges.		
c. Cut perpendicular to the base again, but at a different location on the pyramid.		
What do you notice about cuts of a triangular pyramid perpendicular to the table?		

4. Make a cylinder and a triangular prism like in exercises #1 and #2 again. Also make a rectangular prism. Orient them as in the figure below:



What is the shape of the base of the cylinder?

What is the shape of the base of the triangular prism?

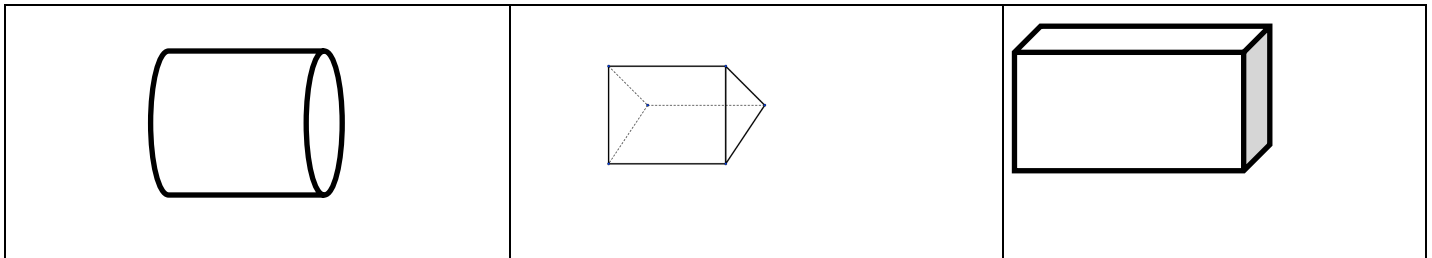
What is the shape of the base of the rectangular prism?

Will all cuts parallel to the base result in the same planar figure for the cylinder?

Will all cuts parallel to the base result in the same planar figure for the triangular prism?

Will all cuts parallel to the base result in the same planar figure for the rectangular prism?

5. Now rotate each 90° as shown in the figure below:



What is the shape of the part of the cylinder resting on the table?

What is the shape of the part of the triangular prism resting on the table?

What is the shape of the part of the rectangular prism resting on the table?

Will all cuts parallel to the table result in the same planar figure for the cylinder?

Will all cuts parallel to the table result in the same planar figure for the triangular prism?

Will all cuts parallel to the table result in the same planar figure for the rectangular prism?

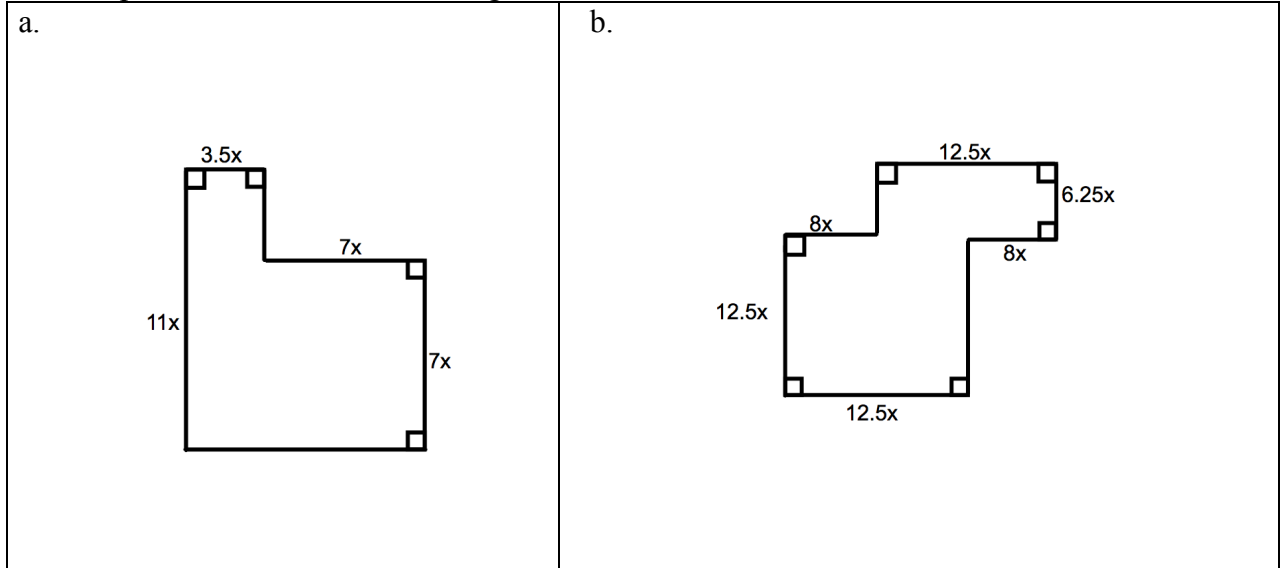
8.2b Homework: Area of Plane Sections

For each plane-section described below, state the shape of the plane-section and its area. Refer to previous class activities, as needed.

1. Imagine cutting a cube, parallel to the base.
 - a. What shape is the plane section?
 - b. If you cut a $5 \times 5 \times 5$ inch cube parallel to any face, what will the area of the plane section be?
2. Imagine a right square-based prism with edge lengths $\frac{3}{4} \times \frac{3}{4} \times 4\frac{1}{2}$ inches.
 - a. If you make a cut parallel to the square base, what will the plane section be?
 - b. What will the area of the plane section described in “a” be?
 - c. What will the plane section be if the cut is made parallel to the lateral face?
 - d. What will the area of the plane section in “c” be?
3. Imagine cutting a sphere with diameter 10 cm parallel to the table through the center.
 - a. What shape will any plane section be?
 - b. What will the area of the plane section be?
4. Imagine cutting a cylinder of diameter 12.62 cm and height 8 cm, parallel to the base.
 - a. What shape is the plane section and what is its area?
 - b. What shape is the plane section if the cut is perpendicular to the base? What do you know about the figure?
5. Imagine a triangular prism:
 - a. What shape is the plane section parallel to the base?
 - b. What shape is a plane section perpendicular to the base?
 - c. If the area of the plane section in “b” is 6.25 cm^2 and the height of the prism 5 cm. What was the length of the cut?
6. Imagine cutting a square based right pyramid parallel to the base.
 - a. What shape is the plane section?
 - b. If the dimensions of the length and the width of the plane section are $\frac{3}{2}$ in. and $\frac{3}{2}$ in., what is the area of the plane section?

Spiral Review

1. Find the perimeter and area of each figure below:



2. Ten percent of the population is left-handed. Otto believes left-handed people have an advantage in boxing. If he observes 3 people boxing, describe a simulation that would show the probability one is left-handed.

3. Solve and graph the following inequality: $-97 \geq 4(19 - 2x) + 3$



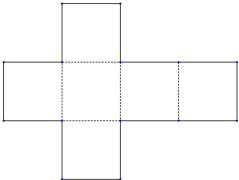
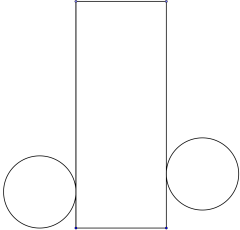
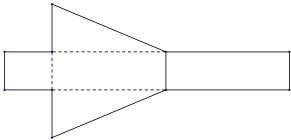
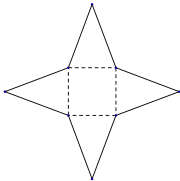
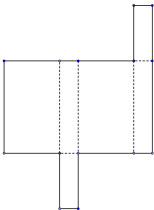
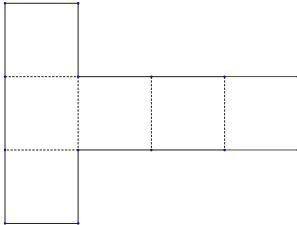
4. Solve $-17 + 5$

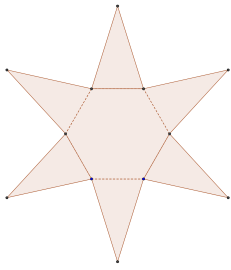
5. Angel owes his mom \$124. Angel made two payments of \$41 to his mom. How much does Angel now owe his mother?

8.2c Class Activity: Nets of 3D Objects

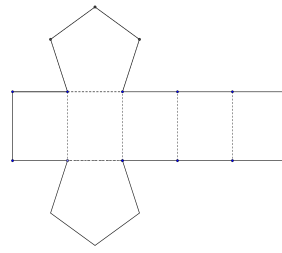
Review from 6th grade: A net is a two dimensional figure that can be folded to make a three dimensional object. You can learn about a 3D object by examining its net.

Look at each net below. Determine if the net can be folded into a three-dimensional object for which you could make equal parallel planar cuts (e.g. opposite sides are the same). Remember, you can orient objects any way you want. If it does not allow for equal parallel planar cuts, explain why.

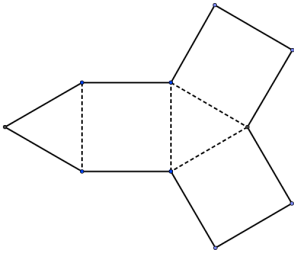
NET	NET
<div></div> <div>1. Can you create equal parallel plane sections?</div>	<div></div> <div>2. Can you create equal parallel plane sections?</div>
<div></div> <div>3. Can you create equal parallel plane sections?</div>	<div></div> <div>4. Can you create equal parallel plane sections?</div>
<div></div> <div>5. Can you create equal parallel plane sections?</div>	<div></div> <div>6. Can you create equal parallel plane sections?</div>



7. Can you create equal parallel plane sections?



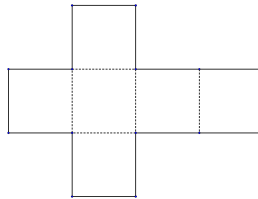
8. Can you create equal parallel plane sections?



9. Can you create equal parallel plane sections?

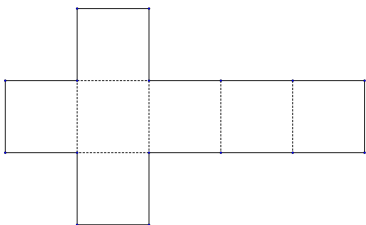
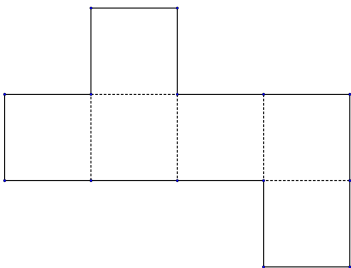
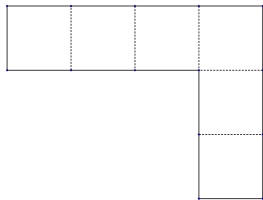

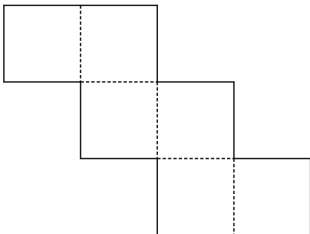
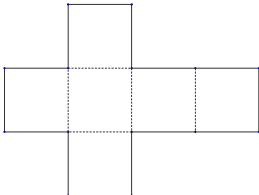
Notes:

10. The net to the right forms a cube.



- If each square face of the cube has a side length of 4.2 cm, what is the surface area of the cube?
- If each square face of the cube has a length of 4.2 cm, what is the volume of the cube?
- If you created a new cube with edge lengths that are 150% of the original cube (edge lengths of 4.2 cm), what would the new surface area and volume be?
- If you created a new cube with edge lengths that are 70% of the original cube (edge lengths of 4.2 cm), what would the new surface area and volume be?

8.2c Homework: Nets of 3D Objects

1. Which of the following nets make a cube?		
a.	b.	c.
		
d.	e.	f.
		

2. Draw a net for the square-based rectangular prism to the right:

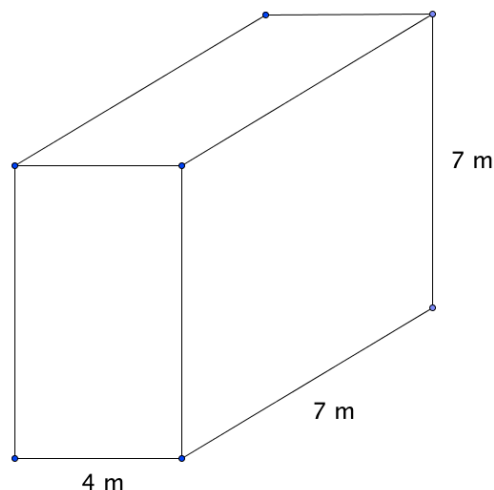
	
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3. Suppose the prism in # 2 above has base edges of $8 \times 8 \times 12$ inches. Find the surface area and volume of the prism. Show all your work.
4. Still using the prism above, find the new dimensions of the prism if you scaled it by $\frac{3}{2}$. Then find the new surface area and volume.
5. Still using the prism above, find the new dimensions of the prism if you scale it by $\frac{2}{3}$. Then find the new surface area and volume.

Spiral Review

- Solve:
 - $3 - 9$
 - $47 + (-65)$
- On a map of New York City, with a scale of 1 inch = $\frac{1}{2}$ mile, Central Park is 1 inch wide and 5 inches long. What is the area of the park?

- Find the surface area and volume of the following rectangular prism:



- Find the additive inverse and multiplicative inverse of each of the following numbers:

Number	Additive Inverse	Multiplicative Inverse
2		
$\frac{17}{7}$		
-2.15		

- Given the following table, find the indicated unit rate:

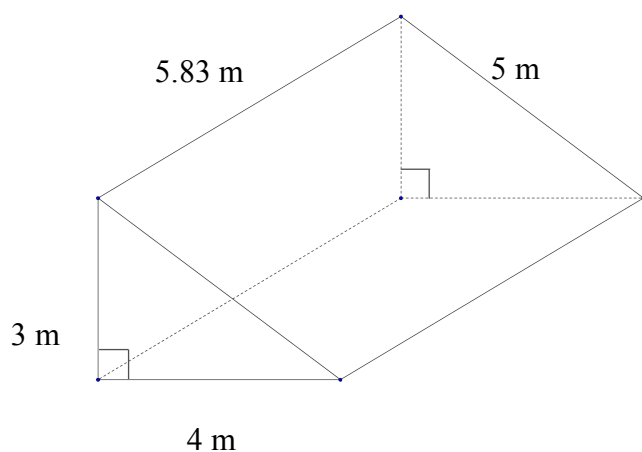
Days	Total Push-ups
2	30
4	60
29	435

_____ push-ups per day

8.2c Additional Practice: Surface Area and Volume



1. Explain your procedure for finding surface area.
2. Explain your procedure for finding volume.
3. Find the surface area and volume of the right prism below. Note that the triangular bases are right triangles with the right angle occurring where the 4m and 3m sides meet.



Find the surface areas and volume for each. You may wish to sketch the figure. Show work. Include units.

Shape and Dimensions	Surface Area	Volume
4. Rectangular prism: <ul style="list-style-type: none"> length 10 in width 8 in height 5 in. 		
5. Rectangular prism: <ul style="list-style-type: none"> length 4 feet width 2 feet height 6.2 feet 		

6. If the base area of a right rectangular prism is 28 cm^2 and the volume is 177.8 cm^3 , what is the height of the prism?
7. If the height of a square based prism is 13 in and its volume is 637 in^3 , what is the length of each side of the base?
8. Give three possible edge lengths for a prism of volume 96 m^3 .

8.2d Class Activity: Growing and Shrinking Stuff

For each of the following problems, find the necessary measurements to answer all the questions.

1. A cube with side lengths of 8 centimeters is enlarged so its side lengths are now 24 centimeters.
 - a. By what scale factor did the side lengths increase? Show your work or justify your answer.
 - b. By what scale factor will the surface area increase? Show your work or justify your answer.
 - c. By what scale factor did the volume increase? Show your work or justify your answer.
2. A rectangular prism with side lengths of 5 yards, 10 yards, and 15 yards is reduced by a scale factor resulting in new sides of 1 yd, 2 yd, and 3 yd.
 - a. What was the scale factor for the reduction?
 - b. What would be the new surface area and volume for the prism?
 - c. By what scale factor did the surface area and volume each change?
3. A rectangular prism with side lengths 2 mm, 3 mm, and 5 mm is enlarged by some scale factor. The new volume of the prism is 240 mm^3 .
 - a. By what scale factor were the side lengths increased?
 - b. What is the new length of each of the sides?
 - c. What is the new surface area of the prism?

8.2d Homework: Growing and Shrinking Stuff

For each of the following problems, find the necessary measurements to answer all the questions.

1. A mini cereal box has the following dimensions: 4.5 in by 6 in by 2 in.
 - a. If all the dimensions are doubled, will it require double the amount of cardboard to make the box? Why or why not?
 - b. If all the dimensions are doubled, will it hold double the amount of cereal? Why or why not?
 - c. If *one* of the dimensions is doubled, will it require double the amount of cardboard to make the box? Why or why not?
 - d. If one of the dimensions is doubled, will it hold double the amount of cereal? Why or why not?
2. A container of chocolate milk mix has the following dimensions: a square base with sides of 6 in. and height 9 in.
 - a. If all the dimensions are reduced by a scale factor of $\frac{1}{3}$, will it require a third the amount of materials to make the container? Why or why not?
 - b. If all the dimensions are reduced by a scale factor of $\frac{1}{3}$, will it hold a third the amount of chocolate milk mix? Why or why not?
 - c. If the length of each side of the base is reduced by a scale factor of $\frac{1}{3}$, how much surface area and volume will it now have?
 - d. What is the percent change of surface area and volume for “c”?

3. Refer back to the mini cereal box with dimensions, 4.5" by 6" by 2".
- If all the dimensions are increased by 4 inches to: 8.5" by 10" by 6" how does this change the amount of cardboard needed to make the box?
 - If all the dimensions are increased by a value of 4 inches, as described above, how will this change the amount of cereal the box can hold?

Spiral Review

1. Mrs. Zamora will not tell you how the class in general did on the last test. You really want to know how you compare, so you survey 10 random students out of 35. Estimate the average score for your class and describe how far off the estimate might be.

77	89	79	100	94
100	80	81	88	87

2. Draw and describe the plane section that results from the following cut: a cylinder cut parallel to the base.
3. Katherine is visiting patients in a hospital. She visits 9 patients in 3 hours. She visits 18 patients in 6 hours. Is this situation proportional? If so, what is the unit rate?
4. A candidate for Congress is trying to decide what to focus her campaign on in order to win the election. Describe how she might obtain a random sample to know what issues are important to her constituents.
5. Kurt puts 70% of his earnings into his savings. Write and solve an equation to find how much money he earned if he had \$165 to spend.

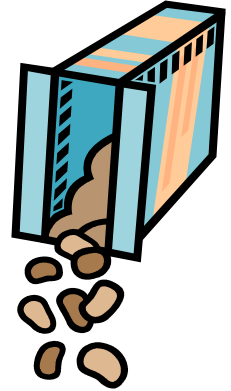
8.2e Project: Packing Packages

Have you ever notice that cereal generally comes in tall thin boxes and that laundry soap generally comes in short wide boxes? Why do you think they come as they do?

Think about your experiences with perimeter, surface area, and with volume in this chapter. What kind of box do you predict might hold the most and take the least amount of cardboard? Explain your thinking.

Suppose your favorite cereal comes in a box that is 24 cm. high, 20 cm. long, and 6 cm. wide. This box of cereal costs \$4.35.

1. Draw a model of the box. Find the surface area and volume for the box. Show work and label answers



The $24\text{ cm} \times 20\text{ cm} \times 6\text{ cm}$ dimensions of the box can also be thought of as a height of 24 cm and a girth (distance around the box) of 52 cm. If we add these two measures (height and girth), we get 76 cm.

For this project you will:

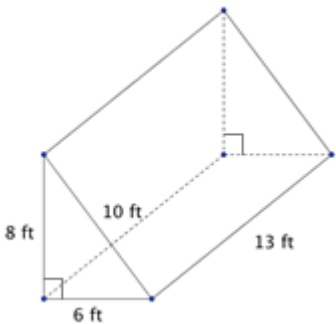
- a. Draw a model of a box with a total girth plus height of no more than 76 cm that holds the most cereal possible with the least surface area possible.
- b. Build the box as described in “a”.
- c. Provide a table, graph or spreadsheet to show how you arrived at your dimensions (remember: use only height and girth for your table, graph, or spreadsheet.)
- d. Find the percent decrease in surface area for your new box from the original box.
- e. Find the percent increase in volume for your new box from the original box.
- f. Write two paragraphs about your project. In the first paragraph, state and justify how much you would charge for cereal in this box. In the second paragraph, explain why you think cereal does not come in the type of box you designed.

8.2f Self-Assessment: Section 8.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems can be found on the following page.

Skill/Concept	Beginning Understanding	Developing Skill and Understanding	Practical Skill and Understanding	Deep Understanding, Skill Mastery
1. Describe the type of plane sections of right prisms and pyramids that result from different cuts.	I struggle to know or describe the type of plane section that will result from different cuts of right prisms and pyramids regardless of how I try to do it.	I know and can describe the type of plane section that will result from different cuts of right prisms and pyramids if I'm able to use objects or apps to make cuts.	I know and can describe the type of plane section that will result from different cuts of right prisms and pyramids with and without manipulatives/apps.	I know and can describe the type of plane section that will result from different cuts of right prisms and pyramids. I can explain how the plane section varies with different cuts.
2. Solve real world and mathematical problems involving volume and surface area of three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	I struggle to solve real world problems involving volume and surface area.	I can find volume and surface area of three-dimensional objects in context with some help.	I can solve real world and mathematical problems involving volume and surface area of three-dimensional objects.	I can solve real world and mathematical problems involving volume and surface area of three-dimensional objects, explain how I got my answer and why it is correct.
3. Find the scale factor and/or percent change in surface area and/or volume when dimensions are changed.	I struggle to find scale factor and/or percent change between measurements.	I can find scale factor and/or percent change between measurements with some help.	I can find scale factor and/or percent change between measurements.	I can find scale factor and/or percent change between measurements, explain my procedure and why my answer is correct.

Sample Problems for Section 8.2

1. Draw and describe the plane section that results from the following cuts.
 - a. A right rectangular prism is cut parallel to the base.
 - b. A cube is cut perpendicular to the base and parallel to a face.
 - c. A right pyramid is cut parallel to the base.
 - d. Describe how each cut above is similar and different from other similar cuts for the same object.
2. Answer each question about volume and surface area.
 - a. Find the volume and surface area of the triangular prism below.
 - b. Find the volume and surface area of a right rectangular prism with dimensions 8 cm by 15 cm by 2 cm.
 - c. If a right rectangular prism with dimensions 8 cm by 15 cm by 2 cm is scaled by a factor of 3, what would the new volume and surface area be?
 - d. What is the length of a box with a surface area of 472 cm^2 and a width of 11 cm and a height of 6 cm?
 - e. Ellie is filling a sandbox for her son. The sandbox dimensions are 4 ft by 4 ft by 1 ft. How many cubic feet of sand does she need to buy to fill the sandbox?
3. The dimensions of Hans' turtle aquarium are 18 in by 18 in by 12 in. The dimensions of Anna's rabbit cage are all doubled.
 - a. Find the surface area and volume of each animal habitat.
 - b. Find the scale factor between the surface areas of each container.
 - c. Find the scale factor between the volumes of each container.
 - d. In general, how are the scale factors of surface areas related? Volumes related?

Play Dough Recipe

Ingredients:

- 2 C flour
- 2 C warm water (food coloring is optional)
- 1 C salt
- 2 T oil
- 1 T cream of tartar

Directions:

In a medium pot, add water and oil together and stir. Then add the flour, salt, and cream of tartar. Keep stirring until all the ingredients are blended together and the mixture is not sticking to the sides of the pot. Then knead the mixture. For storage, keep play dough in plastic bags or sealed containers.

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