Preface

The Common Core State Standards represent a fundamental change in thinking about school mathematics. In terms of procedural skills, there is little change, and that change in the direction of broadening the uses of those skills. Along with the necessity of developing fluency in basic skills,the most significant difference is that the emphasis is on understanding the actual mathematical content. Algebraic thinking permeates the curriculum from the primary grades, and develops in abstraction through grades 6,7,8. Geometry in 7th and 8th grades is now based on the development of intuition on rigid motions and dilations, as a lead-in to the deeper study in high school. In the elementary grades students learn how to gather and work with data; then in grades 7 and 8, students begin to apply statistical concepts to the analysis of data. Most importantly, emphasis is on deepening student understanding of the central strands of ideas, their flexibility and their applicability, while continuing to require proficiency in technique and procedures.

That emphasis is at the core of the "fundamental change": from that of viewing mathematics as a collection of tools and protocols for solving problems involving one or more of number, geometry and data, to that of seeing mathematics as a small collection of "strands of knowledge" that develop more or less continuously through the curriculum, providing the user with a source of basic understanding that can be called on and adapted in a wide variety of circumstances.

The first six years of school education are devoted to developing the core of mathematical knowledge that is expected of all functioning adults. The last four years, grades 9-12, develop critical thinking skills in the context of that knowledge and its extensions, in more and more complex situations. Grades 7 and 8 provide the transition from basic knowledge to creative application of that knowledge by consolidating the themes of the previous 6 years in a form that allows an expansion of those ideas, and then beginning the development of that expansion, preparing students for the versatile use to be developed in high school. Success in this transition is critical to the purposes of school education: students must be willing to extend themselves through experimentation and imagination without fear (in high school, in one's profession, in life); and for that they need a strong foundation, and the experience of moving through unknown terrain with exploratory zeal.

The intent of this text is to introduce tools and guiding ideas to be used by students, teachers and parents as they maneuver this transition; to ground students with mathematical life skills; and to give them the background needed to succeed in high school mathematics and science. These tools are to be made available on a variety of platforms. This was the vision of the Utah State Legislature when they provided funding (through SB 217 2012) for the development of a mathematics text for grades 7 and 8. In 2012, a group consisting of outstanding experienced teachers, teacher educators, and mathematicians proposed a response to this vision. These materials are the result of our work over the subsequent two years, including testing in pilot schools and revision based on their feedback.

The guiding principle for this work is that what happens in the classroom is education, and the teacher is its agent. Success hinges on student engagement and the influence of outside agents (primarily parents). It is the teacher who encourage the engagement of the students, and central to this is a solid and flexible understanding of the material of the course. The goal of these materials is to provide students, teachers and parents with the means to do their part in the process of education.

More specifically we feel that our role is to provide teachers, students and parents with materials that are varied enough so that they can learn from them, adapt them, and use them in a way that makes them comfortable with the

authority they have. The controversy over direct instruction vs. inquiry is, in our view, a distraction from the goal of interconnecting subject matter content and pedagogy. The teacher will organize the classroom around learning in the way that is most suitable to the teacher and students. Our role is to provide materials that are adaptable by the teacher whatever the pedagogical approach.

Components of the Text

A. Student Workbook. This is organized into chapters around content objectives, and in each chapter, a sequence of lessons. Each lesson covers classroom activity and homework for a 50 minute class. Sometmes the demands of the material exceed this limitation; when we recognize this, we say so; but some teachers may see different time constraints, and we defer to the teacher to decide how much time to devote to a lesson, how much of it is essential to the demands of the relevant standard. What is important are the proportions dedicated to the various divisions, so that it all fits into a year's work. Within a lesson, the activities for the students are graduated, so that, in working the problems, students can arrive at an understanding of a concept or procedure. In most cases there is an abundancee of problems, providing the teacher with an opportunity to adapt to specific needs.

B. Teacher Edition of the Workbook. Here, the student workbook is annotated with information of value to the teacher (and parent) giving guides through the activities, solutions to the problems, and cautions about what misunderstandings might arise. The intention is that the teacher will read this in advance, and use it to inform the classroom experience.

C. Glossary. It is a principle that to proceed to understanding, we must first agree on the meaning of words we use. Mathematical definitions provide a test for the appropriateness of the use of a term, but do not explain what the term is about. The CCSS suggests that at first students develop an intuitive grasp of the meaning of a term, and as that intuition deepens, the need for precision becomes clear and then the process culminates in the mathematical formulation. In eighth grade, for many terms (proportionality, change, number) we are at the end of that process, but for many, we are in the intuitive stage of developing intuition (function - both in analyzing a functional relation and in synthesizing one from a table of data, geometric transformation, congruence).In this context, we see the point of a *Glossary* as describing how terms are to be understood, indicating the areas of their usefulness Thus our glossary should not be viewed as part of the codification of mathematics; where understanding a term has reached its end, the text will provide the definition.

D. Mathematical Foundations. Here we provide a contextual development of the mathematics, in terms of what has preceded and what is to come, and provide illustrative examples of the problems students are expected to attack and the procedures to deal with them. This material is essential to the understanding of the teacher, and useful to the parent working to help the student grasp the deeper meaning of any problem. Taken as background, it is valuable to the teacher preparing the lesson. For the parent , the mathematical foundations provide examples of problems and solutions while, for the student, it is a text to ground their understanding, and a source from which to obtain it.

Here we find many worked-out examples to illustrate what the students will do in the workbooks, and to illustrate connections with other parts of the curriculum. Throughout this text one finds passages denoted as **Extension**. The intent here is to provide the teacher with tools to answer questions that might come up, but go beyond the standard-based curriculum. Indeed, these extensions provide tools to raise questions, whose exploration leads the student to a richer understanding of the material (while in many cases previewing what is to come in future years). In this sense, these passages are also designed to maintain and promote the interest of those students who grasp the fundamental ideas and techniques quickly.

In summary, in 7th and 8th grade, the curriculum must affect a transition from the acquisition and consolidation of fundamental mathematical concepts and procedures, to an ability and flexibility in their use to solve problems. There are several critical parts to this transition:

a. Number sense: from counting to measuring;

b. Algebraic thinking: from unknown to variable, from equation to relation to function;

c. Geometry: from a study and classification of shape and size to a study of attributes of geometric figures that remain the same under transformation;

Through these areas an important transition is begun: from *static* to *dynamic* - from understanding how things are to understanding how they change, We see this in:

- Number: from the sense of counting objects, or parts of objects, to moving along a line in tenths and hundredths and so on to assign a numerical expression to a point;

- Algebra: from the idea that we have expressed relations among specific numbers, some of which are unknown to the idea that we are expressing a relation between two variables, and that we sweep along explicit values of this relation to discern a pattern of change

- Geometry: from the study of the shape of figures to the comparison of figures and ways to move one figure onto the other, if possible.

This is a tall order; too much to be accomplished in two grades. But the Core Standards account for this and do not ask for *accomplishment*, but an expansion of thinking prerequisite to accomplishment. Much has happened in prior grades to prepare students for these transitions, and the goal by the end of eighth grade is that students are eager to make them.

These materials were created over a two year period, 2012-2014, tested at pilot schools in Utah during the academic year 2013-2014 with a final revision completed in Fall, 2014. Fundamental to this work was a functional collaboration among teachers, mathematics educators and mathematicians. Assumptions, priorities and techniques ranged widely from group to group and person to person. We were, however, united in our conviction that the new Core Standards presented mathematics in a functionally different way that is a vast improvement over the past. This conviction led us (sometimes over months) to agreement on the materials; often that agreement was that there are indeed several ways to proceed.

Structure of the Text: Seventh Grade

The text is organized around the central concepts of seventh grade mathematics: consolidation of numbers and operations as understood on the number line; proportion and ratio in the context of variables; understanding perimeter and area of polygonal figures, and volumes of special polyhedra; introduction to probability and statistics as a basis of the analysis of data.

We begin with *Probability* as the vehicle for students to engage in a new mathematical topic while reviewing and practicing whole number and rational number arithmetic and in preparing the way for the study of statistical inference (Chapter 7). In the second chapter, *The Number System*, we extend and formalize students' understanding of the number system, including negative rational numbers. The main work of the chapter is in understanding how the arithmetic operations extend from fractions to all rational numbers. Making connections from concrete (*specific / numeric*) thinking to algebraic (*unknown / variables*) thinking is a challenging but essential step in the mathematical progression of every student. Chapter 3, *Expressions and Equations*, focuses on facilitating this transition by making connections to patterns (mathematical properties) already experienced for numbers and through repeated problems involving basic real-life examples.

Because of their prevalence and importance in science, commerce and daily life, proportional relationships, ratios and their applications are essential for students to understand so they become informed consumers and critical thinkers. Chapter 4, *Proportional Relationships*, focuses on building that understanding through the concept of unit rates. Chapter 5, *Geometric Figures and Scale Drawings*, merges student thinking by connecting concepts

developed about ratio and proportion in the previous four chapters to concepts in geometry. We then transition efficiently into Chapter 6, *Equations and Inequalities*, where we will use the geometric relationships that we explored in the last chapter to combine them with the algebra we have learned in the prior chapters.

In Chapter 7, *Probability and Statistics*, students develop an understanding of data sampling and making inferences from representations of the sample data, with attention to both measures of central tendency and variability.

We close the textbook with Chapter 8, *Geometric and Spatial Thinking*, connecting mathematics with the physical world that plays an important role in modeling phenomena whose origins are not necessarily 'physical,' emphasizing key ideas that assist students to develop a deeper understanding of numbers in the broadest possible sense: from understandings of algebra, geometry, measurement and data from previous grades.

Let us now briefly scan the conceptual high points of the text chapter by chapter.

Chapter 1 and 2: The Number System: Operations in Context; Extend and Formalize the Number System

Traditionally, seventh grade starts by gathering up everything students have learned about numbers and arithmetic, in a way that increases their flexibility with operations while illuminating the underlying algebraic structure of the number system. Experience suggests that this traditional approach does not work: students, weak as well as strong, find this an uninteresting review of either what they already know or what they are unlikely to learn through repetition. For this reason, we approach this review with a new topic, probability, with the idea that its extrinsic interest will attract the students' attention, while exhibiting the importance of arithmetic operations in context. Another reason for starting the year with probabilistic activities is to develop a culture of thinking about mathematics as a way to investigate real world situations. A third reason is that activities at the beginning of the year can help foster a classroom culture of discussion and collaboration.

Chapter 1. In this chapter, probability is a vehicle for students to engage in a new mathematical topic while reviewing and practicing whole number and rational number arithmetic. It is only an introduction to probability to which students will return in Chapter 7. The concepts studied in 7th grade around chance processes (theoretical and experimental probabilities) lay a foundation for work in later years when students study conditional probability, compound events, evaluate outcomes of decisions, use probabilities to make fair decisions, etc.

We are also preparing the way for the study of statistical inference (Chapter 7), given that probability provides a mathematical description of randomness, such as the chance variation observed in the outcomes of randomized experiments and random samples. This development occurs as students explore outcomes of a variety of probabilistic situations.

This is students' first introduction to probability. In the first section students will study chance processes, experiments or situations for which they know the possible outcomes but do not know which outcome will occur at any run of the experiment. Students will look at probabilities as ratios (part:whole) expressed as fractions, decimals, or percents thus developing their understanding in new contexts.

Probabilities will be determined from the results or outcomes of experiments. Students identify the set of all possible outcomes for an experiment as the *sample space* and an *event* as a subset of that space. They will learn that the probability of an event is expressed by a number between 0 and 1, and that that number is the sum of the probabilities of the outcomes in that event. They will understand the commonalities and differences between experimental and theoretical probability in given situations. This will conclude the first section.

While studying probability, students continue their study of rational numbers. Throughout this chapter students are provided with opportunities to review and build, based on knowledge from previous grades, fluency with fractions, percents, and decimals and recognize equivalent forms of rational numbers. They will convert rational numbers to decimals and percents and will look at their placement on the number line. This lays the foundation

for 8th grade where students study irrational numbers to complete the real number system. The two key objectives of the second section are 1) students articulate relationships among equivalent fractions, decimals, and percents using words, models, and symbols and 2) students understand and use models to find portions of different wholes.

The concept of equivalent fractions naturally leads students to the issues of ordering and estimation. Ordering positive and negative fractions will be connected to the number line. The chapter concludes with a section in which students begin to transition from relying solely on models to writing numeric expressions, specifically about solving percent and fraction problems, especially percent increase or decrease such as discounts, interest, taxes and tips.

Chapter 2. In this chapter, students extend and formalize their understanding of the number system, including negative rational numbers. Students first develop and explain arithmetic operations with integers, using both tiles and the number line as models. Then, they move, in both models, to the rational numbers by observing that the unit represented by the tile, or the first hash mark on the number line, can be changed. For example it could be the qth part of the unit originally represented by the first hash mark on the line. As the laws of arithmetic remain the same, they extend directly to the arithmetic for the qth part of the original unit. It all holds together from the simple observation that q copies of the qth part of the unit returns us to the original unit. In this way, the arithmetic of rational numbers is a consequence of the properties of the four basic operations of addition, subtraction, multiplication and division.

By applying these properties, and by viewing negative numbers in terms of everyday contexts (i.e. money in an account or yards gained or lost on a football field), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. Students re-examine equivalent forms of expressing rational numbers (fractions of integers, complex fractions, and decimals) and interpret decimal expansions in terms of successive estimates of the number representing a point on the number line. They also increase their proficiency with mental arithmetic by articulating strategies based on properties of operations.

The main work of this chapter is in understanding how the arithmetic operations extend from fractions to all rational numbers. At first addition is defined for the integers by adjunction of lengths, and subtraction as the addition of the opposite: in symbols: $a - b = a + (-b)$. This idea is a prelude to vector concepts to be developed in high school mathematics. Students' understanding of multiplication and division is extended to integers so that their properties continue to hold (using the understanding that the a number is the opposite of its opposite (e.g., $-(-1) = 1$).

Sections 2.1 builds intuition and comfort with integer addition and subtraction so that by the end of the section students can reason through addition and subtraction of integers without a model, and then will be able to extend those operations to all rational numbers. The goal of Section 2.2 is twofold: 1) fluency with multiplication and division of integers and 2) understanding how multiplication and division with integers is an extension of the rules of arithmetic as learned in previous grades. Section 2.3 moves on to rational numbers: placing them as points on the number line, and then extending the understanding of arithmetic operations to all rational numbers.

Chapter 3 and 4: Algebraic Thinking: Expressions and Equations, Proportional Relationships to Solving Problems

Chapter 3. Making connections from concrete (specific / numeric) thinking to algebraic (involving unknown quantities / variables) thinking is a challenging but essential step in the mathematical progression of every student. Chapter 3 focuses on facilitating this transition by making connections to patterns (mathematical properties) already experienced for numbers and through repeated problems involving basic real-life examples. After developing understanding and procedural fluency with arithmetic properties of rational numbers, students learn how to manipulate equations to find solutions. Physical representations, including algebra tiles and area models, aid in understanding these operations for integer values and then extending them to include rational numbers. Using the distributive property "in reverse" helps students begin to master the important skill of factoring.

Once students have understood and achieved fluency with the algebraic processes, they then take real world situations, model them with algebraic equations, and use properties of arithmetic to solve them. The chapter concludes by having students model and solve percent increase and percent decrease problems that involve a little more algebraic thinking than the set of problems at the end of Chapter 1.

The objective of Chapter 3 is to facilitate students' transition from concrete representations and manipulations of arithmetic and algebraic thinking to abstract representations. Each section supports this transition by asking students to model problem situations, construct arguments, look for and make sense of structure, and reason abstractly as they explore various representations of situations. Throughout this chapter, students work with fairly simple expressions and equations to build a strong intuitive understanding of structure. For example, students should understand the difference between $2x$ and x^2 and why $3(2x - 1)$ is equivalent (equal in value) to $6x - 3$ and $6x + (-3)$.

Another major theme throughout this chapter is the identification and use in argument of the arithmetic properties. The goal is for students to understand that they have used the commutative, associative, additive and multiplicative inverse, and distributive properties informally throughout their education. They are merely naming and more formally defining them now for use in justification of mathematical (quantitative) arguments.

Section 3.1 reviews and builds on students' skills with arithmetic from previous courses, as previously noted, to write basic numeric and algebraic expressions in various ways. In this section, students work on understanding the difference between an expression and an equation. Further, they should understand how to represent an unknown in either an expression or equation. Students will connect manipulations with numeric expressions to manipulations with algebraic expressions. In connecting the way arithmetic works with integers to working with algebraic expressions, students name and formalize the properties of arithmetic. By the end of this section students should be proficient at simplifying expressions and justifying their work with properties of arithmetic.

Section 3.2 uses the skills developed in the previous section to solve equations. Students will need to distribute and combine like terms to solve equations. In Grade 7, students only solve linear equations in the form of $ax + b = c$ or $a(x + b) = c$, where a, b and c are rational numbers. This section will rely heavily on the use of models to solve equations, but students should be encouraged to move to abstract representation when they are ready and fluent with the concrete models.

We close the chapter with section 3.3 which is about converting contextual (story) problems into algebraic equations and solving them. Contexts involve simple equations with rational numbers so the focus will be on concept formation and abstraction (solving algebraic formulations).

Chapter 4. Because of their prevalence and importance, proportional relationships, ratios and their applications are essential for students to understand so they become informed consumers and critical thinkers. This chapter focuses on building that understanding through the concept of unit rates. Students will learn to determine, using various tools and models, when two quantities are proportional. Knowing whether two quantities are proportional allows us to compare and evaluate options in the market place.

We begin Section 4.1 by understanding and applying unit rates. In Section 4.2, students use models, tables, and graphs to determine whether two quantities are proportional. Then they analyze stories, including comparing unit rates, and begin to create equations to model proportional situations (further developed in section 4.3). Finally, students put it all together by taking a given scenario and representing it using both a table and a graph, determining whether the two quantities are proportional, and if so, creating a formula to express the proportional relationship.

As a continuation from the previous section, in Section 4.3 students take data from a table, determine the pattern and decide whether there is a proportional relationship, then create an equation to model the situation; recognizing that the equation is an explicit way to show the relationship between two quantities. The importance of the unit rate for proportional data is reinforced in the creation of the modeling equation.

The concepts studied in the previous sections will be applied throughout Section 4.4. Students will set up and solve proportions for real-world problems, including problems with percentages of increase and decrease.

Chapter 5 and 6: Geometric Relationships combine with Algebra: Geometric Applications and Word Problems

Chapter 5. Here we connect concepts developed about ratio and proportion in the previous four chapters to concepts in geometry. In the first section we start by exploring conditions necessary, in both angle measure and side length, to construct unique triangles with ruler and protractor. The concept of 'uniqueness' is discussed as an introduction to the idea of equivalence under a rigid motion. Students will distinguish with more precision than in previous years that two figures can be exactly the same size and shape, or can be the same shape, but different size, or can be of different shape. The focus of the second section is on polygons that are the same shape but different size. Students construct scaled drawings of triangles first, and then other figures, and through explorations, note that objects that are the same shape but different size have angle measures that are the same and side lengths that are proportional. Essential here is the notion of scale. Students will connect ideas of scale to ratio and proportion to reproduce images. We note that side lengths change by the same factor but area changes by the square of the factor. In the third section, we turn to circles and observe that all circles are scaled drawings of each other; from which it follows that for any circle, the circumference (length of the perimeter) is proportional to the length of the radius, and the area is proportional to the square of the radius. Students will discover the remarkable fact that the constant of proportionality for the circumference is twice that of the area. In fact, we have $A = \pi r^2$ and $C = 2\pi r$; where π is the area (in square units) of a circle of radius 1 unit. The chapter ends with students examining angle relations as a means to solve problems, a theme to be further explored in the next chapter.

Students will also observe that there are many triangles with given angle measures at the vertices, and that they are all scale drawings of one another. This is a significant characteristic of similarity that will be further explored in Grades 8 and Secondary II; in Grade 7 we simply observe that it is true for the triangles that we construct with ruler and protractor. In section 4, we gather together, through exploration, other statements that appear to be true: for example that the sum of the angles of a triangle is a straight angle, and use that fact to solve problems involving angles.

Chapter 6. In this chapter we will use the geometric relationships that we explored in the last chapter and combine them with the algebra we have learned in prior chapters.

This chapter brings together several ideas. The theme throughout however is writing equations or inequalities to represent contexts. In the first section, students work with ideas in geometry and represent their thinking with equations. Also in that section, students solidify their understanding of the relationship between measuring in one-, two-, and three-dimensions. In the second section, students will be writing equations for a variety of real life contexts and then finding solutions. The last section explores inequalities. This is the first time students think about solutions to situations as having a range of answers.

In Chapter 3 students learned how to solve one-step and simple multi-step equations using models. In this chapter students extend that work to more complex contexts. In particular they build on understandings developed in Chapter 5 about geometric figures and their relationships. Work on inequalities in this chapter builds on Grade 6 understandings where students were introduced to inequalities represented on a number line. The goal is Grade 7 is to move to solving simple one-step inequalities, representing ideas symbolically rather than with models.

Throughout mathematics, students need to be able to model a variety of contexts with algebraic expressions and equations. Further, algebraic expressions help shed new light on the structure of the context. Thus the work in this chapter helps to move students to thinking about concrete situations in more abstract terms. Lastly, by understanding how an unknown in an expression or an equation can represent a "fixed" quantity, students will be able to move to contexts where the unknown can represent variable amounts (i.e. functions in Grade 8).

Chapter 7 and 8: The Integration of Probability, Statistics, Geometry and Algebra

Chapter 7. In this chapter, students develop an understanding of data sampling and making inferences from representations of the sample data, with attention to both measures of central tendency and variability. They will do this by gathering samples, creating plots, representing the data in a variety of ways and by comparing sample data sets, building on the familiarity with the basic statistics of data sampling developed over previous years. Students find probabilities, including those for compound events, using organized lists, tables, and tree diagrams to display and analyze compound events to determine their probabilities. Activities are designed to help students move from experiences to general conjectures about probability and number. They compare graphic representations of data from different populations to make comparisons of center and spread of the populations, through both calculations and observation.

Section 1 begins with an exploration of basic probability and notation, using objects such as dice and cards. Students will develop modeling strategies to make sense of different contexts and then move to generalizations. In order to perform the necessary probability calculations, students work with fraction and decimal equivalents. These exercises should strengthen students' abilities with rational number operations. Some theoretical probabilities are not known, but can be estimated by repeating a trial many times, thus estimating the probability from a large number of trials. This is known as the Law of Large Numbers, and will be explored by tossing a Hershey's Kiss many times and calculating the proportion of times the Kiss lands on its base.

Section 2 investigates the basics of gathering samples randomly in order to learn about characteristics of populations, in other words, the basics of inferential statistics. Typically, population values are not knowable because most populations are too large and their characteristics too difficult to measure. "Inferential statistics" means that samples from the population are collected, and then analyzed, in order to make judgments about the population. The key to obtaining samples that represent the population is to select samples randomly. This is not always easy to do, and an important part of this chapter is to think about what "random sample" means. Students will gather samples from real and pretend populations, plot the data, perform calculations on the sample results, and then use the information from the samples to make decisions about characteristics of the population.

Section 3 uses inferential statistics to compare two or more populations. In this section, students use data from existing samples and also gather their own data. They compare plots from the different populations, and then make comparisons of center and spread of the populations, through both calculations and visual comparisons. This unit introduces the importance of fairness in random sampling, and of using samples to draw inferences about populations. Some of the statistical tools used in Grade 6 will be practiced and expanded upon as students continue to work with measures of center and spread to make comparisons between populations. Students will investigate chance processes as they develop, use, and evaluate probability models. Compound events will be explored through simulation, and by multiple representations such as tables, lists, and tree diagrams.

Chapter 8. Geometric and spatial thinking connect mathematics with the physical world and play an important role in modeling phenomena whose origins are not necessarily 'physical.' An example of this is the use in 6th grade of Nets in the context of area and volume. Geometric thinking is also important because it supports the development of number and arithmetic concepts and skills, by providing students with a context for intuitive understanding. The sections in this chapter emphasize key ideas that assist students to develop a deeper understanding of numbers. In grades K-6 students learned to work with basic two-dimensional geometric shapes: triangles, squares, rectangles, and others. In addition, students learned specific parts and properties of shapes allowing them to identify and classify them into categories, and to determine how the categories of shapes are related. In this chapter students will be engaged in using what they have previously learned about drawing geometric figures (using rulers and protractor, coordinate grids and technology) to solve problems involving area and circumference of a circle, and real-world mathematical problems involving area and perimeter of two-dimensional objects composed of triangles and quadrilaterals. Furthermore, students will explore 3D geometric figures and circles and apply their mathematical knowledge of rational numbers, proportionality and algebra to solve problems involving surface areas and volumes, and to express meaningful formulas and recognize equivalent expressions.

More specifically, section 8.1 builds from understandings of geometry, measurement and data from grades 3-6. It

utilizes the scope of the number system and is a review and extension of previously learned skills. For example, in sixth grade students learned how to find area by composing and decomposing into rectangles especially with spatial structuring of rectangular arrays. Using the shape composition and decomposition skills, students learned to develop area formulas for parallelograms and triangles. They also learned how to address three different cases for triangles: a height that is a side of a right angle, a height that lies over the base and a height that is outside the triangle. Composition and decomposition of regions continues to be important for solving a wide variety of area problems, including justifications of formulas and solving real world problems. We will further see that composition and decomposition of shapes is important since it is used throughout geometry from Grade 6 to high school and beyond. Previously, in Chapter 5, students learned how to find the circumference and area of circles, whereas the focus of section 8.1 will be to extend and apply the area and perimeter of circles, triangles, rectangles, parallelograms, and trapezoids to various real-world and mathematical problems. The main goal of section 8.1 is to help students understand the relationship between perimeter and area. Thus we work with problems in context that involve both area and perimeter of various polygons and circles.

Our focus for section 8.2 centers on 3D figures. Students begin by examining plane sections of 3D figures. The point of work in the elementary grades with plane sections was to develop the ability to use drawings and physical models to identify and construct parallel and perpendicular lines and planes in 3D For this reason, in the elementary grades, plane sections were actually cross sections: special plane sections parallel to a face of the object, or perpendicular to an axis of symmetry of the object (we note that it has become customary to use these names interchangeably). In Grade 8 students go more deeply in the detailed visualization of 3D objects, and for that reason we consider all sorts of plane sections.

Furthermore, in the elementary grades, students study volume and surface area of special objects in a descriptive way. In 7th grade we want to go further, in order to understand the distinctions and relations between surface area and volume. As the volume of an object grows, does its surface area grow? This is the analog in 3D of the study of perimeter and area of figures in the plane. Here we introduce the ideas involved in computation of volumes, and then relate that to the determination of surface area using nets (as in 6th grade). Students will then differentiate between surface area and volume and use their understanding to solve various problems. One of the tools introduced at this point is Cavalieri's principle: that the volume of a figure developed around a particular axis is determined by the area of the section of the object by planes perpendicular to the axis. This is not a Grade 8 core topic, but it seems to fit naturally and easily in the discussion of sections, to provide an added intuition into area calculations.

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